

restart;

# *Mixed triopoly with a no negative profit constraint and a transfer*

$$Q := q[a] + q[b] + q[c];$$

$$Q := q_a + q_b + q_c \quad (1.1)$$

$$P := 1 - Q;$$

$$P := 1 - q_a - q_b - q_c \quad (1.2)$$

$$TC[a] := m \cdot q[a] + F;$$

$$TC_a := m q_a + F \quad (1.3)$$

$$TC[b] := m \cdot q[b] + F;$$

$$TC_b := m q_b + F \quad (1.4)$$

$$TC[c] := m \cdot q[c] + t \cdot q[c] + F;$$

$$TC_c := m q_c + t q_c + F \quad (1.5)$$

$$profit[a] := q[a] * P - TC[a];$$

$$profit_a := (1 - q_a - q_b - q_c) q_a - m q_a - F \quad (1.6)$$

$$profit[b] := q[b] * P - TC[b];$$

$$profit_b := (1 - q_a - q_b - q_c) q_b - m q_b - F \quad (1.7)$$

$$profit[c] := q[c] * P - TC[c];$$

$$profit_c := (1 - q_a - q_b - q_c) q_c - m q_c - t q_c - F \quad (1.8)$$

$$\#SS := \frac{Q^2}{2} + profit[a] + profit[b] + profit[c] + t \cdot q[c];$$

$$SS := factor\left(\frac{Q^2}{2} + profit[a] + profit[b] + profit[c] + t \cdot q[c]\right);$$

$$SS := -\frac{1}{2} q_a^2 - q_a q_b - q_a q_c - \frac{1}{2} q_b^2 - q_b q_c - \frac{1}{2} q_c^2 + q_a - m q_a - 3 F + q_b - m q_b + q_c - m q_c \quad (1.9)$$

$$P > 0, q[a] > 0, q[b] > 0, q[c] > 0, m > 0;$$

$$0 < 1 - q_a - q_b - q_c, 0 < q_a, 0 < q_b, 0 < q_c, 0 < m \quad (1.10)$$

#*Reaction functions*

$$FOC[q[a]] := diff(profit[a], q[a]);$$

$$FOC_{q_a} := -2 q_a + 1 - q_b - q_c - m \quad (1.11)$$

$$q[a](q[b], q[c]) := solve(FOC[q[a]], q[a]);$$

$$q_a(q_b, q_c) := \frac{1}{2} - \frac{q_b}{2} - \frac{q_c}{2} - \frac{m}{2} \quad (1.12)$$

$$FOC[q[b]] := diff(profit[b], q[b]);$$

$$FOC_{q_b} := -2 q_b + 1 - q_a - q_c - m \quad (1.13)$$

$$q[b](q[a], q[c]) := solve(\%, q[b]);$$

$$q_b(q_a, q_c) := \frac{1}{2} - \frac{q_a}{2} - \frac{q_c}{2} - \frac{m}{2} \quad (1.14)$$

$$SS;$$

$$-\frac{1}{2} q_a^2 - q_a q_b - q_a q_c - \frac{1}{2} q_b^2 - q_b q_c - \frac{1}{2} q_c^2 + q_a - m q_a - 3 F + q_b - m q_b + q_c - m q_c \quad (1.15)$$

$$FOC2[q[c]] := diff(SS, q[c]);$$

$$FOC2_{q_c} := -q_a - q_b - q_c + 1 - m \quad (1.16)$$

$$FOC[q[c]] := profit[c];$$

$$FOC_{q_c} := (1 - q_a - q_b - q_c) q_c - m q_c - t q_c - F \quad (1.17)$$

$$q[c](q[a], q[b]) := isolate(\%, q[c]);$$

$$q_c(q_a, q_b) := q_c = -\frac{m}{2} - \frac{t}{2} - \frac{q_a}{2} - \frac{q_b}{2} + \frac{1}{2} \quad (1.18)$$

$$-\frac{1}{2} \left( m^2 + 2 m t + 2 m q_a + 2 m q_b + t^2 + 2 t q_a + 2 t q_b + q_a^2 + 2 q_a q_b + q_b^2 - 4 F \right. \\ \left. - 2 m - 2 t - 2 q_a - 2 q_b + 1 \right)^{1/2}$$

$$q[c] = piecewise(0 < rhs(\%), rhs(\%));$$

$$q_c = \begin{cases} -\frac{m}{2} - \frac{t}{2} - \frac{q_a}{2} - \frac{q_b}{2} + \frac{1}{2} - \frac{\sqrt{m^2 + 2 m t + 2 m q_a + 2 m q_b + t^2 + 2 t q_a + 2 t q_b + q_a^2 + 2 q_a q_b + q_b^2 - 4 F - 2 m - 2 t - 2 q_a - 2 q_b + 1}}{2} \\ 0 \end{cases}$$

