- Matching Market
  - if the search engine knew all the advertisers' valuations for clicks
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  - the advertisers' valuations are not known
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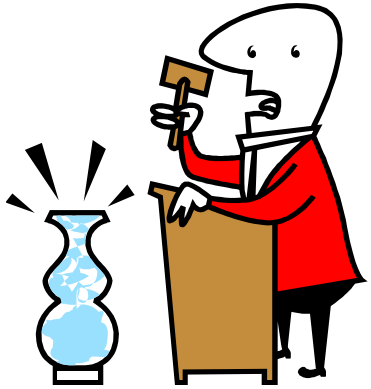
  - the design of a price-setting procedure for matching markets
- Generalized Second-Price Auction (GSP)
  - the procedure that the search industry adopted

Setting the prices

- Search engines determine prices using an auction procedure, in which they solicit bids from the advertisers.



The Bidding Prices

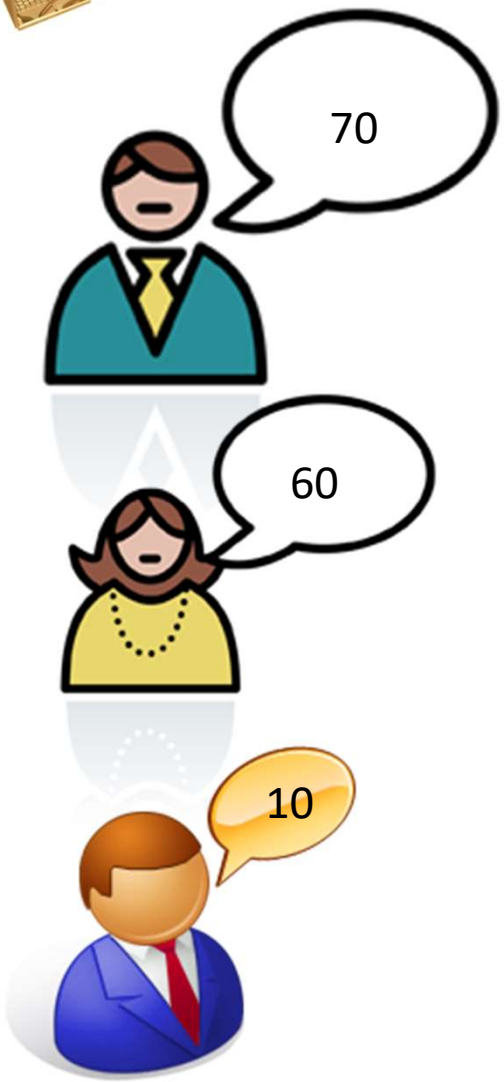
- In the early days
  - advertisers were simply asked to report their revenues per click in the form of bids, and then they were assigned slots in decreasing order of these bids.
  - advertisers are under-report, small increments to experiment with the outcome

Four main types of auctions

- Ascending-bid auctions (English auctions)
- Descending-bid auctions (Dutch auctions)
- First-price sealed auctions
- Second-price sealed-bid auctions (Vickrey auctions)

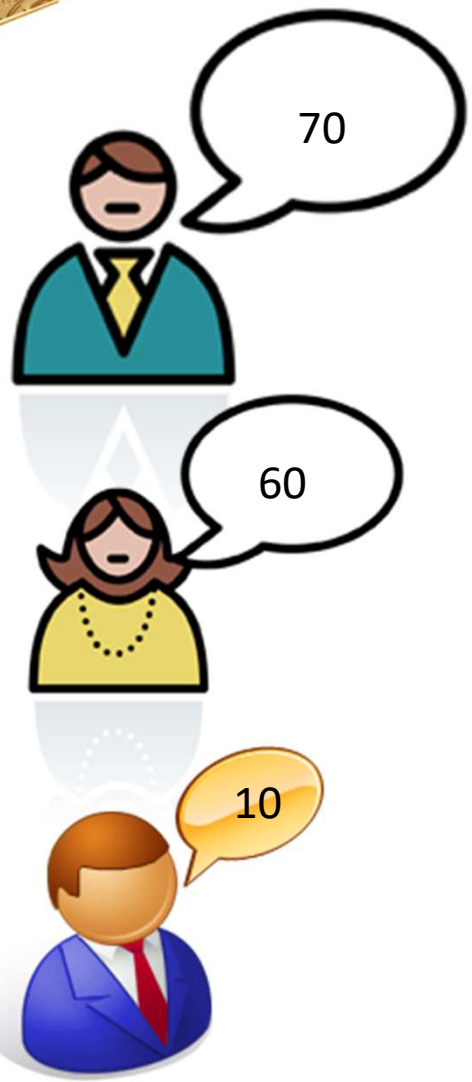
- First-price auctions
  - bidders underreport, non-truthful bidding, very complex bidding for everyone
- Second-price auctions
  - truthful bidding is a dominant strategy for all advertisers

The Second-Price Auction (single item)



- The bidders' values for the item are $v_1, v_2, v_3, \ldots, v_n$ in decreasing order.

