```
restart;
```

#Mixed triopoly with a no negative profit constraint and social costs

```
Q := q[a] + q[b] + q[c];
```

$$Q := q_a + q_b + q_c \tag{1.1}$$

```
P := 1 - Q;
```

$$P := 1 - q_a - q_b - q_c \tag{1.2}$$

```
TC[a] := m·q[a] + F;
```

$$TC_a := m\,q_a + F \tag{1.3}$$

```
TC[b] := m·q[b] + F;
```

$$TC_b := m\,q_b + F \tag{1.4}$$

```
TC[c] := m·q[c] + s·q[c] + F;
```

$$TC_c := m\,q_c + s\,q_c + F \tag{1.5}$$

```
profit[a] := q[a]*P - TC[a];
```

$$profit_a := \left(1 - q_a - q_b - q_c\right) q_a - m\,q_a - F \tag{1.6}$$

```
profit[b] := q[b]*P - TC[b];
```

$$profit_b := \left(1 - q_a - q_b - q_c\right) q_b - m\,q_b - F \tag{1.7}$$

```
profit[c] := q[c]*P - TC[c];
```

$$profit_c := \left(1 - q_a - q_b - q_c\right) q_c - m\,q_c - s\,q_c - F \tag{1.8}$$

```
SS := factor( Q^2/2 + profit[a] + profit[b] + profit[c] );
```

$$SS := -\frac{1}{2}\,q_a^2 - q_a\,q_b - q_a\,q_c - \frac{1}{2}\,q_b^2 - q_b\,q_c - \frac{1}{2}\,q_c^2 + q_a - m\,q_a - 3\,F + q_b - m\,q_b + q_c - m\,q_c - s\,q_c \tag{1.9}$$

```
P > 0, q[a] > 0, q[b] > 0, q[c] > 0, m > 0;
```

$$0 < 1 - q_a - q_b - q_c,\ 0 < q_a,\ 0 < q_b,\ 0 < q_c,\ 0 < m \tag{1.10}$$

#Reaction functions

```
FOC[q[a]] := diff(profit[a], q[a]);
```

$$FOC_{q_a} := -2\,q_a + 1 - q_b - q_c - m \tag{1.11}$$

```
q[a](q[b], q[c]) := isolate(FOC[q[a]], q[a]);
```

$$q_a\left(q_b, q_c\right) := q_a = \frac{1}{2} - \frac{q_b}{2} - \frac{q_c}{2} - \frac{m}{2} \tag{1.12}$$

```
FOC[q[b]] := diff(profit[b], q[b]);
```

$$FOC_{q_b} := -2\,q_b + 1 - q_a - q_c - m \tag{1.13}$$

```
q[b](q[a], q[c]) := isolate(%, q[b]);
```

$$q_b(q_a, q_c) := q_b = \frac{1}{2} - \frac{q_a}{2} - \frac{q_c}{2} - \frac{m}{2} \tag{1.14}$$

$SS;$

$$-\frac{1}{2} q_a^2 - q_a q_b - q_a q_c - \frac{1}{2} q_b^2 - q_b q_c - \frac{1}{2} q_c^2 + q_a - m q_a - 3 F + q_b - m q_b + q_c - m q_c - s q_c \tag{1.15}$$

$\#FOC[q[c]] := diff(SS, q[c]);$
$\#FOC[q[c]] := diff(profit[c], q[c]);$
$FOC[q[c]] := profit[c];$

$$FOC_{q_c} := (1 - q_a - q_b - q_c) q_c - m q_c - s q_c - F \tag{1.16}$$

$(solve(\%, q[c]));$

$$-\frac{m}{2} - \frac{s}{2} - \frac{q_a}{2} - \frac{q_b}{2} + \frac{1}{2} + \frac{1}{2}\left(m^2 + 2 m s + 2 m q_a + 2 m q_b + s^2 + 2 s q_a + 2 s q_b + q_a^2 + 2 q_a q_b + q_b^2 - 4 F - 2 m - 2 s - 2 q_a - 2 q_b + 1\right)^{1/2}, -\frac{m}{2} - \frac{s}{2} - \frac{q_a}{2} - \frac{q_b}{2} + \frac{1}{2} - \frac{1}{2}\left(m^2 + 2 m s + 2 m q_a + 2 m q_b + s^2 + 2 s q_a + 2 s q_b + q_a^2 + 2 q_a q_b + q_b^2 - 4 F - 2 m - 2 s - 2 q_a - 2 q_b + 1\right)^{1/2} \tag{1.17}$$

$q[c](q[a], q[b]) := \%[1];$

$$q_c(q_a, q_b) := -\frac{m}{2} - \frac{s}{2} - \frac{q_a}{2} - \frac{q_b}{2} + \frac{1}{2} + \frac{1}{2}\left(m^2 + 2 m s + 2 m q_a + 2 m q_b + s^2 + 2 s q_a + 2 s q_b + q_a^2 + 2 q_a q_b + q_b^2 - 4 F - 2 m - 2 s - 2 q_a - 2 q_b + 1\right)^{1/2} \tag{1.18}$$