*#Reaction functions in a mixed triopoly,  $m=0, t=0.2, F=0.005$*

*subs( $m=0, t=0.2, F=0.005, (q[a](q[b], q[c]) + q[b](q[a], q[c]))$ );*

$$1 - \frac{q_b}{2} - q_c - \frac{q_a}{2} \quad (1.1.1)$$

$$isolate(\%, q[c]);$$

$$q_c = 1 - \frac{q_b}{2} - \frac{q_a}{2} \quad (1.1.2)$$

*subs( $m=0, t=0.2, F=0.005, (q[c](q[a], q[b]))$ );*

$$q_c = 0.4000000000 - \frac{q_a}{2} - \frac{q_b}{2} - \frac{\sqrt{q_a^2 + 2 q_a q_b + q_b^2 + 0.620 - 1.6 q_a - 1.6 q_b}}{2} \quad (1.1.3)$$

$$convert(\%, rational);$$

$$q_c = \frac{2}{5} - \frac{q_a}{2} - \frac{q_b}{2} - \frac{\sqrt{q_a^2 + 2 q_a q_b + q_b^2 + \frac{31}{50} - \frac{8}{5} q_a - \frac{8}{5} q_b}}{2} \quad (1.1.4)$$

*simplify*(%, *sqrt*, *symbolic*);

$$q_c = \frac{2}{5} - \frac{q_a}{2} - \frac{q_b}{2} - \frac{\sqrt{100 q_a^2 + (200 q_b - 160) q_a + 100 q_b^2 - 160 q_b + 62}}{20} \quad (1.1.5)$$

*factor*(%, *method* = "Wang");

$$q_c = \frac{2}{5} - \frac{q_a}{2} - \frac{q_b}{2} - \frac{\sqrt{100 q_a^2 + 200 q_a q_b + 100 q_b^2 - 160 q_a - 160 q_b + 62}}{20} \quad (1.1.6)$$

*sys* := {*FOC*[*q*[*a*]], *FOC*[*q*[*b*]], *FOC*[*q*[*c*]]};

$$\text{sys} := \left\{ (1 - q_a - q_b - q_c) q_c - m q_c - t q_c - F, -2 q_a + 1 - q_b - q_c - m, -2 q_b + 1 - q_a - q_c - m \right\} \quad (1.20)$$

*solve*( *sys*, {*q*[*a*], *q*[*b*], *q*[*c*]} );

$$\left\{ q_a = \frac{1}{3} - \frac{\text{RootOf}(\_Z^2 + (m + 3 t - 1) \_Z + 3 F)}{3} - \frac{m}{3}, q_b = \frac{1}{3} - \frac{\text{RootOf}(\_Z^2 + (m + 3 t - 1) \_Z + 3 F)}{3} - \frac{m}{3}, q_c = \text{RootOf}(\_Z^2 + (m + 3 t - 1) \_Z + 3 F) \right\} \quad (1.21)$$

*convert*(%, *radical*);

$$\left\{ q_a = \frac{1}{6} - \frac{m}{6} + \frac{t}{2} - \frac{\sqrt{m^2 + 6 m t + 9 t^2 - 12 F - 2 m - 6 t + 1}}{6}, q_b = \frac{1}{6} - \frac{m}{6} + \frac{t}{2} - \frac{\sqrt{m^2 + 6 m t + 9 t^2 - 12 F - 2 m - 6 t + 1}}{6}, q_c = -\frac{m}{2} - \frac{3 t}{2} + \frac{1}{2} + \frac{\sqrt{m^2 + 6 m t + 9 t^2 - 12 F - 2 m - 6 t + 1}}{2} \right\} \quad (1.22)$$

*qastar* := *rhs*(%[1]);

$$qastar := \frac{1}{6} - \frac{m}{6} + \frac{t}{2} - \frac{\sqrt{m^2 + 6 m t + 9 t^2 - 12 F - 2 m - 6 t + 1}}{6} \quad (1.23)$$

*qbstar* := *rhs*(%%[2]);

$$qbstar := \frac{1}{6} - \frac{m}{6} + \frac{t}{2} - \frac{\sqrt{m^2 + 6 m t + 9 t^2 - 12 F - 2 m - 6 t + 1}}{6} \quad (1.24)$$

*qcstar* := *rhs*(%%[3]);

$$qcstar := -\frac{m}{2} - \frac{3 t}{2} + \frac{1}{2} + \frac{\sqrt{m^2 + 6 m t + 9 t^2 - 12 F - 2 m - 6 t + 1}}{2} \quad (1.25)$$

