

🔭 Assignment: Measuring Cosmological Parameters Using Type Ia Supernovae

In this assignment, you'll analyze observational data from the Pantheon+SH0ES dataset of Type Ia supernovae to measure the Hubble constant H_0 and estimate the age of the universe. You will:

- Plot the Hubble diagram (distance modulus vs. redshift)
- Fit a cosmological model to derive H_0 and Ω_m
- Estimate the age of the universe
- Analyze residuals to assess the model
- Explore the effect of fixing Ω_m
- Compare low-z and high-z results

Let's get started!

📦 Getting Started: Setup and Libraries

Before we dive into the analysis, we need to import the necessary Python libraries:

- `numpy`, `pandas` — for numerical operations and data handling
- `matplotlib` — for plotting graphs
- `scipy.optimize.curve_fit` and `scipy.integrate.quad` — for fitting cosmological models and integrating equations
- `astropy.constants` and `astropy.units` — for physical constants and unit conversions

Make sure these libraries are installed in your environment. If not, you can install them using:

```
```bash pip install numpy pandas matplotlib scipy astropy
```

```
In [624]: import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
from scipy.optimize import curve_fit
from scipy.integrate import quad
from astropy.constants import c
from astropy import units as u
```

## Load the Pantheon+SH0ES Dataset

We now load the observational supernova data from the Pantheon+SH0ES sample. This dataset includes calibrated distance moduli  $\mu$ , redshifts corrected for various effects, and uncertainties.

### Instructions:

- Make sure the data file is downloaded from [Pantheon dataset](https://github.com/PantheonPlusSH0ES/DataRelease/blob/main/Pantheon%2B_Data/4_DISTANCES_AND_Calibration.csv) ([https://github.com/PantheonPlusSH0ES/DataRelease/blob/main/Pantheon%2B\\_Data/4\\_DISTANCES\\_AND\\_Calibration.csv](https://github.com/PantheonPlusSH0ES/DataRelease/blob/main/Pantheon%2B_Data/4_DISTANCES_AND_Calibration.csv)) and available locally.
- We use `delim_whitespace=True` because the file is space-delimited rather than comma-separated.
- Commented rows (starting with `#`) are automatically skipped.

We will extract:

- `zHD` : Hubble diagram redshift
- `MU_SH0ES` : Distance modulus using SH0ES calibration
- `MU_SH0ES_ERR_DIAG` : Associated uncertainty

More detailed column names and the meanings can be referred here:

Finally, we include a combined file of all the fitted parameters for each SN, before and after light-curve cuts are applied. This is in the format of a .FITRES file and has all the meta-information listed above along with the fitted SALT2 parameters. We show a screenshot of the release in [Figure 7](#). Here, we give brief descriptions of each column. **CID** – name of SN. **CIDint** – counter of SNe in the sample. **IDSURVEY** – ID of the survey. **TYPE** – whether SN Ia or not – all SNe in this sample are SNe Ia. **FIELD** – if observed in a particular field. **CUTFLAG\_SNANA** – any bits in light-curve fit flagged. **ERRFLAG\_FIT** – flag in fit. **zHEL** – heliocentric redshift. **zHELEERR** – heliocentric redshift error. **zCMB** – CMB redshift. **zCMBERR** – CMB redshift error. **zHD** – **Hubble** Diagram redshift. **zHDERR** – **Hubble** Diagram redshift error. **VPEC** – peculiar velocity. **VPECERR** – peculiar-velocity error. **MWEBV** – MW extinction. **HOST\_LOGMASS** – mass of host. **HOST\_LOGMASS\_ERR** – error in mass of host. **HOST\_sSFR** – sSFR of host. **HOST\_sSFR\_ERR** – error in sSFR of host. **PKMJDINI** – initial guess for PKMJD. **SNRMAX1** – First highest signal-to-noise ratio (SNR) of light curve. **SNRMAX2** – Second highest SNR of light curve. **SNRMAX3** – Third highest SNR of light curve. **PKMJD** – Fitted PKMJD. **PKMJDERR** –

```
In [625]: # Local file path
file_path = "Pantheon+SH0ES.dat"

Load the file
df = pd.read_csv(file_path, delim_whitespace=True, comment='#', header=None,)
See structure
print(df.head())
print(len(df))
print(df.shape)
```

## 2\_hubble\_parameter

	0	1	2	3	4	5	6	7	\					
0	CID	IDSURVEY	zHD	zHDERR	zCMB	zCMBERR	zHEL	zHELERR						
1	2011fe	51	0.00122	0.00084	0.00122	2e-05	0.00082	2e-05						
2	2011fe	56	0.00122	0.00084	0.00122	2e-05	0.00082	2e-05						
3	2012cg	51	0.00256	0.00084	0.00256	2e-05	0.00144	2e-05						
4	2012cg	56	0.00256	0.00084	0.00256	2e-05	0.00144	2e-05						
	8	9	10	11	12	13	14	15	16	17	18			
0	m_b_corr	m_b_corr_err_DIAG	MU_SH0ES	MU_SH0ES_ERR_DIAG	CEPH_DIST	IS_CALIBRATOR	USED_IN_SH0ES_HF	c	cERR	x1	x1ERR			
1	9.74571	1.51621	28.9987	1.51645	29.177	1	0	-0.1076	0.04008	-0.548188	0.13373			
2	9.80286	1.51723	29.0559	1.51747	29.177	1	0	-0.032895	0.038463	-0.380481	0.0861			
3	11.4703	0.781906	30.7233	0.782372	30.8433	1	0	0.10073	0.018231	0.49196	0.023545			
4	11.4919	0.798612	30.7449	0.799068	30.8433	1	0	0.12247	0.03903	0.71261	0.083554			
	19	20	21	22	23	24	25	26	27	28	29	30	31	\
0	mB	mBERR	x0	x0ERR	COV_x1_c	COV_x1_x0	COV_c_x0	RA	DEC	HOST_RA	HOST_DEC	HOST_ANGSEP	VPEC	
1	9.58436	0.0327221	2.63181	0.0793177	0.00011378	-0.00052525	-0.00272765	210.774	54.2737	-999	-999	-9	0	
2	9.78448	0.0352442	2.1888	0.0710511	-0.000443845	-0.00150198	-0.00220084	210.774	54.2737	-999	-999	-9	0	
3	11.8161	0.0237119	0.33695	0.00735879	-6.06025e-05	9.35054e-06	-0.000110842	186.803	9.4203	-999	-999	-9	0	
4	11.8801	0.0359611	0.31765	0.0105211	0.000222656	-0.000136637	-0.000344022	186.803	9.4203	-999	-999	-9	0	
	32	33	34	35	36	37	38	39	40	41	42	\		
0	VPECERR	MWEBV	HOST_LOGMASS	HOST_LOGMASS_ERR	PKMJD	PKMJDERR	NDOF	FITCHI2	FITPROB	m_b_corr_err_RAW	m_b_corr_err_VPEC			
1	250	0.00758935	10.677	-9	55815	0.1071	36	26.8859	0.86447	0.0991	1.496			
2	250	0.00758935	10.677	0	55815.2	0.0579	101	88.3064	0.81222	0.0971	1.496			
3	250	0.0177724	9.633	0.002	56082.4	0.0278	165	233.5	0.000358347	0.0399	0.7134			
4	250	0.0177724	9.633	0.004	56082.4	0.0667	55	100.122	0.000193186	0.0931	0.7134			
	43	44	45	46										
0	biasCor_m_b	biasCorErr_m_b	biasCor_m_b_COVSCALE	biasCor_m_b_COVADD										
1	0.0381	0.005	1	0.003										
2	-0.0252	0.003	1	0.004										
3	0.0545	0.019	1	0.036										
4	0.0622	0.028	1	0.04										

```
1702
(1702, 47)

C:\Users\asus\AppData\Local\Temp\ipykernel_8580\3109395580.py:5: FutureWarning:
g: The 'delim_whitespace' keyword in pd.read_csv is deprecated and will be re-
moved in a future version. Use ``sep='\s+'`` instead
 df = pd.read_csv(file_path, delim_whitespace=True, comment='#', header=None)
```



## Preview Dataset Columns

Before diving into the analysis, let's take a quick look at the column names in the dataset. This helps us verify the data loaded correctly and identify the relevant columns we'll use for cosmological modeling.

