GNSS Positioning: Theoretical Groundwork for Adaptive Kalman Filter using MLE and Implementation of Moving Window Averages

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Durvesh Sunil Baharwal

Abstract

For navigation, surveying, and other uses that call for precise location data, the "Global Navigation Satellite System" (GNSS) plays the crucial tool. The Satellites constellations known as the Global Navigation Satellite System, (GNSS) send signals to GNSS receivers on the ground or in orbit. GNSS receivers to calculate their position, speed and time use these signals. GNSS finds many applications in the position's determination of satellites in LEO and GEO orbits also. In our study focusing on enhancing Global Navigation Satellite Systems (GNSS) precision, we forged a theoretical foundation for implementing Maximum Likelihood Estimation (MLE) within the Extended Kalman Filter (EKF). Our groundwork involved a meticulous exploration of MLE's pivotal role in refining positioning accuracy by maximizing the likelihood function, specifically concerning GNSS residuals. While direct implementation awaited, our efforts concentrated on laying robust groundwork, including the practical implementation of a complex moving window average in MATLAB. This moving window average function serves as a critical component in our envisioned MLE equations for updating CNN (Covariance of the Clock Noise) and CEE (Covariance of the Code and Carrier Phase) matrices within the Extended Kalman Filter framework, thereby advancing the prospects of achieving superior GNSS-based positioning accuracy.

List of acronyms

1. DGPS:	Differential GPS
2. ECEF:	Earth-Centered Earth-Fixed
3. EGNOS:	European Geostationary Navigation Overlay Service
4. EKF:	Extended Kalman Filter
5. GLONASS:	Global Navigation Satellite System
6. GNSS:	Global Navigation Satellite System
7. GPS:	Global Positioning System
8. IC:	Integrated Circuits
9. KF:	Kalman Filter
10. RINEX:	Receiver Independent Exchange format
11. RTK:	Real-Time Kinematic
12. SA:	Selective Availability
13. SBAS:	Satellite Based Augmentation System
14. SNR:	Signal-to-Noise Ratio
15. UKF:	Unscented Kalman Filter
16. MLE:	Maximum Likelihood Estimation

Introduction

Prologue:

In an era driven by technological advancements, the importance of precise and reliable Global Navigation Satellite System (GNSS) positioning is undeniable. Applications ranging from autonomous vehicles to geodetic surveys rely on GNSS data to function efficiently. However, in resource-constrained hardware environments, obtaining high-precision GNSS data can be challenging.

Motivation:

Navigation systems are the backbone of diverse applications, relying on precise state estimations in dynamic environments. Our exploration dives deep into the fusion of Maximum Likelihood Estimation (MLE) within the Extended Kalman Filter (EKF), a powerful tandem in addressing nonlinear complexities in estimation models. However, our pursuit isn't solely theoretical. We aim to augment this framework's adaptability by introducing a novel approach: leveraging the moving window average technique directly within the MLE equation. By incorporating this refined residual computation from satellite data epochs and normalizing it by satellite appearances in prior epochs, our goal is to significantly refine and fortify the accuracy of estimations. This endeavor isn't just about advancing navigation—it's about revolutionizing state estimation methodologies, offering a more resilient and precise solution for intricate real-world scenarios.

Objective:

The primary objective of this internship project is to implement GNSS positioning into a MATLAB programming environment, to increase GNSS positioning accuracy by Integrating (MLE) Maximum Likelihood Estimation on (EKF) Extended Kalman Filter based differential GNSS module in MATLAB environment. The specific goals are as follows:

1. **Incorporate Least Squares:** Implement least squares algorithms by using Time-Differenced Pseudo Range method for Velocity estimation of GNSS receiver positions and clock offsets.
2. **Implement Kalman Filtering:** Integrate Kalman filtering techniques to improve the real-time GNSS positioning accuracy and robustness, especially in environments with signal disruptions.
3. **Moving Window Average:** Employ a moving window average to derive the mean of residuals from available satellite data at each epoch, normalizing by the count of appearances of satellites in previous epochs, facilitating their integration into the Maximum likelihood Estimation equations within the Extended Kalman Filter Framework.
4. **Study of Maximum Likelihood Estimation (MLE) and MLE within the Extended Kalman Filter:** Conducted an in-depth study and analysis of the principles, methodologies, and applications of Maximum Likelihood Estimation (MLE) and its adaptation within the

Extended Kalman Filter Framework, laying the groundwork for future implementation and application in navigation and estimation systems.