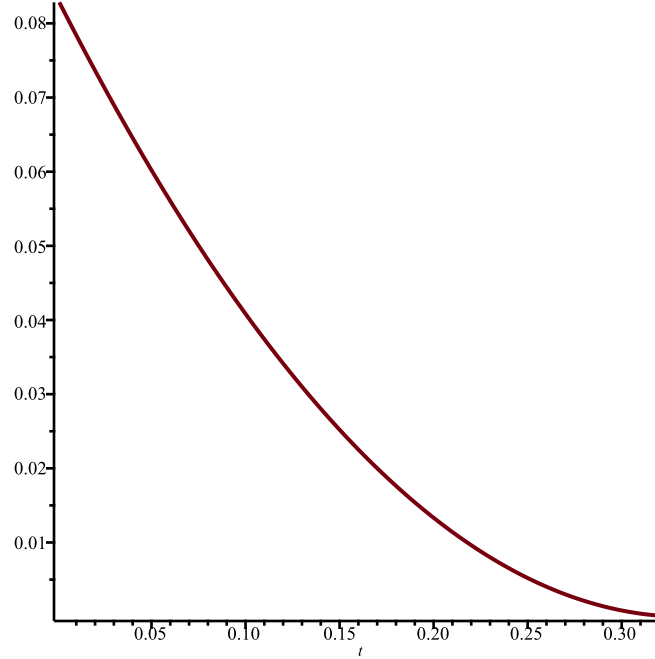
$$(1 - 3 t)^2 - 12 F = 0;$$

$$(1 - 3 t)^2 - 12 F = 0 \quad (1.26)$$

`solve(%o, F);`

$$\frac{(-1 + 3 t)^2}{12} \quad (1.27)$$

`plot(%o, t=0.001 .. 0.32)`



$$Qstarsmixed000 := \text{subs}(q[a] = (qastar), q[b] = (qbstar), q[c] = (qcstar), m=0, t=0, F=0, Q);$$

$$Qstarsmixed000 := 1 \quad (1.28)$$

$$Qstarsmixed := \text{subs}(q[a] = (qastar), q[b] = (qbstar), q[c] = (qcstar), Q);$$

$$Qstarsmixed := \frac{5}{6} - \frac{5 m}{6} - \frac{t}{2} + \frac{\sqrt{m^2 + 6 m t + 9 t^2 - 12 F - 2 m - 6 t + 1}}{6} \quad (1.29)$$

$$CS := \frac{(\%o)^2}{2} + qcstar \cdot t;$$

$$CS := \frac{\left( \frac{5}{6} - \frac{5 m}{6} - \frac{t}{2} + \frac{\sqrt{m^2 + 6 m t + 9 t^2 - 12 F - 2 m - 6 t + 1}}{6} \right)^2}{2} + \left( -\frac{m}{2} - \frac{3 t}{2} \right. \\ \left. + \frac{1}{2} + \frac{\sqrt{m^2 + 6 m t + 9 t^2 - 12 F - 2 m - 6 t + 1}}{2} \right) t \quad (1.30)$$

`simplify(%o, sqrt, symbolic);`

$$\frac{(-5 m + 15 t + 5) \sqrt{m^2 + (6 t - 2) m + 9 t^2 - 12 F - 6 t + 1}}{36} + \frac{13 m^2}{36} - \frac{5 t^2}{4} - \frac{F}{6} \\ - \frac{13 m}{18} + \frac{13}{36} \quad (1.31)$$

$$Pfinal := \text{subs}(q[a] = (qastar), q[b] = (qbstar), q[c] = (qcstar), P);$$

$$Pfinal := \frac{1}{6} + \frac{5 m}{6} + \frac{t}{2} - \frac{\sqrt{m^2 + 6 m t + 9 t^2 - 12 F - 2 m - 6 t + 1}}{6} \quad (1.32)$$

*simplify*(%, *sqrt*, *symbolic*);

$$\frac{1}{6} + \frac{5m}{6} + \frac{t}{2} - \frac{\sqrt{m^2 + (6t-2)m + 9t^2 - 12F - 6t + 1}}{6} \quad (1.33)$$