```
In [626]: print(df[[2, 10, 11]].to_string())
```

	2	10	11
0	zHD	MU_SH0ES	MU_SH0ES_ERR_DIAG
1	0.00122	28.9987	1.51645
2	0.00122	29.0559	1.51747
3	0.00256	30.7233	0.782372
4	0.00256	30.7449	0.799068
5	0.00299	30.7757	0.881212
6	0.00317	30.7946	0.614535
7	0.00331	30.4604	0.594683
8	0.00331	30.5528	0.580251
9	0.00331	30.4013	0.578445
10	0.00331	30.5107	0.578546
11	0.00333	31.5011	0.591006
12	0.00349	31.7933	0.55272
13	0.00349	31.4967	0.545492
14	0.00359	31.6969	0.550993
15	0.00384	30.9999	0.566505
16	0.00407	31.3933	0.476404
17	0.00407	31.3779	0.479276
18	0.00432	31.499	0.514259
19	0.00432	31.5611	0.530838
20	0.00465	31.9309	0.442017
21	0.00465	32.033	0.420403
22	0.00482	31.2476	0.558603
23	0.00483	31.1331	0.446882
24	0.00483	31.6525	0.564724
25	0.00483	31.2234	0.410577
26	0.00483	31.34	0.562589
27	0.00483	31.7635	0.574442
28	0.00488	31.4473	0.448264
29	0.00488	31.539	0.407028
30	0.00538	31.6512	0.514401
31	0.00548	31.7282	0.501431
32	0.0057	31.9117	0.469628
33	0.0057	31.8985	0.483668
34	0.00571	31.6799	0.375913
35	0.00571	31.6292	0.385756
36	0.00587	31.7403	0.491367
37	0.00588	32.1822	0.339643
38	0.00592	32.4685	0.487055
39	0.00602	31.9404	0.344335
40	0.00602	32.0666	0.343707
41	0.00602	31.9102	0.350249
42	0.00616	30.9079	0.524992
43	0.00616	31.1857	0.452235
44	0.00625	32.0667	0.32593
45	0.00625	32.0023	0.36156
46	0.00625	32.0999	0.322811
47	0.00625	32.0337	0.339166
48	0.0063	32.0754	0.533991
49	0.0063	32.0843	0.537335
50	0.0063	32.0203	0.538159
51	0.00631	32.8333	0.562902
52	0.00634	32.4001	0.482567
53	0.00648	32.4608	0.318474
54	0.00648	32.4338	0.32229
55	0.00665	31.7321	0.531169

56	0.00668	31.2217	0.420706
57	0.00673	32.3593	0.359446
58	0.00676	31.9733	0.415464
59	0.00677	32.1951	0.34053
60	0.00677	32.1729	0.371125
61	0.00677	32.2048	0.34795
62	0.00692	32.4405	0.439634
63	0.0071	32.3193	0.325586
64	0.0071	32.3327	0.348317
65	0.0071	32.3221	0.322655
66	0.00719	32.4423	0.300309
67	0.00719	32.4642	0.300269
68	0.00747	32.7943	0.289134
69	0.00747	32.7423	0.294759
70	0.00747	32.7685	0.292411
71	0.0075	32.3312	0.373843
72	0.0075	32.2998	0.396652
73	0.00795	32.6418	0.362719
74	0.00814	32.7164	0.317031
75	0.00814	32.6151	0.317457
76	0.00814	32.7092	0.367554
77	0.00815	32.3711	0.371926
78	0.00815	32.7434	0.280148
79	0.00815	32.6707	0.286975
80	0.00817	32.4455	0.490787
81	0.00818	32.484	0.342662
82	0.00821	33.2138	0.462834
83	0.00838	32.521	0.465611
84	0.00838	32.6922	0.484865
85	0.00855	32.6928	0.359218
86	0.00855	32.852	0.318032
87	0.00857	32.7527	0.26758
88	0.00861	32.7223	0.383433
89	0.00863	32.3744	0.338325
90	0.00863	32.3945	0.331296
91	0.00863	32.335	0.337879
92	0.00864	32.4198	0.425219
93	0.00883	32.8616	0.327035
94	0.00906	32.799	0.368859
95	0.00906	32.7498	0.409525
96	0.00925	32.8914	0.286425
97	0.00925	32.8973	0.282867
98	0.00925	32.9159	0.285505
99	0.00935	32.3353	0.401367
100	0.00954	33.0399	0.268981
101	0.00954	33.2135	0.253629
102	0.00954	33.2309	0.251906
103	0.00954	33.2469	0.254885
104	0.00957	32.901	0.309548
105	0.00957	32.8654	0.334488
106	0.00966	32.8029	0.35789
107	0.00966	32.9165	0.372831
108	0.00993	33.4753	0.372879
109	0.00996	32.6457	0.409088
110	0.00996	32.7458	0.360296
111	0.00996	32.9015	0.334006
112	0.01016	32.7794	0.348547

113	0.01017	32.9182	0.365223
114	0.01017	32.9674	0.341878
115	0.01026	33.3378	0.306294
116	0.01026	33.2253	0.299086
117	0.01028	33.2788	0.247952
118	0.01042	33.1236	0.317227
119	0.01044	33.0869	0.343916
120	0.01061	33.3806	0.309009
121	0.01061	33.2931	0.27457
122	0.01073	33.1081	0.364668
123	0.01079	33.0683	0.324016
124	0.01079	33.1809	0.377969
125	0.01096	33.5709	0.314858
126	0.01114	33.7337	0.342487
127	0.01114	33.6369	0.29522
128	0.01122	33.8347	0.353909
129	0.01122	33.4336	0.283424
130	0.01122	33.4947	0.325172
131	0.01155	33.5833	0.281169
132	0.01195	33.4974	0.301502
133	0.01213	33.8783	0.346381
134	0.0122	33.7023	0.266666
135	0.01233	33.7403	0.254135
136	0.01246	33.8286	0.265115
137	0.01258	33.708	0.24076
138	0.01258	33.7246	0.250097
139	0.01259	33.5874	0.26006
140	0.01259	33.5178	0.26619
141	0.01279	33.7492	0.416658
142	0.01283	33.6365	0.287379
143	0.01283	33.5028	0.331124
144	0.01303	33.7355	0.239783
145	0.01303	33.7607	0.28722
146	0.01304	33.5752	0.292564
147	0.01304	33.5673	0.284786
148	0.01312	33.52	0.273536
149	0.01312	33.4717	0.261601
150	0.01325	33.9682	0.303693
151	0.01325	33.9846	0.299399
152	0.01325	33.9247	0.304606
153	0.01375	33.797	0.277855
154	0.01375	33.7683	0.232778
155	0.01375	33.772	0.235755
156	0.01376	33.7649	0.288791
157	0.01386	33.6741	0.345367
158	0.01388	33.6968	0.233531
159	0.01389	33.8053	0.288803
160	0.01389	33.6651	0.296207
161	0.01411	33.8206	0.250873
162	0.01424	33.8444	0.265489
163	0.01442	34.0187	0.307194
164	0.01442	34.0249	0.352338
165	0.01442	34.3366	0.370715
166	0.01442	34.1781	0.396908
167	0.01442	34.252	0.397235
168	0.01446	33.9078	0.240515
169	0.0145	34.1727	0.365029

170	0.01453	33.7526	0.2163
171	0.0146	33.8826	0.249316
172	0.01462	33.9253	0.243928
173	0.01463	33.7553	0.390506
174	0.01463	33.7189	0.39282
175	0.01467	33.8916	0.240694
176	0.01472	33.7894	0.397352
177	0.01484	34.0109	0.270222
178	0.01492	34.1902	0.372674
179	0.01493	33.8524	0.277558
180	0.01499	33.974	0.229564
181	0.01499	33.9261	0.231708
182	0.01515	33.8877	0.294985
183	0.01519	34.1793	0.26677
184	0.01525	33.7706	0.30219
185	0.01529	34.0334	0.344305
186	0.01542	34.3426	0.372888
187	0.01543	34.0744	0.314862
188	0.01546	34.0331	0.226465
189	0.01549	34.2732	0.34036
190	0.0155	34.2581	0.272528
191	0.0155	34.2112	0.258071
192	0.0155	33.9994	0.28632
193	0.0155	33.8852	0.313932
194	0.01557	33.7902	0.290577
195	0.01557	33.8072	0.290395
196	0.01562	34.1732	0.264363
197	0.01562	34.1302	0.33431
198	0.01565	34.3317	0.278713
199	0.01576	34.1657	0.28688
200	0.01578	33.9986	0.219937
201	0.01578	34.0697	0.290382
202	0.01581	33.9926	0.239289
203	0.01581	33.9004	0.218994
204	0.01587	34.3916	0.270341
205	0.01588	34.0287	0.224824
206	0.0159	34.1663	0.242893
207	0.0159	34.1323	0.209471
208	0.0159	34.0329	0.272511
209	0.01603	34.1062	0.281582
210	0.01652	34.4137	0.34637
211	0.01652	33.9999	0.334729
212	0.01656	34.1608	0.218117
213	0.01657	33.9726	0.41528
214	0.01662	34.2793	0.261916
215	0.01666	34.3408	0.198023
216	0.01671	33.8985	0.245383
217	0.01678	34.1679	0.246475
218	0.01682	34.4649	0.358327
219	0.01682	34.5341	0.341619
220	0.01682	33.9673	0.284408
221	0.0169	33.841	0.289311
222	0.0169	33.8975	0.264562
223	0.01692	34.4894	0.310314
224	0.01698	34.3409	0.228387
225	0.01699	34.0755	0.297815
226	0.01705	34.1973	0.214463

227	0.01718	34.022	0.308995
228	0.01718	34.2003	0.325329
229	0.0172	34.6084	0.219814
230	0.0173	34.1436	0.28269
231	0.0173	34.1517	0.257136
232	0.01733	34.4009	0.221698
233	0.01733	34.5101	0.231239
234	0.01734	34.361	0.220285
235	0.01737	34.31	0.263557
236	0.01737	34.2669	0.262425
237	0.01743	33.7799	0.330633
238	0.01747	34.3228	0.200241
239	0.01747	34.3133	0.19782
240	0.01752	34.1465	0.231417
241	0.01776	34.2221	0.320412
242	0.01778	34.3364	0.210115
243	0.01778	34.2464	0.213428
244	0.01784	34.2939	0.246602
245	0.01784	34.2418	0.22287
246	0.0179	34.2975	0.318912
247	0.01802	34.2661	0.234211
248	0.01808	34.6078	0.20147
249	0.01826	34.9792	0.21848
250	0.01826	34.2005	0.280568
251	0.01839	34.4393	0.226901
252	0.01855	34.492	0.255362
253	0.01855	34.576	0.296243
254	0.01865	34.6171	0.238984
255	0.01865	34.5716	0.239989
256	0.01866	34.6066	0.277471
257	0.01875	34.5466	0.215514
258	0.01875	34.4866	0.321023
259	0.01905	34.4286	0.281407
260	0.01947	34.7747	0.195403
261	0.01947	34.8085	0.195531
262	0.01975	34.757	0.466611
263	0.01975	34.7394	0.433516
264	0.01976	34.6865	0.406241
265	0.01995	34.3648	0.295725
266	0.02001	34.6854	0.204275
267	0.02006	34.5121	0.199486
268	0.02019	34.5072	0.233893
269	0.02019	34.702	0.19246
270	0.02023	34.5995	0.188309
271	0.02023	34.5012	0.295566
272	0.02023	34.576	0.189903
273	0.02023	34.6482	0.187832
274	0.02024	34.9988	0.187715
275	0.02034	34.8165	0.241825
276	0.02034	34.5749	0.375822
277	0.02035	34.5709	0.187708
278	0.02035	34.5656	0.177716
279	0.02035	34.4931	0.19075
280	0.02044	34.6552	0.214591
281	0.02049	35.1263	0.375783
282	0.02052	34.556	0.333904
283	0.02056	34.4168	0.217156

284	0.02056	34.5647	0.249505
285	0.02081	34.4863	0.330445
286	0.02082	34.674	0.230899
287	0.02082	34.6349	0.254511
288	0.0209	34.8118	0.300223
289	0.02096	34.6956	0.216494
290	0.02106	34.6693	0.230687
291	0.02116	34.375	0.203089
292	0.02116	34.8194	0.196425
293	0.02118	35.118	0.310686
294	0.02118	35.2056	0.304103
295	0.02131	34.7561	0.180584
296	0.02131	34.7344	0.177177
297	0.02131	34.7419	0.187753
298	0.02134	34.7627	0.339971
299	0.02137	34.3468	0.19901
300	0.02151	34.5899	0.199177
301	0.02153	34.6114	0.393324
302	0.0217	34.7709	0.195863
303	0.02183	34.7583	0.283448
304	0.02183	34.7742	0.253646
305	0.02197	34.8589	0.246447
306	0.02198	34.9654	0.216973
307	0.02203	34.6739	0.247785
308	0.02205	34.7477	0.199095
309	0.02207	34.8847	0.186029
310	0.02215	35.0481	0.288632
311	0.02219	35.0136	0.216159
312	0.02228	34.8059	0.18018
313	0.02228	34.8234	0.25811
314	0.02231	34.9245	0.169837
315	0.02234	34.7727	0.199004
316	0.02234	34.8704	0.204479
317	0.02236	34.7814	0.224663
318	0.02239	34.7714	0.302654
319	0.02239	34.8603	0.24312
320	0.0224	34.759	0.288719
321	0.0224	34.7997	0.25527
322	0.02241	35.0392	0.214526
323	0.02255	34.7306	0.243141
324	0.02266	34.8438	0.220547
325	0.0227	34.8693	0.2033
326	0.02273	34.6849	0.358325
327	0.02295	35.2685	0.304874
328	0.02295	34.8584	0.20127
329	0.02298	34.9992	0.267367
330	0.02298	34.9712	0.237284
331	0.02303	34.985	0.19074
332	0.02307	34.7666	0.340338
333	0.02313	35.0225	0.271221
334	0.02316	34.883	0.27497
335	0.02321	34.9168	0.178524
336	0.02325	35.0537	0.229702
337	0.02331	34.6478	0.266962
338	0.02331	34.4301	0.279423
339	0.02342	34.9851	0.282579
340	0.02342	35.0081	0.221692

341	0.02342	34.9783	0.225403
342	0.02343	34.8582	0.260273
343	0.02343	34.8582	0.170636
344	0.02344	34.8331	0.263771
345	0.02352	34.8391	0.21588
346	0.02354	34.9998	0.217718
347	0.02357	35.0406	0.206572
348	0.02357	34.9509	0.259128
349	0.02357	34.5715	0.261746
350	0.02365	34.9543	0.274039
351	0.02369	34.943	0.2098
352	0.02388	34.9949	0.180801
353	0.0239	34.889	0.403697
354	0.0239	34.994	0.277383
355	0.02391	34.8993	0.190331
356	0.02401	35.1018	0.195014
357	0.02411	35.1049	0.31033
358	0.02411	34.4711	0.398476
359	0.02412	34.8782	0.224067
360	0.02417	34.9421	0.208286
361	0.02417	35.084	0.183555
362	0.02428	35.1803	0.232251
363	0.02429	35.0771	0.195416
364	0.02432	34.9871	0.184147
365	0.02432	34.953	0.198064
366	0.02432	35.2317	0.235205
367	0.02434	35.0102	0.201948
368	0.02453	35.008	0.162015
369	0.02453	35.0746	0.209952
370	0.02453	34.9493	0.170871
371	0.02457	35.1784	0.189112
372	0.02462	35.1061	0.235556
373	0.02462	35.0521	0.231261
374	0.02462	34.8898	0.233972
375	0.02464	34.9528	0.213795
376	0.02464	34.807	0.18246
377	0.02466	34.8112	0.317595
378	0.02491	35.0862	0.240547
379	0.02494	35.1973	0.228493
380	0.02509	35.2152	0.268527
381	0.0251	34.8369	0.288042
382	0.0251	34.8482	0.181918
383	0.0251	35.079	0.205996
384	0.0251	35.009	0.176889
385	0.02512	35.0276	0.190198
386	0.02513	34.4583	0.333265
387	0.02517	34.9117	0.306484
388	0.02517	35.1989	0.238281
389	0.02517	35.2277	0.204374
390	0.02519	35.1139	0.15531
391	0.02519	35.1145	0.174825
392	0.02521	34.9842	0.185284
393	0.02525	34.9441	0.240723
394	0.02525	34.9063	0.269066
395	0.02534	35.1882	0.176701
396	0.02534	35.1974	0.185937
397	0.02556	35.1828	0.233724

398	0.02557	35.0621	0.190177
399	0.02585	34.7664	0.344692
400	0.02591	35.1181	0.200181
401	0.02596	35.4014	0.200491
402	0.02598	35.2931	0.179545
403	0.02598	35.1081	0.198242
404	0.02598	34.835	0.348314
405	0.02598	35.3111	0.17873
406	0.02626	35.2855	0.288909
407	0.02626	35.3429	0.323231
408	0.02632	35.0223	0.372346
409	0.02669	35.308	0.179587
410	0.02691	35.2688	0.183373
411	0.02691	35.2892	0.205883
412	0.02695	35.1184	0.181369
413	0.02713	35.2513	0.18676
414	0.02714	35.3224	0.216057
415	0.0273	35.2892	0.298907
416	0.02731	35.2673	0.172357
417	0.02731	35.3437	0.230789
418	0.02731	35.1118	0.346591
419	0.02731	35.227	0.261645
420	0.02736	35.313	0.220137
421	0.0274	35.3042	0.352852
422	0.02741	35.5236	0.225337
423	0.02748	35.113	0.172724
424	0.02748	35.1538	0.186856
425	0.02748	35.1117	0.170295
426	0.02773	35.3467	0.289875
427	0.02803	35.7258	0.321699
428	0.02805	35.1306	0.184771
429	0.0281	35.6739	0.216353
430	0.02816	35.3613	0.190628
431	0.02819	35.3174	0.173415
432	0.0282	35.2874	0.292934
433	0.02823	35.5061	0.163093
434	0.02823	35.5563	0.160218
435	0.02823	35.558	0.160985
