restart; #Pure triopoly capacity constraint

$Q := q[a] + q[b] + q[c]$;

$$Q := q_a + q_b + q_c \tag{1}$$

$P := 1 - Q$;

$$P := 1 - q_a - q_b - q_c \tag{2}$$

$TC[a] := m \cdot q[a] + (q[a] - x[a])^2 + F$;

$$TC_a := m\, q_a + (q_a - x_a)^2 + F \tag{3}$$

$TC[b] := m \cdot q[b] + (q[b] - x[b])^2 + F$;

$$TC_b := m\, q_b + (q_b - x_b)^2 + F \tag{4}$$

$TC[c] := m \cdot q[c] + (q[c] - x[c])^2 + F$;

$$TC_c := m\, q_c + (q_c - x_c)^2 + F \tag{5}$$

$profit[a] := q[a] * P - TC[a]$;

$$profit_a := (1 - q_a - q_b - q_c)\, q_a - m\, q_a - (q_a - x_a)^2 - F \tag{6}$$

$profit[b] := q[b] * P - TC[b]$;

$$profit_b := (1 - q_a - q_b - q_c)\, q_b - m\, q_b - (q_b - x_b)^2 - F \tag{7}$$

$profit[c] := q[c] * P - TC[c]$;

$$profit_c := (1 - q_a - q_b - q_c)\, q_c - m\, q_c - (q_c - x_c)^2 - F \tag{8}$$

$SS := \frac{Q^2}{2} + profit[a] + profit[b] + profit[c]$;

$$SS := \frac{(q_a + q_b + q_c)^2}{2} + (1 - q_a - q_b - q_c)\, q_a - m\, q_a - (q_a - x_a)^2 - 3\, F + (1 - q_a - q_b - q_c)\, q_b - m\, q_b - (q_b - x_b)^2 + (1 - q_a - q_b - q_c)\, q_c - m\, q_c - (q_c - x_c)^2 \tag{9}$$

$a > 0$;

$$0 < a \tag{10}$$

$P > 0$;

$$0 < 1 - q_a - q_b - q_c \tag{11}$$

$q[a] \geq 0$;

$$0 \leq q_a \tag{12}$$

$q[b] \geq 0$;

$$0 \leq q_b \tag{13}$$

$q[c] \geq 0$;

$$0 \leq q_c \tag{14}$$

$m \geq 0$;

$$0 \leq m \tag{15}$$

#Second stage: simultaneous quantity choice

$FOC[q[a]] := diff(profit[a], q[a]);$

$$FOC_{q_a} := -4\,q_a + 1 - q_b - q_c - m + 2\,x_a \tag{16}$$

$q[a](q[b], q[c], x[a]) := solve(FOC[q[a]], q[a]);$

$$q_a(q_b, q_c, x_a) := \frac{1}{4} - \frac{q_b}{4} - \frac{q_c}{4} - \frac{m}{4} + \frac{x_a}{2} \tag{17}$$

$FOC[q[b]] := diff(profit[b], q[b]);$

$$FOC_{q_b} := -4\,q_b + 1 - q_a - q_c - m + 2\,x_b \tag{18}$$

$q[b](q[a], q[c], x[b]) := solve(\%, q[b]);$

$$q_b(q_a, q_c, x_b) := \frac{1}{4} - \frac{q_a}{4} - \frac{q_c}{4} - \frac{m}{4} + \frac{x_b}{2} \tag{19}$$

$FOC[q[c]] := diff(profit[c], q[c]);$

$$FOC_{q_c} := -4\,q_c + 1 - q_a - q_b - m + 2\,x_c \tag{20}$$

$q[c](q[a], q[b], x[c]) := solve(\%, q[c]);$

$$q_c(q_a, q_b, x_c) := \frac{1}{4} - \frac{q_a}{4} - \frac{q_b}{4} - \frac{m}{4} + \frac{x_c}{2} \tag{21}$$

