HISTOGRAM MATCHING/SPECIFICATON PROOF

- Method that is used to generate a processed image that has a specified histogram is called **histogram matching/ specification.**

- Let r and z denote the intensity levels of input and output images respectively.
- Let $p_r(r)$ and $p_z(z)$ denote their corresponding continuous probability density functions.
- We can estimate $p_r(r)$ from given input image while $p_z(z)$ is specified probability density function that we wish the output image to have.

- Let s be a random variable with the property:-

$s=T(r)$

= (L-1) p_r �� ���� ��$_0$---------------(1) where

w->dummy variable of integration

- Suppose we define a random variable z with property

��

G(z)= (L-1) p_z�� ����

0

=s ---------------------(2)

where t->dummy variable of integration

- From eq (1) and (2) we can conclude that G(z)=T(r) and therefore z must satisfy the condition

$$z = G^{-1}[T(r)]$$
$$= G^{-1}(s) \text{ --------------------(3)}$$

- Image whose intensity levels have a specified probability density function can be obtained from the given image by using the following procedure:

i. Obtain transformation function T(r) using eq 1. ii.

Use eq 2 to obtain transformation function G(z). iii.

Obtain inverse transformation $z=G^{-1}(s)$.

iv. Obtain output image by first equalizing the input image.

- Discrete formulation of histogram specification is

$s_k=T(r_k)$

�� ��=0

= (L-1) P_r(����)

=(L-1) �� ��/��

�� ��=0, k=0,1,...,L-1----(4)

where n->total number of pixels

n_j->number of pixels with gray level r_j

�� ��=0-------------- (5)

$G(z_k)$=(L-1) ��$_{��}$(����)

for a value of k so that

$$G(z_k)=s_k \text{---------------} (6)$$

- We find the desired z_k by obtaining inverse transformation:-

$$z_k = G^{-1}(s_k)$$