436	0.02833	35.2665	0.177545
437	0.02844	35.372	0.160301
438	0.02844	35.3421	0.156527
439	0.02849	35.1049	0.181734
440	0.02849	35.4368	0.180322
441	0.02849	35.2913	0.178243
442	0.02849	35.359	0.180243
443	0.02849	35.1963	0.366604
444	0.02857	35.5996	0.235009
445	0.02858	35.6596	0.251383
446	0.02866	35.3449	0.211378
447	0.02866	35.4105	0.201911
448	0.02873	34.9487	0.227749
449	0.02888	35.2763	0.655117
450	0.02898	35.4779	0.275376
451	0.02902	35.0558	0.218663
452	0.02913	35.5152	0.250819
453	0.02915	35.6948	0.183345
454	0.02917	35.2391	0.185823

455	0.02919	35.6024	0.169987
456	0.02938	35.6222	0.299966
457	0.02938	35.472	0.225755
458	0.02944	35.2761	0.180915
459	0.02947	35.0939	0.272694
460	0.02949	35.6079	0.224233
461	0.0295	35.3638	0.283742
462	0.02953	35.4654	0.177866
463	0.02956	35.5064	0.190895
464	0.02969	35.2781	0.194122
465	0.02978	35.5015	0.190369
466	0.02978	35.5116	0.190788
467	0.0299	35.4354	0.185335
468	0.02996	35.861	0.192522
469	0.03012	35.5195	0.196088
470	0.03012	35.4838	0.199296
471	0.03012	35.4761	0.233488
472	0.03023	35.5185	0.166012
473	0.03031	35.5542	0.216273
474	0.03036	35.4803	0.173804
475	0.03047	35.5345	0.258248
476	0.03059	35.3586	0.164518
477	0.03075	35.3522	0.343157
478	0.03076	35.6084	0.212671
479	0.03083	35.4648	0.278746
480	0.03086	35.4164	0.184841
481	0.03091	35.4926	0.166476
482	0.03096	35.6481	0.206672
483	0.03108	35.9224	0.399302
484	0.03108	35.406	0.332119
485	0.03108	35.3374	0.375292
486	0.0311	35.565	0.183722
487	0.03114	35.67	0.236835
488	0.03114	35.5409	0.184896
489	0.03116	35.5877	0.169094
490	0.03117	35.5304	0.283582
491	0.03118	35.6675	0.174531
492	0.0312	35.4957	0.175954
493	0.0312	35.5793	0.14868
494	0.0312	35.4841	0.153803
495	0.03121	35.772	0.224228
496	0.03121	35.7428	0.226764
497	0.03121	35.7867	0.216659
498	0.03132	35.6231	0.320886
499	0.03143	35.6461	0.196214
500	0.03143	35.5602	0.185563
501	0.03147	35.9119	0.273901
502	0.03148	35.7017	0.164289
503	0.03148	35.707	0.22057
504	0.03162	35.6474	0.19424
505	0.03162	35.6847	0.188076
506	0.03166	35.7279	0.17805
507	0.03192	35.4592	0.276064
508	0.03201	35.4896	0.148675
509	0.03236	35.6746	0.20211
510	0.03255	35.576	0.196814
511	0.0326	35.8562	0.258808

512	0.0326	35.8577	0.294619
513	0.03264	35.8808	0.19004
514	0.03264	35.6656	0.274525
515	0.03264	35.9082	0.198168
516	0.03272	35.6638	0.156049
517	0.03272	35.6366	0.169859
518	0.03272	35.6431	0.148421
519	0.03282	35.6329	0.204026
520	0.03289	35.8381	0.178659
521	0.0329	35.7248	0.15355
522	0.03294	35.7992	0.326022
523	0.03296	35.7324	0.22019
524	0.03296	35.8907	0.225741
525	0.03298	35.6164	0.168015
526	0.03333	35.6643	0.150947
527	0.03333	35.7268	0.162633
528	0.03335	35.8159	0.198961
529	0.03341	35.8647	0.150987
530	0.03341	35.8516	0.143619
531	0.03371	35.7797	0.159699
532	0.03373	35.829	0.273673
533	0.03375	36.0229	0.243018
534	0.03378	35.8276	0.259126
535	0.03393	35.9089	0.177213
536	0.03403	35.7587	0.312966
537	0.03428	35.9188	0.207416
538	0.03428	35.9378	0.211316
539	0.03437	35.7837	0.217075
540	0.03441	35.825	0.340606
541	0.03451	35.9757	0.358083
542	0.03459	35.7348	0.155656
543	0.03479	35.8566	0.205773
544	0.03491	35.9118	0.165632
545	0.03512	35.8816	0.169326
546	0.03515	35.8937	0.211664
547	0.03516	35.8012	0.48333
548	0.03516	35.8038	0.482057
549	0.03522	35.8349	0.334212
550	0.03529	35.861	0.1686
551	0.03551	35.9905	0.374822
552	0.03568	36.234	0.188455
553	0.03576	35.9591	0.225173
554	0.03576	36.1545	0.230318
555	0.03583	35.9325	0.204232
556	0.03585	35.5515	0.204326
557	0.03599	35.9909	0.187185
558	0.03616	35.8586	0.223482
559	0.03624	36.1845	0.187245
560	0.03635	36.0035	0.185251
561	0.03682	35.9282	0.1566
562	0.03682	35.8836	0.185109
563	0.03697	36.0748	0.289284
564	0.03702	35.8056	0.308166
565	0.03702	35.9287	0.318471
566	0.03702	36.0695	0.175025
567	0.03702	36.08	0.336812
568	0.03705	35.9637	0.166565

569	0.03707	36.0475	0.166339
570	0.03725	36.0067	0.135213
571	0.03725	35.9927	0.149174
572	0.0373	35.8698	0.18969
573	0.0374	35.9789	0.163683
574	0.03753	35.949	0.262609
575	0.03756	36.3333	0.171248
576	0.03756	36.2185	0.171341
577	0.03787	36.0063	0.206656
578	0.0379	36.0624	0.163008
579	0.03796	36.0889	0.198876
580	0.03818	35.8938	0.151724
581	0.03818	35.8617	0.152703
582	0.03828	36.1141	0.236153
583	0.03834	36.5177	0.245806
584	0.03847	36.2254	0.311188
585	0.03848	36.2564	0.149974
586	0.03873	36.0125	0.17035
587	0.03884	36.026	0.219507
588	0.03902	36.0125	0.204712
589	0.03909	36.1962	0.220822
590	0.0391	36.2125	0.18352
591	0.04014	36.284	0.147901
592	0.04035	36.5486	0.234635
593	0.04056	36.0167	0.143473
594	0.04056	36.0818	0.14498
595	0.04077	35.9918	0.210464
596	0.04089	36.1263	0.168835
597	0.04095	36.4104	0.307187
598	0.04099	36.0403	0.147245
599	0.04101	36.3153	0.150794
600	0.04167	36.5601	0.280122
601	0.04222	36.2101	0.208042
602	0.04258	36.5106	0.426487
603	0.04268	36.2931	0.151018
604	0.04268	35.9138	0.29449
605	0.04285	36.2521	0.175538
606	0.04285	36.2071	0.166515
607	0.04289	36.2018	0.162522
608	0.04306	36.2457	0.207056
609	0.04349	36.4165	0.166064
610	0.04349	36.2137	0.152187
611	0.04349	36.2882	0.166626
612	0.04369	36.2594	0.177777
613	0.04408	36.3298	0.181208
614	0.04438	36.5469	0.231201
615	0.04439	36.2916	0.249078
616	0.0444	36.551	0.270525
617	0.04464	36.5114	0.286272
618	0.04487	36.4565	0.162334
619	0.04521	36.6866	0.304921
620	0.04545	36.4541	0.196266
621	0.04575	36.4634	0.173532
622	0.04575	36.5139	0.151933
623	0.04583	36.1579	0.182756
624	0.0459	36.3811	0.196005
625	0.04625	36.2636	0.218105

626	0.04631	36.5394	0.159223
627	0.04643	36.3247	0.195021
628	0.04656	36.5081	0.209562
629	0.04664	36.4713	0.19409
630	0.04682	36.4786	0.148197
631	0.04682	36.4857	0.182928
632	0.04691	36.7582	0.280677
633	0.04738	36.5642	0.229149
634	0.0476	36.6009	0.161688
635	0.04777	36.5946	0.145385
636	0.04777	36.5627	0.208806
637	0.04819	36.4976	0.165873
638	0.04837	36.3735	0.154741
639	0.0486	36.3303	0.135835
640	0.04865	36.641	0.181462
641	0.04934	36.5997	0.202684
642	0.0494	36.4691	0.176995
643	0.04944	36.5828	0.168097
644	0.04985	36.6402	0.244395
645	0.04989	36.4714	0.18553
646	0.05018	36.5693	0.155812
647	0.05023	36.8583	0.163226
648	0.05032	36.5403	0.453411
649	0.05044	36.4621	0.201896
650	0.05049	36.5461	0.190961
651	0.05052	36.7036	0.133364
652	0.05066	36.8523	0.302557
653	0.05106	36.8804	0.16836
654	0.05126	36.393	0.201448
655	0.05151	36.5862	0.156832
656	0.05194	36.7759	0.228182
657	0.05202	36.8679	0.208308
658	0.05202	36.5433	0.298829
659	0.05222	36.8411	0.218249
660	0.05262	36.9144	0.207509
661	0.05309	36.7752	0.161766
662	0.05316	36.8474	0.172204
663	0.05346	36.8631	0.133817
664	0.05468	36.8327	0.178902
665	0.05469	36.8581	0.149593
666	0.05502	36.6433	0.175423
667	0.05541	36.8676	0.194374
668	0.05614	36.8109	0.193162
669	0.05653	36.9098	0.185051
670	0.05693	37.0393	0.176308
671	0.05708	36.9903	0.171451
672	0.05728	37.0687	0.221323
673	0.05824	36.9821	0.138149
674	0.05824	36.9324	0.14674
675	0.0583	36.9545	0.182842
676	0.05886	36.9802	0.215566
677	0.05886	37.146	0.252804
678	0.05974	36.3001	0.617516
679	0.06092	37.032	0.17759
680	0.06099	37.1631	0.123921
681	0.06099	37.1925	0.16057
682	0.06121	37.1383	0.198009

683	0.06137	36.9608	0.196145
684	0.06137	36.9691	0.141986
685	0.06153	37.0747	0.183316
686	0.06372	37.3517	0.206518
687	0.06384	37.0197	0.155855
688	0.06446	37.0786	0.154429
689	0.06533	37.0851	0.153034
690	0.06627	37.3617	0.203312
691	0.06654	37.1627	0.296189
692	0.06711	37.4237	0.194412
693	0.06725	37.2344	0.203006
694	0.06786	37.4935	0.190499
695	0.06844	37.2381	0.191221
696	0.06886	37.4213	0.159324
697	0.06897	37.2817	0.174144
698	0.07023	37.2944	0.277681
699	0.07056	37.2961	0.226908
700	0.07056	37.2775	0.212322
701	0.07073	37.4787	0.176182
702	0.07083	37.4814	0.447068
703	0.07089	37.4543	0.24781
704	0.0709	37.7291	0.501827
705	0.07091	37.3712	0.228257
706	0.07116	37.3552	0.196012
707	0.07158	37.2494	0.16107
708	0.07167	37.4155	0.208957
709	0.07193	37.5618	0.180815
710	0.07222	37.6319	0.199695
711	0.07252	37.6225	0.304764
712	0.07393	37.5535	0.176502
713	0.0744	37.4346	0.222054
714	0.07446	37.4501	0.192118
715	0.0752	37.4763	0.179761
716	0.0752	37.5656	0.154994
717	0.0756	37.8142	0.289148
718	0.07575	37.5461	0.217171
719	0.07588	37.6342	0.198325
720	0.07845	37.5715	0.150392
721	0.07859	37.8315	0.146078
722	0.07875	38.0154	0.239252
723	0.07875	38.0942	0.180119
724	0.07895	37.5988	0.503338
725	0.07906	37.7843	0.238199
726	0.07931	37.53	0.147217
727	0.07991	37.6003	0.157805
728	0.08009	37.6668	0.119289
729	0.08167	37.7215	0.219648
730	0.08245	37.8192	0.217606
731	0.08282	37.6774	0.265638
732	0.08309	37.9939	0.195118
733	0.0841	37.9686	0.221594
734	0.08572	37.8406	0.128563
735	0.08704	37.8484	0.21681
736	0.0887	37.7772	0.253181
737	0.09039	37.9202	0.21704
738	0.09089	38.0269	0.167553
739	0.09205	38.1602	0.266341

740	0.09293	38.2226	0.493293
741	0.0995	38.4377	0.194713
742	0.10165	38.2982	0.165557
743	0.10221	38.4085	0.187412
744	0.10246	38.2511	0.199553
745	0.10294	38.1815	0.18625
746	0.10361	38.3338	0.12638
747	0.10374	38.2079	0.196456
748	0.10507	38.4295	0.236183
749	0.10661	38.2611	0.195537
750	0.10707	38.4624	0.151122
751	0.10711	38.4295	0.210466
752	0.10713	38.7801	0.468541
753	0.10774	38.4984	0.180059
754	0.10794	38.4017	0.187108
755	0.10908	38.4016	0.133183
756	0.11001	38.5701	0.179741
757	0.11259	38.6214	0.163201
758	0.11388	38.3876	0.194923
759	0.1165	38.5408	0.159897
760	0.11653	38.6302	0.185742
761	0.1176	38.5727	0.239635
762	0.11792	38.6088	0.214419
763	0.11818	38.4157	0.127058
764	0.11901	38.5629	0.212381
765	0.12014	38.5958	0.200195
766	0.12058	38.7926	0.252257
767	0.12086	38.6348	0.302627
768	0.12207	38.5945	0.157915
769	0.12231	38.6119	0.231142
770	0.12278	38.6853	0.144584
771	0.12316	38.5967	0.274641
772	0.12357	38.6164	0.162163
773	0.12377	38.8009	0.333367
774	0.12383	38.8242	0.150324
775	0.12393	38.6504	0.145797
776	0.12405	38.835	0.221531
777	0.12472	38.7616	0.12587
778	0.1256	39.1417	0.298944
779	0.12623	38.4956	0.293348
780	0.12706	38.8125	0.259679
781	0.1271	38.6849	0.280884
782	0.1272	38.7429	0.159508
783	0.12732	38.7841	0.176913
784	0.1292	38.7547	0.150979
785	0.12921	38.8403	0.212795
786	0.1295	38.833	0.250442
787	0.12958	38.8429	0.142748
788	0.13169	38.802	0.134576
789	0.13281	38.7841	0.122177
790	0.13366	38.8266	0.283718
791	0.13372	38.8434	0.157696
792	0.13477	39.2366	0.199357
793	0.13535	39.0775	0.31333
794	0.13614	38.9771	0.206627
795	0.13658	38.8475	0.195569
796	0.137	39.0095	0.224296

797	0.13713	38.9616	0.259136
798	0.13745	39.3511	0.230391
799	0.13822	38.8857	0.128482
800	0.13826	38.9503	0.15974
801	0.1384	39.0229	0.12425
802	0.13851	39.0401	0.319084
803	0.13875	38.9999	0.176022
804	0.1388	39.0467	0.130742
805	0.13955	39.0213	0.145767
806	0.14082	39.1775	0.184019
807	0.14104	39.2835	0.203514
808	0.14123	39.0407	0.212429
809	0.14134	39.1237	0.260864
810	0.14325	39.047	0.160602
811	0.14345	39.5154	0.37065
812	0.14359	39.1135	0.129494
813	0.14404	39.0756	0.170594
814	0.14423	39.016	0.24096
815	0.14542	39.1518	0.210163
816	0.14638	39.1857	0.207408
817	0.14652	39.158	0.187578
818	0.14737	39.0154	0.197495
819	0.14747	38.9672	0.171803
820	0.14771	39.134	0.188785
821	0.14778	39.161	0.170177
822	0.14803	39.2285	0.184985
823	0.14806	39.0382	0.157931
824	0.14834	39.0239	0.142683
825	0.14898	39.1606	0.17494
826	0.1494	39.1104	0.175216
827	0.1505	39.1058	0.170038
828	0.15084	39.1687	0.134119
829	0.15195	39.0477	0.158769
830	0.15291	39.3239	0.173198
831	0.15318	39.2175	0.127754
832	0.15336	39.0991	0.159928
833	0.15341	39.2564	0.179373
834	0.15357	39.3639	0.175468
835	0.15511	39.2076	0.167881
836	0.15526	39.278	0.157843
837	0.15536	39.3994	0.175858
838	0.15601	38.9704	0.179204
839	0.15671	39.2087	0.210288
840	0.15856	39.0794	0.163715
841	0.15921	39.4267	0.283686
842	0.15931	38.9503	0.143914
843	0.15948	39.0908	0.143045
844	0.16004	39.0428	0.270648
845	0.16008	39.2286	0.167715
846	0.16053	39.2057	0.217053
847	0.16061	39.4242	0.26036
848	0.16077	39.2162	0.20927
849	0.16122	39.3106	0.246322
850	0.16236	39.3252	0.224064
851	0.16375	39.2807	0.200998
852	0.16448	39.2634	0.199319
853	0.16471	39.3293	0.139452

854	0.16495	39.3182	0.150918
855	0.16603	39.4938	0.241887
856	0.16606	39.1817	0.193496
857	0.16828	39.2757	0.165826
858	0.16924	39.5155	0.182871
859	0.16971	39.6721	0.274897
860	0.17042	39.4742	0.132684
861	0.17124	39.5424	0.275502
862	0.17169	39.4603	0.214715
863	0.17256	39.5021	0.177409
864	0.1727	39.2676	0.152344
865	0.17297	39.7263	0.252568
866	0.17331	39.3871	0.131849
867	0.17374	39.5403	0.18318
868	0.17378	39.3493	0.151501
869	0.17392	39.5986	0.318528
