1) Bisection $x^3 - x^2 + 2$

2) N.R $x^3 - x^2 + 2$

3) secant $x^3 + x - 1$

4) Take an Array input, find it's max.

5) Add 2 matrix

6) multiply 2 matrix

7) Determinent of a matrix

8) complex

9) Derivative with array

10) $\int_0^{\pi} \sin x$ using trap, simp, simp 3/8

11) $\int_0^{\pi/2} \cos x$ using romburg

12) 100 pseudo random number.

13) calculate pi using rand()

14) calculate $\int_0^{\pi/2} \sin x$ → monte carlo.

15) $\frac{dy}{dt} + 2y = 2 - e^{-4t}$; $y(0) = 1$

16) $\frac{d^2\theta}{dt^2} + \frac{g}{L}\theta = 0$ → euler

18) $\frac{d^2x}{dt^2} = -kx/m$

19) integrate x^2 using gaussian quadr.

20) system of linear eqn by gaussian elimination with backward substution.

21) same → using L.U factor.

22) **Example-7** The period of a simple pendulum for large angle amplitude (θ_M) is given in terms of the complete elliptic integral of the first kind $K(m)$ as

$$T = 4\Big(\frac{L}{g}\Big)^{1/2} K\Big(sin^2(\frac{\theta_M}{2})\Big) = 4\Big(\frac{L}{g}\Big)^{1/2} \int_0^{\pi/2} \Big(1 - sin^2(\frac{\theta_M}{2}) sin^2\phi\Big)^{-1/2} d\phi$$

where $0 \le \theta_M < \pi$. (a) Using Simpson's 1/3 rule, write a code in C++ function double pendulumT(double thetaM), that evaluates the period T for a given θ_M as the arument using the above definition. Choose the value of L/g such that, as $\theta_M \to 0$, T=1s. Call the function pendulumT() from the main() function with different values of θ_M as input. (b) Using the above function, plot T vs θ_M for $\theta_M \in [0, \pi/2]$

23) 4. Compute the following integrals correct to 7 decimal places

$$I_1 = \int_{\pi/4}^{\pi/2} \frac{\cos x \ \ln \sin x}{1 + \sin x} dx \qquad (Ans: \ -0.02657996319303578798.....)$$

$$I_2 = \int_0^{3\pi/2} \tan x \ dx \qquad (Ans: 0.96054717892973049468........)$$

24)

3. The time dependent temperature at a point of a long bar is given by

$$T(t) = 100\left(1 - \frac{2}{\sqrt{\pi}}\int_0^{8/\sqrt{t}} e^{-\tau^2}\, d\tau\right)$$

(a) Write a C++ function double temp(double t) that evaluates the temperature at a time t using any suitable numerical integration technique.

(b) Write a complete C++ program to save data for $T(t)$ in the range $t\epsilon[10,200]$ with an increment of 0.01. Plot the data Tvs. t using gnuplot and save the figure as temp.png.

#include<iostream>

25,26)

4. The spherical Bessel's functions have the integral representation

$$j_n(z) = \frac{z^n}{2^{n+1}n!}\int_0^{\pi} cos(zcos\theta)(sin\theta)^{2n+1}d\theta$$

(a) (i) Write a C++ function double bessel (int n, double z), that evaluates the spherical Bessel's functions$j_n(z)$, using the above definition, at some point z (Use any suitable integration technique).

(ii) Plot the first four Bessel's function using gnuplot. Save the plot as sp_bessel.png.

(b) The Fresnel integral of the second kind may be defined in terms of the spherical Bessel's function by

$$S(x) = \frac{1}{2}\int_0^{x} j_0(z)\sqrt(z)dz$$

(i) Define a C++ function double fresnel (double x) using the above definition and the function bessel (n,z) in part (a) that will calculate $S(x)$.

(ii) Plot $S(x)$ for$x\in$ [0, 20] with an increment of 0.1 and by using gnuplot. Save your plot as fresnel.png.

27)

5. The Fresnel sine and cosine integrals are defined respectively as

$S(x) = \int_0^x sin\left(\frac{\pi u^2}{2}\right) du$ and $C(x) = \int_0^x cos\left(\frac{\pi u^2}{2}\right) du$

(a) (i) Define two C++ functions double Fr_sin(double x) and Fr_cos(double x) that will calculate the above given integrals by using any suitable numerical technique.

