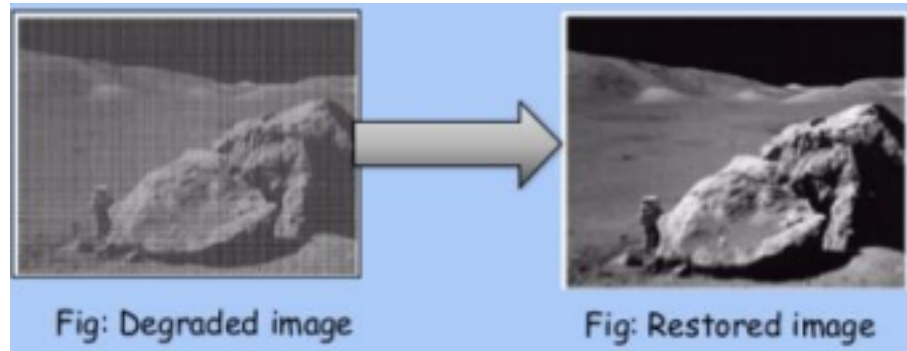


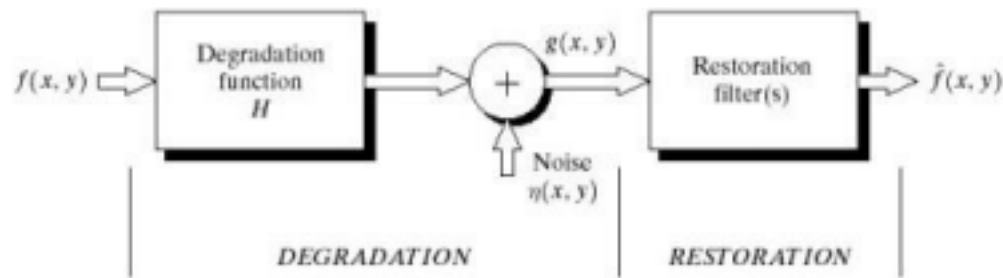
# IMAGE RESTORATION <sup>1</sup>

## IMAGE RESTORATION

- It restores the images that have been degraded with prior knowledge of degradation function.



# IMAGE DEGRADATION/RESTORATION MODEL



- Degradation function  $H$ , along with additive noise  $\eta(x,y)$  operates on input image  $f(x,y)$  to produce degraded image  $g(x,y)$ .
- Given  $g(x,y)$  along with some knowledge of degradation function  $H$  and some knowledge about noise term  $\eta(x,y)$ , objective of restoration is to obtain an estimate  $\hat{f}(x,y)$  of original image.  $\hat{f}$
- The more information we know about  $H$  and  $\eta$ , more closer  $\hat{f}(x,y)$  will be to  $f(x,y)$ .

- Degradation function in spatial domain is given by:-

$$g(x,y)=h(x,y)*f(x,y)+ \text{N}(x,y)$$

where  $h(x,y)$   $\rightarrow$  spatial representation of degradation function

\*  $\rightarrow$  convolution

- Degradation function in frequency domain is given by:-

$$G(u,v)=H(u,v)F(u,v)+ N(u,v)$$

where capital letters are Fourier transforms of corresponding terms.

# INVERSE FILTERING

- Simplest approach of restoration.
- Idea is to recover original image from blurred image. •

We compute the estimate  $\hat{F}(u,v)$  of the transform of original image by dividing transform of degraded image  $G(u,v)$  by degradation function:-

$$\hat{F}(u,v) = \frac{G(u,v)}{H(u,v)} \quad (1)$$

- Degradation function in frequency domain is given by the equation:-

$$G(u,v) = H(u,v)F(u,v) + N(u,v) \quad (2)$$

- Substituting eq 2 in eq 1

$$\hat{F}(u,v) = \frac{1}{2} \left( F(u,v) + \frac{1}{2} (F(u,v) + F(u,v)) \right) + \frac{1}{2} (F(u,v) + F(u,v)) \quad (3)$$

## Disadvantage:-

To recover the undegraded image, we should know degradation function plus noise.

If degradation function has zero or very small values than the ratio  $\frac{\hat{F}}{D(u,v)}$

$\frac{\hat{F}}{D(u,v)}$  will dominate the estimate  $(u,v)$ .

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(a) The blur image



(b) The result image by using inverse filtering

# WIENER FILTERING

- Aim is to find best estimate  $\hat{f}$  of the uncorrupted image  $f$  such that mean square error between them is minimized.
- This error is given by:-

$$E[(f - \hat{f})^2] \text{----- (1)}$$

where  $E\{.\}$  -> expected value of the argument • It is assumed that the noise and the image are uncorrelated that is, one or the other has zero mean and intensity levels in the estimate are linear function of levels in degraded image.



- Based on these conditions, minimum of error function in equation 1 is given in frequency domain by the expression

$$\hat{F}(u, v) = \left[ \frac{H^*(u, v) S_f(u, v)}{S_f(u, v) |H(u, v)|^2 + S_\eta(u, v)} \right] G(u, v) \text{-----}(2)$$

$$= \left[ \frac{H^*(u, v)}{|H(u, v)|^2 + S_\eta(u, v) / S_f(u, v)} \right] G(u, v) \text{-----}(3)$$

$$= \left[ \frac{1}{H(u, v)} \frac{|H(u, v)|^2}{|H(u, v)|^2 + S_\eta(u, v) / S_f(u, v)} \right] G(u, v) \text{-----}(4)$$

- Where we have used the fact that product of complex quantity with its conjugate is equal to the magnitude of

complex quantity squared. This result is known as Wiener Filter.

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- Terms inside the brackets are referred to as minimum mean square error filter or least square error filter.
- From equation 3 terms are as follows:-

$H(u,v)$  = degradation function

$|H(u,v)|^2 = H^*(u,v)H(u,v)$

$H^*(u,v)$  = complex conjugate of  $H(u,v)$

$S_n(u,v) = |N(u,v)|^2$  power spectrum of noise

$S_f(u,v) = |F(u,v)|^2$  power spectrum of undegraded image

$G(u,v)$  is the transform of the degraded image.

- Wiener filter does not have same problem as that of inverse filter with the zeros in the degradation function, unless the entire denominator is zero for same values of

u and v.

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