MATHEMATICAL MORPHOLOGY

- Mathematical Morphology is a tool for extracting image components that are useful for representation and description.
- It can provide boundaries of objects, their skeletons etc.

- It is also useful for many pre and post processing techniques especially in edge thinning, pruning etc.

BASIC CONCEPTS OF SET THEORY

Sets and Elements

- A set A is a collection of elements having the same property.
- An element a of A is denoted as $a \in A$.

- If an element x does not belong to the set A, it is denoted as $x \notin A$.
- An empty set ∅ is a set that contains null element. • Example : Let Z be the set of all integers, then $5 \in Z$, but $2.5 \notin Z$.

Relationships Between Two Sets

- A set A is equal to another set B if A and B consist of exactly the same elements. This relationship is written as $A = B$; otherwise, $A \neq B$.
- A is known as a subset of B if for all $a \in A$, $a \in B$. • A

is said to be contained in B and is denoted as $A \subseteq B$ or B contains A and is denoted as $B \supseteq A$. • Suppose $A \subseteq B$ and $A \neq B$, A is called a proper subset of B and is denoted as $A \subset B$ or $B \supset A$.

Operations Involving Sets

Given two sets A and B, which are contained in the universal set S:

(a) $A \cup B$, the union of A and B, is a new set defined as follows:

$$A \cup B = \{x \mid x \in A \quad \text{or} \quad x \in B\}$$

(b) $A \cap B$, the intersection of A and B, is a new set given by

$$A \cap B = \{x \mid x \in A \quad \text{and} \quad x \in B\}$$

(c) $A - B$, the difference of A and B, is defined as

$$A - B = \{x \mid x \in A \quad \text{and} \quad x \notin B\}$$

(d) A^c, the complement of A, is defined as

$$\begin{aligned} A^c &= \{x \mid x \in S \quad \text{and} \quad x \notin A\} \\ &= S - A \end{aligned}$$

MORPHOLOGY FOR BINARY IMAGES

- In mathematical morphology, a binary image is treated as a set consisting of the ordered pairs of coordinates of pixel points in that image.

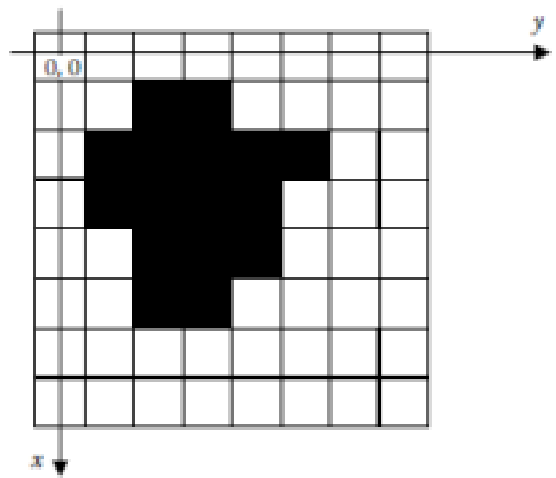


FIGURE 5.1 A binary image as a set.

- Consider the image given above. The top left pixel point is the origin, and its coordinates are given as (0,0).

- The coordinates of the pixel points in the first row from left to right are (0,0), (0,1), ..., (0,7). Similarly, the coordinates of the bottom right pixel point are (7,7).

- The given image is considered as a set of ordered pairs of coordinates denoted as $S = \{(i, j) | 0 \leq i \leq 7, 0 \leq j \leq 7\}$

- The object in the image consists of black points whose corresponding coordinates are

- $(1,2),(1,3)$
- $(2,1),(2,2),(2,3),(2,4),(2,5)$
- $(3,1),(3,2),(3,3),(3,4)$
- $(4,2),(4,3),(4,4)$
- $(5,2),(5,3)$

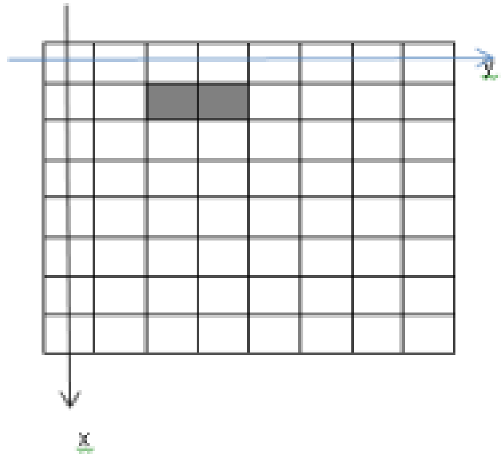
- The set of coordinates corresponding to the object is $A = \{(1,2),(1,3),(2,1),(2,2),(2,3),(2,4),(2,5),(3,1),(3,2),(3,3),(3,4),(4,2),(4,3),(4,4),(5,2),(5,3)\}$

- Suppose the whole image is considered as a universal set, then the background is the complement of A.

$$A^c = S - A$$

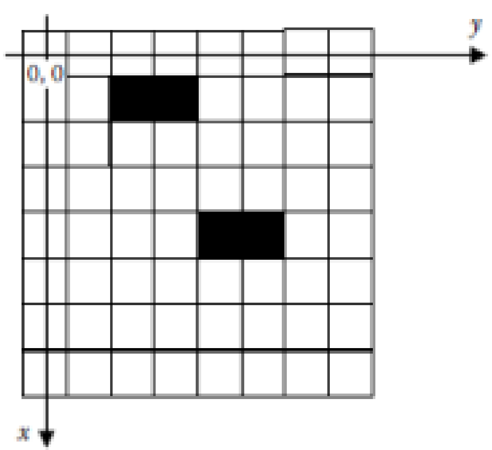
- Based on the addition of coordinates, the translation A_h of the set A by the point $h \in s$ is defined as

$$A_h = A + h$$
$$= \{x+h \in S \mid x \in A\}$$



- The above figure shows the universal set $S = \{(i, j) | 0 \leq i \leq 7, 0 \leq j \leq 7\}$ and an object $C = \{(1,2),(1,3)\}$. The translation of C by $h = (3,2) \in S$ can be calculated as

$C_h=\{(1,2)+(3,2),(1,3)+(3,2)\}=\{(4,4),(4,5)\}$



- A structuring element E is a set consisting of a local origin o, known as the representative point, and its neighbouring points.
- Figure shown below are some typical structuring elements given by

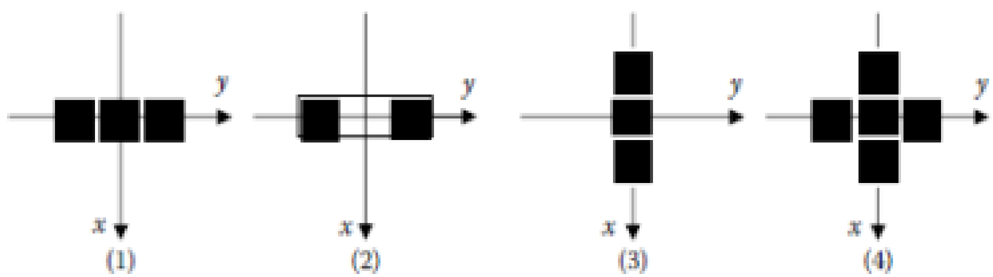


FIGURE 5.3 Some typical structuring elements.

- (1) E1 = {(0,-1),(0,0),(0,1)}

- (2) E2 = {(0,-1),(0,1)}
- (3) E3 = {(-1,0),(0,0),(1,0)}
- (4) E4 = {(-1,0),(0,-1,),(0,0),(0,1),(1,0)}

In these structuring elements, the representative point is (0,0).