(ii) Write the values of the integrals in different columns by using the defined functions in (a) in a file for $x\epsilon[0,5]$. Plot the Fresnel integrals and save the plot as fresnel.png.

(b) Consider the following diffraction pattern

$$I = 0.5I_0\{[C(u_0) + 0.5]^2 + [S(u_0) + 0.5]^2\}$$

Here $I_0$is the incident intensity, I is the diffracted intensity and u_0 is proportional to the distance away from the knife edge.

(i) Write a C++ function double diffraction (double u0, double I0) that calls the functions Fr_sin() and Fr_cos and returns the diffracted intensity

(ii) Calculate I/I_0 for u_0 varying from −1.0 to +4.0 in steps of 0.1 and write them in a file. Plot your results of I/I_0 vs u_0 from the datafile using gnuplot and save the file. [Check your answer: at u0=1,2,3 and 4 the values of I/I0 are respectively 1.25923,0.843997,1.10763 and 0.922073]

28)

6. In the scattering of neutrons (A = 1) with a nucleus of mass number A > 1, the average of the cosine of the scattering angle ψ (i.e. $u = \langle cos\psi \rangle$) is given by

$$u(A) = \langle cos\psi \rangle = \frac{1}{2}\int_0^{\pi} \frac{Acos\theta + 1}{(A^2 + 2Acos\theta + 1)^2} sin\theta d\theta$$

(a) Write a C++ function double scatt_angle (double A) that evaluates the average $u(A)$ for a mass number A using any suitable numerical integration technique.
(7)

(b) Save data from your program for u (A) in the range $A\epsilon[2,20]$. Plot the data using gnuplot and save the plot as angle.png.

(30)

10. The Bessel's function has the integral representation

$$J_n(x) = \frac{2}{\pi^{\frac{1}{2}}\left(n - \frac{1}{2}\right)!}\left(\frac{x}{2}\right)^n \int_0^{\pi/2} cos(xsin\theta)cos^{2n}\theta \, d\theta \quad for \quad n > \frac{-1}{2}$$

(a) (i) Write a C++ function double bessel(int n, double x), that evaluates the Bessel's functions Jn(x), using the above definition, at some point x. [3]

(ii) Write a complete C++ program to plot the first four Bessel's functions in a single plot and save the plot as bessel.png

(b) The fraction of light incident normally on a circular aperture that is transmitted is given

$$T = 1 - \frac{1}{2kq}\int_0^{2kq} J_0(x)dx$$

where a is the radius of the aperture and $k = 2\pi/\lambda$ is the wavenumber.

(i) Using the function bessel(n,x), define a C++ function double transmitted (double q) that evaluates the fraction above.

(ii)Plot transmitted(q) for ($k = 2\pi$) vs. q for $a\epsilon[0,4]$. Save the plot as transmitted.png.

31

11. The spherical Bessel's functions have the following integral representation

$$j_n(x) = \frac{x^n}{2^{n+1}n!}\int_0^{\pi} \cos(xcos\theta)\,(sin\theta)^{2n+1}d\theta$$

(a) Write a C++ function **double Bessel (double x, int n)**, that evaluates the Bessel's functions $j_n(x)$, using the above definition, at some point x by using any suitable integration technique. Then plot the first four Bessel's function for $x\epsilon[0,25]$ using gnuplot. Save the plot as **bessel.png**.

(b) An analysis of the antenna radiation pattern for a system of circular aperture involves the function

$$g(u) = \int_0^1 f(r)j_0(ur)rdr$$

where $f(r) = 1 - r^2$
Write a C++ function **double ant_radiation (double u)** from the above definition and plot $g(u)$ Vs. u for $u\epsilon[0,25]$. Save the plot as **radiation.png**.