#Reaction functions in a mixed triopoly, $m = 0, t = 0.2, F = 0.005$
$subs(m = 0, t = 0.2, F = 0.005, (q[a](q[b], q[c]) + q[b](q[a], q[c])));$

$$q_a + q_b = 1 - \frac{q_b}{2} - q_c - \frac{q_a}{2} \tag{1.1.1}$$

$isolate(\%, q[c]);$

$$q_c = -\frac{3 q_a}{2} - \frac{3 q_b}{2} + 1 \tag{1.1.2}$$

$subs(m = 0, t = 0.2, F = 0.005, (q[c](q[a], q[b])));$

$$\frac{1}{2} - \frac{s}{2} - \frac{q_a}{2} - \frac{q_b}{2} + \frac{\sqrt{s^2 + 2 s q_a + 2 s q_b + q_a^2 + 2 q_a q_b + q_b^2 - 2 s - 2 q_a - 2 q_b + 0.980}}{2} \tag{1.1.3}$$

$convert(\%, rational);$

$$\frac{1}{2} - \frac{s}{2} - \frac{q_a}{2} - \frac{q_b}{2} + \frac{\sqrt{s^2 + 2\,s\,q_a + 2\,s\,q_b + q_a^2 + 2\,q_a\,q_b + q_b^2 - 2\,s - 2\,q_a - 2\,q_b + \frac{49}{50}}}{2} \quad \textbf{(1.1.4)}$$

simplify(%, sqrt, *symbolic*);

$$\frac{1}{2} - \frac{s}{2} - \frac{q_a}{2} - \frac{q_b}{2} + \frac{1}{20}\sqrt{100\,q_a^2 + (200\,s + 200\,q_b - 200)\,q_a + 100\,q_b^2 + (200\,s - 200)\,q_b + 100\,s^2 - 200\,s + 98} \quad \textbf{(1.1.5)}$$

factor(%, *method* = "Wang");

$$\frac{1}{2} - \frac{s}{2} - \frac{q_a}{2} - \frac{q_b}{2} + \frac{1}{20}\left(100\,s^2 + 200\,s\,q_a + 200\,s\,q_b + 100\,q_a^2 + 200\,q_a\,q_b + 100\,q_b^2 - 200\,s - 200\,q_a - 200\,q_b + 98\right)^{1/2} \quad \textbf{(1.1.6)}$$

#*q*[*c*](*q*[*a*], *q*[*c*], *x*[*b*])
#*q*[*c*] := *piecewise*(0 < *rhs*(%), *rhs*(%));
#*FOC*[*q*[*c*]];
sys := { *FOC*[*q*[*a*]], *FOC*[*q*[*b*]], *FOC*[*q*[*c*]]};

$$sys := \left\{\left(1 - q_a - q_b - q_c\right) q_c - m\,q_c - s\,q_c - F,\ -2\,q_a + 1 - q_b - q_c - m,\ -2\,q_b + 1 - q_a - q_c - m\right\} \quad \textbf{(1.19)}$$

solve(*sys*, {*q*[*a*], *q*[*b*], *q*[*c*]});

$$\left\{q_a = \frac{1}{3} - \frac{RootOf\left(\_Z^2 + (m + 3\,s - 1)\,\_Z + 3\,F\right)}{3} - \frac{m}{3},\ q_b = \frac{1}{3} - \frac{RootOf\left(\_Z^2 + (m + 3\,s - 1)\,\_Z + 3\,F\right)}{3} - \frac{m}{3},\ q_c = RootOf\left(\_Z^2 + (m + 3\,s - 1)\,\_Z + 3\,F\right)\right\} \quad \textbf{(1.20)}$$

convert(%, *radical*);

$$\left\{q_a = \frac{1}{6} - \frac{m}{6} + \frac{s}{2} - \frac{\sqrt{m^2 + 6\,m\,s + 9\,s^2 - 12\,F - 2\,m - 6\,s + 1}}{6},\ q_b = \frac{1}{6} - \frac{m}{6} + \frac{s}{2} - \frac{\sqrt{m^2 + 6\,m\,s + 9\,s^2 - 12\,F - 2\,m - 6\,s + 1}}{6},\ q_c = -\frac{m}{2} - \frac{3\,s}{2} + \frac{1}{2} + \frac{\sqrt{m^2 + 6\,m\,s + 9\,s^2 - 12\,F - 2\,m - 6\,s + 1}}{2}\right\} \quad \textbf{(1.21)}$$

#*evala*(*Simplify*(%));
qastar := *rhs*(%[1]);

$$qastar := \frac{1}{6} - \frac{m}{6} + \frac{s}{2} - \frac{\sqrt{m^2 + 6\,m\,s + 9\,s^2 - 12\,F - 2\,m - 6\,s + 1}}{6} \tag{1.22}$$

$qbstar := rhs(\%\%[2]);$

$$qbstar := \frac{1}{6} - \frac{m}{6} + \frac{s}{2} - \frac{\sqrt{m^2 + 6\,m\,s + 9\,s^2 - 12\,F - 2\,m - 6\,s + 1}}{6} \tag{1.23}$$

$qcstar := rhs(\%\%\%[3]);$

$$qcstar := -\frac{m}{2} - \frac{3\,s}{2} + \frac{1}{2} + \frac{\sqrt{m^2 + 6\,m\,s + 9\,s^2 - 12\,F - 2\,m - 6\,s + 1}}{2} \tag{1.24}$$

$simplify(qastar, \mathrm{sqrt}, symbolic);$

$$\frac{1}{6} - \frac{m}{6} + \frac{s}{2} - \frac{\sqrt{m^2 + (6\,s - 2)\,m + 9\,s^2 - 12\,F - 6\,s + 1}}{6} \tag{1.25}$$

