ENGG 1130 Multivariable Calculus for Engineers

Assignment 2 (Term 2, 2019-2020)

Assigned Date: 24 Jan 2020 (Friday) 10:00 am

Deadline: **7 Feb 2020 (Friday) 12 noon**

- Show **ALL** your steps and details for each question unless otherwise specified.
- Make a copy of your homework before submitting the original!
- Please submit the hard copy of your HW to **Box C03, SEEM Department, ERB 5/F** before the prescribed deadline.

* means harder questions.

Notation: $\langle a, b, c\rangle$ represents the vector $a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$

Full score: 100

1. **(10 %)** Consider the set of points in a plane whose polar coordinates satisfy the following equation:

$$r = \frac{1}{(c + d\cos\theta)}, \qquad \text{where } c, d > 0, \qquad c - d > 0$$

(a) Upon transforming back to rectangular coordinates (in 2D), show that $c\sqrt{x^2+y^2} + dx$ is a constant. Find that constant.

(b) Based on the constant you obtain in (a), show that such equation will be an equation of an ellipse.

2. **(10 %)** Find the distance between a point $P = (5,3,3)$ and the line l with symmetric equations:

$$x - 1 = \frac{y+8}{-4} = \frac{z-7}{3}$$

by the following approach (i.e. **NO marks** will be given if you use other approaches).

Approach: Parametrizing the line with parameter t, then express the distance between P and a desired point on l in terms of t.

3. **(10 %)** Find an equation in rectangular coordinates for the following surface, and sketch the corresponding graphs. Label all necessary points (for example intercepts) as well.

(a) $r + 5z = 0$

(b) $\phi = \frac{\pi}{4}$

4. **(10 %)** An object with mass m that moves in an elliptical path with constant angular speed ω has position vector $\mathbf{r}(t) = \langle a\cos\omega t, b\sin\omega t\rangle$.

Find the force acting on the object, and show that it is directed towards the origin.

5. **(20 %)** We define the curvature of a path by $\|\mathbf{r}''(s)\|$, where $\mathbf{r}(s)$ is the arc-length parametrization of the path.

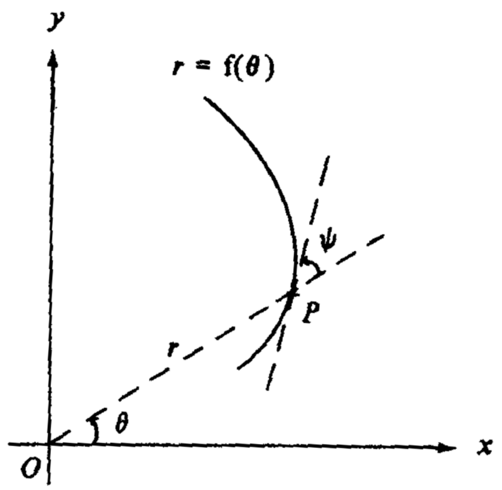
Given a path $\mathbf{r}(t)$, we let $\mathbf{r}(s)$ be its arc-length parametrization so that $s = \int_0^t \|\mathbf{r}'(\tau)\| d\tau$.

(a) Show that $\mathbf{r}'(t) \times \mathbf{r}''(t) = \left(\frac{ds}{dt}\right)^3 \mathbf{r}'(s) \times \mathbf{r}''(s)$.

(b) Hence, or otherwise, show that the curvature can be expressed in terms of t.

Give the explicit form of the curvature function.

*6. **(20 %)** Given a curve C on the xy-plane with equation $r = f(\theta)$, where r and θ are the polar coordinates used to describe any point lying on C, in particular point P in the following figure. Let O be the origin and ψ be the angle from the line OP to the tangent line at point P. We assume f is continuously differentiable and non-negative.



(a) Express $\tan\psi$ in terms of r and derivative of r with respect to θ.

(b) Given two curves C_1: $r = 2 - 2\cos\theta$ (where $0 \le \theta < 2\pi$) and C_2: $r = 2$.

(i) Find all points of intersection of these two curves.

(ii) Find the angle between the tangent lines at each point of intersection you obtained in (i).

(c) Some part of C_1 is inside C_2, find the arc length of such part.

7. **(10 %)** Find the limit of the following vector-valued functions (if it exists).

Show your steps and any formula / theorem that you applied.

(a) $\lim_{t \to 0} \left(|t|^t \mathbf{i} + \cos^{\frac{1}{t}}(t) \mathbf{j} + (1-t)^t \mathbf{k} \right)$

(b) $\lim_{t \to \infty} \left(e^{-t} \mathbf{i} + \frac{1}{t} \mathbf{j} + t^{\frac{1}{t}} \mathbf{k} \right)$

*8. **(10 %)** Let $\mathbf{r}(t) = \langle x(t), y(t), z(t) \rangle$, solve the following initial-value problem (i.e. solve for $\mathbf{r}(t)$).

$$\begin{cases} \dfrac{d\mathbf{r}(t)}{dt} = \mathbf{k} \times \mathbf{r}(t) \\ \mathbf{r}(0) = \mathbf{i} + \mathbf{k} \end{cases}$$

(**Hint:** If $\frac{d^2 x(t)}{dt^2} = -x(t)$, then we have $x(t) = A \cos t + B \sin t$ for some constants A and B.)

END OF ASSIGNMENT 2