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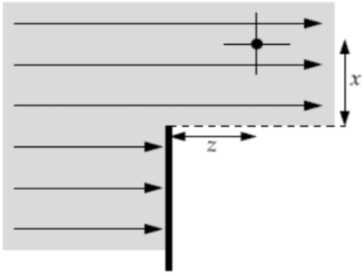
13. Suppose a plane wave of wavelength λ such as light or a sound wave is blocked by an object with a straight edge, represented by the solid line at the bottom of this figure:
The wave will be diffracted at the edge and the resulting intensity at the position (x, z) marked by the dot is given by near-field diffraction theory to be

$$I = \frac{I_0}{8}([2C(u) + 1]^2 + [2S(u) + 1]^2)$$

Where I_0 is the intensity of the wave before diffraction and

$u = x\sqrt{\frac{2}{\lambda z}}$ and the Fresnel integrals are defined as

$C(u) = \int_0^u \cos(\frac{\pi t^2}{2})dt$ and $S(u) = \int_0^u \sin(\frac{\pi t^2}{2})dt$



(a) Define two functions **Fr_cos(double u)** and **double Fr_sin(double u)** to evaluate the above integrals by using any suitable numerical technique.

(b) Define another function **double diffraction(double x)** that will calculate I/I_0 by using the above definition and for a given value of x (as shown in the figure).

(c) Write a complete C++ program to calculate I/I_0 and make a plot of it as a function of x in the range $-5m$ to $5m$ for the case of a sound wave with wavelength $\lambda = 1m$, measured $z = 3m$ past the straight edge. Save the plot as **intensity.png**.

33

15. Period of an anharmonic oscillator is given by

$$T = \sqrt{8m}\int_0^A \frac{dx}{\sqrt{V(A) - V(x)}}$$

where A is the amplitude of oscillation.

(a) Suppose the potential is $V(x) = x^4$ and the mass of the particle is $m = 1\,kg$. Write a C++ function **double tperiod(double A)** that evaluates the time period of the oscillator for the given amplitude **A** using Gaussian quadrature with 20 points.

[The required relations for Gaussian quadrature method are as follows

$$nP_n(x) = (2n-1)xP_{n-1}(x) - (n-1)P_{n-2}(x)$$

$$P_n'(x) = \frac{n}{x^2 - 1}\{xP_n(x) - P_{n-1}(x)\}$$

Where $P_n(x)$ the Legendre polynomial of degree n and $P_n'(x)$ is the derivative of $P_n(x)$]

(b) Use the function to make a graph of the period for the amplitudes ranging from $A = 0\ to\ A = 2$. You should find that the oscillator gets faster as the amplitude increases.

34

16. Debye's theory of solids gives the heat capacity of a solid at temperature T to be

$$C_v = 9V\rho K_B \left(\frac{T}{\theta_D}\right)^3 \int_0^{\theta_D/T} \frac{x^4 e^x}{(e^x - 1)^2} dx$$

where V is the volume of the solid, ρ is the number density of atoms, k_B is Boltzmann's constant, and θ_D is the so-called Debye temperature, a property of solids that depends on their density and speed of sound.

a) Write a C++ function **double sp_heat(double T)** that calculates C_V for a given value of the temperature, for a sample consisting of 1000 cubic centimeters of solid aluminum, which has a number density of $\rho = 6.022 \times 10^{28}\, m^{-3}$ and a Debye temperature of $\theta_D = 428K$. Use Gaussian quadrature to evaluate the integral, with 50 sample points.

b) Use your function to make a graph of the heat capacity as a function of temperature from $T = 5K \text{ to } T = 500K$. Save the graph as **spheat.png**.

35

18. The eigenfunctions of the one-dimensional simple harmonic oscillator are

$$u_n(x) = \frac{1}{\sqrt{2^n n!\,\sqrt{\pi}}} H_n(x)\, e^{-x^2/2}$$

where $H_n(x)$ is the Hermite polynomial of order n and we have used a system of units in which $\sqrt{\hbar/m\omega} = 1$.

(a) Define a function double hermite (double x, int N) that will evaluates the Hermite polynomial of order n at a point x by using the following recursion relation

$$H_n(x) = 2xH_{n-1}(x) - 2(n-1)H_{n-2}(x)$$

(b) define a function double f (double x, int N) that will evaluates the eigenfunctions by using the above given definition. Plot the eigenfunctionds for n= 0,1,2,3 and for $x\epsilon[-5,5]$ with an increment of 0.001.

(C) Evaluate $\int_{-\infty}^{+\infty} u_n(x) u_m(x) dx$ for $n, m\epsilon[0,5]$. Can you interpret the result?

37

3. Determine the smallest positive root of $x - e^{-x}$ up to three significant figure.

40

- Calculate the integral

$$I = \int_{-\pi}^{\pi} \frac{(1 + cosx)sin|2x|}{1 + |sin(2x)|} dx$$

Random number