$Qstarsmixed := subs(q[a] = (qastar), q[b] = (qbstar), q[c] = (qcstar), m = 0, t = 0, F = 0, Q);$

$$Qstarsmixed := \frac{5}{6} - \frac{s}{2} + \frac{\sqrt{9\,s^2 - 6\,s + 1}}{6} \tag{1.26}$$

$Pfinal := subs(q[a] = (qastar), q[b] = (qbstar), q[c] = (qcstar), P);$

$$Pfinal := \frac{1}{6} + \frac{5\,m}{6} + \frac{s}{2} - \frac{\sqrt{m^2 + 6\,m\,s + 9\,s^2 - 12\,F - 2\,m - 6\,s + 1}}{6} \tag{1.27}$$

$simplify(\%, \mathrm{sqrt}, symbolic);$

$$\frac{1}{6} + \frac{5\,m}{6} + \frac{s}{2} - \frac{\sqrt{m^2 + (6\,s - 2)\,m + 9\,s^2 - 12\,F - 6\,s + 1}}{6} \tag{1.28}$$

$profitafinal := subs(q[a] = (qastar), q[b] = (qbstar), q[c] = (qcstar), profit[a]);$

$$profitafinal := \left(\frac{1}{6} + \frac{5\,m}{6} + \frac{s}{2} - \frac{\sqrt{m^2 + 6\,m\,s + 9\,s^2 - 12\,F - 2\,m - 6\,s + 1}}{6}\right)\left(\frac{1}{6} - \frac{m}{6} + \frac{s}{2} - \frac{\sqrt{m^2 + 6\,m\,s + 9\,s^2 - 12\,F - 2\,m - 6\,s + 1}}{6}\right) - m\left(\frac{1}{6} - \frac{m}{6} + \frac{s}{2} - \frac{\sqrt{m^2 + 6\,m\,s + 9\,s^2 - 12\,F - 2\,m - 6\,s + 1}}{6}\right) - F \tag{1.29}$$

$simplify(\%, \mathrm{sqrt}, symbolic);$

$$\frac{(m - 3\,s - 1)\sqrt{m^2 + (6\,s - 2)\,m + 9\,s^2 - 12\,F - 6\,s + 1}}{18} + \frac{m^2}{18} + \frac{s^2}{2} - \frac{4\,F}{3} - \frac{m}{9} + \frac{1}{18} \tag{1.30}$$

$profitcfinal := subs(q[a] = (qastar), q[b] = (qbstar), q[c] = (qcstar), profit[c]);$

$$profitcfinal := \left(\frac{1}{6} + \frac{5\,m}{6} + \frac{s}{2} - \frac{\sqrt{m^2 + 6\,m\,s + 9\,s^2 - 12\,F - 2\,m - 6\,s + 1}}{6}\right)\left(-\frac{m}{2} - \frac{3\,s}{2} + \frac{1}{2} + \frac{\sqrt{m^2 + 6\,m\,s + 9\,s^2 - 12\,F - 2\,m - 6\,s + 1}}{2}\right) - m\left(-\frac{m}{2} - \frac{3\,s}{2} + \frac{1}{2} + \frac{\sqrt{m^2 + 6\,m\,s + 9\,s^2 - 12\,F - 2\,m - 6\,s + 1}}{2}\right) - s\left(-\frac{m}{2} - \frac{3\,s}{2} + \frac{1}{2}\right) \tag{1.31}$$

$$+\frac{\sqrt{m^2+6\,m\,s+9\,s^2-12\,F-2\,m-6\,s+1}}{2}\Bigg)-F$$

simplify(%, sqrt, *symbolic*);

$$0 \tag{1.32}$$

Overallprofitfinal := *subs*(*q*[*a*] = (*qastar*), *q*[*b*] = (*qbstar*), *q*[*c*] = (*qcstar*), (*profit*[*a*] + *profit*[*b*] + *profit*[*c*]));

$$\begin{aligned}\mathit{Overallprofitfinal} := 2\Bigg(&\frac{1}{6}+\frac{5\,m}{6}+\frac{s}{2}\\&-\frac{\sqrt{m^2+6\,m\,s+9\,s^2-12\,F-2\,m-6\,s+1}}{6}\Bigg)\Bigg(\frac{1}{6}-\frac{m}{6}+\frac{s}{2}\\&-\frac{\sqrt{m^2+6\,m\,s+9\,s^2-12\,F-2\,m-6\,s+1}}{6}\Bigg)-2\,m\Bigg(\frac{1}{6}-\frac{m}{6}+\frac{s}{2}\\&-\frac{\sqrt{m^2+6\,m\,s+9\,s^2-12\,F-2\,m-6\,s+1}}{6}\Bigg)-3\,F+\Bigg(\frac{1}{6}+\frac{5\,m}{6}+\frac{s}{2}\\&-\frac{\sqrt{m^2+6\,m\,s+9\,s^2-12\,F-2\,m-6\,s+1}}{6}\Bigg)\Bigg(-\frac{m}{2}-\frac{3\,s}{2}+\frac{1}{2}\\&+\frac{\sqrt{m^2+6\,m\,s+9\,s^2-12\,F-2\,m-6\,s+1}}{2}\Bigg)-m\Bigg(-\frac{m}{2}-\frac{3\,s}{2}+\frac{1}{2}\\&+\frac{\sqrt{m^2+6\,m\,s+9\,s^2-12\,F-2\,m-6\,s+1}}{2}\Bigg)-s\Bigg(-\frac{m}{2}-\frac{3\,s}{2}+\frac{1}{2}\\&+\frac{\sqrt{m^2+6\,m\,s+9\,s^2-12\,F-2\,m-6\,s+1}}{2}\Bigg)\end{aligned} \tag{1.33}$$

simplify(%, sqrt, *symbolic*);

$$\frac{(m-3\,s-1)\sqrt{m^2+(6\,s-2)\,m+9\,s^2-12\,F-6\,s+1}}{9}+\frac{m^2}{9}+s^2-\frac{8\,F}{3}-\frac{2\,m}{9}+\frac{1}{9} \tag{1.34}$$

