Assignment 1

Deadline: April 15

1 Problem 1 (20%)

The following graph (i.e., Figure 1) is a simple social network where each node represents a user. The color of the node stands for the type of each user (i.e., normal or malicious). Please answer the following questions.

- **Q1:** (6%) Use the *adjacency matrix* to describe this graph.
- **Q2:** (6%) List in-degree and out-degree of each node.
- **Q3:** (8%) List all *simple paths* from node A to node F.

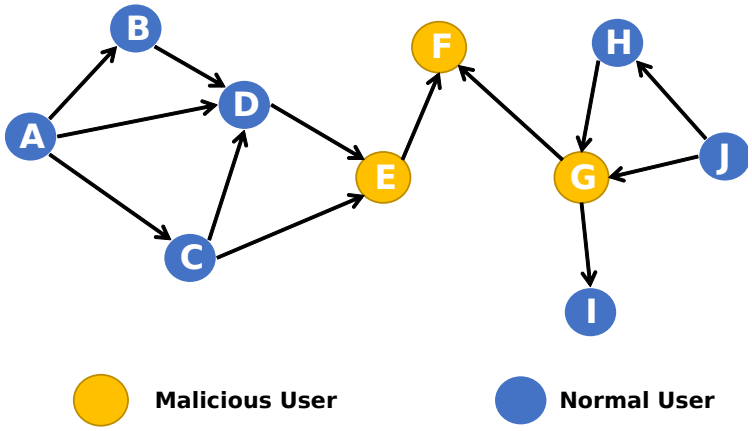

Figure 1: Figure 1: Graph of Problem 1.

2 Problem 2 (20%)

Reconsider the graph in *Problem 1*. Now, we replace all the directed edges in that graph by **undirected edges** (i.e., Figure 2). Please answer the following questions.

- **Q1:** (5%) Use the *adjacency matrix* to describe this undirected graph.

- **Q2:** (5%) Compute the cluster coefficient of each node.
- **Q3:** (5%) Find out all bridges and local bridges in this graph
- **Q4:** (5%) According to the distribution of normal users or malicious users in the graph, measure the homophily of the graph by normal-normal, normal-malicious, malicious-malicious. And figure out if there is evidence of homophily in this graph.

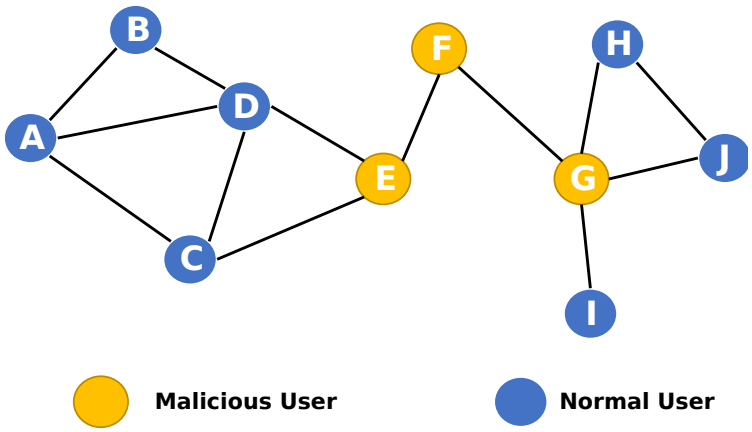

Figure 2: Figure 2: Graph of Problem 2.

3 Problem 3 (20%)

In the following figure (i.e., Figure 3), $+$ and $-$ represent friendship and antagonism respectively. Please answer the following questions.

- **Q1:** (6%) Is the graph in Figure 3 structurally balanced?
- **Q2:** (7%) Add another node F and build either positive or negative connections with existing five nodes (i.e., A, B, C, D and E), so that the new network satisfies **Structural Balance Property**.
- **Q3:** (7%) Add another node F and build either positive or negative connections with existing five nodes (i.e., A, B, C, D and E), so that the new network *only* satisfies **Weak Structural Balance Property** but *does not* satisfy **Structural Balance Property**.

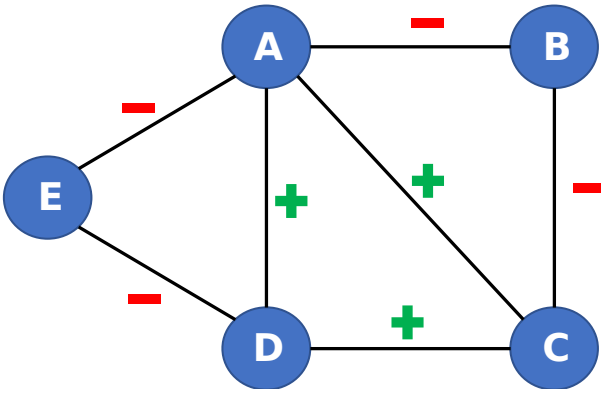

Figure 3: Figure 3: Graph of Problem 3.

4 Problem 4 (20%)

In this problem, we consider a model regarding to the economy with network effects. Assume the $r(x)$ denotes the intrinsic interest of consumer x in good, $r(x) = 1 - x^2$. z represents the fraction of the population using the good, suppose function $f(z)$ measures the benefit to each consumer from those who use the good. Let $f(z) = \frac{z}{2}$.

- **Q1:** (6%) Suppose p represents the reservation price. Plot the reservation price with the change of fraction of customers z. And find the corresponding fraction of customers when $p = 0.1$.
- **Q2:** (7%) Highlight the downward and upward pressure regions for the reservation price plot. And explain why there is downward and upward pressure in these regions.
- **Q3:** (7%) Now consider the dynamic case of economy. Set the current price $p^* = 0.1$. Please plot the fraction of population who buy the product versus the expected fraction of population who will use the product.

5 Problem 5 (20%)

In this problem we consider the information cascades model in *P38 of lecture notes E* with specific values for the probabilities. Let's suppose that the probability that **Accept** (A) is a good idea is $p = \frac{1}{2}$, and the probability of a High signal if Good is true (as well as the probability of a Low signal if Bad is true) is $q = \frac{3}{4}$. Finally, let's assume that Good is actually true.

- **Q1:** (7%) What is the probability that the first person to decide will choose **Accept**? What is the probability that this person will choose **Reject**?
- **Q2:** (6%) What is the probability of observing each of the four possible pairs of choices by the first two people: $(A, A), (A, R), (R, A)$ and (R, R)? [A pair of choices such as (A, R) means that the first person chose **Accept** and the second person chose **Reject**.]
- **Q3:** (7%) What is the probability of an **Accept** or a **Reject** cascade emerging with the decision by the third person to choose? Explain why a cascade emerges with this probability.