Overview of GPS positioning

The Global Positioning System (GPS) was created in the 70s by the U.S. Department of Defense (DOD). It is the only fully functional Global Navigation Satellite System (GNSS) among those currently operational (the others are the Russian GLONASS and the Chinese regional geostationary BEIDOU). Other GNSS are being developed by the European Union (GALILEO), China (COMPASS or BEIDOU 2), India (IRNSS) and Japan (QZSS).

The GPS is composed by three parts: the space segment, the control segment and the user segment. The space segment is currently formed by a constellation of 32 satellites, orbiting at an altitude of approximately 20000 km on six orbital planes, approximately tilted of 55° with respect to Earth's equatorial plane. The system was designed to always guarantee the visibility of at least 4 satellites from every point on the Earth surface with clear sky view.

The control segment includes a Master Control Station located near Colorado Springs and several monitoring stations located around the world. The purpose of the control segment is to monitor the satellite transmissions continuously, to predict the satellite ephemerides, to calibrate the satellite clocks and to periodically update the navigation information that the satellites carry onboard.

Absolute positioning

GPS positioning is based on the hypothesis that the signal travel time from a satellite to the receiver is known. This would imply that the signal transmission and reception epochs with respect to a reference GPS time are known exactly. Nevertheless, since a perfect synchronism between either satellite or receiver clocks and GPS time is impossible, the offset between each clock and GPS time must be taken into account. For satellite and receiver clocks respectively the offset is defined as

$$dt^s = t^s - t_{GPS}$$

$$dt_r = t_r - t_{GPS}$$

where t^s is the satellite clock, t_r is the receiver clock and t_{GPS} is the reference GPS time.

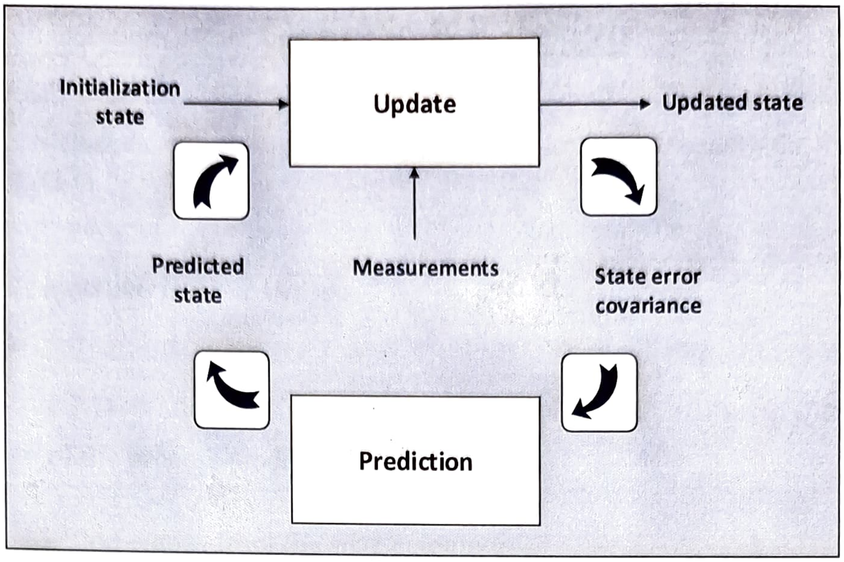
Relative positioning

GPS observations are affected by various kinds of errors, such as clock errors on both receiver and satellites, signal delays due to its transition through the atmosphere, signal reflections (i.e. multipath), just to name a few. Some of them can be reduced or totally removed by applying a relative positioning between two receivers. By differentiating the observations of two receivers (coming from satellites they both can "see") it is possible to remove satellite and receiver clock errors and to reduce atmospheric errors (if the two receivers are close enough to consider the atmospheric effect to be the same).

Relative positioning can be either static or kinematic. In both cases one of the two receivers is stationing at a location of known coordinates (master station), while the other one is respectively static or moving (rover).

Kalman Filtering

Kalman filtering (KF) is a recursive algorithm used for optimal parametric estimation in linear dynamic systems. It combines measurements with past information about the system's evolution to estimate unknown parameters. This process involves state variables, which are updated at each epoch. The KF cycle begins with initializing state variables and their error covariance matrix, followed by the updating of state estimations based on observation equations. The KF is often applied in the context of GPS positioning and navigation, including signal tracking, multi-sensor fusion, ambiguity resolution, and trajectory smoothing.



