

Gradient Descent - Group 3

$X \quad Y$

1 3

Initial $m = -1$

learning rate: 0.1

3 6

Initial $b = 1$

Step 1: $\hat{Y} = mx + b$

$$\hat{y}_1 = -1 \cdot 1 + 1 = 0$$

$$\hat{y}_2 = -1 \cdot 3 + 1 = -2$$

Step 2: Cost = $\frac{1}{2} \sum (y - \hat{y})^2$

$$cost_1 = 3 - 0 = 3$$

$$cost_2 = 6 - (-2) = 8$$

Step 3: $M_{\text{new}} = M_{\text{old}} - (\text{learning rate} \cdot \frac{\partial \text{MSE}}{\partial m})$

$$\frac{\partial \text{MSE}}{\partial m} = \frac{1}{2} \sum (3 - (m \cdot 1 + b))^2 + (6 - (m \cdot 3 + b))^2$$

$$\begin{aligned} 3.1 \quad \frac{\partial \text{MSE}}{\partial m} &= \frac{1}{2} \sum (3 - (m \cdot 1 + b))^2 + (6 - (m \cdot 3 + b))^2 \\ &= (3 - m - b)^2 + (6 - 3m - b)^2 \end{aligned}$$

Chain rule

$$\text{let } u' = 3 - m - b = -1 \quad \text{let } u' = 6 - 3m - b = -3$$

$$\text{let } v' = u'^2 = 2u \quad \text{let } v' = u'^2 = 2u$$

$$2u \cdot -1 = -2u \quad 2u \cdot -3 = -6u$$

$$\text{Now } -2(3 - m - b) \quad \text{Now } -6(6 - 3m - b)$$

$$= -2(3 - m - b) - 6(6 - 3m - b)$$

$$-6 + 2m + 2b - 36 + 18m + 6b$$

$$-6 - 36 + 20m + 18m + 2b + 6b$$

$$-42 + 20m + 8b / 2$$

$$\frac{-42 + 20(-1) + 8}{2} = \frac{-54}{2} = -27$$

$$\frac{\partial \text{MSE}}{\partial m} = -27$$

NLJ

$$m_{\text{new}} = -1 - 0.1 \cdot -27 = 1.7$$

Step 4: $b_{\text{new}} = b_{\text{old}} - (\text{learning rate} \cdot \frac{\partial \text{mse}}{\partial b})$

$$\frac{\partial \text{mse}}{\partial b} = \frac{1}{2} \sum [(3 - (mx + b))^2 + (6 - (mx + b))^2]$$

$$= (3 - m - b)^2 + (6 - 3m - b)^2$$

$$U' = 3 - m - b = -1$$

$$V' = U^2 = 2U \quad \text{Chain rule}$$

$$2U + 1 = -2U$$

$$-2(3 - m - b) - 2(6 - 3m - b)$$

$$-6 + 2m + 2b - 12 + 6m + 2b$$

$$-6 - 12 + 8m + 6m + 2b + 2b$$

$$-18 + 8m + 4b$$

$$-18 - 8 \cdot (-1) + 4$$

$$\frac{\partial \text{mse}}{\partial b} = \frac{-22}{2} = -11 //$$

$$b_{\text{new}} = 1 - (0.1 \cdot -11) = 2.1$$

First Iteration

$$m_{\text{new}} = 1.7$$

$$b_{\text{new}} = 2.1$$

by Miracle Nanen

$$m_{\text{new}} = 3(mx + b)^2 + 6 - (mx + b)^2$$

$$\stackrel{!}{=} -42 + 20m - 8b$$

$$= -42 + 34 + 16 \cdot 8 = 112 - 8 \cdot 8 = 44$$

$$m_{\text{new}} = 1.7 - 0.1 \cdot \frac{29}{16} = 1.26$$

$$\frac{29}{16}$$

$$m_{\text{new}} = 1.26$$

NL

$$\begin{aligned}
 b_{\text{new}} &= -18 + 8m + 4b \\
 &= -18 + 8(1.7) + 4(2.1) \\
 &= -18 + 13.6 + 8.4 \\
 &= \underline{\underline{4 = 2}}
 \end{aligned}$$

$$\begin{aligned}
 b_{\text{new}} &= 2.1 - 0.1 \cdot 2 \\
 &= 1.9
 \end{aligned}$$

