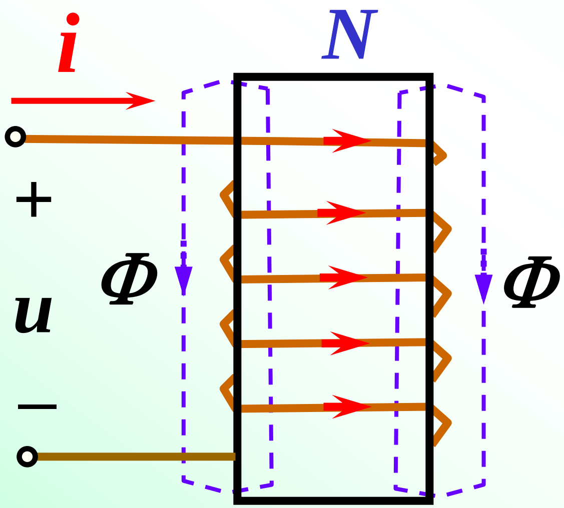
第11章 耦合电感和理想变压器

第十一章 耦合电感和理想变压器

§ 11-1 基本概念

回顾电感元件

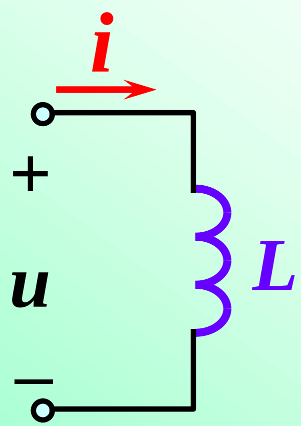


磁链 $\psi = N\Phi$

自电感 $L = \dfrac{N\Phi}{i}$

单位：Φ—韦伯（Wb）

L— 亨利（H）



u和 i 关联参考方向，自感电压为

$$u = \frac{\mathrm{d}\Psi}{\mathrm{d}t} = L\frac{\mathrm{d}i}{\mathrm{d}t}$$

一、耦合电感

自感磁链

$$\psi_{L1}=N_1\Phi_{11}=L_1 i_1$$

互感磁链

$$\psi_M=N_2\Phi_{21}=M i_1$$

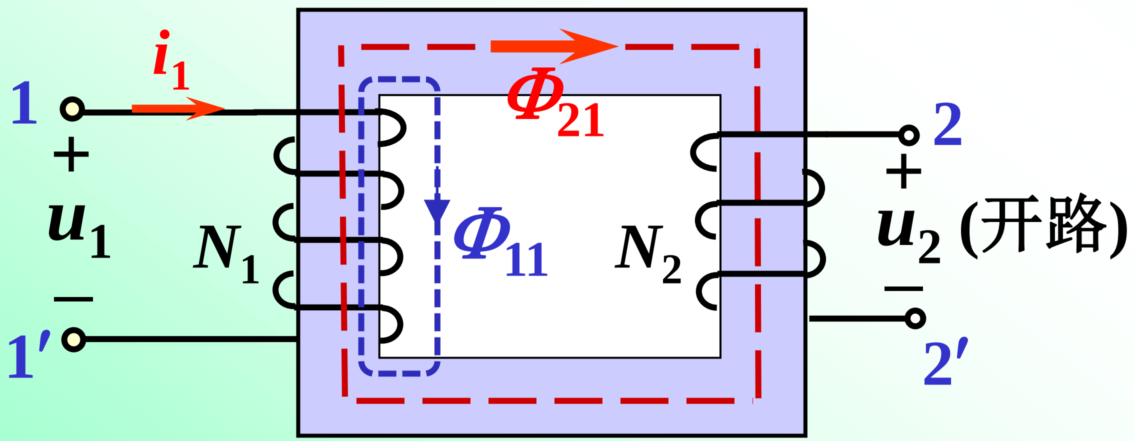
自感系数（自感）

$$L_1=\frac{N_1\Phi_{11}}{i_1}$$

互感系数（互感）

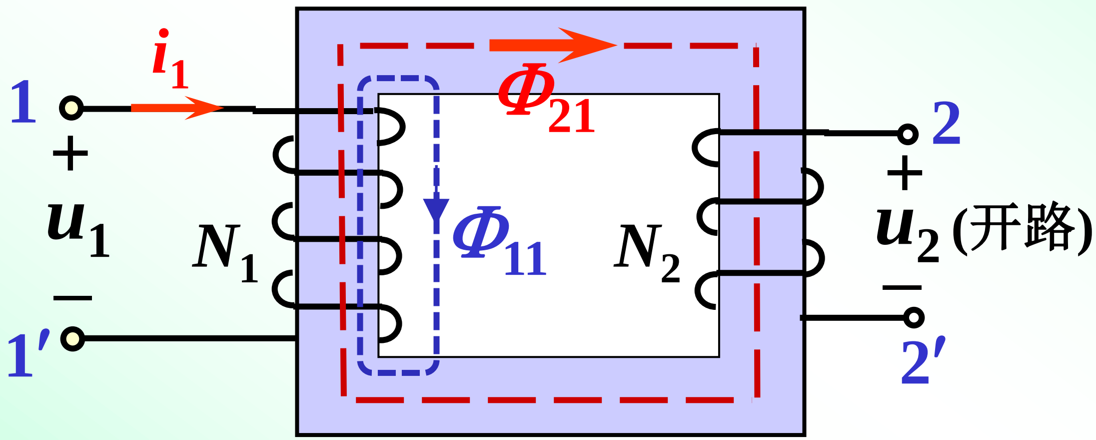
$$M=\frac{N_2\Phi_{21}}{i_1}$$

单位: H (亨利)

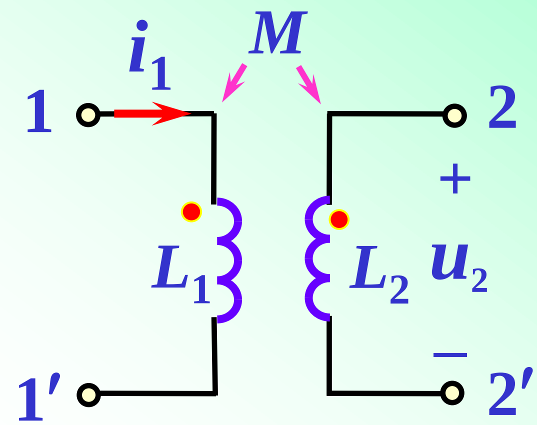


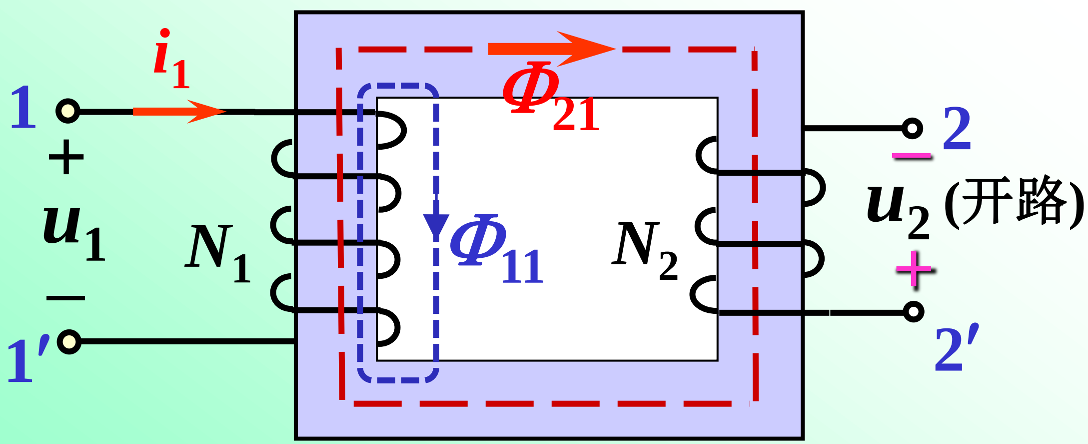
同名端：

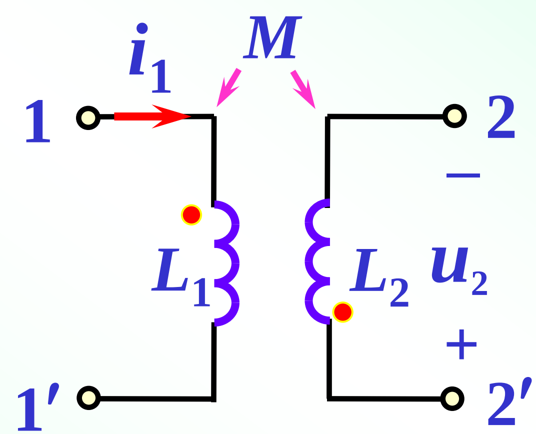
- 产生互感电压的电流流入端
- 互感电压的"+"极端



电路符号



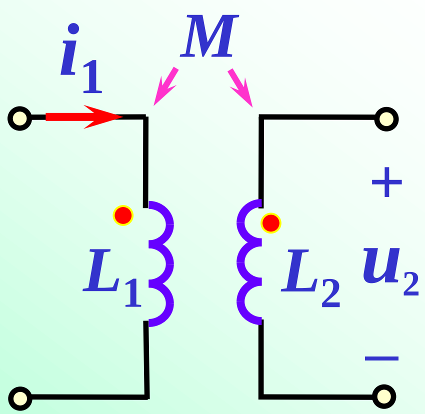




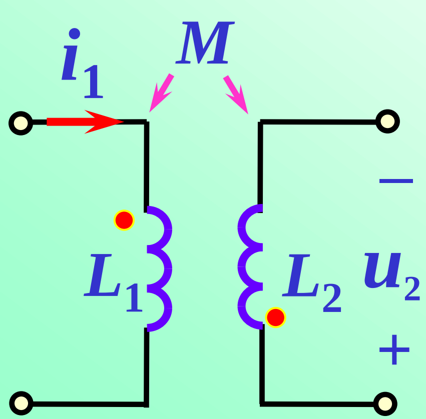
二、 根据同名端确定互感电压的正负

同名端一致： 电流由同名端流入，且互感电压"+ "参考方向在同名端侧

同名端不一致： 电流由同名端流入，且互感电压"+ "参考方向在不同名端侧

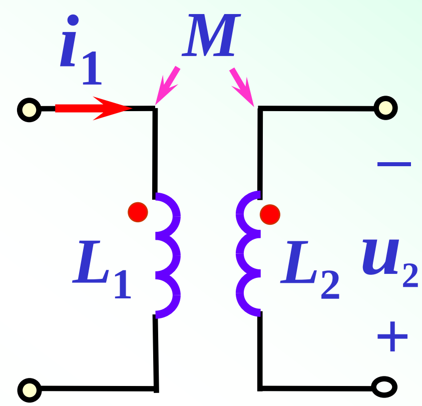


$$u_2 = \frac{\mathrm{d}\psi_M}{\mathrm{d}t} = \frac{\mathrm{d}\,Mi_1}{\mathrm{d}t} = M\frac{\mathrm{d}i_1}{\mathrm{d}t}$$



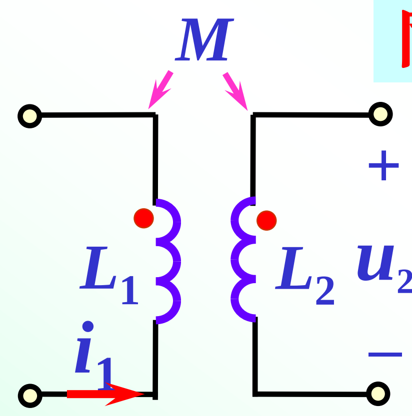
同名端一致

$$u_2 = M\frac{\mathrm{d}i_1}{\mathrm{d}t}$$



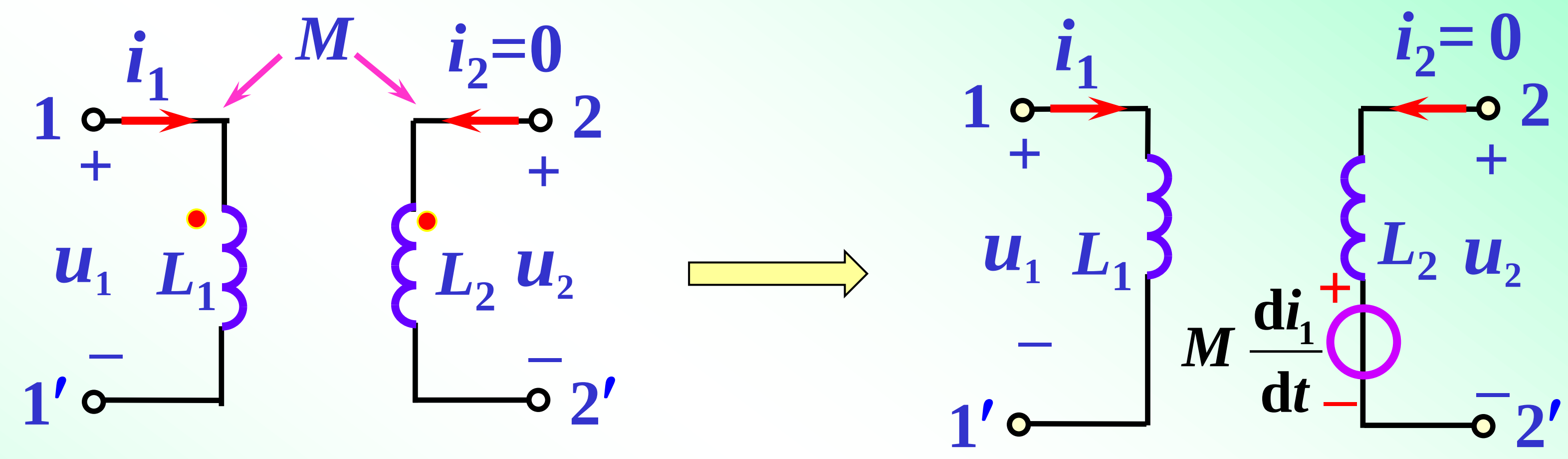
$$u_2 = -M\frac{\mathrm{d}i_1}{\mathrm{d}t}$$

