CSCI 5350 Assignment 1

Due date: 12 October 2020

1. Consider the game of Prisoner's Dilemma $G = \langle N, (A_i), (u_i) \rangle$ shown below.

		Player 2	
		C	D
Player 1	C	3, 3	0, 5
	D	5, 0	1, 1

a) What is N in this game G?
b) What is (A_i) in this game G?

Define the social welfare $S(a)$ of an outcome $a \in A = \times_{i \in N} A_i$ to be $S(a) = \sum_{i \in N} u_i(a)$. The two players find it difficult to play Prisoner's Dilemma because both players are motivated to play an outcome that is not optimal in terms of social welfare. So, they come to John, a game theory student, for help. John suggests that they modify the game by adding one new action F to each of A_i, so that $A_i' = A_i \cup \{F\}$, and play the new <u>symmetric</u> game $G' = \langle N, (A_i'), (u_i') \rangle$ instead. John says that the intuitive idea is that F should be a compromising action, so that by playing F both players will receive the same utility f, that is, $u_1'(F, F) = u_2'(F, F) = f$, while $u_1'(a) = u_1(a)$ and $u_2'(a) = u_2(a)$ for all $a \in A' \cap A$, where $A' = \times_{i \in N} A_i'$.

c) How many outcomes are there in the new game $G' = \langle N, (A_i'), (u_i') \rangle$?

John does not say what $u_1'(a)$ and $u_2'(a)$ should be if $a \in A'$ but $a \notin A$. However, John says that if u_1' and u_2' are appropriately defined, it will be possible that (F, F) is the <u>only</u> pure strategy Nash equilibrium in the new game $G' = \langle N, (A_i'), (u_i') \rangle$.

d) Is John correct that it is possible that (F, F) is the only pure strategy Nash equilibrium in the new game $G' = \langle N, (A_i'), (u_i') \rangle$? Justify your answer.

e) John can be right or wrong. Answer one of the following two questions, <u>but not both</u>.

- If John is correct, what are the <u>general</u> conditions to be imposed on the definitions of u_1' and u_2', and the range of values of f, for (F, F) to be the only pure strategy Nash equilibrium in the new game $G' = \langle N, (A_i'), (u_i') \rangle$? Justify your answer.

- If John is not correct, explain whether John can be correct by relaxing the constraint that $u_1'(a) = u_1(a)$ and $u_2'(a) = u_2(a)$ for all a that is in both $A = \times_{i \in N} A_i$ and $A' = \times_{i \in N} A_i'$.

f) Give a correlated equilibrium of the game $G' = \langle N, (A_i'), (u_i') \rangle$, or justify that none exists.

2. Peter and Mary are going to have dinner together. There are two options: the Italian restaurant *Spasso*, or the Cantonese restaurant *Tin Tak Heen*. Both Peter and Mary know that if each of them goes to a different restaurant, then each of them will be unhappy and will receive a utility of 0 (zero). Mary does not really like *Spasso*, so Peter and Mary will receive a utility of 5 (five) and 1 (one) respectively if they go to *Spasso* together; but each will still receive a utility of 1 (one) if they go to *Tin Tak Heen*.

   a) Model the situation as a strategic game $\langle N, (A_i), (u_i) \rangle$ with 2 players, with Peter being player 1 and Mary being player 2.
      i) Write down explicitly what N, (A_i), and (u_i) are.
      ii) Find all pure strategy Nash equilibria of the game.
      iii) Find all completely mixed strategy Nash equilibria of the game.

   b) For one of the completely mixed strategy Nash equilibria in a)iii), find a corelated equilibrium $< (\Omega, \pi), (\mathcal{P}_i), (\sigma_i) >$ in which the players effectively play the completely mixed strategy Nash equilibrium.

   c) Is the completely mixed strategy Nash equilibria you use in b) an ESS? Justify your answer.

3. Now consider the scenario in Question 2 again. Peter believes that there are actually two possible situations. First, Peter believes that there is a probability of ¼ that the scenario described in Question 2 is the real situation. In fact, Peter also believes that there is still a probability of ¾ that both Peter

and Mary will be very happy and each of them will receive a utility of 5 (five) if they go to *Spasso* (action S) together; or each will receive a utility of 1 (one) if they go to *Tin Tak Heen* (action T) together. On the other hand, Mary believes that the probability of the first situation is only ⅞, while the probability of the second situation is ⅛.

Model the situation as a Bayesian game $\langle N, \Omega, (A_i), (T_i), (\tau_i), (p_i), (u_i) \rangle$ with 2 players, with Peter being player 1 and Mary being player 2, and 2 states:

- In state A, the utility profile will be (5, 5) for outcome (S, S), (1, 1) for outcome (T, T), and (0, 0) for all other outcomes.
- In state B, the utility profile will be (5, 1) for outcome (S, S), (1, 1) for outcome (T, T), and (0, 0) for all other outcomes.

Unfortunately both players 1 and 2 have no hint, at all, what the real state is.

a) What is Ω in this game?
b) What is (T_i) in this game?
c) What is (τ_i) in this game?
d) What is (p_i) in this game?
e) What is u_1 (utility function of player 1, i.e., Peter) in this game?
f) Consider the following action profile: type A player 1 plays S, type B player 1 plays S, type A player 2 plays S, type B player 2 plays S. Is this action profile a Nash equilibrium? Justify your answer.

4. Give an example of a finite symmetric game that has only asymmetric equilibria, or prove that there is no such case.

5. Let G be a strictly competitive game that has a Nash equilibrium.
   a) Show that if some of player 1's payoff in G are increased in such a way that the resulting game G' is strictly competitive then G' has no equilibrium in which player 1 is worse off than she was in an equilibrium of G. (Note that G' may have no equilibrium at all.)
   b) Show that the game that results if player 1 is prohibited from using one of her actions in G does not have an equilibrium in which player 1's payoff is higher than it is in an equilibrium of G.
   c) Give examples to show that neither of the above properties necessarily holds for a game that is not strictly competitive.

— End of Assignment —