$SS;$

$$\frac{(q_a + q_b + q_c)^2}{2} + (1 - q_a - q_b - q_c)\,q_a - m\,q_a - (q_a - x_a)^2 - 3\,F + (1 - q_a - q_b - q_c)\,q_b - m\,q_b - (q_b - x_b)^2 + (1 - q_a - q_b - q_c)\,q_c - m\,q_c - (q_c - x_c)^2 \tag{22}$$

$\#\#FOC[q[c]] := diff(profit[c], q[c]);$
$\#\#q[c](q[a], q[c], x[b]) := solve(\%, q[c]);$
$\#\#FOCN := piecewise(0 \le \%, FOC[q[c]]);$
$sys := \{ FOC[q[a]], FOC[q[b]], FOC[q[c]], \};$

$dsolve(\ sys, \{q[a], q[b], q[c]\});$
$sys := \{-4\,q_a + 1 - q_b - q_c - m + 2\,x_a, -4\,q_b + 1 - q_a - q_c - m + 2\,x_b, -4\,q_c + 1 - q_a - q_b - m + 2\,x_c\}$

$$\left\{q_a = \frac{1}{6} + \frac{5\,x_a}{9} - \frac{x_b}{9} - \frac{m}{6} - \frac{x_c}{9}, q_b = \frac{1}{6} - \frac{x_a}{9} + \frac{5\,x_b}{9} - \frac{m}{6} - \frac{x_c}{9}, q_c = \frac{1}{6} - \frac{x_a}{9} - \frac{x_b}{9} - \frac{m}{6} + \frac{5\,x_c}{9}\right\} \tag{23}$$

$q[a](x[a], x[b], x[c]) := rhs(\%[1]);$

$$q_a(x_a, x_b, x_c) := \frac{1}{6} + \frac{5\,x_a}{9} - \frac{x_b}{9} - \frac{m}{6} - \frac{x_c}{9} \tag{24}$$

$q[b](x[a], x[b], x[c]) := rhs(\%\%[2]);$

$$q_b(x_a, x_b, x_c) := \frac{1}{6} - \frac{x_a}{9} + \frac{5\,x_b}{9} - \frac{m}{6} - \frac{x_c}{9} \tag{25}$$

$q[c](x[a], x[b], x[c]) := rhs(\%\%\%[3]);$

$$q_c(x_a, x_b, x_c) := \frac{1}{6} - \frac{x_a}{9} - \frac{x_b}{9} - \frac{m}{6} + \frac{5\,x_c}{9} \tag{26}$$

#First stage: simultaneous capacity choice

profit[*a*];

$$(1 - q_a - q_b - q_c)\,q_a - m\,q_a - (q_a - x_a)^2 - F \tag{27}$$

subs(*q*[*a*] = *q*[*a*](*x*[*a*], *x*[*b*], *x*[*c*]), *q*[*b*] = *q*[*b*](*x*[*a*], *x*[*b*], *x*[*c*]), *q*[*c*] = *q*[*c*](*x*[*a*], *x*[*b*], *x*[*c*]), *profit*[*a*]);

$$\left(\frac{1}{2} - \frac{x_a}{3} - \frac{x_b}{3} + \frac{m}{2} - \frac{x_c}{3}\right)\left(\frac{1}{6} + \frac{5\,x_a}{9} - \frac{x_b}{9} - \frac{m}{6} - \frac{x_c}{9}\right) - m\left(\frac{1}{6} + \frac{5\,x_a}{9} - \frac{x_b}{9} - \frac{m}{6} - \frac{x_c}{9}\right) - \left(\frac{1}{6} - \frac{4\,x_a}{9} - \frac{x_b}{9} - \frac{m}{6} - \frac{x_c}{9}\right)^2 - F \tag{28}$$