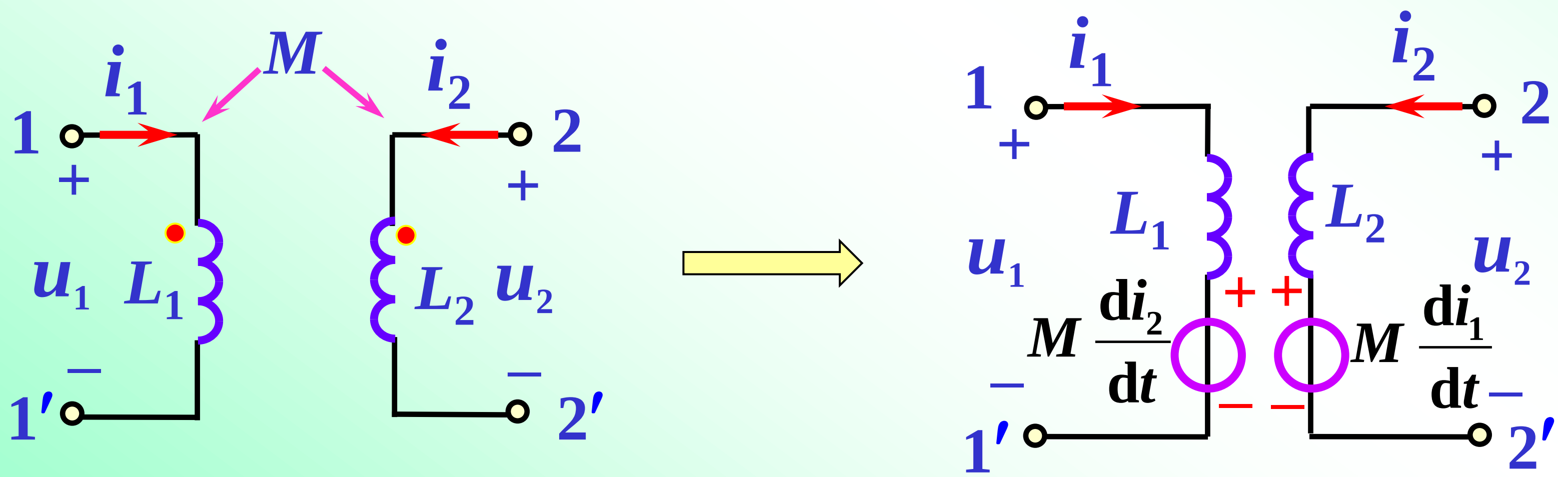
同名端不一致



$$u_2 = -M\frac{\mathrm{d}i_1}{\mathrm{d}t}$$

三、互感电压用附加的电压源代替



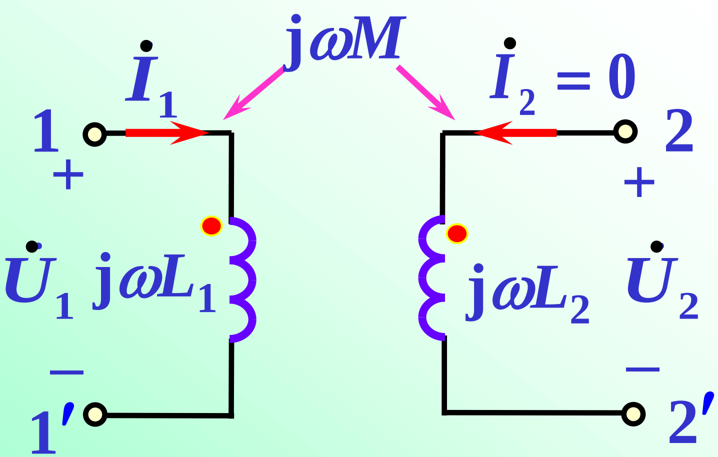


四、耦合电感及附加电压源的相量模型

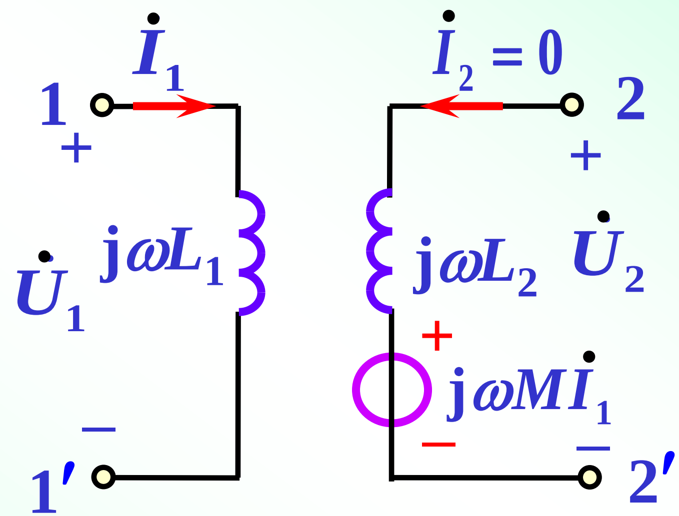
对于正弦稳态电路, 可采用相量模型。

在相量模型中，电路参数 $L \rightarrow \mathrm{j}\omega L$, $M \rightarrow \boxed{\mathrm{j}\omega M}$

$$u_1 = L\frac{di_1}{dt} \rightarrow \dot{U}_1 = \mathrm{j}\omega L\dot{I}_1, \quad u_2 = M\frac{di_1}{dt} \rightarrow \boxed{\dot{U}_2 = \mathrm{j}\omega M\dot{I}_1}$$



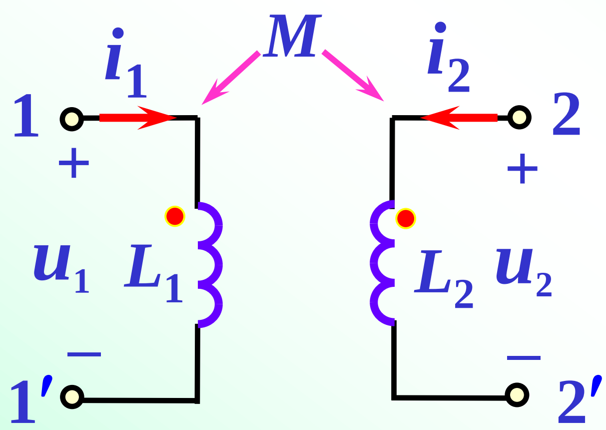
耦合电感相量模型



用附加电压源计及互感的相量模型

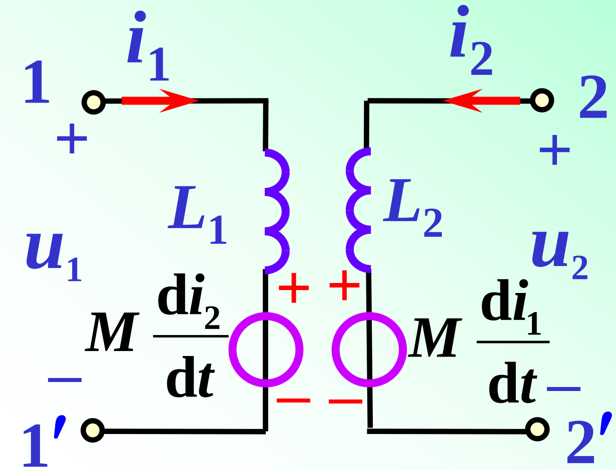
§ 11-2 耦合电感的VCR　耦合系数

一、耦合电感的VCR

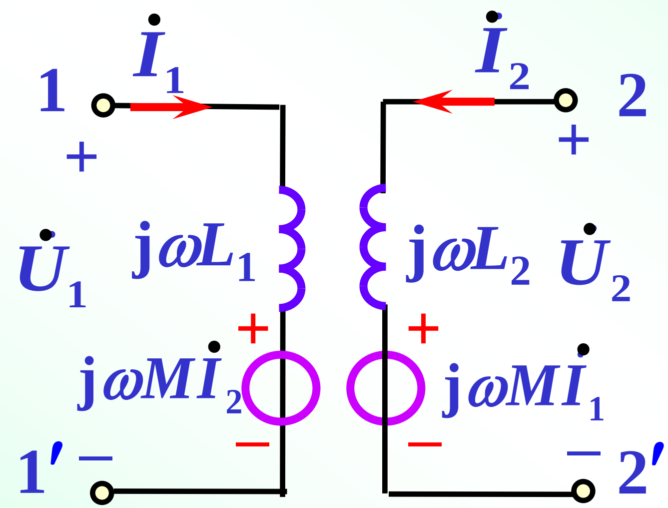


互感电压用附加

的电压源代替



相量模型



$$u_1 = L_1\frac{di_1}{dt} + M\frac{di_2}{dt}$$

$$u_2 = L_2\frac{di_2}{dt} + M\frac{di_1}{dt}$$

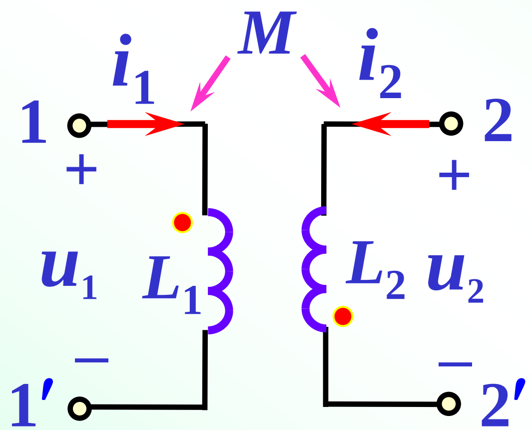
耦合电感的VCR

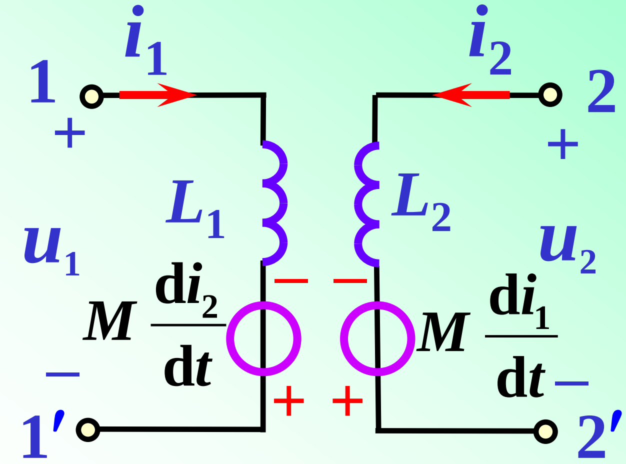
正弦稳态相量形式VCR

$$\dot U_1 = j\omega L_1\dot I_1 + j\omega M\dot I_2$$

$$\dot U_2 = j\omega \dot L_2 I_2 + j\omega M\dot I_1$$

同名端不一致时

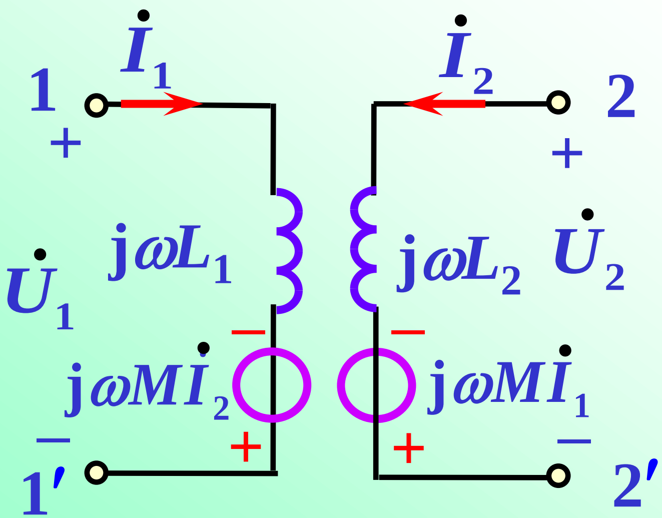




VCR

$$u_1 = L_1\frac{\mathrm{d}i_1}{\mathrm{d}t} - M\frac{\mathrm{d}i_2}{\mathrm{d}t}$$

$$u_2 = L_2\frac{\mathrm{d}i_2}{\mathrm{d}t} - M\frac{\mathrm{d}i_1}{\mathrm{d}t}$$