Overallprofitfinal := *subs*(*q*[*a*] = (*qastar*), *q*[*b*] = (*qbstar*), *q*[*c*] = (*qcstar*), (*profit*[*a*] + *profit*[*b*] + *profit*[*c*]));

$$\begin{aligned}\mathit{Overallprofitfinal} := 2\Bigg(&\frac{1}{6}+\frac{5\,m}{6}+\frac{s}{2}\\&-\frac{\sqrt{m^2+6\,m\,s+9\,s^2-12\,F-2\,m-6\,s+1}}{6}\Bigg)\Bigg(\frac{1}{6}-\frac{m}{6}+\frac{s}{2}\\&-\frac{\sqrt{m^2+6\,m\,s+9\,s^2-12\,F-2\,m-6\,s+1}}{6}\Bigg)-2\,m\Bigg(\frac{1}{6}-\frac{m}{6}+\frac{s}{2}\\&-\frac{\sqrt{m^2+6\,m\,s+9\,s^2-12\,F-2\,m-6\,s+1}}{6}\Bigg)-3\,F+\Bigg(\frac{1}{6}+\frac{5\,m}{6}+\frac{s}{2}\end{aligned} \tag{1.35}$$

$$-\frac{\sqrt{m^2+6\,m\,s+9\,s^2-12\,F-2\,m-6\,s+1}}{6}\Bigg)\left(-\frac{m}{2}-\frac{3\,s}{2}+\frac{1}{2}+\frac{\sqrt{m^2+6\,m\,s+9\,s^2-12\,F-2\,m-6\,s+1}}{2}\right)-m\left(-\frac{m}{2}-\frac{3\,s}{2}+\frac{1}{2}+\frac{\sqrt{m^2+6\,m\,s+9\,s^2-12\,F-2\,m-6\,s+1}}{2}\right)-s\left(-\frac{m}{2}-\frac{3\,s}{2}+\frac{1}{2}+\frac{\sqrt{m^2+6\,m\,s+9\,s^2-12\,F-2\,m-6\,s+1}}{2}\right)$$

simplify(%, sqrt, *symbolic*);

$$\frac{(m-3\,s-1)\sqrt{m^2+(6\,s-2)\,m+9\,s^2-12\,F-6\,s+1}}{9}+\frac{m^2}{9}+s^2-\frac{8\,F}{3}-\frac{2\,m}{9}+\frac{1}{9} \quad \textbf{(1.36)}$$

$Qstarsmixed := subs(q[a]=(qastar), q[b]=(qbstar), q[c]=(qcstar), Q);$

$$Qstarsmixed := \frac{5}{6}-\frac{5\,m}{6}-\frac{s}{2}+\frac{\sqrt{m^2+6\,m\,s+9\,s^2-12\,F-2\,m-6\,s+1}}{6} \quad \textbf{(1.37)}$$

$CS := \frac{(\%)^2}{2};$

$$CS := \frac{\left(\frac{5}{6}-\frac{5\,m}{6}-\frac{s}{2}+\frac{\sqrt{m^2+6\,m\,s+9\,s^2-12\,F-2\,m-6\,s+1}}{6}\right)^2}{2} \quad \textbf{(1.38)}$$

simplify(%, sqrt, *symbolic*);

$$\frac{\left(5-5\,m-3\,s+\sqrt{m^2+(6\,s-2)\,m+9\,s^2-12\,F-6\,s+1}\right)^2}{72} \quad \textbf{(1.39)}$$

$Qstarsmixm := subs(q[a]=(qastar), q[b]=(qbstar), q[c]=(qcstar), F=0, Q);$

$$Qstarsmixm := \frac{5}{6}-\frac{5\,m}{6}-\frac{s}{2}+\frac{\sqrt{m^2+6\,m\,s+9\,s^2-2\,m-6\,s+1}}{6} \quad \textbf{(1.40)}$$

$Qprivatestarsmixm := subs(q[a]=(qastar), q[b]=(qbstar), q[c]=(qcstar), F=0, Q-q[c]);$

$$Qprivatestarsmixm := \frac{1}{3}-\frac{m}{3}+s-\frac{\sqrt{m^2+6\,m\,s+9\,s^2-2\,m-6\,s+1}}{3} \quad \textbf{(1.41)}$$

$Qpublicstarsmixm := subs(q[a]=(qastar), q[b]=(qbstar), q[c]=(qcstar), F=0, q[c]);$

$$Qpublicstarsmixm := -\frac{m}{2}-\frac{3\,s}{2}+\frac{1}{2}+\frac{\sqrt{m^2+6\,m\,s+9\,s^2-2\,m-6\,s+1}}{2} \quad \textbf{(1.42)}$$