*profitafinal* := *subs*(*q*[*a*] = (*qastar*), *q*[*b*] = (*qbstar*), *q*[*c*] = (*qcstar*), *profit*[*a*]);

$$\begin{aligned} \text{profitafinal} := & \left( \frac{1}{6} + \frac{5m}{6} + \frac{t}{2} - \frac{\sqrt{m^2 + 6mt + 9t^2 - 12F - 2m - 6t + 1}}{6} \right) \left( \frac{1}{6} - \frac{m}{6} \right. \\ & + \frac{t}{2} - \frac{\sqrt{m^2 + 6mt + 9t^2 - 12F - 2m - 6t + 1}}{6} \Big) - m \left( \frac{1}{6} - \frac{m}{6} + \frac{t}{2} \right. \\ & \left. \left. - \frac{\sqrt{m^2 + 6mt + 9t^2 - 12F - 2m - 6t + 1}}{6} \right) - F \end{aligned} \quad (1.34)$$

*simplify*(%, *sqrt*, *symbolic*);

$$\frac{(m-3t-1)\sqrt{m^2 + (6t-2)m + 9t^2 - 12F - 6t + 1}}{18} + \frac{m^2}{18} + \frac{t^2}{2} - \frac{4F}{3} - \frac{m}{9} + \frac{1}{18} \quad (1.35)$$

*subs*(*m*=0, %)

$$\frac{(-1-3t)\sqrt{9t^2 - 12F - 6t + 1}}{18} + \frac{1}{18} + \frac{t^2}{2} - \frac{4F}{3} \quad (1.36)$$

*profitcfinal* := *subs*(*q*[*a*] = (*qastar*), *q*[*b*] = (*qbstar*), *q*[*c*] = (*qcstar*), *profit*[*c*]);

$$\begin{aligned} \text{profitcfinal} := & \left( \frac{1}{6} + \frac{5m}{6} + \frac{t}{2} - \frac{\sqrt{m^2 + 6mt + 9t^2 - 12F - 2m - 6t + 1}}{6} \right) \left( -\frac{m}{2} \right. \\ & - \frac{3t}{2} + \frac{1}{2} + \frac{\sqrt{m^2 + 6mt + 9t^2 - 12F - 2m - 6t + 1}}{2} \Big) - m \left( -\frac{m}{2} - \frac{3t}{2} + \frac{1}{2} \right. \\ & + \frac{\sqrt{m^2 + 6mt + 9t^2 - 12F - 2m - 6t + 1}}{2} \Big) - \left( -\frac{m}{2} - \frac{3t}{2} + \frac{1}{2} \right. \\ & \left. \left. + \frac{\sqrt{m^2 + 6mt + 9t^2 - 12F - 2m - 6t + 1}}{2} \right) t - F \end{aligned} \quad (1.37)$$

*simplify*(%, *sqrt*, *symbolic*);

$$0 \quad (1.38)$$

*Overallprofitfinal* := *subs*(*q*[*a*] = (*qastar*), *q*[*b*] = (*qbstar*), *q*[*c*] = (*qcstar*), (*profit*[*a*] + *profit*[*b*] + *profit*[*c*]));

$$\begin{aligned} \text{Overallprofitfinal} := & 2 \left( \frac{1}{6} + \frac{5m}{6} + \frac{t}{2} \right. \\ & - \frac{\sqrt{m^2 + 6mt + 9t^2 - 12F - 2m - 6t + 1}}{6} \Big) \left( \frac{1}{6} - \frac{m}{6} + \frac{t}{2} \right. \\ & - \frac{\sqrt{m^2 + 6mt + 9t^2 - 12F - 2m - 6t + 1}}{6} \Big) - 2m \left( \frac{1}{6} - \frac{m}{6} + \frac{t}{2} \right. \\ & \left. - \frac{\sqrt{m^2 + 6mt + 9t^2 - 12F - 2m - 6t + 1}}{6} \right) - 3F + \left( \frac{1}{6} + \frac{5m}{6} + \frac{t}{2} \right. \end{aligned} \quad (1.39)$$

$$\begin{aligned}
& - \frac{\sqrt{m^2 + 6 m t + 9 t^2 - 12 F - 2 m - 6 t + 1}}{6} \Bigg) \left( -\frac{m}{2} - \frac{3 t}{2} + \frac{1}{2} \right. \\
& + \frac{\sqrt{m^2 + 6 m t + 9 t^2 - 12 F - 2 m - 6 t + 1}}{2} \Bigg) - m \left( -\frac{m}{2} - \frac{3 t}{2} + \frac{1}{2} \right. \\
& + \frac{\sqrt{m^2 + 6 m t + 9 t^2 - 12 F - 2 m - 6 t + 1}}{2} \Bigg) - \left( -\frac{m}{2} - \frac{3 t}{2} + \frac{1}{2} \right. \\
& + \frac{\sqrt{m^2 + 6 m t + 9 t^2 - 12 F - 2 m - 6 t + 1}}{2} \Bigg) t
\end{aligned}$$