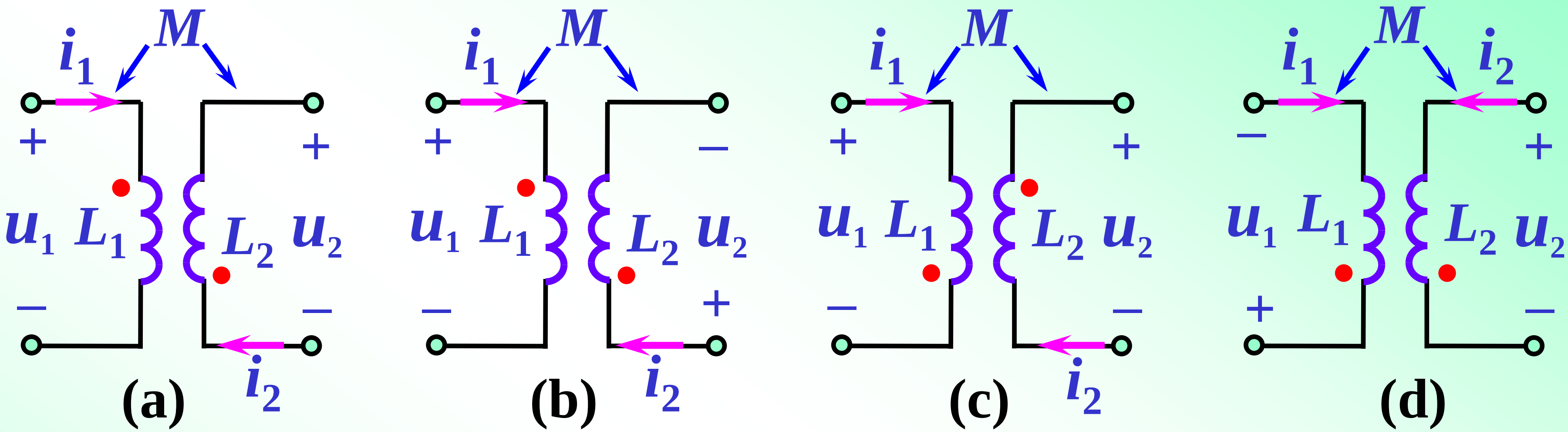


相量VCR

$$\dot{U}_1 = j\omega L_1\dot{I}_1 - j\omega M\dot{I}_2$$

$$\dot{U}_2 = j\omega L_2\dot{I}_2 - j\omega M\dot{I}_1$$

练习题11-4 试写出下列4个电路的VCR。



(a) $u_1 = L_1 \dfrac{di_1}{dt} + M \dfrac{di_2}{dt}$

$u_2 = -L_2 \dfrac{di_2}{dt} - M \dfrac{di_1}{dt}$

(b) $u_1 = L_1 \dfrac{di_1}{dt} + M \dfrac{di_2}{dt}$

$u_2 = L_2 \dfrac{di_2}{dt} + M \dfrac{di_1}{dt}$

(c) $u_1 = L_1 \dfrac{di_1}{dt} + M \dfrac{di_2}{dt}$

$u_2 = -L_2 \dfrac{di_2}{dt} - M \dfrac{di_1}{dt}$

(d) $u_1 = -L_1 \dfrac{di_1}{dt} - M \dfrac{di_2}{dt}$

$u_2 = L_2 \dfrac{di_2}{dt} + M \dfrac{di_1}{dt}$

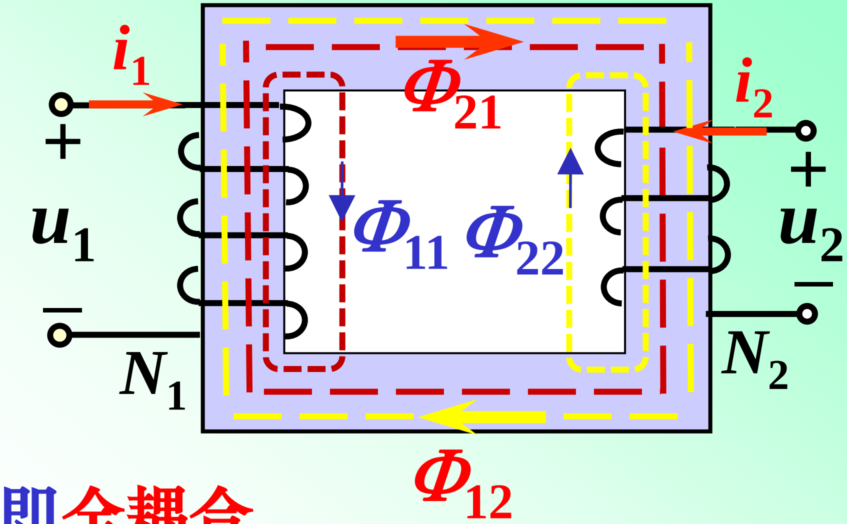
二、耦合系数

i_1：$L_1 \rightarrow$自感磁链 $\Psi_{11}=N_1\Phi_{11}=L_1 i_1$

$L_2 \rightarrow$互感磁链 $\Psi_{21}=N_2\Phi_{21}=Mi_1$

i_2：$L_2 \rightarrow$自感磁链 $\Psi_{22}=N_2\Phi_{22}=L_2 i_2$

$L_1 \rightarrow$互感磁链 $\Psi_{12}=N_1\Phi_{12}=Mi_2$



极限情况：$\Phi_{21}=\Phi_{11}$，$\Phi_{12}=\Phi_{22}$，即全耦合

此时互感M值最大，$M=M_{max}$。

$$L_1L_2=\left(\frac{N_1\Phi_{11}}{i_1}\right)\left(\frac{N_2\Phi_{22}}{i_2}\right)=\left(\frac{N_2\Phi_{21}}{i_1}\right)\left(\frac{N_1\Phi_{12}}{i_2}\right)=M^2 \Rightarrow \boxed{M_{max}=\sqrt{L_1L_2}}$$

定义：M值与M_{max}值之比k称为**耦合系数**。

$$k=\frac{M}{\sqrt{L_1L_2}}$$

用来衡量两线圈耦合程度。

$$0\le k\le 1\begin{cases} k=1 \text{ 全耦合，} k>0.5 \text{ 紧耦合} \\ k<0.5 \text{ 松耦合，} k=0 \text{ 无耦合} \end{cases}$$

三、耦合电感的储能

- 电感的储能 $w(t)=\frac{1}{2}L\,i^2(t)$
- 正弦稳态电路中电感的平均储能 $W=\frac{1}{2}L\,I^2$
- **含互感M的两耦合电感的储能**

L_1自感储能 $\frac{1}{2}L_1i_1^{\ 2}=\frac{1}{2}\psi_{11}i_1$， L_2自感储能 $\frac{1}{2}L_2i_2^{\ 2}$

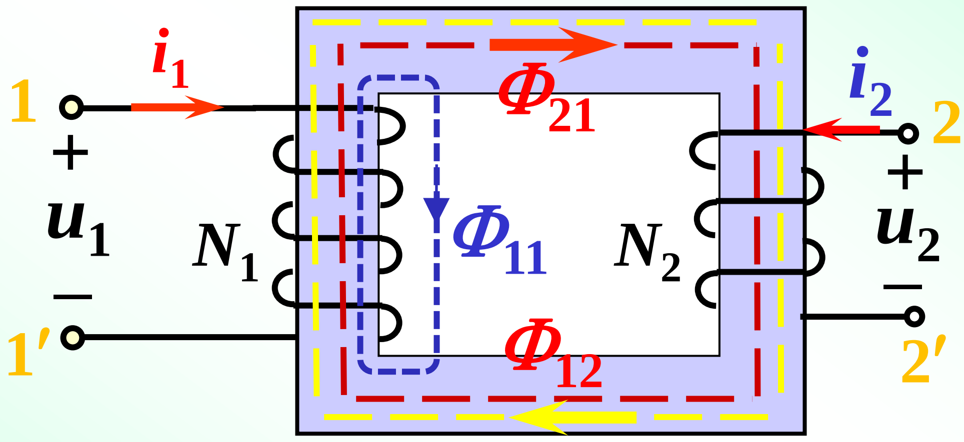
L_1互感储能 $\frac{1}{2}\psi_{12}i_1=\frac{1}{2}(Mi_2)i_1=\frac{1}{2}Mi_1i_2$

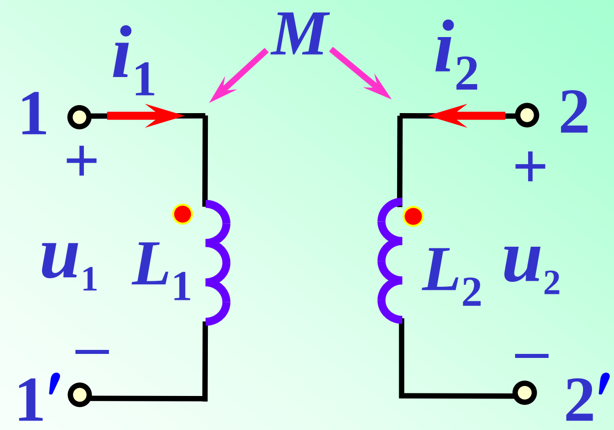
L_2互感储能 $\frac{1}{2}\psi_{21}i_2=\frac{1}{2}(Mi_1)i_2=\frac{1}{2}Mi_1i_2$

