IMAGE SMOOTHING USING FREQUENCY DOMAIN METHODS

1) IDEAL LOW-PASS FILTER (ILPF)

- The main idea of an ideal low-pass filtering is to preserve the signal with low frequency and cut off the high-frequency signal whose frequency is greater than a pre assigned value. • Transfer Function of this filter is

$$H(u,v)=\begin{cases}1, & D(u,v)\leq D_0\\ 0, & D(u,v)>D_0\end{cases}$$

where $D(u,v)$-> is the distance between the point (u,v) and the origin of the frequency domain.

D_0-> is positive constant(threshold) given in advance, called the cut-off frequency.

$$D(u,v)=\sqrt{u^2+v^2}$$

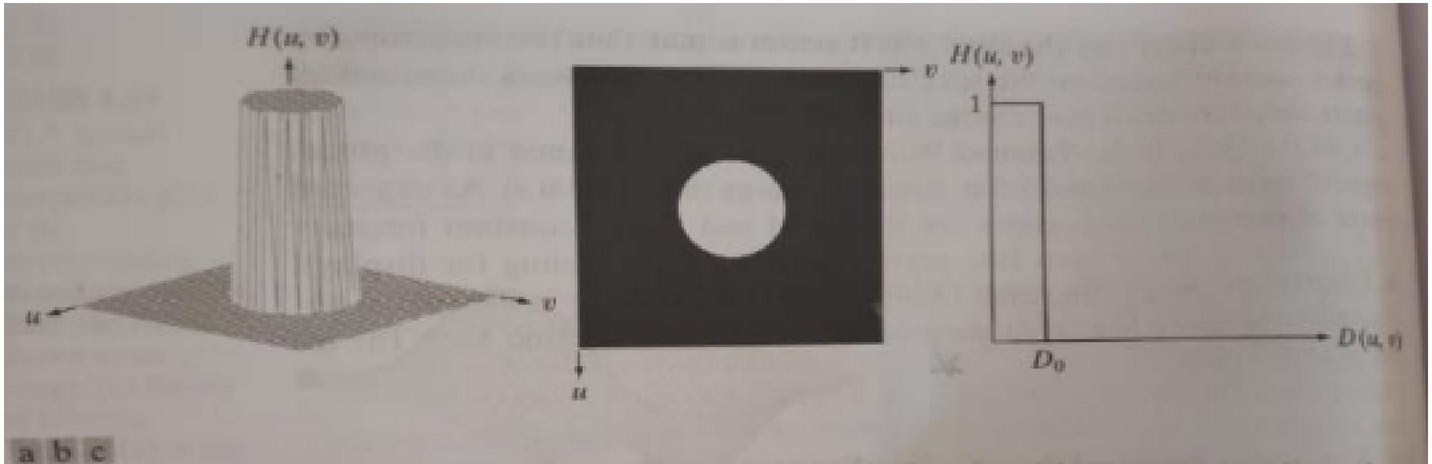


FIGURE 4.40 (a) Perspective plot of an ideal lowpass-filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross section.

- Figure 4.40 (a) shows a plot of H(u,v) and figure 4.40 (b) shows a filter displayed as an image.
- Ideal means all frequencies on or inside a circle of radius $D_0$are passed without attenuation whereas all frequencies outside the circle are completely attenuated(filtered out).
- Ideal low pass filter is radially symmetric about the

origin. Radial cross section of the filter is shown in figure 4.40 (c)

- Point of transition between H(u,v)=1 and H(u,v)=0 is called cut-off frequency. In the figure 4.40,cut-off frequency is D_{0}.
- Problem that occurs with this filter is known as Ringing Effect.

2) TRAPEZOIDAL LOW-PASS FILTER (TLPF)

- Its transfer function is defined by

$$H(u,v)=\begin{cases} 1, & D(u,v)\le D_0 \\ \dfrac{D(u,v)-D_1}{D_0-D_1}, & D_0<D(u,v)<D_1 \\ 0, & D(u,v)\ge D_1 \end{cases}$$

where $D(u,v)$-> is the distance between the point (u,v) and the origin of the frequency domain.

$$D(u,v)=\sqrt{u^2+v^2}$$

D_0-> is positive constant(threshold) given in advance, called the cut-off frequency.

D_1->is a constant satisfying $D_1 > D_0$

3) BUTTERWORTH LOW-PASS FILTER (BLPF)

