CSCI 3230

Fundamentals of Artificial Intelligence

Chapter 18

LEARNING FROM EXAMPLES

2 types of AI: HI=>AI; importance of learning; data mining; knowledge acquisition; computer/machine learning; KDD- knowledge discovery and data mining; applications of Data mining: supermarket, marketing, financial predictions; DNA diagnoses, etc.; Big Data analytics; Crowd/Internet sourcing; sentiment analysis

Fall 2019

Outline

- A General Model of Learning Agents
- Inductive Learning
- Learning Decision Trees
- Using Information Theory
- Learning General Logical Descriptions
- Why Learning Works: Computational Learning Theory

Learning from observation

- Learning: Precepts not only for acting, but also for improving the agent's ability to act
- Learning involves the interaction between the agent and the world through observation (examples) by the agent of its own decision-making processes.
- To improve their behavior through study of their own/others experience.
- To acquire new knowledge or refine existing knowledge

Data→ Information→ Knowledge
?Applications
Predictions, classification:
improvement in performance for finance, marketing, sales and medical applications

A General Model of Learning Agents

- A learning agent has four conceptual components, as shown in Fig 18.1. (See chap.2 for details)

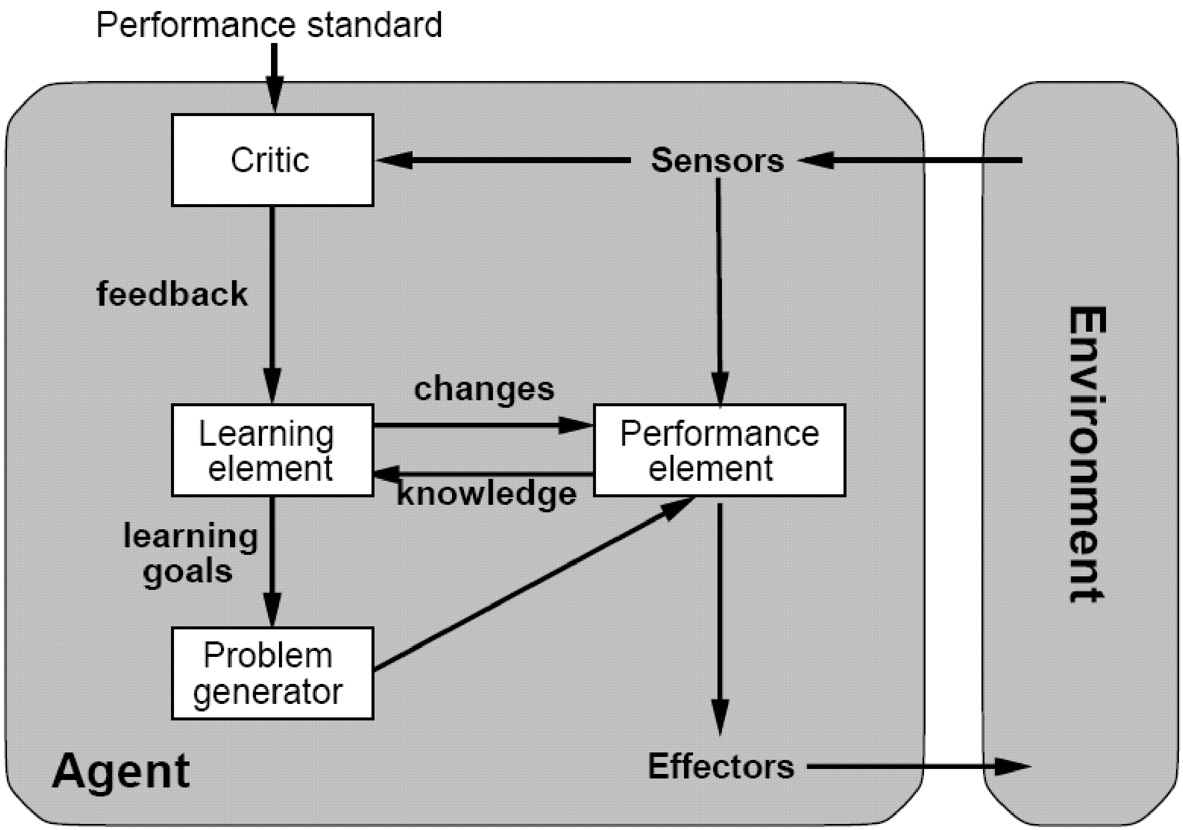


Fig. 18.1 A general model of learning agents

A General Model of Learning Agents

The design of the learning element is affected by **4** major issues:

- Which components of the performance element are to be improved or learnt. (p.6), e.g. NN: weights or structure
- What representation is used for those components (p.7)

  e.g. regression or classification, models…
- What feedback is available (pp.8-9) supervised or un…
- What prior information is available (p.10) landscape, variable types…

A General Model of Learning Agents
– Components of the performance element

Some of the information/knowledge or components (of the KB) are:

1. A direct mapping from conditions on the current state to actions. E.g. functions
2. A means to infer relevant properties of the world from the percepts sequence. E.g. inference, prediction model
3. Information about the way the world evolves. (states) model
4. Utility information indicating the desirability of world states.
5. Action-value information indicating the desirability of particular actions in particular states.
   - (evaluation function)
6. Goals that describe classes of states whose achievement maximizes the agent's utility.
7. Constraints. Complexity, rules

A General Model of Learning Agents
– Representation of the components

- These components can be represented using any of the representation schemes in this book and learnt
- E.g.
  - deterministic descriptions such as linear weighted polynomials for utility functions in game-playing programs regression
  - propositional and first-order logical sentences for all of the components in a logical agent; classifiers and
  - probabilistic descriptions such as belief (Bayesian) networks for the inferential components of a decision-theoretic agent.
  - … Rules; decision trees; random forest; NN; SVM; Non-linear integrals; semantic & hierarchy networks; OO

A General Model of Learning Agents
– Classification of learning by Available feedback (c.f. human learning)

- **Unsupervised learning**
  - No hint at all about the correct outputs.
  - It learns patterns in the inputs. E.g. attendance
  - It learns what to do based on a utility function, e.g. Nearest Neighbor
  - E.g. reinforcement learning is a form of unsupervised learning; clustering; discovery learning; robot discovers the concept of "door" itself; anomaly detection; Generative Adversarial Networks (GAN); Expectation-maximization algorithm (EM)
- **Reinforcement learning**
  - In learning the condition-action component, the agent receives some evaluation of its action but is not told the correct action.
  - The hefty bill (penalty, e.g. braking, hit the car in front) is called a **reinforcement** – Rewards or punishments; used in AlphaGo Zero,
  - E.g. Q-learning: model-free reinforcement learning

A General Model of Learning Agents
– Available feedback (c.f. human learning)

- Supervised learning
  - In predicting the outcome of an action, the available feedback generally tells the agent what the correct outcome is.
  - Both the inputs & outputs of a component can be perceived (Often, the outputs are provided by a friendly teacher.)
  - E.g. car braking – guess stopping in 10m; supervisor: actually should be 15m. Classification problems– given training examples with class labels, learning by examples. SVM, decision tree, Random Forrest, etc.

