ENGG1410-E: Short Test 3

Name: **Student ID:**

Write all your answers on this sheet, and use the back if necessary.

Problem 1 (40%). Consider the surface S: $z^2 + 2y = 3$ with $0 \leq x \leq 1$. Calculate $\iint_S (x^2 + y^2)\, dydz$.

Answer: S is perpendicular to the yz-plane. Therefore, $\iint_S (x^2 + y^2)\, dydz = 0$.

Problem 2 (60%). Let C be the curve that is the intersection of the following two surfaces:

$$\begin{aligned} x^2 + y^2 + z^2 &= 9 \\ z &= 0. \end{aligned}$$

Orient C so that it passes points $(3, 0, 0), (0, 3, 0), (-3, 0, 0)$ in this order. Calculate $\int_C 2y\, dx + 3x\, dy - z^2\, dz$. (Hint: apply Stokes' Theorem on a properly chosen surface.)

Answer: Let S be the circle enclosed by C, oriented with its upper side taken. Introduce

$$\begin{aligned} f_1 &= 2y \\ f_2 &= 3x \\ f_3 &= -z^2. \end{aligned}$$

Hence:

$$\begin{aligned} \frac{\partial f_3}{\partial y} - \frac{\partial f_2}{\partial z} &= 0 \\ \frac{\partial f_1}{\partial z} - \frac{\partial f_3}{\partial x} &= 0 \\ \frac{\partial f_2}{\partial x} - \frac{\partial f_1}{\partial y} &= 1 \end{aligned} \tag{1}$$

By Stokes Theorem, we have:

$$\int_C 2y\, dx + 3x\, dy - z^2\, dz = \iint_S 1\, dxdy = 9\pi.$$