总储能 $w=\frac{1}{2}L_1i_1^{\ 2}+\frac{1}{2}L_2i_2^{\ 2}\pm Mi_1i_2$

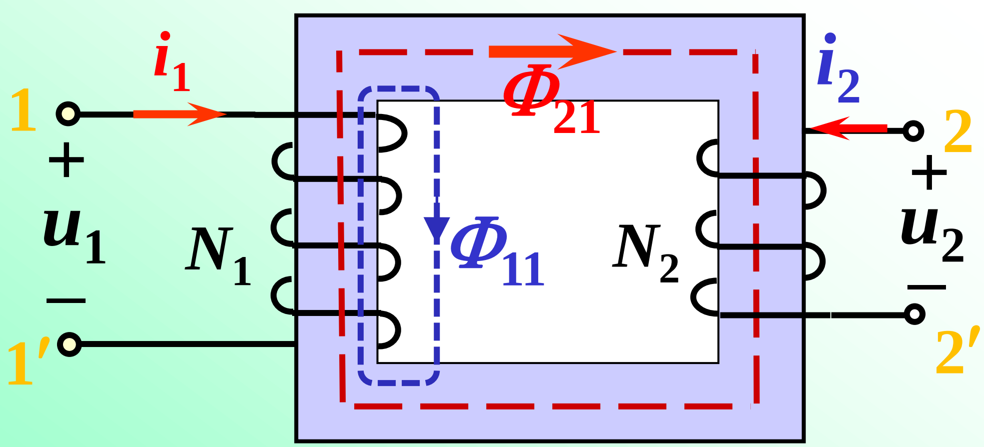
电流均指向同名端时取正号，否则取负号。

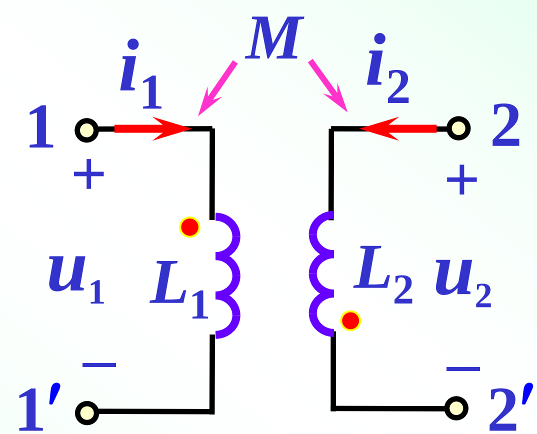
若电流均指向同名端，则自感磁通与互感磁通方向一致。





总储能 $w = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 + M i_1 i_2$

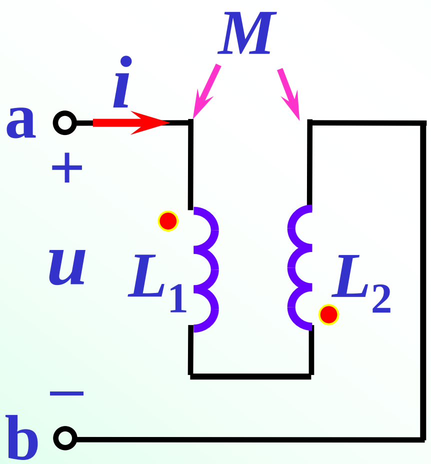


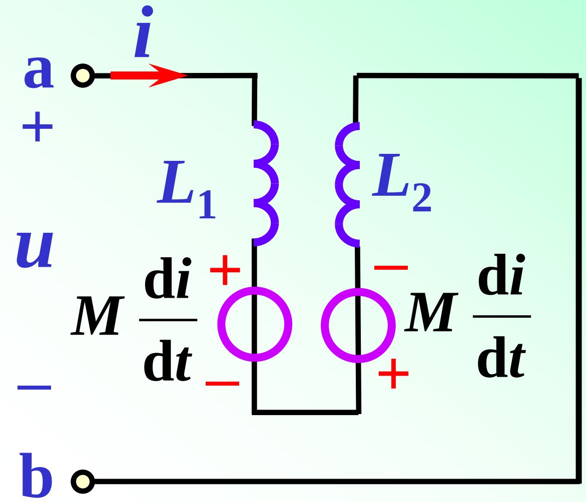


总储能 $w = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 - M i_1 i_2$

四、耦合电感线圈的串联

1. 顺接串联：异名端相接





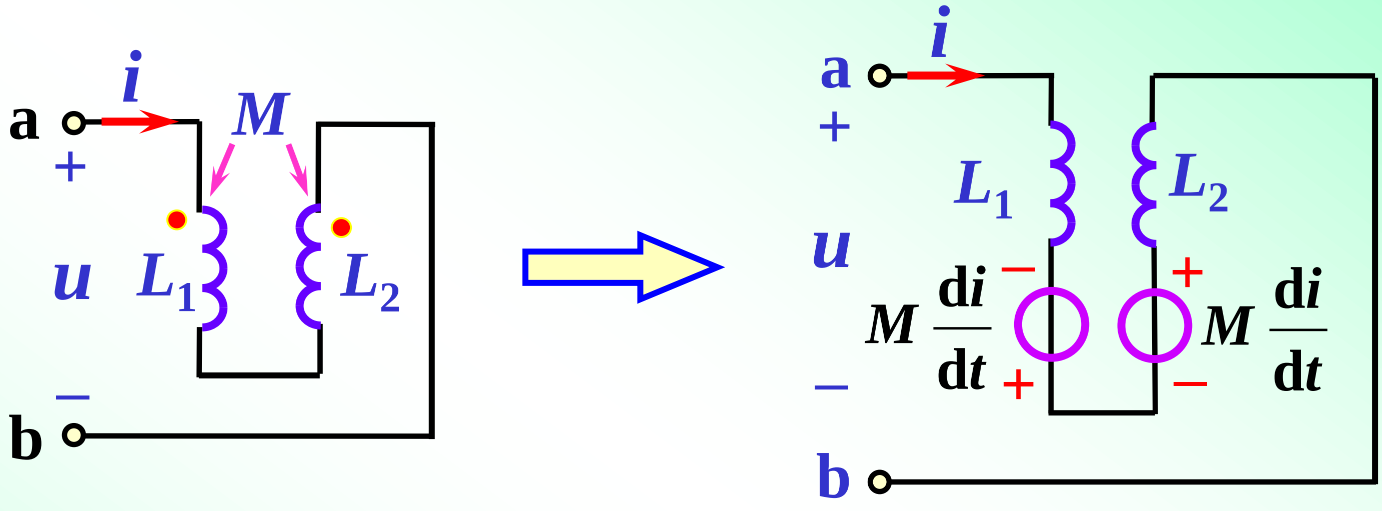
$$u_{ab}=L_1\frac{\mathrm{d}i}{\mathrm{d}t}+M\frac{\mathrm{d}i}{\mathrm{d}t}+L_2\frac{\mathrm{d}i}{\mathrm{d}t}+M\frac{\mathrm{d}i}{\mathrm{d}t}=(L_1+L_2+2M)\frac{\mathrm{d}i}{\mathrm{d}t}$$

$$u_{ab}=L\frac{\mathrm{d}i}{\mathrm{d}t}$$

等效电感 $L=L_1+L_2+2M$

正弦稳态时，顺接等效阻抗 $Z=\mathrm{j}\omega(L_1+L_2+2M)$

2. 反接串联：同名端相接



$$u_{ab}=L_1\frac{\mathrm{d}i}{\mathrm{d}t}-M\frac{\mathrm{d}i}{\mathrm{d}t}+L_2\frac{\mathrm{d}i}{\mathrm{d}t}-M\frac{\mathrm{d}i}{\mathrm{d}t}=(L_1+L_2-2M)\frac{\mathrm{d}i}{\mathrm{d}t}$$

$$u_{ab}=L\frac{\mathrm{d}i}{\mathrm{d}t}$$

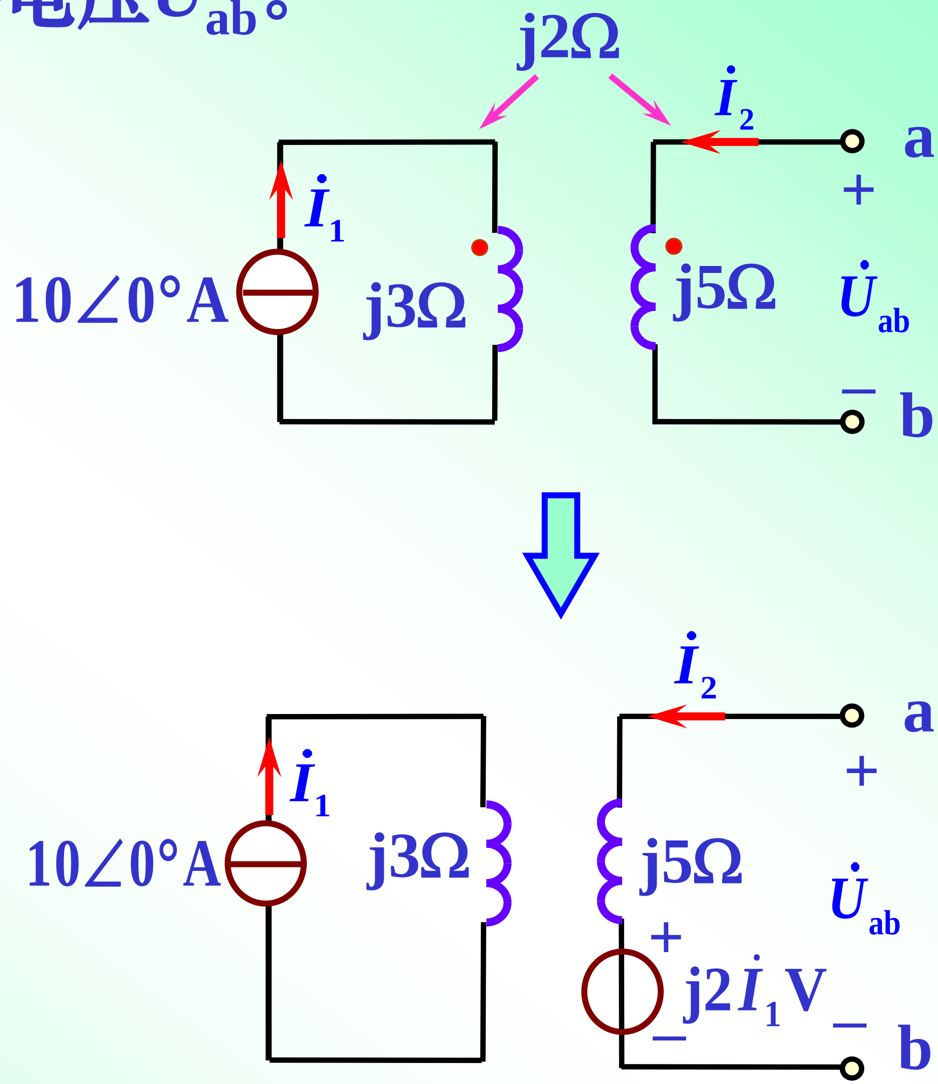
等效电感　$L=L_1+L_2-2M$

正弦稳态时，反接等效阻抗　$Z=\mathrm{j}\omega(L_1+L_2-2M)$

例：求图示电路中的开路电压$\dot{U}_{ab}$。

解：

画出电路的附加电压源的相量模型



由于 $\dot{I}_2 = 0$

有 $\dot{U}_{ab} = \mathrm{j}2\dot{I}_1$

$= \mathrm{j}2 \times 10\underline{/0^\circ}$

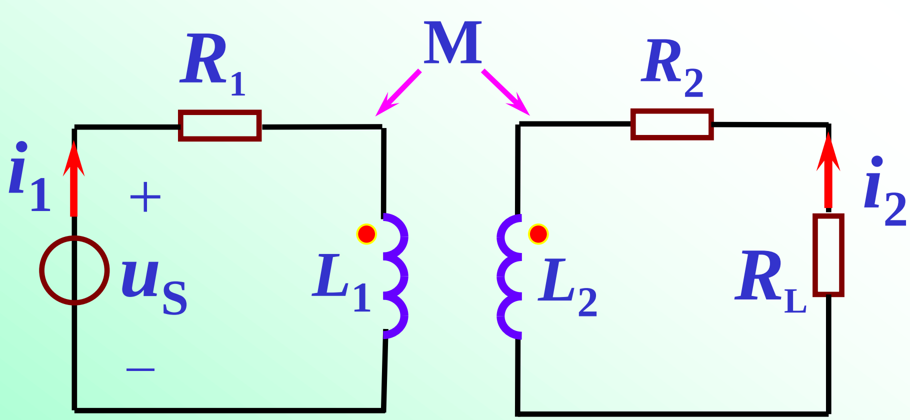
$= \mathrm{j}20\mathrm{V}$

$= 20\underline{/90^\circ}\ \mathrm{V}$

§11-3 空心变压器电路的分析 反映阻抗

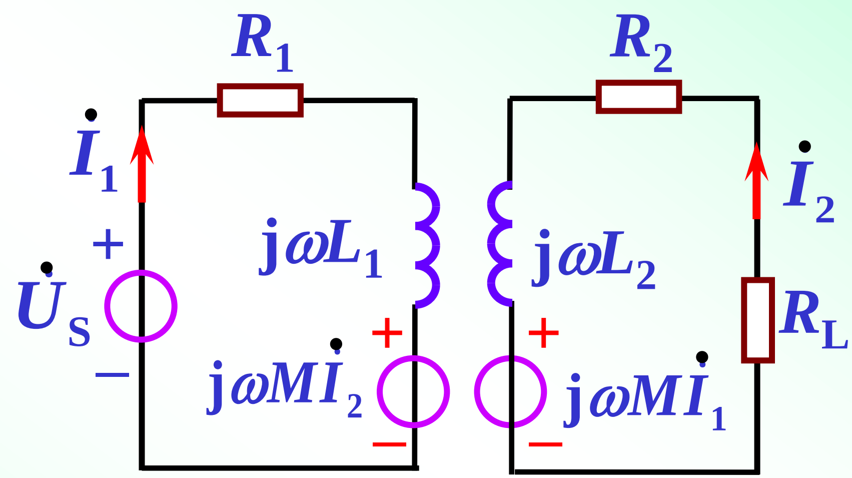
一. 空心变压器电路模型

变压器是利用电磁感应原理而制作的。通常一次线圈接电源，二次线圈接负载。能量通过磁场由电源耦合给负载。



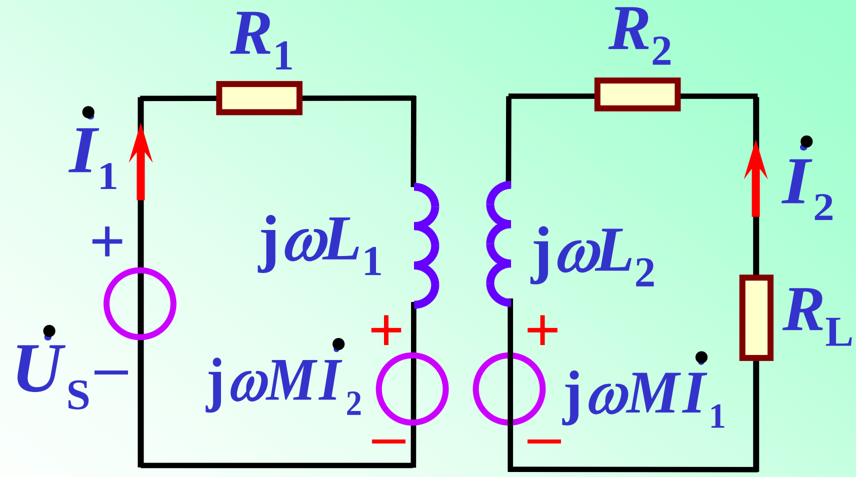
一次回路
(初级回路)

二次回路
(次级回路)



相量模型

二. 空心变压器电路的分析



1. 回路电流法

$$\begin{cases}(R_1 + j\omega L_1)\dot{I}_1 + j\omega M\dot{I}_2 = \dot{U}_S \\ j\omega M\dot{I}_1 + (R_2 + R_L + j\omega L_2)\dot{I}_2 = 0\end{cases}$$

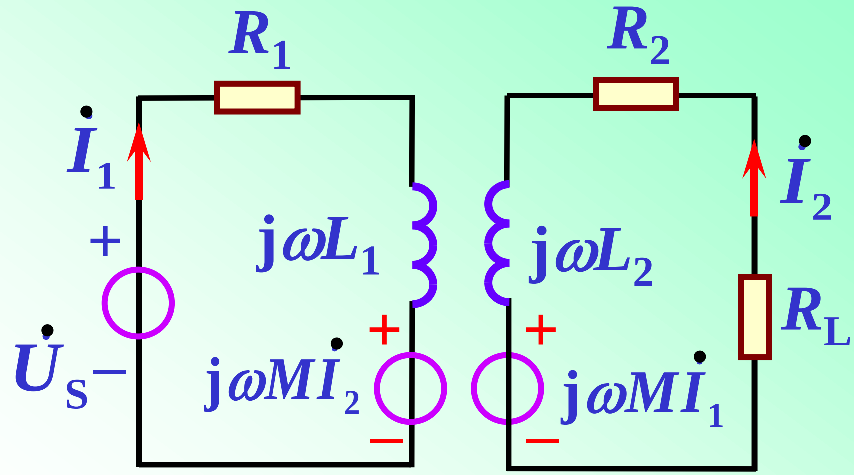
$$\begin{cases}Z_{11}\dot{I}_1 + Z_{12}\dot{I}_2 = \dot{U}_S \\ Z_{21}\dot{I}_1 + Z_{22}\dot{I}_2 = 0\end{cases}$$

其中
$Z_{11} = R_1 + j\omega L_1$ 自阻抗
$Z_{22} = R_2 + R_L + j\omega L_2$ 自阻抗
$Z_{12} = Z_{21} = j\omega M$ 互阻抗

依据克莱姆法则得

$$\dot{I}_1 = \frac{Z_{22}\dot{U}_S}{Z_{11}Z_{22} - Z_{12}Z_{21}}, \qquad \dot{I}_2 = \frac{-Z_{21}\dot{U}_S}{Z_{11}Z_{22} - Z_{12}Z_{21}}$$