No 2nd Iteration

$m_{\text{new}} = 1.26$ by klagner Musayeva

$$b_{\text{new}} = 1.9$$

$$1.6 + 1.26 + 0.1 \cdot 1.9 + 0.1 \cdot 1.9 = 4.04$$

$$1.6 + 1.26 + 0.1 \cdot 1.9 + 0.1 \cdot 1.9 = 4.04$$

$$1.6 + 1.26 + 0.1 \cdot 1.9 + 0.1 \cdot 1.9 = 4.04$$

$$1.6 + 1.26 + 0.1 \cdot 1.9 + 0.1 \cdot 1.9 = 4.04$$

minimizing residual error

1.6 + 1.26 + 0.1 · 1.9 + 0.1 · 1.9 = 4.04

1.6 + 1.26 + 0.1 · 1.9 + 0.1 · 1.9 = 4.04

1.6 + 1.26 + 0.1 · 1.9 + 0.1 · 1.9 = 4.04

1.6 + 1.26 + 0.1 · 1.9 + 0.1 · 1.9 = 4.04

1.6 + 1.26 + 0.1 · 1.9 + 0.1 · 1.9 = 4.04

1.6 + 1.26 + 0.1 · 1.9 + 0.1 · 1.9 = 4.04

1.6 + 1.26 + 0.1 · 1.9 + 0.1 · 1.9 = 4.04

1.6 + 1.26 + 0.1 · 1.9 + 0.1 · 1.9 = 4.04

1.6 + 1.26 + 0.1 · 1.9 + 0.1 · 1.9 = 4.04

Iteration 3

Using Iteration 2 parameters

$$M_{\text{new}} = 1.26$$

$$B_{\text{new}} = 1.9$$

$$\frac{\partial \text{MSE}}{\partial M} = \frac{42 + 20M + 8b}{2}$$

$$= \frac{-42 + 20(1.26) + 8(1.9)}{2}$$

$$= \frac{-42 + 25.2 + 15.2}{2} = \frac{-1.6}{2}$$

$$= -0.8$$

$$M_{\text{new}} = 1.26 - 0.1 \times -0.8 = 1.26 + 0.08$$

$$= \underline{\underline{1.34}}$$

$$\frac{\partial \text{MSE}}{\partial b} = \frac{-18 + 8 + 4b}{2}$$

$$= \frac{-18 + 8(1.26) + 4(1.9)}{2}$$

$$= \frac{-18 + 10.08 + 7.6}{2}$$

$$= \frac{-18 + 17.68}{2} = -\frac{0.32}{2}$$

$$= -0.16$$

$$B_{\text{new}} = 1.9 - 0.1 \times -0.16$$

$$= 1.9 + 0.016$$

$$= 1.916$$

Iteration # By Duchime Paulutte.

$$\begin{aligned}
 \frac{dmse}{dm} &= -42 + 20m + 8b \\
 &= -42 + 20(1.34) + 8(1.9) \\
 &= -42 + 26.8 + 15.2 \\
 &= \frac{0}{2} \\
 &= \underline{\underline{0}}
 \end{aligned}$$

$$\begin{aligned}
 m_{\text{new}} &= 1.34 - (0.1 \times 0) \\
 &= 1.34 - 0 \\
 &= \underline{\underline{1.34}}
 \end{aligned}$$

$$\begin{aligned}
 b_{\text{new}} &= -18 + 8m + 4b \\
 \frac{dmse}{db} &= -18 + (8 \times 1.34) + 4(1.9) \\
 &= -18 + 10.7 + 7.6 \\
 &= \frac{0.3}{2} \\
 &= \underline{\underline{0.15}}
 \end{aligned}$$

$$\begin{aligned}
 b_{\text{new}} &= 1.9 - (0.1 \times 0.15) \\
 &= 1.9 - 0.015 \\
 &= \underline{\underline{1.8}}
 \end{aligned}$$