$Qstarsmixzero := subs(q[a]=(qastar), q[b]=(qbstar), q[c]=(qcstar), m=0, F=0, Q);$

$$Qstarsmixzero := \frac{5}{6}-\frac{s}{2}+\frac{\sqrt{9\,s^2-6\,s+1}}{6} \quad \textbf{(1.43)}$$

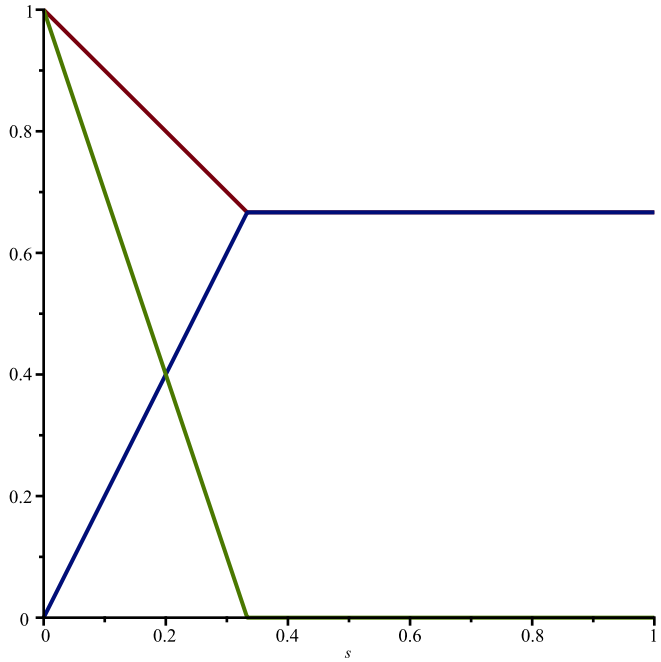
$Qprivatestarsmixzero := subs(q[a]=(qastar), q[b]=(qbstar), q[c]=(qcstar), m=0, F=0, Q-q[c]);$

$$Qprivatestarsmixzero := \frac{1}{3}+s-\frac{\sqrt{9\,s^2-6\,s+1}}{3} \quad \textbf{(1.44)}$$

$Qpublicstarsmixzero := subs(q[a] = (qastar), q[b] = (qbstar), q[c] = (qcstar), m = 0, F = 0, q[c]);$

$$Qpublicstarsmixzero := \frac{1}{2} - \frac{3\,s}{2} + \frac{\sqrt{9\,s^2 - 6\,s + 1}}{2} \tag{1.45}$$

$plot([Qstarsmixzero, Qprivatestarsmixzero, Qpublicstarsmixzero], s = 0 .. 1);$



$\#evala(Simplify(\%));$
$subs(q[a] = qastar, q[b] = qbstar, q[c] = qcstar, SS);$

$$\begin{aligned}
&-2\left(\frac{1}{6} - \frac{m}{6} + \frac{s}{2} - \frac{\sqrt{m^2 + 6\,m\,s + 9\,s^2 - 12\,F - 2\,m - 6\,s + 1}}{6}\right)^2 - 2\left(\frac{1}{6} - \frac{m}{6} + \frac{s}{2}\right. \\
&\left. - \frac{\sqrt{m^2 + 6\,m\,s + 9\,s^2 - 12\,F - 2\,m - 6\,s + 1}}{6}\right)\left(-\frac{m}{2} - \frac{3\,s}{2} + \frac{1}{2}\right. \\
&\left. + \frac{\sqrt{m^2 + 6\,m\,s + 9\,s^2 - 12\,F - 2\,m - 6\,s + 1}}{2}\right) \\
&- \frac{\left(-\frac{m}{2} - \frac{3\,s}{2} + \frac{1}{2} + \frac{\sqrt{m^2 + 6\,m\,s + 9\,s^2 - 12\,F - 2\,m - 6\,s + 1}}{2}\right)^2}{2} + \frac{5}{6} - \frac{5\,m}{6} \\
&- \frac{s}{2} + \frac{\sqrt{m^2 + 6\,m\,s + 9\,s^2 - 12\,F - 2\,m - 6\,s + 1}}{6} - 2\,m\left(\frac{1}{6} - \frac{m}{6} + \frac{s}{2}\right. \\
&\left. - \frac{\sqrt{m^2 + 6\,m\,s + 9\,s^2 - 12\,F - 2\,m - 6\,s + 1}}{6}\right) - 3\,F - m\left(-\frac{m}{2} - \frac{3\,s}{2} + \frac{1}{2}\right. \\
&\left. + \frac{\sqrt{m^2 + 6\,m\,s + 9\,s^2 - 12\,F - 2\,m - 6\,s + 1}}{2}\right) - s\left(-\frac{m}{2} - \frac{3\,s}{2} + \frac{1}{2}\right. \\
&\left. + \frac{\sqrt{m^2 + 6\,m\,s + 9\,s^2 - 12\,F - 2\,m - 6\,s + 1}}{2}\right)
\end{aligned} \tag{1.46}$$