FOC[*x*[*a*]] := *diff*(%, *x*[*a*]);

$$FOC_{x_a} := \frac{10}{27} - \frac{62\,x_a}{81} - \frac{20\,x_b}{81} - \frac{10\,m}{27} - \frac{20\,x_c}{81} \tag{29}$$

x[*a*](*x*[*b*], *x*[*c*]) := *rhs*(*isolate*(%, *x*[*a*]));

$$x_a(x_b, x_c) := \frac{15}{31} - \frac{10\,x_b}{31} - \frac{15\,m}{31} - \frac{10\,x_c}{31} \tag{30}$$

profit[*b*];

$$(1 - q_a - q_b - q_c)\,q_b - m\,q_b - (q_b - x_b)^2 - F \tag{31}$$

subs(*q*[*a*] = *q*[*a*](*x*[*a*], *x*[*b*], *x*[*c*]), *q*[*b*] = *q*[*b*](*x*[*a*], *x*[*b*], *x*[*c*]), *q*[*c*] = *q*[*c*](*x*[*a*], *x*[*b*], *x*[*c*]), *profit*[*b*]);

$$\left(\frac{1}{2} - \frac{x_a}{3} - \frac{x_b}{3} + \frac{m}{2} - \frac{x_c}{3}\right)\left(\frac{1}{6} - \frac{x_a}{9} + \frac{5\,x_b}{9} - \frac{m}{6} - \frac{x_c}{9}\right) - m\left(\frac{1}{6} - \frac{x_a}{9} + \frac{5\,x_b}{9} - \frac{m}{6} - \frac{x_c}{9}\right) - \left(\frac{1}{6} - \frac{x_a}{9} - \frac{4\,x_b}{9} - \frac{m}{6} - \frac{x_c}{9}\right)^2 - F \tag{32}$$

FOC[*x*[*b*]] := *diff*(%, *x*[*b*]);

$$FOC_{x_b} := \frac{10}{27} - \frac{20\,x_a}{81} - \frac{62\,x_b}{81} - \frac{10\,m}{27} - \frac{20\,x_c}{81} \tag{33}$$

evala(*Simplify*(%))

$$\frac{10}{27} - \frac{20\,x_a}{81} - \frac{62\,x_b}{81} - \frac{10\,m}{27} - \frac{20\,x_c}{81} \tag{34}$$

x[*b*](*x*[*a*], *x*[*c*]) := *rhs*(*isolate*(%, *x*[*b*]));

$$x_b(x_a, x_c) := \frac{15}{31} - \frac{10\,x_a}{31} - \frac{15\,m}{31} - \frac{10\,x_c}{31} \tag{35}$$

profit[*c*];

$$(1 - q_a - q_b - q_c)\,q_c - m\,q_c - (q_c - x_c)^2 - F \tag{36}$$

subs(*q*[*a*] = *q*[*a*](*x*[*a*], *x*[*b*], *x*[*c*]), *q*[*b*] = *q*[*b*](*x*[*a*], *x*[*b*], *x*[*c*]), *q*[*c*] = *q*[*c*](*x*[*a*], *x*[*b*], *x*[*c*]),

$profit[c]);$

$$\left(\frac{1}{2}-\frac{x_a}{3}-\frac{x_b}{3}+\frac{m}{2}-\frac{x_c}{3}\right)\left(\frac{1}{6}-\frac{x_a}{9}-\frac{x_b}{9}-\frac{m}{6}+\frac{5\,x_c}{9}\right)-m\left(\frac{1}{6}-\frac{x_a}{9}-\frac{x_b}{9}-\frac{m}{6}+\frac{5\,x_c}{9}\right)-\left(\frac{1}{6}-\frac{x_a}{9}-\frac{x_b}{9}-\frac{m}{6}-\frac{4\,x_c}{9}\right)^2-F \tag{37}$$

$FOC[x[c]] := diff(\%, x[c]);$

$$FOC_{x_c} := \frac{10}{27}-\frac{20\,x_a}{81}-\frac{20\,x_b}{81}-\frac{10\,m}{27}-\frac{62\,x_c}{81} \tag{38}$$

$evala(Simplify(\%))$

$$\frac{10}{27}-\frac{20\,x_a}{81}-\frac{20\,x_b}{81}-\frac{10\,m}{27}-\frac{62\,x_c}{81} \tag{39}$$