2. 用反映阻抗计算



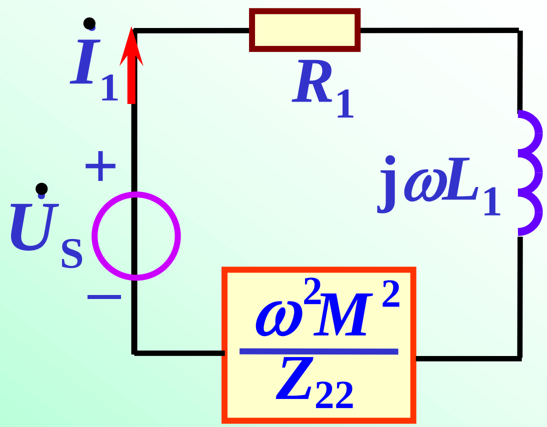
电源端的输入阻抗

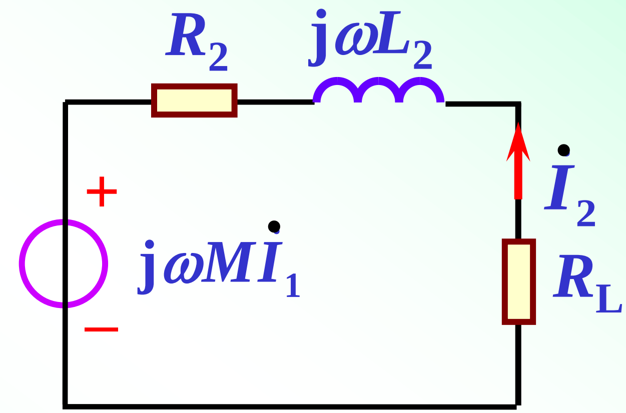
$$Z_i=\frac{\dot{U}_S}{\dot{I}_1}=\frac{Z_{11}Z_{22}-Z_{12}Z_{21}}{Z_{22}}$$

$$=Z_{11}-\frac{Z_{12}Z_{21}}{Z_{22}}=Z_{11}+\frac{\omega^2M^2}{Z_{22}}$$

二次回路在一次回路的反映阻抗

等效一次回路





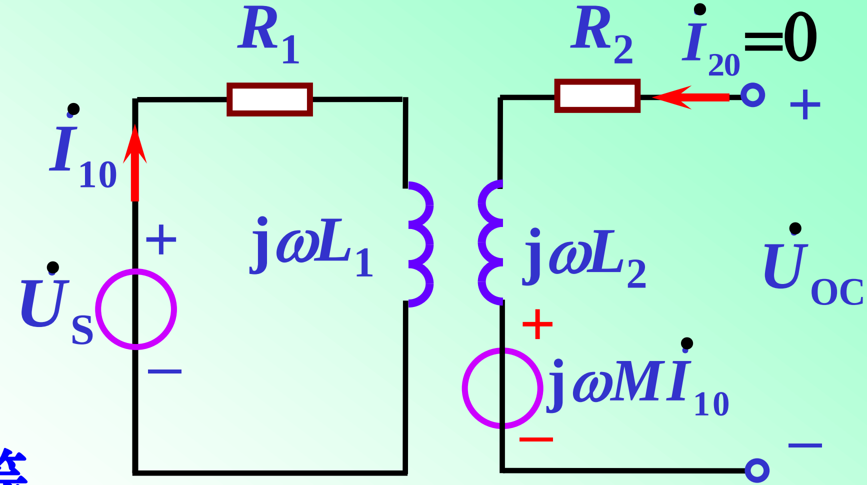
$$\dot{I}_1=\frac{\dot{U}_S}{R_1+\mathrm{j}\omega L_1+\dfrac{\omega M^2}{Z_{22}}}$$

$$\dot{I}_2=\frac{-\mathrm{j}\omega M\dot{I}_1}{R_2+R_L+\mathrm{j}\omega L_2}$$

3. 用戴维南定理分析

将R_L断开，求戴维南等效电路

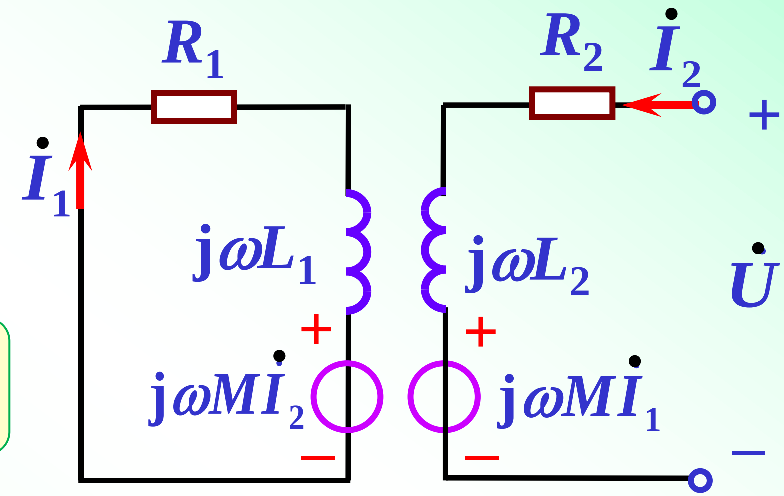
$$\dot{I}_{10}=\frac{\dot{U}_S}{R_1+j\omega L_1} \Rightarrow \dot{U}_{OC}=j\omega M\dot{I}_{10}$$



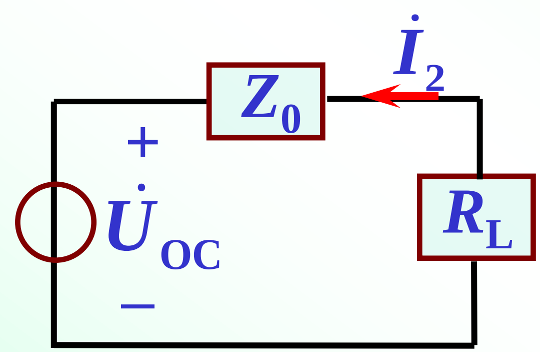
将u_S置零, 加压求流求等效阻抗, 可等效看作一次与二次颠倒，则

$$Z_0=R_2+j\omega L_2+\frac{\omega^2M^2}{Z_{11}}$$

一次回路在二次回路的反映阻抗



$$\dot{I}_2=\frac{\dfrac{-j\omega M\dot{U}_S}{R_1+j\omega L_1}}{R_2+j\omega L_2+\dfrac{\omega^2M^2}{Z_{11}}+R_L}$$



等效二次回路

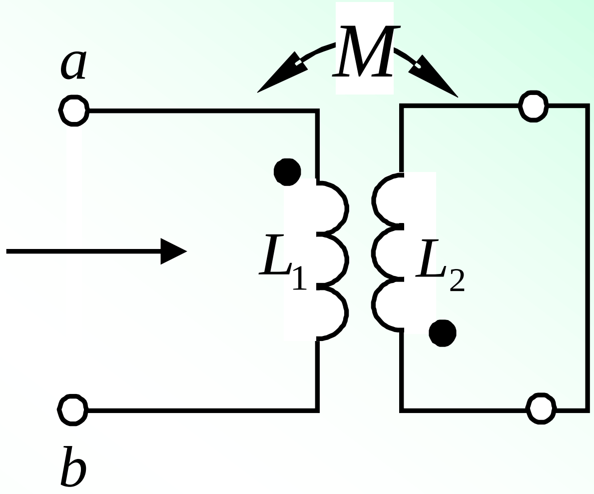
例： 图中电路二次侧短路，已知：L_1=0.1H，L_2=0.4H，M=0.12H 求ab端的等效电感L。

解：

二次回路在一次回路的反映阻抗为 $\dfrac{\omega^2 M^2}{\mathrm{j}\omega L_2}$

则一次侧等效阻抗为

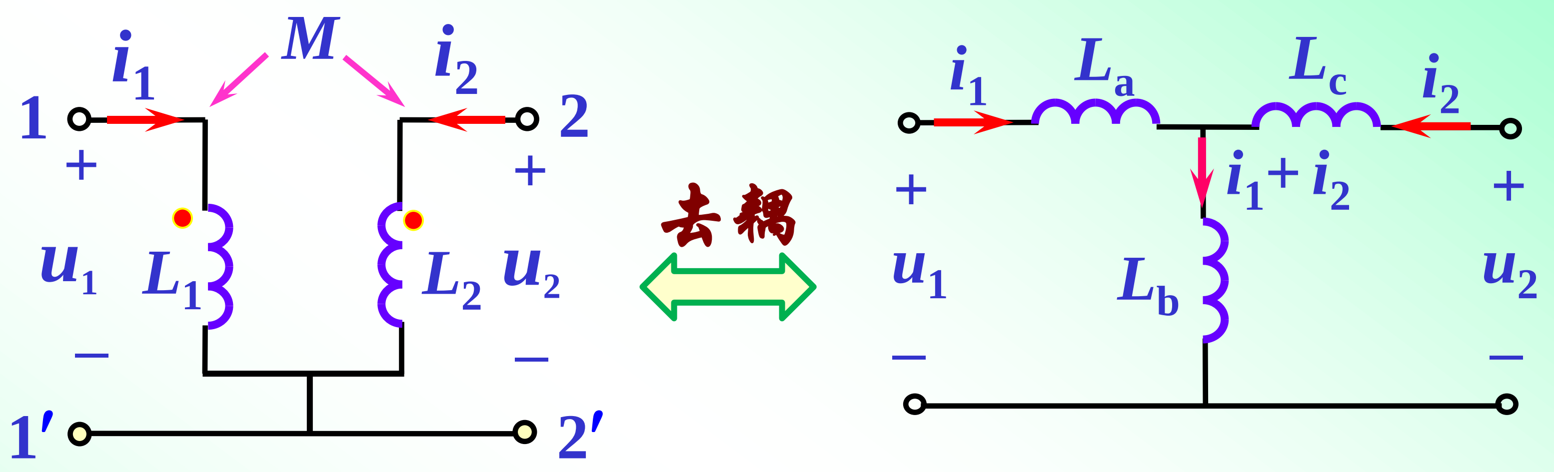
$$Z = \mathrm{j}\omega L_1 + \frac{\omega^2 M^2}{\mathrm{j}\omega L_2} = \mathrm{j}\omega\left(L_1 - \frac{M^2}{L_2}\right)$$



则等效电感

$$L = L_1 - \frac{M^2}{L_2} = 0.064\,\mathrm{H} = 64\,\mathrm{mH}$$

§11-4 耦合电感的去耦等效电路



$$\begin{cases} u_1 = L_1\dfrac{di_1}{dt} + M\dfrac{di_2}{dt} \\ u_2 = L_2\dfrac{di_2}{dt} + M\dfrac{di_1}{dt} \end{cases}$$

$$\begin{cases} u_1 = L_a\dfrac{di_1}{dt} + L_b\dfrac{d(i_1+i_2)}{dt} \\ u_2 = L_c\dfrac{di_2}{dt} + L_b\dfrac{d(i_1+i_2)}{dt} \end{cases}$$

$$\begin{cases} u_1 = (L_a+L_b)\dfrac{di_1}{dt} + L_b\dfrac{di_2}{dt} \\ u_2 = (L_c+L_b)\dfrac{di_2}{dt} + L_b\dfrac{di_1}{dt} \end{cases}$$

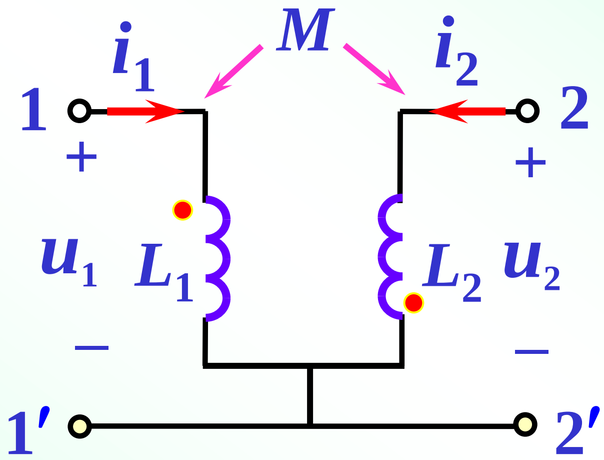
同名端连接时

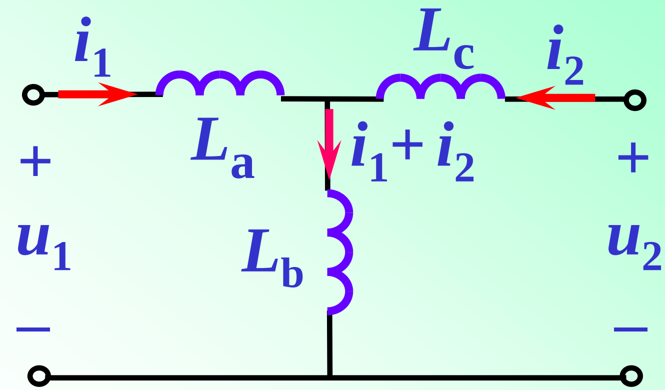
去耦等效电感

$L_a = L_1 - M$

$L_b = M$

$L_c = L_2 - M$





$$\begin{cases} u_1 = L_1 \dfrac{\mathrm{d}i_1}{\mathrm{d}t} - M \dfrac{\mathrm{d}i_2}{\mathrm{d}t} \\ u_2 = L_2 \dfrac{\mathrm{d}i_2}{\mathrm{d}t} - M \dfrac{\mathrm{d}i_1}{\mathrm{d}t} \end{cases}$$

$$\begin{cases} u_1 = L_a \dfrac{\mathrm{d}i_1}{\mathrm{d}t} + L_b \dfrac{\mathrm{d}(i_1 + i_2)}{\mathrm{d}t} \\ u_2 = L_c \dfrac{\mathrm{d}i_2}{\mathrm{d}t} + L_b \dfrac{\mathrm{d}(i_1 + i_2)}{\mathrm{d}t} \end{cases}$$

$$\begin{cases} u_1 = (L_a + L_b) \dfrac{\mathrm{d}i_1}{\mathrm{d}t} + L_b \dfrac{\mathrm{d}i_2}{\mathrm{d}t} \\ u_2 = (L_c + L_b) \dfrac{\mathrm{d}i_2}{\mathrm{d}t} + L_b \dfrac{\mathrm{d}i_1}{\mathrm{d}t} \end{cases}$$

