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Question 1:

Here, $f(z) = \log_e(1+z)$

and $z = x^T x$, $x \in R^d$

let,

$$f'(z) = \frac{d}{dz} \log_e(1+z)$$

$$= \frac{1}{1+z} \cdot \frac{d}{dx}(x^T x)$$

$$= \frac{1}{1+x^T x} \cdot 2x$$

$$= \frac{2x}{1+x^T x}$$

Ans:

Question 2 :

$$f(z) = e^{-z/2}$$

where, $z = g(y) = y^T S^{-1} y$

$$y = h(x) = x - \mu$$

$$(x, \mu \in R^d, \; S \in R^{d \times d})$$

Now, $f'(z) = \frac{d}{dz}\left(e^{-z/2}\right)$

$$= e^{-z/2} \cdot \frac{d}{dz}\left(-z/2\right)$$

$$= e^{-z/2} \cdot \left(-\frac{1}{2}\right) \cdot \frac{d}{dy} y^T S^{-1} y \cdot \frac{d}{dx}(x - \mu)$$

Here, $\frac{d}{dy}\left(y^T S^{-1} y\right) = \lim_{h \to 0} \frac{g(y+h) - g(y)}{h}$

$$\Rightarrow \lim_{h \to 0} \frac{(y^T + h) \cdot S^{-1} \cdot (y+h) - y^T S^{-1} y}{h}$$

$$\Rightarrow \lim_{h \to 0} \frac{\cancel{y^T S^{-1} y} + y^T S^{-1} h + h S^{-1} y + S^{-1} h^2 - \cancel{y^T S^{-1} y}}{h}$$

$$\Rightarrow \lim_{h \to 0} \frac{\cancel{h}\left(y^T S^{-1} + S^{-1} y + S^{-1} h\right)}{\cancel{h}}$$

$$\Rightarrow y^T s^{-1} + s^{-1} y + \lim_{h \to 0} s^{-1} h$$

$$\Rightarrow y^T s^{-1} + s^{-1} y$$

and, $\frac{d}{dx}(x - \mu) = 1$

$$\therefore f'(z) = -\frac{1}{2} \cdot e^{-z/2} \cdot \left(y^T s^{-1} + s^{-1} y\right)$$

Ans.