Data Structure & Algorithms　Homework 3

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#All scripts tested under Python 3.5 environment#

Q1 Develop an implementation of the basic symbol-table API that uses 2-3 trees that are not necessarily balanced as the underlying data structure. Allow 3-nodes to lean either way. Hook the new node onto the bottom with a black link when inserting into a 3-node at the bottom.

Solution:

Instead of using a 2-3 tree, I choose to apply a Red-Black tree for implementation. Reason being: a typical 2-3 tree has no such property as "lean either way"(because they already are) and "black link"(this makes sense only in RB tree). In my implementation, RB-tree is implemented as a lite version:

```
class RBT_Lite(object):
```

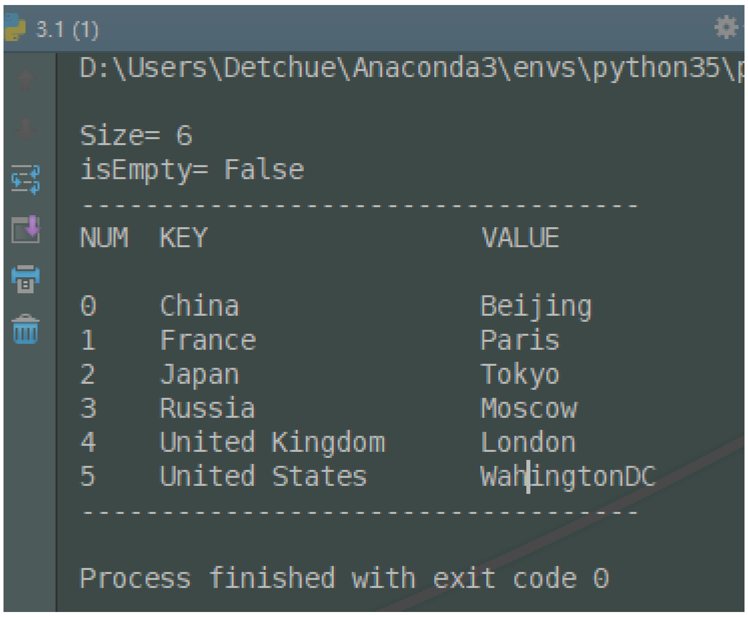
It does not rotate when inserting, does not check if red links are properly placed as in a real RB tree. It is so a unbalanced tree, but still works.

To implement API for symbol table, I wrote another class:

```
class ST(object):
```

which generates a table for itself while initializing.

Several functions have been implemented for symbol table, and finally a summary function is used to easily check out most functions.



```
D:\Users\Detchue\Anaconda3\envs\python35\

Size= 6
isEmpty= False
-----------------------------------
NUM  KEY                 VALUE

0    China               Beijing
1    France              Paris
2    Japan               Tokyo
3    Russia              Moscow
4    United Kingdom      London
5    United States       WahingtonDC
-----------------------------------

Process finished with exit code 0
```

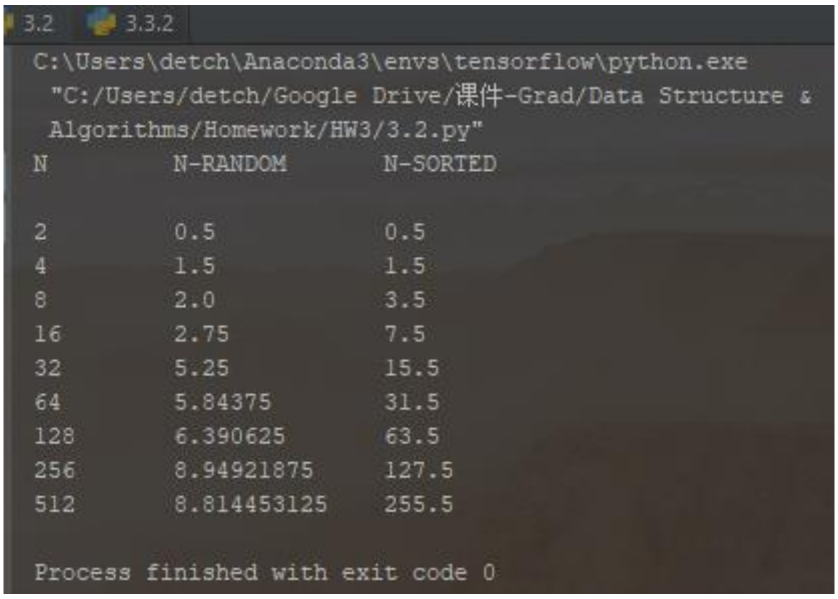
Q2. Run experiments to develop a hypothesis estimating the average path length in a tree built from (i) N-random insertions. (ii) N-sorted insertions?