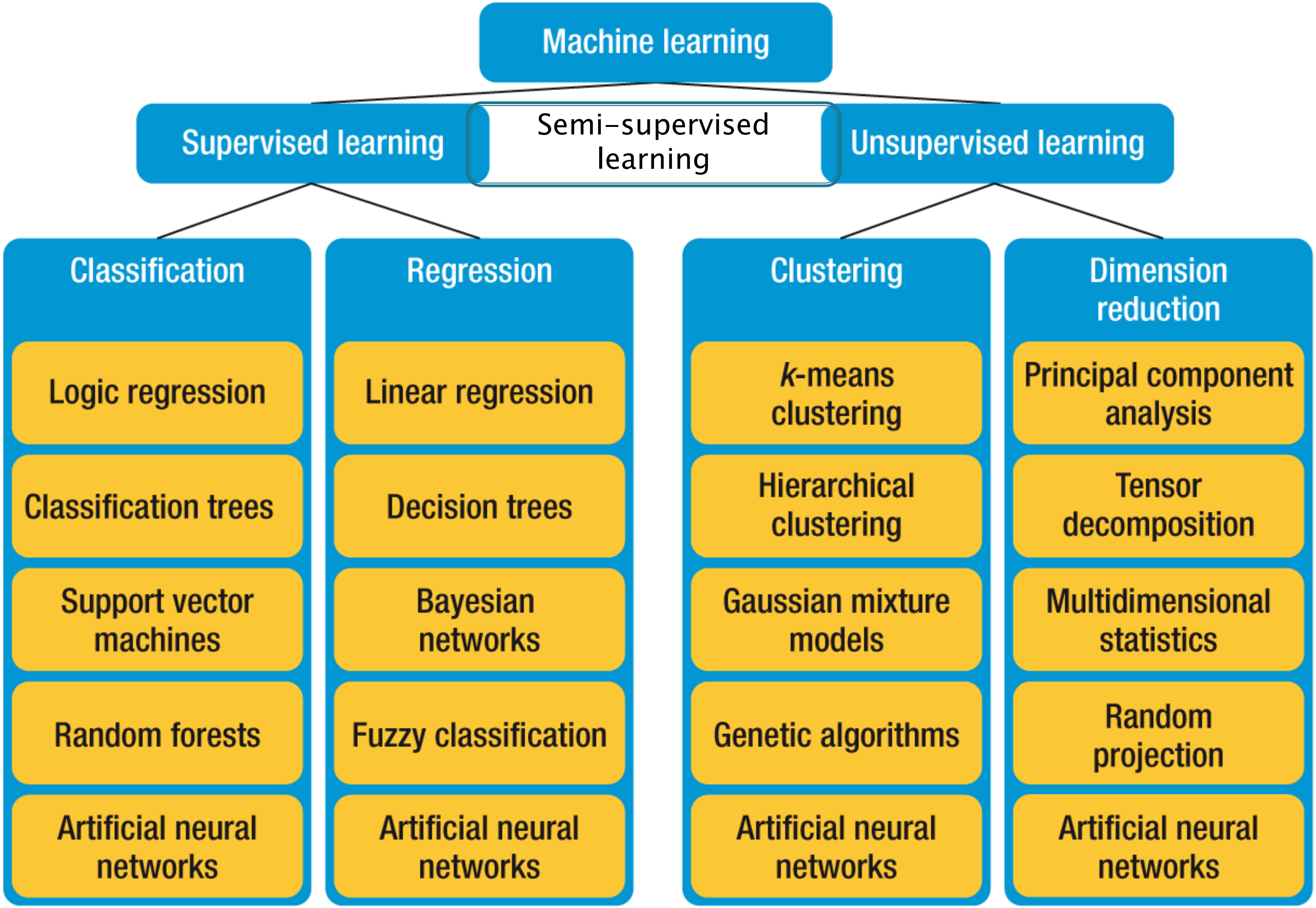
- Semi-supervised learning
  - Given some labeled examples and some un-labeled examples.
  - Clustering + guess for labels?

A General Model of Learning Agents
– Available feedback (c.f. human learning) / Prior knowledge

Prior knowledge

- The majority of learning research in AI, CS, and psychology, the agent begins with no knowledge about what it is trying to learn. It only has access to the examples presented by its experience.
- Most human learning takes place with background knowledge. Some psychologists and linguists claim that even newborn babies exhibit knowledge of the world.
- E.g. Transfer Learning, Analogy, meta knowledge, problem nature

  e.g. discrete vs. continuous; deterministic vs. stochastic; landscape;


Machine learning
Supervised learning
Semi-supervised learning
Unsupervised learning
Classification
Regression
Clustering
Dimension reduction
Logic regression
Classification trees
Support vector machines
Random forests
Artificial neural networks
Linear regression
Decision trees
Bayesian networks
Fuzzy classification
Artificial neural networks
k-means clustering
Hierarchical clustering
Gaussian mixture models
Genetic algorithms
Artificial neural networks
Principal component analysis
Tensor decomposition
Multidimensional statistics
Random projection
Artificial neural networks

Inductive Learning 歸納法

- In supervised learning, the learning element is given the correct (or approximately correct) value of the function for particular inputs, and changes its representation (h) of the function (f) to try to match the information provided by the feedback ($\delta = h - f$).

- Formally, an example is a pair (x, $f(x)$), where x is the input and $f(x)$ is the output of the function applied to x. *$f(x)$: ground truth or Tag*

- The task of pure inductive inference (or induction):

  Given a collection of examples of f, return a function h that approximates f. The function h (representations e.g.??) is called a hypothesis.

Inductive Learning

Data mining

- The true f is unknown, so there are many choices for h, but without further knowledge, we have no way to prefer (b), (c), or (d). (see Fig 18.2)

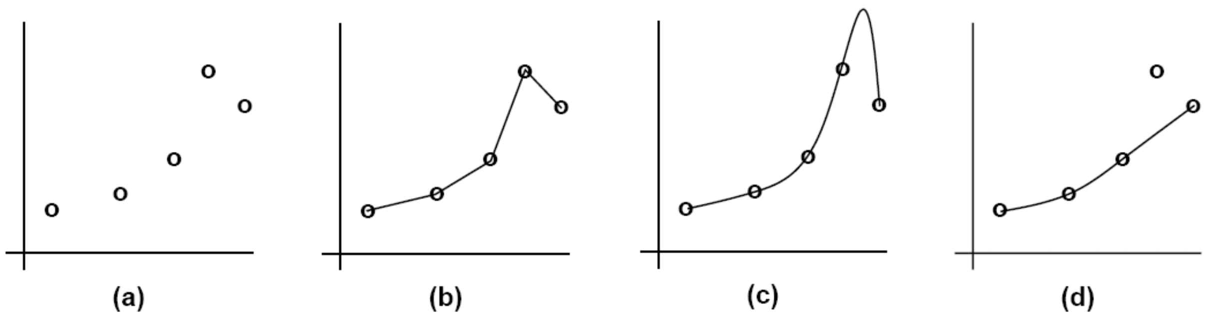


Fig. 18.2 in (a) we have some example (*input, output*) pairs. In (b), (c) and (d) we have 3 hypotheses for function from which there example could be drawn. (which one better?)

- Any preference for one hypothesis over another, beyond mere consistency with the examples, is called a bias.
- Because of a large number of possible consistent hypotheses, all learning algorithms exhibit some sort of *bias*:
  - E.g. simplest hypothesis, avoid over-fitting, noise, & outliners.
  - regularized learning

Inductive Learning

- The *choice* of *representation* for the desired function is the most important issue facing the designer of a learning agent.