异名端连接时

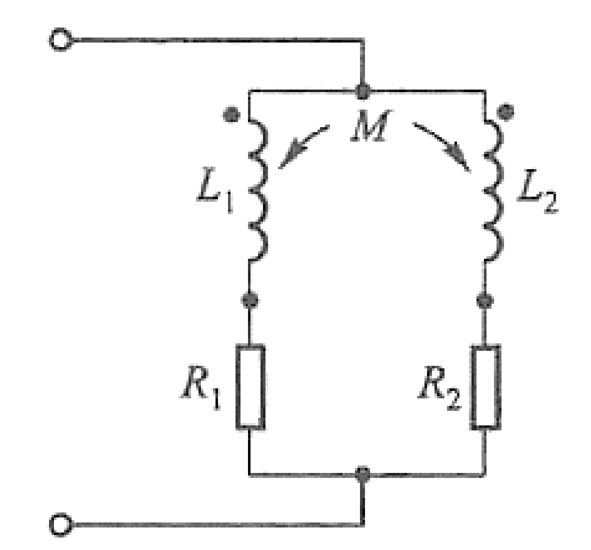
去耦等效电感

$L_a = L_1 + M$

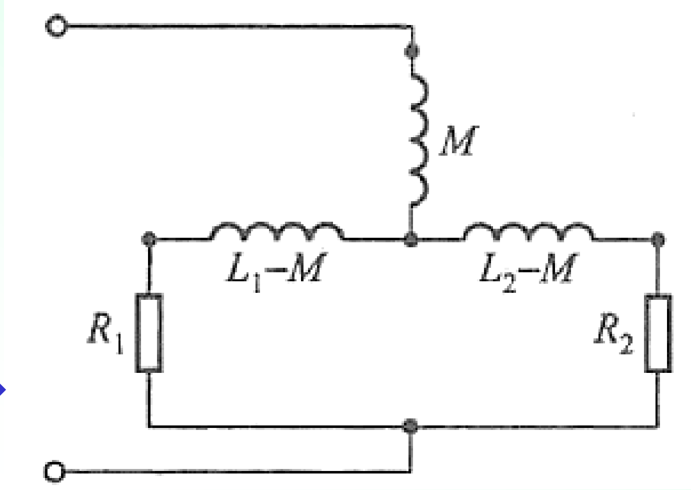
$L_b = -M$

$L_c = L_2 + M$

例：求图示电路的输入阻抗

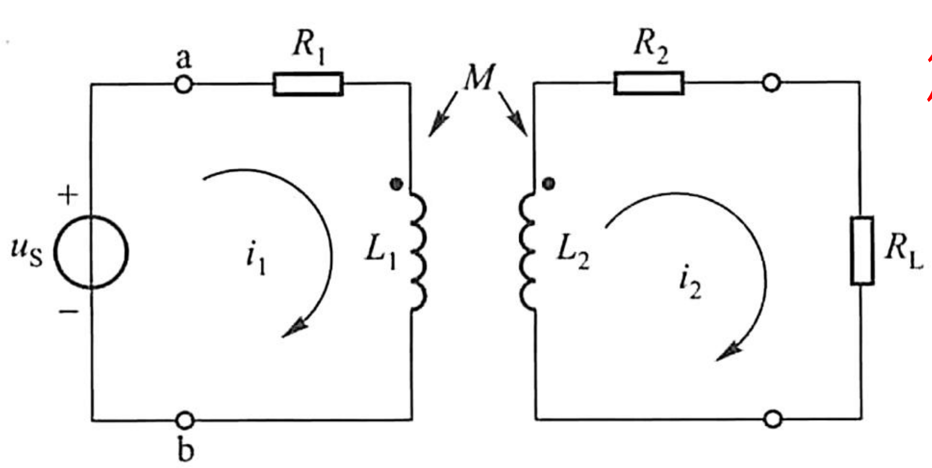


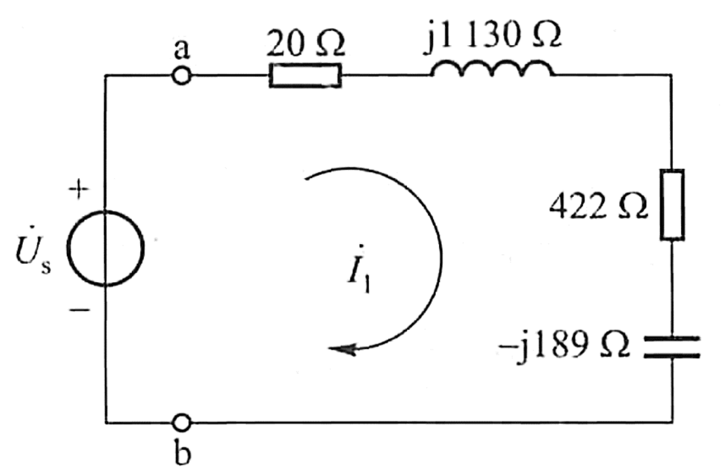
去耦等效电路



$$Z_i = j\omega M + \frac{[R_1 + j\omega(L_1 - M)][R_2 + j\omega(L_2 - M)]}{R_1 + R_2 + j\omega(L_1 + L_2 - 2M)}$$

例11-5： 已知L_1=3.6H, L_2=0.06H, M=0.465H, R_1=20Ω, R_2=0.08Ω, R_L=42Ω， $u_S = 115\sqrt{2}\cos(314t)$。求初级电流$\dot{I}_1$





等效一次回路

解： 用反映阻抗

$$Z_{11} = R_1 + j\omega L_1 = 20 + j1130\ \Omega$$

$$Z_{22} = R_2 + R_L + j\omega L_2 = 42 + j19\ \Omega$$

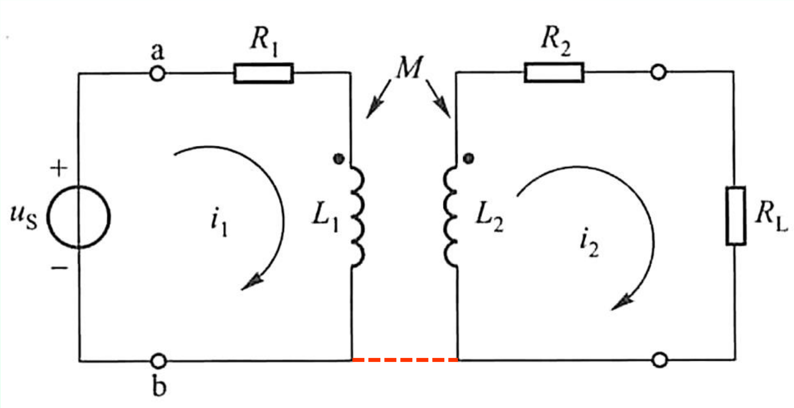
次级在初级的反映阻抗为

$$Z_{ref} = \frac{\omega^2 M^2}{Z_{22}} = 422 - j189\ \Omega$$

$$\dot{I}_1 = \frac{\dot{U}_S}{Z_{11} + Z_{ref}} = \frac{115\angle 0^\circ}{442 + j941}$$

$$= 110.6\angle -64.8^\circ\ \text{mA}$$

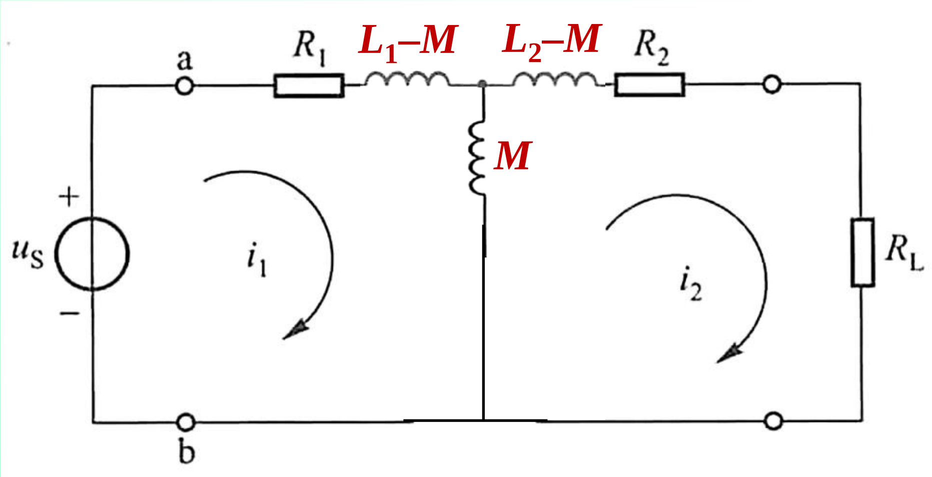
例11-5： 已知L_1=3.6H, L_2=0.06H, M=0.465H, R_1=20Ω, R_2=0.08Ω, R_L=42Ω， $u_s = 115\sqrt{2}\cos(314t)$。求初级电流$\dot{I}_1$



解： 用T形去耦电路

$$\begin{cases}\left[R_1 + j\omega(L_1 - M) + j\omega M\right]\dot{I}_1 - j\omega M\dot{I}_2 = \dot{U}_S \\ -j\omega M\dot{I}_1 + \left[R_2 + R_L + j\omega(L_2 - M) + j\omega M\right]\dot{I}_2 = 0\end{cases}$$

化简得
$$\begin{cases}\left(R_1 + j\omega L_1\right)\dot{I}_1 - j\omega M\dot{I}_2 = \dot{U}_S \\ -j\omega M\dot{I}_1 + \left(R_2 + R_L + j\omega L_2\right)\dot{I}_2 = 0\end{cases}$$



T形去耦电路

$$\begin{cases}\left(20 + j1130\right)\dot{I}_1 - j146\dot{I}_2 = 115 \\ -j146\dot{I}_1 + \left(42 + j19\right)\dot{I}_2 = 0\end{cases}$$

$$\dot{I}_1 = 110.6\angle -64.8^\circ \text{ mA}$$

§11-5 理想变压器的VCR

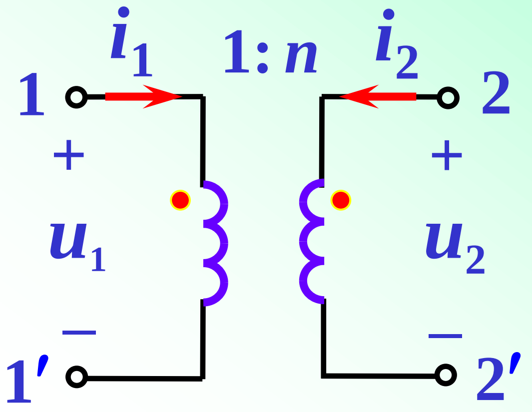
理想变压器是一种双口电阻元件，它也是一种耦合元件。

1. 电路模型

初级匝数为N_1, 次级匝数为N_2

则参数匝比（变比）为：

$$n = \frac{N_2}{N_1}$$



理想变压器的电路模型

2. 理想变压器的VCR

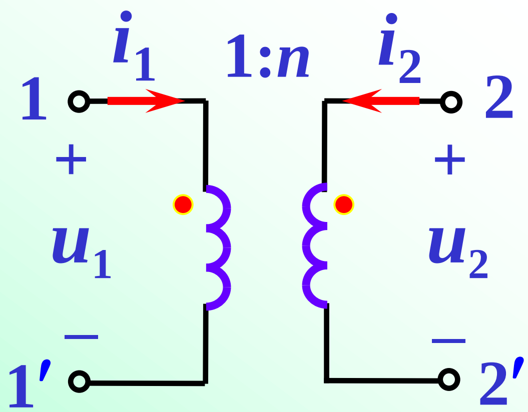
当两电感趋于无穷且全耦合情况下，根据耦合电感VCR可推得：

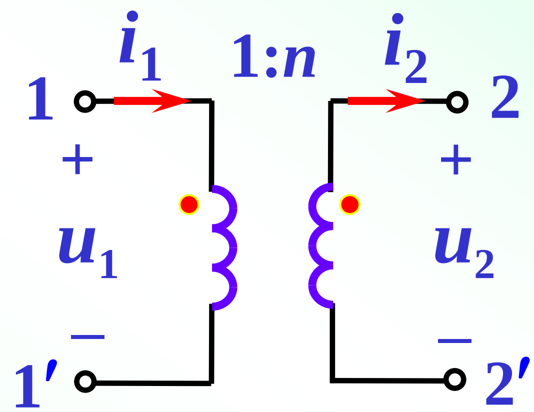
$$u_2 = n u_1 \qquad i_2 = -(1/n)\, i_1$$