In scenarios where nonlinearity is introduced due to system dynamics or observations, Extended Kalman Filters (EKF) or Unscented Kalman Filters (UKF) are employed. The EKF linearizes dynamic model equations around the previous state, while the UKF uses the unscented transform for nonlinear transformations. The KF is versatile and can integrate various features, such as accelerometers, gyroscopes, and odometers, to aid GPS positioning and navigation. A common application involves integrating GPS with an Inertial Navigation System (INS) to minimize the impact of signal outages. This integration can be loosely coupled, using GPS positions as inputs, or tightly coupled, utilizing raw data (e.g., pseudoranges and carrier phases). Tightly coupled GPS/INS systems are preferred due to their robustness and accuracy.

For achieving accurate positioning with low-cost devices, a tightly coupled KF configuration can replace the costly INS with a dynamic model. Utilizing double difference observations, additional accuracy can be obtained by incorporating information sources relevant to the navigation context. This implementation supports both code and phase observations (single or double frequency) and continuously updates ambiguities when phase data is available. The algorithm is flexible and can easily accommodate additional optional observations.

Generic KF design

Step 1: Prediction

$$X_{k+1}^{-} = f(X_k) + \epsilon_{k+1}$$

The linearized state equation is described as:

$$A_{k+1} = \frac{\partial f(X_k)}{\partial X}$$

A_{k+1} is the Jacobian of the process model.

The covariance prediction comes from the state equation:

$$P_{k+1}^{-} = A_{k+1} P_k A_{k+1}^T + Q_{k+1}$$

Q_{k+1} is the process covariance matrix.

Step 2: Update

The linearized observation equation is defined as:

$$H_{k+1} = \frac{\partial h(X_{k+1}^{-})}{\partial X}$$

Where H is the Jacobian of the measurement model.

The Kalman Gain can be defined as:

$$K = P_{k+1}^{-} H_{k+1}^T (H_{k+1} P_{k+1}^{-} H_{k+1}^T + R_{k+1})^{-1}$$

R_{k+1} is the measurement covariance matrix.

The state X_{k+1} and covariance P_{k+1} are updated as the output of the EKF:

$$X_{k+1} = X_{k+1}^{-} + K(Z_{k+1} - h(X_{k+1}^{-}))$$

$$P_{k+1} = (I - KH_{k+1}) * P_{k+1}^{-}$$

Extended Kalman Filter Equations:

Prediction Step:

1. **State Prediction:**

$\hat{x}_k^- = f(\hat{x}_{k-1}, u_k)$

- **Explanation:** Predicts the next state ($\hat{x}_k^-$) based on the previous estimated state ($\hat{x}_{k-1}$) and the control input (u_k), using a nonlinear state transition function f.

2. **Covariance Prediction:**

$P_k^- = F_k P_{k-1} F_k^T + Q_k$

- **Explanation:** Predicts the covariance (P_k^-) of the predicted state using the Jacobian matrix of the state transition function (F_k), the previous covariance (P_{k-1}), and the process noise covariance (Q_k).

Update Step:

1. **Measurement Prediction:**

$\hat{z}_k = h(\hat{x}_k^-)$

- **Explanation:** Predicts the expected measurement ($\hat{z}_k$) based on the predicted state ($\hat{x}_k^-$) using a nonlinear measurement function h.

2. **Measurement Residual:**

$y_k = z_k - \hat{z}_k$

- **Explanation:** Calculates the measurement residual (y_k) as the difference between the actual measurement (z_k) and the predicted measurement ($\hat{z}_k$).

3. **Jacobian of Measurement Model:**

$H_k = \left.\frac{\partial h}{\partial x}\right|_{\hat{x}_k^-}$

- **Explanation:** Calculates the Jacobian matrix (H_k) of the measurement function h with respect to the state at $\hat{x}_k^-$.

4. **Innovation Covariance:**

$S_k = H_k P_k^- H_k^T + R_k$

- **Explanation:** Calculates the innovation covariance (S_k) using the Jacobian (H_k), the predicted covariance (P_k^-), and the measurement noise covariance (R_k).

5. **Kalman Gain:**

$$K_k = P_k^- H_k^T (S_k)^{-1}$$

- **Explanation:** Computes the Kalman Gain (K_k) to weight the prediction and measurement information based on their respective covariances.

6. **State Update:**

$$\hat{x}_k = \hat{x}_k^- + K_k y_k$$

- **Explanation:** Updates the estimated state ($\hat{x}_k$) using the Kalman Gain and the measurement residual.

