Aprendizaje Reforzado

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Preface

This is a Quarto book.

To learn more about Quarto books visit https://quarto.org/docs/books.

```
import numpy as np

x = 10
```

Ejercicio 4

Para el caso infinito. Estamos considerando

$$c\left(x,a\right)=a^{1-\gamma}$$

Entonces

$$\nu\left(x\right)=\max_{a\in A(x)}\left\{a^{1-\gamma}+\beta\nu\left(\left(1+i\right)\left(x-a\right)\right)\right\},$$

considerando $\nu\left(x\right)=cx^{1-\gamma}$. Entonces

$$\nu\left(x\right)=\max_{a\in A(x)}\left\{a^{1-\gamma}+\beta c\left[\left(1+i\right)\left(x-a\right)\right]^{1-\gamma}\right\},$$

definimos

$$q\left(x,a\right)=a^{1-\gamma}+\beta c\left[\left(1+i\right)\left(x-a\right)\right]^{1-\gamma},$$

entonces

$$\partial_{a}q=\left(1-\gamma\right)a^{-\gamma}+\beta c\left(1-\gamma\right)\left(1+i\right)^{1-\gamma}\left(-1\right)\left(x-a\right)^{-\gamma}.$$

Si $\partial_{a}q=0$. Entonces

$$\begin{aligned}
a^{-\gamma} &= \beta c\left(1+i\right)^{1-\gamma}\left(x-a\right)^{-\gamma}\\
\left(\beta c\left(1+i\right)^{1-\gamma}\right)^{-1} &= \left(\frac{x-a}{a}\right)^{-\gamma}\\
\beta^{-1}c^{-1}\left(1+i\right)^{\gamma-1} &= \left(\frac{x}{a}-1\right)^{-\gamma}\\
\left[\beta^{-1}c^{-1}\left(1+i\right)^{\gamma-1}\right]^{-\frac{1}{\gamma}}+1 &= \frac{x}{a}
\end{aligned}$$

Por lo tanto

$$\begin{aligned} a &= \frac{x}{\left[\beta^{-1}c^{-1}\left(1+i\right)^{\gamma-1}\right]^{-\frac{1}{\gamma}}+1} \\ &= \frac{x}{\left[\beta c\left(1+i\right)^{1-\gamma}\right]^{\frac{1}{\gamma}}+1} \end{aligned}$$

Ahora remplazamos en q

$$\begin{aligned} \nu\left(x\right) &= \left(\frac{x}{\left[\beta c\left(1+i\right)^{1-\gamma}\right]^{\frac{1}{\gamma}}+1}\right)^{1-\gamma}+\beta c\left[\left(1+i\right)\left(x-\frac{x}{\left[\beta c\left(1+i\right)^{1-\gamma}\right]^{\frac{1}{\gamma}}+1}\right)\right]^{1-\gamma} \\ &= x^{1-\gamma}\left(\frac{1}{\left[\beta c\left(1+i\right)^{1-\gamma}\right]^{\frac{1}{\gamma}}+1}\right)+x^{1-\gamma}\left(1+i\right)^{1-\gamma}\beta c\left(1-\frac{1}{\left[\beta c\left(1+i\right)^{1-\gamma}\right]^{\frac{1}{\gamma}}+1}\right)^{1-\gamma} \\ &= x^{1-\gamma}\left[\left(\frac{1}{\left[\beta c\left(1+i\right)^{1-\gamma}\right]^{\frac{1}{\gamma}}+1}\right)+\left(1+i\right)^{1-\gamma}\beta c\left(1-\frac{1}{\left[\beta c\left(1+i\right)^{1-\gamma}\right]^{\frac{1}{\gamma}}+1}\right)^{1-\gamma}\right]. \end{aligned}$$

Entonces

$$
\begin{aligned}
c &= \left(\frac{1}{\left[\beta c\left(1+i\right)^{1-\gamma}\right]^{\frac{1}{\gamma}}+1}\right)^{1-\gamma}+\left(1+i\right)^{1-\gamma}\beta c\left(1-\frac{1}{\left[\beta c\left(1+i\right)^{1-\gamma}\right]^{\frac{1}{\gamma}}+1}\right)^{1-\gamma} \\
&= \left(\frac{1}{\left[\beta c\left(1+i\right)^{1-\gamma}\right]^{\frac{1}{\gamma}}+1}\right)^{1-\gamma}+\left(1+i\right)^{1-\gamma}\beta c\left(\frac{\left[\beta c\left(1+i\right)^{1-\gamma}\right]^{\frac{1}{\gamma}}}{\left[\beta c\left(1+i\right)^{1-\gamma}\right]^{\frac{1}{\gamma}}+1}\right)^{1-\gamma} \\
&= \left(\frac{1}{\left[\beta c\left(1+i\right)^{1-\gamma}\right]^{\frac{1}{\gamma}}+1}\right)^{1-\gamma}\left(1+\left(1+i\right)^{1-\gamma}\beta c\left(\left[\beta c\left(1+i\right)^{1-\gamma}\right]^{\frac{1}{\gamma}}\right)^{1-\gamma}\right) \\
&= \left(\frac{1}{\left[\beta c\left(1+i\right)^{1-\gamma}\right]^{\frac{1}{\gamma}}+1}\right)^{1-\gamma}\left(1+\left(1+i\right)^{1-\gamma}\beta c\left[\beta c\left(1+i\right)^{1-\gamma}\right]^{\frac{1}{\gamma}-1}\right) \\
&= \left(\frac{1}{\left[\beta c\left(1+i\right)^{1-\gamma}\right]^{\frac{1}{\gamma}}+1}\right)^{1-\gamma}\left(1+\left[\beta c\left(1+i\right)^{1-\gamma}\right]^{\frac{1}{\gamma}}\right) \\
&= \left(\left[\beta c\left(1+i\right)^{1-\gamma}\right]^{\frac{1}{\gamma}}+1\right)^{\gamma-1}\left(1+\left[\beta c\left(1+i\right)^{1-\gamma}\right]^{\frac{1}{\gamma}}\right) \\
c &= \left(\left[\beta c\left(1+i\right)^{1-\gamma}\right]^{\frac{1}{\gamma}}+1\right)^{\gamma}
\end{aligned}
$$