理想变压器电压、电流比符号的判断

(1)两电压高电位端与同名端一致时，电压比取正，反之取负。

(2)两电流都从同名端流入，电流比取负，反之取正。



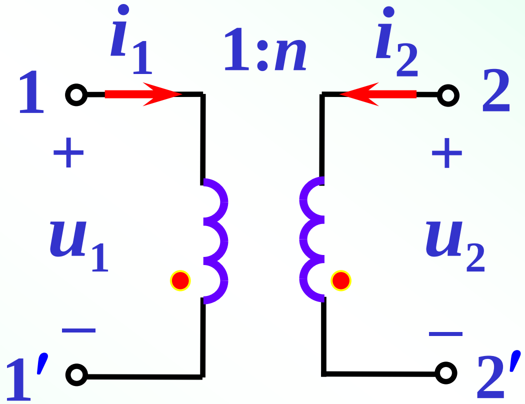


$$u_2 = nu_1$$

$$i_2 = -(1/n)i_1$$

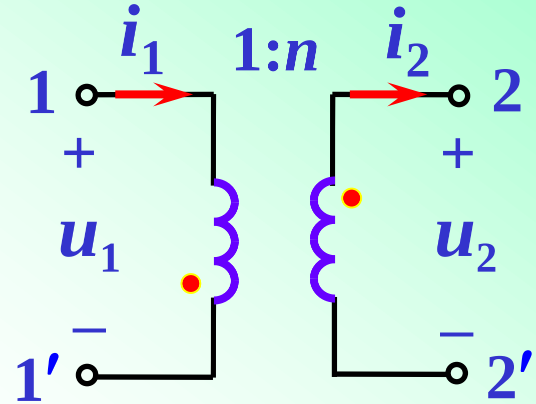
$$u_2 = nu_1$$

$$i_2 = (1/n)i_1$$



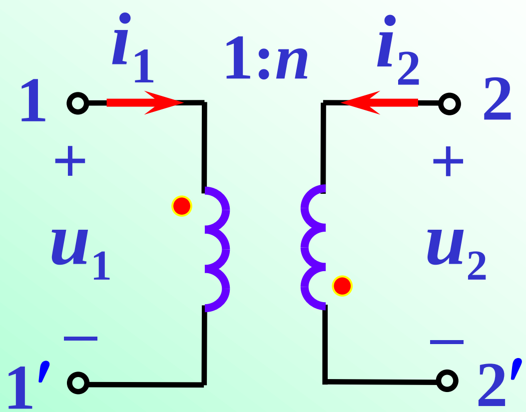
$$u_2 = nu_1$$

$$i_2 = -(1/n)i_1$$



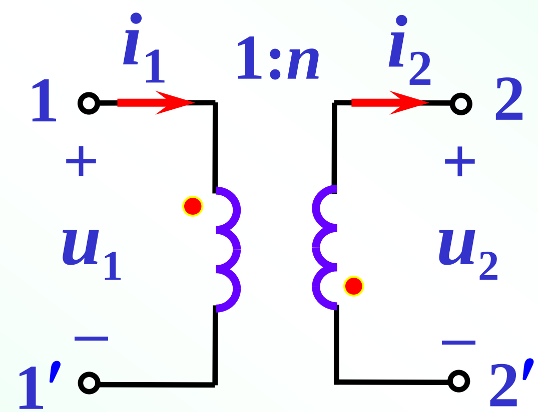
$$u_2 = -nu_1$$

$$i_2 = -(1/n)i_1$$



$$u_2 = -nu_1$$

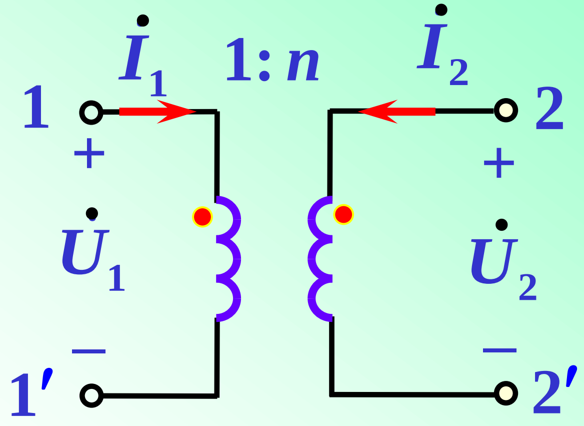
$$i_2 = (1/n)i_1$$



$$u_2 = -nu_1$$

$$i_2 = -(1/n)i_1$$

理想变压器的VCR的相量形式



$u_2=nu_1$， $i_2=-(1/n)\,i_1$

$$\dot{U}_2 = n\dot{U}_1 \qquad \dot{I}_2 = -\frac{1}{n}\dot{I}_1$$

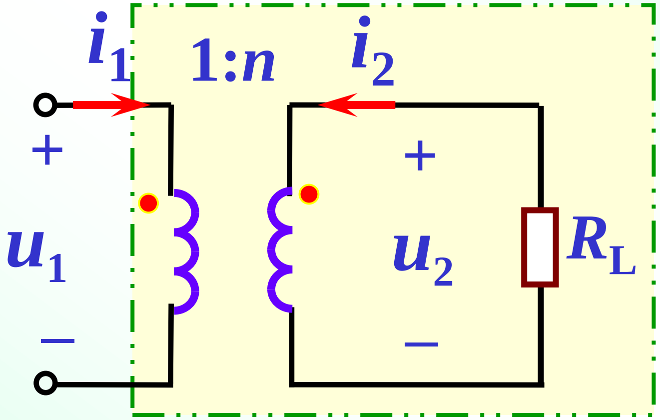
3. 功率

$$p = u_2 i_2 + u_1 i_1 = n u_1\left(-\frac{1}{n} i_1\right) + u_1 i_1 = 0$$

结论：理想变压器既不消耗能量也不储存能量。

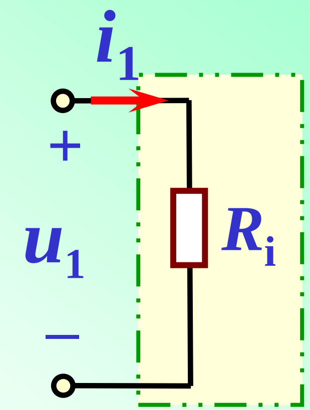
理想变压器是无记忆元件。

§11-6 理想变压器的阻抗变换性质



$$u_2 = n u_1$$

$$i_2 = -(1/n) i_1$$



理想变压器不仅可以实现电压变换，电流变换，而且能够实现**阻抗变换**。

二次电阻对一次侧的折合电阻

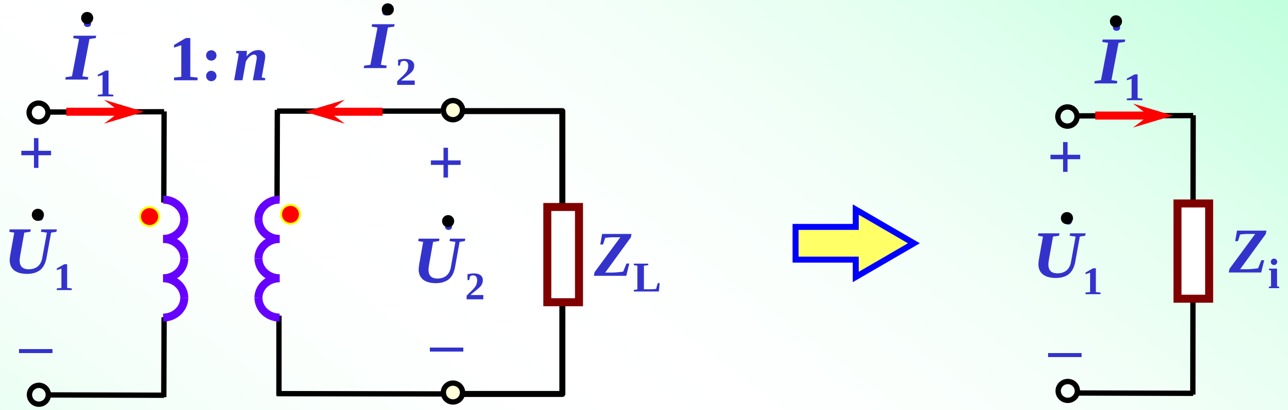
$$R_i = \frac{u_1}{i_1} = \frac{\frac{1}{n} u_2}{-n i_2} = -\frac{1}{n^2} \cdot \frac{u_2}{i_2} = \frac{1}{n^2} R_L$$

$$\boxed{\therefore R_i = \frac{1}{n^2} R_L}$$

$n > 1$ 电阻折合到初级变小： $R_i < R_L$

$n < 1$ 电阻折合到初级变大： $R_i > R_L$

在正弦稳态，可由对应的相量模型进行分析



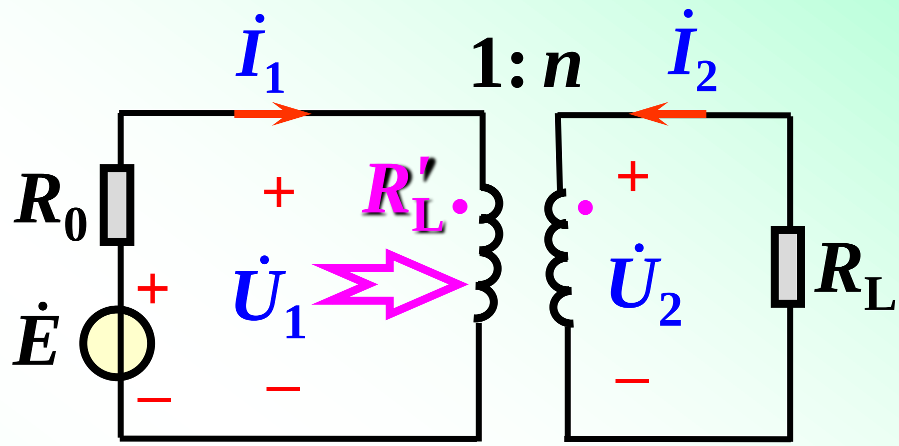
二次阻抗对一次侧的折合阻抗：

$$Z_i = \frac{\dot{U}_1}{\dot{I}_1} = \frac{\frac{1}{n}\dot{U}_2}{-n\dot{I}_2} = -\frac{1}{n^2}\frac{\dot{U}_2}{\dot{I}_2} = \frac{1}{n^2}Z_L$$

理想变压器具有变换阻抗的性质，可以利用理想变压器实现最大功率匹配。

例： 电路如图，交流信号源$E = 120\text{V}$，内阻$R_0 = 800\Omega$，负载 $R_L = 8\,\Omega$。(1) 当R_L折算到原边的等效电阻 $R_L' = R_0$ 时，求变压器的匝数比和信号源输出的功率；(2)若将负载直接与信号源联接时，信号源输出多大功率?

解： （1）$R_L' = \dfrac{1}{n^2}R_L = R_0$



$$\frac{1}{n} = \sqrt{\frac{R_0}{R_L}} = \sqrt{\frac{800}{8}} = 10$$

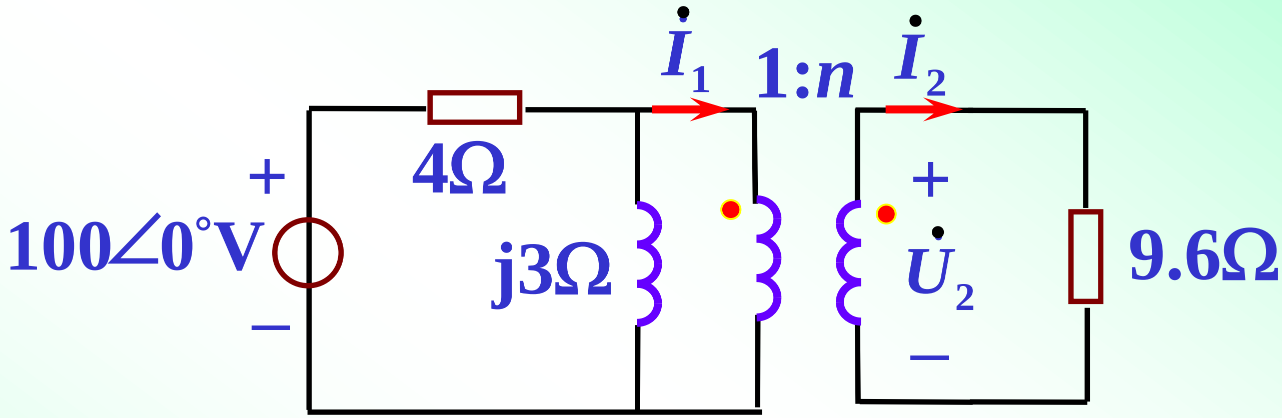
$$n = 0.1$$

$$P = \left(\frac{E}{R_0 + R_L'}\right)^2 R_L' = \left(\frac{120}{800+800}\right)^2 \times 800 = 4.5\text{W}$$