870	0.17417	39.5734	0.137128
871	0.1742	39.3317	0.154309
872	0.17438	39.5053	0.183971
873	0.17443	39.5546	0.230974
874	0.17444	39.4849	0.162645
875	0.17498	39.7434	0.182014
876	0.17666	39.5164	0.135467
877	0.17713	39.5095	0.175623
878	0.17737	39.4771	0.138264
879	0.17786	39.3394	0.257355
880	0.17791	39.6477	0.131546
881	0.17852	39.9669	0.207864
882	0.17901	39.641	0.157339
883	0.17911	39.7672	0.201468
884	0.17931	39.5355	0.229828
885	0.17952	39.5509	0.280575
886	0.17973	39.5519	0.174634
887	0.17975	39.5962	0.208929
888	0.18004	39.9979	0.268081
889	0.1805	39.3688	0.13762
890	0.18073	39.5035	0.133745
891	0.18078	39.7166	0.179158
892	0.18082	39.6391	0.133983
893	0.18086	39.6151	0.246207
894	0.18092	39.6213	0.207991
895	0.18126	39.6585	0.225961
896	0.18136	39.4868	0.134057
897	0.18145	39.3953	0.235174
898	0.18217	39.5863	0.201062
899	0.18229	39.5259	0.149858
900	0.18231	39.6885	0.152439
901	0.18252	39.7821	0.177319
902	0.1836	39.5995	0.294191
903	0.18375	39.68	0.151568
904	0.18389	39.752	0.19743
905	0.18402	39.796	0.218377
906	0.18458	39.9384	0.282465
907	0.18459	39.7891	0.142656
908	0.18484	39.6548	0.27682
909	0.18493	39.613	0.134864
910	0.18506	39.7435	0.159325

911	0.18515	39.6347	0.160441
912	0.18522	39.5427	0.172189
913	0.18527	39.7882	0.139358
914	0.18667	39.7559	0.144508
915	0.18728	39.7694	0.13671
916	0.18809	39.7451	0.191467
917	0.18906	39.4384	0.149571
918	0.18908	39.8089	0.256269
919	0.18929	39.9359	0.228953
920	0.18929	40.1408	0.200523
921	0.18948	39.7413	0.209197
922	0.18952	39.7624	0.238646
923	0.18995	39.7286	0.224725
924	0.19025	39.8807	0.15799
925	0.19042	39.6892	0.294972
926	0.19044	39.9182	0.154841
927	0.19413	39.7983	0.218857
928	0.19434	39.8787	0.222002
929	0.19546	40.1343	0.403955
930	0.19553	39.7111	0.156617
931	0.19557	39.9082	0.195934
932	0.19568	39.958	0.212325
933	0.19616	39.7288	0.124812
934	0.19621	39.7747	0.157962
935	0.19627	39.8397	0.254651
936	0.19635	39.8927	0.29342
937	0.19733	39.8018	0.14158
938	0.19741	39.6932	0.130873
939	0.19782	39.492	0.189171
940	0.19817	39.7561	0.182544
941	0.19828	39.8879	0.174409
942	0.19861	40.3246	0.238597
943	0.19877	39.8881	0.211487
944	0.19878	39.889	0.174502
945	0.19908	40.0216	0.217638
946	0.19953	39.8511	0.254227
947	0.19961	39.6764	0.246154
948	0.1997	39.8965	0.17155
949	0.20077	39.6837	0.194976
950	0.20083	39.9838	0.252956
951	0.20087	39.7672	0.316127
952	0.20128	39.8575	0.216953
953	0.20231	39.6184	0.198124
954	0.20249	39.9949	0.159703
955	0.20279	39.8373	0.184965
956	0.2032	39.9658	0.146626
957	0.20329	39.8822	0.166845
958	0.20331	39.9277	0.173799
959	0.20334	39.9335	0.216127
960	0.20382	39.9041	0.171481
961	0.20469	39.8809	0.160012
962	0.20483	39.8427	0.202322
963	0.20523	40.1661	0.227714
964	0.20562	39.8126	0.174944
965	0.20691	39.9225	0.279831
966	0.20748	40.316	0.333666
967	0.20751	40.0725	0.148972

968	0.2084	39.7299	0.272182
969	0.20856	39.8758	0.146778
970	0.20868	40.1034	0.239508
971	0.2087	39.8639	0.197696
972	0.20879	39.8798	0.15965
973	0.20948	39.8413	0.262138
974	0.21015	40.1479	0.148877
975	0.21037	39.4543	0.186032
976	0.21084	39.9632	0.223017
977	0.21095	39.806	0.155718
978	0.21114	39.9552	0.163264
979	0.21134	40.1415	0.266119
980	0.21174	39.9676	0.230498
981	0.212	39.9136	0.194498
982	0.21225	39.9374	0.243726
983	0.2135	40.0218	0.16528
984	0.21365	40.0084	0.19033
985	0.21398	39.8874	0.197488
986	0.2144	40.0179	0.298904
987	0.21507	39.8825	0.477887
988	0.21521	40.4641	0.22943
989	0.21578	40.2253	0.258716
990	0.2165	39.8253	0.141535
991	0.21689	40.0672	0.190384
992	0.21692	40.0932	0.185301
993	0.21742	40.1395	0.269611
994	0.21794	39.9942	0.174837
995	0.21845	40.2216	0.204417
996	0.21881	40.1416	0.247596
997	0.21897	40.1925	0.240729
998	0.21898	40.1292	0.218822
999	0.21899	40.0296	0.130815
1000	0.21941	40.0048	0.208962
1001	0.2195	40.1098	0.160031
1002	0.21972	40.0616	0.124319
1003	0.21978	40.0882	0.173376
1004	0.2199	40.2387	0.220195
1005	0.22017	39.9228	0.242946
1006	0.22066	40.1876	0.225274
1007	0.22083	40.0798	0.266821
1008	0.22132	40.0441	0.143254
1009	0.22132	40.0588	0.204756
1010	0.2215	40.1937	0.166162
1011	0.22299	40.3257	0.281229
1012	0.22349	40.1511	0.167278
1013	0.22385	40.0162	0.172818
1014	0.22449	40.1293	0.19721
1015	0.22449	39.9449	0.245775
1016	0.22493	40.1126	0.160616
1017	0.22542	40.1491	0.174035
1018	0.22543	40.1329	0.188923
1019	0.22551	40.1856	0.197444
1020	0.22578	39.9865	0.175963
1021	0.22645	40.3912	0.201078
1022	0.22726	40.097	0.228488
1023	0.22818	40.2236	0.183395
1024	0.22865	40.1604	0.308241

1025	0.22928	40.2229	0.202405
1026	0.22978	40.1351	0.188066
1027	0.2302	40.1283	0.227444
1028	0.23041	40.3062	0.260109
1029	0.23082	40.0717	0.296847
1030	0.23086	40.1049	0.204707
1031	0.23092	40.2958	0.226289
1032	0.23125	40.3169	0.189753
1033	0.23162	40.201	0.226906
1034	0.23164	40.0875	0.237265
1035	0.23205	40.2045	0.159507
1036	0.23282	39.8341	0.180906
1037	0.23367	40.1857	0.255059
1038	0.23378	40.4148	0.19637
1039	0.23391	40.1788	0.292139
1040	0.23395	40.1389	0.214099
1041	0.23472	40.1912	0.32799
1042	0.23481	40.2784	0.167703
1043	0.23494	40.158	0.183242
1044	0.2352	40.1629	0.275154
1045	0.23556	40.2336	0.214202
1046	0.23639	40.3805	0.155735
1047	0.23662	40.2636	0.156727
1048	0.23664	40.4674	0.193131
1049	0.23685	40.119	0.236651
1050	0.23693	40.2279	0.210237
1051	0.23721	40.2245	0.203948
1052	0.23746	40.7365	0.305716
1053	0.23789	40.2317	0.172272
1054	0.2395	39.9059	0.208409
1055	0.23981	40.13	0.180786
1056	0.2402	40.3847	0.264226
1057	0.24085	40.4016	0.166736
1058	0.24122	40.3291	0.130654
1059	0.24151	40.1214	0.184136
1060	0.24164	40.172	0.261675
1061	0.2422	40.564	0.221356
1062	0.2428	40.4217	0.219858
1063	0.24284	40.5734	0.259119
1064	0.24329	40.2516	0.143114
1065	0.24354	40.538	0.306318
1066	0.2441	40.2956	0.242814
1067	0.24444	40.2064	0.221704
1068	0.24459	40.1263	0.186144
1069	0.24511	40.3336	0.209543
1070	0.24523	40.4957	0.181643
1071	0.24553	40.3582	0.219468
1072	0.24585	40.4318	0.167359
1073	0.24617	40.3473	0.149416
1074	0.2465	40.378	0.17135
1075	0.24656	40.2829	0.214739
1076	0.2466	40.3907	0.15515
1077	0.24711	40.1983	0.137434
1078	0.24846	40.203	0.17717
1079	0.2486	40.4912	0.215929
1080	0.24861	40.4276	0.195634
1081	0.24862	40.3657	0.261564

1082	0.24865	40.3256	0.178325
1083	0.24896	40.427	0.180619
1084	0.24909	40.3167	0.167358
1085	0.24972	40.6733	0.220606
1086	0.24983	40.2961	0.206796
1087	0.24986	40.3915	0.163714
1088	0.25019	40.5965	0.224495
1089	0.25022	40.3764	0.218218
1090	0.25043	40.4145	0.179616
1091	0.25051	40.5692	0.188725
1092	0.25063	40.3009	0.188277
1093	0.2509	40.3873	0.202604
1094	0.25199	40.4698	0.207347
1095	0.2526	40.4622	0.166497
1096	0.2526	40.4066	0.259149
1097	0.2533	40.3688	0.15705
1098	0.25358	40.0303	0.262822
1099	0.25407	40.4546	0.167762
1100	0.25616	40.6458	0.194176
1101	0.2562	40.5593	0.271675
1102	0.25637	40.3729	0.207312
1103	0.25683	40.5023	0.251739
1104	0.25731	40.5776	0.249962
1105	0.25736	40.4941	0.193606
1106	0.25772	40.6088	0.22531
1107	0.25813	40.528	0.231214
1108	0.25859	40.4663	0.209281
1109	0.25865	40.51	0.149785
1110	0.25904	40.5418	0.163699
1111	0.25919	40.2879	0.178603
1112	0.25923	40.4255	0.162943
1113	0.25948	40.5238	0.233841
1114	0.25952	40.442	0.169977
1115	0.26062	40.5033	0.171288
1116	0.26094	40.6128	0.185932
1117	0.26141	40.447	0.207526
1118	0.26162	40.4182	0.225542
1119	0.26172	40.6874	0.258941
1120	0.26173	40.681	0.233293
1121	0.26175	40.392	0.226027
1122	0.26184	40.6935	0.249126
1123	0.262	40.7216	0.216893
1124	0.26303	41.2641	0.514634
1125	0.26323	40.6388	0.170092
1126	0.2636	40.3997	0.177345
1127	0.26393	40.5588	0.167704
1128	0.26397	40.3035	0.205843
1129	0.26408	40.6342	0.381046
1130	0.26419	40.5074	0.238455
1131	0.2646	40.5153	0.197965
1132	0.26463	40.6719	0.256474
1133	0.26582	40.6087	0.219728
1134	0.26583	40.3244	0.206109
1135	0.2664	40.2901	0.264978
1136	0.267	40.5463	0.204902
1137	0.26791	40.5103	0.202065
1138	0.26818	40.4623	0.232043

1139	0.26859	40.579	0.191142
1140	0.2686	40.4553	0.240404
1141	0.26887	40.6039	0.217156
1142	0.2702	40.6363	0.197002
1143	0.27021	40.5089	0.164866
1144	0.27057	40.5139	0.20931
1145	0.27094	40.4887	0.166124
1146	0.27141	40.5677	0.123404
1147	0.27276	40.5415	0.173203
1148	0.27351	40.4721	0.208686
1149	0.27456	40.7399	0.293068
1150	0.27509	40.7769	0.239421
1151	0.27516	40.5286	0.241865
1152	0.27585	40.6631	0.213576
1153	0.27631	40.6749	0.180061
1154	0.27668	40.8504	0.272235
1155	0.27776	40.6186	0.24474
1156	0.27782	40.9187	0.17803
1157	0.27935	40.7037	0.164335
1158	0.2799	40.5056	0.19932
1159	0.27995	40.5664	0.153331
1160	0.2806	40.6663	0.153096
1161	0.28132	40.6224	0.174763
1162	0.28207	40.5986	0.166629
1163	0.28266	40.744	0.297355
1164	0.28344	40.6596	0.184378
1165	0.28405	40.5735	0.154134
1166	0.28446	40.5431	0.186562
1167	0.28446	40.6919	0.20709
1168	0.28478	41.1135	0.351325
1169	0.28557	40.7046	0.185989
1170	0.28588	40.671	0.173206
1171	0.28665	40.6389	0.217509
1172	0.28734	40.5691	0.250397
1173	0.28758	40.5379	0.238886
1174	0.28775	40.8753	0.227224
1175	0.28814	40.7075	0.188011
1176	0.28844	40.749	0.137848
1177	0.28854	40.7458	0.165419
1178	0.28894	40.9433	0.226038
1179	0.28901	40.7428	0.228497
1180	0.28905	40.6763	0.168052
1181	0.28907	40.702	0.277224
1182	0.28949	40.9765	0.256114
1183	0.28995	40.7393	0.271437
1184	0.29008	40.7298	0.234493
1185	0.29045	40.7861	0.23085
1186	0.29123	40.484	0.219145
1187	0.29193	40.7606	0.177707
1188	0.29196	40.7653	0.172199
1189	0.29215	40.7505	0.483728
1190	0.29221	40.4229	0.292471
1191	0.29251	40.6378	0.228825
1192	0.29256	40.6952	0.224836
1193	0.29293	40.7458	0.174302
1194	0.29298	40.8268	0.203556
1195	0.29365	40.857	0.204834

1196	0.29504	40.762	0.173778
1197	0.2958	41.0209	0.307913
1198	0.29587	40.9989	0.165779
1199	0.29647	40.9938	0.366054
1200	0.29692	40.6595	0.202621
1201	0.29695	40.7505	0.210501
1202	0.29853	41.2705	0.232731
1203	0.29855	40.6539	0.272883
1204	0.29893	40.9195	0.25505
1205	0.29933	41.0113	0.341681
1206	0.29945	41.0213	0.195981
1207	0.29947	40.7335	0.186464
1208	0.3002	40.9837	0.198158
1209	0.3003	40.8795	0.247188
1210	0.30089	41.1729	0.294726
1211	0.30142	40.8585	0.191702
1212	0.30152	41.1086	0.206046
1213	0.30251	40.8894	0.22475
1214	0.30304	40.8204	0.274703
1215	0.30352	40.9204	0.142521
1216	0.30365	41.0778	0.197526
1217	0.30404	40.719	0.22209
1218	0.30406	40.7052	0.182517
1219	0.30476	41.0411	0.242802
1220	0.30486	40.9018	0.140115
1221	0.30526	40.5924	0.306711
1222	0.30538	40.9054	0.133839
1223	0.30548	41.0041	0.226225
1224	0.30581	40.9451	0.165493
1225	0.30652	40.8959	0.201565
1226	0.30681	41.16	0.239095
1227	0.30691	40.9573	0.150058
1228	0.30818	41.2018	0.139651
1229	0.30834	40.9533	0.254065
1230	0.30878	40.9954	0.152417
1231	0.30922	41.046	0.250639
1232	0.30938	40.9757	0.270379
1233	0.30949	41.0177	0.300429
1234	0.30962	40.72	0.257177
1235	0.31052	40.6197	0.336421
1236	0.31061	40.9962	0.221391
1237	0.31088	41.046	0.182638
1238	0.31144	41.0123	0.141744
1239	0.31179	41.0377	0.210925
1240	0.31203	40.9503	0.160446
1241	0.31262	40.9565	0.149709
1242	0.31267	41.0352	0.279475
1243	0.3127	40.6239	0.2062
1244	0.31296	41.0195	0.203499
1245	0.31303	41.0061	0.176794
1246	0.31387	41.0228	0.214434
1247	0.31453	41.1251	0.155711
1248	0.31506	41.0194	0.227804
1249	0.3152	40.8837	0.219964
1250	0.31588	41.2033	0.233304
1251	0.31593	40.933	0.227638
1252	0.31623	41.1979	0.274091

1253	0.31782	41.0738	0.195921
1254	0.31804	41.0772	0.167213
1255	0.31822	41.0199	0.153489
1256	0.3185	41.3345	0.327925
1257	0.31901	41.141	0.177674
1258	0.31901	40.8941	0.203876
1259	0.32096	41.2315	0.207296
1260	0.32104	40.9546	0.156373
1261	0.32118	41.1921	0.293444
1262	0.32206	40.7271	0.199265
1263	0.32223	40.8628	0.158186
1264	0.32223	40.8774	0.138773
1265	0.32276	40.9316	0.211904
1266	0.32446	41.0699	0.126468
1267	0.32548	41.2273	0.193265
1268	0.3256	41.2625	0.255286
1269	0.3258	41.1639	0.271274
1270	0.32581	41.0478	0.141716
1271	0.32632	41.0504	0.157201
1272	0.32804	41.0002	0.171833
1273	0.32842	41.3079	0.308428
1274	0.32848	41.351	0.278699
1275	0.32851	40.934	0.295651
1276	0.32868	41.0716	0.150485
1277	0.32868	41.1	0.144172
1278	0.32871	40.9276	0.222195
1279	0.32907	41.0566	0.165152
1280	0.32941	41.4109	0.310571
1281	0.32952	41.2391	0.162498
1282	0.32968	41.1031	0.114803
1283	0.32995	40.8444	0.209813
1284	0.33047	41.2306	0.131607
1285	0.33056	41.1674	0.214426
1286	0.33063	41.1689	0.23109
1287	0.33089	41.1586	0.239453
1288	0.3309	40.996	0.212048
1289	0.33103	41.0098	0.137116
1290	0.33147	41.0301	0.218692
1291	0.33147	41.2335	0.285481
1292	0.33147	41.1012	0.166036
1293	0.33189	41.0149	0.143999
1294	0.33237	40.977	0.159233
1295	0.33298	41.3299	0.201675
1296	0.3335	41.0025	0.141896
1297	0.3335	41.0366	0.176123
1298	0.3345	41.1204	0.23572
1299	0.33498	41.0251	0.171864
1300	0.3351	41.0242	0.177927
1301	0.33671	41.0367	0.18424
1302	0.33749	41.1678	0.278653
1303	0.33751	41.0098	0.23724
1304	0.33762	41.1477	0.176101
1305	0.33778	41.1537	0.233862
1306	0.33818	41.145	0.211242
1307	0.33848	41.1131	0.200412
1308	0.33897	41.0555	0.356511
1309	0.34061	41.1737	0.192422

1310	0.34098	41.0587	0.1943
1311	0.34111	41.0641	0.255103
1312	0.34169	41.2122	0.321008
1313	0.34446	41.3081	0.152468
1314	0.34448	41.2914	0.299203
1315	0.34487	41.2333	0.171671
1316	0.34545	41.2033	0.23021
1317	0.34602	41.3318	0.136683
1318	0.34696	41.4163	0.370346
1319	0.34717	41.1174	0.244272
1320	0.34881	41.3721	0.218415
1321	0.3491	41.2499	0.158604
1322	0.34912	41.282	0.206036
1323	0.34932	41.1406	0.157954
1324	0.34939	41.2033	0.168604
1325	0.34949	41.005	0.226318
1326	0.34949	41.3756	0.171432
1327	0.34949	41.1526	0.205831
1328	0.34968	41.4873	0.287684
1329	0.35015	41.1867	0.147015
1330	0.35059	41.2342	0.163121
1331	0.35099	41.2532	0.161378
1332	0.35149	41.4798	0.209505
1333	0.35149	41.3343	0.138963
1334	0.35372	41.3657	0.191944
1335	0.35519	41.4647	0.223042
1336	0.35774	41.3574	0.291964
1337	0.358	41.3539	0.215252
1338	0.35842	41.2423	0.215992
1339	0.3585	41.1484	0.288995
1340	0.35909	41.1532	0.305677
1341	0.35909	41.4429	0.171109
1342	0.35946	41.4213	0.15447
1343	0.36017	41.187	0.191161
1344	0.36034	41.0791	0.230289
1345	0.36091	41.4328	0.197529
1346	0.36147	41.1159	0.288022
1347	0.3628	41.2452	0.209997
1348	0.36325	41.364	0.215101
1349	0.36499	41.4626	0.291566
1350	0.36623	41.136	0.226225
1351	0.36656	41.2327	0.155657
1352	0.36713	41.2088	0.151911
1353	0.3679	41.2627	0.165575
1354	0.36799	41.4699	0.158375
1355	0.36839	41.5479	0.336428
1356	0.36857	41.463	0.345551
1357	0.36962	41.4498	0.244353
1358	0.36987	41.2185	0.255045
1359	0.37023	41.3794	0.188778
1360	0.37052	41.3759	0.163241
1361	0.37053	41.3833	0.160025
1362	0.37057	41.6395	0.256631
1363	0.37062	41.273	0.282407
1364	0.37062	41.0698	0.135854
1365	0.37095	41.4098	0.39052
1366	0.37138	41.334	0.168104

1367	0.37153	41.3842	0.116639
1368	0.37153	41.4289	0.165418
1369	0.37353	41.3114	0.133039
1370	0.37436	41.4722	0.234786
1371	0.37676	41.4155	0.15471
1372	0.37805	41.3598	0.223183
1373	0.37852	41.4271	0.182401
1374	0.37991	41.2959	0.219919
1375	0.37997	41.4154	0.163795
1376	0.38061	41.4614	0.188044
1377	0.38092	41.4511	0.201434
1378	0.38129	41.4549	0.20413
1379	0.38153	41.6775	0.218983
1380	0.38185	41.5615	0.198898
1381	0.38248	41.3726	0.265174
1382	0.38291	41.3678	0.206585
1383	0.38402	41.5182	0.20184
1384	0.386	41.5217	0.152114
1385	0.38672	41.3791	0.270807
1386	0.38744	41.5567	0.273474
1387	0.38781	41.6302	0.283892
1388	0.38895	41.383	0.199578
1389	0.38949	41.544	0.192176
1390	0.39255	41.4742	0.237288
1391	0.3951	41.4677	0.203648
1392	0.39706	41.259	0.168535
1393	0.39944	41.7119	0.28782
1394	0.4005	41.4918	0.158339
1395	0.40221	41.4795	0.199524
1396	0.40293	41.5478	0.290402
1397	0.40368	41.616	0.213828
1398	0.40463	41.6747	0.166047
1399	0.40483	41.3373	0.202033
1400	0.4055	41.7807	0.135518
1401	0.40646	41.6914	0.168815
1402	0.40895	41.7002	0.240024
1403	0.4092	41.7405	0.363654
1404	0.40935	41.7061	0.274957
1405	0.40949	41.6784	0.172193
1406	0.41004	41.6836	0.279968
1407	0.41123	41.662	0.318089
1408	0.4114	41.8335	0.242677
1409	0.41161	41.8131	0.149106
1410	0.41266	41.7893	0.330128
1411	0.41657	41.7295	0.149208
1412	0.41857	41.6174	0.202099
1413	0.41936	41.6863	0.192599
1414	0.41939	41.4604	0.22264
1415	0.4196	41.8662	0.224621
1416	0.41965	41.7981	0.235757
1417	0.41995	41.4394	0.294415
1418	0.42036	41.8343	0.344188
1419	0.42088	41.6178	0.17643
1420	0.42094	41.728	0.250117
1421	0.42195	41.5336	0.270821
1422	0.42247	41.6483	0.231707
1423	0.42249	41.6796	0.275449

1424	0.42348	41.5955	0.186465
1425	0.4236	41.8403	0.400817
1426	0.42399	41.7264	0.13417
1427	0.42459	41.7483	0.195831
1428	0.42568	41.7039	0.195468
1429	0.42602	41.7517	0.175449
1430	0.427	41.813	0.229539
1431	0.42703	41.8348	0.198324
1432	0.42793	42.2757	0.23861
1433	0.42859	41.8825	0.403543
1434	0.42895	41.6443	0.193398
1435	0.42903	41.8266	0.250757
1436	0.42976	41.9219	0.19807
1437	0.4302	41.8354	0.165783
1438	0.43494	41.7828	0.183674
1439	0.43546	41.5839	0.13365
1440	0.43683	42.0051	0.1818
1441	0.4376	41.6253	0.16614
1442	0.43783	41.882	0.356877
1443	0.4389	41.4679	0.211194
1444	0.43891	41.6682	0.279961
1445	0.43935	41.8262	0.202784
1446	0.44013	42.1401	0.441915
1447	0.44067	41.6364	0.160177
1448	0.44149	41.6823	0.183572
1449	0.44245	41.8523	0.172891
1450	0.44263	41.9039	0.139699
1451	0.44361	41.793	0.182219
1452	0.44391	41.8069	0.290347
1453	0.44764	41.8552	0.223954
1454	0.44886	41.8853	0.221814
1455	0.44891	41.9875	0.215648
1456	0.45068	41.6446	0.157686
1457	0.45106	41.9281	0.213431
1458	0.45162	41.5892	0.225346
1459	0.45162	41.9923	0.154715
1460	0.45163	41.7162	0.305385
1461	0.45218	41.9959	0.320076
1462	0.45328	41.7953	0.207786
1463	0.45346	41.8863	0.14411
1464	0.45458	41.7873	0.209236
1465	0.45477	41.8839	0.284149
1466	0.45885	41.9679	0.252185
1467	0.46064	42.0021	0.190943
1468	0.46104	42.0162	0.230251
1469	0.46164	41.9839	0.138441
1470	0.46643	42.3424	0.396492
1471	0.46654	42.048	0.135922
1472	0.46842	41.9733	0.206146
1473	0.46887	41.7631	0.234035
1474	0.47042	42.2151	0.404516
1475	0.47065	42.0135	0.180637
1476	0.47163	41.9798	0.210913
1477	0.47204	42.0374	0.25691
1478	0.47565	41.938	0.177112
1479	0.47786	42.2269	0.228278
1480	0.47941	41.8695	0.203883

1481	0.47946	42.0107	0.198987
1482	0.47988	41.8956	0.237796
1483	0.48064	42.0041	0.160362
1484	0.48065	41.9748	0.139221
1485	0.48164	42.3055	0.210887
1486	0.48888	42.0842	0.199732
1487	0.49244	42.2783	0.335403
1488	0.49499	42.0821	0.165011
1489	0.49566	42.1453	0.120919
1490	0.49579	42.0817	0.153945
1491	0.49739	42.1268	0.225856
1492	0.50067	42.257	0.253816
1493	0.50098	42.1165	0.283133
1494	0.50282	42.161	0.218549
1495	0.50285	42.0703	0.312874
1496	0.50306	42.1484	0.311282
1497	0.50316	42.1444	0.228687
1498	0.50593	42.079	0.166703
1499	0.50615	42.1043	0.151709
1500	0.50725	42.2358	0.180974
1501	0.50739	42.2419	0.288822
1502	0.50825	42.0193	0.184312
1503	0.51016	42.0945	0.144405
1504	0.51095	42.0583	0.239085
1505	0.51169	42.3885	0.227281
1506	0.51387	42.2567	0.35966
1507	0.51437	42.2259	0.196162
1508	0.51469	42.3259	0.402924
1509	0.51726	42.0932	0.190441
1510	0.51883	42.1949	0.250421
1511	0.51885	42.1824	0.209558
1512	0.51941	42.2438	0.315111
1513	0.51968	42.2905	0.196455
1514	0.52271	42.3496	0.273927
1515	0.524	42.1774	0.369236
1516	0.5262	42.1702	0.223271
1517	0.52859	42.0855	0.188829
1518	0.52937	42.2913	0.141023
1519	0.53099	41.8802	0.23431
1520	0.53111	42.5109	0.289268
1521	0.53124	42.5253	0.38093
1522	0.53225	41.9821	0.221979
1523	0.53271	42.2472	0.294354
1524	0.53372	42.1649	0.273819
1525	0.53435	42.1433	0.225295
1526	0.53534	42.3205	0.204703
1527	0.53572	42.2134	0.209507
1528	0.53942	42.3338	0.244476
1529	0.54097	42.2791	0.233447
1530	0.54152	42.53	0.24247
1531	0.54545	42.5633	0.191404
1532	0.54834	42.0235	0.363222
1533	0.54881	42.1695	0.378555
1534	0.55021	42.3253	0.19588
1535	0.55033	42.3728	0.208456
1536	0.55046	42.4104	0.376261
1537	0.55267	42.1938	0.244151

1538	0.55375	42.49	0.207133
1539	0.55384	42.0827	0.177409
1540	0.55782	42.4389	0.195189
1541	0.55882	42.7331	0.3833
1542	0.55882	42.2975	0.356215
1543	0.56181	42.2937	0.227829
1544	0.56269	42.6016	0.265242
1545	0.56283	42.4296	0.234712
1546	0.56437	42.3928	0.221111
1547	0.56437	42.4881	0.207587
1548	0.56437	42.3969	0.296515
1549	0.56481	42.4873	0.19275
1550	0.56879	42.2407	0.262337
1551	0.57542	42.1912	0.452867
1552	0.57577	42.5915	0.213158
1553	0.57619	42.4887	0.202103
1554	0.57668	42.4263	0.166843
1555	0.57737	42.645	0.290949
1556	0.5783	42.4174	0.24485
1557	0.57831	42.4512	0.241176
1558	0.57882	42.4272	0.214391
1559	0.57891	42.5317	0.231686
1560	0.57967	42.3923	0.23116
1561	0.58067	42.573	0.262496
1562	0.5808	42.7061	0.239821
1563	0.58301	42.5408	0.168029
1564	0.5833	42.7974	0.186201
1565	0.58382	42.4765	0.198486
1566	0.58382	42.6489	0.184752
1567	0.5839	42.4199	0.199369
1568	0.58582	42.707	0.37297
1569	0.5873	42.6241	0.203038
1570	0.58879	42.5021	0.220032
1571	0.59029	42.5156	0.248253
1572	0.59179	42.4508	0.255828
1573	0.60067	42.6116	0.278838
1574	0.60155	42.7562	0.394779
1575	0.60328	42.7503	0.2001
1576	0.60626	42.6158	0.275076
1577	0.60827	42.5904	0.228388
1578	0.60876	42.4282	0.350327
1579	0.60878	42.6179	0.230751
1580	0.60981	42.6876	0.321483
1581	0.61071	42.6686	0.184238
1582	0.61127	42.9049	0.247686
1583	0.61177	42.701	0.16444
1584	0.61577	42.7085	0.347272
1585	0.61875	42.7164	0.388756
1586	0.61941	42.6948	0.2753
1587	0.61983	42.7557	0.154862
1588	0.6207	42.6988	0.268136
1589	0.62183	42.5894	0.275623
1590	0.62525	42.6577	0.215148
1591	0.62725	42.585	0.244041
1592	0.63077	42.6632	0.202856
1593	0.63183	42.3111	0.32695
1594	0.63225	42.8493	0.323662

1595	0.63399	42.5972	0.147609
1596	0.63777	42.5298	0.442879
1597	0.63794	42.8637	0.31082
1598	0.63824	42.6594	0.367516
1599	0.63873	42.7804	0.261646
1600	0.63934	42.79	0.171844
1601	0.64185	42.8476	0.256941
1602	0.64311	42.3609	0.407842
1603	0.64371	42.7992	0.309841
1604	0.64371	42.8034	0.316528
1605	0.6477	42.819	0.288435
1606	0.64852	43.0397	0.272369
1607	0.6487	42.7335	0.310678
1608	0.64962	42.7805	0.149571
1609	0.66213	42.9793	0.279772
1610	0.6647	42.8698	0.233989
1611	0.67072	43.0677	0.274795
1612	0.67773	43.2251	0.311797
1613	0.6819	42.8376	0.239367
1614	0.68372	43.1716	0.407734
1615	0.68519	43.0166	0.366173
1616	0.68872	42.8137	0.226508
1617	0.68935	42.8535	0.15677
1618	0.68936	43.0444	0.271991
1619	0.69072	42.9143	0.425857
1620	0.69318	42.9162	0.318024
1621	0.69718	42.9692	0.212738
1622	0.69818	42.9671	0.342807
1623	0.69918	42.9402	0.338462
1624	0.69918	43.0691	0.215345
1625	0.69932	43.1107	0.155898
1626	0.69992	43.1147	0.29538
1627	0.7007	42.917	0.223194
1628	0.70117	43.0968	0.261582
1629	0.70192	42.8962	0.462713
1630	0.70192	43.0433	0.252753
1631	0.71571	42.9944	0.245932
1632	0.71874	42.8739	0.18843
1633	0.71968	43.1638	0.204549
1634	0.71969	42.97	0.342226
1635	0.72015	43.1612	0.243162
1636	0.72074	43.0459	0.305174
1637	0.72674	43.0811	0.248363
1638	0.72774	43.008	0.257091
1639	0.72866	43.005	0.283498
1640	0.73014	42.9186	0.321961
1641	0.73161	43.1799	0.196424
1642	0.73368	42.9845	0.195902
1643	0.73395	43.22	0.388289
1644	0.73495	43.2537	0.224615
1645	0.73568	43.1807	0.25711
1646	0.73596	43.1675	0.425639
1647	0.73675	42.992	0.21489
1648	0.74196	43.4813	0.32514
1649	0.74276	43.0825	0.327179
1650	0.74573	43.1675	0.195179
1651	0.74607	42.9817	0.25474

1652	0.74911	42.9396	0.262488
1653	0.7493	43.1772	0.163225
1654	0.75074	42.9012	0.318716
1655	0.75899	43.2469	0.233153
1656	0.76076	43.0667	0.201161
1657	0.7611	43.1767	0.236628
1658	0.76165	43.131	0.239872
1659	0.76566	43.2369	0.225434
1660	0.76666	43.4224	0.250162
1661	0.76674	43.3623	0.27025
1662	0.76709	43.5587	0.25715
1663	0.76865	43.2929	0.311675
1664	0.76932	43.2168	0.162741
1665	0.77309	43.3568	0.295081
1666	0.77929	43.0395	0.311092
1667	0.78807	43.3042	0.232049
1668	0.78907	43.3837	0.271177
1669	0.78928	43.1995	0.165559
1670	0.79662	42.9026	0.372096
1671	0.79863	43.4338	0.288144
1672	0.83981	43.36	0.211845
1673	0.83981	43.2012	0.390423
1674	0.85482	43.6168	0.299913
1675	0.93585	43.5331	0.239939
1676	0.97423	44.1111	0.201519
1677	1.01242	44.1177	0.367085
1678	1.01988	44.2757	0.225903
1679	1.02088	43.9706	0.47547
1680	1.02789	44.5364	0.353385
1681	1.04817	44.4229	0.488758
1682	1.12092	44.4405	0.22363
1683	1.23225	44.3207	0.610496
1684	1.23597	44.905	0.273735
1685	1.29911	44.8641	0.275291
1686	1.3041	44.7206	0.316517
1687	1.30611	45.0197	0.445162
1688	1.31317	44.7867	0.316349
1689	1.3291	44.8745	0.238344
1690	1.34101	44.7541	0.306395
1691	1.35136	44.5492	0.31846
1692	1.35608	44.8094	0.238282
1693	1.39103	44.7944	0.374345
1694	1.41633	44.4579	0.735776
1695	1.5429	45.0902	0.371014
1696	1.54901	45.293	0.233771
1697	1.61505	45.1595	0.333024
1698	1.69706	45.2863	0.38048
1699	1.80119	45.4865	0.281981
1700	1.91165	45.4233	0.358642
1701	2.26137	46.1828	0.281309

## 🧹 Clean and Extract Relevant Data

To ensure reliable fitting, we remove any rows that have missing values in key columns:

- zHD : redshift for the Hubble diagram
- MU\_SH0ES : distance modulus
- MU\_SH0ES\_ERR\_DIAG : uncertainty in the distance modulus

We then extract these cleaned columns as NumPy arrays to prepare for analysis and modeling.

```
In []: # Filter for entries with usable data based on the required columns
z = df[2].values[1:][1:].astype(float) # skip header row and converted into float type
m = df[10].values[1:][1:].astype(float)
m_e = df[11].values[1:][1:].astype(float)
```

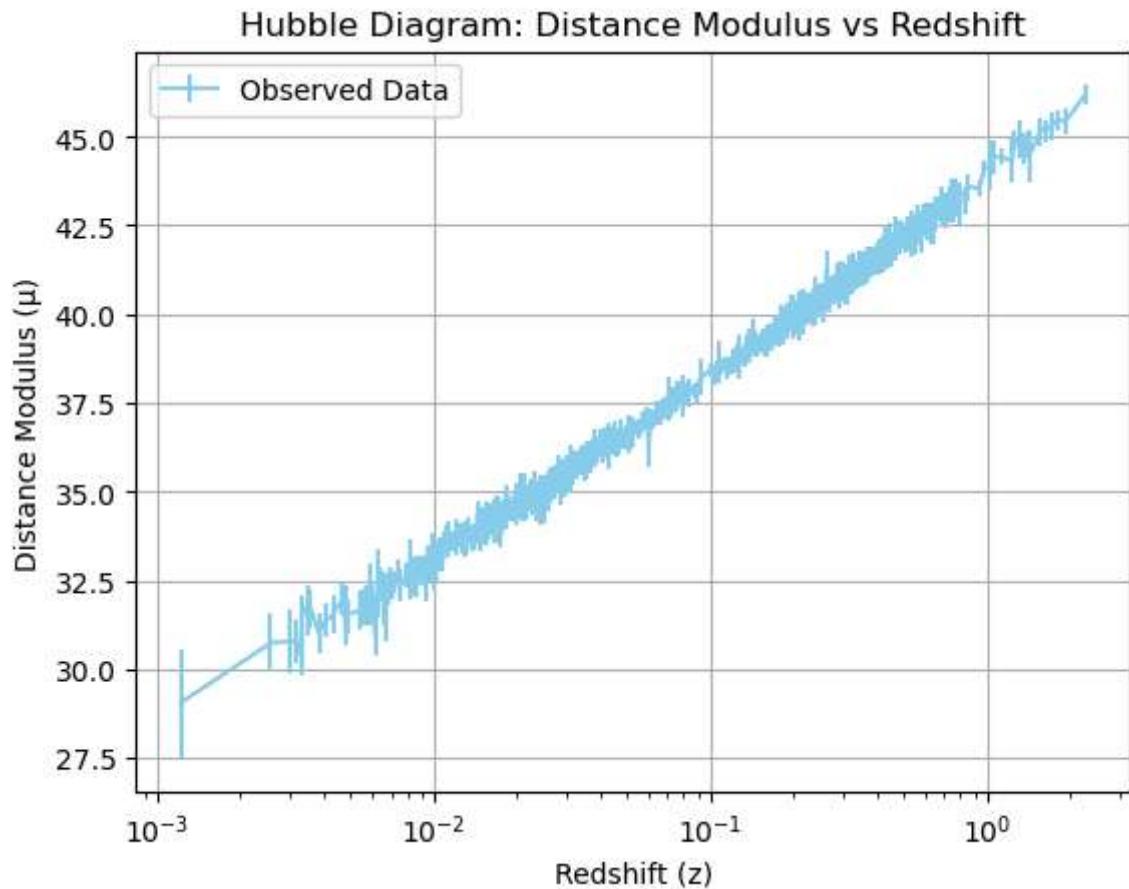
## 📈 Plot the Hubble Diagram

Let's visualize the relationship between redshift  $z$  and distance modulus  $\mu$ , known as the Hubble diagram. This plot is a cornerstone of observational cosmology—it allows us to compare supernova observations with theoretical predictions based on different cosmological models.

We use a logarithmic scale on the redshift axis to clearly display both nearby and distant supernovae.

```
In [686]: # Write a code to plot the distance modulus and the redshift (x-axis), Label them accordingly.
plt.figure()
plt.errorbar(z, m, m_e, label='Observed Data', color='skyblue')
plt.xlabel('Redshift (z)')
plt.ylabel('Distance Modulus (μ)')
plt.title('Hubble Diagram: Distance Modulus vs Redshift')
plt.grid(True)

#Try using log scale in x-axis
plt.xscale('log')
plt.legend()
plt.show()
```



## Define the Cosmological Model

We now define the theoretical framework based on the flat  $\Lambda$ CDM model (read about the model in wikipedia if needed). This involves:

- The dimensionless Hubble parameter:

$$E(z) = \sqrt{\Omega_m(1+z)^3 + (1-\Omega_m)}$$

- The distance modulus is:

$$\mu(z) = 5 \log_{10}(d_L/\text{Mpc}) + 25$$

- And the corresponding luminosity distance :

$$d_L(z) = (1+z) \cdot \frac{c}{H_0} \int_0^z \frac{dz'}{E(z')}$$

These equations allow us to compute the expected distance modulus from a given redshift  $z$ , Hubble constant  $H_0$ , and matter density parameter  $\Omega_m$ .

```
In []: # Define the E(z) for flat LCDM
def E(z, Omega_m):
 return np.sqrt(Omega_m * (1+z)**3 + (1 - Omega_m))

Luminosity distance in Mpc, try using scipy quad to integrate.
def luminosity_distance(z, H0, Omega_m):
 integral,_ = quad(lambda z_: 1 / E(z_, Omega_m), 0, z)
 d_L = (1 + z) * (c.to('km/s').value / H0) * integral
 return d_L # in Mpc

Theoretical distance modulus, use above function inside mu_theory to compute
Luminosity distance
def mu_theory(z, H0, Omega_m):
 return 5 * np.log10([luminosity_distance(z_, H0, Omega_m) for z_ in z]) +
25
```

## Fit the Model to Supernova Data

We now perform a non-linear least squares fit to the supernova data using our theoretical model for  $\mu(z)$ . This fitting procedure will estimate the best-fit values for the Hubble constant  $H_0$  and matter density parameter  $\Omega_m$ , along with their associated uncertainties.

We'll use:

- `curve_fit` from `scipy.optimize` for the fitting.
- The observed distance modulus (`mu`), redshift (`z`), and measurement errors.

The initial guess is:

- $H_0 = 70 \text{ km/s/Mpc}$
- $\Omega_m = 0.3$

```
In [630]: # Initial guess: H0 = 70, Omega_m = 0.3
p0 = [70, 0.3]

Write a code for fitting and taking error out of the parameters
p_optimal, covariance = curve_fit(mu_theory, z, m, p0=p0, sigma=m_e, absolute_
sigma=True)
H0_fit, Omega_m_fit = p_optimal
H0_err, Omega_m_err = np.sqrt(np.diag(covariance))

print(f"Fitted H0 = {H0_fit:.2f} ± {H0_err:.2f} km/s/Mpc")
print(f"Fitted Omega_m = {Omega_m_fit:.3f} ± {Omega_m_err:.3f}")
```

Fitted H0 = 72.97 ± 0.26 km/s/Mpc  
Fitted Omega\_m = 0.351 ± 0.019

## Estimate the Age of the Universe

Now that we have the best-fit values of  $H_0$  and  $\Omega_m$ , we can estimate the age of the universe. This is done by integrating the inverse of the Hubble parameter over redshift:

$$t_0 = \int_0^\infty \frac{1}{(1+z)H(z)} dz$$

We convert  $H_0$  to SI units and express the result in gigayears (Gyr). This provides an independent check on our cosmological model by comparing the estimated age to values from other probes like Planck CMB measurements.

In [687]: # Write the function for age of the universe as above

```
def age_of_universe(H0, Omega_m):
 integral = lambda z_: 1 / ((1 + z_) * E(z_, Omega_m))
 integral, _ = quad(integral, 0, np.inf) # Using quad function to integrate the equation
 H0_si = H0 * u.km / (u.s * u.Mpc) # Convert H0 to SI units
 t0 = integral / H0_si.to('1/s').value / (3600 * 24 * 365.25 * 1e9) # Convert to Gyr
 return t0 # in Gyr

t0 = age_of_universe(H0_fit, Omega_m_fit)
print(f"Estimated age of Universe: {t0:.2f} Gyr")
```

Estimated age of Universe: 12.36 Gyr

## Analyze Residuals

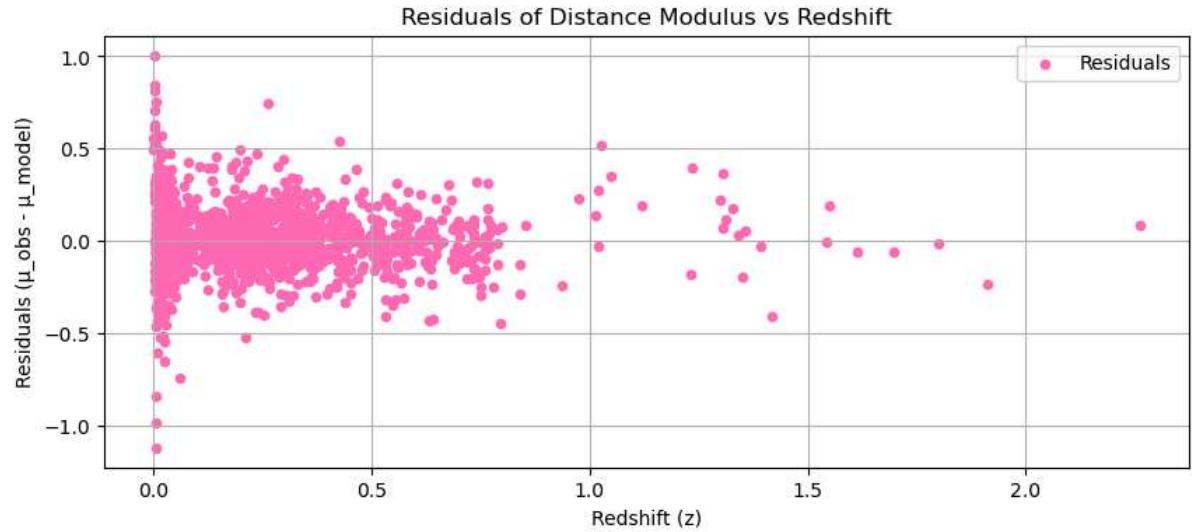
To evaluate how well our cosmological model fits the data, we compute the residuals:

$$\text{Residual} = \mu_{\text{obs}} - \mu_{\text{model}}$$

Plotting these residuals against redshift helps identify any systematic trends, biases, or outliers. A good model fit should show residuals scattered randomly around zero without any significant structure.

```
In [632]: # Write the code to find residual by computing mu_theory and then plot
mu_model = mu_theory(z, H0_fit, Omega_m_fit) # Compute the theoretical distance modulus

Calculate residuals
residuals = m - mu_model
Plot residuals
plt.figure(figsize=(10, 4))
plt.grid(True)
plt.scatter(z, residuals, label='Residuals', color='hotpink', s=20)
plt.legend()
plt.xlabel('Redshift (z)')
plt.ylabel('Residuals ($\mu_{\text{obs}} - \mu_{\text{model}}$)')
plt.title('Residuals of Distance Modulus vs Redshift')
plt.show()
```



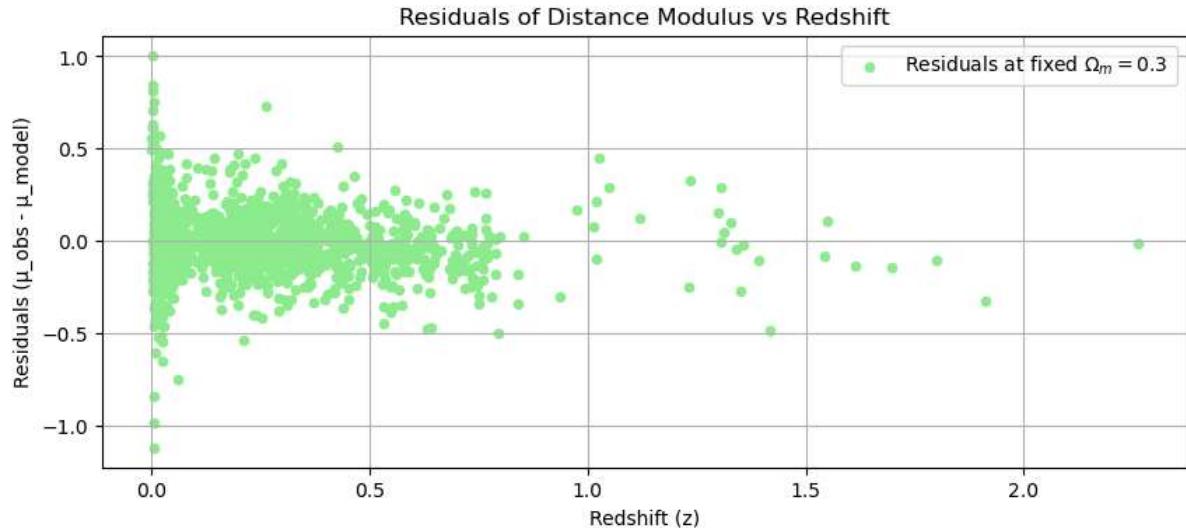
## 🔧 Fit with Fixed Matter Density

To reduce parameter degeneracy, let's fix  $\Omega_m = 0.3$  and fit only for the Hubble constant  $H_0$ .

```
In [688]: def mu_fixed_Om(z, H0):
 return mu_theory(z, H0, Omega_m=0.3)
residuals = m - mu_fixed_Om(z, H0_fit)

Try fitting with this fixed value 'Omega_m=0.3'
plt.figure(figsize=(10, 4))
plt.grid(True)
plt.scatter(z, residuals, label='Residuals at fixed $ \Omega_m = 0.3 $ ', color='lightgreen', s=20)
plt.legend()
plt.xlabel('Redshift (z)')
plt.ylabel('Residuals ($\mu_{\text{obs}} - \mu_{\text{model}}$)')
plt.title('Residuals of Distance Modulus vs Redshift')
plt.show()

<>:8: SyntaxWarning: invalid escape sequence '\0'
<>:8: SyntaxWarning: invalid escape sequence '\0'
C:\Users\asus\AppData\Local\Temp\ipykernel_8580\263392893.py:8: SyntaxWarning: invalid escape sequence '\0'
 plt.scatter(z, residuals, label='Residuals at fixed $ \Omega_m = 0.3 $ ', color='lightgreen', s=20)
```



## 🔍 Compare Low-z and High-z Subsamples

Finally, we examine whether the inferred value of  $H_0$  changes with redshift by splitting the dataset into:

- **Low-z** supernovae ( $z < 0.1$ )
- **High-z** supernovae ( $z \geq 0.1$ )

We then fit each subset separately (keeping  $\Omega_m = 0.3$ ) to explore any potential tension or trend with redshift.

```
In []: # Split the data for the three columns and do the fitting again and see
z_split = 0.1

Calculate H0 for low and high redshift by using curve_fit function
H0_low, cov_low = curve_fit(mu_theory, z[z < z_split], m[z < z_split], p0=p0,
sigma=m_e[z < z_split], absolute_sigma=True)
H0_high, cov_high = curve_fit(mu_theory, z[z >= z_split], m[z >= z_split], p0=p0,
sigma=m_e[z >= z_split], absolute_sigma=True)

Calculate the errors(standard deviation) for the low and high redshift
H0_low_err = np.sqrt(cov_low[0][0])
H0_high_err = np.sqrt(cov_high[0][0])

print the results
print(f"Low-z (z < {z_split}): H0 = {H0_low[0]:.2f} ± {H0_low_err:.2f} km/s/Mpc")
print(f"High-z (z ≥ {z_split}): H0 = {H0_high[0]:.2f} ± {H0_high_err:.2f} km/
s/Mpc")
```

Low-z ( $z < 0.1$ ):  $H_0 = 72.74 \pm 0.59$  km/s/Mpc  
 High-z ( $z \geq 0.1$ ):  $H_0 = 73.18 \pm 0.50$  km/s/Mpc

You can check your results and potential reasons for different values from accepted constant using this paper by authors of the [Pantheon+ dataset](https://arxiv.org/pdf/2310.11727.pdf) (<https://arxiv.org/pdf/2310.11727.pdf>).

You can find more about the dataset in the paper too

```
In [677]: # Hubble diagram with model fit
plt.figure()
plt.errorbar(z, m, m_e ,color='orange', label='Observed Data')
mu_model = mu_theory(z, H0_fit, Omega_m_fit)
plt.plot(z, mu_model, label='Model Fit', color='black')
plt.grid(True)
plt.xlabel('Redshift (z)')
plt.ylabel('Distance Modulus')
plt.title('Hubble diagram with Model fit')
plt.xscale('log')
plt.legend()
plt.show()
```