$SSstar := evala(Simplify(\%));$

$$SSstar := -\frac{s}{2} + \frac{m\,s}{2} + \frac{17\,m^2}{36} - \frac{m\sqrt{m^2+6\,m\,s+9\,s^2-12\,F-2\,m-6\,s+1}}{36} - \frac{5\,s\sqrt{m^2+6\,m\,s+9\,s^2-12\,F-2\,m-6\,s+1}}{12} + \frac{5\,s^2}{4} - \frac{17\,F}{6} - \frac{17\,m}{18} + \frac{17}{36} + \frac{\sqrt{m^2+6\,m\,s+9\,s^2-12\,F-2\,m-6\,s+1}}{36} \tag{1.47}$$

simplify(%, sqrt, *symbolic*);

$$\frac{(-m-15\,s+1)\sqrt{m^2+(6\,s-2)\,m+9\,s^2-12\,F-6\,s+1}}{36} + \frac{17\,m^2}{36} + \frac{(18\,s-34)\,m}{36} + \frac{5\,s^2}{4} - \frac{17\,F}{6} - \frac{s}{2} + \frac{17}{36} \tag{1.48}$$

SSstar000 := *subs*(*q*[*a*] = (*qastar*), *q*[*b*] = (*qbstar*), *q*[*c*] = (*qcstar*), *m* = 0, *F* = 0, *SSstar*);

$$SSstar000 := -\frac{s}{2} + \frac{17}{36} - \frac{5\,s\sqrt{9\,s^2-6\,s+1}}{12} + \frac{5\,s^2}{4} + \frac{\sqrt{9\,s^2-6\,s+1}}{36} \tag{1.49}$$

% = $\frac{4}{9}$;

$$-\frac{s}{2} + \frac{17}{36} - \frac{5\,s\sqrt{9\,s^2-6\,s+1}}{12} + \frac{5\,s^2}{4} + \frac{\sqrt{9\,s^2-6\,s+1}}{36} = \frac{4}{9} \tag{1.50}$$

isolate(%, *s*);

$$s = \frac{1}{15} \tag{1.51}$$

SSstar000 = $\frac{15}{32}$;

$$-\frac{s}{2} + \frac{17}{36} - \frac{5\,s\sqrt{9\,s^2-6\,s+1}}{12} + \frac{5\,s^2}{4} + \frac{\sqrt{9\,s^2-6\,s+1}}{36} = \frac{15}{32} \tag{1.52}$$

solve(%, *s*);

$$RootOf\left(-120\,\_Z\sqrt{(-1+3\,\_Z)^2} + 360\,\_Z^2 + 8\sqrt{(-1+3\,\_Z)^2} - 144\,\_Z + 1\right) \tag{1.53}$$

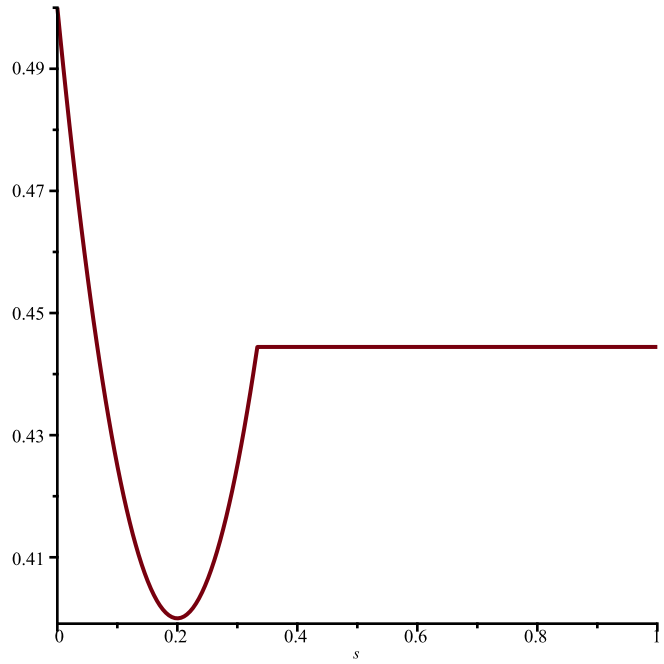
allvalues(%);

$$\frac{1}{5} - \frac{\sqrt{11}}{20} \tag{1.54}$$

SSstar00 := *subs*(*q*[*a*] = (*qastar*), *q*[*b*] = (*qbstar*), *q*[*c*] = (*qcstar*), *m* = 0, *SSstar*);

$$SSstar00 := -\frac{s}{2} + \frac{17}{36} - \frac{5\,s\sqrt{9\,s^2-12\,F-6\,s+1}}{12} + \frac{5\,s^2}{4} - \frac{17\,F}{6} + \frac{\sqrt{9\,s^2-12\,F-6\,s+1}}{36} \tag{1.55}$$

plot(*SSstar000*, *s* = 0..1);



$isolate(SSstar, s);$

$$s = -\frac{1}{10\left(-2m^2+9F+4m-2\right)}\left(-4m^3+23Fm+12m^2-23F\right. \tag{1}$$
$$-\left(-64m^6+1136Fm^4+384m^5-6681F^2m^2-4544Fm^3-960m^4+13005F^3+13362F^2m+6816F\right.$$
$$\left.\left.+1280m^3-6681F^2-4544Fm-960m^2+1136F+384m-64\right)^{1/2}-12m+4\right)$$

$mbreak[c] := rhs(evala(Simplify(\%)));$

$$mbreak_c := -\frac{1}{10\left(-2m^2+9F+4m-2\right)}\left(-4m^3+23Fm+12m^2-23F\right. \tag{1.57}$$
$$\left.-\sqrt{\left(-m^2+5F+2m-1\right)\left(-8m^2+51F+16m-8\right)^2}-12m+4\right)$$