7. **Covariance Update:**

$$P_k = (I - K_k H_k) P_k^-$$

- **Explanation:** Updates the covariance (P_k) using the Kalman Gain to incorporate the information from the measurement.

These equations represent the core steps of the Extended Kalman Filter for nonlinear systems, involving prediction and update steps to estimate the state of a system based on nonlinear models and measurements.

When implementing the EKF, these equations are iteratively applied for each time step to predict the state and update the estimate using measurements, improving the accuracy of the state estimation in nonlinear systems.

Employing Moving Window Averages for Enhanced Maximum Likelihood Estimation in the Extended Kalman Filter Framework

Introduction

One of the primary objectives of this study is to enhance the precision of state estimation within the Extended Kalman Filter (EKF) framework by integrating a moving window average methodology. This method aims to compute the mean of residuals derived from satellite data at each epoch. These residuals, reflecting the differences between observed and estimated measurements, encapsulate crucial information essential for accurate estimation in navigation systems.

Moving Window Average Technique

The utilization of a moving window average involves computing the average of residuals within a specified window size at each epoch. By adopting this approach, we aim to mitigate potential outlier effects and variability in the observed data. This calculated average serves as a representative measure of the discrepancies between actual and predicted satellite measurements, providing a smoother and more stable estimation basis.

Normalization by Satellite Appearances

Moreover, the mean of residuals obtained through the moving window average is normalized by the count of appearances of satellites in previous epochs. This normalization factor accounts for varying satellite availability, ensuring a balanced consideration of satellite data while deriving the mean residual values. This adaptive normalization process aims to refine the estimation accuracy by appropriately weighing the contributions of different satellite signals across epochs.

Integration into Maximum Likelihood Estimation (MLE) Equations

The derived mean residuals, normalized by satellite appearances, serve as pivotal inputs within the Maximum Likelihood Estimation (MLE) equations. By integrating these refined residuals into the estimation process, the objective is to enhance the adaptability and accuracy of the estimation models within the Extended Kalman Filter framework. These adjustments empower the system to iteratively learn and refine estimation parameters, thereby advancing the reliability of state estimation in dynamic and challenging environments.

Conclusion

The employment of a moving window average technique, coupled with normalization by satellite appearances, represents a crucial step towards refining state estimation methodologies. This approach forms a key foundation for the subsequent integration of refined residuals into the MLE equations within the Extended Kalman Filter. Ultimately, this objective aims to enhance the robustness and accuracy of navigation systems, vital for their effective operation across diverse and challenging scenarios.

Maximum Likelihood Estimation (MLE) in GNSS Positioning

Introduction

In the domain of Global Navigation Satellite Systems (GNSS), Maximum Likelihood Estimation (MLE) plays a pivotal role in optimizing parameters to enhance positioning accuracy. Understanding MLE involves maximizing the likelihood function, especially when applied to GNSS residuals the differences between observed and estimated satellite measurements.

Understanding the Likelihood Function

In MLE, the likelihood function $L(\theta|X)$ determines the probability of observing the data (X) given specific parameter values (θ). For GNSS positioning, let's consider a simple linear model:

Y = MX + C

Where:

- y represents observed measurements.
- x represents predictor values (e.g., time, position).
- m is the slope.
- c is the intercept.
- ϵ is the measurement noise.

The likelihood function for this model, assuming Gaussian noise, is:

$$L(m, c|y, x) = \prod_{i=1}^{N} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(y_i-(mx_i-c))^2}{2\sigma^2}}$$

Where:

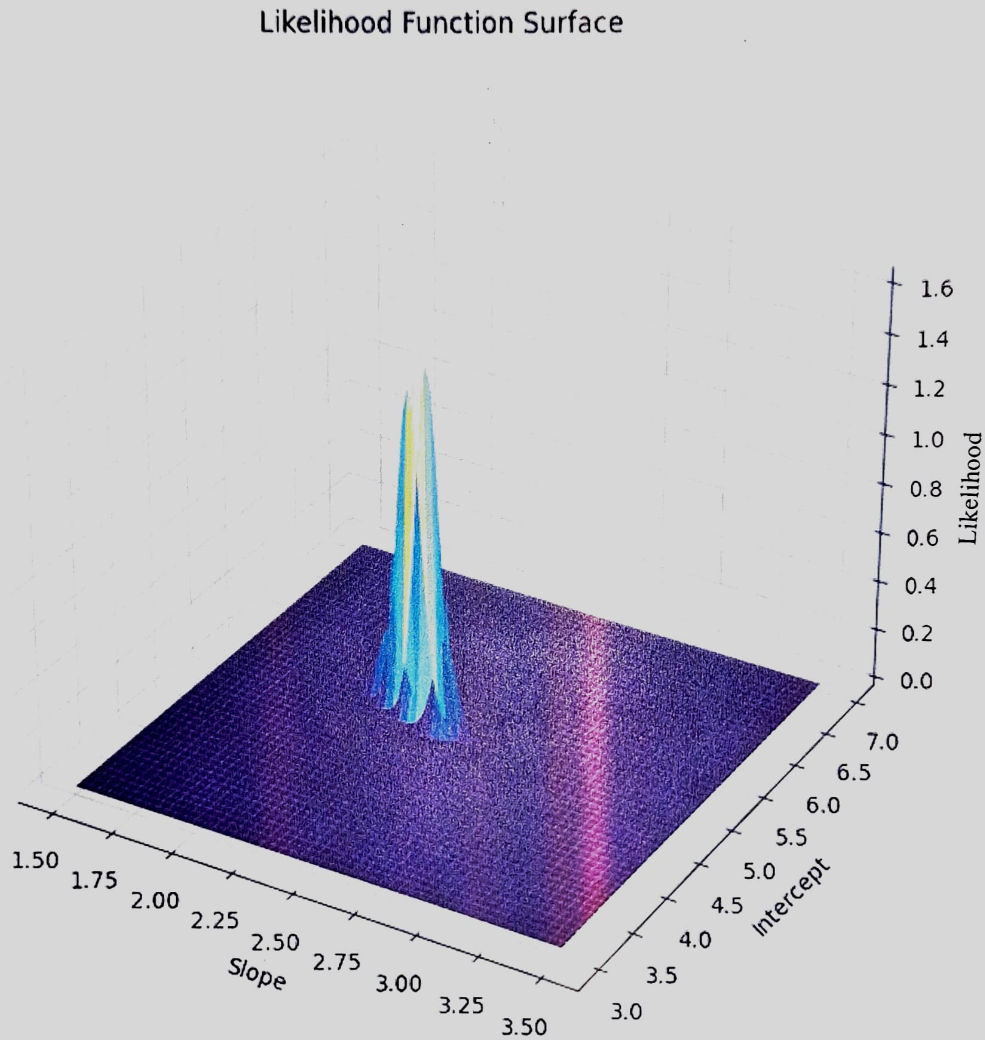
- N is the number of observations.
- σ2 is the variance of the measurement noise.

MLE: Finding Optimal Parameters

The goal of MLE is to find the values of m and c that maximize the likelihood function. This involves differentiating the logarithm of the likelihood function with respect to the parameters and solving for their optimal values:

$$\frac{\partial \log L}{\partial m} = 0$$
$$\frac{\partial \log L}{\partial c} = 0$$

This process involves iterative optimization techniques such as gradient descent or numerical optimization algorithms to converge towards the optimal parameter values.



Explanation of the Likelihood Function Surface

Axes Interpretation:

- **X-axis (Slope):** Represents the range of possible slope values within the specified range.
- **Y-axis (Intercept):** Depicts the range of intercept values.
- **Z-axis (Likelihood):** Demonstrates the likelihood value corresponding to each combination of slope and intercept values.

Surface Interpretation:

- **Surface Shape:** The surface represents the likelihood function's behavior concerning changes in slope and intercept values.
- **Peak/Maximum:** The highest point on the surface indicates the combination of slope and intercept values that maximize the likelihood function.
- **Contours:** Contours on the surface outline regions of similar likelihood values. Closer contours denote areas with relatively similar likelihood values.

Optimization Insight:

- **Optimal Parameter Values:** The point where the surface peaks signifies the optimal or best-fit slope and intercept values that maximize the likelihood of observing the given dataset.
- **Parameter Space Exploration:** By visualizing this surface, one can understand how different combinations of slope and intercept affect the likelihood, aiding in identifying the parameter values that best describe the observed data.

Application in MLE:

- **Maximization Objective:** In the context of Maximum Likelihood Estimation (MLE), the goal is to find the parameters (slope and intercept) that correspond to the peak of the likelihood surface, as these values offer the best-fit model to the observed data.

