Assignment 1

Sunday, August 25, 2024 6:37 PM

1.3 Determine whether or not each of the following signals is periodic. In case a signal is periodic, specify its fundamental period.

a) $x_a(t) = 3\cos(5t + \pi/6) = x_a(t+T_p) = 3\cos(5(t+T_p)+\pi/6)$

$x_a(t+T_p) = x_a(t)$ $\quad = 3\cos(5t + 2\pi + \pi/6) = x_a \rightarrow$ because offset does not affect periodicity

This function is periodic because it is given in continuous time. Therefore the fundamental period is found by $\Omega = 2\pi F$, where $\Omega = 5$ so:

$$5 = 2\pi F$$

fundamental frequency $\rightarrow$ $F = \frac{5}{2\pi}$ and $T_p = \frac{1}{F}$

$T_p = \frac{2\pi}{5}$ which is the fundamental period

b) $x(n) = 3\cos(5n + \pi/6)$

$\omega = 2\pi f.$

- Periodic only if f is a rationale number $\rightarrow$ find N, an integer, the period in samples

$2\pi f_0 N = 2\pi k$

$5N = 2\pi k$

$N = \frac{2\pi k}{5}$ let $k = 5$, it must be an integer

$N = 2\pi$ $\rightarrow$ this is not rationale, therefore f is not rationale either

the signal $x(n)$ is not periodic

c) $x(n) = 2e^{j(n/6 - \pi)}$

using Euler's identity: $\theta = n/6 - \pi$

$$x(n) = 2\left[\cos(n/6 - \pi) + j\sin(n/6 - \pi)\right]$$

$\omega = 2\pi f_0 = \frac{1}{6}$

$\frac{1}{6}N = 2\pi k$

$N = 12\pi k$, this is not rationale for any k integer, f is not rationale

the signal $x(n)$ is not periodic

d)

the signal $x(n)$ is not periodic

d) $x(n) = \cos(n/8)\cos(\pi n/8)$

each component must be periodic

$$\omega_1 = 2\pi f_0 = \frac{1}{8}$$

$$\frac{1}{8}N = 2\pi k$$

$N = 16\pi k$, this is not rationale for any k integer, f is not rationale

the signal $x(n)$ is not periodic

e) $x(n) = \cos\left(\frac{\pi n}{2}\right) - \sin\left(\frac{\pi n}{8}\right) + 3\cos\left(\frac{\pi n}{4} + \pi/3\right)$

$\omega_1 = 2\pi f_{01} = \frac{\pi}{2}$	$\omega_2 = 2\pi f_{02} = \frac{\pi}{8}$	$\omega_3 = \frac{\pi}{4}$
$\frac{\pi}{2}N_1 = 2\pi k_1$	$\frac{\pi}{8}N_2 = 2\pi k_2$	$\frac{\pi}{4}N_3 = 2\pi k_3$
$N_1 = 4k_1$	$N_2 = 16k_2$	$N_3 = 8k_3$
let $k_1 = 1$, $N_1 = 4$	let $k_2 = 1$, $N_2 = 16$	let $k_3 = 1$, $N_3 = 8$

$N_2 > N_3 > N_1$ so $N_2 = 16$ is the fundamental period

1.7 An analog signal contains frequencies up to 10 kHz.

a) What range of sampling frequencies allows exact reconstruction of this signal from its samples?

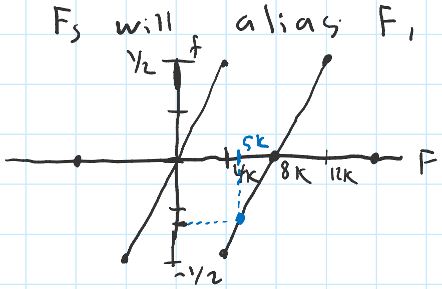
$F_n > 20\,kHz$ according to the Nyquist Rate

b) Suppose that we sample this signal with a sampling frequency Fs = 8 kHz. Examine what happens to the frequency F1 = 5 kHz.

F_s will alias F_1 because: $F_s \ngtr 2 \cdot F_1$

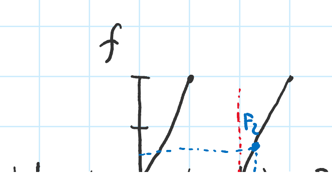
$8\,kHz \ngtr 10\,kHz$

$$\boxed{F_{alias} = |F_1 - 1 \cdot F_s| = 3\,kHz}$$



c) Repeat part (b) for a frequency F2 = 9 kHz.

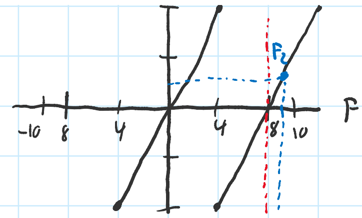
F_s undersamples F_2 so F_2 is aliased



F_s undersamples F_2 so F_2 is aliased

$F_s \not> 2 \cdot F_2$

$8k \not> 18\,kHz$ $\qquad F_{alias} = |F_2 - 1 \cdot F_s| = 1\,kHz$



1.9 An analog signal xa(t) = sin(480 * pi * t) + 3sin(720 * pi * t) is sampled 600 times per second.

a) Determine the Nyquist sampling rate for xa(t).