$subs(F=0, \%);$

$$-\frac{-4m^3+4+12m^2-\sqrt{\left(-m^2+2m-1\right)\left(-8m^2+16m-8\right)^2}-12m}{10\left(-2m^2+4m-2\right)} \tag{1.58}$$

$SSdiffmc := diff(SSstar, s);$

$$SSdiffmc := -\frac{1}{2}+\frac{m}{2}-\frac{m(6m+18s-6)}{72\sqrt{m^2+6ms+9s^2-12F-2m-6s+1}} \tag{1.59}$$
$$-\frac{5\sqrt{m^2+6ms+9s^2-12F-2m-6s+1}}{12}$$
$$-\frac{5s(6m+18s-6)}{24\sqrt{m^2+6ms+9s^2-12F-2m-6s+1}}+\frac{5s}{2}$$
$$+\frac{6m+18s-6}{72\sqrt{m^2+6ms+9s^2-12F-2m-6s+1}}$$

$subs(m=0.2, s=0.3, F=0, SSstar);$

$$0.2844444444 \tag{1.60}$$

$subs(m=0.1, s=0.2, F=0, SSstar);$

$$0.3250000000 \tag{1.61}$$

$subs(m=0.1, s=0.3, F=0, SSstar);$

$$0.3600000000 \tag{1.62}$$

$subs(m=0.1, s=0.28, F=0, SSstar);$

$$0.3490000000 \tag{1.63}$$

$subs(m=0.1, s=0.11, F=0, SSstar);$

$$0.3362500000 \tag{1.64}$$

$SSstarplot01 := subs(m=0.1, SSstar);$

$$SSstarplot01 := -0.4500000000\, s + 0.3825000000 + 0.02500000000 \sqrt{9\, s^2 - 12\, F - 5.4\, s + 0.81} - \frac{5\, s \sqrt{9\, s^2 - 12\, F - 5.4\, s + 0.81}}{12} + \frac{5\, s^2}{4} - \frac{17\, F}{6} \tag{1.65}$$

$convert(\%, rational);$

$$-\frac{9\, s}{20} + \frac{153}{400} + \frac{\sqrt{9\, s^2 - 12\, F - \frac{27}{5}\, s + \frac{81}{100}}}{40} - \frac{5\, s \sqrt{9\, s^2 - 12\, F - \frac{27}{5}\, s + \frac{81}{100}}}{12} + \frac{5\, s^2}{4} - \frac{17\, F}{6} \tag{1.66}$$

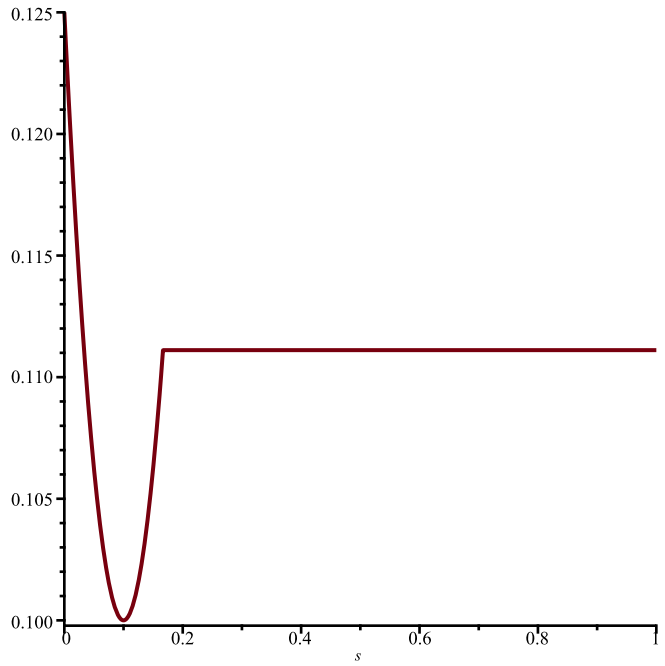
$SSstarplot050 := subs(m=0.5, F=0, SSstar);$

$$SSstarplot050 := -0.2500000000\, s + 0.1180555556 + 0.01388888889 \sqrt{9\, s^2 + 0.25 - 3.0\, s} - \frac{5\, s \sqrt{9\, s^2 + 0.25 - 3.0\, s}}{12} + \frac{5\, s^2}{4} \tag{1.67}$$

$convert(\%, rational);$

$$-\frac{s}{4} + \frac{17}{144} + \frac{\sqrt{9\, s^2 + \frac{1}{4} - 3\, s}}{72} - \frac{5\, s \sqrt{9\, s^2 + \frac{1}{4} - 3\, s}}{12} + \frac{5\, s^2}{4} \tag{1.68}$$

$plot(SSstarplot050,\ s=0..1);$



$SSstarplot00 := subs(m = 0, SSstar);$

$$SSstarplot00 := -\frac{s}{2} + \frac{17}{36} - \frac{5\,s\sqrt{9\,s^2 - 12\,F - 6\,s + 1}}{12} + \frac{5\,s^2}{4} - \frac{17\,F}{6} + \frac{\sqrt{9\,s^2 - 12\,F - 6\,s + 1}}{36} \tag{1.69}$$