$x[c](x[a], x[b]) := rhs(isolate(\%, x[c]));$

$$x_c(x_a, x_b) := \frac{15}{31}-\frac{10\,x_a}{31}-\frac{10\,x_b}{31}-\frac{15\,m}{31} \tag{40}$$

$sys := \{FOC[x[a]], FOC[x[b]], FOC[x[c]]\};$

$$sys := \left\{\frac{10}{27}-\frac{62\,x_a}{81}-\frac{20\,x_b}{81}-\frac{10\,m}{27}-\frac{20\,x_c}{81}, \frac{10}{27}-\frac{20\,x_a}{81}-\frac{62\,x_b}{81}-\frac{10\,m}{27}-\frac{20\,x_c}{81}, \frac{10}{27}-\frac{20\,x_a}{81}-\frac{20\,x_b}{81}-\frac{10\,m}{27}-\frac{62\,x_c}{81}\right\} \tag{41}$$

$solve(sys, \{x[a], x[b], x[c]\});$

$$\left\{x_a = \frac{5}{17}-\frac{5\,m}{17}, x_b = \frac{5}{17}-\frac{5\,m}{17}, x_c = \frac{5}{17}-\frac{5\,m}{17}\right\} \tag{42}$$

$xstar[a] := rhs(\%[1]);$

$$xstar_a := \frac{5}{17}-\frac{5\,m}{17} \tag{43}$$

$xstar[b] := rhs(\%\%[2]);$

$$xstar_b := \frac{5}{17}-\frac{5\,m}{17} \tag{44}$$

$xstar[c] := rhs(\%\%\%[3]);$

$$xstar_c := \frac{5}{17}-\frac{5\,m}{17} \tag{45}$$

$qstar[a] := subs(x[a]=xstar[a], x[b]=xstar[b], x[c]=xstar[c], q[a](x[a], x[b], x[c]));$

$$qstar_a := \frac{9}{34}-\frac{9\,m}{34} \tag{46}$$

$q[b](x[a], x[b], x[c]);$

$$\frac{1}{6}-\frac{x_a}{9}+\frac{5\,x_b}{9}-\frac{m}{6}-\frac{x_c}{9} \tag{47}$$

$qstar[b] := subs(x[a]=xstar[a], x[b]=xstar[b], x[c]=xstar[c], q[b](x[a], x[b], x[c]));$

$$qstar_b := \frac{9}{34}-\frac{9\,m}{34} \tag{48}$$

$q[c](x[a], x[b], x[c]);$

$$\frac{1}{6} - \frac{x_a}{9} - \frac{x_b}{9} - \frac{m}{6} + \frac{5\,x_c}{9} \tag{49}$$

$qstar[c] := subs(x[a]=xstar[a], x[b]=xstar[b], x[c]=xstar[c], q[c](x[a], x[b], x[c]))$;

$$qstar_c := \frac{9}{34} - \frac{9\,m}{34} \tag{50}$$

$Qstar := qstar[a] + qstar[b] + qstar[c]$;

$$Qstar := \frac{27}{34} - \frac{27\,m}{34} \tag{51}$$

$Pstar := subs(q[a]=qstar[a], q[b]=qstar[b], q[c]=qstar[c], P)$;

$$Pstar := \frac{7}{34} + \frac{27\,m}{34} \tag{52}$$

$Xstar := xstar[a] + xstar[b] + xstar[c]$;

$$Xstar := \frac{15}{17} - \frac{15\,m}{17} \tag{53}$$

$subs(q[a]=qstar[a], q[b]=qstar[b], q[c]=qstar[c], x[a]=xstar[a], x[b]=xstar[b], x[c]=xstar[c], SS)$;

$$\frac{\left(\frac{27}{34} - \frac{27\,m}{34}\right)^2}{2} + 3\left(\frac{7}{34} + \frac{27\,m}{34}\right)\left(\frac{9}{34} - \frac{9\,m}{34}\right) - 3\,m\left(\frac{9}{34} - \frac{9\,m}{34}\right) - 3\left(-\frac{1}{34} + \frac{m}{34}\right)^2 - 3\,F \tag{54}$$

