Artificial Intelligence Masterclass

Linear Algebra for AI

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A matrix is a collection of numbers by rows and columns.

$$\boldsymbol{A} = \begin{bmatrix} x_{11} & \cdots & x_{1n} \\ \vdots & \ddots & \vdots \\ x_{m1} & \cdots & x_{mn} \end{bmatrix}_{m \times n}$$

Types of matrices

- A vector is a matrix which has only one row and only one column. If there is only one row it is called as a row vector and if there is only one column it is called as a column vector.

$$\boldsymbol{a} = \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix} \quad \boldsymbol{b} = \begin{bmatrix} -2 & 7 & 4 \end{bmatrix}$$

- A scalar matrix is a matrix with only one row and only one column.

$$\boldsymbol{a} = 5$$

- A square matrix is a matrix in which the number of rows equal to the number of columns.

$$\boldsymbol{A} = \begin{bmatrix} 1 & 6 \\ 3 & 2 \end{bmatrix}$$

Types of matrices

- A null matrix which is also called a zero matrix is any matrix in which all the elements are 0.

$$\boldsymbol{B} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

- A diagonal matrix is a square matrix where all the off diagonal elements are 0.

$$\boldsymbol{C} = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 7 \end{bmatrix}$$

- An identity matrix is a diagonal matrix where all the diagonal elements are 1. It is generally denoted by I_n when the dimension is $n \times n$.

$$\boldsymbol{I}_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Matrix addition & subtraction

$$\boldsymbol{A}_{m\times n} \pm \boldsymbol{B}_{m\times n} = \boldsymbol{R}_{m\times n}$$

where $r_{ij} = a_{ij} + b_{ij}$

Ex:-

$$\begin{bmatrix} 2 & 4 & 6 \\ -1 & 3 & 2 \\ 1 & 6 & 2 \end{bmatrix} + \begin{bmatrix} 4 & 6 & 9 \\ 10 & -5 & 2 \\ 3 & 4 & 0 \end{bmatrix} =$$

Ex:-

$$\boldsymbol{A} = \begin{bmatrix} 2 & 3 \\ -2 & 6 \end{bmatrix} \; \boldsymbol{B} = \begin{bmatrix} 4 & 3 \\ 1 & 5 \end{bmatrix} \; \boldsymbol{C} = \begin{bmatrix} 6 & 2 \\ 1 & 3 \end{bmatrix}$$

Find **A**+**B-C** =

Answers

Ex:-

$$\begin{bmatrix} 2 & 4 & 6 \\ -1 & 3 & 2 \\ 1 & 6 & 2 \end{bmatrix} + \begin{bmatrix} 4 & 6 & 9 \\ 10 & -5 & 2 \\ 3 & 4 & 0 \end{bmatrix} = \begin{bmatrix} 6 & 10 & 15 \\ 9 & -2 & 4 \\ 4 & 10 & 2 \end{bmatrix}$$

Ex:-

$$\boldsymbol{A} = \begin{bmatrix} 2 & 3 \\ -2 & 6 \end{bmatrix} \ \boldsymbol{B} = \begin{bmatrix} 4 & 3 \\ 1 & 5 \end{bmatrix} \ \boldsymbol{C} = \begin{bmatrix} 6 & 2 \\ 1 & 3 \end{bmatrix}$$

Find **A**+**B**-**C** = $\begin{bmatrix} 0 & 4 \\ -2 & 8 \end{bmatrix}$

Matrix multiplication

- Scalar multiplication

$$k\boldsymbol{A} = \boldsymbol{B}$$

such that for all i, j $\quad b_{ij} = ka_{ij}$

Ex:-

$$2 \times \begin{bmatrix} 1 & 2 \\ -3 & 0 \end{bmatrix} =$$

- Multiplying 2 matrices

$$\boldsymbol{C}_{m\times p} = \boldsymbol{A}_{m\times n} \times \boldsymbol{B}_{n\times p}$$

such that,

$$c_{ij} = \sum_{k=1}^{p} a_{ik} \times b_{kj}$$

Answers

Ex:-

$$2 \times \begin{bmatrix} 1 & 2 \\ -3 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ -6 & 0 \end{bmatrix}$$

Ex:-

$$\begin{bmatrix} 2 & 8 & 1 \\ 3 & 6 & 4 \end{bmatrix} \times \begin{bmatrix} 1 & 7 \\ 9 & -2 \\ 6 & 3 \end{bmatrix} =$$

$$\boldsymbol{A} = \begin{bmatrix} 1 & 2 \\ 3 & 2 \\ -1 & 0 \end{bmatrix} \quad \boldsymbol{B} = \begin{bmatrix} 4 & 1 & 1 \\ 2 & 2 & 1 \end{bmatrix}$$

- Find **AB** and **BA**

Ex:-

$$\begin{bmatrix} 2 & 8 & 1 \\ 3 & 6 & 4 \end{bmatrix} \times \begin{bmatrix} 1 & 7 \\ 9 & -2 \\ 6 & 3 \end{bmatrix} = \begin{bmatrix} 80 & 1 \\ 81 & 21 \end{bmatrix}$$

$$\boldsymbol{A} = \begin{bmatrix} 1 & 2 \\ 3 & 2 \\ -1 & 0 \end{bmatrix} \; \boldsymbol{B} = \begin{bmatrix} 4 & 1 & 1 \\ 2 & 2 & 1 \end{bmatrix}$$

- $\boldsymbol{AB} = \begin{bmatrix} 8 & 5 & 3 \\ 16 & 7 & 5 \\ -4 & -1 & -1 \end{bmatrix}$
- $\boldsymbol{BA} = \begin{bmatrix} 6 & 10 \\ 7 & 8 \end{bmatrix}$

Matrix transpose

Transpose of a matrix is where the rows and columns are interchanged.

$$\boldsymbol{B} = \boldsymbol{A}^T \longrightarrow\ b_{ij} = a_{ji}$$

Ex:-

$$\boldsymbol{A} = \begin{bmatrix} 2 & 7 & 1 \\ 8 & 6 & 4 \end{bmatrix} \qquad\qquad \boldsymbol{A}^T =$$

Ex:-

$$\boldsymbol{A} = \begin{bmatrix} 2 & 7 & 1 \\ 8 & 6 & 4 \end{bmatrix} \qquad \boldsymbol{A}^T = \begin{bmatrix} 2 & 8 \\ 7 & 6 \\ 1 & 4 \end{bmatrix}$$

Determinant of a matrix

Determinant is defined only for a square matrix.

$$|\boldsymbol{A}| = det(\boldsymbol{A})$$

- For a scalar matrix,

$$\boldsymbol{A} = a \dashrightarrow |\boldsymbol{A}| = a$$

- In 2×2 case,

$$\boldsymbol{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \dashrightarrow |\boldsymbol{A}| = ad - bc$$

- In 3×3 case

$$\boldsymbol{A} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \dashrightarrow |\boldsymbol{A}| = a(ei - fh) - b(di - gf) + c(dh - ge)$$

Ex:- Find the determinant of following matrices.

$$\boldsymbol{A} = \begin{bmatrix} 2 & 4 \\ -3 & 2 \end{bmatrix}$$

$$\boldsymbol{B} = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 0 & 1 \\ 1 & -1 & 3 \end{bmatrix}$$

Ex:- Find the determinant of following matrices.

$$\boldsymbol{A} = \begin{bmatrix} 2 & 4 \\ -3 & 2 \end{bmatrix} -----\rightarrow \det(A) = 16$$

$$\boldsymbol{B} = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 0 & 1 \\ 1 & -1 & 3 \end{bmatrix} -----\rightarrow \det(B) = 1(0+1) - 2(9-1) + 1(-1-0) = -16$$

Inverse of a matrix

The inverse of a matrix A is denoted by $\boldsymbol{A^{-1}}$, such that,

$$\boldsymbol{AA^{-1} = A^{-1}A = I}$$

Ex:-

$$\boldsymbol{A} = \begin{bmatrix} 1 & -1 & 2 \\ 4 & 0 & 6 \\ 0 & 1 & -1 \end{bmatrix}$$

Ex:-

$$\boldsymbol{A} = \begin{bmatrix} 1 & -1 & 2 \\ 4 & 0 & 6 \\ 0 & 1 & -1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 3 & -0.5 & 2 \\ -2 & 0.5 & -1 \\ -2 & 0.5 & -2 \end{bmatrix}$$