$SSstarplot00 = \frac{15}{32} - 3\,F;$

$$-\frac{s}{2} + \frac{17}{36} - \frac{5\,s\sqrt{9\,s^2 - 12\,F - 6\,s + 1}}{12} + \frac{5\,s^2}{4} - \frac{17\,F}{6} + \frac{\sqrt{9\,s^2 - 12\,F - 6\,s + 1}}{36} = \frac{15}{32} - 3\,F \tag{1.70}$$

$solve(\%, s);$

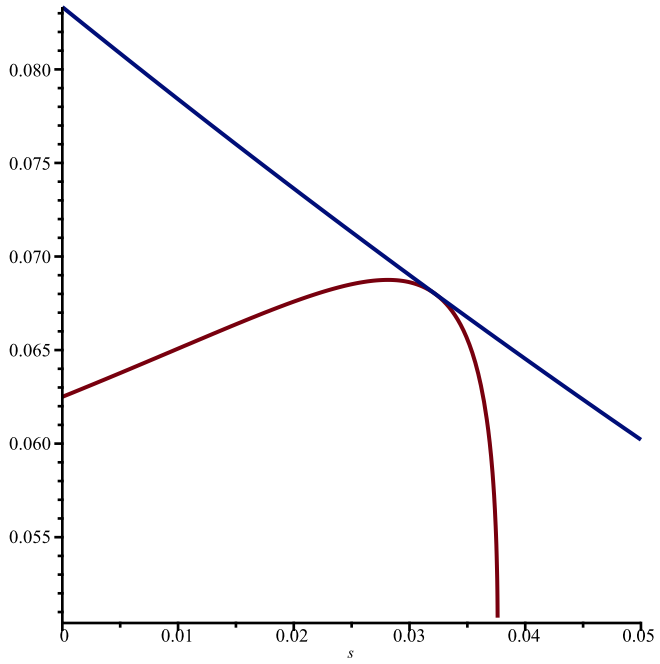
$$\frac{512\,F - 28 + \sqrt{-368640\,F^3 + 132864\,F^2 - 15232\,F + 539}}{20\,(288\,F - 7)}, \frac{512\,F - 28 - \sqrt{-368640\,F^3 + 132864\,F^2 - 15232\,F + 539}}{20\,(288\,F - 7)} \tag{1.71}$$

$solve(\%\%, F);$

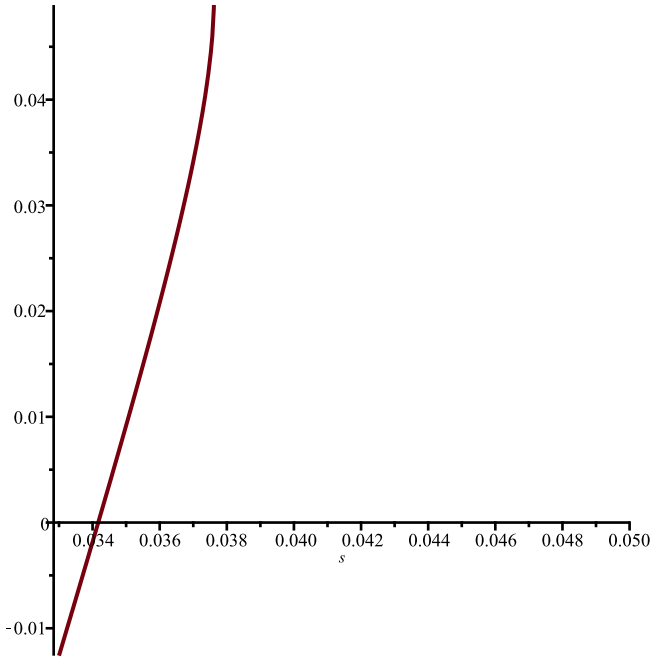
$$-\frac{15\,s^2}{2} + \frac{5\left(-15\,s + 1 + \frac{3\sqrt{144\,s^2 - 32\,s + 1}}{2}\right)s}{2} + \frac{11\,s}{2} - \frac{3}{16} - \frac{\sqrt{144\,s^2 - 32\,s + 1}}{4}, -\frac{15\,s^2}{2} + \frac{5\left(-15\,s + 1 - \frac{3\sqrt{144\,s^2 - 32\,s + 1}}{2}\right)s}{2} + \frac{11\,s}{2} \tag{1.72}$$

$$-\frac{3}{16}+\frac{\sqrt{144\,s^2-32\,s+1}}{4}$$

$$plot\left(\left[\%[2],\ \frac{(1-3\,s)^2}{12}\right],\ s=0.0\ ..0.05\right);$$



$$plot(\%\%[1],\ s=0.033\ ..0.05);$$



$$\#plot(SSstarplot00,\ \ s=0\ ..1);$$

$$SSstarplot00=\frac{4}{9}-2\,F;$$

$$-\frac{s}{2}+\frac{17}{36}-\frac{5\,s\sqrt{9\,s^2-12\,F-6\,s+1}}{12}+\frac{5\,s^2}{4}-\frac{17\,F}{6}+\frac{\sqrt{9\,s^2-12\,F-6\,s+1}}{36}=\frac{4}{9} \qquad \textbf{(1.73)}$$

$$-2\,F$$

```
solve(%, s);
```

$$-\frac{5\,F}{4}+\frac{1}{15} \tag{1.74}$$

```
solve(%%, F);
```

$$-\frac{4\,s}{5}+\frac{4}{75},\ 0 \tag{1.75}$$

```
plot([%[1], (1-3*s)^2/12], s=0.0..0.1);
```