Solution:

Since no tree type has been appointed here, I turn to choose BST for implementation.

Apparently, N-sorted case is the worst case for a BST, because it will generate a whole list of items which requires iteration from root (very beginning) to find a specific item. And because of that, the average path length equals to **half of tree size**, which can be told from my result shown below.

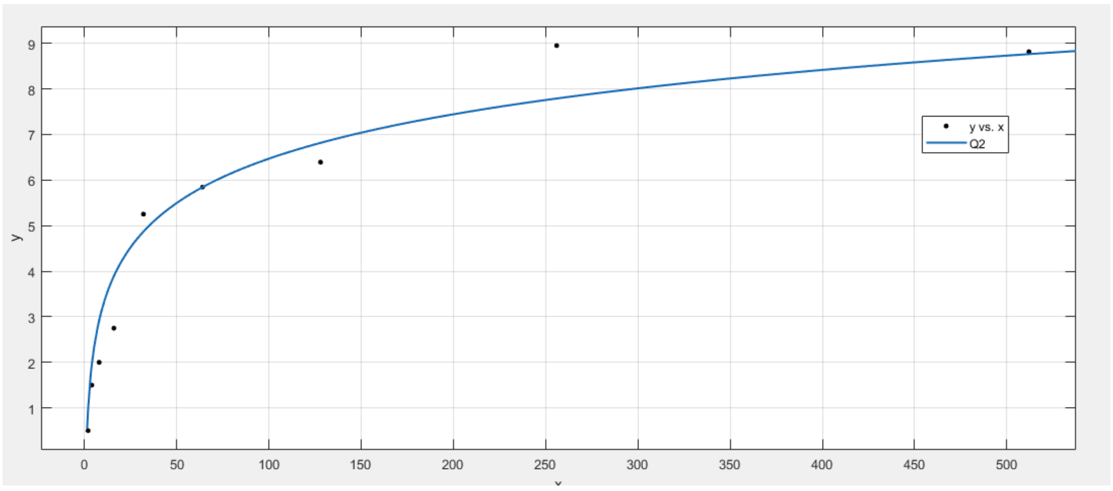
One more thing to be mentioned: the implementation entails recursion for tree traversal, and because the tree generated in N-sorted case is so bad, the tree size can not be more than 999, or python interpreter will raise error. This is the reason why my test data size is so small.



```
C:\Users\detch\Anaconda3\envs\tensorflow\python.exe
 "C:/Users/detch/Google Drive/课件-Grad/Data Structure &
 Algorithms/Homework/HW3/3.2.py"
N          N-RANDOM       N-SORTED

2          0.5            0.5
4          1.5            1.5
8          2.0            3.5
16         2.75           7.5
32         5.25           15.5
64         5.84375        31.5
128        6.390625       63.5
256        8.94921875     127.5
512        8.814453125    255.5

Process finished with exit code 0
```



Curve fitting is based on Matlab-R2017b Curve Fitting Tool.

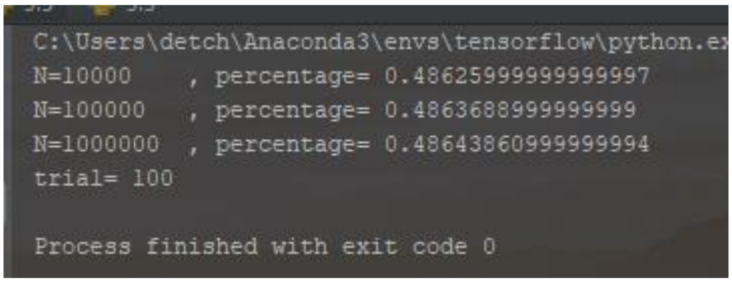
y(N-random) ≈ 1.405log(x)

y(N-sorted) = 0.5N

Q3. Write a program that computes the percentage of red nodes in a given red-black tree. Test program by running at least 100 trials of the experiment of increasing N random keys into an initially empty tree for N=10^4, 10^5 and 10^6 and formulate a hypothesis.

Solution:

In a typical RBT, the ratio of red and black nodes is be generally similar according to experience. (Not counting nil sons of course.) So the most possible percentage in most common cases should be around 0.5. This is well demonstrated in my first screenshot, where random list is used as data.