（2）$P = \left(\dfrac{120}{800+8}\right)^2 \times 8 = 0.176\text{W}$

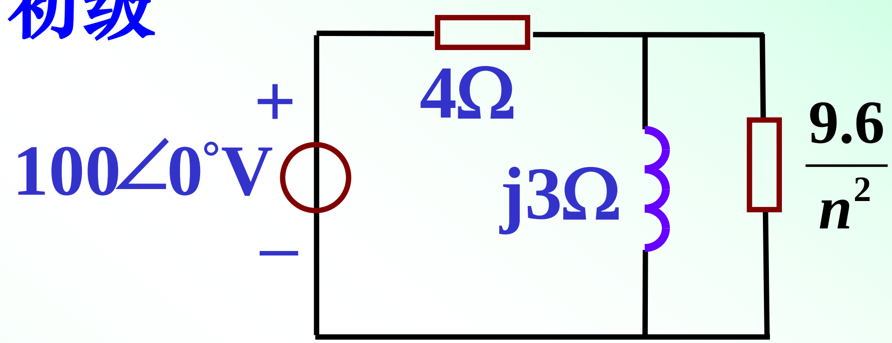
利用变压器可达到阻抗匹配

例: 求负载获得最大功率时的匝比 n，并求最大功率 $P_{L\max}$。



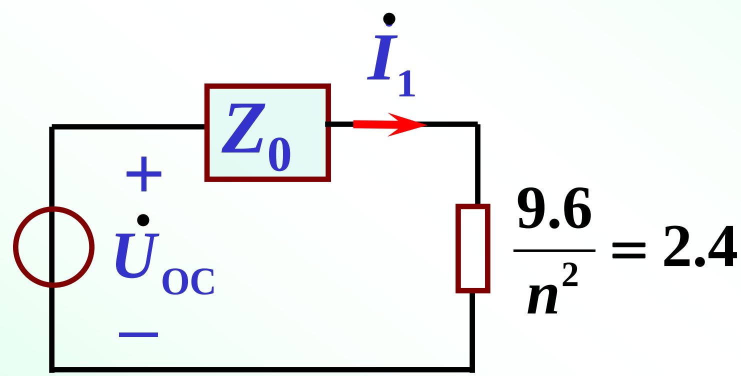
$$\boxed{Z_i = \frac{1}{n^2} Z_L}$$

解: 将变压器的次级折算到初级



$$Z_0 = \frac{4 \times j3}{4 + j3} = \frac{j12}{5\angle 36.9^\circ}$$

$$= 2.4\angle 53.1^\circ \Omega$$



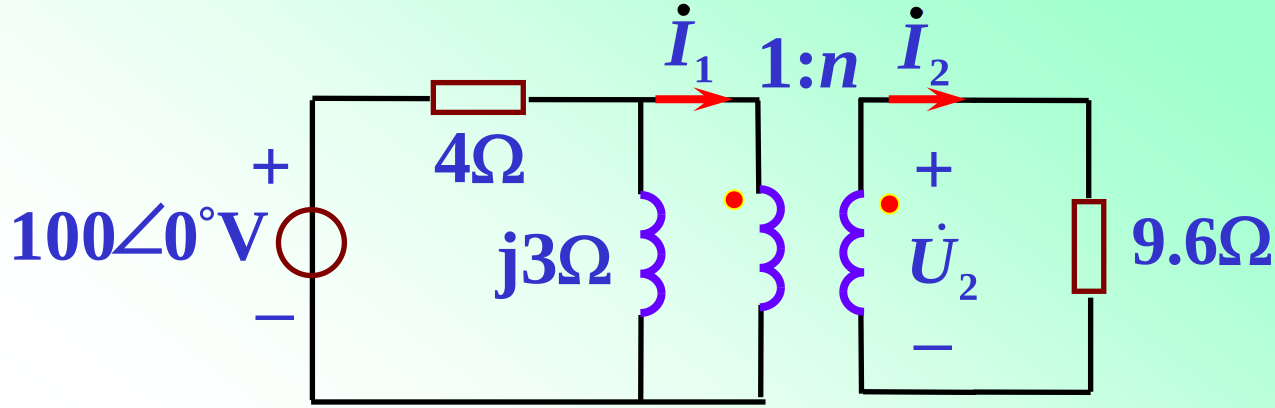
由模匹配 $\dfrac{9.6}{n^2} = 2.4$

得 $n^2 = 4$ $\quad n = 2$

$n=2$

$Z_0 = 2.4\angle 53.1^\circ\,\Omega$

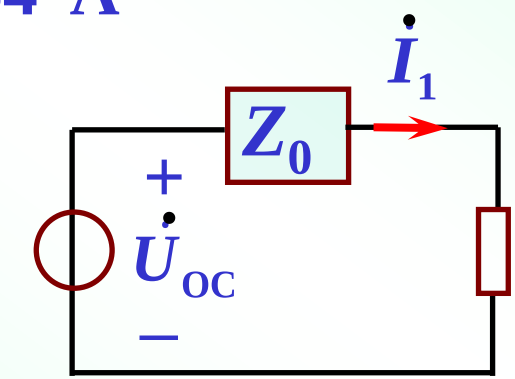
用戴维南定理对初级进行化简。



$$\dot{U}_{OC} = \frac{j3}{4+j3} \times 100\angle 0^\circ = 60\angle 53.1^\circ\ \text{V}$$

$$\dot{I}_1 = \frac{60\angle 53.1^\circ}{2.4\angle (53.1)^\circ + 2.4} = 13.986\angle 26.54^\circ\ \text{A}$$

$$P_{Lmax} = 13.986^2 \times 2.4 = \underline{496.46\ \text{W}}$$

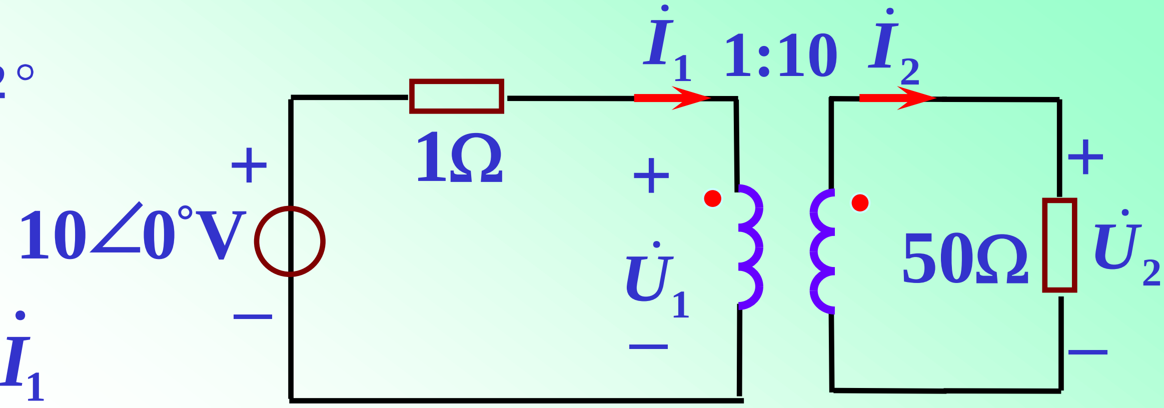


$$\frac{9.6}{n^2} = 2.4$$

或 $\dot{I}_2 = \frac{1}{2}\dot{I}_1 = 6.993\angle 26.54^\circ\ \text{A}$

$$P_{Lmax} = 6.993^2 \times 9.6 = \underline{496.46\ \text{W}}$$

例： 求图示电路中U_2。



解：(1) 回路法

$$\dot{U}_2 = 10\dot{U}_1 \qquad \dot{I}_2 = \frac{1}{10}\dot{I}_1$$

$$\dot{I}_1 + \dot{U}_1 = 10\angle 0^\circ$$

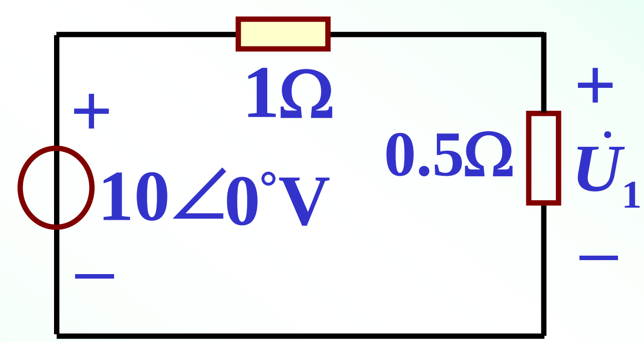
$$50\dot{I}_2 = \dot{U}_2$$

$$\dot{U}_1 = \frac{10}{3}\text{V} \qquad \dot{U}_2 = 10\dot{U}_1 = \frac{100}{3}\text{V}$$

(2) 把负载阻抗折合到初级$R_L' = 50/10^2 = 0.5\ \Omega$

$$\dot{U}_1 = \frac{\frac{1}{2}}{1+\frac{1}{2}} \times 10\angle 0^\circ = \frac{10}{3}\text{V}$$

$$\dot{U}_2 = \frac{100}{3}\text{V}$$

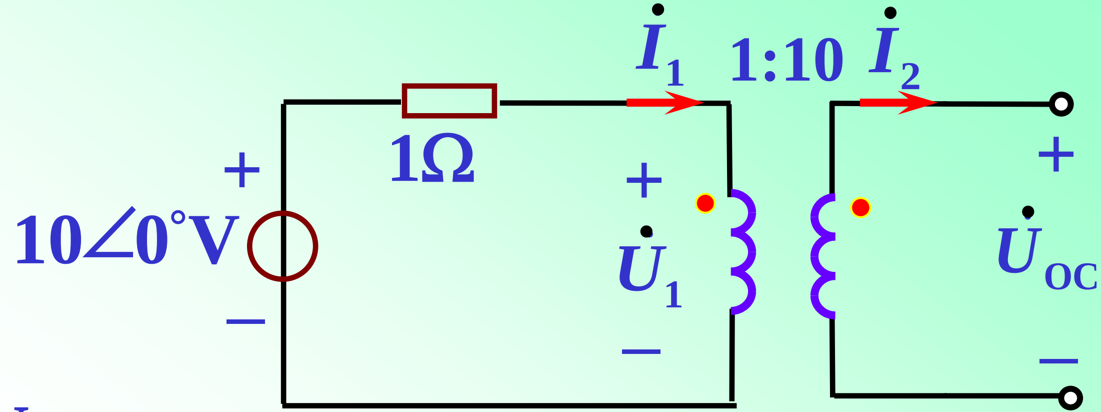


(3) 用戴维南定理

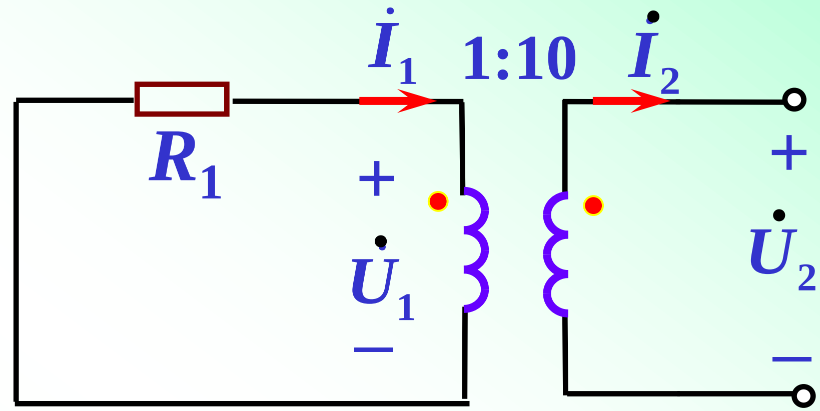
$\dot{I}_2 = 0, \quad \dot{I}_1 = 10\dot{I}_2 = 0$

$\dot{U}_1 = 10\angle 0^\circ\ \text{V}$

$\dot{U}_{OC} = 10\dot{U}_1 = 100\angle 0^\circ \text{V}$



把初级回路阻抗折合到次级回路

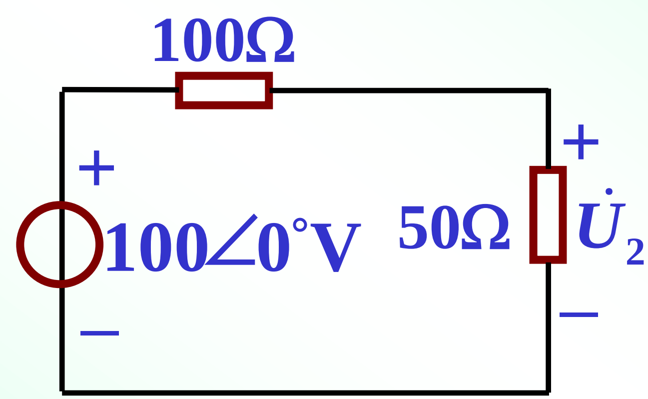


$$\frac{\dot{U}_1}{\dot{I}_1} = -R_1$$

$$R_0 = -\frac{\dot{U}_2}{\dot{I}_2} = -\frac{n\dot{U}_1}{\frac{1}{n}\dot{I}_1} = n^2 R_1$$

$$R_0 = 10^2 \times 1 = 100\,\Omega$$

$$\dot{U}_2 = \frac{50}{100+50} \times 100\angle 0^\circ = \frac{100}{3}\ \text{V}$$



第11章　小结

1. **基本概念：** 互感，同名端，耦合系数，反映阻抗，变比 n（匝比），折合阻抗

2. **电路模型：**
耦合电感（用附加电压源计及互感，去耦等效电路）
空心变压器（等效一次电路，等效二次电路）
理想变压器

3. **VCR：** 耦合电感 $u_1 = L_1 \dfrac{\mathrm{d}i_1}{\mathrm{d}t} + M\dfrac{\mathrm{d}i_2}{\mathrm{d}t}$　　$u_2 = L_2 \dfrac{\mathrm{d}i_2}{\mathrm{d}t} + M\dfrac{\mathrm{d}i_1}{\mathrm{d}t}$

理想变压器 $u_2 = nu_1$，$i_2 = -(1/n)i_1$