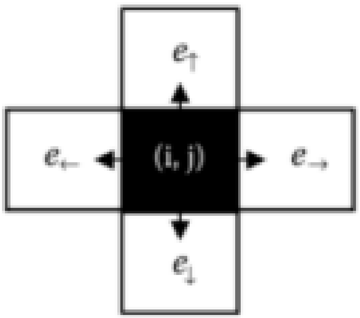
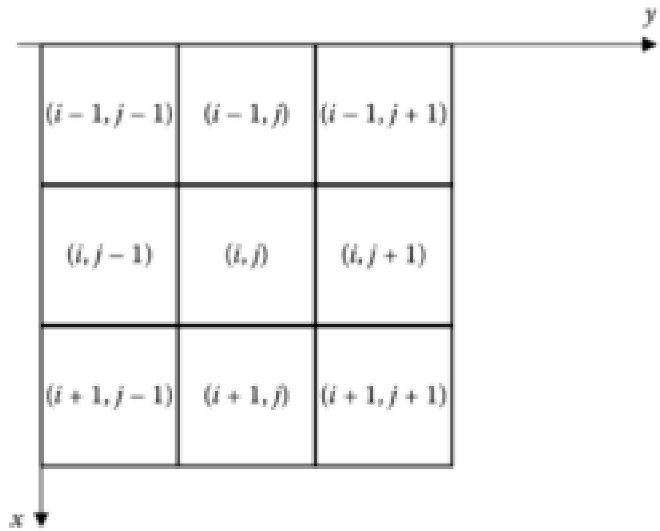
EDGE RELAXATION

- Edge relaxation is a method based on the concept of crack edges.
- There are four crack edges attached to the pixel at (i, j) that are defined by its relation to its 4-neighbours:


$e_{\uparrow}$
$e_{\leftarrow}$
(i, j)
$e_{\rightarrow}$
$e_{\downarrow}$

y
$(i-1, j-1)$
$(i-1, j)$
$(i-1, j+1)$
$(i, j-1)$
(i, j)
$(i, j+1)$
$(i+1, j-1)$
$(i+1, j)$
$(i+1, j+1)$
x

$e_{\rightarrow}(i,j)$:

$$M(e_{\rightarrow}(i,j)) = \left| f(i,j+1) - f(i,j) \right|$$

$$\theta(e_{\rightarrow}(i,j)) = \begin{cases} 0, & \text{if} \quad f(i,j+1) \geq f(i,j) \\ \pi, & \text{if} \quad f(i,j+1) < f(i,j) \end{cases}$$

$e_{\uparrow}(i,j)$:

$$M(e_{\uparrow}(i,j)) = \left| f(i-1,j) - f(i,j) \right|$$

$$\theta(e_{\uparrow}(i,j)) = \begin{cases} \dfrac{\pi}{2}, & \text{if} \quad f(i-1,j) \geq f(i,j) \\ \dfrac{3}{2}\pi, & \text{if} \quad f(i-1,j) < f(i,j) \end{cases}$$

$e_{\leftarrow}(i,j)$:

$$M(e_{\leftarrow}(i,j)) = \left| f(i,j-1) - f(i,j) \right|$$

$$\theta(e_{\leftarrow}(i,j)) = \begin{cases} \pi, & \text{if} \quad f(i,j-1) \geq f(i,j) \\ 0, & \text{if} \quad f(i,j-1) < f(i,j) \end{cases}$$

$e_{\downarrow}(i,j)$:

$$M(e_{\downarrow}(i,j)) = \left| f(i+1,j) - f(i,j) \right|$$

$$\theta(e_{\downarrow}(i,j)) = \begin{cases} \dfrac{3\pi}{2}, & \text{if} \quad f(i+1,j) \geq f(i,j) \\ \dfrac{1}{2}\pi, & \text{if} \quad f(i+1,j) \geq f(i,j) \end{cases}$$

- The pixel positions related to the crack edges are called end vertices. For example, the end vertices of the crack edge $e_{\rightarrow}(i , j)$ are (i, j) and (i, j + 1), and the end vertices of the crack edge $e_{\uparrow}(i ,j)$ are (i, j) and (i - 1, j).
- The main idea of edge relaxation is to decide whether a crack edge can be used to extend a continuous border based on the properties of its neighbors.
- Each crack edge e is assigned a confidence, represent ing its strength of being a border part.
- The main steps are described as follows:-

1.

- Crack edges are partitioned into different patterns according to the types of their two end vertices. The type of a vertex is the number of crack edges that originate from it. The type of an edge is represented by a pair of numbers consisting of the types of its vertices.
- The type of a vertex u is computed according to its other three crack edges, excluding the one currently being dealt with.
- Let (a, b, c) be the current confidences of the three crack edges and assume that $a \geq b \geq c$. Let q be a constant usually chosen as 0.1, and $m = \max(a, b, c, q)$.
- Four types of confidences can be computed in association with the vertex u:

$$conf(0) = (m-a)(m-b)(m-c)$$

$$conf(1) = a(m-b)(m-c)$$

$$conf(2) = ab(m-c)$$

$$conf(3) = abc$$

- Then the type of the vertex u is defined as

$$type(u) = j$$

2.

- Every crack edge is assigned with a confidence value as a part of the border, and the edge types are used to modify this confidence. By symmetry, only the following edge types need to be considered:

0-0: Isolated edge; the edge confidence needs to be decreased.

0-1: Uncertain; no influence on the edge confidence.

0-2, 0-3: Dead end; the edge confidence needs to be decreased.

1-1: Continuation; the edge confidence needs to be increased.

1-2, 1-3: Continuation; the edge confidence needs to be increased.

2-2, 2-3, 3-3: Bridge between borders; no influence on the edge confidence.

3. For each crack edge e, an iterative update may be applied to obtain its confidence, denoted as $c(e)$. Superscript (k) is used to denote the k-th iterative update. The $(k+1)$-th iterative update of edge confidence is based on the edge type and the previous confidence $c^{(k)}(e)$ according to the following choices:

   Confidence increases (according to the edge type):
   $c^{(k+1)}(e) = \min\{1, c^{(k)}(e) + \delta\}$
   Confidence decreases (according to the edge type):
   $c^{(k+1)}(e) = \max\{0, c^{(k)}(e) - \delta\}$
   No influence (according to the edge type):
   $c^{(k+1)}(e) = c^{(k)}(e)$

Here δ denotes a constant chosen in the range from 0.1 to 0.3 [1], which stands for the influence on the edge confidence.

Algorithm 4.3: Edge relaxation based on crack edges

For the given image $f(i,j): 0 \le i \le m-1, 0 \le j \le n-1$

Preparing two thresholds $\tau_1 < \tau_2$ for convergence estimation;

Computing all crack edges for all pixels and initialize edge confidence:

$c^{(0)}(e) = \text{normalize}(M(e))$ for every crack edge e;

$k = 0$;

Repeat

{ count=0;

For each crack edge e:-

If $(c^{(k)}(e) \neq 0)$ and $c^{(k)}(e) \neq 1$ then

{ count=count + 1;

Find the edge type according to the confidences of its neighbouring crack edges;

Update the confidence $c^{(k+1)}(e)$ according to the edge type and $c^{(k)}(e)$;

If $c^{(k+1)}(e) > \tau_2$ then $c^{(k+1)}(e) = 1$;

If $c^{(k+1)}(e) < \tau_1$ then $c^{(k+1)}(e) = 0$;

$k = k + 1$;

}

} Until (count =0)

End-Algorithm