- In learning there is a fundamental trade-off between expressiveness – is the desired function representable in the representation language? – and efficiency –is the learning problem tractable for a given choice of representation.
  - Search space (complexity)? Free lunch? Optimal?
  - (E.g. straight line Vs polynomial).

- i.e. Trade-off among:
  - Model/algorithm complexity – effectiveness – efficiency – memory

Learning Decision Trees
– Decision trees as performance elements

- Decision tree induction is one of the simplest and yet most successful forms of learning algorithm in *inductive learning*.

Decision trees as performance elements

- A **decision tree** – input: a set of properties (*attributes*), outputs: a yes/no "decision". It represents **Boolean function.**
- Functions with a larger range of outputs okay too, but for simplicity usually stick to the Boolean case.
- internal node = a test ; branches are labeled with possible values of the test. Each leaf node specifies the Boolean value result if reached. C4.5; See 5;

Learning Decision Trees
– Decision trees as performance elements

- *Example:* To learn a definition for the **goal predicate** (concept) WillWait. 1st: decide *attributes* available to describe the problem domain:

1. Alternate: whether there is a suitable alternative restaurant nearby.
2. Bar: whether the restaurant has a comfortable bar area to wait in.
3. Fri/Sat: true on Fridays and Saturdays.
4. Hungry: whether we are hungry.
5. Patrons: how many people are in the restaurant (values are *None*, *Some*, *Full*).
6. Price: the restaurant's price range ($, $$, $$$).
7. Raining: whether it is raining outside.
8. Reservation: whether we made a reservation.
9. Type: the kind of restaurant (*French*, *Italian*, *Thai or Burger*).
10. WaitEstimate: the wait estimated by the host (0-10 minutes, 10-30, 30-60, $>$60).

- The decision tree is given in Fig 18.4

Learning Decision Trees
– Inducing decision trees from examples

Example	Attributes										Goal
	Alt	*Bar*	*Fri*	*Hun*	*Pat*	*Price*	*Rain*	*Res*	*Type*	*Est*	*WillWait*
X_1	*Yes*	*No*	*No*	*Yes*	*Some*	*$$$*	*No*	*Yes*	*French*	*0–10*	*Yes*
X_2	*Yes*	*No*	*No*	*Yes*	*Full*	*$*	*No*	*No*	*Thai*	*30–60*	*No*
X_3	*No*	*Yes*	*No*	*No*	*Some*	*$*	*No*	*No*	*Burger*	*0–10*	*Yes*
X_4	*Yes*	*No*	*Yes*	*Yes*	*Full*	*$*	*No*	*No*	*Thai*	*10–30*	*Yes*
X_5	*Yes*	*No*	*Yes*	*No*	*Full*	*$$$*	*No*	*Yes*	*French*	*>60*	*No*
X_6	*No*	*Yes*	*No*	*Yes*	*Some*	*$$*	*Yes*	*Yes*	*Italian*	*0–10*	*Yes*
X_7	*No*	*Yes*	*No*	*No*	*None*	*$*	*Yes*	*No*	*Burger*	*0–10*	*No*
X_8	*No*	*No*	*No*	*Yes*	*Some*	*$$*	*Yes*	*Yes*	*Thai*	*0–10*	*Yes*
X_9	*No*	*Yes*	*Yes*	*No*	*Full*	*$*	*Yes*	*No*	*Burger*	*>60*	*No*
X_{10}	*Yes*	*Yes*	*Yes*	*Yes*	*Full*	*$$$*	*No*	*Yes*	*Italian*	*10–30*	*No*
X_{11}	*No*	*No*	*No*	*No*	*None*	*$*	*No*	*No*	*Thai*	*0–10*	*No*
X_{12}	*Yes*	*Yes*	*Yes*	*Yes*	*Full*	*$*	*No*	*No*	*Burger*	*30–60*	*Yes*

Fig. 18.5 Examples for the restaurant domain. Price? discretization

Learning Decision Trees
– Decision trees as performance elements

White box: internal node: a test of an attribute;
Line: branch: possible values of the attribute;
Shaded box: leave node: **answer**

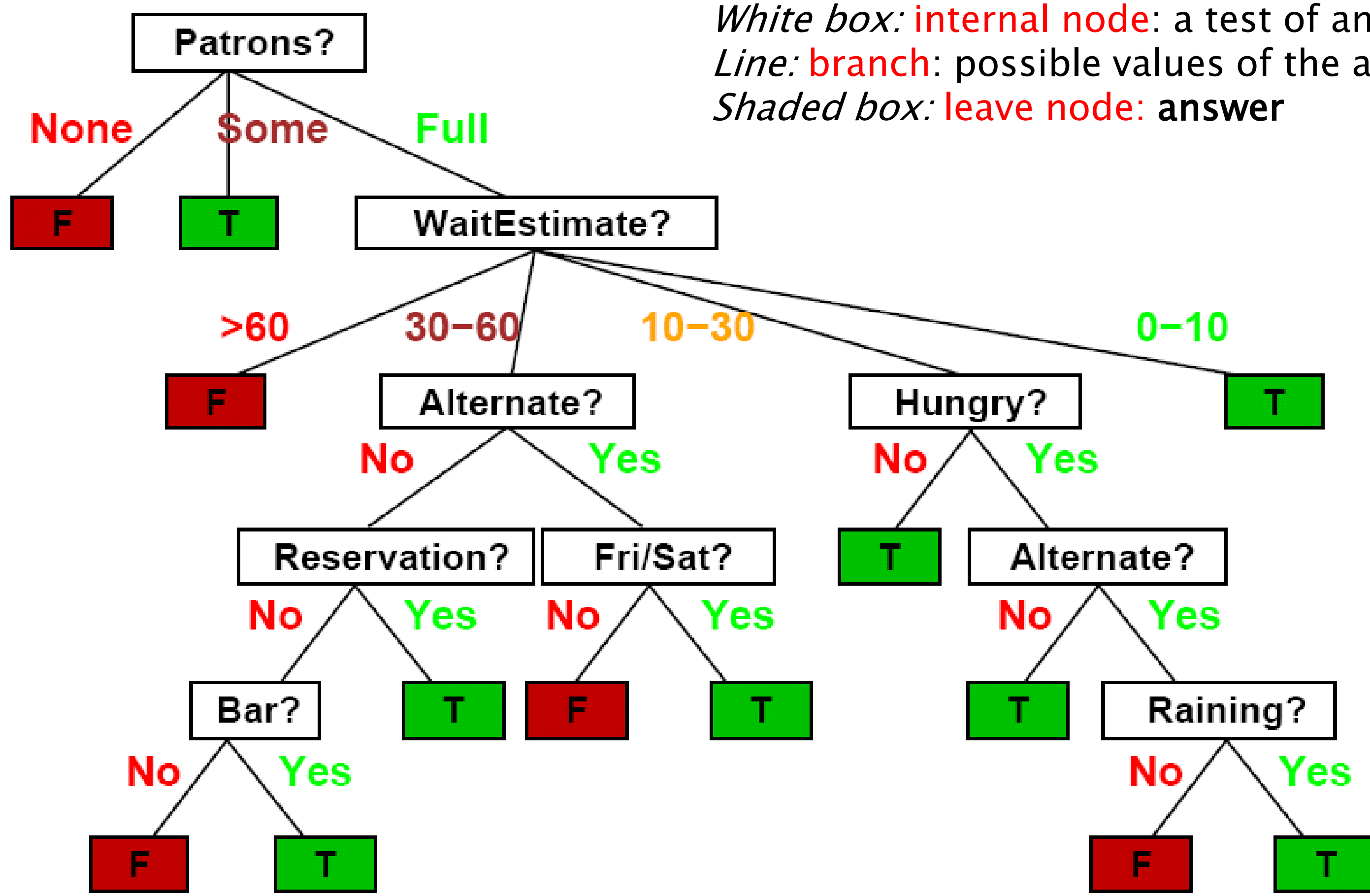


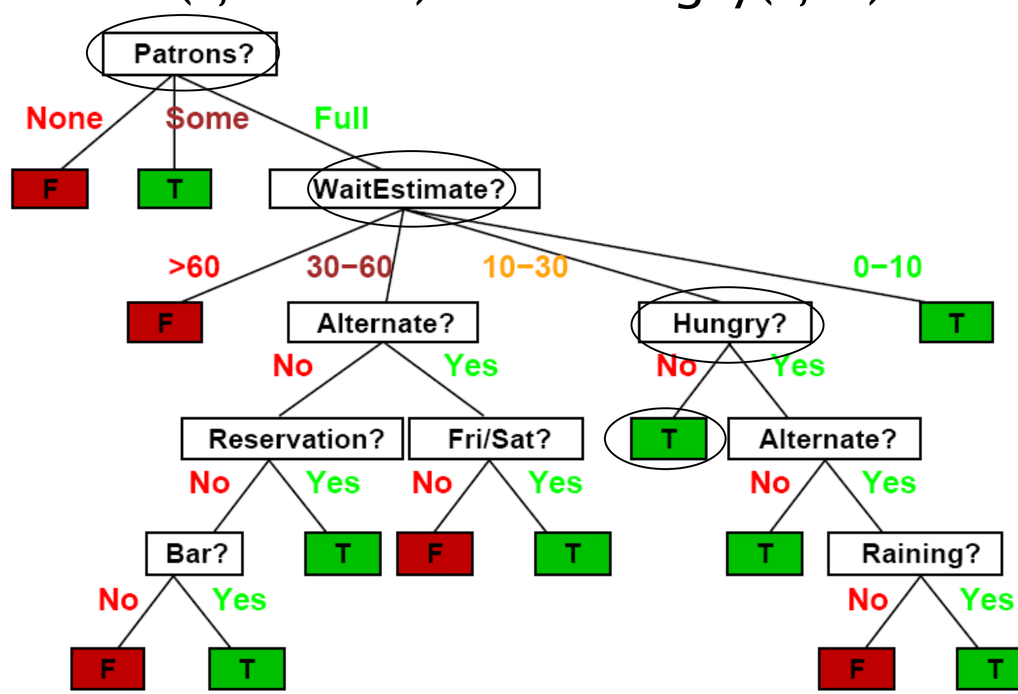
Fig. 18.4 A decision tree for deciding whether to wait for a table

Learning Decision Trees
– Decision trees as performance elements

- A path to a *Yes*–node can be expressed by a conjunction of tests implication.
- E.g., the path for a restaurant full of patrons, with an estimated wait of 10–30 minutes when the agent is not hungry is expressed by the logical sentence:

$\forall$r Patrons(r, Full) $\wedge$ WaitEstimate(r, 10–30) $\wedge$ $\neg$ Hungry(r, N) $\Rightarrow$ WillWait(r)

r: variable for restaurant



Learning Decision Trees
– Expressiveness of decision trees

- Decision trees are fully expressive within the class of propositional languages, i.e. any Boolean function can be written as a decision tree.
- Trivially done by having each row in the truth table for the function correspond to a path in the tree.
- Not a good way to represent the function, because the truth table is exponentially large in the number of attributes (2^n)
- Clearly, decision trees can represent many functions with much smaller trees.

Learning Decision Trees
– Inducing decision trees from examples

- An **example** is described by the *values* of the *attributes* and the value of the *goal* predicate – called the **classification** Tag of the example. If *true*, we call it a **positive** example; otherwise, a **negative** example.

- A set of examples $X_1, \ldots, X_{12}$ for the restaurant domain is shown in Figure 18.5.
  - Positive examples – the goal WillWait is *true* (X_1, X_3, ...)
  - Negative examples – the goal WillWait is *false* (X_2, X_5,...).
  - The complete set of examples is called the **training set.**

Learning Decision Trees
– Inducing decision trees from examples

Example	Attributes										Goal
	Alt	*Bar*	*Fri*	*Hun*	*Pat*	*Price*	*Rain*	*Res*	*Type*	*Est*	*WillWait*
X_1	*Yes*	*No*	*No*	*Yes*	*Some*	*$$$*	*No*	*Yes*	*French*	*0–10*	*Yes*
X_2	*Yes*	*No*	*No*	*Yes*	*Full*	*$*	*No*	*No*	*Thai*	*30–60*	*No*
X_3	*No*	*Yes*	*No*	*No*	*Some*	*$*	*No*	*No*	*Burger*	*0–10*	*Yes*
X_4	*Yes*	*No*	*Yes*	*Yes*	*Full*	*$*	*No*	*No*	*Thai*	*10–30*	*Yes*
X_5	*Yes*	*No*	*Yes*	*No*	*Full*	*$$$*	*No*	*Yes*	*French*	*>60*	*No*
X_6	*No*	*Yes*	*No*	*Yes*	*Some*	*$$*	*Yes*	*Yes*	*Italian*	*0–10*	*Yes*
X_7	*No*	*Yes*	*No*	*No*	*None*	*$*	*Yes*	*No*	*Burger*	*0–10*	*No*
X_8	*No*	*No*	*No*	*Yes*	*Some*	*$$*	*Yes*	*Yes*	*Thai*	*0–10*	*Yes*
X_9	*No*	*Yes*	*Yes*	*No*	*Full*	*$*	*Yes*	*No*	*Burger*	*>60*	*No*
X_{10}	*Yes*	*Yes*	*Yes*	*Yes*	*Full*	*$$$*	*No*	*Yes*	*Italian*	*10–30*	*No*
X_{11}	*No*	*No*	*No*	*No*	*None*	*$*	*No*	*No*	*Thai*	*0–10*	*No*
X_{12}	*Yes*	*Yes*	*Yes*	*Yes*	*Full*	*$*	*No*	*No*	*Burger*	*30–60*	*Yes*

Fig. 18.5 Examples for the restaurant domain. Price? discretization

Learning Decision Trees
– Inducing decision trees from examples

- Extracting a pattern is to describe a large number of cases in a concise way. Not just a correct decision tree fits the examples, but a concise one.
- A general principle of inductive learning: **Ockham's razor**. The most likely hypothesis is the simplest one that is consistent with all observations. (?)
- Far fewer simple hypotheses than complex ones, so only a small chance for any wildly incorrect simple hypothesis to be consistent with all observations. ∴ other things being equal, a simple hypothesis consistent with the observations is more likely to be correct than a complex one.
- Overfitting?
- Finding the smallest decision tree is intractable, but with some **simple heuristics**, can find a smallish one.

f(x1,x2) vs f(x1, x2, x3, x4); feature selection; overfitting problems; regularized learning

Learning Decision Trees
– Inducing decision trees from examples

- Figure 18.6 shows how the algorithm gets started. Given 12 training examples, classified into +ve and –ve sets. Then decide which attribute to use as the first test in the tree. (?how)
- Patrons is a fairly important attribute, because if the value is None or Some, example sets left can answer definitively (No and Yes, respectively).
- Type is a poor attribute, because it leaves 4 possible outcomes, with the same number of +ve and –ve answers. (?so)

Example	Attributes										Goal
	Alt	*Bar*	*Fri*	*Hun*	*Pat*	*Price*	*Rain*	*Res*	*Type*	*Est*	*WillWait*
X_1	*Yes*	*No*	*No*	*Yes*	*Some*	*$$$*	*No*	*Yes*	*French*	*0–10*	*Yes*
X_2	*Yes*	*No*	*No*	*Yes*	*Full*	*$*	*No*	*No*	*Thai*	*30–60*	*No*
X_3	*No*	*Yes*	*No*	*No*	*Some*	*$*	*No*	*No*	*Burger*	*0–10*	*Yes*
X_4	*Yes*	*No*	*Yes*	*Yes*	*Full*	*$*	*No*	*No*	*Thai*	*10–30*	*Yes*
X_5	*Yes*	*No*	*Yes*	*No*	*Full*	*$$$*	*No*	*Yes*	*French*	*>60*	*No*
X_6	*No*	*Yes*	*No*	*Yes*	*Some*	*$$*	*Yes*	*Yes*	*Italian*	*0–10*	*Yes*
X_7	*No*	*Yes*	*No*	*No*	*None*	*$*	*Yes*	*No*	*Burger*	*0–10*	*No*
X_8	*No*	*No*	*No*	*Yes*	*Some*	*$$*	*Yes*	*Yes*	*Thai*	*0–10*	*Yes*
X_9	*No*	*Yes*	*Yes*	*No*	*Full*	*$*	*Yes*	*No*	*Burger*	*>60*	*No*
X_{10}	*Yes*	*Yes*	*Yes*	*Yes*	*Full*	*$$$*	*No*	*Yes*	*Italian*	*10–30*	*No*
X_{11}	*No*	*No*	*No*	*No*	*None*	*$*	*No*	*No*	*Thai*	*0–10*	*No*
X_{12}	*Yes*	*Yes*	*Yes*	*Yes*	*Full*	*$*	*No*	*No*	*Burger*	*30–60*	*Yes*

Learning Decision Trees
– Inducing decision trees from examples

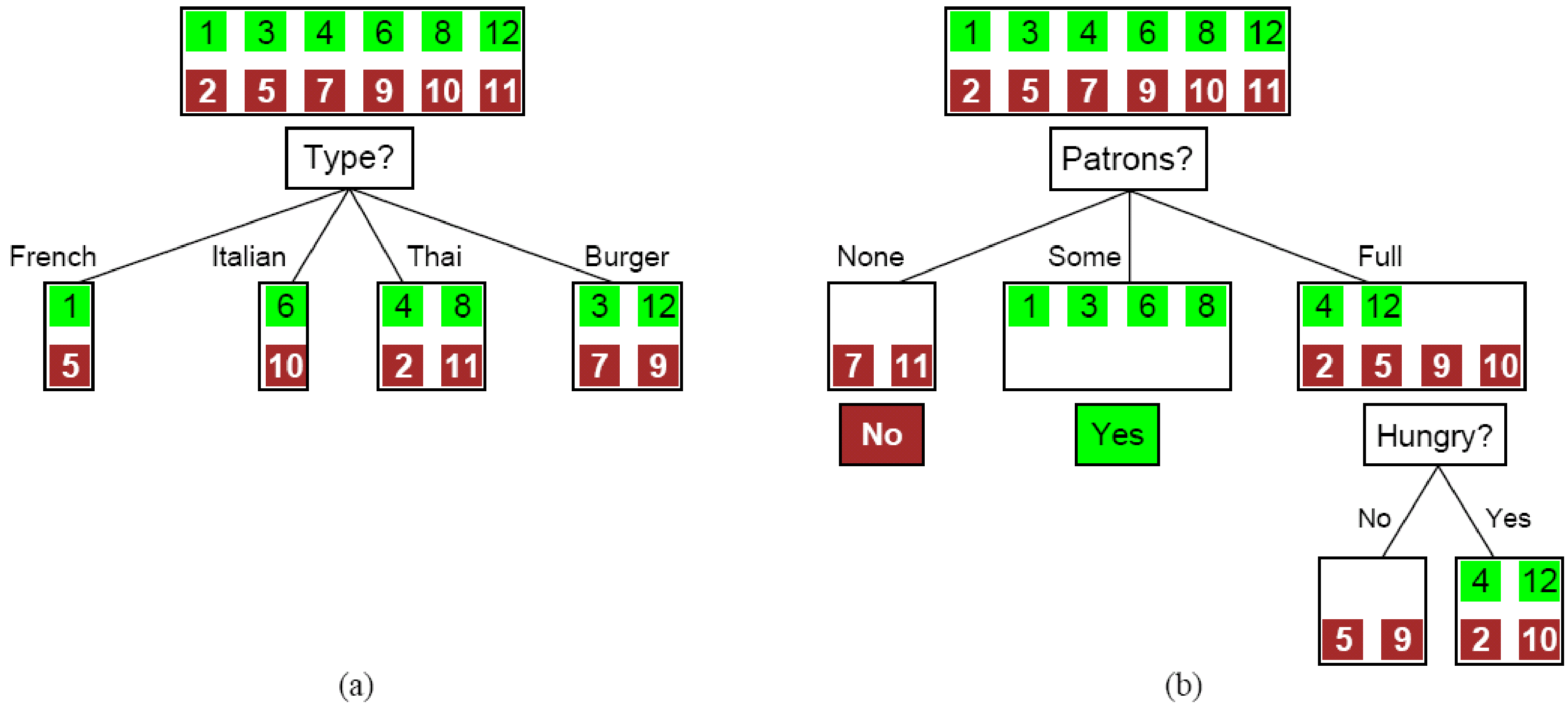
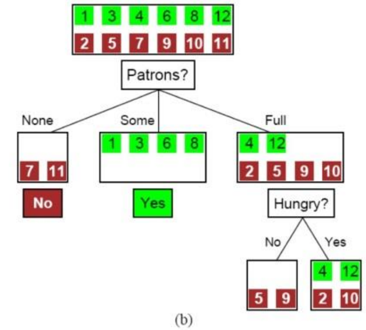


Fig 18.6 Splitting the examples by testing on attributes. (a) Type is a poor choice, no distinction between +ve and –ve examples, and (b) Patrons is a good attribute to test first, and Hungry is a fairly good second test, given that Patrons is the first test.

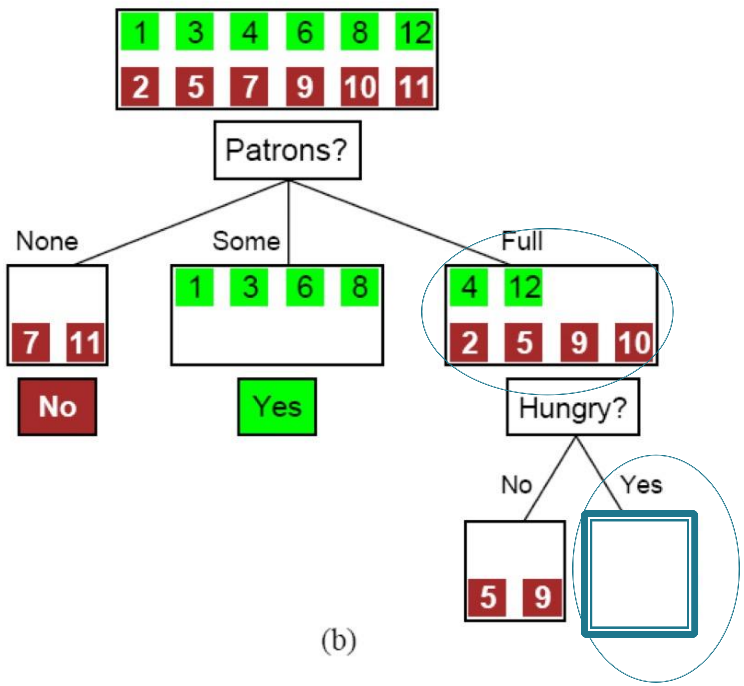
Learning Decision Trees
– Inducing decision trees from examples



- After the first attribute test splits up the examples, each outcome is a new decision tree learning problem in itself, with fewer examples and one fewer attribute. 4 possible cases to consider for these recursive problems: (see Fig. 18.6(b))

1. If there are some +ve and some −ve examples, then choose the best attribute to split them.
2. If all the remaining examples are +ve (or all −ve), then done: we can answer Yes or No.

3. If there are no examples left in this path, it means that no such example has been observed, and we return the majority classification of the node's parent.



Case 3, e.g.: No examples left in this path i.e. Hungry-Yes ⇒ Majority of parent node, *default* (Patron-Full) = “No” - not wait

Learning Decision Trees
– Inducing decision trees from examples

4. If there are no attributes left, but with both +ve and −ve examples, which means these examples have exactly the same description, but different classifications.

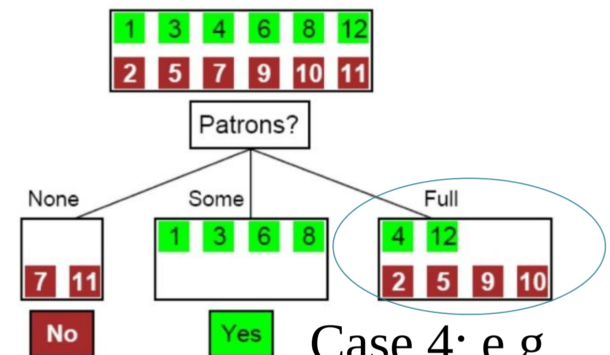
This happens when

(i) some of the data are incorrect, i.e. noise in the data;

(ii) the attributes do not give enough information to fully describe the situation; or

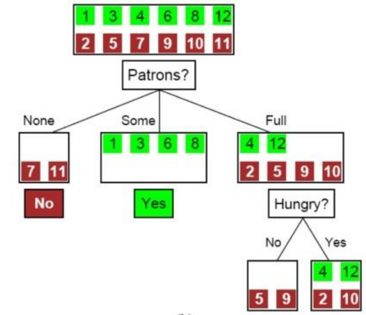
(iii) the domain is truly nondeterministic.

One simple way out: use a majority vote.



Case 4: e.g. No attributes left here ⇒ Majority vote = “No”

Learning Decision Trees
– Inducing decision trees from examples



(b)

```
function Decision-Tree-Learning(examples, attributes, default) returns a decision tree
    inputs: examples, set of examples
            attributes, set of attributes
            default, default value for the goal predicate
    if examples is empty then return default  //majority-value of parent (case 3)
    else if all examples have the same classification then return the classification // leaf node (case 2)
    else if attributes is empty then return Majority-Value(examples) // (case 4)
    else                                                        // (case 1)
       best ← Choose-Attribute(attributes, examples) //e.g. info gain
       tree ← a new decision tree with root test best //sub-tree root
       for each value v_i of best do
          examples_i ← {elements of examples with best = v_i}  // e.g. best=Patron; v_i =None: (7, 11)
          subtree ← Decision-Tree-Learning(examples_i, attributes - best, Majority-Value(examples))
          add a branch to tree with label v_i and subtree subtree
       end
       return tree
```

Fig 18.7 The decision tree learning algorithm

Is this algo optimal??

Learning Decision Trees

– Inducing decision trees from examples

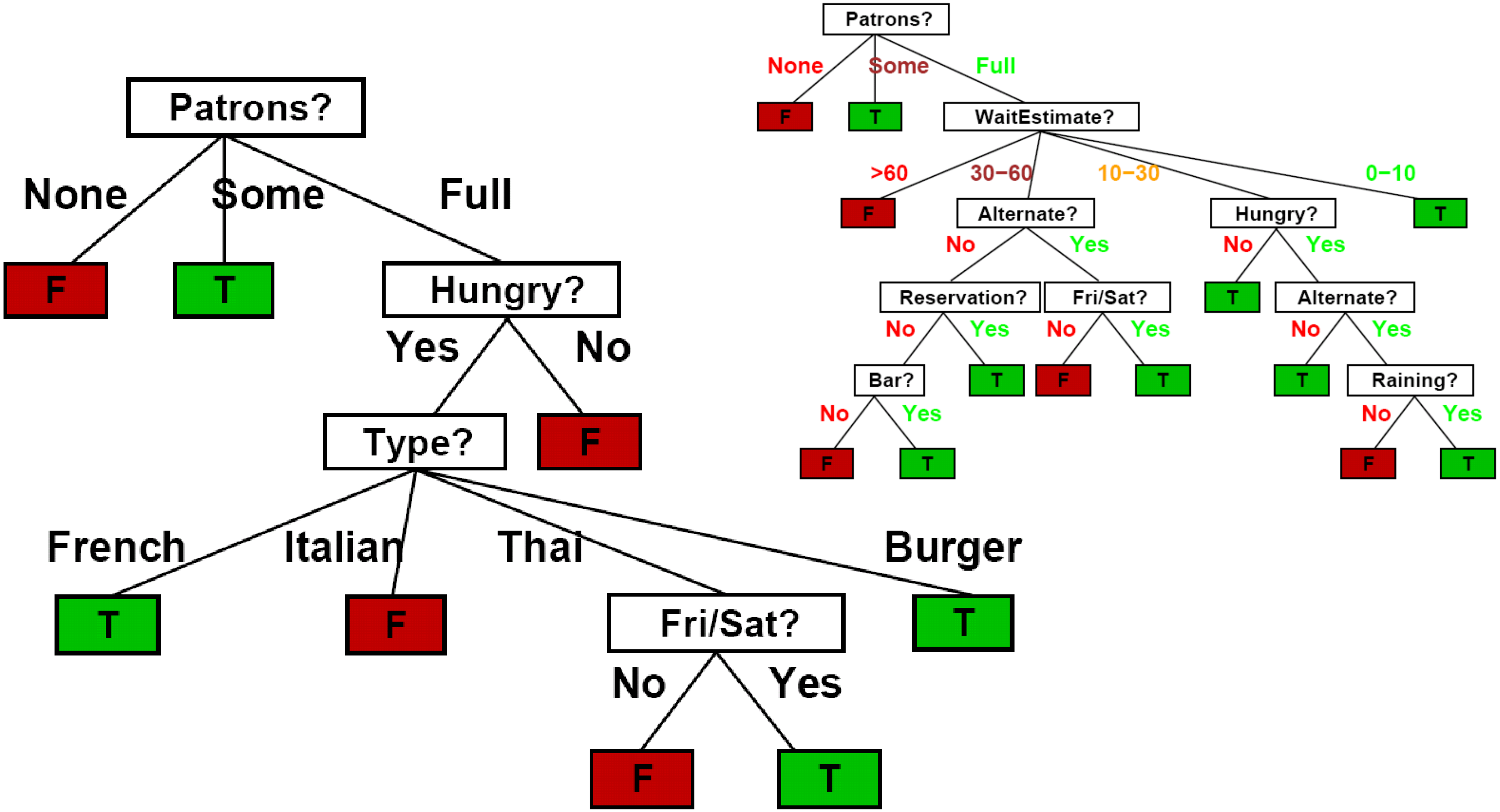


Fig 18.8 The decision tree induced from the 12-example training set.

(different from Fig 18.4 why?)

Learning Decision Trees
– Assessing the performance of the learning algorithm

- Good if it predicts accurately the classifications of unseen examples.
- Assess the quality of a hypothesis by checking its predictions against the correct classification.
- We do this on a set of examples called **test set**. **Methodology**:

1. Collect a large set of examples.(may not be possible?)
2. Divide it into two disjoint sets: the **training set** and the **test set.**
3. Use the learning algorithm with the training set as examples to generate a hypothesis h. Then test with test set.
4. Repeat steps 1 to 4 for different sizes of training sets and different randomly selected training sets (say, 20) of each size:

Learning Decision Trees
– Assessing the performance of the learning algorithm

- Take the average prediction quality of these trials as a function of the size of the training set
- Plot the **learning curve** for the algorithm on DECISION-TREE-LEARNING with the restaurant examples shown in Figure 18.9.
- Notice (next page) that as the training set grows, the prediction quality increases. (Hence, also called **happy graphs**.) A good sign that there is some pattern in the data and the learning algorithm is picking it up.

Learning Decision Trees
– Assessing the performance of the learning algorithm

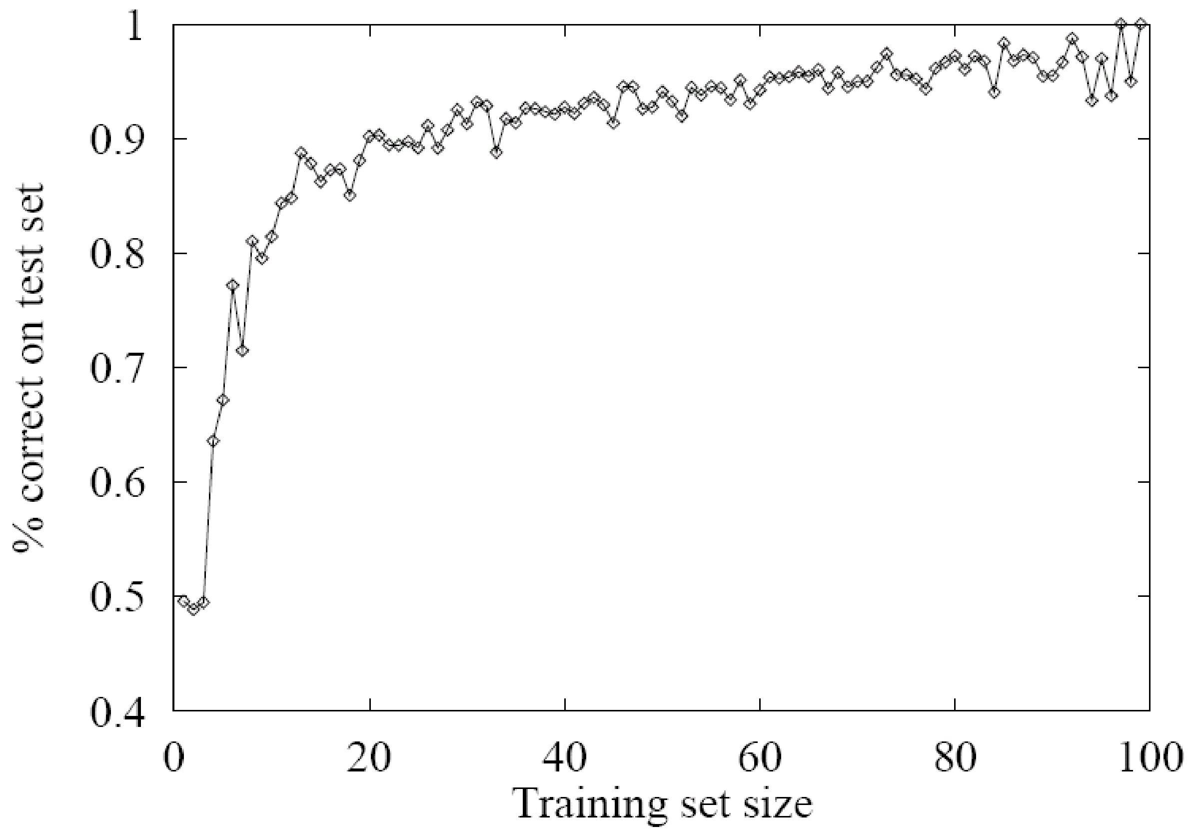


Fig 18.9. A **learning curve** for the decision tree algorithm on 100 randomly generated examples in the restaurant domain. The graph summarizes 20 trials of each size.

Using Information Theory

- In general, for an event to happen, if the possible answers v_i have probabilities $P(v_i)$, then the information content I of the actual answer is given by ??

$$I(P(v_1),\ldots,P(v_n)) = \sum_{i=1}^{n} -P(v_i)\log_2 P(v_i)$$

- This is just the average information content of the n events (the $-\log_2 P$ terms) weighted by the probabilities of the events. To check this equation, for the tossing of a fair coin we get

probability of it happening

$$I(\frac{1}{2},\frac{1}{2}) = -\frac{1}{2}\log_2\frac{1}{2} - \frac{1}{2}\log_2\frac{1}{2} = 1bit$$

0.01x6.64+0.99x0.01=.0664+.0099

- If the coin is loaded to give 99% heads we get $I(1/100, 99/100)$ = *0.08* bit, and as the probability of heads go to 1, the information of the actual answer goes to *0*. $\log 1 = 0$

$\log_2$ of Probabilities

$\log_2(P)$										
P	**0**	**0.01**	**0.02**	**0.03**	**0.04**	**0.05**	**0.06**	**0.07**	**0.08**	**0.09**
0	- inf.	-6.64	-5.64	-5.06	-4.64	-4.32	-4.06	-3.84	-3.64	-3.47
0.1	-3.32	-3.18	-3.06	-2.94	-2.84	-2.74	-2.64	-2.56	-2.47	-2.4
0.2	-2.32	-2.25	-2.18	-2.12	-2.06	-2	-1.94	-1.89	-1.84	-1.79
0.3	-1.74	-1.69	-1.64	-1.6	-1.56	-1.51	-1.47	-1.43	-1.4	-1.36
0.4	-1.32	-1.29	-1.25	-1.22	-1.18	-1.15	-1.12	-1.09	-1.06	-1.03
0.5	-1	-0.97	-0.94	-0.92	-0.89	-0.86	-0.84	-0.81	-0.79	-0.76
0.6	-0.74	-0.71	-0.69	-0.67	-0.64	-0.62	-0.6	-0.58	-0.56	-0.54
0.7	-0.51	-0.49	-0.47	-0.45	-0.43	-0.42	-0.4	-0.38	-0.36	-0.34
0.8	-0.32	-0.3	-0.29	-0.27	-0.25	-0.23	-0.22	-0.2	-0.18	-0.17
0.9	-0.15	-0.14	-0.12	-0.1	-0.09	-0.07	-0.06	-0.04	-0.03	-0.01

e.g. $\log_2(0.12) = -3.06$; $\log_2(1) = 0$;

the smaller the *P*, the higher the information content log *P*;

