Summary paper 34: Wavelets in wandering subspaces

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1 Introduction

This paper by T. Goodman extends Mallat's construction to orthonormal wavelets generated by a finite set of functions.

Definition 1. *A* multiresolution approximation *is a sequence of closed subspaces* $(V_m)_{m\in\mathbb{Z}}$ *of* L^2 *such that*

1. $V_m \subset V_{m+1}$
2. $\bigcup_{m\in\mathbb{Z}} V_m$ *is dense in* L^2 *and* $\bigcap_{m\in\mathbb{Z}} V_m = \{0\}$
3. $f \in V_m \implies D_2 f \in V_{m+1}$ *with* $D_a f(x) = f(ax)$
4. $f \in V_m \implies T_{2^{-m}n} f \in V_m$ *for all* $n \in \mathbb{Z}$ *with* $T_\tau f(x) = f(x-\tau)$
5. $\exists$ *isomorphism* $\mathcal{I} : V_0 \to l^2(\mathbb{Z})$ *which commutes with the action of* $\mathbb{Z}$ *i.e* $\mathcal{I}T_k = t_k\mathcal{I}$ *with* t_k *the translation by* k *on sequences of* $l^2(\mathbb{Z})$.

<u>Notations</u>

- $\mathcal{H}$ a complex Hilbert space
- T a unitary operator on $\mathcal{H}$ i.e $T : \mathcal{H} \to \mathcal{H}$ s.t its adjoint $T^* : \mathcal{H} \to \mathcal{H}$ satisfies $TT^* = T^*T = I$.
- $\mathcal{S} \subset \mathcal{H}$ is a *wandering* subspace if $T^m(\mathcal{S}) \perp T^n(\mathcal{S})$ if $m \neq n$.
- $\mathcal{S}$ is a *complete* wandering subpsace V of $\mathcal{H}$ if

$$V = \bigoplus_{n\in\mathbb{Z}} T^n(\mathcal{S})$$