*simplify*(%, *sqrt*, *symbolic*);

$$\frac{(m - 3 t - 1) \sqrt{m^2 + (6 t - 2) m + 9 t^2 - 12 F - 6 t + 1}}{9} + \frac{m^2}{9} + t^2 - \frac{8 F}{3} - \frac{2 m}{9} + \frac{1}{9} \quad (1.40)$$

*subs*(*m*=0, %)

$$\frac{(-1 - 3 t) \sqrt{9 t^2 - 12 F - 6 t + 1}}{9} + \frac{1}{9} + t^2 - \frac{8 F}{3} \quad (1.41)$$

*#subs*(*m*=0, *t*=0.2, *F*=0.005, *qastar*);

*Qstarsmixm* := *subs*(*q*[*a*] = (*qastar*), *q*[*b*] = (*qbstar*), *q*[*c*] = (*qcstar*), *Q*);

$$Qstarsmixm := \frac{5}{6} - \frac{5 m}{6} - \frac{t}{2} + \frac{\sqrt{m^2 + 6 m t + 9 t^2 - 12 F - 2 m - 6 t + 1}}{6} \quad (1.42)$$

*Qprivatestarsmixm* := *subs*(*q*[*a*] = (*qastar*), *q*[*b*] = (*qbstar*), *q*[*c*] = (*qcstar*), *F*=0, *Q* - *q*[*c*]);

$$Qprivatestarsmixm := \frac{1}{3} - \frac{m}{3} + t - \frac{\sqrt{m^2 + 6 m t + 9 t^2 - 2 m - 6 t + 1}}{3} \quad (1.43)$$

*Qpublicstarsmixm* := *subs*(*q*[*a*] = (*qastar*), *q*[*b*] = (*qbstar*), *q*[*c*] = (*qcstar*), *F*=0, *q*[*c*]);

$$Qpublicstarsmixm := -\frac{m}{2} - \frac{3 t}{2} + \frac{1}{2} + \frac{\sqrt{m^2 + 6 m t + 9 t^2 - 2 m - 6 t + 1}}{2} \quad (1.44)$$

*Qstarsmix* := *subs*(*q*[*a*] = (*qastar*), *q*[*b*] = (*qbstar*), *q*[*c*] = (*qcstar*), *m*=0.1, *F*=0, *Q*);

$$Qstarsmix := 0.7500000000 - \frac{t}{2} + \frac{\sqrt{9 t^2 + 0.81 - 5.4 t}}{6} \quad (1.45)$$

% =  $\frac{2}{3}$ ;

$$0.7500000000 - \frac{t}{2} + \frac{\sqrt{9 t^2 + 0.81 - 5.4 t}}{6} = \frac{2}{3} \quad (1.46)$$

*isolate*(%, *t*);

$$t = 0.2333333333 \quad (1.47)$$

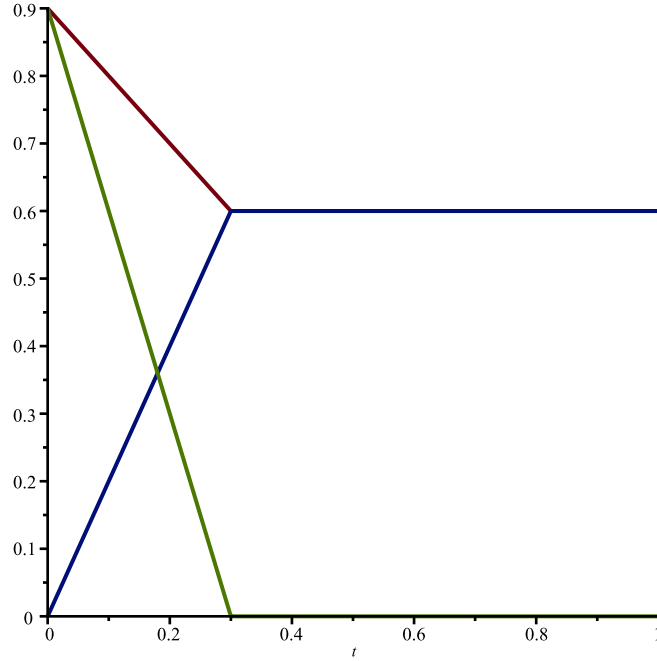
*Qprivatestarsmix* := *subs*(*q*[*a*] = (*qastar*), *q*[*b*] = (*qbstar*), *q*[*c*] = (*qcstar*), *m*=0.1, *F*=0, *Q* - *q*[*c*]);

$$Qprivatestarsmix := 0.3000000000 + t - \frac{\sqrt{9 t^2 + 0.81 - 5.4 t}}{3} \quad (1.48)$$