Integration into the Explanation:

The visualization of the likelihood function surface using the 3D plot helps demonstrate how MLE aims to find the optimal parameters. This explanation of the plot emphasizes the search for the parameter values that maximize the likelihood, improving the understanding of MLE's application in GNSS positioning.

Updating CNN and Cee Matrices using Residuals

CNN and Cee matrices encapsulate the uncertainties associated with the clock noise and code/carrier phase measurements, respectively. By utilizing residuals derived from GNSS observations within the MLE framework, these matrices can be dynamically updated. The incorporation of MLE-derived residuals allows for an adaptive adjustment of the CNN and Cee matrices, capturing the evolving nature of the system and improving the precision of the estimation process.

Impact on Positioning Accuracy

The integration of MLE with residuals for updating CNN and Cee matrices holds immense potential in enhancing positioning accuracy. This approach enables the system to learn from observed discrepancies, iteratively refining the estimation process. By dynamically adjusting critical matrices based on real-time observations, the accuracy and reliability of GNSS-based positioning can be significantly improved, especially in challenging scenarios with varying environmental conditions and signal interferences.

Conclusion

The utilization of MLE in conjunction with residuals for updating CNN and Cee matrices represents a promising avenue for advancing GNSS-based positioning accuracy. This methodology provides a means to adaptively refine key estimation parameters, empowering navigation systems to offer more precise and reliable positioning solutions across diverse and challenging real-world scenarios.

Summary

1. **Least Square Techniques:** This framework encompasses various features, such as Least Squares calculations for rover position estimation, design matrix generation, and Pseudorange measurements. The framework also includes functions for converting between global and local coordinate systems, computing topocentric satellite coordinates, and handling ionospheric delay corrections. Overall, your project aims to enable accurate GNSS-based positioning and navigation for hardware applications using MATLAB programming.
2. **Kalman Filter:** The implementation of a Kalman filter-based single point/differential GNSS technique in a MATLAB programming environment for hardware applications involves utilizing key components of the Kalman filter algorithm. This includes the transition matrix, state vector, predicted covariance matrix, distance of rover position, distance of master position, measurement matrix, design matrix, observation vector, the inversion of the measurement covariance matrix, Kalman gain, and the identity matrix. By effectively utilizing these elements, the Kalman filter algorithm updates the state estimate, providing accurate estimations of position, velocity, and acceleration in the context of Global Navigation Satellite Systems (GNSS) for hardware applications. This approach enhances GNSS accuracy and robustness, making it valuable for various real-world navigation and positioning tasks.
3. **Associated Modules:** The primary features and variables include state transition matrix, state estimate, state covariance estimate, process noise covariance, identity matrix, measurement matrix, kalman gain, measurement noise covariance, innovation covariance, system noise covariance, innovation residual, maximum innovation residual threshold, and state covariance propagation. The implementation of this framework will enable accurate and real-time GNSS positioning and navigation for various hardware applications.
4. **Satellite Position and Satellite Orbit:** These include satellite position and orbit information, time-related parameters like and satellite clock corrections, satellite position and velocity coordinates, and considerations for Earth's rotation correction. The framework's primary purpose is to process and optimize the GNSS data to accurately determine the user's position and provide reliable navigation information, making it valuable for various hardware applications that rely on GNSS positioning.
5. **Moving Window For Enhance Estimation**: Our study employs a moving window average technique to derive smoothed residuals from satellite data at each epoch. This method computes a stable mean of discrepancies, normalized by satellite appearances in previous epochs. By refining these

residuals, our aim is to bolster the accuracy and adaptability of estimation models within the Extended Kalman Filter. Integrating these refined residuals into the Maximum Likelihood Estimation equations advances the precision of navigation systems, crucial for reliable state estimation in varying environmental conditions.

6. **Maximum Likelihood Estimation on Extended Kalman Filter**: Despite not implementing it directly, our groundwork involved comprehensively understanding and articulating the theoretical underpinnings of Maximum Likelihood Estimation (MLE) in GNSS positioning. This encompassed elucidating the fundamental role of MLE in refining positioning accuracy by maximizing the likelihood function, particularly applied to GNSS residuals. Furthermore, our focus extended to the pivotal objective of utilizing MLE to update crucial matrices CNN and Cee. While direct implementation awaited, our efforts were directed towards laying a strong foundation by exploring intricate concepts such as the complex moving window average. This approach aimed to fortify the theoretical groundwork, essential for subsequent practical implementation in refining GNSS-based positioning precision.

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