restart; # *Mixed triopoly with capacity constraint and a social costs*

$Q := q[a] + q[b] + q[c];$

$$Q := q_a + q_b + q_c \tag{1}$$

$P := 1 - Q;$

$$P := 1 - q_a - q_b - q_c \tag{2}$$

$TC[a] := m \cdot q[a] + (q[a] - x[a])^2 + F;$

$$TC_a := m\, q_a + \left(q_a - x_a\right)^2 + F \tag{3}$$

$TC[b] := m \cdot q[b] + (q[b] - x[b])^2 + F;$

$$TC_b := m\, q_b + \left(q_b - x_b\right)^2 + F \tag{4}$$

$TC[c] := m \cdot q[c] + s \cdot q[c] + (q[c] - x[c])^2 + F;$

$$TC_c := m\, q_c + s\, q_c + \left(q_c - x_c\right)^2 + F \tag{5}$$

$profit[a] := q[a] * P - TC[a];$

$$profit_a := \left(1 - q_a - q_b - q_c\right) q_a - m\, q_a - \left(q_a - x_a\right)^2 - F \tag{6}$$

$profit[b] := q[b] * P - TC[b];$

$$profit_b := \left(1 - q_a - q_b - q_c\right) q_b - m\, q_b - \left(q_b - x_b\right)^2 - F \tag{7}$$

$profit[c] := q[c] * P - TC[c];$

$$profit_c := \left(1 - q_a - q_b - q_c\right) q_c - m\, q_c - s\, q_c - \left(q_c - x_c\right)^2 - F \tag{8}$$

$SS := \frac{Q^2}{2} + profit[a] + profit[b] + profit[c];$

$$SS := \frac{\left(q_a + q_b + q_c\right)^2}{2} + \left(1 - q_a - q_b - q_c\right) q_a - m\, q_a - \left(q_a - x_a\right)^2 - 3\, F + \left(1 - q_a - q_b - q_c\right) q_b - m\, q_b - \left(q_b - x_b\right)^2 + \left(1 - q_a - q_b - q_c\right) q_c - m\, q_c - s\, q_c - \left(q_c - x_c\right)^2 \tag{9}$$

$a > 0;$

$$0 < a \tag{10}$$

$P > 0;$

$$0 < 1 - q_a - q_b - q_c \tag{11}$$

$q[a] > 0;$

$$0 < q_a \tag{12}$$

$q[b] > 0;$

$$0 < q_b \tag{13}$$

$m[a] > 0;$

$$0 < m_a \tag{14}$$

#*Fourth Stage: simultaneous quantity choice*

$FOC[q[a]] := diff(profit[a], q[a]);$

$$FOC_{q_a} := -4\,q_a + 1 - q_b - q_c - m + 2\,x_a \tag{15}$$

$q[a](q[b], q[c], x[a]) := isolate(FOC[q[a]], q[a]);$

$$q_a(q_b, q_c, x_a) := q_a = \frac{1}{4} - \frac{q_b}{4} - \frac{q_c}{4} - \frac{m}{4} + \frac{x_a}{2} \tag{16}$$

$FOC[q[b]] := diff(profit[b], q[b]);$

$$FOC_{q_b} := -4\,q_b + 1 - q_a - q_c - m + 2\,x_b \tag{17}$$

$q[b](q[a], q[c], x[c]) := isolate(\%, q[b]);$

$$q_b(q_a, q_c, x_c) := q_b = \frac{1}{4} - \frac{q_a}{4} - \frac{q_c}{4} - \frac{m}{4} + \frac{x_b}{2} \tag{18}$$

$SS;$

$$\frac{(q_a + q_b + q_c)^2}{2} + (1 - q_a - q_b - q_c)\,q_a - m\,q_a - (q_a - x_a)^2 - 3\,F + (1 - q_a - q_b - q_c)\,q_b \tag{19}$$
$$- m\,q_b - (q_b - x_b)^2 + (1 - q_a - q_b - q_c)\,q_c - m\,q_c - s\,q_c - (q_c - x_c)^2$$

$FOC[q[c]] := diff(SS, q[c]);$

$$FOC_{q_c} := -q_a - q_b - 3\,q_c + 1 - m - s + 2\,x_c \tag{20}$$

$FOC2[q[c]] := diff(\%, q[c]);$

$$FOC2_{q_c} := -3 \tag{21}$$

$\#FOC[q[c]] := diff(profit[c], q[c]);$
$q[c](q[a], q[c], x[b]) := rhs(isolate(\%\%, q[c]));$

$$q_c(q_a, q_c, x_b) := -\frac{q_a}{3} - \frac{q_b}{3} + \frac{1}{3} - \frac{m}{3} - \frac{s}{3} + \frac{2\,x_c}{3} \tag{22}$$

$FOCN := piecewise(0 \le \%, FOC[q[c]]);$

$FOCN :=$ (23)

$$\begin{cases} -q_a - q_b - 3\,q_c + 1 - m - s + 2\,x_c & 0 \le -\frac{q_a}{3} - \frac{q_b}{3} + \frac{1}{3} - \frac{m}{3} - \frac{s}{3} + \frac{2\,x_c}{3} \\ 0 & otherwise \end{cases}$$

$sys := \{FOC[q[a]], FOC[q[b]], FOC[q[c]]\};$

$dsolve(sys, \{q[a], q[b], q[c]\});$

$sys := \{-4\,q_a + 1 - q_b - q_c - m + 2\,x_a, -4\,q_b + 1 - q_a - q_c - m + 2\,x_b, -q_a - q_b - 3\,q_c + 1$
$- m - s + 2\,x_c\}$

$$\left\{q_a = \frac{22\,x_a}{39} - \frac{4\,x_b}{39} + \frac{2}{13} - \frac{2\,m}{13} + \frac{s}{13} - \frac{2\,x_c}{13}, q_b = -\frac{4\,x_a}{39} + \frac{22\,x_b}{39} + \frac{2}{13} - \frac{2\,m}{13}\right. \tag{24}$$
$$\left. + \frac{s}{13} - \frac{2\,x_c}{13}, q_c = -\frac{2\,x_a}{13} - \frac{2\,x_b}{13} + \frac{3}{13} - \frac{3\,m}{13} - \frac{5\,s}{13} + \frac{10\,x_c}{13}\right\}$$

$q[a](x[a], x[b], x[c]) := rhs(\%[1]);$

$$q_a(x_a, x_b, x_c) := \frac{22\,x_a}{39} - \frac{4\,x_b}{39} + \frac{2}{13} - \frac{2\,m}{13} + \frac{s}{13} - \frac{2\,x_c}{13} \tag{25}$$

$q[b](x[a], x[b], x[c]) := rhs(\%\%[2]);$

$$q_b(x_a, x_b, x_c) := -\frac{4\,x_a}{39} + \frac{22\,x_b}{39} + \frac{2}{13} - \frac{2\,m}{13} + \frac{s}{13} - \frac{2\,x_c}{13} \tag{26}$$

$q[c](x[a], x[b], x[c]) := rhs(\%\%\%[3]);$

$$q_c(x_a, x_b, x_c) := -\frac{2\,x_a}{13} - \frac{2\,x_b}{13} + \frac{3}{13} - \frac{3\,m}{13} - \frac{5\,s}{13} + \frac{10\,x_c}{13} \tag{27}$$

$sys2 := \{ FOC[q[a]], FOC[q[b]]\};$

$dsolve(\ sys2, \{q[a], q[b]\}\ );$

$$sys2 := \{-4\,q_a + 1 - q_b - q_c - m + 2\,x_a,\ -4\,q_b + 1 - q_a - q_c - m + 2\,x_b\}$$

$$\left\{q_a = \frac{1}{5} - \frac{q_c}{5} - \frac{m}{5} + \frac{8\,x_a}{15} - \frac{2\,x_b}{15},\ q_b = \frac{1}{5} - \frac{q_c}{5} - \frac{m}{5} - \frac{2\,x_a}{15} + \frac{8\,x_b}{15}\right\} \tag{28}$$

$subs(q[a] = rhs(\%[1]), q[b] = rhs(\%[2]), FOC[q[c]]);$

$$\frac{3}{5} - \frac{13\,q_c}{5} - \frac{3\,m}{5} - \frac{2\,x_a}{5} - \frac{2\,x_b}{5} - s + 2\,x_c \tag{29}$$

$isolate(\%, q[c]);$

$$q_c = -\frac{2\,x_a}{13} - \frac{2\,x_b}{13} + \frac{3}{13} - \frac{3\,m}{13} - \frac{5\,s}{13} + \frac{10\,x_c}{13} \tag{30}$$

$sys2 := \{ FOC[q[a]], FOC[q[b]]\};$

$dsolve(\ sys2, \{q[a], q[b]\}\ );$

$$sys2 := \{-4\,q_a + 1 - q_b - q_c - m + 2\,x_a,\ -4\,q_b + 1 - q_a - q_c - m + 2\,x_b\}$$

$$\left\{q_a = \frac{1}{5} - \frac{q_c}{5} - \frac{m}{5} + \frac{8\,x_a}{15} - \frac{2\,x_b}{15},\ q_b = \frac{1}{5} - \frac{q_c}{5} - \frac{m}{5} - \frac{2\,x_a}{15} + \frac{8\,x_b}{15}\right\} \tag{31}$$

$q[ab] = rhs(\%[1]) + rhs(\%[2]);$

$$q_{ab} = \frac{2}{5} - \frac{2\,q_c}{5} - \frac{2\,m}{5} + \frac{2\,x_a}{5} + \frac{2\,x_b}{5} \tag{32}$$

$solve(\%, q[c]);$

$$1 - m + x_a + x_b - \frac{5\,q_{ab}}{2} \tag{33}$$

#Third Stage: capacity choice of private firms

$profit[a];$

$$\left(1 - q_a - q_b - q_c\right) q_a - m\,q_a - \left(q_a - x_a\right)^2 - F \tag{34}$$

$subs(q[a] = q[a](x[a], x[b], x[c]), q[b] = q[b](x[a], x[b], x[c]), q[c] = q[c](x[a], x[b], x[c]),$
$profit[a]);$

$$\left(\frac{6}{13} - \frac{4\,x_a}{13} - \frac{4\,x_b}{13} + \frac{7\,m}{13} + \frac{3\,s}{13} - \frac{6\,x_c}{13}\right)\left(\frac{22\,x_a}{39} - \frac{4\,x_b}{39} + \frac{2}{13} - \frac{2\,m}{13} + \frac{s}{13}\right. \tag{35}$$

$$-\frac{2\,x_c}{13}\Bigg)-m\left(\frac{22\,x_a}{39}-\frac{4\,x_b}{39}+\frac{2}{13}-\frac{2\,m}{13}+\frac{s}{13}-\frac{2\,x_c}{13}\right)-\left(-\frac{17\,x_a}{39}-\frac{4\,x_b}{39}+\frac{2}{13}-\frac{2\,m}{13}+\frac{s}{13}-\frac{2\,x_c}{13}\right)^2-F$$

$FOC[x[a]] := diff(\%, x[a]);$

$$FOC_{x_a} := -\frac{1106\,x_a}{1521}-\frac{352\,x_b}{1521}+\frac{176}{507}-\frac{176\,m}{507}+\frac{88\,s}{507}-\frac{176\,x_c}{507} \tag{36}$$

$x[a](x[b], x[c]) := rhs(isolate(\%, x[a]));$

$$x_a(x_b, x_c) := -\frac{176\,x_b}{553}+\frac{264}{553}-\frac{264\,m}{553}+\frac{132\,s}{553}-\frac{264\,x_c}{553} \tag{37}$$

$profit[b];$

$$(1-q_a-q_b-q_c)\,q_b-m\,q_b-(q_b-x_b)^2-F \tag{38}$$

$subs(q[a]=q[a](x[a], x[b], x[c]), q[b]=q[b](x[a], x[b], x[c]), q[c]=q[c](x[a], x[b], x[c]), profit[b]);$

$$\left(\frac{6}{13}-\frac{4\,x_a}{13}-\frac{4\,x_b}{13}+\frac{7\,m}{13}+\frac{3\,s}{13}-\frac{6\,x_c}{13}\right)\left(-\frac{4\,x_a}{39}+\frac{22\,x_b}{39}+\frac{2}{13}-\frac{2\,m}{13}+\frac{s}{13}-\frac{2\,x_c}{13}\right)-m\left(-\frac{4\,x_a}{39}+\frac{22\,x_b}{39}+\frac{2}{13}-\frac{2\,m}{13}+\frac{s}{13}-\frac{2\,x_c}{13}\right)-\left(-\frac{4\,x_a}{39}-\frac{17\,x_b}{39}+\frac{2}{13}-\frac{2\,m}{13}+\frac{s}{13}-\frac{2\,x_c}{13}\right)^2-F \tag{39}$$

$FOC[x[b]] := diff(\%, x[b]);$

$$FOC_{x_b} := -\frac{352\,x_a}{1521}-\frac{1106\,x_b}{1521}+\frac{176}{507}-\frac{176\,m}{507}+\frac{88\,s}{507}-\frac{176\,x_c}{507} \tag{40}$$

$evala(Simplify(\%))$

$$-\frac{352\,x_a}{1521}-\frac{1106\,x_b}{1521}+\frac{176}{507}-\frac{176\,m}{507}+\frac{88\,s}{507}-\frac{176\,x_c}{507} \tag{41}$$

$x[b](x[a], x[c]) := rhs(isolate(\%, x[b]));$

$$x_b(x_a, x_c) := -\frac{176\,x_a}{553}+\frac{264}{553}-\frac{264\,m}{553}+\frac{132\,s}{553}-\frac{264\,x_c}{553} \tag{42}$$

$evala(Simplify(\%));$

$$-\frac{176\,x_a}{553}+\frac{264}{553}-\frac{264\,m}{553}+\frac{132\,s}{553}-\frac{264\,x_c}{553} \tag{43}$$

#excourse of simultanios choice

$subs(q[a]=q[a](x[a], x[b], x[c]), q[b]=q[b](x[a], x[b], x[c]), q[c]=q[c](x[a], x[b], x[c]), SS);$

$$\frac{\left(\frac{4\,x_a}{13}+\frac{4\,x_b}{13}+\frac{7}{13}-\frac{7\,m}{13}-\frac{3\,s}{13}+\frac{6\,x_c}{13}\right)^2}{2}+\left(\frac{6}{13}-\frac{4\,x_a}{13}-\frac{4\,x_b}{13}+\frac{7\,m}{13}+\frac{3\,s}{13}-\frac{6\,x_c}{13}\right)\left(\frac{22\,x_a}{39}-\frac{4\,x_b}{39}+\frac{2}{13}-\frac{2\,m}{13}+\frac{s}{13}-\frac{2\,x_c}{13}\right)-m\left(\frac{22\,x_a}{39}-\frac{4\,x_b}{39}+\frac{2}{13}-\frac{2\,m}{13}+\frac{s}{13}-\frac{2\,x_c}{13}\right)-\left(-\frac{17\,x_a}{39}-\frac{4\,x_b}{39}+\frac{2}{13}-\frac{2\,m}{13}+\frac{s}{13}-\frac{2\,x_c}{13}\right)^2-3\,F+\left(\frac{6}{13}-\frac{4\,x_a}{13}-\frac{4\,x_b}{13}+\frac{7\,m}{13}+\frac{3\,s}{13}-\frac{6\,x_c}{13}\right)\left(-\frac{4\,x_a}{39}+\frac{22\,x_b}{39}+\frac{2}{13}-\frac{2\,m}{13}+\frac{s}{13}-\frac{2\,x_c}{13}\right)-m\left(-\frac{4\,x_a}{39}+\frac{22\,x_b}{39}+\frac{2}{13}-\frac{2\,m}{13}+\frac{s}{13}-\frac{2\,x_c}{13}\right)-\left(-\frac{4\,x_a}{39}-\frac{17\,x_b}{39}+\frac{2}{13}-\frac{2\,m}{13}+\frac{s}{13}-\frac{2\,x_c}{13}\right)^2+\left(\frac{6}{13}-\frac{4\,x_a}{13}-\frac{4\,x_b}{13}+\frac{7\,m}{13}+\frac{3\,s}{13}-\frac{6\,x_c}{13}\right)\left(-\frac{2\,x_a}{13}-\frac{2\,x_b}{13}+\frac{3}{13}-\frac{3\,m}{13}-\frac{5\,s}{13}+\frac{10\,x_c}{13}\right)-m\left(-\frac{2\,x_a}{13}-\frac{2\,x_b}{13}+\frac{3}{13}-\frac{3\,m}{13}-\frac{5\,s}{13}+\frac{10\,x_c}{13}\right)-s\left(-\frac{2\,x_a}{13}-\frac{2\,x_b}{13}+\frac{3}{13}-\frac{3\,m}{13}-\frac{5\,s}{13}+\frac{10\,x_c}{13}\right)-\left(-\frac{2\,x_a}{13}-\frac{2\,x_b}{13}+\frac{3}{13}-\frac{3\,m}{13}-\frac{5\,s}{13}-\frac{3\,x_c}{13}\right)^2 \tag{1.1}$$

$tert := diff(\%, x[c]);$

$$tert := -\frac{134\,s}{169}-\frac{70\,x_c}{169}-\frac{64\,x_b}{169}-\frac{64\,x_a}{169}-\frac{70\,m}{169}+\frac{70}{169} \tag{1.2}$$

$isolate(\%, x[c]);$

$$x_c = -\frac{67\,s}{35}-\frac{32\,x_b}{35}-\frac{32\,x_a}{35}-m+1 \tag{1.3}$$

$ses := \{FOC[x[a]], FOC[x[b]], tert\};$

$$ses := \left\{-\frac{134\,s}{169}-\frac{70\,x_c}{169}-\frac{64\,x_b}{169}-\frac{64\,x_a}{169}-\frac{70\,m}{169}+\frac{70}{169}, -\frac{1106\,x_a}{1521}-\frac{352\,x_b}{1521}+\frac{176}{507}-\frac{176\,m}{507}+\frac{88\,s}{507}-\frac{176\,x_c}{507}, -\frac{352\,x_a}{1521}-\frac{1106\,x_b}{1521}+\frac{176}{507}-\frac{176\,m}{507}+\frac{88\,s}{507}-\frac{176\,x_c}{507}\right\} \tag{1.4}$$

$solve(\ ses,\ \{x[a], x[b], x[c]\});$

$$\left\{x_a = \frac{44\,s}{17}, x_b = \frac{44\,s}{17}, x_c = 1-\frac{113\,s}{17}-m\right\} \tag{1.5}$$

$sys := \{\ FOC[x[a]], FOC[x[b]]\};$

$solve(\ sys,\ \{x[a], x[b]\});$

$$sys := \left\{ -\frac{1106\,x_a}{1521} - \frac{352\,x_b}{1521} + \frac{176}{507} - \frac{176\,m}{507} + \frac{88\,s}{507} - \frac{176\,x_c}{507}, -\frac{352\,x_a}{1521} - \frac{1106\,x_b}{1521} + \frac{176}{507} - \frac{176\,m}{507} + \frac{88\,s}{507} - \frac{176\,x_c}{507} \right\}$$

$$\left\{ x_a = \frac{88}{243} - \frac{88\,m}{243} + \frac{44\,s}{243} - \frac{88\,x_c}{243}, x_b = \frac{88}{243} - \frac{88\,m}{243} + \frac{44\,s}{243} - \frac{88\,x_c}{243} \right\} \quad (44)$$

$x[a](x[c]) := rhs(\%[1]);$

$$x_a(x_c) := \frac{88}{243} - \frac{88\,m}{243} + \frac{44\,s}{243} - \frac{88\,x_c}{243} \quad (45)$$

$x[b](x[c]) := rhs(\%\%[2]);$

$$x_b(x_c) := \frac{88}{243} - \frac{88\,m}{243} + \frac{44\,s}{243} - \frac{88\,x_c}{243} \quad (46)$$

$q[a](x[a], x[b], x[c]);$

$$\frac{22\,x_a}{39} - \frac{4\,x_b}{39} + \frac{2}{13} - \frac{2\,m}{13} + \frac{s}{13} - \frac{2\,x_c}{13} \quad (47)$$

$q[a](x[c]) := subs(x[a] = (x[a](x[c])), x[b] = (x[b](x[c])), \%);$

$$q_a(x_c) := \frac{26}{81} - \frac{26\,m}{81} + \frac{13\,s}{81} - \frac{26\,x_c}{81} \quad (48)$$

$q[b](x[a], x[b], x[c]);$

$$-\frac{4\,x_a}{39} + \frac{22\,x_b}{39} + \frac{2}{13} - \frac{2\,m}{13} + \frac{s}{13} - \frac{2\,x_c}{13} \quad (49)$$

$q[b](x[c]) := subs(x[a] = (x[a](x[c])), x[b] = (x[b](x[c])), \%);$

$$q_b(x_c) := \frac{26}{81} - \frac{26\,m}{81} + \frac{13\,s}{81} - \frac{26\,x_c}{81} \quad (50)$$

$q[c](x[a], x[b], x[c]);$

$$-\frac{2\,x_a}{13} - \frac{2\,x_b}{13} + \frac{3}{13} - \frac{3\,m}{13} - \frac{5\,s}{13} + \frac{10\,x_c}{13} \quad (51)$$

$q[c](x[c]) := subs(x[a] = (x[a](x[c])), x[b] = (x[b](x[c])), \%);$

$$q_c(x_c) := \frac{29}{243} - \frac{29\,m}{243} - \frac{107\,s}{243} + \frac{214\,x_c}{243} \quad (52)$$

#Second Stage: capacity choice of public firm

$SS;$

$$\frac{(q_a + q_b + q_c)^2}{2} + (1 - q_a - q_b - q_c)\,q_a - m\,q_a - (q_a - x_a)^2 - 3\,F + (1 - q_a - q_b - q_c)\,q_b - m\,q_b - (q_b - x_b)^2 + (1 - q_a - q_b - q_c)\,q_c - m\,q_c - s\,q_c - (q_c - x_c)^2 \quad (53)$$

$subs(q[a] = q[a](x[c]), q[b] = (q[b](x[c])), q[c] = q[c](x[c]), x[a] = (x[a](x[c])), x[b] = (x[b](x[c])), \%);$

$$\frac{\left(\frac{185}{243}-\frac{185\,m}{243}-\frac{29\,s}{243}+\frac{58\,x_c}{243}\right)^2}{2}+2\left(\frac{58}{243}+\frac{185\,m}{243}+\frac{29\,s}{243}-\frac{58\,x_c}{243}\right)\left(\frac{26}{81}-\frac{26\,m}{81}+\frac{13\,s}{81}-\frac{26\,x_c}{81}\right)-2\,m\left(\frac{26}{81}-\frac{26\,m}{81}+\frac{13\,s}{81}-\frac{26\,x_c}{81}\right)-2\left(-\frac{10}{243}+\frac{10\,m}{243}-\frac{5\,s}{243}+\frac{10\,x_c}{243}\right)^2-3\,F+\left(\frac{58}{243}+\frac{185\,m}{243}+\frac{29\,s}{243}-\frac{58\,x_c}{243}\right)\left(\frac{29}{243}-\frac{29\,m}{243}-\frac{107\,s}{243}+\frac{214\,x_c}{243}\right)-m\left(\frac{29}{243}-\frac{29\,m}{243}-\frac{107\,s}{243}+\frac{214\,x_c}{243}\right)-s\left(\frac{29}{243}-\frac{29\,m}{243}-\frac{107\,s}{243}+\frac{214\,x_c}{243}\right)-\left(\frac{29}{243}-\frac{29\,m}{243}-\frac{107\,s}{243}-\frac{29\,x_c}{243}\right)^2 \tag{54}$$

#subs(q[a]=q[a](x[c]), q[b]=(q[b](x[c])), q[c]=q[c](x[c]), %)
#subs(x[a]=(x[a](x[c])), x[b]=(x[b](x[c])), %)
SSx[c] := evala(Simplify(%));

$$SSx_c := -\frac{2723}{59049}\,s+\frac{5446}{59049}\,x_c-3\,F-\frac{53603}{59049}\,m+\frac{53603}{118098}\,m^2+\frac{2723}{59049}\,m\,s-\frac{5446}{59049}\,m\,x_c+\frac{28163}{118098}\,s^2-\frac{56326}{59049}\,s\,x_c-\frac{2723}{59049}\,x_c^2+\frac{53603}{118098} \tag{55}$$

FOC[x[c]] := diff(%, x[c]);

$$FOC_{x_c} := \frac{5446}{59049}-\frac{5446\,m}{59049}-\frac{56326\,s}{59049}-\frac{5446\,x_c}{59049} \tag{56}$$

FOC2[x[c]] := diff(%, x[c]);

$$FOC2_{x_c} := -\frac{5446}{59049} \tag{57}$$

xstar[c] := rhs(isolate(%%, x[c]));

$$xstar_c := 1-m-\frac{28163\,s}{2723} \tag{58}$$

qstar[a] := subs(x[c]=xstar[c], q[a](x[c]));

$$qstar_a := \frac{9477\,s}{2723} \tag{59}$$

qstar[b] := subs(x[c]=xstar[c], q[b](x[c]));

$$qstar_b := \frac{9477\,s}{2723} \tag{60}$$

qstar[c] := subs(x[c]=xstar[c], q[c](x[c]));

$$qstar_c := 1-m-\frac{26001\,s}{2723} \tag{61}$$

xstar[a] := subs(x[c]=xstar[c], x[a](x[c]));

$$xstar_a := \frac{10692\,s}{2723} \tag{62}$$

xstar[b] := subs(x[c]=xstar[c], x[b](x[c]));

$$xstar_b := \frac{10692\,s}{2723} \tag{63}$$

$Qstar := qstar[a] + qstar[b] + qstar[c];$

$$Qstar := -\frac{7047\,s}{2723} + 1 - m \tag{64}$$

$Pstar := subs(q[a] = qstar[a], q[b] = qstar[b], q[c] = qstar[c], P);$

$$Pstar := \frac{7047\,s}{2723} + m \tag{65}$$

$Xstar := xstar[a] + xstar[b] + xstar[c];$

$$Xstar := -\frac{6779\,s}{2723} + 1 - m \tag{66}$$

$subs(q[a] = qstar[a], q[b] = qstar[b], q[c] = qstar[c], x[a] = xstar[a], x[b] = xstar[b], x[c] = xstar[c], SS);$

$$\frac{\left(-\frac{7047\,s}{2723} + 1 - m\right)^2}{2} + \frac{18954\left(\frac{7047\,s}{2723} + m\right)s}{2723} - \frac{18954\,m\,s}{2723} - \frac{7626694\,s^2}{7414729} - 3F + \left(\frac{7047\,s}{2723} + m\right)\left(1 - m - \frac{26001\,s}{2723}\right) - m\left(1 - m - \frac{26001\,s}{2723}\right) - s\left(1 - m - \frac{26001\,s}{2723}\right) \tag{67}$$

$SSfinal := evala(Simplify(\%));$

$$SSfinal := \frac{28163}{5446}\,s^2 - s + m\,s + \frac{1}{2} - m + \frac{1}{2}\,m^2 - 3F \tag{68}$$

$profitfinala := subs(q[a] = qstar[a], q[b] = qstar[b], q[c] = qstar[c], x[a] = xstar[a], x[b] = xstar[b], x[c] = xstar[c], profit[a]);$

$$profitfinala := \frac{9477\left(\frac{7047\,s}{2723} + m\right)s}{2723} - \frac{9477\,m\,s}{2723} - \frac{1476225\,s^2}{7414729} - F \tag{69}$$

$simplify(\%, symbolic);$

$$\frac{9329742\,s^2}{1059247} - F \tag{70}$$

$profitfinalc := subs(q[a] = qstar[a], q[b] = qstar[b], q[c] = qstar[c], x[a] = xstar[a], x[b] = xstar[b], x[c] = xstar[c], profit[c]);$

$$profitfinalc := \left(\frac{7047\,s}{2723} + m\right)\left(1 - m - \frac{26001\,s}{2723}\right) - m\left(1 - m - \frac{26001\,s}{2723}\right) - s\left(1 - m - \frac{26001\,s}{2723}\right) - \frac{4674244\,s^2}{7414729} - F \tag{71}$$

$simplify(\%, symbolic);$

$$\frac{4324}{2723}\,s - \frac{4324}{2723}\,m\,s - \frac{117102568}{7414729}\,s^2 - F \tag{72}$$

$Totalprofit = 2 \cdot profitfinala + profitfinalc;$

$$Totalprofit = \frac{18954\left(\frac{7047\,s}{2723} + m\right)s}{2723} - \frac{18954\,m\,s}{2723} - \frac{7626694\,s^2}{7414729} - 3F + \left(\frac{7047\,s}{2723}\right. \tag{73}$$

$$+m\left(1-m-\frac{26001\,s}{2723}\right)-m\left(1-m-\frac{26001\,s}{2723}\right)-s\left(1-m-\frac{26001\,s}{2723}\right)$$

simplify(%, *symbolic*);

$$Totalprofit=\frac{13513820}{7414729}\,s^2-\frac{4324}{2723}\,m\,s-3\,F+\frac{4324}{2723}\,s \tag{74}$$

$$CS:=\frac{(Qstar)^2}{2};$$

$$CS:=\frac{\left(-\frac{7047\,s}{2723}+1-m\right)^2}{2} \tag{75}$$

simplify(%, *symbolic*);

$$\frac{(7047\,s-2723+2723\,m)^2}{14829458} \tag{76}$$

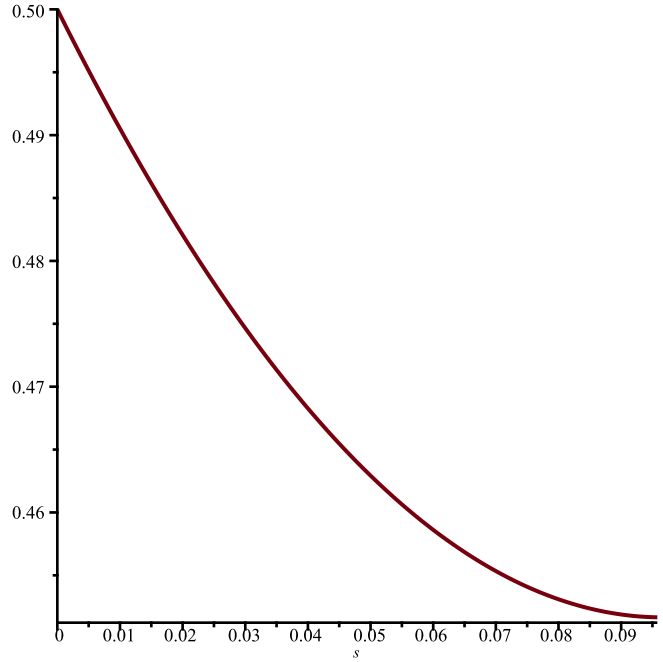
evala(*Simplify*(%));

$$\frac{(7047\,s-2723+2723\,m)^2}{14829458} \tag{77}$$

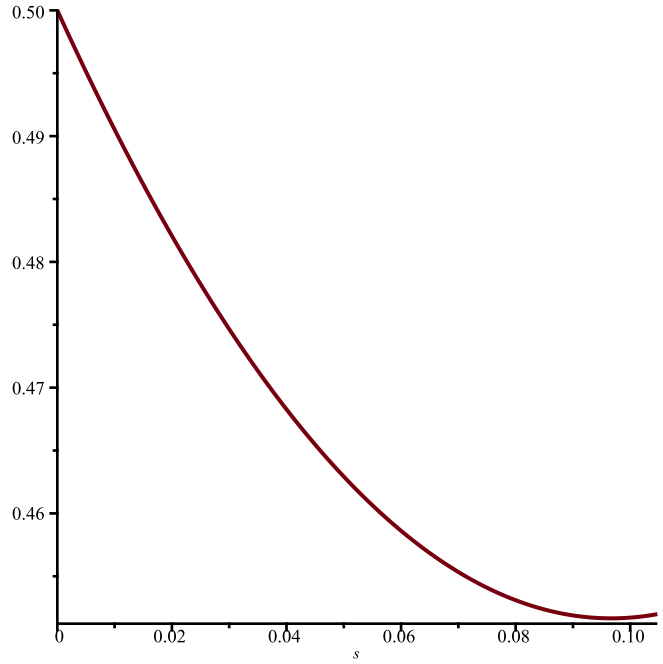
subs($m=0,\ F=0$, *SSfinal*)

$$\frac{28163}{5446}\,s^2-s+\frac{1}{2} \tag{78}$$

plot(%, $s=0..(0.095775737)$);



plot(%%, $s=0..(0.104726741)$);



##*plot*(%%%, $s = 0 .. 0.386405562$);

$SSfinal = \frac{838}{1849} - 2\,F$;

$$\frac{28163}{5446}\,s^2 - s + m\,s + \frac{1}{2} - m + \frac{1}{2}\,m^2 - 3\,F = \frac{838}{1849} - 2\,F \tag{79}$$

subs($m = 0$, %);

$$\frac{28163}{5446}\,s^2 - s + \frac{1}{2} - 3\,F = \frac{838}{1849} - 2\,F \tag{80}$$

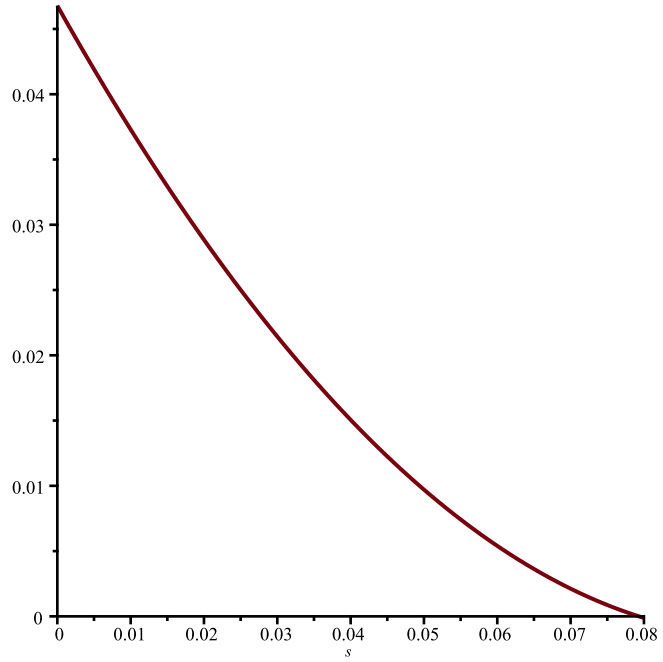
solve(%, s);

$$\frac{2723}{28163} + \frac{\sqrt{442836044 + 283591665602\,F}}{1211009},\ \frac{2723}{28163} - \frac{\sqrt{442836044 + 283591665602\,F}}{1211009} \tag{81}$$

solve(%%, F);

$$\frac{173}{3698} + \frac{28163}{5446}\,s^2 - s \tag{82}$$

plot(%, $s = 0.0 ..0.08$);



$SSfinal = \frac{1101}{2312} - 3\,F;$

$$\frac{28163}{5446}\,s^2 - s + m\,s + \frac{1}{2} - m + \frac{1}{2}\,m^2 - 3\,F = \frac{1101}{2312} - 3\,F \tag{83}$$

$subs(m = 0, \%);$

$$\frac{28163}{5446}\,s^2 - s + \frac{1}{2} - 3\,F = \frac{1101}{2312} - 3\,F \tag{84}$$

$solve(\%, s);\ evalf(\%, 10);$

$$\frac{2723}{28163} + \frac{3\sqrt{483732781}}{957542},\ \frac{2723}{28163} - \frac{3\sqrt{483732781}}{957542}$$

$$0.1655945932,\ 0.02777969217 \tag{85}$$