The Second-Price Auction (single item)



- What is the price ?

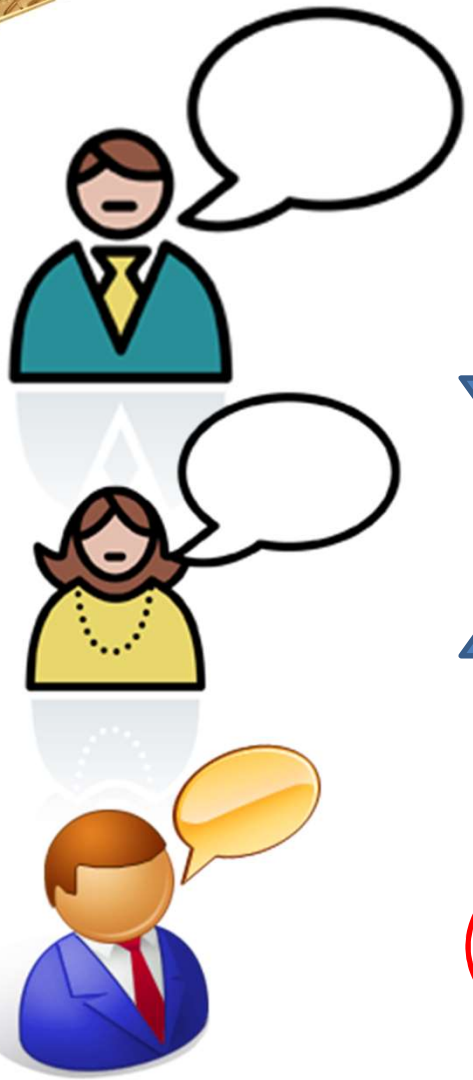
Each individual is charged the harm they cause to the rest of the world

The Second-Price Auction (single item)

