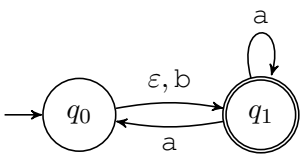
Problem 1 (25 points)

Let $\Sigma = \{\mathtt{a}, \mathtt{b}\}$. Consider the language L of following NFA M:



(a) (14 points) Give a minimal DFA for the language L. Prove that your DFA is minimal (for every pair of states, give a string to distinguish them).

If your DFA is not minimal you will get partial credits.

(b) (6 points) Write a regular expression for L. *Hint: There is a simple solution.*

(c) (5 points) Write a regular expression for the complement of L.

Problem 2 (20 points)

Consider the language

$$L = \{w \in \{\mathtt{a}, \mathtt{b}\}^* \mid w \text{ contains an even number of } \mathtt{a}\text{'s and ends with the symbol } \mathtt{b}\}.$$

(a) (10 points) Give a DFA for the language L. Briefly explain how you obtain your DFA (insufficient explanation will get no points).

(b) (10 points) Give a context-free grammar for L. Briefly explain how your context-free grammar works (insufficient explanation will get no points).

Problem 3 (20 points)

Consider the language

$$L = \{w \in \{\texttt{a}, \texttt{b}\}^* \mid w \text{ contains no more } \texttt{a}\text{'s than } \texttt{b}\text{'s}\}.$$

For example, `ab` and `bab` are in L, but `baa` is not.

(a) (10 points) Prove that L is irregular.

(b) (10 points) Give a pushdown automaton for the language L. Briefly explain how your pushdown automaton works (insufficient explanation will get no points).

Problem 4 (35 points)

Consider language

$$L = \{w \in \{\mathtt{a}, \mathtt{b}\}^* \mid w \text{ contains the same number of } \mathtt{a}\text{'s and } \mathtt{b}\text{'s}\}.$$

The language L may be described by the following context-free grammar G:

$$S \to \mathtt{a}S\mathtt{b}S \mid \mathtt{b}S\mathtt{a}S \mid \varepsilon$$

(a) (7 points) Show that the grammar G is ambiguous. *Hint: Some string of length at most four will help you.*

(b) (8 points) Convert the grammar to Chomsky Normal Form.

Problem 4 (continued)

(c) (10 points) Using the grammar from part (b), apply the Cocke–Younger–Kasami on input `abba`. Show the table of partial derivations. Draw a parse tree derived by the algorithm.

(d) (10 points) Consider the context-free language

$$L' = \{w \in \{\mathtt{a}, \mathtt{b}\}^* \mid w \text{ is a palindrome}\}.$$

Recall that a string w is a palindrome if it reads the same forward and backward.

What is the intersection language $L \cap L'$? Show that $L \cap L'$ is not context-free. (Note: This shows that context-free languages are not closed under intersection.)