HISTOGRAM EQUALIZATION PROOF

• It is used to create an image with equally distributed brightness level and also improves the contrast. • Idea behind this technique is to find a transformation s=T(r) to be applied to each pixel of the input image f(x,y), such that uniform distribution of gray levels in the entire range results for the output image g(x,y).

- Let r be the intensities of an image to be processed. • We assume that r is in the range from [0,L-1], where 0->black and L-1->white.
- Transformation $s=T(r)$,$0 \leq r \leq L-1$produces an output intensity level s for every pixel in the input image

having intensity r.

- We assume that

a) T (r) is monotonically increasing function in the interval $0 \leq r \leq L-1$.

b) $0 \leq T(r) \leq L-1$ for $0 \leq r \leq L-1$.

- We also use the inverse

$r=T^{-1}(s)$ $0 \leq s \leq L-1$ in which condition a will

change to

a') T (r) is strictly monotonically increasing function in the interval $0 \leq r \leq L-1$.

- Condition (a) guarantees that output intensity values will never be less than corresponding input values. • Condition (b) guarantees that range of output intensities is same as input.
  - Condition (a’) guarantees that mappings from s back to r will be one-to-one ,thus preventing ambiguities.

a b

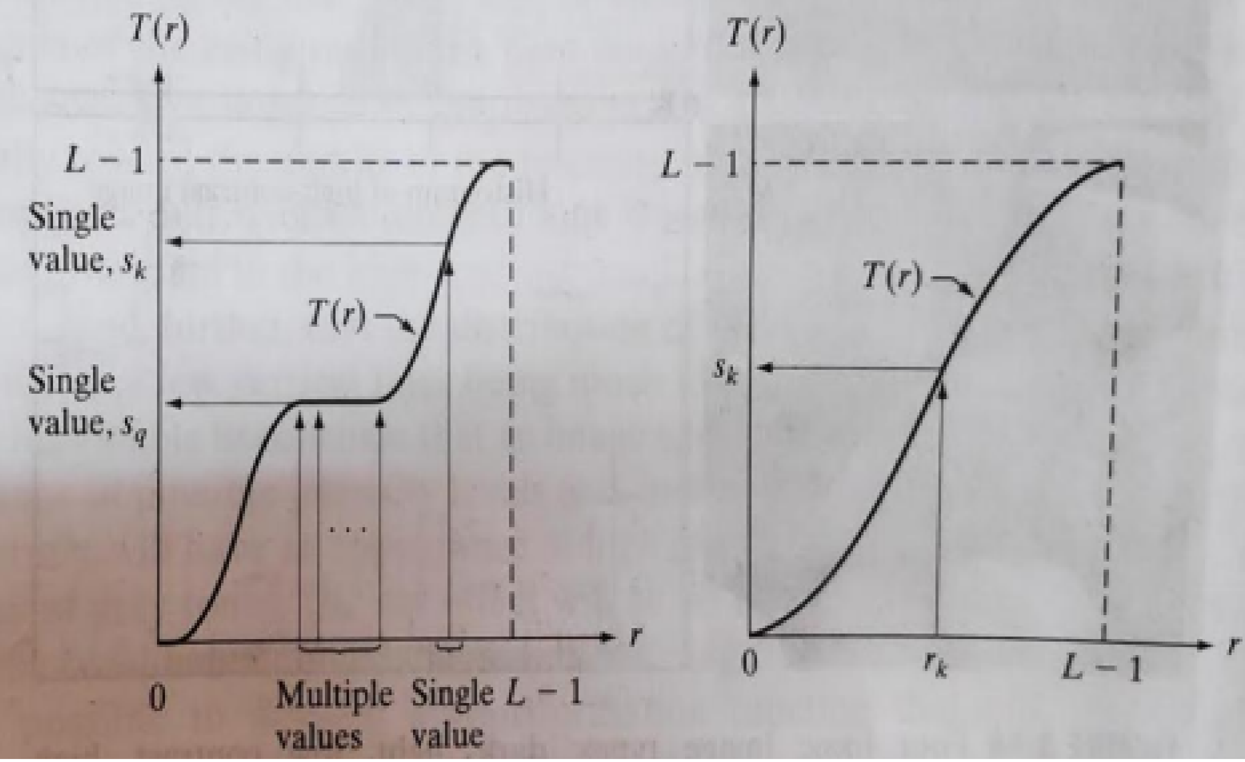

FIGURE 3.17 (a) Monotonically increasing function, showing how multiple values can map to a single value. (b) Strictly monotonically increasing function. This is a one-to-one mapping, both ways.

- Let $p_r(r)$ and $p_s(s)$ denote probability density function of variable r and s respectively.

- Transformation function is of the form

$$s=T(r)$$

=(L-1) p_r �� ���� $^{��}{}_{0}$ ---------- (1)

where w->dummy variable of integration

- Basic probability theory is that if $p_r(r)$ and T(r)are known and T(r) is continuous and differentiable, then probability density function of transformed variable s can be obtained by using the formula

$p_s(s)$=$p_r(r)$ ����

���� -------------- (2)

����

���� = ����(��)

����

= (L-1) �� ���� [redacted] r [redacted] �� 0 =(L-1) p_r��

------------ (3)

- Substituting eq (3) in eq (2)

$p_s(s)= p_r(r)$ ����

����

= $p_r(r)$ 1

(��−1)$p_r(r)$

= 1

(��−1) $0 \leq s \leq L-1$

- Probability of occurrence of gray level r_k in an image is given by:-

$P_r(r_k) = n_k/n$ -------- (4) • Discrete approximation of transformation function for histogram equalization is:-

$s_k = T(r_k)$

$= \sum_{��=0}^{��} P_r(����)$ -------------- (5)

• Substituting equation 1 in 2

$s_k = \sum_{��=0}^{��} ��_{��}/��$ k=0,1,……..L-1

where r_k->input intensity

s_k-> processed intensity

n_j -> frequency of intensity j

n-> sum of all frequencies