The term $3\sin(720\pi t)$ will require the highest sampling rate

$\Omega_2 = 720\pi = 2\pi F_2 \qquad F_2 = 360\,Hz$

$F_{max} = 2 \cdot F_2 = \boxed{720\,Hz}$ is the Nyquist rate

b) Determine the folding frequency (using sampling rate 600 Hz).

$$F_{folding} = \frac{F_s}{2} = \frac{600}{2} = \boxed{300\,Hz}$$

c) What are the frequencies, in radians, in the resulting discrete time signal x(n)? (using sampling rate 600 Hz)

$\sin \frac{6\pi}{5} = -\sin\frac{4\pi}{5}$

$f_1 = \frac{F_1}{F_s} = \frac{480\pi}{600} = \omega_1 \qquad f_2 = \frac{F_2}{F_s} = \frac{720\pi}{600} = \omega_2$

$\omega_1 = \frac{4\pi}{5}$ rad/sample $\qquad \omega_2 = \frac{6\pi}{5} \frac{rad}{sample}$

$-\pi \leq \omega_0 \leq +\pi$

$\Rightarrow \omega_2 = -\frac{4\pi}{5} \frac{rad}{sample}$

d) If x(n) is passed through an ideal D/A converter, what is the reconstructed signal ya(t)? (using sampling rate 600 Hz)

$f_{01} = \frac{480\pi}{2\pi} = \underline{240\,Hz} \qquad f_{02} = \frac{720\pi}{2\pi} = 360\,Hz \rightarrow$ this will be aliased

$$f_{02alias} = |f_{02} - F_s| = \underline{240\,Hz}$$

both components are 240Hz -

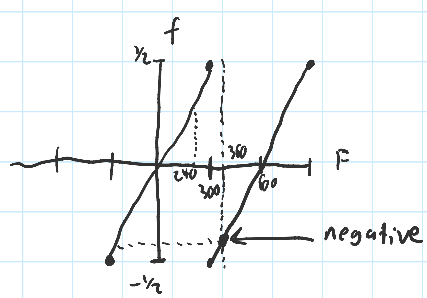
~~$y_a(t) = \sin(480\pi t) + 3\sin(480\pi t)$~~

~~$y_a(t) = 4\sin(480\pi t)$~~

from part c, second component is actually negative

so $y_a(t) = \sin(480\pi t) - 3\sin(480\pi t)$

$= -2\sin(480\pi t)$



1.10 A digital communication link carries binary-coded words representing samples of an input signal:

xa(t) = 3cos(600 *pi * t) + 2cos(1800 * pi * t)

The link is operated at 10,000 bits/sec and each input sample is quantized into 1024 different voltage levels.

a) What are the sampling frequency and the folding frequency?

1 Sample = 2^{10} levels

$\Rightarrow$ requires 10 bits

$$F_s = \frac{bits/sec}{sample/bits} = \frac{10{,}000}{10} = 1K\,\frac{Sample}{sec} = 1\,KHz$$

1 Sample = 2^{10} levels
$\Rightarrow$ requires 10 bits

$F_s = \frac{\text{bits/sec}}{\text{Sample/bits}} = \frac{10{,}000}{10} = 1\text{K} \frac{\text{Sample}}{\text{sec}} = 1\text{KHz}$

$\boxed{F_{folding} = \frac{F_s}{2} = 500\text{ Hz}}$ $\boxed{F_s = 1\text{KHz}}$

b) What is the Nyquist rate for the signal xa(t)?

Second component has higher frequency

$\Omega_2 = 1800\pi = 2\pi F_2$, $F_2 = 900\text{ Hz}$ $F_{max} = 2 \cdot F_2 = \boxed{1800\text{Hz}}$

c) What are the frequencies in the resulting discrete-time signal x(n)?
(use sampling rate from part a)

$f_{t1} = \frac{600\pi}{2\pi} = 300\text{Hz}$ $f_{t2} = \frac{1800\pi}{2\pi} = 900\text{Hz}$

$\boxed{f_{n1} = 300\text{ Hz}}$ because $f_{t1} \cdot 2 = 600\text{Hz} < F_s = 1000\text{ Hz}$

f_{t2} is aliased because $f_{t2} \cdot 2 = 1800\text{Hz} > F_s = 1000\text{ Hz}$

So

$f_{n2\,alias} = |f_{t2} - 1 \cdot F_s| = \boxed{100\text{ Hz}}$

d) What is the resolution delta? (use sampling rate from part a)

resolution delta is the difference between two samples,
scaled by the amplitude range.
Since there are 1024 levels, the difference between
two samples is $\frac{1}{1024-1}$

So $\Delta = \frac{1}{1023} A$, where A is the amplitude range

$3\cos\left(\frac{600}{1000}\pi t\right) = 3\cos\left(\frac{3\pi}{5} t\right)$

$2\cos\left(\frac{1800}{1000}\pi t\right) = 2\cos\left(\frac{3\pi}{5} t\right)$

both terms have a minimum and maximum for the same input
so -5 = min, and $+5$ = max

$\Delta = \frac{5-(-5)}{1023} = \frac{10}{1023}$