$Q_{publicstarmix} := \text{subs}(q[a] = (qastar), q[b] = (qbstar), q[c] = (qcstar), m = 0.1, F = 0, q[c]);$

$$Q_{publicstarmix} := 0.4500000000 - \frac{3t}{2} + \frac{\sqrt{9t^2 + 0.81 - 5.4t}}{2} \quad (1.49)$$

$\text{plot}([Q_{starmix}, Q_{privatestarmix}, Q_{publicstarmix}], t = 0 .. 1);$



$Q_{starmixzero} := \text{subs}(q[a] = (qastar), q[b] = (qbstar), q[c] = (qcstar), m = 0, F = 0, Q);$

$$Q_{starmixzero} := \frac{5}{6} - \frac{t}{2} + \frac{\sqrt{9t^2 - 6t + 1}}{6} \quad (1.50)$$

$$\% = \frac{2}{3};$$

$$\frac{5}{6} - \frac{t}{2} + \frac{\sqrt{9t^2 - 6t + 1}}{6} = \frac{2}{3} \quad (1.51)$$

$\text{solve}(\%, t);$

$$\text{RootOf}(-1 + 3\_Z - \sqrt{(-1 + 3\_Z)^2}) \quad (1.52)$$

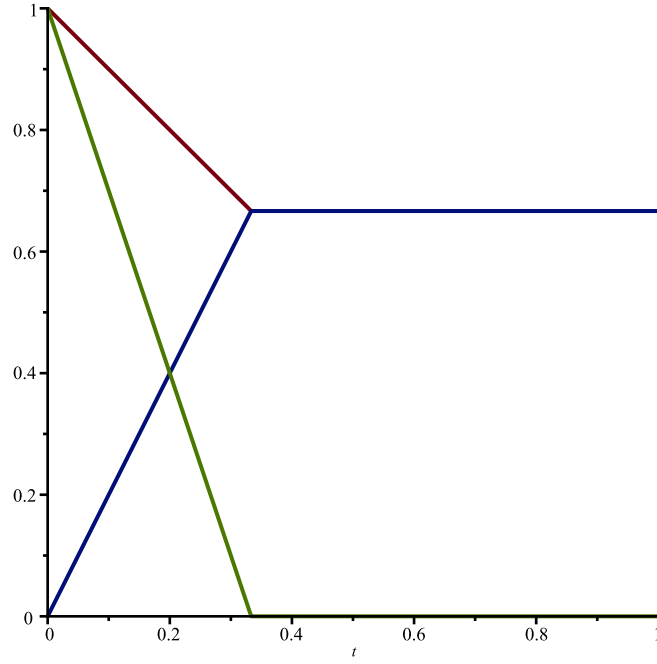
$Q_{privatestarmixzero} := \text{subs}(q[a] = (qastar), q[b] = (qbstar), q[c] = (qcstar), m = 0, F = 0, Q - q[c]);$

$$Q_{privatestarmixzero} := \frac{1}{3} + t - \frac{\sqrt{9t^2 - 6t + 1}}{3} \quad (1.53)$$

$Q_{publicstarmixzero} := \text{subs}(q[a] = (qastar), q[b] = (qbstar), q[c] = (qcstar), m = 0, F = 0, q[c]);$

$$Q_{publicstarmixzero} := \frac{1}{2} - \frac{3t}{2} + \frac{\sqrt{9t^2 - 6t + 1}}{2} \quad (1.54)$$

$\text{plot}([Q_{starmixzero}, Q_{privatestarmixzero}, Q_{publicstarmixzero}], t = 0 .. 1);$



`#evala(Simplify(%));`  
`subs(q[a]=qastar, q[b]=qbstar, q[c]=qcstar, SS);`

$$\begin{aligned}
 & -2 \left( \frac{1}{6} - \frac{m}{6} + \frac{t}{2} - \frac{\sqrt{m^2 + 6mt + 9t^2 - 12F - 2m - 6t + 1}}{6} \right)^2 - 2 \left( \frac{1}{6} - \frac{m}{6} + \frac{t}{2} \right. \\
 & \quad \left. - \frac{\sqrt{m^2 + 6mt + 9t^2 - 12F - 2m - 6t + 1}}{6} \right) \left( -\frac{m}{2} - \frac{3t}{2} + \frac{1}{2} \right. \\
 & \quad \left. + \frac{\sqrt{m^2 + 6mt + 9t^2 - 12F - 2m - 6t + 1}}{2} \right) \\
 & \quad - \frac{\left( -\frac{m}{2} - \frac{3t}{2} + \frac{1}{2} + \frac{\sqrt{m^2 + 6mt + 9t^2 - 12F - 2m - 6t + 1}}{2} \right)^2}{2} + \frac{5}{6} - \frac{5m}{6} \\
 & \quad - \frac{t}{2} + \frac{\sqrt{m^2 + 6mt + 9t^2 - 12F - 2m - 6t + 1}}{6} - 2m \left( \frac{1}{6} - \frac{m}{6} + \frac{t}{2} \right. \\
 & \quad \left. - \frac{\sqrt{m^2 + 6mt + 9t^2 - 12F - 2m - 6t + 1}}{6} \right) - 3F - m \left( -\frac{m}{2} - \frac{3t}{2} + \frac{1}{2} \right. \\
 & \quad \left. + \frac{\sqrt{m^2 + 6mt + 9t^2 - 12F - 2m - 6t + 1}}{2} \right)
 \end{aligned} \tag{1.55}$$

`SSstar := evala(Simplify(%));`

$$\begin{aligned}
 SSstar := & -\frac{17F}{6} - \frac{17m}{18} + \frac{17m^2}{36} - \frac{t^2}{4} - \frac{m\sqrt{m^2 + 6mt + 9t^2 - 12F - 2m - 6t + 1}}{36} \\
 & + \frac{t\sqrt{m^2 + 6mt + 9t^2 - 12F - 2m - 6t + 1}}{12} + \frac{17}{36}
 \end{aligned} \tag{1.56}$$



$$+ \frac{\sqrt{m^2 + 6 m t + 9 t^2 - 12 F - 2 m - 6 t + 1}}{36}$$

*simplify(% , sqrt, symbolic);*

$$\frac{(-m + 3 t + 1) \sqrt{m^2 + (6 t - 2) m + 9 t^2 - 12 F - 6 t + 1}}{36} + \frac{17 m^2}{36} - \frac{t^2}{4} - \frac{17 F}{6} \quad (1.57)$$

$$- \frac{17 m}{18} + \frac{17}{36}$$

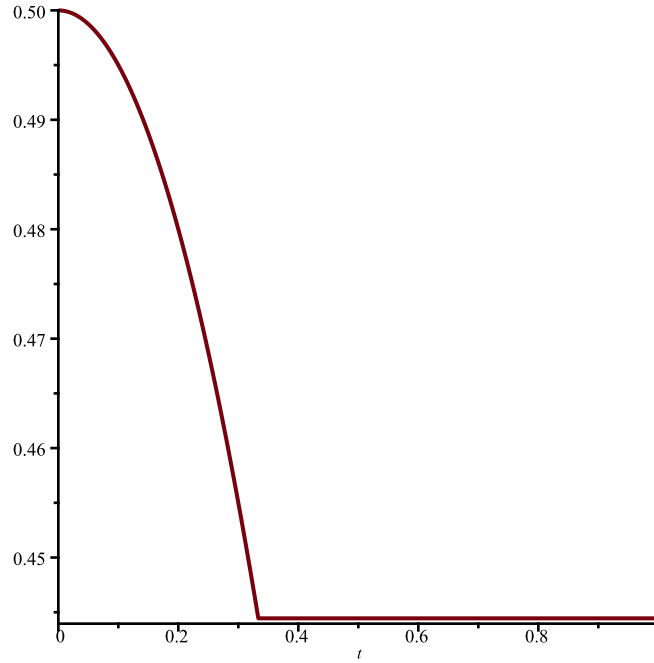
*SSstar00 := subs(q[a] = (qastar), q[b] = (qbstar), q[c] = (qcstar), m = 0, SSstar);*

$$SSstar00 := -\frac{17 F}{6} + \frac{17}{36} - \frac{t^2}{4} + \frac{t \sqrt{9 t^2 - 12 F - 6 t + 1}}{12} + \frac{\sqrt{9 t^2 - 12 F - 6 t + 1}}{36} \quad (1.58)$$

*SSstar000 := subs(q[a] = (qastar), q[b] = (qbstar), q[c] = (qcstar), m = 0, F = 0, SSstar);*

$$SSstar000 := \frac{17}{36} - \frac{t^2}{4} + \frac{t \sqrt{9 t^2 - 6 t + 1}}{12} + \frac{\sqrt{9 t^2 - 6 t + 1}}{36} \quad (1.59)$$

*plot(SSstar000, t = 0 .. 1);*



$$SSstar00 = \frac{15}{32} - 3 F;$$

$$- \frac{17 F}{6} + \frac{17}{36} - \frac{t^2}{4} + \frac{t \sqrt{9 t^2 - 12 F - 6 t + 1}}{12} + \frac{\sqrt{9 t^2 - 12 F - 6 t + 1}}{36} = \frac{15}{32} - 3 F \quad (1.60)$$

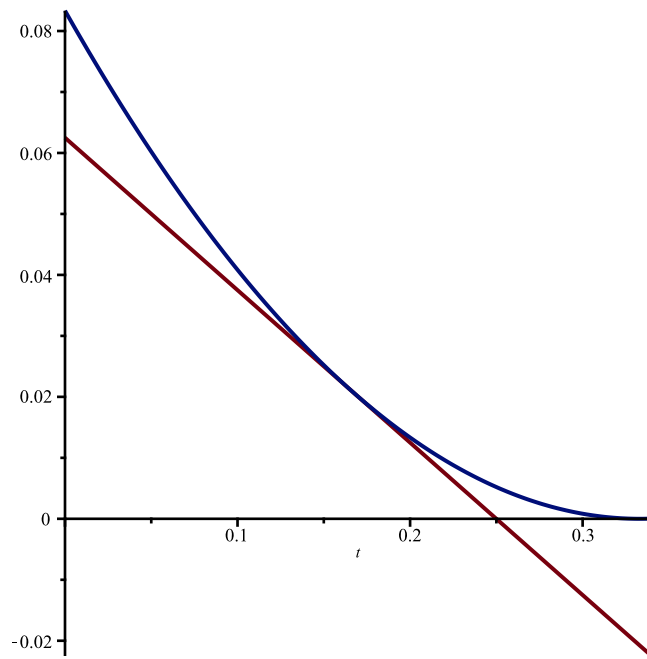
*solve(% , t);*

$$-4 F + \frac{1}{4}, -\frac{4 F}{7} - \frac{1}{4} \quad (1.61)$$

*solve(%% , F);*

$$-\frac{7 t}{4} - \frac{7}{16}, -\frac{t}{4} + \frac{1}{16} \quad (1.62)$$

*plot([%[2], (1-3 t)^2 / 12], t = 0.0 .. 0.34);*



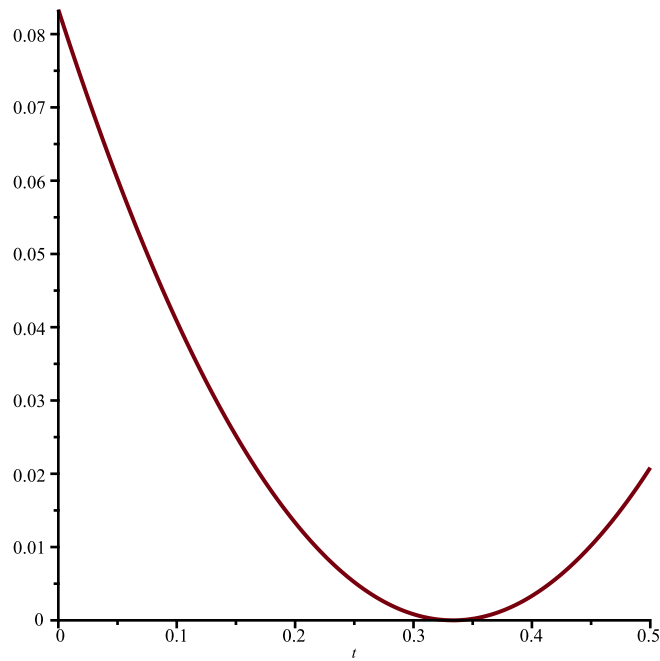
$$(1 - 3 t)^2 - 12 F = 0;$$

$$(1 - 3 t)^2 - 12 F = 0 \quad (1.63)$$

`solve(%0, F);`

$$\frac{(-1 + 3 t)^2}{12} \quad (1.64)$$

`plot(%0, t=0 .. 0.5)`



$$SS_{star00} = \frac{4}{9} - 2 F;$$

$$-\frac{17 F}{6} + \frac{17}{36} - \frac{t^2}{4} + \frac{t \sqrt{9 t^2 - 12 F - 6 t + 1}}{12} + \frac{\sqrt{9 t^2 - 12 F - 6 t + 1}}{36} = \frac{4}{9} - 2 F \quad (1.65)$$

`solve(%0, t);`

$$-\frac{1}{18}+\frac{5\sqrt{1-18\,F}}{18},-\frac{1}{18}-\frac{5\sqrt{1-18\,F}}{18}\tag{1.66}$$

```
solve(%%,F);
```

$$-\frac{18}{25}\,t^2-\frac{2}{25}\,t+\frac{4}{75},0\tag{1.67}$$

```
plot([ %[1], (1-3 t)^2/12 ],t=0.0..0.34);
```