Ahora, nos queda despejar c.

$$
\begin{aligned}
c^{\frac{1}{\gamma}} &= \beta^{\frac{1}{\gamma}}c^{\frac{1}{\gamma}}\left(1+i\right)^{\frac{1}{\gamma}-1}+1 \\
1 &= \beta^{\frac{1}{\gamma}}\left(1+i\right)^{\frac{1}{\gamma}-1}+c^{-\frac{1}{\gamma}} \\
c^{-\frac{1}{\gamma}} &= 1-\beta^{\frac{1}{\gamma}}\left(1+i\right)^{\frac{1}{\gamma}-1} \\
c &= \left(1-\beta^{\frac{1}{\gamma}}\left(1+i\right)^{\frac{1}{\gamma}-1}\right)^{-\gamma}
\end{aligned}
$$

Ejercicio 6

$$
\begin{aligned}
\nu\left(x\right) &= \max_{a\in A(x)}\left\{c\left(x,a\right)+\nu\left(f\left(x,a\right)\right)\right\} \\
&= \max_{a\in A(x)}\left\{a^2+x^2+E\left[\nu\left(x+a+\xi\right)\right]\right\}
\end{aligned}
$$

Para $\nu(x) = ax^2 + b$

$$\begin{aligned}
\nu(x) &= \max_{a\in A(x)} \{c(x,a) + \beta E[\nu(f(x,a))]\} \\
&= \max_{a\in A(x)} \left\{A^2 + x^2 + \beta\left(E\left[a\left(f^2(x,a)\right)\right] + b\right)\right\} \\
&= \max_{a\in A(x)} \left\{A^2 + x^2 + \beta\left(aE\left[f^2(x,a)\right] + b\right)\right\}
\end{aligned}$$

Notemos que

$$\begin{aligned}
E\left[f^2(x,a)\right] &= E\left[(x + A + \xi)^2\right] \\
&= E\left[x^2 + A^2 + \xi^2 + 2xA + 2x\xi + 2\xi A\right] \\
&= x^2 + A^2 + E\left[\xi^2\right] + 2xA + 2xE[\xi] + 2AE[\xi] \\
&= x^2 + A^2 + d + 2xA
\end{aligned}$$

Entonces

$$\begin{aligned}
ax^2 + b &= \max_{a\in A(x)} \left\{A^2 + x^2 + \beta\left[a\left(x^2 + A^2 + d + 2xA\right) + b\right]\right\} \\
&= \max_{a\in A(x)} \left\{A^2 + x^2 + \beta a\left(x^2 + A^2 + d + 2xA\right) + \beta b\right\} \\
&= \max_{a\in A(x)} \left\{A^2 + x^2 + \beta ax^2 + \beta aA^2 + \beta ad + 2\beta axA + \beta b\right\} \\
&= \max_{a\in A(x)} \left\{A^2(\beta a + 1) + 2\beta axA + x^2 + \beta ax^2 + \beta ad + \beta b\right\}
\end{aligned}$$

Definimos

$$w(x,A) = A^2(\beta a + 1) + 2\beta axA + x^2 + \beta ax^2 + \beta ad + \beta b,$$

entonces

$$\partial_A w = 2A(\beta a + 1) + 2\beta ax.$$

Si $\partial_A w = 0$, entonces

$$A = -\frac{\beta ax}{\beta a + 1}$$

Entonces

$$\begin{aligned}
\nu(x) &= (\beta ax)^2 - 2\frac{(\beta ax)^2}{\beta a + 1} + x^2 + \beta ax^2 + \beta ad + \beta b \\
&= x^2\left([\beta a]^2 - 2\frac{(\beta a)^2}{\beta a + 1} + 1 + \beta a\right) + \beta ad + \beta b
\end{aligned}$$

Entonces

$$
\begin{aligned}
a &= [\beta a]^2 - 2\frac{(\beta a)^2}{\beta a + 1} + 1 + \beta a \\
b &= \beta a d + \beta b,
\end{aligned}
$$

de forma rapida

$$
b = \frac{\beta a d}{1 - \beta},
$$

entonces queda pendiente calcular a

$$
\begin{aligned}
a &= [\beta a]^2 - 2\frac{(\beta a)^2}{\beta a + 1} + 1 + \beta a. \\
0 &= (\beta a)^2 \left(1 - \frac{2}{\beta a + 1}\right) + 1 + (\beta - 1)\, a \\
&= (\beta a)^2 (\beta a + 1 - 2) + \beta a + 1 + (a\beta - a)(\beta a + 1) \\
&= (\beta a)^2 (\beta a - 1) + \beta a + 1 + \left[(a\beta)^2 + a\beta - \beta a^2 - a\right] \\
&= (\beta a)^3 + 2a\beta + 1 - \beta a^2 - a \\
&= \beta^3 a^3 - \beta a^2 + (2\beta - 1)\, a + 1
\end{aligned}
$$

1 Introduction

This is a book created from markdown and executable code.

See Knuth (1984) for additional discussion of literate programming.

2 Summary

In summary, this book has no content whatsoever.

References

Knuth, Donald E. 1984. "Literate Programming." *Comput. J.* 27 (2): 97–111. https://doi.org/10.1093/comjnl/27.2.97.