MORPHOLOGY FOR GREY SCALE IMAGES

- **Grey Scale dilation Operation**
- Grey Scale Erosion Operation

GREY SCALE DILATION OPERATION

- The dilation of a grey-scale image f (i, j), $0 \le i \le n_0$-1, 0 $\le j \le m_0$-1 , and the structuring element h(s,t),$m_1 \le$ s

$\leq m_2$, $n_1 \leq t \leq n_2$, denoted by $f \oplus h$ is defined as

$$f \oplus h(i,j) = \max\{f(i-s, j-t) + h(s,t) \mid n_1 \leq s \leq n_2, 0 \leq (i-s) \leq n_0 - 1, \quad m_1 \leq t \leq m_2, 0 \leq (j-t) \leq m_0 - 1\}$$

where $0 \leq i \leq n_0 - 1, 0 \leq j \leq m_0 - 1.$

- Similar to convolution operation.

- Compute the new signal generated by the dilation $f \oplus h$ given one-dimensional signal as follows

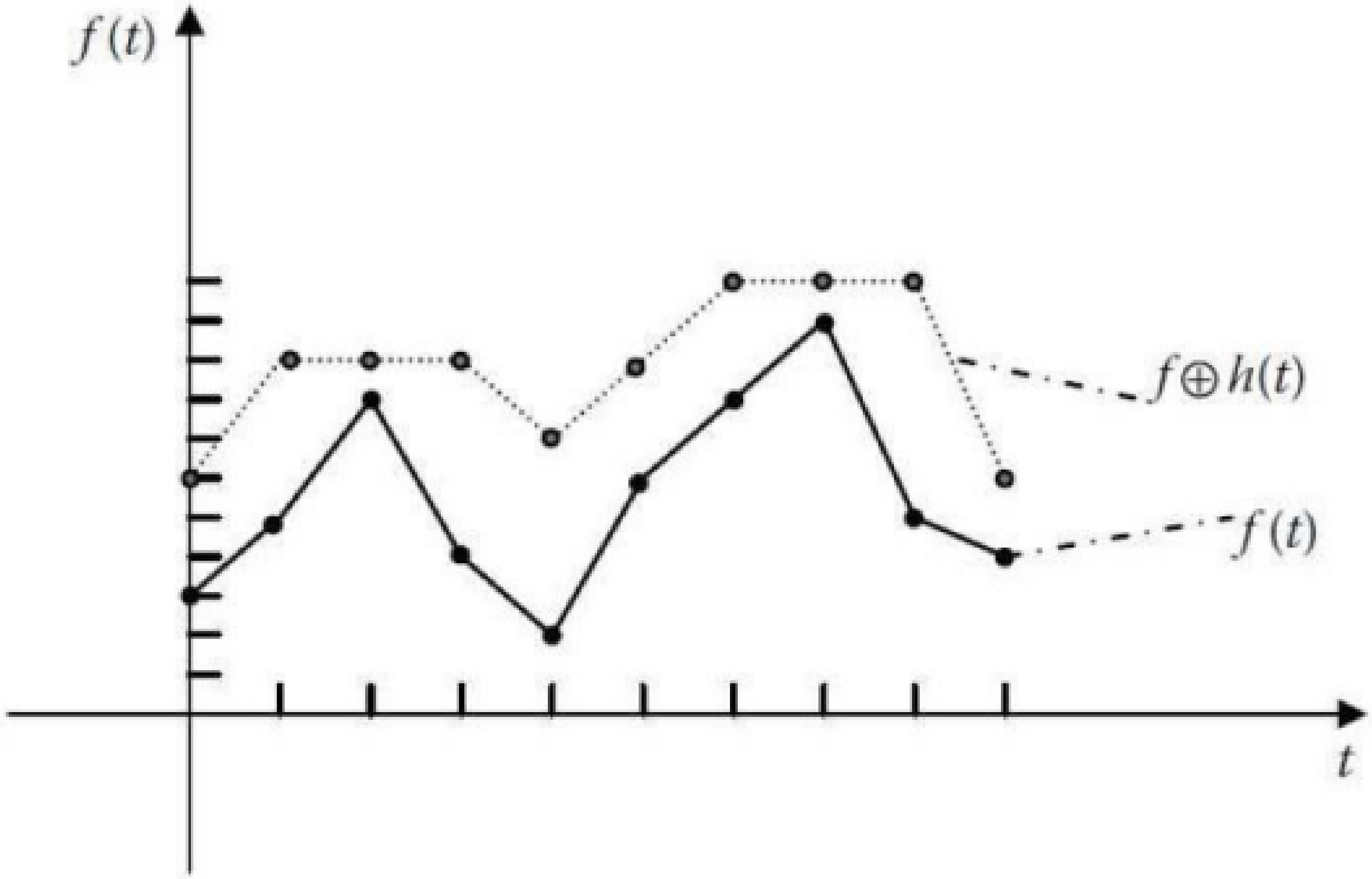
$f(t) = \{ f(0), f(1), f(2), f(3), f(4), f(5), f(6), f(7),$

f(8), f(9)}

= {3, 5, 8, 4, 2, 6, 8, 10, 5, 4}

With structuring element: h(t)= {h(-1), h(0), h(1)}={1,1,1}

FOR SOLUTION REFER THE VIDEO ATTACHED



An illustration of $f \oplus h(t)$.