- The harm to the second bidder is 60
- No harm to the third bidder

Assign the item to the highest bidder at the second-highest price

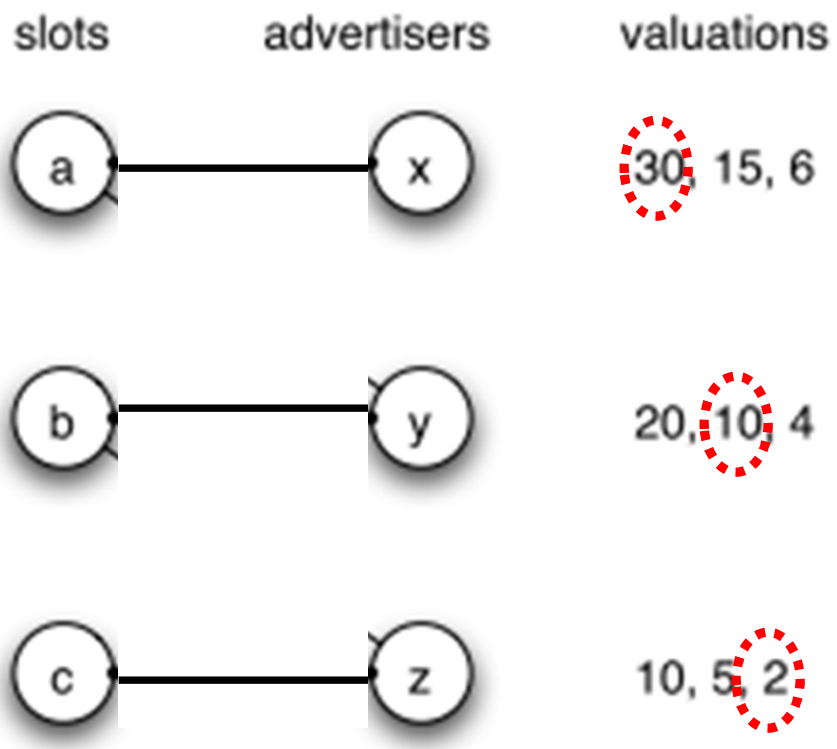
Multiple items



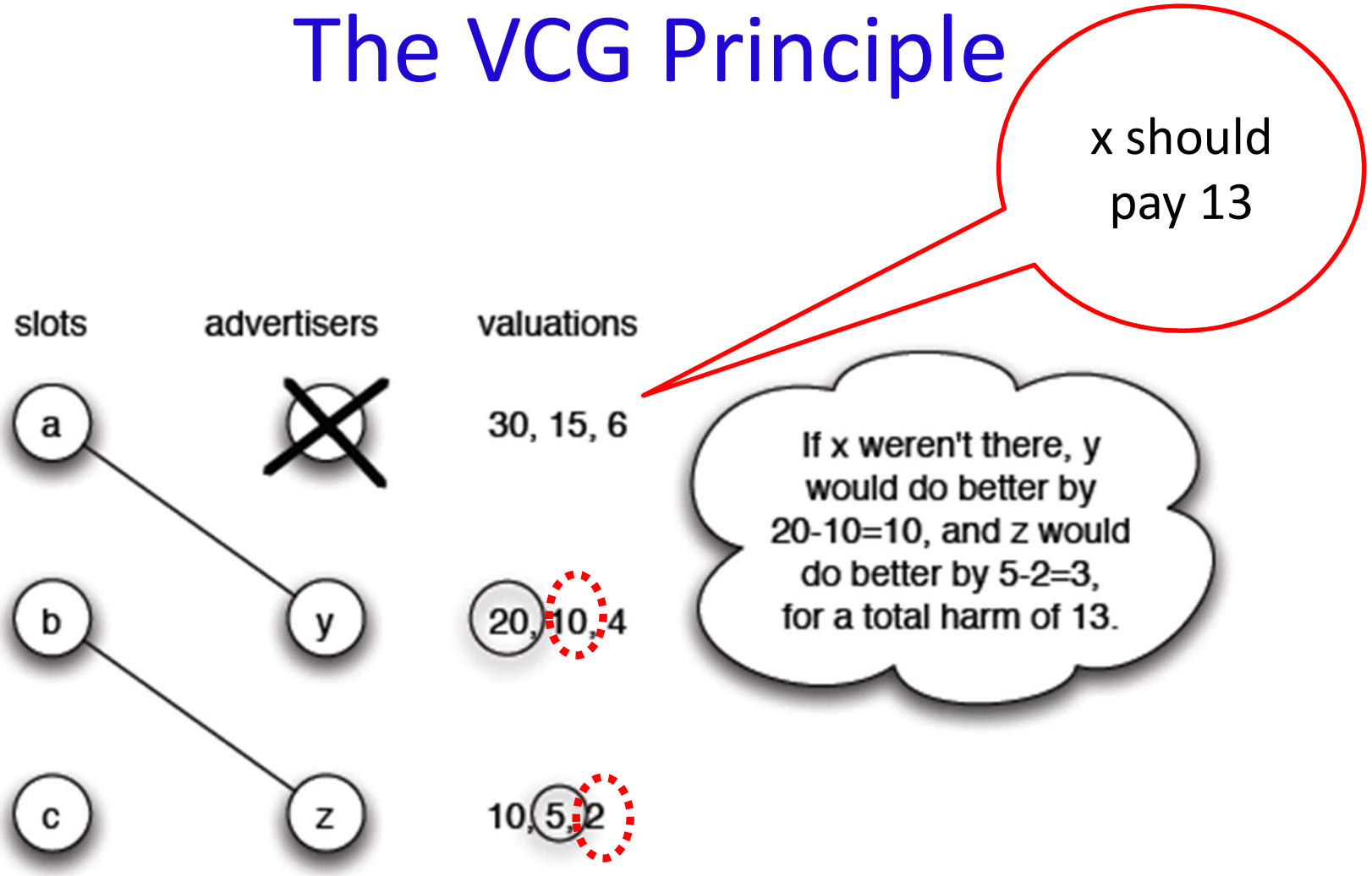
Each individual is charged the harm they cause to the rest of the world

How do we determine the harm ?

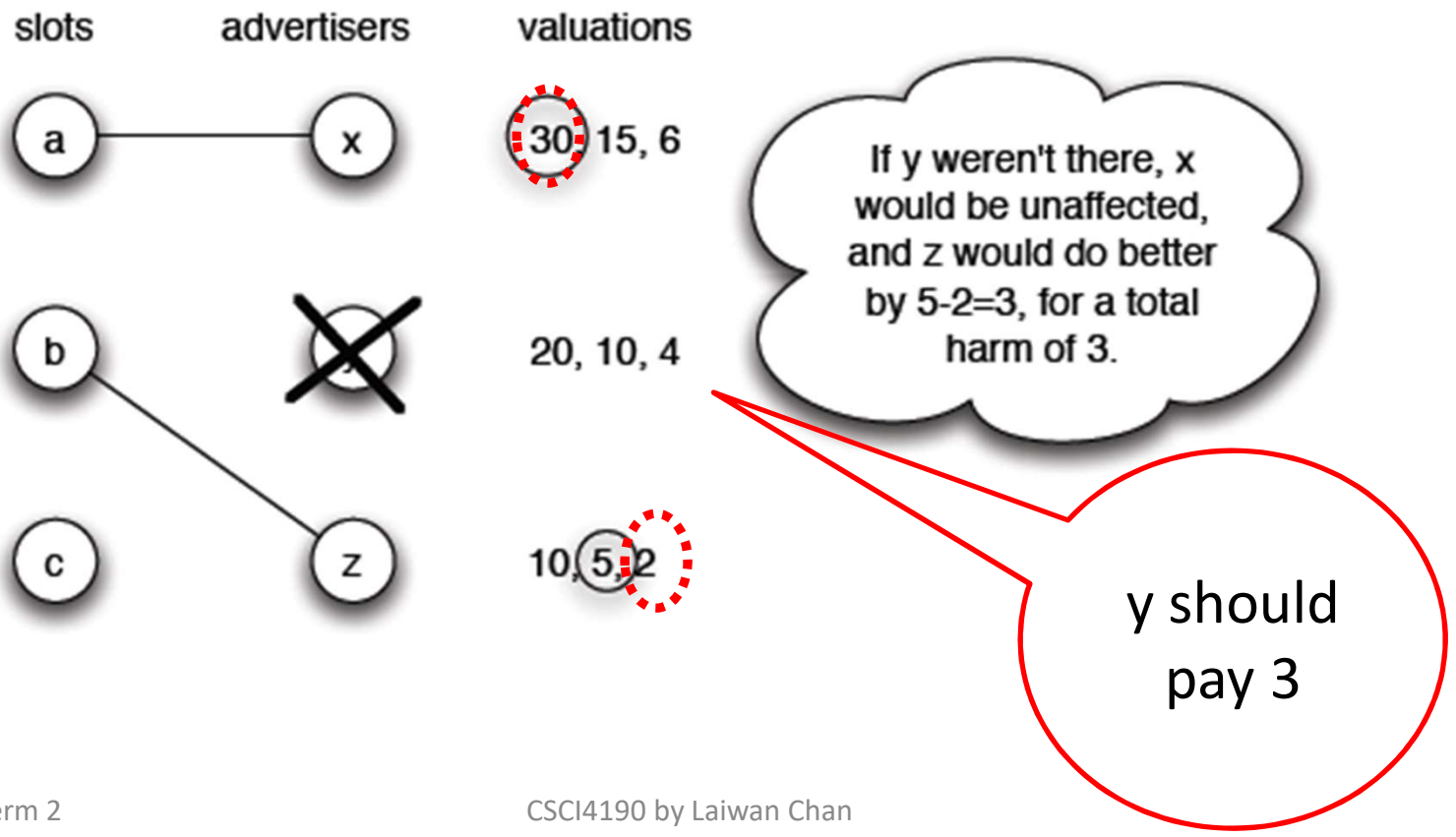
The VCG Principle

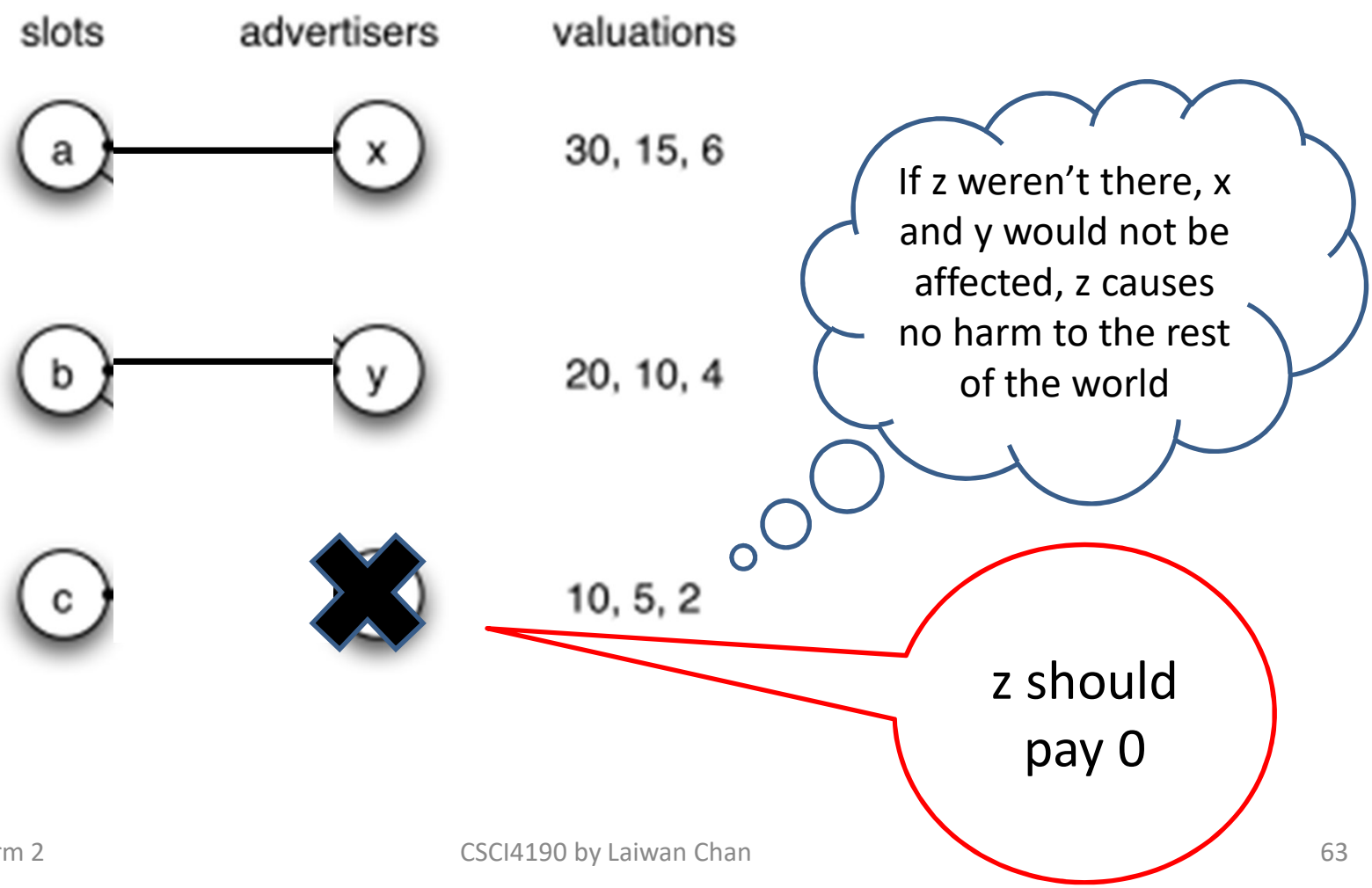


The VCG Principle



The VCG Principle

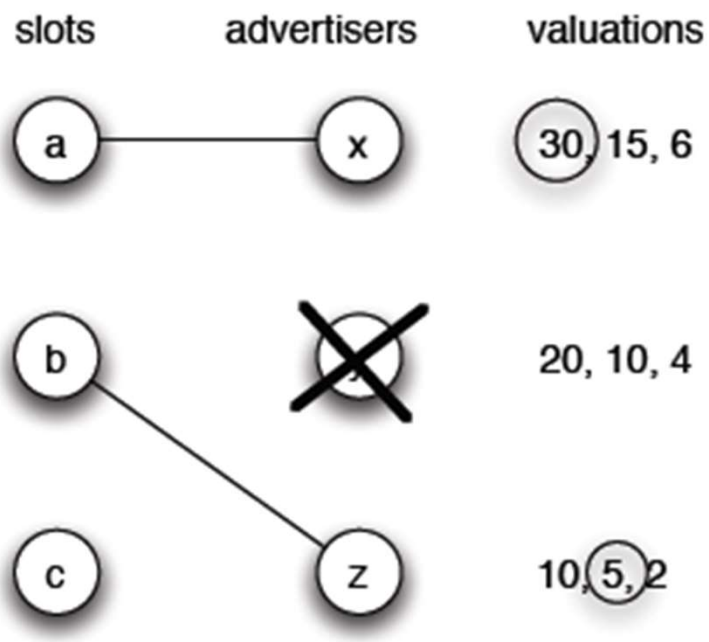



slots
advertisers
valuations
a
x
30, 15, 6
b
y
20, 10, 4
c
10, 5, 2
If z weren't there, x and y would not be affected, z causes no harm to the rest of the world
z should pay 0

- S = the set of sellers
- B = the set of buyers
- V_B^S = the maximum total valuation over all possible perfect matching of sellers and buyers
- *S-i* = the set of sellers with seller i removed
- *B-j* = the set of buyers with buyer j removed

The VCG Principle

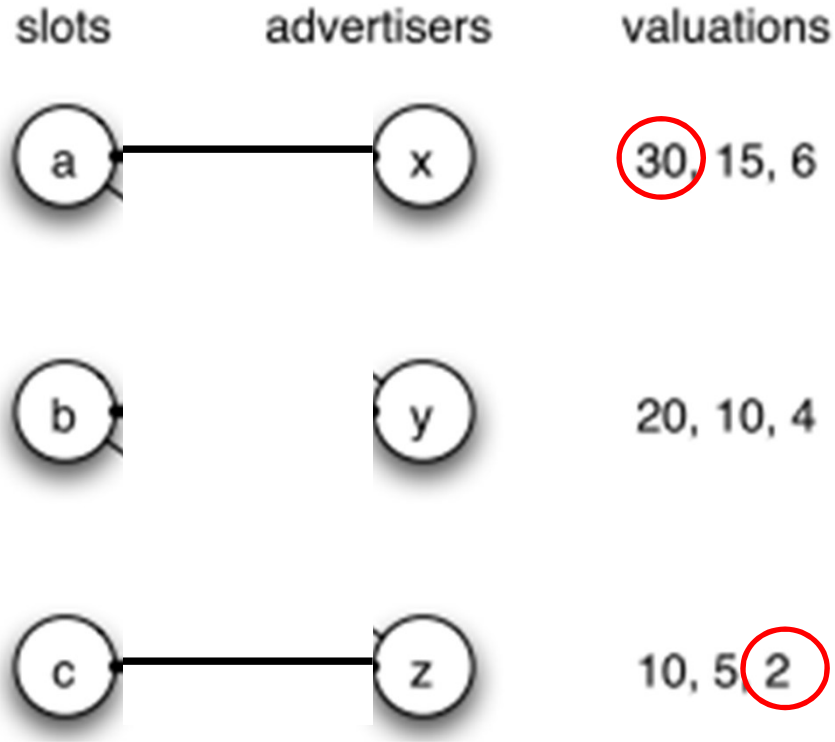
- V_{B-j}^{S} = total valuation if buyer j does not exist



V_{B-y}^{S}= 30+5=35

The VCG Principle

- V_{B-j}^{S-i} = total valuation if buyer j and seller i do not exist



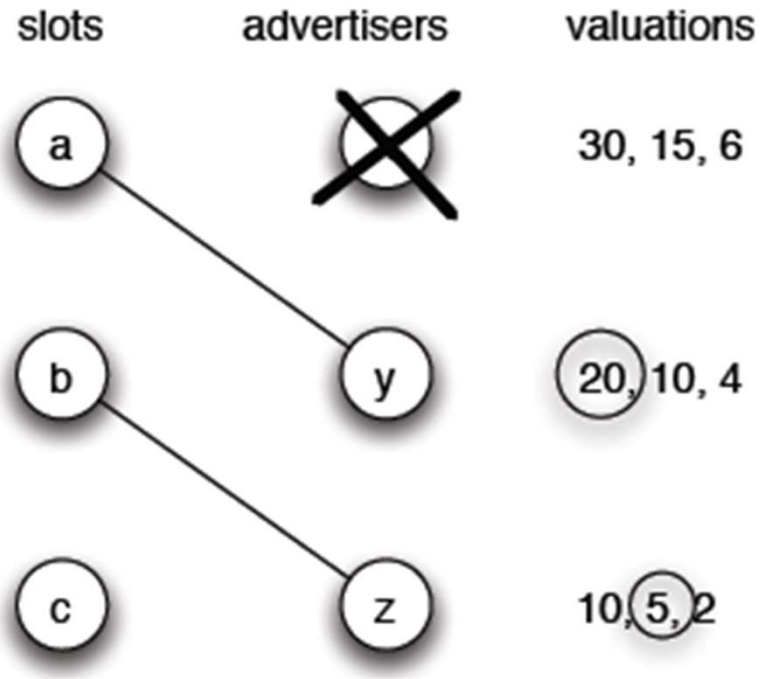
V_{B-j}^{S-i}= 30+2=32

The VCG price $p_{ij} = V_{B-j}^{S} - V_{B-j}^{S-i}$

The VCG price p_{by}= 35-32=3

The VCG Principle

The VCG price $p_{ij} = V_{B-j}^{S} - V_{B-j}^{S-i}$

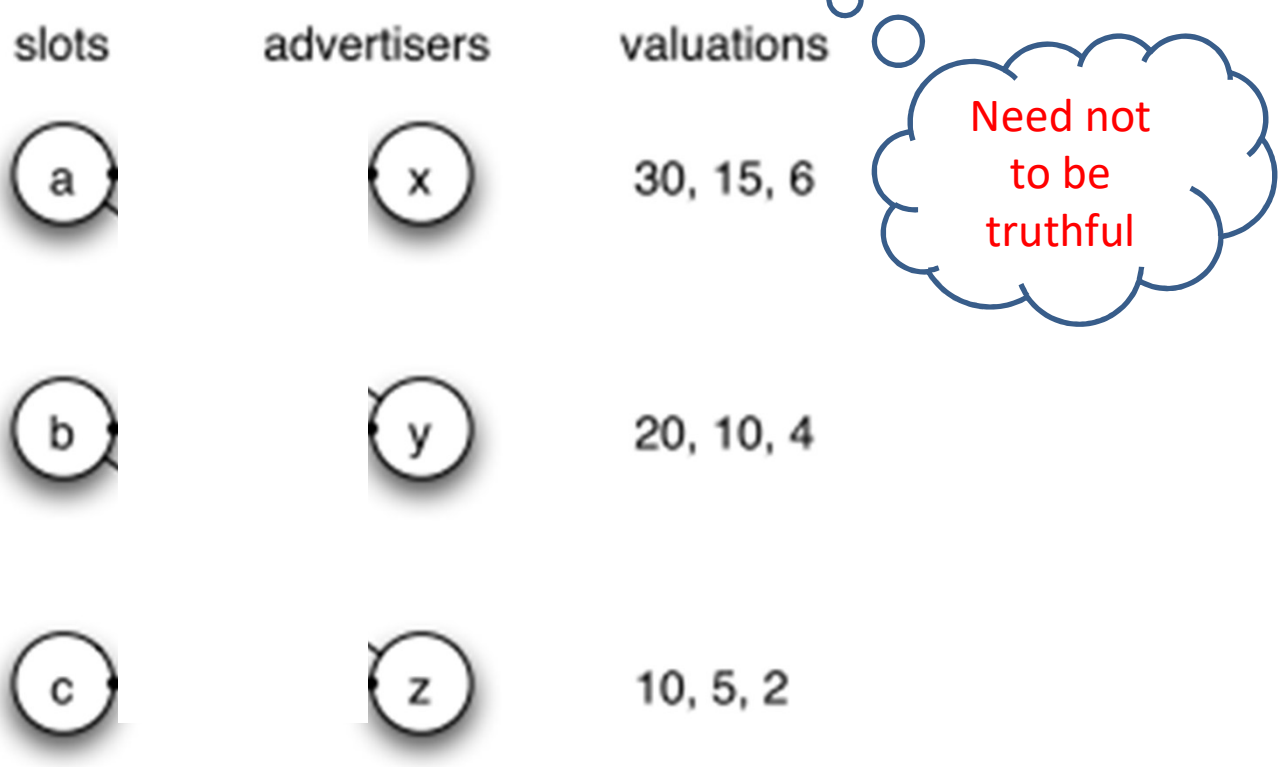


The VCG price $p_{ax} = V_{B-x}^{S} - V_{B-x}^{S-a}$

=(20+5)-(10+2)=13

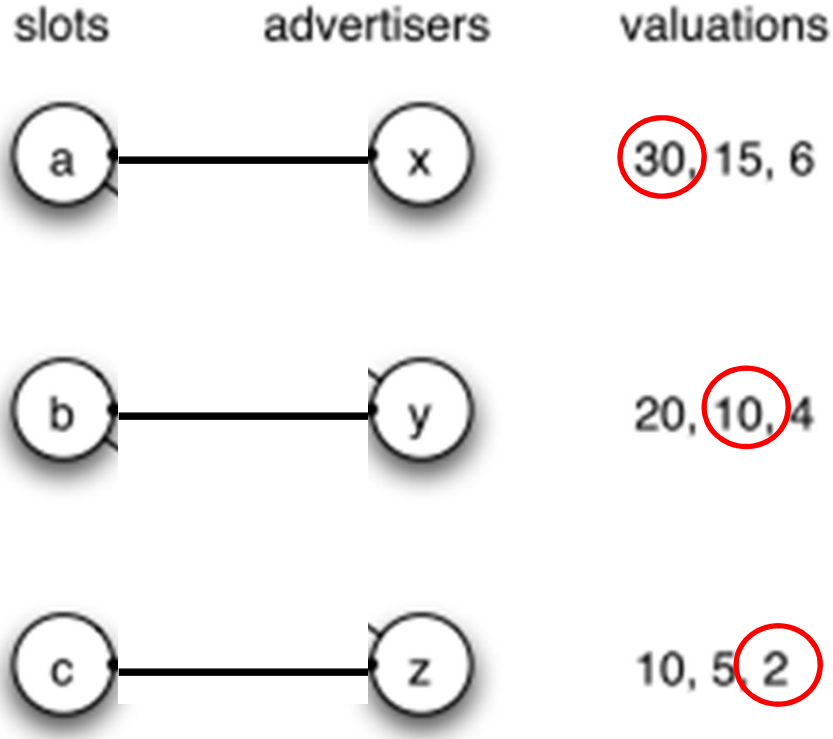
The VCG Price-setting procedure

1. Ask buyers to announce valuations for the items



The VCG Price-setting procedure

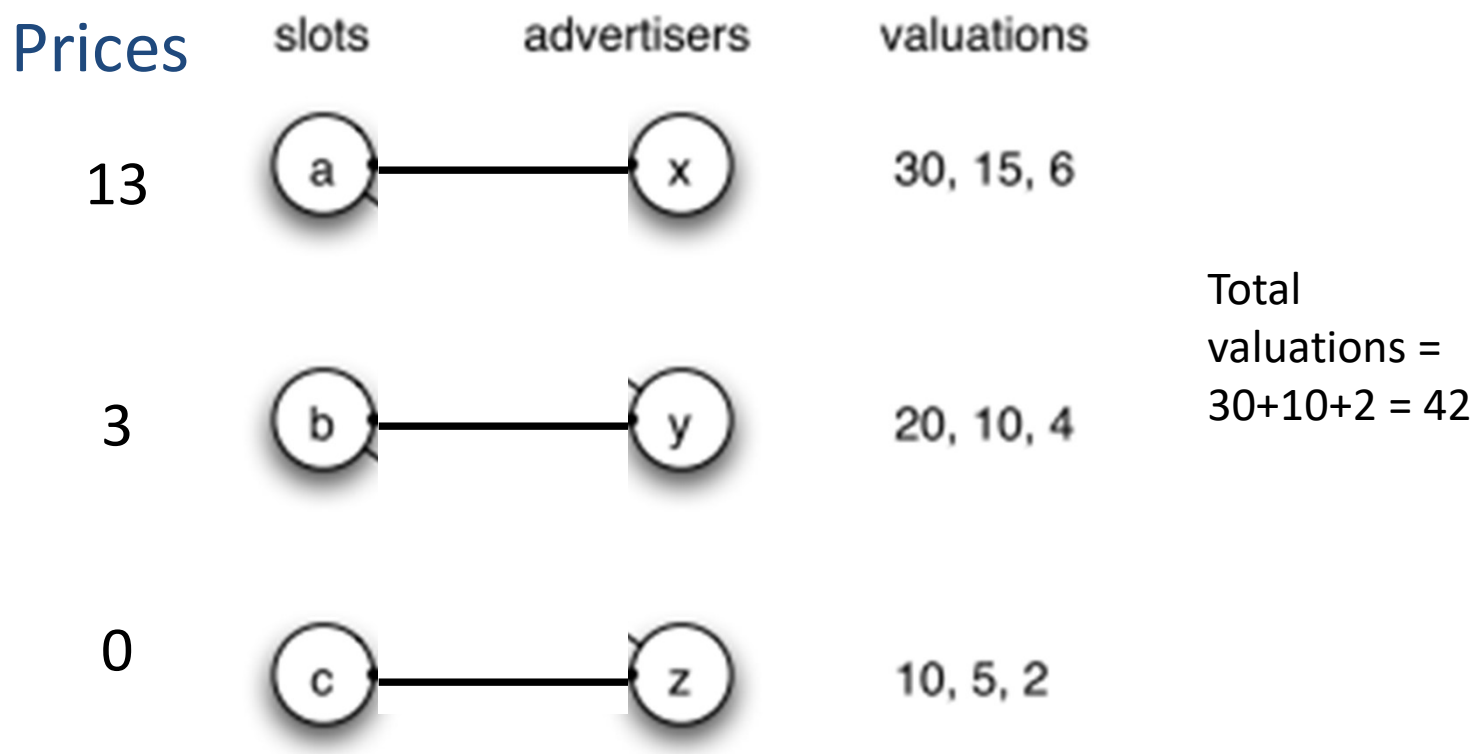
2. Choose a perfect matching that maximizes the total valuations of each buyer for what they get.