$SSfinal := simplify(\%, \text{sqrt}, symbolic)$;

$$SSfinal := \frac{1101}{2312} - \frac{1101}{1156}\,m + \frac{1101}{2312}\,m^2 - 3\,F \tag{55}$$

$profitfinal := subs(q[a]=qstar[a], q[b]=qstar[b], q[c]=qstar[c], x[a]=xstar[a], x[b]=xstar[b], x[c]=xstar[c], profit[a])$;

$$profitfinal := \left(\frac{7}{34} + \frac{27\,m}{34}\right)\left(\frac{9}{34} - \frac{9\,m}{34}\right) - m\left(\frac{9}{34} - \frac{9\,m}{34}\right) - \left(-\frac{1}{34} + \frac{m}{34}\right)^2 - F \tag{56}$$

$simplify(\%, symbolic)$;

$$\frac{31}{578} - \frac{31}{289}\,m + \frac{31}{578}\,m^2 - F \tag{57}$$

$PS := \%\cdot 3$;

$$PS := \frac{93}{578} - \frac{93}{289}\,m + \frac{93}{578}\,m^2 - 3\,F \tag{58}$$

$CS := \frac{(Qstar)^2}{2}$;

$$CS := \frac{\left(\frac{27}{34} - \frac{27\,m}{34}\right)^2}{2} \tag{59}$$

$simplify(\%, symbolic)$;

$$\frac{729\,(-1+m)^2}{2312} \tag{60}$$

$CS + PS;$

$$\frac{\left(\frac{27}{34} - \frac{27\,m}{34}\right)^2}{2} + \frac{93}{578} - \frac{93\,m}{289} + \frac{93\,m^2}{578} - 3\,F \tag{61}$$

$simplify(\%, symbolic);$

$$\frac{1101}{2312} - \frac{1101}{1156}\,m + \frac{1101}{2312}\,m^2 - 3\,F \tag{62}$$

$subs(m=0, F=0, \%);\ evalf(\%, 10);$

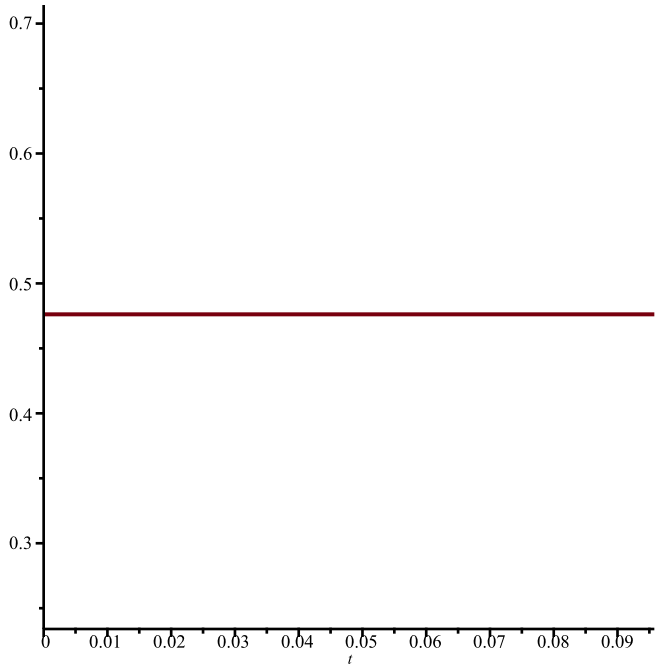
$$\frac{1101}{2312}$$

$$0.4762110727 \tag{63}$$

$subs(m=0, F=0, SSfinal);$

$$\frac{1101}{2312} \tag{64}$$

$plot(\%,\ t=0\ ..\ (0.095775737));$



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