EXAMPLE 2

The following matrix defines an 8-bit grey scale image with the size 8 × 8.

$$f = \begin{bmatrix} f(0,0) & f(0,1) & \cdots & f(0,7) \\ f(1,0) & f(1,1) & \cdots & f(1,7) \\ \cdots & \cdots & \cdots & \cdots \\ f(7,0) & f(7,1) & \cdots & f(7,7) \end{bmatrix}$$

The following matrix defines an 8-bit grey scale image with the size 8 × 8.

$$
= \begin{bmatrix}
200 & 201 & 202 & 202 & 203 & 202 & 200 & 198 \\
202 & 203 & 205 & 204 & 204 & 202 & 200 & 197 \\
205 & 210 & 211 & 212 & 210 & 209 & 208 & 205 \\
205 & 208 & 210 & 212 & 214 & 210 & 211 & 208 \\
210 & 212 & 215 & 218 & 217 & 219 & 220 & 218 \\
212 & 214 & 218 & 220 & 220 & 219 & 218 & 218 \\
210 & 212 & 213 & 215 & 216 & 216 & 210 & 212 \\
208 & 208 & 210 & 211 & 212 & 214 & 210 & 210
\end{bmatrix}
$$

- Structuring Element

$$h = \begin{bmatrix} h(-1,-1) & h(-1,0) & h(-1,1) \\ h(0,-1) & h(0,0) & h(0,1) \\ h(1,-1) & h(1,0) & h(1,1) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

- Compute the intensities of pixels located at (0,0) and (3,2) in the resulting image after performing the dilation f $\oplus$ h.

Dilation Problem

$$f \oplus h(0,0) = \max\left\{ \begin{bmatrix} 200 & 201 \\ 202 & 203 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \right\}$$

Convolve both

$$= \max\left\{ \begin{bmatrix} 203 & 202 \\ 201 & 200 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \right\}$$

$$= \max \begin{bmatrix} 203 & 203 \\ 202 & 201 \end{bmatrix}$$

$$= 203$$

Convolve both

$$f \oplus h(3,2) = \max\left\{ \begin{bmatrix} 210 & 211 & 212 \\ 208 & 210 & 212 \\ 212 & 215 & 218 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix} \right\}$$

$$= \max\left\{ \begin{bmatrix} 218 & 215 & 212 \\ 212 & 210 & 208 \\ 212 & 211 & 210 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix} \right\}$$

$$= \max\left\{ \begin{bmatrix} 218 & 216 & 212 \\ 213 & 211 & 209 \\ 212 & 212 & 210 \end{bmatrix} \right\}$$

$$= 218$$

Algorithm 5.3: Grey-scale dilation algorithm

For the given 8-bit grey-scale image $f(i, j)$ $0 \le i, j \le n-1$ with the given structure element matrix $e(s, t)$ $0 \le s, t \le m-1$:-

For $i = 0$ to $n-1$

For $j = 0$ to $n-1$

  $g(i, j) = f(i, j)$; // initialise the result image;

End-for

For $i = m$ to $n - m - 1$

  For $j = m$ to $n - m - 1$ do //exclude border rows and columns;

    $g(i, j) = 255$; max $= f(i, j)$;

    For $s = -m$ to m

    For $t = -m$ to m do

      $temp = f(i - s, j - t) + e(s, t)$;

      If ($temp >$ max) then max $= temp$; End-If

    End-For

    $g(i, j) = max$;

    If $((g(i, j) > 255))$ then $g(i, j) = 255$; End-If

End-For

Output of the resulting image: $g(i, j)$, $0 \le i, j \le n-1$

End-Algorithm

- Grey Scale dilation Operation
- **Grey Scale Erosion Operation**

GREY SCALE EROSION OPERATION

- The erosion of a grey-scale image f (i, j),$0 \le i \le n_0-1$, $0 \le j \le m_0-1$, and the structuring element h(s,t),$m_1 \le s \le m_2$, $n_1 \le t \le n_2$, denoted by $f \Theta h$ is defined as

$$f \Theta h(i,j) = \min\{ f(i+s, j+t) - h(s,t) \mid n_1 \le s \le n_2, 0 \le (i+s) \le n_0 - 1, \\ m_1 \le t \le m_2, 0 \le (j+t) \le m_0 - 1 \}$$

$$\text{where } 0 \le i \le n_0 - 1, 0 \le j \le m_0 - 1.$$

- Similar to correlation operation.