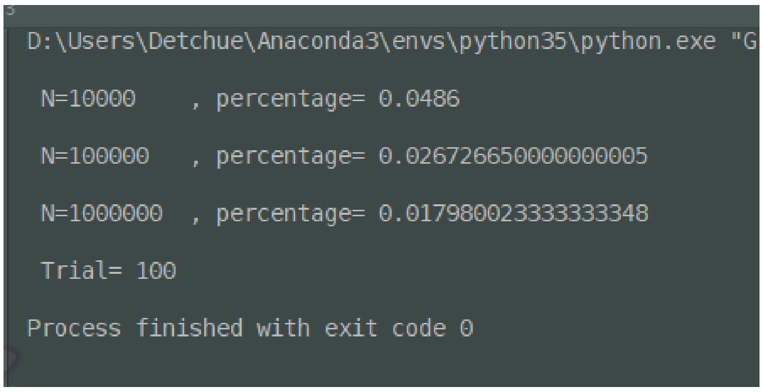


```
C:\Users\detch\Anaconda3\envs\tensorflow\python.ex
N=10000    , percentage= 0.48625999999999997
N=100000   , percentage= 0.4863688999999999
N=1000000  , percentage= 0.48643860999999994
trial= 100

Process finished with exit code 0
```

Running with randomly-generated data set

Then again since we are requested to use given data from **Sakai** for calculation, I used it with slight modification in formats. Because of the **highly frequent duplication** in given file, the result looks rather **uncommon**. I choose to ignore this piece of result, believing that it shows no valid trends.



```
D:\Users\Detchue\Anaconda3\envs\python35\python.exe "G

 N=10000    , percentage= 0.0486

 N=100000   , percentage= 0.026726650000000005

 N=1000000  , percentage= 0.017980023333333348

 Trial= 100

Process finished with exit code 0
```

Running with data set provided (with more trials and higher accuracy)

#not taken into consideration when forming hypothesis

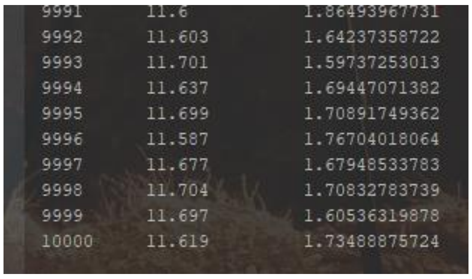
hypothesis:

In a Red-Black Tree, the quantity of red and black nodes are most likely similar to each other in numbers. But black nodes are slightly more than red nodes, around 3% more.

Q4. Run empirical studies to compute the average and std deviation of the average length of a path to a random node (internal path length divided by tree size) in a red-black BST built by insertion of N random keys into an initially empty tree, for N from 1 to 10,000. Do at least 1,000 trials for each size.

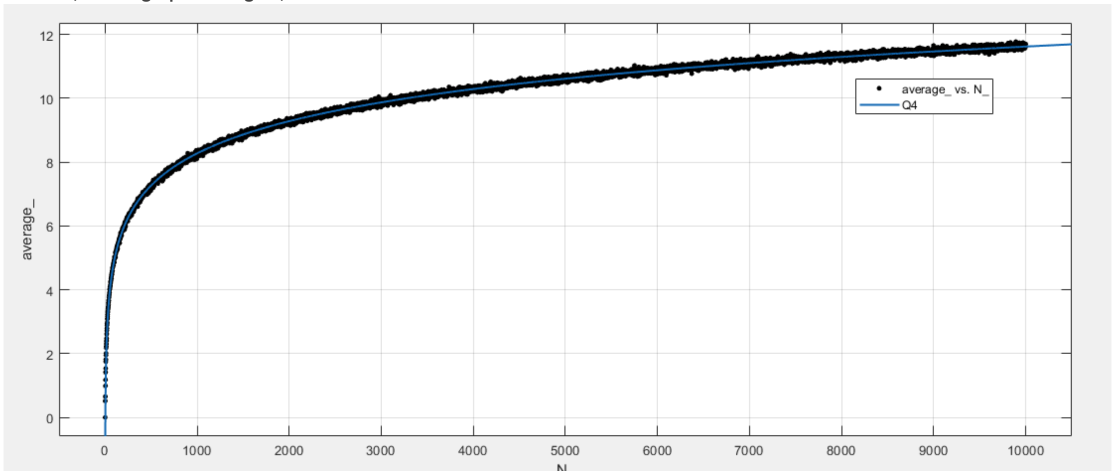
Solution:

Results are printed in txt file (3.4_result_1-10000.txt) attached. A screenshot of last several results are shown below.



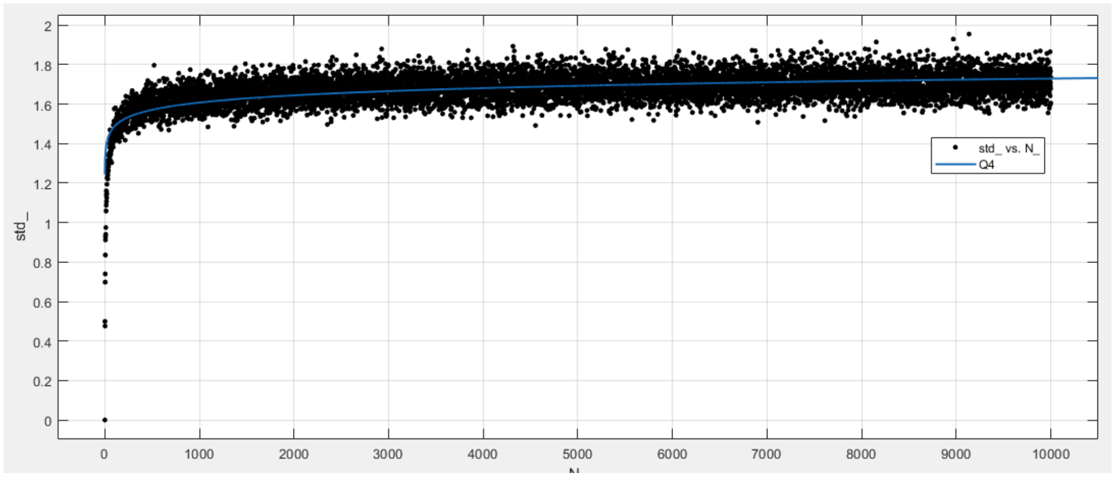
```
9991      11.6          1.86493967731
9992      11.603        1.64237358722
9993      11.701        1.59737253013
9994      11.637        1.69447071382
9995      11.699        1.70891749362
9996      11.587        1.76704018064
9997      11.677        1.67948533783
9998      11.704        1.70832783739
9999      11.697        1.60536319878
10000     11.619        1.73488875724
```

'N-size', 'Average path length', 'STD Deviation'



Average vs N

Average ≈ 1.453*ln(x) - 1.768



std-deviation vs N

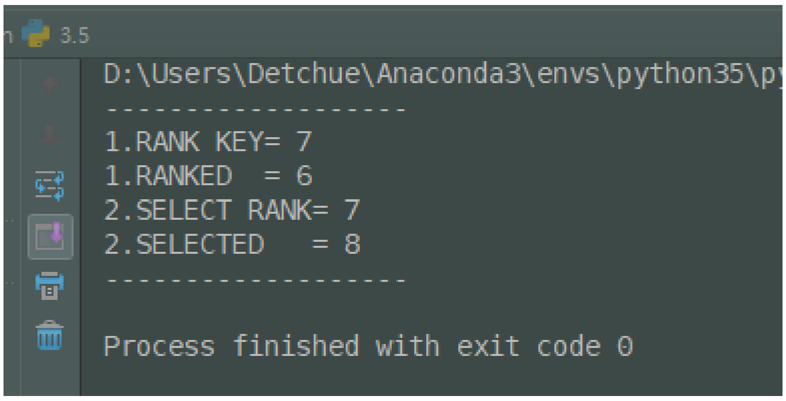
std goes quite stable after N goes beyond 1000.

Q5. Implement the rank() and select() ordered operations for a BST. Use data set linked below. (i) What is the value of select(7) for the data set? (ii) What is the value of rank(7) for the data set?

Data Set for Q3 is: https://sakai.rutgers.edu/access/content/group/f73f2fd4-280d-4e7c-8cf2-9cc34bcffcff/HW-DataSet/select-data.txt

Solution :

Since the given data file does not show high variety (but with high duplication), the final tree is pretty much like a range() list. This explains why whatever rank-key or select-key is given, the corresponding value is close to the given key. (e.g. here rank(7)=6, select(7)=8)



```
D:\Users\Detchue\Anaconda3\envs\python35\p
-------------------
1.RANK KEY= 7
1.RANKED   = 6
2.SELECT RANK= 7
2.SELECTED    = 8
-------------------

Process finished with exit code 0
```

rank(7)=6 and select(7)=8