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Strategic market intervention by a public firm: Public housing as an instrument to boost competition

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Abstract

A regulator considers intervening in an oligopolistic market with a welfare maximizing public firm. The public firm has to pay a premium to its workers or has additional marginal costs. Two models are revised: First, a Cournot triopoly where the public firm is not allowed to make negative profits. Second, a sequential game with capacity choices is constructed. Market intervention in an oligopolistic market is welfare improving if the cost asymmetry is not too large.

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1. Introduction

Affordable housing is a topic that is becoming increasingly urgent, especially in conurbations. The global trend that housing prices in metropolises are increasing faster than incomes is worrying.

> In recent decades, house prices have escalated far above household incomes in many parts of the world. In some metropolitan markets house prices have doubled, tripled, or even quadrupled relative to household incomes. Cox and Pavletich (2020)

Policies to make housing affordable for lower- and middle- income households differ all over the world and OECD (2018) acknowledged a coherence: "The decline in the social housing stock and increased migration have also contributed to a lack of affordable housing units". International and German media are taking up the issue frequently (The Economist 2020a, Tagesspiegel 2018) and consequently governments are reacting with new policies (Thomschke 2019). One reason that spurs this development is that there is not enough activity in the housing construction of metropolitan areas (Dullien and Krebs 2020, Wissenschaftlicher Beirat BMWi 2018). In sight of growing demand and lack of supply this leads to exceptional price increases in German cities (Voigtländer 2019). However, is it an ongoing public discussion whether an intervention by a public firm is helpful to overcome the shortage of affordable housing. Some argue against new non-profit orientated firms in the housing sector as firms that do not maximize profit have a controlling problem and are potentially less efficient(Voigtländer 2016), while others argue that more public housing construction is needed to solve the problem (Dullien and Krebs 2020, Holm et al. 2017). Vienna is regularly mentioned as a positive example as a large part of the housing space is administered by the city's administration and new tenancy agreements are possible to get for an affordable price (Kapeller 2017). Through the existing public housing stock is preventing house price bubbles (Dullien et al. 2016). However, others argue that housing is not more affordable in Vienna than in most German cities in real terms (Simons 2020).

In 2015, the German Bundesrat even passed a law on a rent brake to counteract the problem of increasing prices. The governments of the federal states in Germany now have the possibility to cap the rent at 10% above the local comparable rent (Thomschke 2019). The effectiveness of the rent brake is however controversial Dullien and Krebs (2020), Wissenschaftlicher Beirat BMWi (2018), Thomschke (2019). In Berlin, the city which has seen the highest rent increases in recent years (Schmid 2020), even had a petition calling for the nationalization of housing associations (Götz 2018). In recent years, many municipalities and cities in Germany have sold their properties to private property groups in order to decrease their debt in the short run (Spars 2017). The performance[1] of the shares of the two largest private property groups Vonovia (137%) and Deutsche Wohnen (94%), has outperformed growth rates of the DAX (47%) over the past five years on a noteworthy level . Hence, the question arises whether the state or local

[1]Performance including dividends, retrieved on 03.02.2021 from `https://www.finanzen.net/`.

authorities should invest more in real estate constructing or nationalization to increase supply, intensify competition among housing providers and shift social surplus from private housing associations to consumers.

As the title of this thesis suggests, it is the goal to explore under which market conditions a regulator wants to intervene. For the sake of simplicity, the focus is on mixed triopolies[2] as there is not only competition between a public and a private firm, but also among two private firms. This leads to the following two main questions: firstly, does the regulator establish a public firm in a market in which a duopoly exists? Secondly, does the regulator transform a private firm into a public firm, if a triopoly exists? To exclude the trivial outcome where market interference is always desirable, the public firm has to bear higher social (real) costs than private firms or pays a welfare neutral premium to their workers. To cover a broad variety of market forces, a Cournot-like game with a no-negative profit constraint is considered as well as a Stackelberg-like game with a capacity choice but without a constraint on positive profit.

This thesis examines the impact of a policy where a public welfare-maximizing firm participates in the housing market. As public-owned firms might be less efficient than private-owned firms, a mixed oligopoly where firms face different cost structures is assumed. The impact of a disadvantageous cost structure of the public firm on the market outcome and the overall efficiency is examined in this study. The structure of this thesis is as follows: In section 2 some main characteristics of global housing markets are described, followed by a brief overview of related literature in section 3. Section 4 considers a mixed triopoly with a public firm that has a no-negative profit constraint. Section 5 considers a Stackelberg-like mixed triopoly with a capacity choice. Results are being drawn and discussed in section 6, and in section 7 a conclusion is made.

[2]A triopoly is an oligopoly formed by three companies. A mixed triopoly is a triopoly composed of companies with different objectives.

2. The housing market

Relevance Housing expenditure is one of the largest single components of household spending, moreover, for many people the share of housing of the household budget is increasing (OECD 2019b). There has been a boom in housing prices for almost 70 years and The Economist (2020b) summarizes:

Figure 1: Global average house price development
Source: The Economist (2020b), Knoll et al. (2017)

> Until the mid-20th century house prices across the rich world were fairly stable (see chart). From then on, however, they boomed both relative to the price of other goods and services as well as relative to incomes. Rents went up, too. The Joint Centre for Housing Studies of Harvard University finds that the median American rent payment rose 61% in real terms between 1960 and 2016 while the median renter's income grew by 5%.

Seeing this relevance in individuals' income share there should be a public discussion in which local authorities might act as corrective.

Special aspects Housing as a good has a peculiar combination of features which makes the real estate market unique as described by Arnott (1987), von Einem (2016), and Voigtländer (2017):

- *necessity*: Housing satisfies a basic human need – shelter – which is not substitutable.
- *importance*: For most households, it is the single most important item of consumption.
- *durability*: Housing is the most durable of major commodities.

- *spatial fixity*: Housing units cannot be transported.
- *heterogeneity*: A housing unit has a great number of complex and multi-dimensional characteristics.
- *non convexities in production*: Construction, conversion, rehabilitation, demolition, and reconstruction involve discontinuous changes.
- *informational asymmetries*: Potential occupants are not fully aware of each housing unit's characteristics, and landlord and tenant do not know each other's traits.
- *high capital commitment*: The initial investment is remarkably high such that additional capital must be borrowed regularly.
- *transactions costs:* Search costs, moving costs, and transaction fees are momentous.

These considerations should be kept in mind when discussing the models of the housing market.

Relevant market In line with Arnott (1987) from these characteristics follows that only a small proportion of housing units are likely to be on the market at a given time and space. Housing is place-bound and people are not very flexible due to their profession or social contacts. Due to the location dependency, a market balance is achieved in local geographic sub-markets. That makes it reasonable to analyze housing markets not on the national level but a metropolitan level, or even with a spatial model (Jones et al. 2005).

As reported by von Einem (2016) housing markets in Germany diverge notably due to demographic changes. Spars and Voigtländer (2015) notice that shrinking and booming cities might be even located close to each other as in the cases of Wuppertal, Cologne, and Dusseldorf. In order to enhance economic efficiency and prevent the demolition of buildings in some cities and the simultaneous erection of new houses and infrastructure in others, it would be favorable to implement incentives using the housing space in shrinking cities. Better transportation facilities, more leisure time facilities, and moderate financial incentives are preferred solutions. This could make shrinking cities more attractive, thereby encouraging price sensitive households to stay or migrate there.

High-income individuals can choose between buying or renting. On the other hand, medium-income individuals without significant wealth can choose between joining a housing cooperative or renting. Moreover, low-income individuals can only choose between renting from a public or a private landlord, and it is often of bad quality. Private landlords often rent to low-income individuals with a high risk of not paying the rent only if they get subsidies. The model with limited competition therefore fits better the worse the income situation of the tenants.

Even if in the short term, due to the workplace and social environment, it can be assumed that there are defined sub-markets in each city, but this is not so clear-cut in the longer term.

In the long period inter-city migration can lead to a higher rate of abandoned properties once booming cities are able to provide more housing (Spars and Voigtländer 2015).

Planning, corruption, and market failure Mentioned by Chiodelli and Moroni (2015) "Corruption in the planning field is largely tied to the opportunities that land-use planning generates by allocating development rights and land uses". This might lead to collusion by the incumbents with the local authorities to prevent market entry of new competitors.

Market failure does not necessarily imply that the government should attempt to solve market failures, because the costs of government failure might be worse than those of the market failure it attempts to fix (Grand 1991). For example, economic crowding out, which leads to investments going excessively to public instead to private projects (Buiter 1977). The general goal of urban housing policy is to provide adequate housing for all residents (Kiepe et al. 2011).

2.1. Mixed housing markets are around the world.

Government interventions in the housing market are not only common in countries with high social security standards or countries that have a communist or socialist background but can also be found in liberal countries such as in North America or Singapore (see table 1). Not only different types of regulations, taxes, and subsidies that influence the housing markets exist, but also in many major European and Asian cities a significant proportion of housing units are allocated by administrative decision. This subsection gives an overview of the current state of public and social housing in some selected countries and cities.

Berlin had many vacant apartments between 2000 and 2010 (Steinig 2016). The stock of public housing decreased significantly after some privatizations and rents increased at an above-average rate (Tagesspiegel 2018). A petition to reverse this development by nationalization is under discussion (Götz 2018).

In 2019, the *Hong Kong* Housing Authority had a stock of 793,428 public rental housing flats, accommodating over two million people, which is 28% of Hong Kong's total population (Hong Kong Housing Authority 2019). The vision of the (Hong Kong Housing Authority 2019) is: "To provide affordable rental housing to low-income families with housing needs, and to help low to middle-income families gain access to subsidized home ownership." According to Census and Statistic Department Hong Kong (2016), citizens living in public housing spend averagely 1,608$ or 12.1% of their income at housing. Meanwhile, citizens of subsidized housing spend 9,806$ or 37.6 % of their income to housing. The most is spent by tenants of private housing with 14,876$ or 40% of their income. That is why even with a high share of public housing the average household spend 35.8 % of their income on housing. The discrepancy between public housing on the one hand and subsidized housing and private housing on the other hand is enormous.

The public housing of the *New York* City Housing Authority (NYCHA) represents 7.8% of the city's rental apartments and houses 4.4% of the city's population (NYCHA 2020).

In *Singapore* 3,247,500 residents live in apartments of the state Housing and Development Board (HDB), which is 81% of the city's population and only Singapore citizens can buy a new subsidized flat from HDB (Singapore 2019).Limited space makes private market housing barely affordable and the price-to-income ratio for a 60m2 (650 sqft) flat is around 12. But Singapore is one of few cities whose affordability has improved over the past decade and the long-term supply is well-stocked (Holzhey and Skoczek 2018).

More than one-quarter of *Vienna's* population lives in apartments owned by the city and another quarter lives in subsidized cooperative housing. "Wiener Wohnen" manages and renovates the facilities owned by the city of Vienna. Having affordable housing in Vienna might also have influence that Vienna is one of the cities with the highest quality of life (Wien 2019). Simons (2020) contradicts the thesis that Vienna is the "capital of affordable housing" and claims that real housing expenses are comparable to German cities. It is not clear that whether "Wiener Wohnen" is allowed to have significant losses as there are no accounts, no balance sheets, no profit and loss accounts available. The rents follow a cost principle that at least all costs connected with the house renting can be passed on to the tenants. Hence, assuming a long-term no-negative profit constraint for the public firm in Vienna is reasonable.

Tokyo Metropolitan Housing (TMH) provides housing for middle-income family households, which are controlled under the THM System by THM Supply Corporation or private landlords and represents 3.1% of all housing in Tokyo. The Municipally owned Housing (MoH) provides housing for low-income households, which are supplied under the Public Housing Law in Tokyo. As well as low-income family households, single residents can also live there but they must be at least 60 years old. In 2018 over 3.6% of Tokyo's households live in MoH (Japanese Government Statistics 2020). Davis (2019) asks in The Wall Street Journal why there is no affordable-housing crisis in Japan? And his answer to that is: "With no rent controls and fewer restrictions on height and density, Tokyo appears to be a city where the market is under control—where supply is keeping home prices from rising as drastically as they have in many other major world cities."

While housing interventions mainly take place in limited regional areas such as cities, it is in general interesting to see how much income is spent on housing in different countries (see table 1 for an overview).

2.2. Housing policy

Policy instruments Countries around the world follow different housing policy instruments, e.g., reforms of the tenancy law, subsidies for property ownership, housing subsidies for lower-income tenants, and construction of government-funded social housing, to name a few. Mostly it is a combination of multiple policies. The impact of introducing a policy is difficult to know a priori. Subsidies for housing ownership and lower-income tenants might lead to a higher overall market price for all tenants. Strict tenancy laws might decrease the supply side of the housing market, as being a landlord gets less attractive. Hence, a focus on government-funded construction as it might increase competition and in consequence lower rents for all tenants

Table 1: Public housing worldwide

City	Public	Public + Subsidized	Spending on Housing	Median Multiple
Germany		12%	16.96%	
Berlin	16%	32%		
Austria		20%	16.91%	
Vienna	26%	51%		
US	0%	3%	15.74%	3.6
New York	4.4%	14.8%		5.4
San Francisco	0%			8.4
UK		17%	20.81%	4.5
London	22%			8.2
Singapore	81%	81%	28.9%	4.6
Hong Kong	28%		35.8%	20.8
Japan	3.5%	5%	19.71%	4.2
Tokyo	3.6%	6.7%		4.8

Sources: Singapore (2019); Wien (2019); Hong Kong Housing Authority (2019); OECD (2019a); Japanese Government Statistics (2020); Cox and Pavletich (2018); Cox and Pavletich (2020); Department of Statistics Singapore (2019); OECD (2020)
Definitions: Public: Housing without profit maximization; Public + Subsidized: Housing without profit maximization and Private Housing with housing allowances for the tentants; Spending on Housing: Household spending on housing in % of disposable income, 2019 or latest available; Median multiple: Median house price divided by median household income.

might be a more efficient policy.

In the United States of America there are mainly two approaches to deliver affordable housing for the lowest income class. Private sector projects exist where tax breaks are given to investors who supply homes for the poor. In addition, often in combination with such private sector projects, local authorities hand out housing vouchers to stimulate the demand side of the housing market (Dreier 2018).

A recap of findings concerning the shortfalls about housing allowances is given by Pero et al. (2016)[p.47]:

> Housing allowances are not always effective in addressing housing affordability as they might be captured by rental prices. Evidence shows that landlords capture a sizeable share of public housing allowances by raising rents in Finland (Kangasharuju 2010, Viren 2011), (Fack 2005, LaFerrère and Le Blanc 2004), the United Kingdom (Gibbons and Manning 2006), and the United States (Susin 2002).

> The capture is estimated to be around 16% in the United States (Susin 2002), but much larger in the United Kingdom and in France, where it is estimated at 50% and 78% respectively (Gibbons and Manning, 2006; Fack, 2006).
>
> There are at least four mechanisms which allowances may drive up rents. First, the allowances may increase housing demand (and thus prices) in a market with limited supply. This occurred when students became eligible to access housing allowances in France in the early 1990s (Fack 2006) . Second, tenants who receive housing allowances – particularly in countries where the housing allowance is not income-based – may choose to consume more housing, i.e. live in more expensive, larger, and higher-quality homes (Turner and Elsinga 2005, Kangasharju 2010) . In markets where rents are freely settled between the landlord and the tenant, landlords can raise rent prices when they know that tenants have housing allowances (Kangasharju 2010) . The landlord and the tenant might also decide to settle for a higher rent and share the maximum rent subsidy (Laferrère and Le Blanc 2004) . Finally, general rents can also increase when landlords know that a significant share of their target population has access to housing allowances (Viren 2013) .

This is why this study focuses on market intervention by a public firm, as an alternative for increasing housing allowances from social security systems and subsidies in social housing construction. Kiepe (2006) reports a market share of 8 - 9% held by municipal housing companies in German largest cities and acknowledges a considerable control potential for local housing market development. He emphasizes that a moderate rent policy on the part of the large municipal housing companies has an overall dampening effect on rents.

Target function of public housing companies There are two types of public housing companies which are associated with other objectives than maximizing their profit. Firstly, housing companies that are owned by the commune, and secondly housing cooperatives which are owned collectively by their inhabitants.

Voigtländer (2018) describes a triad of targets for German housing companies which are owned by the communes: return on investment, social policy, and urban development. This poses a major challenge for municipal enterprises since only one goal can be prioritized at a time (Voigtländer 2018). The goal of return on investment is maximizing profit just as privately owned firms do it. The secondly named goal – social policy – can be linked to be a maximizing social surplus, as social policies often intend to increase overall welfare. The goal of urban development is also interesting but more difficult to quantitate in a micro-theoretical oligopoly model. In some cases, the goal of public housing companies owned by the cities might change over time, for instance after changes in political power or public opinion. Still, for example, the statutes of the municipal housing association "Wohnbau Gießen" are stating: "The primary purpose of the company is to ensure an adequate and socially responsible supply of housing

for the population of Giessen (non-profit purpose)" (Gießen 2012). Other municipal housing associations have defined their goals in a similar way, such as Hanover: "The focus of its business activities is the sustainable provision of housing for all of Hanover's population groups"(Kiepe 2006). Hence, housing companies owned by communes focus on welfare maximization, through a non-profit purpose.

The goal of housing cooperatives is more ambiguous. Cooperatives highlight in general that their goal is not to maximize profit. Wohnungsbaugenossenschaften Deutschland e.V. (2020) reports that in addition to the democratic form of organization, the cooperative form is characterized by non-profit-oriented management. Further, generated surpluses are invested in the maintenance and modernization of the stock, in new construction, and in the expansion of service offerings. Both types of these public housing companies are not maximizing profits but general welfare. As a result, they offer housing to a price that is close to the actual costs of the property.

2.3. Affordable Housing in Germany

In Germany, the state has a duty of care to ensure that equal living conditions are provided all over the country. This includes social services in a narrow sense, as well as health, education, and transport infrastructure, but also housing in a broader sense. The basic provision of "Hartz IV", as well as the housing benefit, aim to support the lowest income groups with housing. The tasks of the welfare state and the understanding of them is a collective question (Lessenich 2015). The lack of unoccupied soil, especially in metropolitan areas, as well as long-term and bureaucratic building permits make the supply relatively inelastic.[3] The less elastic the supply side is, the more social transfers go to the landlords without improving the actual housing situation, both in terms of flatness and quality Pero et al. (2016). To stop rapidly fast increasing rent prices compared to overall inflation without improving the housing quality a price control, which is called "Mietpreisbremse", was introduced. In addition to consumer subsidies, there are also subsidies for social housing, which are limited in time. Investors get it for constructing dwellings and giving "Belegungsrechte", which is a time-limited commitment to rent to tenants that are pre-selected by the municipality as they are difficult to place (Voigtländer 2007). There is also a third way: the possibility of communal or non-profit housing projects.

The Second Industrial Revolution of the nineteenth century and the accompanying urbanization led to housing shortages. Initiated by a civil movement, a solution to the housing problem was provided by the state in the form of a legal tax exemption for affordable public housing (Kuhnert and Leps 2017). The statutory tax exemption was granted if the criteria were met in terms of "Wohnungsgemeinnützigkeit", which implied housing non-profit. Kuhnert and Leps (2017) summarized (Jenkis 1973) description of the housing non-profit status in the seventies of the twentieth century in four criteria: 1. Non-profit: It is the target to recover cost and the

[3] A recent paper by Aastveit et al. (2020) finds that American housing supply elasticities – the extent to which construction responds to higher demand – have fallen since the pre-crisis housing boom.

renunciation of profit maximization. 2. Focus on the most vulnerable: The historical starting point of the activity of the non-profit housing companies is to provide inexpensive and good housing to the low-income segments of the population, i.e., industrial workers. 3. Building commitment: Unlike a free housing enterprise, a nonprofit housing company is not only morally, but also legally obligated to continuously build housing in the sense of meeting the needs of the national economy to create housing units. 4. Commitment of funds: The private housing enterprise may freely dispose of its assets and funds; a nonprofit housing enterprise, on the other hand, is subject to legal bindings.

According to Bundesregierung (2017), the housing non-profit status has made an important contribution to overcome the great housing shortage of the post-war years. However, it was abolished by the Tax Reform Act in 1990. The reasons given by the governing coalition at the time included the reduction of subsidies and tax advantages. About at the same time, a turnaround in the housing and real estate markets became apparent. After years of relaxed housing markets, demand for housing rose sharply. This was due to high income increases, the increase in the number of households, and the influx of "Aussiedler" and migrants from Eastern Europe. The supply of housing could not keep pace so that rents rose as a result of the pressure of demand. To take account of the special housing market situation, federal and state subsidies were greatly increased. In this context, funds for social housing construction were also increased which helped to raise the total number of housing completions steadily.

Bundesregierung (2017) argues, that in the following years, the conventional promotion of social housing reached the limits of its financial viability due to the high promotion costs. A considerable part of the social housing stock was unoccupied. The tenancy law which was in force made it difficult to save costs and led to considerable rent distortions within the social housing stock. Against this background, a fundamental reform of the housing promotion system was initiated. As a first step, the 1994 Housing Promotion Act significantly raised the income thresholds in social housing and introduced a new income-oriented promotion. As a second step, in 2001 a fundamental revision of the entire housing promotion law took place with a reform of the housing construction law. The Housing Promotion Act modernized the social housing promotion scheme to better take into account the existing housing stock. Now greater consideration has been given to modernization, the establishment of occupancy rights, and the acquisition of existing housing. According to Spars (2018), the structure of housing providers has changed significantly since 2000. Large housing companies are increasingly shaping events on the housing market. His study describes how municipalities sold residential properties in large "bundles", leading to concentration and consolidation processes in the private-owned housing market.

Bundesregierung (2017) notes that the target group of the promotion also changed. Previously, the purpose of the promotion of housing construction had been to provide affordable housing for broad sections of the population. However, by the turn of the millennium, the housing market had eased and the population was largely provided with housing. Nevertheless, there

were still households that were disadvantaged in their search for housing due to their financial or social situation and had difficulties in obtaining adequate housing on the general housing market without help. Social housing promotion was concentrated on these households. The Federalism Reform in 2006 shifted the regulatory power for housing from the national to the state level. Since then, the number of newly constructed housing has fallen significantly in the following years. Social housing promotion also focused more strongly on improving the quality of housing stock. The share of subsidized new housing construction in the number of completed dwellings fell from 15% in 2009 to around 6% in 2013 and 2014 respectively. This decline in housing construction activity resulted in a shrinking housing supply (Bundesregierung 2017).

The turnaround in the housing market came with the global financial and economic crisis of 2007/2008, as many capital investors started looking for secure forms of investment. The demand for apartments as investment properties has risen significantly. With the economic recovery from 2009 onwards, private demand for residential space in the economically strong conurbations has also continued to grow. In some regions, even middle-income groups are increasingly finding it difficult to afford housing close to the city center or the workplace (Bundesregierung 2017).

In the face of rising rents, the demand for social housing has been rising since 2012, especially in the conurbations, and it is likely to keep increasing. In addition, the stock of rent- and occupancy-linked housing is declining due to the successive expiry of the contracts. The annual budget of the federal government to support the states to make social housing accessible increased from 500 million prior to 2015, to 1.5 billion Euro in 2017 (Bundesregierung 2017).

2.4. Aspects of modeling

In this study, to examine the modeling of housing markets, some but not all features are taken into account as follows:

(i) *Oligopoly* – Many features described previously, especially spatial fixity but also heterogeneity, the thinness of market, high capital commitment, and transaction costs lead to the conclusion that there is imperfect competition in the housing sector. For the sake of simplicity, an oligopoly is used in the following models to aggregate all characteristics of the housing market which give market power to the supply side.

(ii) *Mixed market* – As we can see in subsection 2.1, in many markets around the world there is not a pure market of only private housing providers, but also public ones. Therefore, mixed markets are a matter of fact in many cities. The target function of public housing companies is discussed before but from now on, a simple welfare maximization approach is used. A triopoly is modeled as a compromise between a situation without competition (monopoly) and a situation with perfect competition.

(iii) *Asymmetry in production* – It is often argued that state-run firms pay comparatively high wages, or their bureaucracy makes them less efficient. Both cases are considered by

assuming at first a premium which is paid to the employees as a transfer and leads to higher wages of public workers. The other case that of inefficiency is represented by higher real marginal costs.

(iv) *No negative profit* – Often public firms are expected to be profitable which can mean that no losses are allowed to be made. Therefore, a model is analyzed in chapter 4 in which the public firm has a no-negative profit constraint.

(v) *Capacity choice* – In chapter 5 companies first choose their capacity and then compete for the quantity of housing that they bring to the market. In chapter 4 and 5, an analysis will be made of the degree to which the cost difference of the transfer and the inefficiency of the state intervenes in the competition through a public firm.

Limitations As a model is an overly simplified version of the world, many aspects of the housing market did not find a way into the model. To name some, here is a non-extensive list of limitations: Instead of modeling market entry barriers, a fixed number of private firms is assumed. The target function might not be welfare maximizing, but instead, quantity maximizing or profit maximizing. Welfare optimization of a public firm as the one which is used in this study is usually not implemented one-to-one in reality. As asymmetric information is not taken into account there is also some uncertainty about the actual costs and the demand. Although the assumption that the housing market is oligopolistic due to the market power of large housing companies is plausible for some large cities with dense populations, it is less true for housing in rural areas. Higher cost of public project cannot be purely an inefficiency or a transfer, but a mix of both. Housing cooperative and municipal housing associations are aggregated to public firms. The model is studying an isolated submarket, instead of a spatial model, which would consider the effect of locality better. An appropriate definition of sub-markets is difficult. Because of the necessity of housing an isoelastic demand instead of linear demand might be closer to the real market conditions. An intervention seems particularly appropriate in social housing for population groups that have particular difficulty in obtaining housing, however, depending on the implementation, it can lead to ghettoization. Further limitations are that the game is static and differentiation in quality is not considered. Political aspects, as well as urban planning and the management of public firms are also omitted.

3. Related Literature

In this section, only related literature is considered which is looking at mixed oligopolies. An early and well recognized contribution to the literature of mixed oligopolies is the work by Merrill and Schneider (1966). They introduced the theoretical concept of mixed oligopolies in the form of strategic interaction of a private profit maximizing firm and a public industry output maximizing firm. Their analysis leads to the conclusion that the entry or existence of a government firm in an oligopolistic industry can improve market performance with lower prices and increased output.

The study of "The public firm as an instrument for regulating an oligopolistic market" by Cremer et al. (1989) investigates if it is socially optimal to have any public firm in a Cournot oligopoly and if so, how many of such firms should there be? They consider a model where public firms face increasing returns to scale as fixed costs are assumed. Public firms face a no-negative profit constraint and may pay a premium to their workers, which is not lowering the efficiency directly but is a pure transfer. They show that it is optimal to have a public firm, but it is less efficient to have more than one public firm. According to the approach of Cremer et al. (1989), the setting of the no-negative profit model of section 4.1 is very similar. In Section 4.1, there is an oligopoly with (i) a public firm competing with private firms, where (ii) the public firm faces a no-negative profit constraint and (iii) a premium to the public firm's workers as a transfer is paid. Keeping (i) and (ii), this paper changes (iii) in Section 4.2 by assuming that the premium paid by the public firm is not a transfer but lowers the welfare, as it is an inefficiency. Note that for a valid welfare analysis as well as for the maximization problem of the public firms it is not sufficient anymore to maximize the industry output. Therefore, the social surplus as the sum of consumer and producer rent is used as a welfare measure.

Another important contribution to the optimal behavior of a public firm in a market where there are also multiple private firms has been made by De Fraja and Delbono (1989). The public firm aims at maximizing a social welfare function containing consumer and producer surplus. The authors compare four possible regimes: (1) the public firm is a welfare maximizing Stackelberg leader; (2) it is a welfare maximizing Cournot-Nash player (mixed oligopoly); (3) it is a profit maximizer (pure oligopoly); and (4) the whole industry is under government control (public monopoly). All firms, public and private, face the same cost function containing fixed and marginal costs. They conclude that full efficiency is reached by a public monopoly in which the government nationalizes the whole industry and decides for the optimum number of firms that operate as public firms. Therefore, this leads to the highest welfare. The ordering of the other three cases is dependent on the parameters. When the number of firms is sufficiently large, the optimal strategy of a welfare maximizing firm is to act as if it wanted to maximize its profit (pure oligopoly).

The impact of a sequential choice of capacity in a mixed oligopoly environment is studied by Lu and Poddar (2005). They analyze the market outcome of a duopoly with four different time

structures: sequential capacity choice and simultaneous quantity choice as well as simultaneous capacity choice and sequential quantity choice with the public firm being the Stackelberg-like leader or the follower. The public firm is less efficient than the private firm and therefore has higher marginal costs to bear. Both types of firms suffer under inefficiency (increasing costs) when not producing the capacity under or over capacity. Their main finding is that in games under mixed oligopoly where capacities and quantities are chosen sequentially, the public firm never chooses excess capacity while the private firm never chooses under-capacity in the equilibrium. Some key aspects of the Lu and Poddar (2005) model are that there is an oligopoly with (i) a public firm competing with a private firm, in which (ii) firms choose capacities and are inefficient when the quantities produced are over or under capacity, and (iii) the public firm has higher marginal costs than the private firm. Section 5.2 of this study is similar to the first case of Lu and Poddar (2005) in which a public firm chooses its capacity before the private firm's choice of capacity and then both choose quantity simultaneously. Keeping the main structure of the sequential game of the first case of Lu and Poddar (2005) model as well as (ii) and (iii), the main difference of this study to Lu and Poddar (2005) work is that (i) is altered as the public firm is now competing with two private firms in a triopoly instead of one in a duopoly. In Section 5.1, the inefficiency of (iii) is replaced by a premium paid to the public firm's workers as a transfer which is similar to Cremer et al. (1989).

Bárcena-Ruiz (2012) alternates the model of De Fraja and Delbono (1989) by assuming that the public firm is as efficient as private firms with the same constant marginal cost of production for all firms instead of a quadratic cost function. Therefore, they obtain a different result: the government does not privatize the public firm regardless of how many private firms are competing in the product market in several relevant cases. They also show that the consumer surplus can be lower in the mixed than in the private oligopoly.

George and La Manna (1996) consider a model where the public firm is less efficient. The public firm can be partially private, and they consider the option that the true cost of private firms might be not disclosed. They can show that public ownership is welfare improving if the cost difference is not too large. It is assumed that the public firm has a first-mover advantage because the government can make a credible announcement of an output target so that it can act as a Stackelberg leader. George and La Manna (1996) conclude: Partial public ownership may be welfare-improving if a publicly owned firm has a higher marginal cost than a private firm and is a Stackelberg leader. Further, a simple transfer function is truth-eliciting if the private firm's marginal cost is private information. And finally, in an efficient stock market the cost of renationalization is "small".

Matsumura and Kanda (2005) allow free entry of private firms and find that in contrast to the case with the fixed number of firms, welfare-maximizing behavior by the public firm is always optimal in mixed markets. They show that mixed markets are better than pure markets involving no public firm if the public firm earns positive profits.

More recent contributions in the field of mixed oligopolies are made by Herr (2011), Correia-

da Silva and Pinho (2018), and Chang et al. (2018). Herr (2011) shows that privatization might be welfare improving in a mixed duopoly with an inefficient public firm, heterogeneous goods, and price regulation. In the spirit of Herr (2011) but in a more general setting, Chang et al. (2018) find that a welfare maximizing price control can make a public firm redundant. In contrast, Correia-da Silva and Pinho (2018) show that a public firm can be desirable if there are many private firms in order to weaken collusion.

4. Mixed triopoly with a no-negative profit constraint

A mixed triopoly with a no-negative profit constraint (NNPC) for the public firm is modeled. The setup is build on Cremer et al. (1989), but additionally, a second case is analyzed in which the public firm pays no premium as a transfer to her workers, but faces real social costs instead.

4.1. The no-negative profit model

Consider a setting with two types of firms: private and public. Private firms are maximizing their individual profit, while public firms maximize total social surplus (TSS), which is the sum of consumer and producer surplus. Both types of firms produce a homogeneous good and they face a linear and normalized market demand (NDF), which results in price:

$$p(Q) = 1 - Q \tag{1}$$

where p is the market price per unit, q_i is the quantity produced by firm i, n is the number of all firms(private and public) in the oligopoly and $Q = \sum_{i=1}^{n} q_i$ is total output of all public and private firms.

Private firms At least two profit maximizing firms in private ownership are assumed. The private firms have cost functions C_i defined by constant marginal cost m and a fixed cost F, corresponding to an irreversible investment in depreciable capital[4].

$$C_i(q_i) = m\ q_i + F. \tag{2}$$

Private firms maximize their profits function (π_i). Revenue is equal to the number of units sold q_i times the price per unit define by equation (1). The profit a business makes is equal to the revenue it takes in, minus what it spends as costs defined by equation (2). Thus the profit function of a private firm i is:

$$\pi_i(q_1, ..., q_n) = q_i\ p(Q) - mq_i - F = q_i(1 - (\sum_{i=1}^{n} q_i)) - mq_i - F. \tag{3}$$

Public firm A welfare maximizing public firm can be established by the government. Either by nationalization of a private firm or by founding a new one. It is assumed that the public firm is maximizing the overall social surplus instead of profits, but has to cover her own costs[5]. This leads to a no-negative profit constraint (NNPC), as the public firm has to assure that she makes no losses. Two cases are taken into consideration. In both cases, the public firm has the same marginal costs m as the private firms. However, in the first case, the public firm is paying a

[4]As land and buildings are cost intensive and have irreversible transaction costs (notary, estate agents, real estate transfer tax) this assumption fits well for the housing market.

[5]This is a political decision which is taken in many, but not all, countries and cities.

premium t as a transfer to their workers besides the marginal costs . The second case considers an inefficiency, which is the public firm has social cost s on top of the marginal costs m of the private firms.

Social Surplus As a measure of welfare, which the public firm wants to maximize, the social surplus is used. The total social surplus is the aggregated consumer surplus(CS) and producer surplus(PS). Because the demand function under consideration is linear, the "rule of halve" can be used to derive the consumer surplus. Producer Surplus are the aggregated profits of all firms. The total social surplus is therefore,

$$TSS(Q) = CS + PS = \frac{Q^2}{2} + \sum_{i=1}^{n} \pi_i(q_1, ..., q_n). \tag{4}$$

Transfer In the transfer case, the public firm is as efficient as the private firms. It has the same marginal costs m and fixed costs F in her cost function as private firms, see (2), but pays a positive premium t per unit output to its workers which is a pure transfer and does not lower the welfare[6]. However, the transfer lowers the profit of the public firm in comparison to the profit of private firms (3), which leads to a tighter NNPC.

$$\pi_i(q_1, ..., q_n)^{transfer} = q_i \; p(Q) - mq_i - tq_i - F \geq 0. \tag{5}$$

The premium t paid by the public firm increases the consumer surplus by the same amount as it decreases the profit of the public firm, respectively the producer surplus. Thus it does not appear in the total surplus function and does not affect its quantity decision:

$$TSS(Q)^{transfer} = Q - \frac{Q^2}{2} - Qm - nF. \tag{6}$$

Social Costs In the second case the public firm is less efficient than the private firms and therefore has higher real social costs. It has costs of s on top of the marginal costs of the private firms per quantity output[7].

$$C_i(q_i)^{socialcosts} = (m + s) \; q_i + F. \tag{7}$$

This has an impact on the social surplus by social costs s times the public firm's quantity q_3, so the total social surplus function is instead:

$$TSS(Q)^{socialcosts} = Q - \frac{Q^2}{2} - Qm - q_i s - nF. \tag{8}$$

[6]For example paying voluntary higher wages per employee, but workers do not have to work more.

[7]Such as having more employees on the payroll to produce the same quantity as the output per worker is on a lower level.

The effect on its profit and its non-negative profit constraint is the same as in the transfer case;

$$\pi_i(q_1, ..., q_n)^{socialcosts} = q_i p(Q) - mq_i - sq_i - F \geq 0. \tag{9}$$

Fixed costs Private firms are already established in the market before the governments intervention and do not deduct fixed costs because they are sunk costs. Hence, while the fixed cost F does not affect the behaviour of the existing private firms in the short run, it does affect the government's decision to nationalize firms. This is because the government has the possibility of not entering the market under consideration. Once the decision of nationalizing or establishing a firm is made, the government forces the now public firm not only to maximize welfare, but also to cover its fixed cost. This is in order to ensure its viability in the long run without further public financing. Accordingly, F is to be taken into account in the public firms's break-even constraint.

4.2. Equilibrium with a transfer

4.2.1. Reaction functions

Firm 1 and firm 2 are private and symmetric with marginal costs m. Firm 3 is a public firm which is as efficient as the private firms with marginal costs m, but pays a transfer t per produced quantity to its workers. Maximizing their profit from equation (3) the private firms face the following problems:

$$\max_{q_1} \pi_1(q_1, q_2, q_3) = q_1 p(q_1 + q_2 + q_3) - q_1 m - F \tag{10}$$

$$\max_{q_2} \pi_2(q_1, q_2, q_3) = q_2 p(q_1 + q_2 + q_3) - q_2 m - F \tag{11}$$

The solution of (10) and (11) gives firm l's reaction function (12) in terms of firm 2's and 3's output, and firm 2's reaction function (13) in terms of firm 1's and 3's output:

$$\bar{q_1}(q_2, q_3) = \frac{1 - q_2 - q_3 - m}{2}, \tag{12}$$

$$\bar{q_2}(q_1, q_3) = \frac{1 - q_1 - q_3 - m}{2}. \tag{13}$$

Notice that fixed cost F are not taken into consideration in the (short term) quantity decision. Given (12) and (13), the public firm faces the summed reaction function:

$$\bar{q}_{12}(q_3) = \frac{2}{3}(1 - q_3 - m), \tag{14}$$

which is represented by the densely dashed red line in figure 2.

The problem of the public firm is maximizing TSS (6), taking NNPC (5) under consideration:

$$\max_{q_3} TSS(q_1+q_2+q_3) = (q_1+q_2+q_3)(1-m) - \frac{(q_1+q_2+q_3)^2}{2} - 3F \tag{15}$$

s.t.

$$\pi_3(q_1,q_2,q_3) = q_3(1-q_1-q_2-q_3) - mq_3 - tq_3 - F \geq 0 \tag{16}$$

From (15) follows that the total social surplus is an increasing function of the public firm's output (up to $1-q_1-q_2-m$) as $\frac{dT}{dq_3} \geq 0$ for all $q_3 \leq 1-q_1-q_2-m$. The unconstrained maximum surplus is obtained when the market price equals marginal cost ($p=m$). For this to be achieved, the public firm must set its quantity $q_3 = 1-q_1-q_2-m$, which is represented by the dotted teal line in figure 2. Thus, because of the costs F and m and the premium t, the public firm's profits are negative at this output $\pi_3(q_1,q_2,1-q_1-q_2-m) = (1-q_1-q_2-m)(-t) - F \leq 0$. But that is precluded by the no-negative profit constraint (16). The optimal rule for the public firm is then to produce as much as allowed by the no-negative profit constraint. This implies that the public firm 3 wants to produce the maximum output allowed by $\pi_3(q_1,q_2,q_3)=0$, which is quadratic in q_3. Hence, the no-negative profit constraint (16) is always binding at the solution.

The reaction function of the public firm $\bar{q_3}(q_1,q_2)$ is thus given by isolating q_3 from a binding NNPC (16). When q_1 and q_2 are large enough ($q_1+q_2 > 1-m-t-2\sqrt{F}$), the market price becomes so low that the public firm cannot break even, and thus q_3 equals zero. Furthermore, assuming that the public firm does not operate when the break-even constraint cannot be satisfied (i.e., there are no real roots because of $(1-m-t-q_1-q_2)^2 - 4F < 0$), obtains firm 3's reaction function in terms of firm 1's and firm 2's output:

$$\bar{q_3}(q_1,q_2) = \begin{cases} \dfrac{1-m-t-q_1-q_2 \pm \sqrt{(1-m-t-q_1-q_2)^2-4F}}{2} & \text{for } (1-m-t-q_1-q_2)^2-4F \geq 0 \\ 0 & \text{else.} \end{cases} \tag{17}$$

which is represented by the solid blue line in figure 2 .

4.2.2. Nash equilibrium

The market solution is given by a (noncooperative) Nash equilibrium. To find a Subgame perfect Nash equilibrium (SPNE), the system of best responses given by (12), (13) and (17) has to be solved[8]:

$$\begin{aligned} 2q_1+q_2+q_3+m-1 &= 0 \\ q_1+2q_2+q_3+m-1 &= 0 \\ (1-m-t-q_1-q_2-q_3)q_3 - F &= 0. \end{aligned} \tag{18}$$

The TSS of the SPNE with the higher q_3 is always bigger than the SPNE of the lower

[8]The code(Maple 2020) that is used to solve the systems and isolate the variables is available on my website: https://dustin-jonak.github.io/research.html

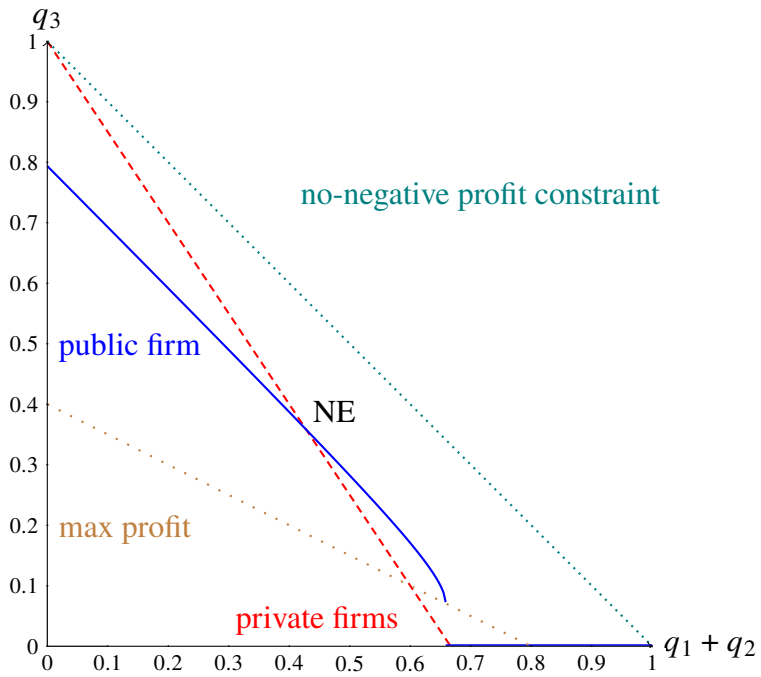


Figure 2: Reaction functions in a mixed triopoly with a no-negative profit constraint

densely dashed red: Private firms reaction function $q_1 + q_2 = \frac{2}{3}(1 - q_3) \Leftrightarrow q_3 = 1 - \frac{3}{2}(q_1 + q_2)$
solid blue: Publics firm reaction function with a no-negative profit constraint
$q_3 = \frac{1-0.2-(q_1+q_2+\sqrt{(1-0.2-(q_1+q_2)^2)-4*0.005})}{2}$ for $q_1 + q_2 = 0$ to 0.658578 and 0 else.
loosely dotted brown: Reaction function if firm 3 would maximize profit $q_3 = \frac{1-0.2-(q_1+q_2)}{2}$
dotted teal: Public's firm reaction function without a NNPC $q_3 = 1 - (q_1 + q_2)$
Parameter values: $F = 0.005, m = 0, t = 0.2, s = 0.2$

public quantity q_3 $TSS^* > TSS^{l}$[9]. To reduce the number of configurations to be investigated, let's assume that the government has a selection mechanism that allows it to enforce the Nash equilibrium yielding the higher total surplus[10]. The solution of system (18) with the higher value of q_3 and higher TSS is yielding the equilibrium quantities:

$$q_1^* = q_2^* = \frac{1 - m + 3t - \sqrt{(1 - m - 3t)^2 - 12F}}{6}, \tag{19}$$

$$q_3^* = \frac{1 - m - 3t + \sqrt{(1 - m - 3t)^2 - 12F}}{2}. \tag{20}$$

Substituting the equilibrium quantities (19) and (20) into Q and (6) obtains

$$Q^* = \frac{5 - 3t - 5m + \sqrt{(1 - m - 3t)^2 - 12F}}{6}, \tag{21}$$

$$TSS^* = \frac{17(1 - 2m + m^2) - 9t^2 + 6F + (1 - m + 3t)\sqrt{(1 - m - 3t)^2 - 12F}}{36} - 3F. \tag{22}$$

The equilibrium total social surplus of a mixed triopoly with a no-negative profit constraint

[9] See 8.1 in the appendix
[10] As assumed in Cremer et al. (1989).

given by equation (22) is represented by the teal line in figure 19. Graphically, (q^*_{1+2}, q^*_3) is an intersection point of the firms' reaction functions (see figure 2 for an illustration).

Normalizing cost to zero ($m = 0$) obtains the same equilibrium like Cremer et al. (1989):

$$\begin{aligned} q_3 &= \frac{1}{2} - \frac{3\,t}{2} + \frac{\sqrt{(1-3t)^2 - 12F}}{2} \\ q_1 = q_2 &= \frac{1}{6} + \frac{t}{2} - \frac{\sqrt{(1-3t)^2 - 12F}}{6} \\ Q &= \frac{5}{6} - \frac{t}{2} + \frac{\sqrt{(1-3t)^2 - 12F}}{6} \\ TSS &= \frac{17 - 9t^2 + 6F + (1+3t)\sqrt{(1-3t)^2 - 12F}}{36} - 3F \end{aligned} \tag{23}$$

For the existence of a mixed triopoly, there has to be a equilibrium with a positive quantity produce for the public and private firms. Quantities of the private firms are positive for a positive transfer and the quantity of the public firm is positive if

$$q_3 = \frac{1}{2} - \frac{3\,t}{2} + \frac{\sqrt{(1-3t)^2 - 12F}}{2} > 0. \tag{24}$$

As the transfer, fixed costs and marginal costs are positive, the following result can be drawn.

Theorem 1. *A mixed Cournot-like triopoly with a welfare maximizig public firm which pays a premium as a transfer to her workers can exist, if the premium t, the fixed costs F and the marginal costs m are small enough* $t < \frac{1-m-2\sqrt{3}\sqrt{F}}{3}$ *and* $F < \frac{(1-m)^2}{12}$. *Then the total social surplus is* $TSS = \frac{17(1-m)^2 - 9t^2 + 6F + (1-m+3t)\sqrt{(1-m-3t)^2 - 12F}}{36} - 3F$.

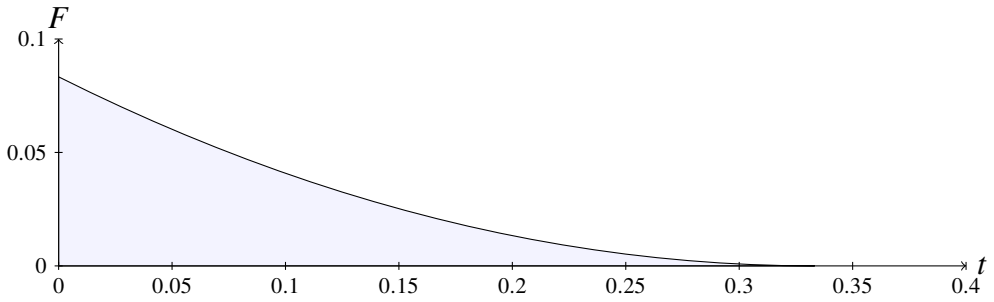


Figure 3: Inequality plot for general equilibrium

Parameter values: $m = 0.00$

Figure 3 shows the area for which a mixed triopoly can exist. Outside this area, there is no stable equilibrium which makes a mixed triopoly possible.

4.2.3. Market intervention by nationalization or entry

Two paths which lead to this market outcome of a mixed triopoly with one public firm and two private firms are considered in illustrated in figure 4. The first one is a Cournot triopoly, and the

government has the possibility to transfer one private firm into a public firm (nationalization). The second one is a Cournot duopoly and the government has the possibility to enter the market by establishing a public firm.

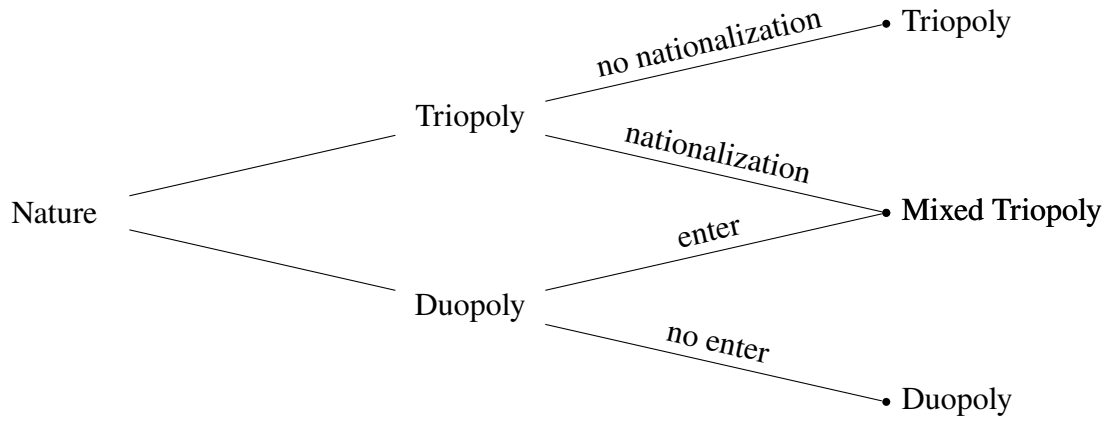


Figure 4: Markets before and after intervention

The optimal strategy of the government is simple. The regulator does not intervene in the market and ends the game if the expected TSS after nationalization (establishing) of a firm is smaller than the initial TSS of a Cournot triopoly (duopoly). For better comparability, the simplified case with marginal cost normalized to zero (m=0) is used.

Cournot triopoly: A symmetric standard Cournot model with three profit maximizing firms which face a NDF with $P = 1 - Q$ and marginal costs m obtains: $q_1 = q_2 = q_3 = \frac{1}{4} - \frac{m}{4}$, $Q = \frac{3(1-m)}{4}$, $P = \frac{1+3m}{4}$, and $TSS = \frac{15}{32}(m-1)^2 - 3F$.[11] For the simplified case $m = 0$, and $F = 0$ it obtains Q= $\frac{3}{4}$, TSS=$\frac{15}{32}$. Total social surplus of a Cournot triopoly is represented by the dotted red line in figure 19.

Nationalization The government will choose to nationalize a firm if overall surplus including the fixed costs is bigger converting a private firm into a public firm.

$$\frac{17 - 9t^2 + 6F + (1+3t)\sqrt{(1-3t)^2 - 12F}}{36} - 3F > \frac{15}{32} - 3F \tag{25}$$

Assuming t is positive, the inequality is true for all:

$$0 < t \leq \frac{1}{6} \quad \wedge \quad 0 < F \leq \frac{1}{12}(1-3t)^2 \tag{26}$$

or

$$\frac{1}{6} < t < \frac{1}{4} \quad \wedge \quad 0 < F < \frac{1}{16}(1-4t) \tag{27}$$

[11] See appendix for derivation of the Nash equilibrium.

which is represented by the light blue region in figure 5. Note equation (26) tells us that for small transfer $t < \frac{1}{6}$ market intervention is limited for fixed costs which allow the existence of real roots $\sqrt{(1-3t)^2 - 12F}$. Equation (27) shows that for bigger transfers $\frac{1}{6} < t < \frac{1}{4}$, the limiting factor is not the existence of real roots, but the existence of a solution of a overall binding NNPC.

Proposition 1. *Suppose a pure private Cournot triopoly, the government has the option to nationalize a profit maximizing private firm into a welfare maximizing public firm which pays a premium t to its workers and has a NNPC. Then the government will establish a firm, if* $t \leq \frac{1}{6} \wedge 0 < F < \frac{1}{12}(1-3t)^2$ *or* $\frac{1}{6} < t < \frac{1}{4} \wedge 0 < F < \frac{1}{16}(1-4t)$.

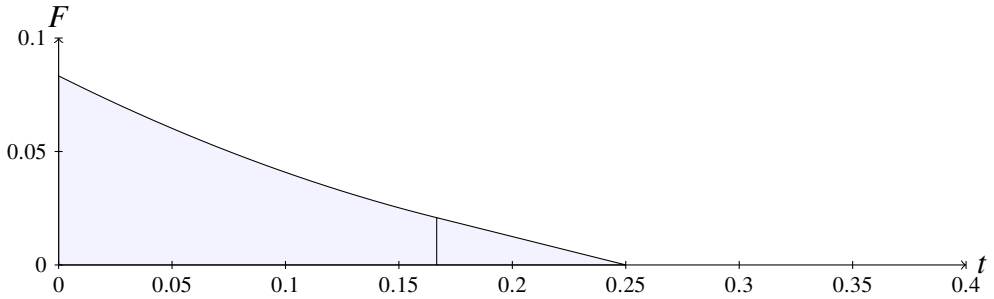


Figure 5: Inequality plot with transfer for Nationalization

Parameter values: $m = 0$

Figure 5 shows the area for which a mixed triopoly can exist. Outside this area, there is no stable equilibrium which makes a mixed triopoly possible. The mixed triopoly is better than the pure triopoly for $t < \frac{1}{4}$, if fixed cost is converging to zero($F \to 0$).

Cournot duopoly: To decide whether a market entry of a public firm in an existing duopoly is welfare improving, it is crucial that neither the fixed costs F nor the transfer t is too high. A symmetric standard Cournot model with two profit maximizing firms, which face a NDF $P = 1-Q$ marginal costs m obtains: $q_1 = q_2 = \frac{1}{3} - \frac{m}{3}$, $Q = \frac{2(1-m)}{3}$, $P = \frac{1+2m}{3}$, and $TSS = \frac{4}{9}(m-1)^2$.[12] For the simplified case $m = 0$, and $F = 0$ it obtains Q= $\frac{2}{3}$, TSS=$\frac{4}{9}$. Total social surplus of a Cournot duopoly is represented by the dotted blue line in figure 19.

Market entry The government will decide to found a firm, if overall welfare including the fixed costs is bigger after the market entry.

$$\frac{17 - 9t^2 + 6F + (1+3t)\sqrt{(1-3t)^2 - 12F}}{36} - 3F > \frac{4}{9} - 2F \tag{28}$$

[12]See appendix for derivation of the Nash equilibrium.

For positive fixed cost $0 \leq F$ and a positive but not too large transfer $0 \leq t \leq 1/3$ the inequality is true for

$$0 < t < \frac{1}{7} \quad \wedge \quad 0 < F \leq \frac{2}{75}(2 - 3t - 27t^2) \tag{29}$$

$$\frac{1}{7} \leq t < \frac{1}{3} \quad \wedge \quad 0 < F \leq \frac{1}{12}(1 - 3t)^2 \tag{30}$$

which is represented by the light blue region in figure 6. Note equation (29) tells us that for small transfer $t < \frac{1}{7}$ market intervention is limited for fixed costs which allow the existence of a solution of a overall binding NNPC. Equation (30) shows that for bigger transfers $\frac{1}{7} < t < \frac{1}{3}$, the limiting factor is not the existence of a solution of a overall binding NNPC, but the existence of real roots $\sqrt{(1-3t)^2 - 12F}$. The mixed triopoly is strictly better than the monopoly, and better or as good as the duopoly. The mixed triopoly is better than the pure duopoly for $t < \frac{1}{3}$, if fixed cost F is converging to zero($F \to 0$).

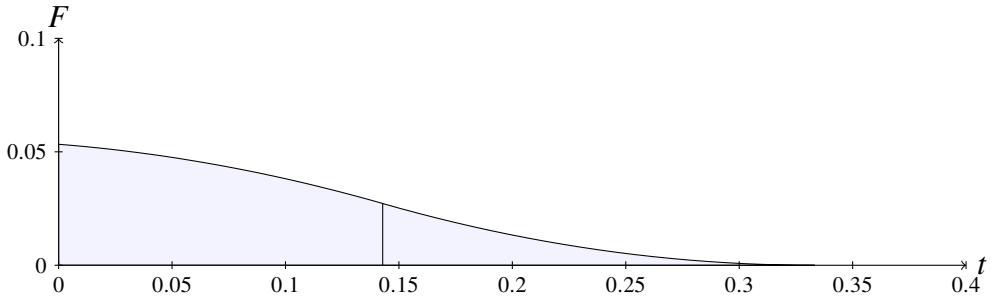


Figure 6: Inequality plot with transfer for market entry

Parameter values: $m = 0$

Proposition 2. *Suppose a pure private Cournot duopoly, the government has the option to establish a welfare maximizing firm, which pays a premium t to its workers and has a NNPC. Then a government will establish a firm, if* $t < \frac{1}{7} \wedge F < \frac{2}{75}(2 - 3t - 27t^2)$ *or* $\frac{1}{7} \leq t < \frac{1}{3} \wedge 0 < F \leq \frac{1}{12}(1 - 3t)^2$.

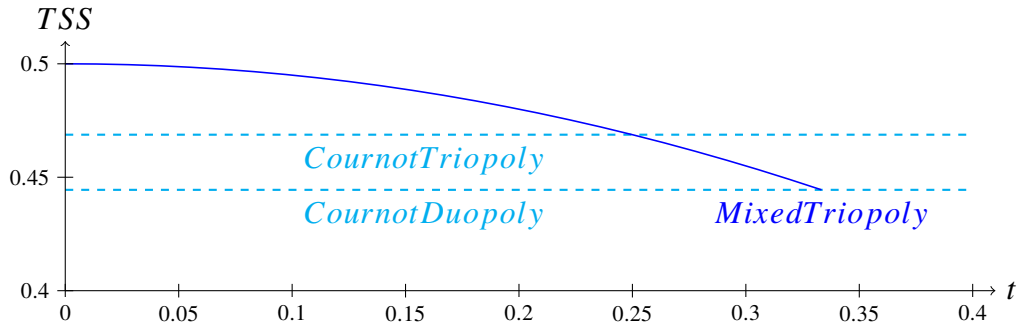


Figure 7: Social surplus with no-negative profit constraint and a transfer

Parameter values: $m = 0, F \to 0$

4.3. Equilibrium with social costs

Up to now, the parameter t has been interpreted as a premium paid to the public firm's workers. As an alternative, let's analyze the mixed triopoly with no-negative profit constraint for the public firms with parameter s which can be interpreted as an inefficiency by a firm, if its owned and operated by the government. Parameter s are additional social costs next to marginal costs m which lowers the public firms profit and the welfare. The market equilibrium up to the decision of nationalization or market entry is unaffected by this alternative interpretation, because the public firm's reaction function is still the zero profit curve.

4.3.1. Reaction functions

As the maximization problem of the private firms are the same as in the case before in 4.2.1, the resulting reaction functions are the same:

$$\bar{q_1}(q_2, q_3) = \frac{1 - q_2 - q_3 - m}{2}, \tag{31}$$

$$\bar{q_2}(q_1, q_3) = \frac{1 - q_1 - q_3 - m}{2}. \tag{32}$$

The public firm now faces a slightly different problem, because the total social surplus is now directly decreasing in s (see (8)):

$$\max_{q_3} T(q_1 + q_2 + q_3) = -\frac{(q_1 + q_2 + q_3)^2}{2} + (q_1 + q_2 + q_3)(1 - m) - q_3 s - 3F \tag{33}$$

s.t.

$$\pi_3(q_1, q_2, q_3) = q_3 p(q_1 + q_2 + q_3) - m q_3 - s q_3 - F \geq 0 \tag{34}$$

As $\frac{dTSS}{dq_3} > 0$ for all $q_3 \leq 1 - q_1 - q_2 - s$, the total surplus is a strictly increasing function of the public firm's output (up to $1 - q_1 - q_2 - s$). The break-even constraint is again always binding at the solution. This implies that firm 3 wants to produce the maximum output allowed by $\pi_3(q_1, q_2, q_3) = 0$, which is quadratic in q_3. The reaction function of the public firm $\bar{q_3}(q_1, q_2)$ is thus given by the larger root of this equation. Furthermore, it is assumed that firm 3 does not operate when the break-even constraint is not satisfied(i.e. there are no real roots). Following the same argumentation as in the transfer case, we get the reaction function of the public firm.

$$\bar{q_3}(q_1, q_2) = \begin{cases} \dfrac{1 - m - s - q_1 - q_2 \pm \sqrt{(1 - m - s - q_1 - q_2)^2 - 4F}}{2} & \text{for } (1 - m - s - q_1 - q_2)^2 - 4F \geq 0 \\ 0 & \text{else.} \end{cases} \tag{35}$$

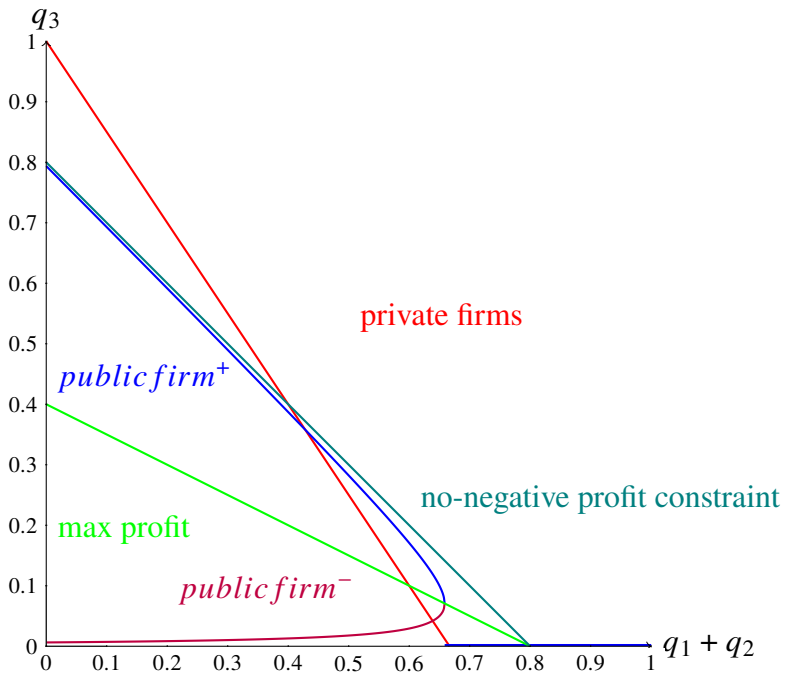


Figure 8: Reaction functions with no-negative profit constraint

Red: $q_1 + q_2 = \frac{2}{3}(1 - q_3) \Leftrightarrow q_3 = 1 - \frac{3}{2}(q_1 + q_2)$
Blue: $q_3 = \frac{1-0.2-(q_1+q_2+\sqrt{(1-0.2-(q_1+q_2)^2)-4F})}{2}$ for $q_1 + q_2 = 0$ to 0.658578 and 0 else.
Purple: $q_3 = \frac{1-0.2-(q_1+q_2-\sqrt{(1-0.2-(q_1+q_2)^2)-4F})}{2}$ for $q_1 + q_2 = 0$ to 0.658578 and 0 else.
Green: Reaction function if firm 3 would maximize profit instead of welfare. $q_3 = \frac{1-0.2-(q_1+q_2)}{2}$
Teal: Reaction function without NNPC $q_3 = 1 - 0.2 - (q_1 + q_2)$
Parameter values: $F \to 0, m = 0, s = 0.2$

4.3.2. Nash equilibrium

As mentioned before, the market solution is given by a (noncooperative) Nash equilibrium. To find a Subgame perfect Nash equilibrium (SPNE), the system of best responses given by (31), (32) and (35) has to be solved.[13] The solution with the higher quantity of the public firm q_3, lower quantity of private firms and higher overall quantity is

$$q_1^h = q_2^h = \frac{1 - m + 3s - \sqrt{(1 - m - 3s)^2 - 12F}}{6}, \tag{36}$$

$$q_3^h = \frac{1 - m - 3s + \sqrt{(1 - m - 3s)^2 - 12F}}{2}, \tag{37}$$

$$Q^h = \frac{5 - 3s - 5m + \sqrt{(1 - m - 3s)^2 - 12F}}{6}, \tag{38}$$

$$TSS^h = \frac{17(1 - m)^2 - 18s(1 - m) + 45s^2 - 102F + (1 - m - 15s)\sqrt{(1 - m - 3s)^2 - 12F}}{36}. \tag{39}$$

[13]The code(Maple 2020) that is used to solve the systems and isolate the variables is available on my website: https://dustin-jonak.github.io/research.html

Due to the quadratic function, there might be a second solution. The solution with the lower quantity of the public firm q_3, higher quantity of private firms and lower overall quantity is:

$$q_1^l = q_2^l = \frac{1 - m + 3s + \sqrt{(1 - m - 3s)^2 - 12F}}{6}, \tag{40}$$

$$q_3^l = \frac{1 - m - 3s - \sqrt{(1 - m - 3s)^2 - 12F}}{2}, \tag{41}$$

$$Q^l = \frac{5 - 3s - 5m - \sqrt{(1 - m - 3s)^2 - 12F}}{6}, \tag{42}$$

$$TSS^l = \frac{17(1-m)^2 - 18s(1-m) + 45s^2 - 102F - (1 - m - 15s)\sqrt{(1 - m - 3s)^2 - 12F}}{36}. \tag{43}$$

Even though $Q^h > Q^l$, because different from the transfer case, more output is not always better. Thus s determines if the higher or lower output equilibrium leads to a higher Social Surplus.

$$TSS^l > TSS^h \quad \Leftrightarrow \quad s > \frac{1 - m}{15} \tag{44}$$

If social costs s are sufficiently low $s < (1 - m)/15$, the gain of a greater overall quantity by the high quantity Nash equilibrium is high enough to compensate for the social costs. But if social costs are above $s > (1 - m)/15$, the lower Nash equilibrium has the greater overall welfare as the increased consumer surplus cannot outweigh the increased social cost of the public firm. Assume that the government has a selection mechanism that allows it to enforce the Nash equilibrium yielding the higher total surplus.

$$TSS^* = \frac{17(1-m)^2 - 18s(1-m) + 45s^2 - 102F + |1 - m - 15s|\sqrt{(1 - m - 3s)^2 - 12F}}{36} \tag{45}$$

which is represented by the blue line in figure 19.

Lemma 1. *If there exists a SPNE of a mixed triopoly with real social costs, the public firm chooses the bigger quantity for $s < (1 - m)/15$ and the lower quantity for $s > (1 - m)/15$.*

With the same argument as well as in the transfer case, it is possible to conclude:

Theorem 2. *A mixed Cournot triopoly with a welfare maximizing public firm with an inefficiency which leads to additional marginal social costs of s can exist, if the inefficiency s, the fixed costs F and the marginal costs m are small enough $s < \frac{1-m-2\sqrt{3}\sqrt{F}}{3}$. Then the total social surplus is $TSS^* = \frac{17(1-m)^2-18s(1-m)+45s^2-102F+|1-m-15s|\sqrt{(1-m-3s)^2-12F}}{36}$.*

4.3.3. Market intervention by nationalization or entry

Like in the section above, consider two paths which might lead to mixed triopoly with one public firm and two private firms. The first one is the Cournot triopoly and the government

has the possibility to nationalize a private firm into a public firm. The second one is a Cournot duopoly and the government can enter the market by establishing a public firm. To decide if entry or nationalization is welfare enhancing, the government has to compare the Social Surplus of the Equilibria. The creation of a new firm gives rise to some additional fixed costs. The corresponding trade-off is clear. On the one hand the existence of a public firm is expected to have a positive effect on total surplus because of the nature of its objective function. On the other hand, a new firm lowers net welfare, everything else equal, due to the increase of social costs s and fixed costs F (in case of entry). For simplicity marginal costs are standardized to zero ($m = 0$).

Nationalization Nationalization of an existing private firm of a symmetric Cournot trio-poly is welfare enhancing if the Nash equilibrium of a mixed triopoly with real social costs has a higher social surplus than the social surplus of a pure triopoly.

$$\frac{17 - 18s + 45s^2 - 102F + (|1 - 15s|)\sqrt{(1-3s)^2 - 12F}}{36} > \frac{15}{32} - 3F. \tag{46}$$

The inequality is true for

$$0 < s < \frac{\sqrt{241} - 14}{54} \quad \wedge \quad 0 < F \leq \frac{(1-3s)^2}{12} \tag{47}$$

and

$$\frac{\sqrt{241} - 14}{54} < s \leq \frac{1}{32} \quad \wedge \quad 0 < F < \frac{4\sqrt{64s^2 - 32s + 1}|15s - 1| - 480s^2 + 128s - 3}{16} \tag{48}$$

and

$$\begin{aligned} \frac{1}{32} < s < \frac{2 - \sqrt{3}}{8} \frac{-4\sqrt{64s^2 - 32s + 1}|15s - 1| - 480s^2 + 128s - 3}{16} \\ < F < \frac{4\sqrt{64s^2 - 32s + 1}|15s - 1| - 480s^2 + 128s - 3}{16} \end{aligned} \tag{49}$$

which is represented by the light blue region in figure 9(a).

Proposition 3. *Suppose a pure private Cournot triopoly, and the goverment has the option to nationalize a profit maximizing private firm into a welfare maximizing public firm which bears a inefficiency s and has a NNPC. Then the government will nationalize a firm, if* $0 < s < \frac{\sqrt{241}-14}{54} \wedge 0 < F \leq \frac{(1-3s)^2}{12}$, *or* $\frac{\sqrt{241}-14}{54} < s \leq \frac{1}{32} \wedge 0 < F < \frac{4\sqrt{64s^2-32s+1}|15s-1|-480s^2+128s-3}{16}$, *or* $\frac{1}{32} < s < \frac{2-\sqrt{3}}{8} \wedge \frac{-4\sqrt{64s^2-32s+1}|15s-1|-480s^2+128s-3}{16} < F < \frac{4\sqrt{64s^2-32s+1}|15s-1|-480s^2+128s-3}{16}$.

Market entry The government will decide to establish a firm, if overall welfare including the fixed costs is bigger after the market entry than the social surplus of a pure duopoly. As positive fixed cost $0 \leq F$ and a positive but not too large social costs $0 \leq s \leq 1/3$ the government will

enter the market if

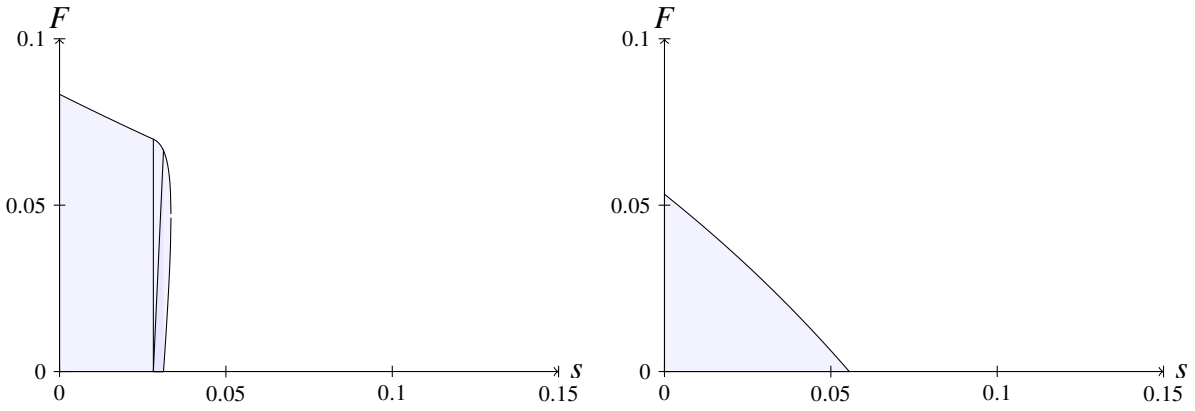
$$\frac{17 - 18s + 45s^2 + 6F + (|1 - 15s|)\sqrt{9\,s^2 - 6\,s + 1 - 12F}}{36} - 3F > \frac{4}{9} - 2F. \tag{50}$$

The inequality is true for

$$0 < s < \frac{1}{18} \quad \wedge \quad 0 < F < \sqrt{225s^2 + 4}|15s - 1| - 225s^2 - 30s + 2 \tag{51}$$

which is represented by the the light blue area in figure 9(b).

Proposition 4. *Suppose a pure private Cournot duopoly, and the option to establish a welfare maximizing firm which bears a inefficiency s and has a NNPC. Then the government will establish a firm, if* $0 < s < \frac{1}{18} \quad \wedge \quad 0 < F < \sqrt{225s^2 + 4}|15s - 1| - 225s^2 - 30s + 2$.



(a) nationalization with social costs

(b) market entry with social costs

Figure 9: Inequality plots

Parameter values: $m = 0$

As you can also see in figure 10, no public firm is established or nationalized for social costs bigger than 1/15, and there exist no equilibrium in which the public choose the smaller quantities for the two cases studied before.

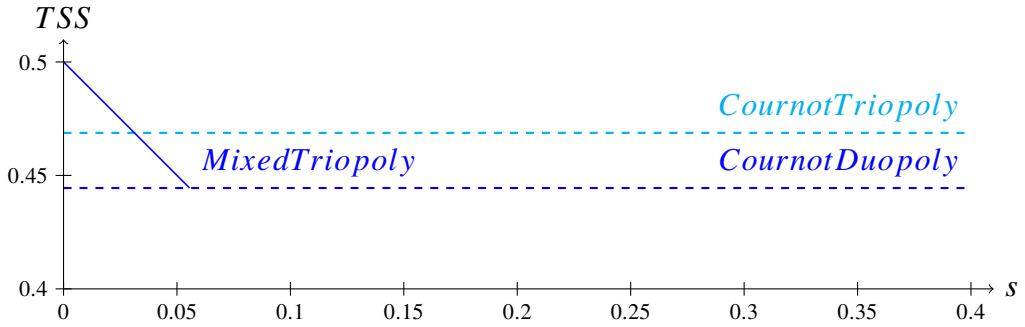


Figure 10: Social Surplus with no-negative profit constraint and social cost

Parameter values: $m = 0, F \to 0$

5. Mixed triopoly with capacity choice

A multi-stage mixed triopoly with capacity choice is modeled. The setup is built on Lu and Poddar (2005), but a triopoly instead of a duopoly is analyzed. Additionally, fixed costs are added and a second case is considered where the public firm has no inefficiency, but pays a premium as a transfer to her employees.

5.1. The capacity model

A mixed triopoly market with two private firms 1 and 2 and one public firms 3 is considered. All three firms produce a homogeneous good and face a normalized inverse demand function given by

$$p(Q) = 1 - Q, \tag{52}$$

where p is the market price and Q is the total output. Total output is the sum of the quantity produced by all three firms

$$Q = \sum_{i=1}^{3} q_i = q_1 + q_2 + q_3, \tag{53}$$

where q_1 denoted the quantity produced by the private firm 1, q_2 denotes the quantity produced by the private firm 2, and q_3 denotes the quantity produced by the public firm 3. Firm 1 and firm 2 are profit-maximizing private firms, and firm 3 is a welfare maximizing public firm.

Capacity choice Firms have to choose a capacity x_i in order to produce a quantity q_i. It is cost efficient to produce at capacity ($x_i = q_i$), and there are penalties[14] for producing over capacity ($x_i < q_i$) or under capacity ($x_i > q_i$).

Private firm The profit π_i of a firm i is a function of their revenue and their costs C_i. Revenue of firm i is generated by selling the quantity q_i at a market price p. The profit function of firm i is the revenue minus costs and therefore given by

$$\pi_i = pq_i - C_i. \tag{54}$$

The cost function C_i of the private firms are

$$C_1(q_1, x_1) = mq_1 + (q_1 - x_1)^2 - F, \tag{55}$$

$$C_2(q_2, x_2) = mq_2 + (q_2 - x_2)^2 - F, \tag{56}$$

where m_i represents the efficiency of firm's i production, x_i the capacity level of firm i, and F fixed cost.

[14] As maintenance of unoccupied houses and flats is costly, and the house owner has to pay heating if she has no dweller. Costs for over-occupied houses are extra costly as well, as costs for maintenance increase.

Costs are increasing linearly at rate m, which is a standard assumption. Firms choose capacity levels x_i such that it is cost-efficient to produce at capacity and costs are increasing quadraticly in deviating from the chosen capacity level $(q_i - x_i)^2$.[15]

Public firm The Social Surplus or Welfare maximized by the public firm is the sum of the consumer rent and the profits of all firms.

$$TSS = \frac{Q^2}{2} + \sum_{i=1}^{3} \pi_i \tag{57}$$

It is assumed that the public firm has additional social costs s or pays a premium t to their employees as a transfer.

Transfer In the transfer case, the public firm is as efficient as the private firms. It has the same marginal costs m and costs for deviating from the capacity. In addition to that, it pays a positive premium t per unit output to workers, which is a pure transfer and does not lower the welfare.

$$C_3(q_3, x_3)^{transfer} = mq_3 + (q_3 - x_3)^2 + tq_3 - F \tag{58}$$

$$\pi_i(q_1, ..., q_n)^{transfer} = q_i\ p(Q) - mq_i - (q_i - x_i)^2 - tq_i - F. \tag{59}$$

The premium t paid by the public firm increases the consumer surplus by the same amount as it decreases the profit of the public firm, respectively the producer surplus. Thus it does not appear in her total surplus function:

$$TSS^{transfer} = Q - \frac{Q^2}{2} - Qm - \sum_{i=1}^{n} (q_i - x_i)^2 - 3F. \tag{60}$$

Social Costs In the second case, the public firm is less efficient than the private firms and it has therefore higher costs. It has costs of s on top of the marginal costs of the private firms per quantity output.

$$C_3(q_3, x_3)^{socialcosts} = mq_3 + (q_3 - x_3)^2 + sq_3 - F. \tag{61}$$

The effect on the public firm's profit is the same as in the transfer case

$$\pi_i(q_1, ..., q_n)^{socialcosts} = q_i p(Q) - mq_i - sq_i - (q_3 - x_3)^2 - F. \tag{62}$$

[15]This cost function was used by Vives (1986); Horiba and Tsutsui (2000); Nishimori and Ogawa (2004) and Lu and Poddar (2005).

However, this has a negative impact on the social surplus by social costs s times public firm's quantity q_3, the total social surplus function is instead:

$$TSS^{socialcosts} = Q - \frac{Q^2}{2} - Qm - q_i s - \sum_{i=1}^{n}(q_i - x_i)^2 - 3F. \tag{63}$$

Time structure The model has four periods. All choices made by firms are observable.

Stage 1. Public firm chooses to enter the market or not.

Stage 2. Public firm chooses capacity x_3 (in case of entry).

Stage 3. Private firms choose capacities x_1 and x_2 simultaneously.

Stage 4. Private and Public firms choose quantity q_1, q_2 and q_3 simultaneously.

We look for subgame perfect Nash equilibria (SPNE) for the sequential game. Following the standard concept, the model is solved backwards, starting with the fourth stage.[16]

Note that the quantity choice and therefore competition of the fourth stage is comparable to a Nash-equilibrium of a Cournot game. The capacity choice of stage two and three in contrast is similar to a Stackelberg game, in which the public firm acts as a Stackelberg leader by having to choose quantities prior to the private firms.

5.2. Equilibrium with a transfer

For now, no inefficiency of the public firm is assumed, but a premium t in addition to the marginal costs m has to be paid as a transfer to public firms workers.

5.2.1. Fourth Stage: simultaneous quantity choice

Given their production capacities x_1, x_2 and x_3 , firms 1 and 2 are maximizing profits (54), and firm 3 is maximizing Social Surplus (57) by choosing their quantities q_1, q_2, q_3

$$\max_{q_1} \pi_1 (q_1, q_2, q_3, x_1) = (1 - q_1 - q_2 - q_3 - m)q_1 - (q_1 - x_1)^2 - F \tag{64}$$

$$\max_{q_2} \pi_2 (q_1, q_2, q_3, x_2) = (1 - q_1 - q_2 - q_3 - m)q_2 - (q_2 - x_2)^2 - F \tag{65}$$

$$\begin{aligned}\max_{q_3} TSS (q_1, q_2, q_3, x_1, x_2, x_3) = &\left(1 - \frac{q_1 + q_2 + q_3}{2} - m\right)(q_1 + q_2 + q_3) \\ &- (q_1 - x_1)^2 - (q_2 - x_2)^2 - (q_3 - x_3)^2 - 3F.\end{aligned} \tag{66}$$

Notice that the public firm does not take the transfer into consideration when maximizing the Social surplus. The premium paid by the public firm to its workers is subtracted of the public firm's profit but it is added to the consumer surplus by the same amount. As the public firm maximizes the premium neutral welfare and not its profit, the premium has no effect on its

[16]The code(Maple 2020) that is used to solve the systems and isolate the variables is available on my website: https://dustin-jonak.github.io/research.html

problem to solve. Hence, parameter t does not appear in the maximization problem. Solving the first order conditions of (64),(65), and (66) for their own corresponding quantities yield

$$q_1(q_2, q_3, x_1) = \frac{1 - q_2 - q_3 - m + 2x_1}{4} \tag{67}$$

$$q_2(q_1, q_3, x_2) = \frac{1 - q_1 - q_3 - m + 2x_2}{4} \tag{68}$$

$$q_3(q_1, q_2, x_3) = \frac{1 - q_1 - q_2 - m + 2x_3}{3}. \tag{69}$$

Combining the reaction functions of the private firms yields:

$$q_{12} = \frac{2}{5}(1 - m - q_3 + x_1 + x_2). \tag{70}$$

In the fourth stage, all firms are choosing the quantity to produce simultaneously. Solving the system (67),(68),(69) yields

$$q_1(x_1, x_2, x_3) = \frac{22x_1 - 4x_2 + 3(2 - 2m - 2x_3)}{39} \tag{71}$$

$$q_2(x_1, x_2, x_3) = \frac{-4x_1 + 22x_2 + 3(2 - 2m - 2x_3)}{39} \tag{72}$$

$$q_3(x_1, x_2, x_3) = \frac{3(-2x_1 - 2x_2 + 3 - 3m + 10x_3)}{39}. \tag{73}$$

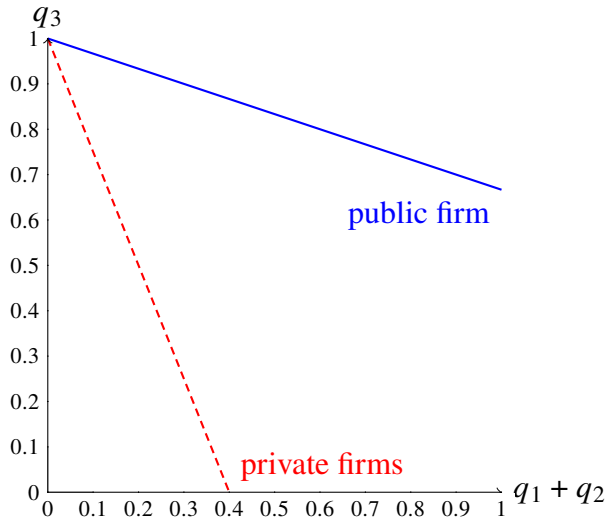


Figure 11: Reaction functions of quantities with capacity choice and a transfer
densely dashed red: $q_3 = 1 - \frac{5}{2}(q_1 + q_2 + x_1 + x_2)$ with $x_1 = x_2 = 0$
solid blue: $q_3 = \frac{1-(q_1+q_2)+2x_3}{3}$ with $x_3 = 1$
Parameter values; $m = 0, F = 0$

5.2.2. Third Stage: capacity choice of private firms

Private firms are observing the capacity chosen by the public firm 3 in stage two, and take it therefore as given. They anticipate the quantity by taking the reaction function of the fourth stage into account. Private firms maximize their profits by choosing capacity x_1 and x_2 simultaneously

$$\max_{x_1} \pi_1 = \left(\frac{22x_1 - 4x_2 + 3(2 - 2m - 2x_3)}{39}\right)\left(\frac{6 - 4x_1 - 4x_2 - 6m - 6x_3}{13}\right) - \left(\frac{-17x_1 - 4x_2 + 3(2 - 2m - 2x_3)}{39}\right) - F \quad (74)$$

$$\max_{x_2} \pi_2 = \left(\frac{-4x_1 + 22x_2 + 3(2 - 2m - 2x_3)}{39}\right)\left(\frac{6 - 4x_1 - 4x_2 - 6m - 6x_3}{13}\right) - \left(\frac{-4x_1 - 17x_2 + 3(2 - 2m - 2x_3)}{39}\right) - F. \quad (75)$$

Solving the problem results in

$$x_1\,(x_2, x_3) = \frac{264 - 176x_2 - 264m - 264x_3}{553} \quad (76)$$

$$x_3\,(x_1, x_3) = \frac{264 - 176x_1 - 264m - 264x_3}{553}. \quad (77)$$

Solving the system yields:

$$x_1(x_3) = x_2(x_3) = \frac{44(2 - 2m - 2x_3)}{243}. \quad (78)$$

The public firm is facing therefore the aggregated reaction function of the private firms:

$$x_{12}(x_2) = \frac{88(2 - 2m - 2x_3)}{243}. \quad (79)$$

Substituting x_1 and x_2 into q_1 and q_1 and q_1, leads to

$$q_1(x_3) = q_2(x_3) = \frac{26}{81}\,(1 - m - x_3) \quad (80)$$

$$q_3(x_3) = \frac{29(1 - m) + 214\,x_3}{243}. \quad (81)$$

5.2.3. Second Stage: capacity choice of public firm

In the second stage, the public firm knows that its capacity choice determines the capacity choice of the private firms in the second stage as well as the quantity choices of the third stages. Therefore, it takes them into account and maximizes the welfare regarding the capacity. Substitute q_1 and q_2 with (80) and q_3 with (81) as well as x_1 and x_2 with (78) into (57) obtains

the maximization problem of the public firm in the second stage:

$$\max_{x_3} TSS = \frac{5446(2x_3(1-m) - x_3^2) + 53603(1-m)^2}{2(59049)} - 3F. \tag{82}$$

The first order condition yields

$$x_3 = 1 - m. \tag{83}$$

Substituting x_3 into x_1, q_1 and q_2, gets

$$x_1 = x_2 = 0 \tag{84}$$

and

$$q_1 = q_2 = 0; \quad q_3 = 1 - m. \tag{85}$$

Both types of firms, private and public, choose exact capacity and therefore

$$Q = 1 - m; \quad X = 1 - m \tag{86}$$

$$TSS^{transfer} = \frac{(1-m)^2}{2} - 3F. \tag{87}$$

Total social surplus in equilibirum is represented by the pink line in figure 19.

Theorem 3. *A mixed triopoly, in which the public firm faces no inefficiency but pays a premium to their workers as a transfer and chooses its capacity before the private firms choose capacity and then quantity is choosen by all firms simultaneously does not exist. In this case there is a public monopoly as the private firms choose no capacity, and the public firm chooses and produces the first best capacity. Hence, the total social surplus is* $TSS = \frac{(1-m)^2}{2} - 3F$.

5.2.4. First Stage: nationalization vs entry

As the capacity constraint is not holding back the public firm to produce the first best quantity and it is because there are no inefficiencies it always enters the market by nationalization of an existing private firm.

Proposition 5. *Suppose a pure private triopoly with capacity choice, and the option to nationalize a private firm to create a welfare maximizing firm which pays a premium as a transfer and choose capacity before private firms choose capacity, and all firms compete over quantities in the last stage. Then the government will establish a public firm chooses the first best capacity and it produces the first best quantity as it acts as a monopolist regardless of the premium paid.*

For establishing a new public firm, the fixed costs F has to be smaller than the improved market welfare.

$$TSS^{transfer} = \frac{(1-m)^2}{2} - 3F > TSS^{Duopoly} = \frac{(1-m)^2 838}{1849} - 2F \tag{88}$$

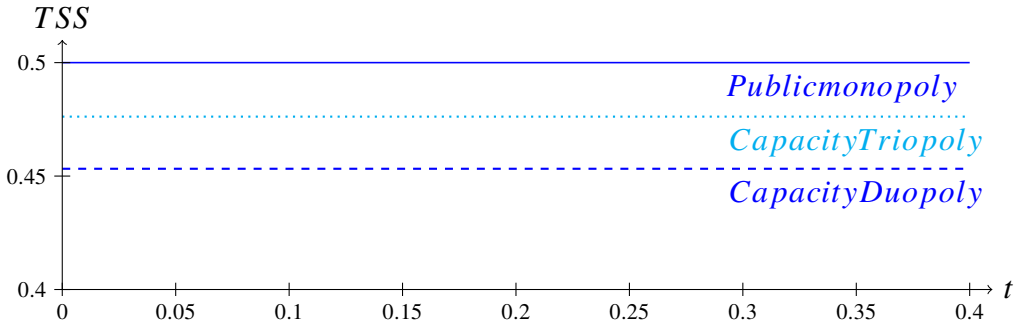


Figure 12: Social Surplus with a capacity constraint and a transfer
solid blue: Public monopoly, dotted cyan: Pure triopoly, dashed blue: Pure duopoly
Parameter values: $m = 0, F \to 0$

This is true for all small $F < 173(1-m)^2/3698 \approx 0.04678(1-m)^2$. In absence of market entry costs $F = 0$ there is no difference between nationalization and entry for the transfer case. It is now optimal for the public firm to produce the whole output by itself $q_3 = 1 - m$ by becoming a monopolist and producing the first best quantity.

Proposition 6. *Suppose a pure private duopoly with capacity choice, and the option of market entry by establishing a welfare maximizing firm which pays a premium as a transfer and chooses capacity before private firms choose capacity, and all firms compete over quantities in the last stage. Then the government will establish a public firm which chooses the first best capacity and produces the first best quantity and acts as a monopolist regardless of the premium paid, if the fixed costs are small enough* $F < 173(1-m)^2/3698$.

5.3. Equilibrium with social costs

5.3.1. Fourth Stage: simultaneous quantity choice

Given their production capacities x_1, x_2 and x_3 , firms 1 and 2 are maximizing profit and firm 3 is maximizing Social Surplus by choosing their quantities:

$$\max_{q_1} \pi_1 = (1 - q_1 - q_2 - q_3 - m)q_1 - (q_1 - x_1)^2 - F, \tag{89}$$

$$\max_{q_2} \pi_2 = (1 - q_1 - q_2 - q_3 - m)q_2 - (q_2 - x_2)^2 - F, \tag{90}$$

$$\begin{aligned} \max_{q_3} TSS = &\left(1 - \frac{q_1 + q_2 + q_3}{2} - m\right)(q_1 + q_2 + q_3) \\ &- (q_1 - x_1)^2 - (q_2 - x_2)^2 - (q_3 - x_3)^2 - sq_3 - 3F. \end{aligned} \tag{91}$$

The first order condition yields:

$$q_1(q_2, q_3, x_1) = \frac{1 - q_2 - q_3 - m + 2x_1}{4}, \tag{92}$$

$$q_2(q_1, q_3, x_3) = \frac{1 - q_1 - q_3 - m + 2x_2}{4}, \tag{93}$$

$$q_3(q_1, q_3, x_2) = \frac{1 - q_1 - q_2 - m - s + 2x_3}{3}. \tag{94}$$

Combining the reaction functions of the private firms yields:

$$q_{12} = \frac{2}{5}(1 - m - q_3 + x_1 + x_2). \tag{95}$$

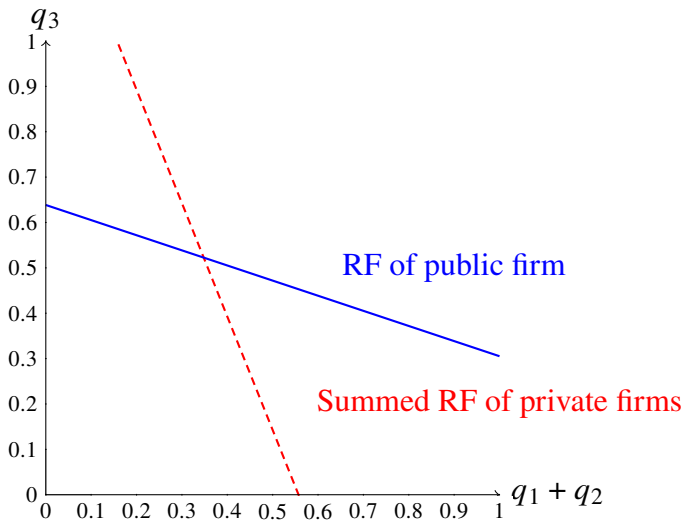


Figure 13: Reaction functions of quantities with capacity choice and social costs
densely dashed red: $q_3 = 1 + x_1 + x_2 - \frac{5}{2}(q_1 + q_2)$ with $x_1 = x_2 = 0.19632758$
solid blue: $q_3 = \frac{1-0.05-(q_1+q_2)+2x_3}{3}$ with $x_3 = 0.48287$
Parameter values: $s = 0.05, F = 0, m = 0$

In the fourth stage, all firms choose the produced quantity simultaneously. Solving the system yields

$$q_1(x_1, x_2, x_3) = \frac{22x_1 - 4x_2 + 3(2 - 2m + s - 2x_3)}{39} \tag{96}$$

$$q_2(x_1, x_2, x_3) = \frac{-4x_1 + 22x_2 + 3(2 - 2m + s - 2x_3)}{39} \tag{97}$$

$$q_3(x_1, x_2, x_3) = \frac{3(-2x_1 - 2x_2 + 3 - 3m - 5s + 10x_3)}{39}. \tag{98}$$

5.3.2. Third Stage: capacity choice of private firms

Private firms are observing the capacity chosen by the public firm 3 in stage two, and take it therefore as given. They anticipate the quantity by taking the reaction function of the fourth stage

into account. Private firms maximize their profits by choosing capacity x_1 and x_2 simultaneously.

$$\max_{x_1} \pi_1 = \left(\frac{22x_1 - 4x_2 + 3(2 - 2m + s - 2x_3)}{39}\right)\left(\frac{6 - 4x_1 - 4x_2 - 6m + 3s - 6x_3}{13}\right) - \left(\frac{-17x_1 - 4x_2 + 3(2 - 2m + s - 2x_3)}{39}\right) - F \quad (99)$$

$$\max_{x_2} \pi_2 = \left(\frac{-4x_1 + 22x_2 + 3(2 - 2m + s - 2x_3)}{39}\right)\left(\frac{6 - 4x_1 - 4x_2 - 6m + 3s - 6x_3}{13}\right) - \left(\frac{-4x_1 - 17x_2 + 3(2 - 2m + s - 2x_3)}{39}\right) - F. \quad (100)$$

Solving the problem (99) and (100) results in

$$x_1(x_2, x_3) = \frac{264 - 176x_2 - 264m + 132s - 264x_3}{553}, \quad (101)$$

$$x_2(x_1, x_3) = \frac{264 - 176x_1 - 264m + 132s - 264x_3}{553}. \quad (102)$$

Solving the system yields the reaction functions of the private firms regarding their capacities

$$x_1(x_3) = x_2(x_3) = \frac{44(2 - 2m - 2x_3 + s)}{243}. \quad (103)$$

The aggregated reaction function is therefore

$$x_{12}(x_3) = \frac{88(2 - 2m - 2x_3 + s)}{243} \quad (104)$$

which is represented by the solid red line in figure 14. Substituting x_1 and x_2 into q_1 and q_2 and q_3, gets

$$q_1(x_3) = q_2(x_3) = \frac{26}{81}\left(1 - m - x_3 + \frac{1}{2}s\right) \quad (105)$$

$$q_3(x_3) = \frac{29(1 - m) + 107(2x_3 - s)}{243}. \quad (106)$$

5.3.3. Second Stage: capacity choice of public firm

In the second stage, the public firm knows that its capacity choice determines the capacity choice of the private firms in the third stage as well as the quantity choices of the fourth stage. Therefore, it takes them into account and maximizes the welfare regarding the capacity. Substitute q_1 and q_2 with (105) and q_3 with (106), as well as x_1 and x_2 with (103) into (57) obtains the maximazation

problem of the public firm in the second stage:

$$\max_{x_3} TSS = \frac{5446(2(1-m)x_3 - x_3^2 + (1-s)) + 53603(1-m)^2 - 2s56326x_3}{2(59049)} - 3F, \quad (107)$$

which yields

$$x_3 = 1 - m - \frac{28163s}{2723}. \quad (108)$$

The Nash Equilibrium can be illustrated graphically(see Fig. 14) by the iso-social-surplus-curve which is tangential to the summed reaction function of the private firms. Right above the iso-socialsurplus curve the social surplus is bigger and left below the curve the social surplus is smaller. As the public firm can only induce a social surplus which is on the summed reaction curve, the optimum is the tangents of the curve and the reaction function of the private firms.

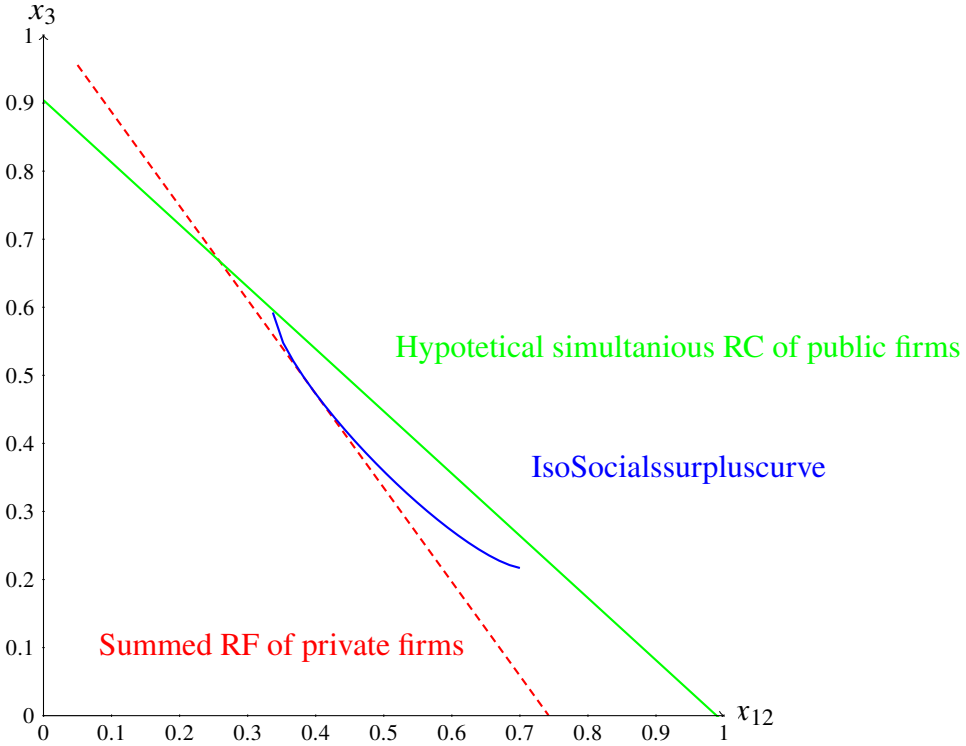


Figure 14: Reaction functions of capacity choice

densely dashed red: $x_1 + x_2 = \frac{2}{243}(88 - 88m + 44s - 88x_3) \Leftrightarrow x_3 = \frac{41}{40} - \frac{243}{176}(x_1 + x_2)$
solid blue: $Socialsurplus(x_1, x_2, x_3) = \frac{1008443}{2178400}$
$\Leftrightarrow x_3 = \frac{633}{700} - \frac{32(x_1+x_2)}{35} - \frac{13}{175}\sqrt{-\frac{3586}{389} + 40(x_1 + x_2) - (75(x_1 + x_2)^2)/2}$
Green: $x_3 = 1 - m - \frac{32(x_1+x_2)-67s}{35}$
Parameter values: $s = 0.05, m = 0, F = 0$

Substituting x_3 into x_1, q_1 and q_2, gets

$$x_1 = x_2 = \frac{10692s}{2723} \quad (109)$$

and

$$q_1 = q_2 = \frac{9477s}{2723}, \quad q_3 = 1 - m - \frac{26001s}{2723} \tag{110}$$

and therefore

$$Q = 1 - m - \frac{7047s}{2723}, \quad X = 1 - m - \frac{6779s}{2723}, \tag{111}$$

$$TSS^{socialcosts} = \frac{(1-m)^2}{2} + ms - s + \frac{28163}{5446}s^2 - 3F. \tag{112}$$

Total social surplus is represented by the red line in figure 19. To maximize the social surplus, it is desirable for a public firm to have total outputs provided by the private firms since the private firms are more efficient. This implies that the public firm tries to make the private firms produce more while it produces less. Since capacity is a strategic substitute and there is a positive relationship between the capacity level of a private firm and its output level, the public firm can improve the social surplus by reducing its own capacity so that the private firms increase their capacity and quantity. On the other hand, enlarging the production share in the market is desirable for the private firm. Hence, the private firm ends up choosing excess capacity ($x_1 = x_2 > q_1 = q_2$) while the public firm chooses under capacity ($x_3 < q_3$).

Theorem 4. *A mixed triopoly, in which the public firm faces inefficiency s and chooses its capacity before the private firms choose capacity and then quantity is chosen simultaneously can exist if $s < (1-m)\frac{2723}{28163}$. In equilibrium private firms choose excess capacity, and the public firm chooses under-capacity $x_3 = 1 - m - \frac{28163s}{2723}$ and produces $q_3 = 1 - m - \frac{26001s}{2723}$.*

5.3.4. First Stage: nationalization vs entry

The public firm decides to enter the market if the market outcome with intervention has a bigger social surplus as without entering the market. Market intervention can be in form of establishing a new welfare maximizing firm next to a duopoly, or by nationalization of an existing private firm of triopoly.

First Case: Capacity choice duopoly The pure duopoly is facing a two stage game, wherefore both firms choose capacity in the first stage and quantities in the second stage. The resulting Nash Equilibrium is as follows:

$$x_1 = x_2 = \frac{16(1-m)}{43} \tag{113}$$

$$q_1 = q_2 = \frac{15(1-m)}{43} \tag{114}$$

$$TSS_{PureDuopoly} = \frac{838(1-m)^2}{1849} - 2F \tag{115}$$

The total social surplus of a two-stage duopoly with capacity choice is represented by the dashed brown line in figure 19.

Market entry Without marginal costs ($m = 0$) it is:

$$TSS_{socialcosts} = \frac{1}{2} - s + \frac{28163}{5446}s^2 - 3F > TSS_{PureDuopoly} = \frac{838}{1849} - 2F. \tag{116}$$

Rearranging yields

$$0 < s < \frac{117089 - 2\sqrt{110709011}}{1211009} \quad \wedge \quad 0 < F < \frac{52073387s^2 - 10069654s + 471079}{10069654}. \tag{117}$$

Proposition 7. *Suppose a pure private duopoly with capacity choice, and the option to establish a welfare maximizing firm which bears an inefficiency s and chooses capacity before private firms choose capacity, and all firms compete over quantities in the last stage. Then the government will establish a firm if the inefficiency and the fixed costs are not too large* $0 \leq s < \frac{117089 - \sqrt{5446}\sqrt{52073387F + 81314}}{1211009} \wedge 0 \leq F \leq \frac{173}{3698}$.

Without fixed costs($F = 0$) it is:

$$s < \frac{2723}{28163} - \frac{2\sqrt{110709011}}{1211009} \approx 0.07931 \tag{118}$$

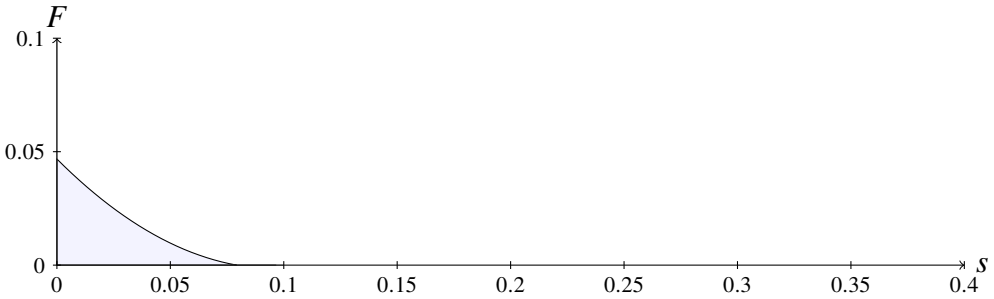


Figure 15: Inequality plot for market entry with capacity choice and social costs
Parameter values: $m = 0$

Second case: Capacity choice triopoly The pure triopoly is facing a two stage game, wherefore all three symmetric private firms choose capacity simultaneously in the first stage and quantities in the second stage. The Nash Equilibrium is as follows:[17]

[17]See Appendix for derivation.

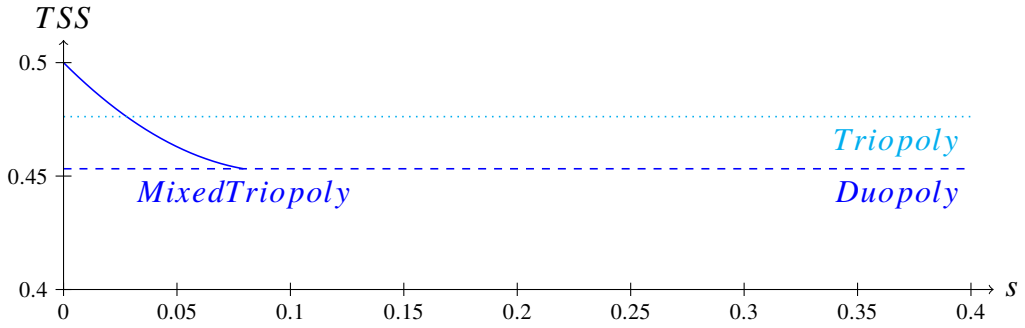


Figure 16: Social Surplus with capacity constraint and social costs
solid blue: Mixed oligopoly, dotted cyan: Pure triopoly, dashed blue: Pure duopoly
Parameter values: $m = 0, F \to 0$

$$x_1 = x_2 = x_3 = \frac{10(1-m)}{34} \tag{119}$$

$$q_1 = q_2 = q_3 = \frac{9(1-m)}{34} \tag{120}$$

$$TSS_{PureTriopoly} = \frac{1101(1-m)^2}{2312} - 3F. \tag{121}$$

Total social surplus of this capacity choice duopoly is represented by the dashed green line in figure 19.

Nationalization The government will nationalize a private firm, if the social surplus is bigger after the nationalization than in a pure triopoly with capacity choice:

$$TSS^{socialcosts} = \frac{1}{2} - s + \frac{28163}{5446}s^2 - 3F > TSS_{PureTriopoly} = \frac{1101}{2312} - 3F. \tag{122}$$

Rearranging yields:

$$s < \frac{(92582 - 3\sqrt{483732781})}{957542} \approx 0.0277797. \tag{123}$$

As the number of firms competing in the oligopoly does not change and the maximization problems of the firms are independend of the fixed costs, the welfare comparison is unaffected by the fixed costs. For the decision to nationalize a firm it is sufficient, if the inefficiency in form of additional social costs is small enough.

Proposition 8. *Suppose a pure private triopoly with capacity choice, and the option to nationalize a private firm to create a welfare maximizing firm which bears an inefficiency and chooses capacity before private firms choose capacity, and all firms compete over quantities in the last stage. Then the government will establish a firm if the inefficiency is not too large* $s < \frac{92582-3\sqrt{483732781}}{957542}$.

6. Results and discussion

6.1. Comparison of consumer surplus

Consumer surpluses (CS) of duopolies are increasing in the number of firms and the stages of games in which firms compete as both increase competition. Competition increases faster in the number of companies than in the number of the game stages. Therefore, a Cournot Duopoly has the lowest, the Capacity Duopoly the second-lowest, the Cournot Triopoly the second-highest, and the Capacity Triopoly the highest consumer surplus:

$$CS^{CournotDuopoly} < CS^{CapacityDuopoly} < CS^{CournotTriopoly} < CS^{CapacityTriopoly}$$

Consumer surplus is decreasing in transfer and social cost which the public firms have to bear, as the firm is decreasing its output in all cases but the one where the firms play a Stackelberg-like game with a capacity choice. In that case, the consumer surplus increases in the same amount as the public firm make losses, because it pays a transfer to its workers. The quantity produced remains at the social optimal first-best level, regardless of the premium paid by the firm. In contrast, the consumer surplus decreases in the case of a mixed Cournot triopoly with an NNPC, as the NNPC forces the public firm to produce less than the welfare-maximizing level.

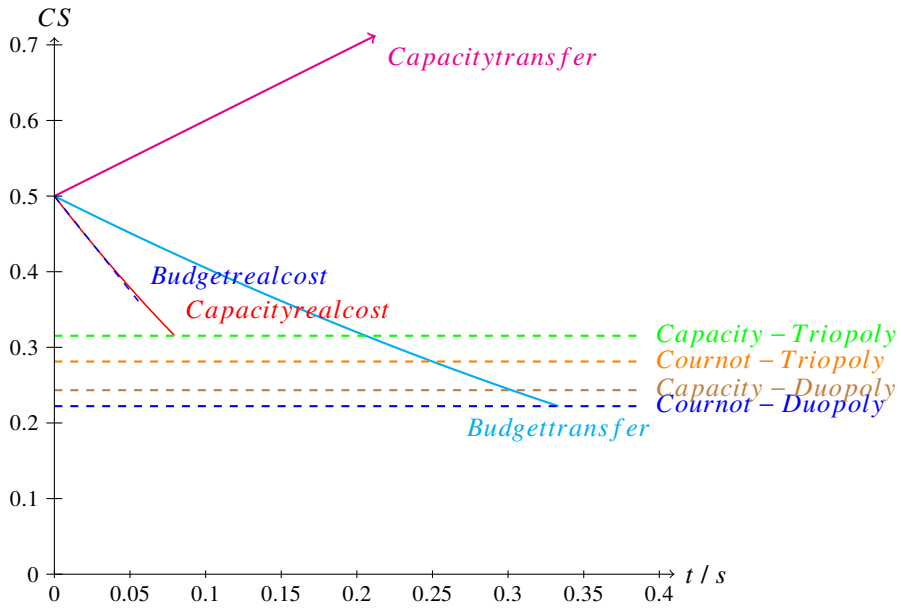


Figure 17: Consumer Surplus over transfer and social costs

Parameter values: $F \to 0, m = 0$

6.2. Comparison of profits

Only in a mixed triopoly with real social costs and a capacity choice the public firm makes a positive profit. Profits in a mixed triopoly with a transfer are the same as for a mixed triopoly with social costs. Naturally, the public firm makes no profit when it is in a Cournot game with

a no-negative profit constraint, which is binding. Growing competition is decreasing profits of the firms, and therefore the ranking is inverse of the consumer surpluses.

$$\Pi^{CapacityTriopoly} < \Pi^{CournotTriopoly} < \Pi^{CapacityDuopoly} < \Pi^{CournotDuopoly}$$

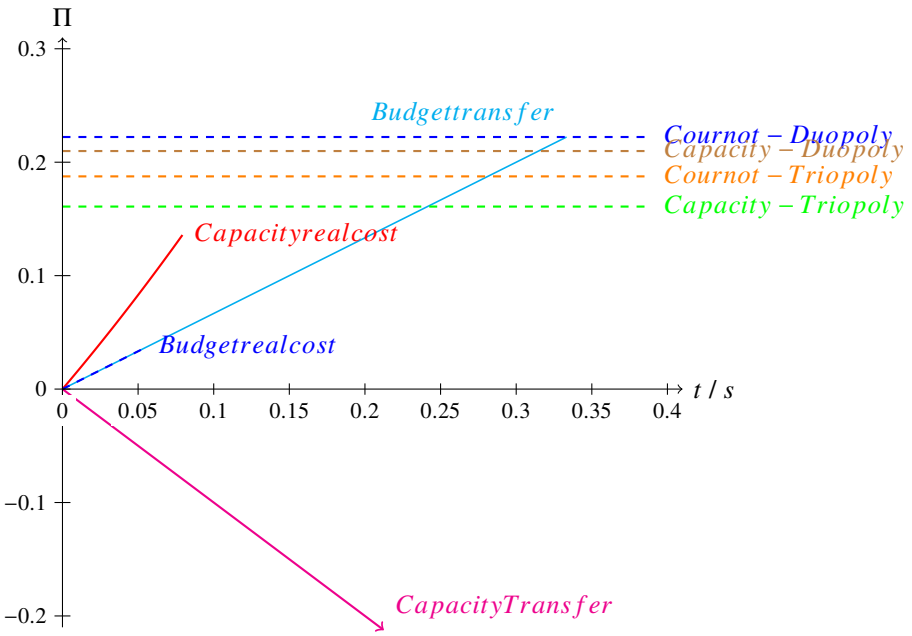


Figure 18: Profits over transfer and social costs

Parameter values: $F \to 0, m = 0$

6.3. Comparison of total social surpluses

The standard case of a private symmetric Cournot duopoly has the lowest welfare level of all cases. The choices of capacities are strategic substitutes and it increases the competition between private firms. Although there are costs of producing over capacity and companies make less profit, the Social Surplus gain on the consumer side is so great that there is a higher total social surplus than in a standard Cournot game. Hence, a pure duopoly in a two stage game in which firms choose simultaneously a capacity level in the first stage and compete over quantities in the second stage leads to higher welfare than a standard Cournot duopoly. An additional third firm increases competition for both the standard Cournot and the capacity model. Welfare is increasing stronger due to a third firm than due to a capacity stage. Therefore the ranking is as follows:

$$TSS^{CournotDuopoly} < TSS^{CapacityDuopoly} < TSS^{CournotTriopoly} < TSS^{CapacityTriopoly}$$

Under mixed triopolies, the case with a no-negative profit constraint and social costs leads to the lowest total welfare. The capacity choice model with social costs has slightly higher welfare

than the pure Cournot game as the additional capacity choice increases competition between firms. Switching from real social costs to a transfer always leads to higher welfare, because the additional costs of the public firm are going welfare neutral to the consumers. But a no-negative profit constraint hinders the public firm to achieve the first best solution. Without the constraint, it is possible to reach the first-best solution in the capacity model as the public firm can announce the first best capacity and produce the first best capacity when allowed to make losses. It becomes a monopolist in that case, but this is still preferred as the losses it makes are paid out to the consumers in form of higher wages.

$$TSS^{MixedCournotTriopolywithinefficiency} < TSS^{MixedCapacityTriopolywithinefficiency} < TSS^{MixedCournotTriopolywithtransfer} < TSS^{PublicCapacityMonopolywithtransfer}$$

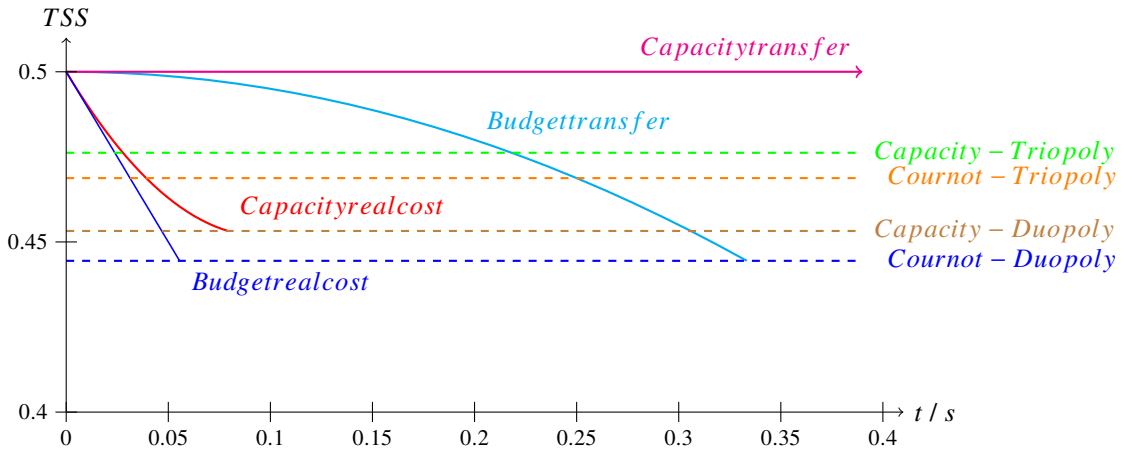


Figure 19: Social surplus over transfer/social costs

Parameter values: $F \to 0, m = 0$

Table 2: Summary of Total Social Surpluses without marginal costs

	NNP constraint	Capacity constraint
transfer	$\frac{17 - 9t^2 - 102F + (3t+1)\sqrt{(1-3t)^2 - 12F}}{36}$	$\frac{1}{2} - 3F$
social cost	$\frac{17 - 45s^2 - 102F + (\lvert 1 - 15s \rvert)\sqrt{(1-3s)^2 - 12F} - 18s}{36}$	$\frac{1}{2} - s + \frac{28163}{5446}s^2 - 3F$
duopoly	$\frac{4}{9} - 2F$	$\frac{838}{1849} - 2F$
triopoly	$\frac{15}{32} - 3F$	$\frac{1101}{2312} - 3F$

Parameter values: m=0

The TSS is bigger for the Capacity constraint than for the no-negative profit constraint, and only equal for $s = 0$ and $F = 0$.

6.4. Quantities

The mixed Cournot Triopoly with an NNPC can lead to a market equilibrium with an output from the public firm between zero and hundred percent of the total output, depending on the transfer (see figure 20). In contrary, the mixed Cournot triopoly with an NNPC and real social cost can exist only with a very high market share of over eighty percent of the public firm or the public firm will not enter the market at all (see figure 21). In the Stackelberg-like game with capacity choice with a transfer there exists no Mixed Triopoly and the public firm will monopolize the market independent of the amount of the transfer (see table 4). With capacity choice and real cost instead, the market share is variable depending on the additional cost. This is similar to the Cournot game with a transfer.

Table 3: Main results no-negative profit constraint

	Pure duopoly	Pure triopoly	Mixed Triopoly with transfer	Mixed triopoly with social costs
q_1, q_2	$\frac{1-m}{3}$	$\frac{1-m}{4}$	$\frac{1-m+3t-\sqrt{(1-m-3t)^2-12F}}{6}$	$\frac{1-m+3s-\sqrt{(1-m-3s)^2-12F}}{6}$
q_3	0	$\frac{1-m}{4}$	$\frac{1-m-3t+\sqrt{(1-m-3t)^2-12F}}{2}$	$\frac{1-m-3s+\sqrt{(1-m-3s)^2-12F}}{2}$
p	$\frac{1+2m}{3}$	$\frac{1+3m}{4}$	$\frac{1+3t+5m-\sqrt{(1-m-3t)^2-12F}}{6}$	$\frac{1+3s+5m-\sqrt{(1-m-3s)^2-12F}}{6}$
π_1, π_2	$\frac{(1-m)^2}{9} - F$	$\frac{(1-m)^2}{16} - F$	$\frac{(1-m)^2+9t^2-6F-(1-m+3t)\sqrt{(1-m+3t)^2-12F}}{18} - F$	$\frac{(1-m)^2+9s^2-6F-(1-m+3t)\sqrt{(1-m+3s)^2-12F}}{18} - F$
π_3	0	$\frac{(1-m)^2}{16} - F$	0	0
Π	$\frac{2(1-m)^2}{9} - 2F$	$\frac{3(1-m)^2}{16} - 3F$	$\frac{(1-m)^2+9t^2+3F-(1-m+3t)\sqrt{(1-m+3t)^2-12F}}{9} - 3F$	$\frac{(1-m)^2+9s^2+3F-(1-m+3s)\sqrt{(1-m+3s)^2-12F}}{9} - 3F$
CS	$\frac{8(1-m)^2}{36}$	$\frac{9(1-m)^2}{32}$	$\frac{5(1-m+3t)\sqrt{(1-m-3t)^2-12F}+13m^2-45t^2-6F-26m+13}{36}$	$\frac{\left(5(1-m)-3s+\sqrt{(1-m-3s)^2-12F}\right)^2}{72}$
TSS	$\frac{4(1-m)^2}{9} - 2F$	$\frac{15(1-m)^2}{32} - 3F$	$\frac{17(1-m)^2-9t^2+(1-m+3t)\sqrt{(1-m-3t)^2-12F}-102F}{36}$	$\frac{17(1-m)^2+18(m-1)s+45s^2+(1-m-15s)\sqrt{(1-m-3s)^2-12F}-102F}{36}$

Notes: NC: necessary condition for SPNE in a mixed triopoly with transfer, $\sqrt{(1-m-3t)^2-12F} > 0$; necessary condition for SPNE in a mixed triopoly with social costs, $\sqrt{(1-m-3s)^2-12F} > 0$.

The code(Maple 2020) that is used to solve the systems and isolate the variables is available on my website: https://dustin-jonak.github.io/research.html

Table 4: Main results Capacity choice

	Pure duopoly	Pure triopoly	Public Monopoly with transfer	Mixed triopoly with real costs
x_1, x_2	$\frac{16(1-m)}{43}$	$\frac{10(1-m)}{34}$	0	$\frac{10692s}{2723}$
x_3	0	$\frac{10(1-m)}{34}$	$1-m$	$1-m-\frac{28163s}{2723}$
q_1, q_2	$\frac{15(1-m)}{43}$	$\frac{9(1-m)}{34}$	0	$\frac{9477s}{2723}$
q_3	0	$\frac{9(1-m)}{34}$	$1-m$	$1-m-\frac{26001s}{2723}$
p	$\frac{13+30m}{43}$	$\frac{7+27m}{34}$	m	$m+\frac{7047s}{2723}$
π_1, π_2	$\frac{194(1-m)^2}{1849}-F$	$\frac{31(1-m)^2}{578}-F$	$-F$	$\frac{9329742s^2}{1059247}-F$
π_3	0	$\frac{31(1-m)^2}{578}-F$	$-t(1-m)-F$	$-\frac{4324s(27082s-2723(1-m))}{7414729}-F$
Π	$\frac{388(1-m)^2}{1849}-2F$	$\frac{93(1-m)^2}{578}-3F$	$-t(1-m)-3F$	$\frac{4s(3378455s+(1-m)2943563)}{7414729}-3F$
CS	$\frac{450(1-m)^2}{1849}$	$\frac{729(1-m)^2}{2312}$	$\frac{(1-m)^2}{2}+t(1-m)$	$\frac{(7047s-2723(1-m))^2}{14829458}$
TSS	$\frac{838(1-m)^2}{1849}-2F$	$\frac{1101(1-m)^2}{2312}-3F$	$\frac{(1-m)^2}{2}-3F$	$\frac{(1-m)^2}{2}+ms-s+\frac{28163}{5446}s^2-3F$

Notes: NC: necessary condition for SPNE in a mixed triopoly with transfer, $1-m>0$; necessary condition for SPNE in a mixed triopoly with social costs, $1-m-\frac{28163s}{2723}>0$.
The code(Maple 2020) that is used to solve the systems and isolate the variables is available on my website: https://dustin-jonak.github.io/research.html

6.5. Discussion

In the models of this study, a static number of private firms is assumed – two or three before market intervention – and only mixed triopolies are considered. A lower number of static firms would lead to less competition and therefore the government would intervene for a higher range of fixed and additional cost. A higher number of private firms would mean that competition is already high before the market intervention. Thus, entry or nationalization of a public firm gets less likely. Market entry of private firms was not considered as market entry barriers were assumed to be high due to regularities and local peculiarities. If these market barriers are low, market-entry gets possible and competition rises so that the need to intervene in the market by a public firm gets lower.

Building land is scarce and regulatory burdens are high, and both together lead to imperfect competition in the housing sector. Bribes to get a building permit, but also to get contracts placed by public authorities are not unusual. But corruption is a general problem in this industry as described by de Jong et al. (2009): "The engineering/construction industry has many forms of corruption. The main ones are kickbacks and bribery, front companies, bid rigging and collusion, fraud, and conflicts of interest". Corruption can, on the one hand, raise barriers to entry and weaken competition, and on the other hand, make the public company less efficient.

It is particularly in densely built-up cities that the capital and regulatory requirements for building new apartments are high. Hence, private homeowners make up only a small percentage of landlords, especially in new constructions. If there are strong restrictions in the housing industry, due to the limitation of building land and long and complicated permit procedures, it is difficult for small participants to enter the market. Therefore, it is reasonable to assume an oligopoly in some urban areas. With costs similar to the private construction industry and it is possible to increase the total output with the help of a non-profit housing association so that the demand can be served better. The result is a positive impact on overall welfare.

A model that allows for a continuous market share of the public firm dependent on a variable is most promising. Notably, as there are only a few cities with over eighty percent market share: Singapore in this study, the Cournot model with real social costs does not fit well with reality.

Inter-dependencies between sub-markets are not considered. In the long run, there is more competition between high demand and low demand regions as employers might choose to migrate the workforce to more affordable regions, and therefore the market to study gets bigger, overall Germany, or even Europe. This lowers the need to increase competition in the submarkets (cities). An increase in competition inside a city might decrease competition between cities, such that cities with high vacancy rates suffer from further decreasing demand. The question arises if public firms are less efficient or just pay higher transfers or a mix of both. Moreover, more empirical insight is needed to evaluate the type and size of inefficiencies of public housing. In addition, a more empirical question is where competition in the housing sector exists as well as an empirical study that examines the proportion of income that is paid for housing as a function of population density and public housing would be interesting.

7. Conclusion

Public housing is used in metropolises worldwide to supply affordable housing. A public firm can increase welfare in a market without perfect competition, in particular oligopolistic markets. This study has analyzed the question of when a government should intervene in the housing sector. In the settings described in this thesis, the government has the option to enter a duopoly market or to nationalize a firm of a triopoly. A public firm maximizes the social surplus instead of its own profit and pays a premium to their workers as a transfer, or has higher marginal costs than private firms which is an inefficiency. The subgame perfect Nash equilibria were derived.

A mixed Cournot triopoly with a welfare-maximizing public firm which must consider a no-negative profit constraint exists for not too large cost asymmetries. A government wants to intervene if the fixed costs and the additional premium is not too large. If the public firm has some additional real social cost, in comparison to the private firms, it wants to intervene when the costs are small, and it takes a major market share. The fixed cost is more restrictive for the decision of a market entry in a duopoly than for nationalization in a triopoly. However, social costs and transfers are more restrictive for nationalization than a market entry. Real costs are more restrictive than a transfer for the decision if a market intervention is taken place, but do not affect the extent of the market intervention.

A mixed Stackelberg-like triopoly with capacity choice and real social costs can exist, and market intervention is desirable for small costs (fixed and marginal). In such a setting, market entry in a duopoly is recommended if additional social costs and fixed costs are small. For the nationalization of a firm in a triopoly, it is sufficient if additional social costs are small, regardless of the fixed costs. Though a mixed Stackelberg-like triopoly and a transfer cannot exist, as the regulator wants to intervene and enters the duopoly with a public firm and it becomes a monopolist if the fixed costs are small enough. In the case of an existing triopoly, the government always nationalizes a firm and becomes a monopolist.

Area-wide affordable housing is crucial for many social and economic aspects. Hence, property usage as well as construction of housing and tenancy contracts are already heavily regulated. However, the housing supply fails to satisfy the growing demand and leads to a market outcome with increasing prices. In highly dense areas such as metropolitan areas, the entrance into the housing market is more restricted as for the development of apartment buildings a much higher budget is needed than for the construction of a single-family house. A publicly-owned firm might be an efficient solution to supply affordable housing and a welfare improving market allocation of living space, despite higher costs in providing it. Therefore, intervention by publicly held firms might help to overcome the occurring problems of an oligopolistic housing market. Neither higher costs nor inefficiency in the production of public companies are sufficient arguments to justify privatization. If the inefficiency of a publicly owned company is not too large and markets suffer under imperfect competition, then public companies can increase welfare even if they are less efficient.

8. Appendix

8.1. Solution with lower quantity of the cournot triopoly with a NNPC and a transfer

Due to the quadratic function, there might be a second solution of system (18) with a lower q_3

$$\begin{aligned} q_1^l = q_2^l &= \frac{1-m+3t+\sqrt{(1-m-3t)^2-12F}}{6}, \\ q_3^l &= \frac{1-m-3t-\sqrt{(1-m-3t)^2-12F}}{2}. \end{aligned} \tag{124}$$

$$\begin{aligned} Q^l &= \frac{5-3t-5m-\sqrt{(1-m-3t)^2-12F}}{6}, \\ TSS^l &= \frac{17(1+m^2-2m)-9t^2+6F-(1-m+3t)\sqrt{(1-m-3t)^2-12F}}{36} - 3F. \end{aligned} \tag{125}$$

8.2. Pure Cournot Duopoly

Firm 1 and firm 2 are private and symmetric and have marginal costs m. Maximizing their profit from equation (3) the private firms face the following problems:

$$\max_{q_1} \pi_1(q_1, q_2) = q_1 p(q_1+q_2) - q_1 m - F \tag{126}$$

$$\max_{q_2} \pi_2(q_1, q_2) = q_2 p(q_1+q_2) - q_2 m - F \tag{127}$$

Solving yields the best responses:

$$\bar{q_1}(q_2) = \frac{1-q_2-m}{2} \tag{128}$$

$$\bar{q_2}(q_1) = \frac{1-q_1-m}{2}. \tag{129}$$

Solving the system of best responses yields the equilibrium quantities:

$$q_1^* = q_2^* = \frac{1-m}{3}. \tag{130}$$

Therefore, the symmetric standard Cournot model with two profit maximizing firms, which face demand $D = 1 - Q$ and marginal costs m obtains: $q_1 = q_2 = \frac{1}{3} - \frac{m}{3}$, $Q = \frac{2(1-m)}{3}$, $P = \frac{1+2m}{3}$, and $TSS = \frac{4}{9}(m-1)^2 - 2F$.

8.3. Pure Cournot Triopoly

Firm 1, firm 2 and firm 3 are private and symmetric and have marginal costs m. Maximizing their profit from equation (3) the private firms face the following problems:

$$\max_{q_1} \pi_1(q_1, q_2, q_3) = q_1 p(q_1 + q_2 + q_3) - q_1 m - F \tag{131}$$

$$\max_{q_2} \pi_2(q_1, q_2, q_3) = q_2 p(q_1 + q_2 + q_3) - q_2 m - F \tag{132}$$

$$\max_{q_3} \pi_3(q_1, q_2, q_3) = q_3 p(q_1 + q_2 + q_3) - q_3 m - F \tag{133}$$

Solving yields the best responses:

$$\bar{q_1}(q_2, q_3) = \frac{1 - q_2 - q_3 - m}{2} \tag{134}$$

$$\bar{q_2}(q_1, q_3) = \frac{1 - q_1 - q_3 - m}{2} \tag{135}$$

$$\bar{q_3}(q_1, q_2) = \frac{1 - q_1 - q_2 - m}{2}. \tag{136}$$

Solving the system of best responses yields the equilibrium quantities:

$$q_1^* = q_2^* = q_3^* = \frac{1 - m}{4}. \tag{137}$$

Therefore, the symmetric standard Cournot model with three profit maximizing firms which face demand $D = 1 - Q$ and marginal costs m obtains: $q_1 = q_2 = q_3 = \frac{1}{4} - \frac{m}{4}$, $Q = \frac{3(1-m)}{4}$, $P = \frac{1+3m}{4}$, and $TSS = \frac{15}{32}(m-1)^2 - 3F$.

8.4. Pure Capacity Duopoly

8.4.1. Second stage: simultaneous quantity choice

Given their production capacities x_1 and x_2 , both private firms are maximizing profits (54) by choosing their quantities q_1 and q_2 simultaneously.

$$\max_{q_1} \pi_1 (q_1, q_2, x_1) = (1 - q_1 - q_2 - m)q_1 - (q_1 - x_1)^2 \tag{138}$$

$$\max_{q_2} \pi_2 (q_1, q_2, x_2) = (1 - q_1 - q_2 - m)q_2 - (q_2 - x_2)^2 \tag{139}$$

The first order conditions of (138) and (139) yield:

$$q_1 (q_2, x_1) = \frac{1 - q_2 - m + 2x_1}{4} \tag{140}$$

$$q_2 (q_1, x_2) = \frac{1 - q_1 - m + 2x_2}{4}. \tag{141}$$

Solving the system (140),(141) yields:

$$q_1(x_1, x_2) = \frac{3(1-m) + 8x_1 - 2x_2}{15} \tag{142}$$

$$q_2(x_1, x_2) = \frac{3(1-m) - 2x_1 + 8x_2}{15}. \tag{143}$$

8.4.2. First stage: simultaneous capacity choice

After substituting the anticipated quantities of the second stage in the profit function and maximizing over capacities, the First Order conditions yield:

$$x_1(x_2) = \frac{48(1-m) - 32x_2}{97} \tag{144}$$

$$x_2(x_1) = \frac{48(1-m) - 32x_1}{97}. \tag{145}$$

Solving the system yields the equilibrium capacities:

$$x_1^* = \frac{16(1-m)}{43} \tag{146}$$

$$x_2^* = \frac{16(1-m)}{43}. \tag{147}$$

The resulting Nash Equilibrium is therefore as follows:

$$x_1 = x_2 = \frac{16(1-m)}{43} \tag{148}$$

$$q_1 = q_2 = \frac{15(1-m)}{43} \tag{149}$$

$$TSS_{PureDuopoly} = \frac{838(1-m)^2}{1849} - 2F \tag{150}$$

8.5. Pure Capacity Triopoly

8.5.1. Second stage: simultaneous quantity choice

Given their production capacities x_1, x_2 and x_3 , all three private firms are maximizing profits (54) by choosing their quantities q_1, q_2, q_3 simultaneously.

$$\max_{q_1} \pi_1(q_1, q_2, q_3, x_1) = (1 - q_1 - q_2 - q_3 - m)q_1 - (q_1 - x_1)^2 \tag{151}$$

$$\max_{q_2} \pi_2(q_1, q_2, q_3, x_2) = (1 - q_1 - q_2 - q_3 - m)q_2 - (q_2 - x_2)^2 \tag{152}$$

$$\max_{q_3} \pi_3(q_1, q_2, q_3, x_3) = (1 - q_1 - q_2 - q_3 - m)q_3 - (q_3 - x_3)^2 \tag{153}$$

The first order conditions of (151),(152), and (153) yield

$$q_1(q_2, q_3, x_1) = \frac{1 - q_2 - q_3 - m + 2x_1}{4} \tag{154}$$

$$q_2(q_1, q_3, x_2) = \frac{1 - q_1 - q_3 - m + 2x_2}{4} \tag{155}$$

$$q_3(q_1, q_2, x_3) = \frac{1 - q_1 - q_2 - m + 2x_3}{4} \tag{156}$$

Solving the system (154),(155),(156) yields

$$q_1(x_1, x_2, x_3) = \frac{3(1-m) + 10x_1 - 2x_2 - 2x_3}{18} \tag{157}$$

$$q_2(x_1, x_2, x_3) = \frac{3(1-m) - 2x_1 + 10x_2 - 2x_3}{18} \tag{158}$$

$$q_3(x_1, x_2, x_3) = \frac{3(1-m) - 2x_1 - 2x_2 + 10x_3}{18}. \tag{159}$$

8.5.2. First stage: simultaneous capacity choice

After substituting the anticipated quantities of the second stage in the profit function and maximizing over capacities, the First Order conditions yield

$$x_1(x_2) = \frac{15(1-m) - 10x_2 - 10x_3}{31} \tag{160}$$

$$x_2(x_1) = \frac{15(1-m) - 10x_1 - 10x_3}{31} \tag{161}$$

$$x_3(x_1) = \frac{15(1-m) - 10x_1 - 10x_2}{31}. \tag{162}$$

Solving the system yields the equilibrium capacities:

$$x_1^* = x_2^* = x_3^* = \frac{5(1-m)}{17} \tag{163}$$

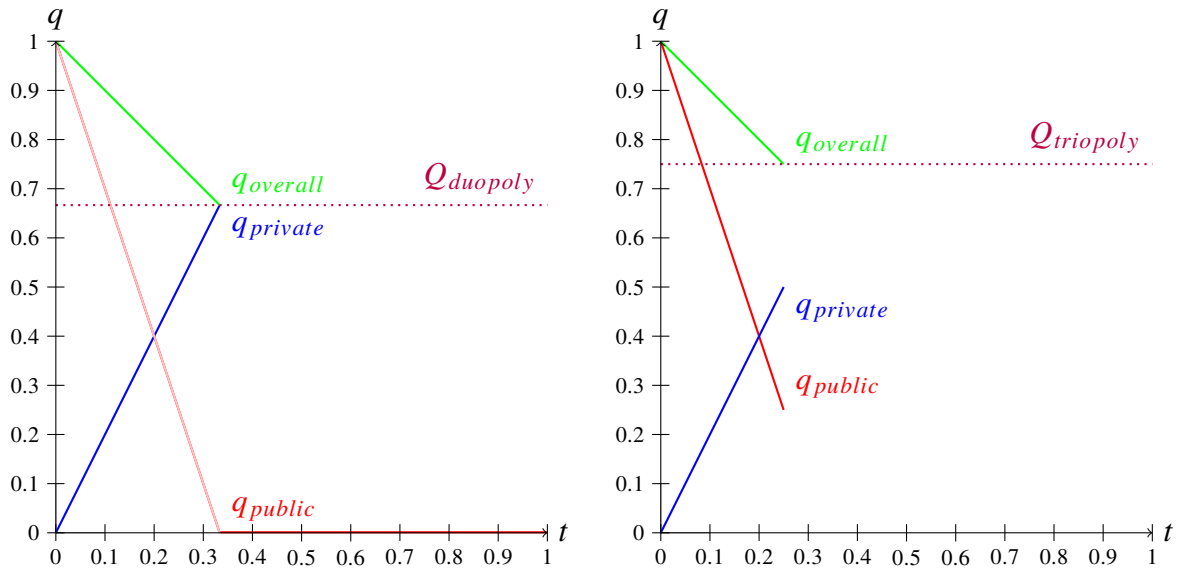
Therefore, the Nash Equilibrium is as follows:

$$x_1 = x_2 = x_3 = \frac{10(1-m)}{34} \tag{164}$$

$$q_1 = q_2 = q_3 = \frac{9(1-m)}{34} \tag{165}$$

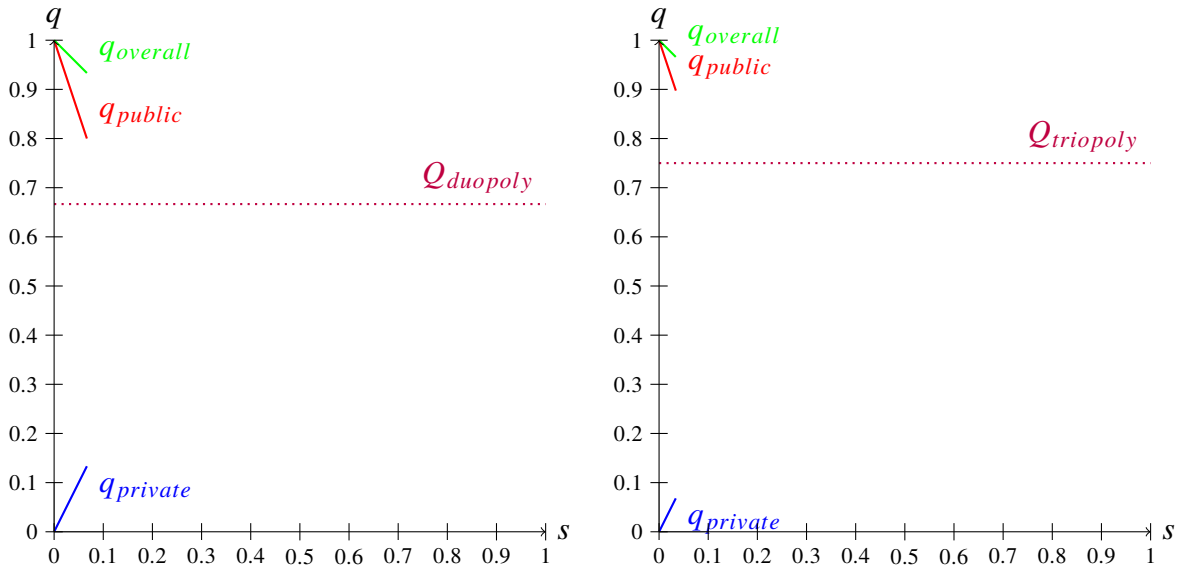
$$TSS_{PureTriopoly} = \frac{1101(1-m)^2}{2312} - 3F \tag{166}$$

8.6. Quantity Plots



(a) Entry in a duopoly market (b) Nationalization of a firm in a triopoly market

Figure 20: Quantity plots with non-negative profit constraint and a transfer



(a) Entry in a duopoly market (b) Nationalization of a firm in a triopoly market

Figure 21: Quantity plots with non-negative profit constraint and social cost

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