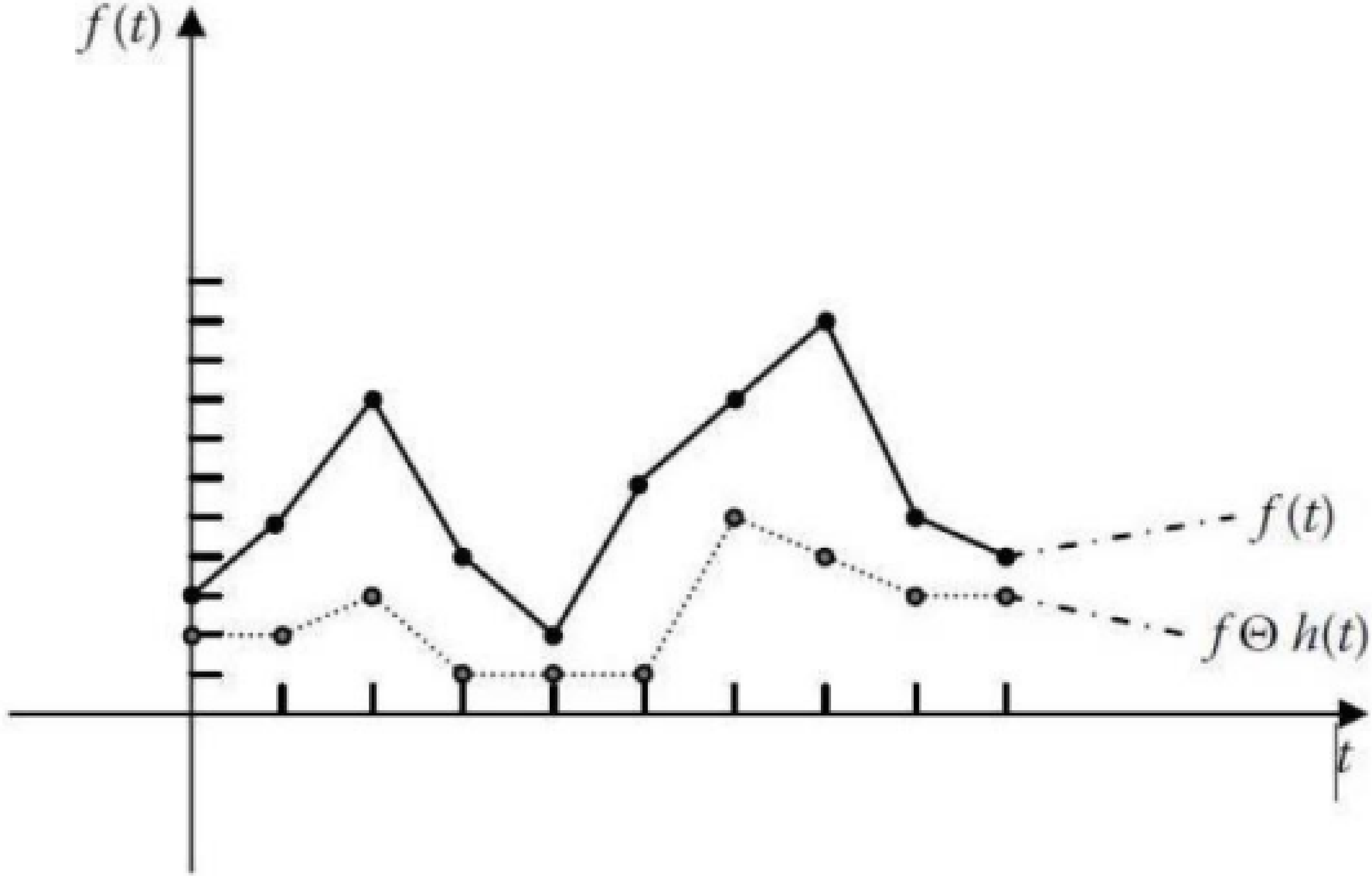
- Compute the new signal generated by the erosion f $\ominus$ h given one-dimensional signal as follows

$f(t) = \{ f(0), f(1), f(2), f(3), f(4), f(5), f(6), f(7), f(8), f(9)\}$

$= \{3, 5, 8, 4, 2, 6, 8, 10, 5, 4\}$

With structuring element: $h(t)= \{h(-1), h(0), h(1)\}=\{1,1,1\}$

FOR SOLUTION REFER THE VIDEO ATTACHED



An illustration of $f \Theta h(t)$.

EXAMPLE 2

- Structuring Element

- Compute the intensities of pixels located at (0,0) and (3,2) in the resulting image after performing the erosion $f \ominus h$.

APPLICATIONS OF GRAY SCALE MORPHOLOGY

- Although formulas for grey scale dilation and erosion are different from that of binary images, definitions are similar.
- Opening and closing operations are similar to that of binary opening and closing operations.
- The opening $f \circ h$ and closing $f \bullet h$ of a grey-scale image f by a structuring element h are defined as follows: