

Gradient Descent - Group 3

X y

1 3

Initial $m = -1$

learning rate: 0.1

3 6

Initial $b = 1$

Step 1: $\hat{y} = mx + b$

$\hat{y}_1 = -1 \cdot 1 + 1 = 0$

$\hat{y}_2 = -1 \cdot 3 + 1 = -2$

Step 2: Cost = $y - \hat{y}$

Cost₁ = $3 - 0 = 3$

Cost₂ = $6 - (-2) = 8$

Step 3: $M_{\text{new}} = M_{\text{old}} - (\text{learning rate} \cdot \frac{\partial \text{MSE}}{\partial m})$

$$\frac{\partial \text{MSE}}{\partial m} = \frac{\partial}{\partial m} \left(\frac{(y - \hat{y})^2 + (y - \hat{y})^2}{2} \right)$$

3.1 $\frac{\partial \text{MSE}}{\partial m} = \frac{\partial}{\partial m} \left(\frac{(3 - (mx + b))^2 + (6 - (mx + b))^2}{2} \right)$

$$= (3 - m - b)^2 + (6 - 3m - b)^2$$

Chain rule

let $u' = 3 - m - b = -1$ | let $u' = 6 - 3m - b = -3$

let $v' = u^2 = 2u$ | let $v' = u^2 = 2u$

$2u \cdot -1 = -2u$ | $2u \cdot -3 = -6u$

Now $-2(3 - m - b)$ | Now $-6(6 - 3m - b)$

$= -2(3 - m - b) - 6(6 - 3m - b)$

$-6 + 2m + 2b - 36 + 18m + 6b$

$-6 - 36 + 2m + 18m + 2b + 6b$

$-42 + 20m + 8b / 2$

$\frac{-42 + 20(-1) + 8}{2} = \frac{-54}{2} = -27$

$\frac{\partial \text{MSE}}{\partial m} = -27$

NLJ

$$m_{\text{new}} = -1 - 0.1 \cdot -27 = 1.7$$

step 4: $b_{\text{new}} = b_{\text{old}} - (\text{learning rate} \cdot \frac{dmse}{db})$

$$\frac{dmse}{db} = \frac{2 \left[(3 - (mx + b))^2 + (6 - (mx + b))^2 \right]}{2}$$

$$= (3 - m - b)^2 + (6 - 3m - b)^2$$

$$u' = 3 - m - b = -1$$

$$u' = 6 - 3m - b = -1$$

$$v' = u'^2 = 24$$

$$v' = u'^2 = 2u$$

$$2u \cdot -1 = -24$$

$$2u \cdot -1 = -24$$

$$-2(3 - m - b) - 2(6 - 3m - b)$$

$$-6 + 2m + 2b - 12 + 6m + 2b$$

$$-6 - 12 + 2m + 6m + 2b + 2b$$

$$-18 + 8m + 4b$$

$$-18 - 8 \cdot (-1) + 4$$

$$\frac{dmse}{db} = \frac{-22}{2} = -11 //$$

$$b_{\text{new}} = 1 - (0.1 \cdot -11) = 2.1$$

First Iteration

by Miracle Nanen

$$m_{\text{new}} = 1.7$$

$$b_{\text{new}} = 2.1$$

$$m_{\text{new}} = 3 - (mx + b)^2 + 6 - (mx + b)^2$$

$$\hat{=} -42 + 20m - 8b$$

$$= -42 + 34 + 16.8 = 8.5$$

$$m_{\text{new}} = 1.7 - 0.1 \cdot \frac{8.5}{2} = 1.26$$

$$m_{\text{new}} = 1.26 //$$

NW

$$b_{\text{new}} = -18 + 8m + 4b$$

$$= -18 + 8(1.7) + 4(2.1)$$

$$= -18 + 13.6 + 8.4$$

$$= 4 = 2$$

$$b_{\text{new}} = 2.1 - 0.1 \cdot 2$$

$$= 1.9$$

2nd iteration

$m_{\text{new}} = 1.26$

$b_{\text{new}} = 1.9$

by Wagner-Murthy

Iteration 3

Using Iteration 2 parameters

$$M_{\text{new}} = 1.26$$

$$B_{\text{new}} = 1.9$$

$$\frac{\partial \text{MSE}}{\partial m} = \frac{42 + 20m + 8b}{2}$$

$$= \frac{-42 + 20(1.26) + 8(1.9)}{2}$$

$$= \frac{-42 + 25.2 + 15.2}{2} = \frac{-1.6}{2}$$

$$= -0.8$$

$$M_{\text{new}} = 1.26 - 0.1 \times -0.8 = 1.26 + 0.08$$

$$= \underline{\underline{1.34}}$$

$$\frac{\partial \text{MSE}}{\partial b} = \frac{-18 + 8 + 4b}{2}$$

$$= \frac{-18 + 8(1.26) + 4(1.9)}{2}$$

$$= \frac{-18 + 10.08 + 7.6}{2}$$

$$= \frac{-18 + 17.68}{2} = \frac{-0.32}{2}$$

$$= -0.16$$

$$B_{\text{new}} = 1.9 - 0.1 \times -0.16$$

$$= 1.9 + 0.016$$

$$= 1.916$$

By Glory Paul

Date: / /

Iteration 4 By Duchime Paulette.

$$\begin{aligned}\frac{dmse}{dm} &= -42 + 20m + 8b \\ &= -42 + 20(1.34) + 8(1.9) \\ &= -42 + 26.8 + 15.2 \\ &= \frac{0}{2}\end{aligned}$$

$$= \underline{\underline{0}}$$

$$\begin{aligned}m_{\text{new}} &= 1.34 - (0.1 \times 0) \\ &= 1.34 - 0 \\ &= \underline{\underline{1.34}}\end{aligned}$$

$$\begin{aligned}b_{\text{new}} &= -18 + 8m + 4b \\ \frac{dmse}{db} &= -18 + (8 \times 1.34) + 4(1.9) \\ &= -18 + 10.7 + 7.6 \\ &= \frac{0.3}{2} \\ &= \underline{\underline{0.15}}\end{aligned}$$

$$\begin{aligned}b_{\text{new}} &= 1.9 - (0.1 \times 0.15) \\ &= 1.9 - 0.015 \\ &= \underline{\underline{1.8}}\end{aligned}$$