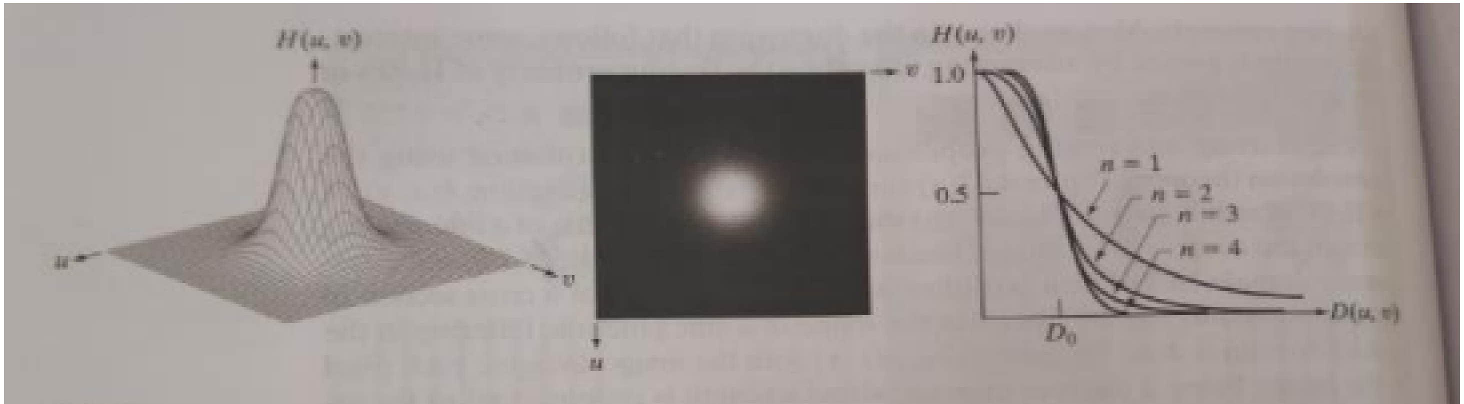
- Its transfer function is given by

$$H(u,v)=\frac{1}{1+\left[\frac{D(u,v)}{D_0}\right]^{2n}}$$

where n ->is the order of filtering

D_0-> is cut-off frequency
D(*u*,*v*)-> is the distance between the point (*u*,*v*) and the origin of the frequency domain.

$$D(u,v)=\sqrt{u^2+v^2}$$



a b c

FIGURE 4.44 (a) Perspective plot of a Butterworth lowpass-filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections of orders 1 through 4.

4) GAUSSIAN LOW-PASS FILTER (GLPF)

- Its transfer function is given by

$$H(u,v) = e^{-D^2(u,v)/2\sigma^2}$$

where D(u,v)-> is the distance between the point (u,v) and the origin of the frequency domain.

$$D(u,v) = \sqrt{u^2 + v^2}$$

σ -> is standard deviation and measure of spread of Gaussian curve

- If we put $\sigma = D_0$, then we get

$$H(u,v) = e^{-D^2(u,v)/2D_0^2}$$

where D_0-> cutoff frequency

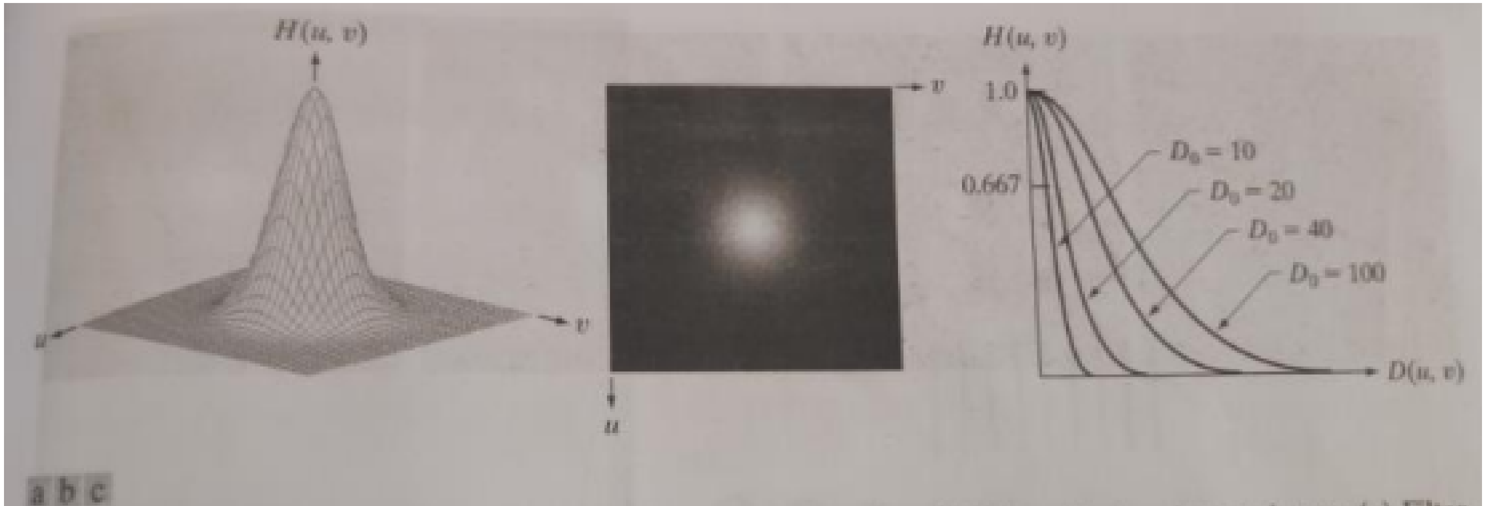


FIGURE 4.47 (a) Perspective plot of a GLPF transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections for various values of D_0.