FOURIER TRANSFORM

- It is use to decompose an image into its sine and cosine components.
- It finds its application in image analysis, image filtering image reconstruction and image compression. • The output of the transformation represents the image in the Fourier or frequency domain, while the input image is the spatial domain equivalent.

<u>One-Dimensional Continuous Fourier Transform</u> Fourier transform F(u) of a continuous function f(t) is given by the equation:-

$$F(u) = \int_{-\infty}^{+\infty} f(t) e^{-i2\pi ut} dt \quad \textbf{(1)}$$

- Given F(u), we can obtain f(t) using Inverse Fourier Transform:-

$$f(t)=\int_{-\infty}^{+\infty}F(u)e^{i2\pi ut}\,du \quad \textbf{(2)}$$

where F(u)-> Fourier Transform

f(t)-> Inverse Fourier transform of F(u)

- Let w=2πu, then eq 1 can be rewritten as follows:

$$F(w) = \int_{-\infty}^{+\infty} f(t) e^{-jwt} dt$$

(3)

- Inverse Fourier Transform from eq (2) will be rewritten as follows:

Let w=2πu, therefore dw= 2π.du

+∞ ������

f(t)= �� �� ��

−∞du

+∞ �� −∞

������ ���� 2π

= �� ��

2π �� �� ��������� ���� +∞

=1

−∞ **(4)** 3

<u>Two-Dimensional Continuous Fourier Transform</u>

For two-dimension function f(x,y), Fourier transform and Inverse Fourier Transform is given by:-

$$F(u,v)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x,y)e^{-i2\pi(ux+vy)}dx\,dy$$

$$f(x,y)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} F(u,v)e^{i2\pi(ux+vy)}du\,dv$$

DISCRETE FOURIER TRANSFORM

- Let a(m) be finite sequence. Discrete Fourier Transform (DFT) of one –dimensional discrete signal is given by:-

��−1

A(u)= �� �� ��^−��2π����/��, �� = 0,1, … �� − 1

��=0 **(1)**

- Inverse Discrete Fourier Transform (IDFT) of one –dimensional discrete signal is given by:-

a(m)= 1 �� �� �� ����2π����/��, �� = 0,1, ... �� − 1
��−1
��=0 **(2)** 5

- For two-dimensional discrete signal, DFT is given by:-

$$A(u,v) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} a(m,n) e^{-i2\pi\left(\frac{um}{M}+\frac{vn}{N}\right)} \quad \textbf{(3)}$$

where $u = 0,1,2,\ldots,M-1,\ \ v = 0,1,2,\ldots,N-1$

- For two-dimensional discrete signal, IDFT is given by:-

$$a(m,n)=\frac{1}{MN}\sum_{u=0}^{M-1}\sum_{v=0}^{N-1}A(u,v)e^{i2\pi\left(\frac{um}{M}+\frac{vn}{N}\right)}$$

$$m=0,1,2,\ldots,M-1;n=0,1,2,\ldots,N-1 \qquad \textbf{(4)}$$

PROPERTIES OF DFT

Fourier transform operation, namely,

For the sake of simplicity, the operator Γ is used to denote the $\Gamma(a(m,n))=A(u,v)$

Let $a_1(m,n), a_2(m,n)$ be two discrete image functions, and $A_1(u,v), A_2(u,v)$

be the corresponding Fourier transforms of $a_1(m,n), a_2(m,n)$

1. Linearity:

$$\begin{aligned}\Gamma\{\alpha a_1(m,n)+\beta a_2(m,n)\} &= \alpha\Gamma(a_1(m,n))+\beta\Gamma(a_2(m,n))\\ &= \alpha A_1(u,v)+\beta A_2(u,v)\end{aligned}$$

where α and β are constants.

2. Separability:

$$\Gamma\{a(m,n)\} = \frac{1}{N}\Gamma_m\{\Gamma_n\{a(m,n)\}\}$$

3. Shift in the spatial domain:

$$\Gamma\{a(m-\alpha, n-\beta)\} = A(u,v)\,e^{-i2\pi(\alpha u+\beta v)}$$

4. Shift in the frequency domain:

$$\Gamma\{a(m,n)\,e^{i2\pi(u_0 m+v_0 n)}\} = A(u-u_0, v-v_0)$$

��=0 =1 ���� |�� ��, �� |

��−1 ��−1 2 ��−1 ��−1 2

|�� ��, �� | ��=0 ��=0

��=0