Total valuations = 30+10+2 = 42

The VCG Price-setting procedure

3. Charge each buyer the appropriate VCG price.



- Claim : If items are assigned and prices computed according to the VCG procedure, then (a) truthfully announcing valuations is a dominant strategy for each buyer, and (b) the resulting assignment maximizes the total valuation of any perfect matching of slots and advertisers.

- Claim : (b) the resulting assignment maximizes the total valuation of any perfect matching of slots and advertisers.

- Proof : if buyers report their valuations truthfully, then the assignment of items is designed to maximize the total valuation by definition.

Please refer to the textbook for the proof of claim (a)

Setting prices through an auction

- Matching Market
  - if the search engine knew all the advertisers' valuations for clicks
- Vickrey-Clarke-Groves (VCG) mechanism
  - the advertisers' valuations are not known
  - encouraging truthful bidding, or to deal with the consequences of untruthful bidding
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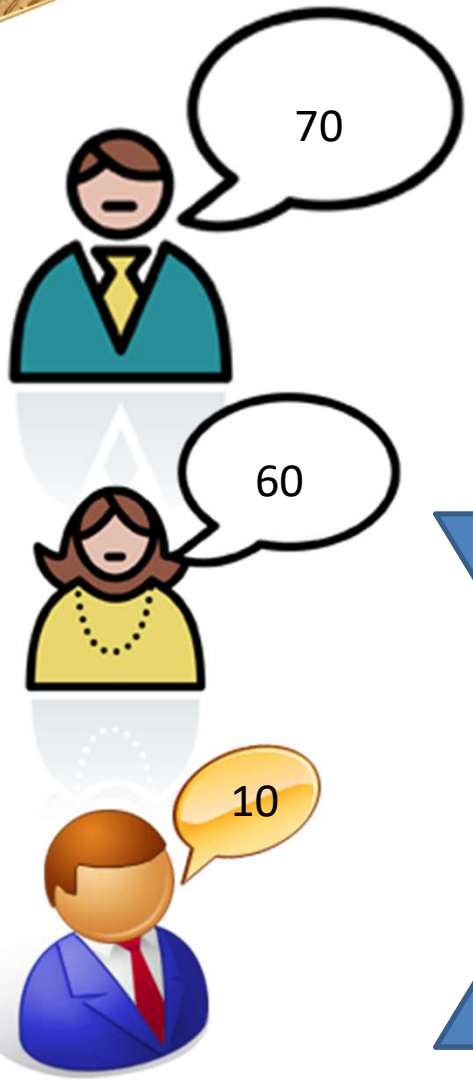
The Generalised Second Price Auction (GSP)



- What is the price ?

awards each slot i to the ith highest bidder, at a price per click equal to the (i + 1)st highest bid.

GSP

- For multiple slots
  - when the bids per click are b_1, b_2, b_3, … in descending order
- GSP charges a cumulative price of $r_i b_{i+1}$ for slot i.
  - the ith highest bidder will get slot i at a price per click of b_{i+1}, multiplying by the clickthrough rate of r_i

If each advertiser bids its true valuation

clickthrough rates	slots	advertisers	revenues per click	valuations
10	a	x	7	70, 28, 0
4	b	y	6	60, 24, 0
0	c	z	1	10, 4, 0

For GSP, p_{ax} = 6x10=60

p_{by} = 1x4=4

Revenue for the publisher = 64

GSP

- GSP was originally developed at Google.
- It had been in use for awhile in the search industry.