Using Information Theory

- For decision tree learning, need to estimate the information needed for (or contained in) a correct classification.
- An estimate of the probabilities of the possible answers *before* any attributes tested is given by the proportions of +ve and −ve examples in the training set.
- Suppose the training set contains *p* +ve examples and *n* −ve examples. Then an estimate of the information contained in a correct answer is

probability

$$I\left(\frac{p}{p+n},\frac{n}{p+n}\right)=-\frac{p}{p+n}\log_2\frac{p}{p+n}-\frac{n}{p+n}\log_2\frac{n}{p+n}$$

e.g. Fig 18.6 $\boldsymbol{p = n = 6,\quad I = 1}$

- A test on a single attribute *A* will not usually give us all, but some information. Can measure exactly how much by looking at information needed before & *after* the attribute test.

Inf given by a test = (Inf needed before) – (Inf needed after the test)

(Inf gain using *A*) $Remiander(A)$

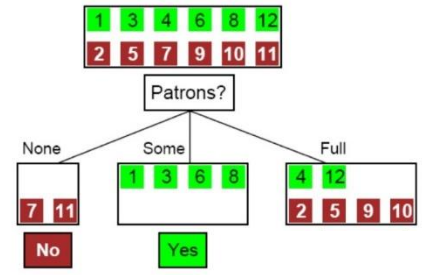
Using Information Theory

To calculate Inf needed after the test of *A*:

- Any attribute *A* divides the training set E into subsets $E_1, \ldots, E_v$ according to their values for *A*, with v distinct values. Each subset E_i has p_i +ve examples and n_i –ve examples, going along that **branch** will need an additional $I(p_i/(p_i+n_i), n_i/(p_i+n_i))$ bits of information to answer the question.
- A random example has the i^{th} value for the attribute with probability $(p_i+n_i) / (p+n)$, so on average, after testing attribute *A*, we will need

$$Remiander(A) = \sum_{i=1}^{v} \frac{p_i+n_i}{p+n} I\left(\frac{p_i}{p_i+n_i}, \frac{n_i}{p_i+n_i}\right)$$

bits of information to classify the examples



Using Information Theory

- The information gain from the attribute test is defined as the difference between the original information requirement and the new requirement:

$$Gain(A) = I\left(\frac{p}{p+n}, \frac{n}{p+n}\right) - Remainder(A)$$

  and the heuristic used in the CHOOSE-ATTRIBUTE function is to choose the attribute with the largest gain.

- (P.T.O.) Looking at the attributes Patrons and Type and their classifying power, as shown in Figure 18.6 we have

?Why = 1?

$$Gain(Patrons) = 1 - \left[\frac{2}{12} I(0,1) + \frac{4}{12} I(1,0) + \frac{6}{12} I\left(\frac{2}{6}, \frac{4}{6}\right)\right] \approx 0.541 bits$$

(1^{st} 2 terms in [ ] above = 0 ➔ no more inf needed)

$$Gain(Type) = 1 - \left[\frac{2}{12} I\left(\frac{1}{2}, \frac{1}{2}\right) + \frac{2}{12} I\left(\frac{1}{2}, \frac{1}{2}\right) + \frac{4}{12} I\left(\frac{2}{4}, \frac{2}{4}\right) + \frac{4}{12} I\left(\frac{2}{4}, \frac{2}{4}\right)\right] = 0 bits$$

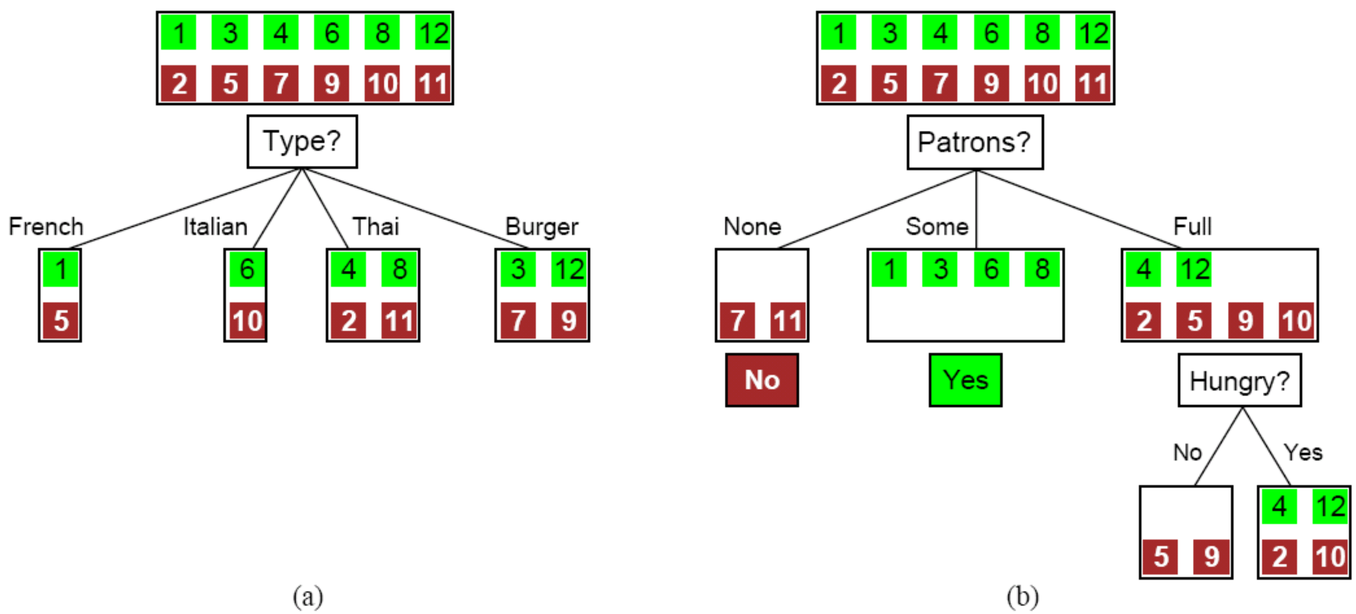
Using Information Theory

- Looking at the attributes Patrons and Type and their classifying power, as shown in Figure 18.6 we have

$$Gain(Patrons) = 1 - [\frac{2}{12} I(0,1) + \frac{4}{12} I(1,0) + \frac{6}{12} I(\frac{2}{6}, \frac{4}{6})] \approx 0.541 bits$$

(1st 2 terms in [ ] above = 0 ➔ no more inf needed)

$$Gain(Type) = 1 - [\frac{2}{12} I(\frac{1}{2}, \frac{1}{2}) + \frac{2}{12} I(\frac{1}{2}, \frac{1}{2}) + \frac{4}{12} I(\frac{2}{4}, \frac{2}{4}) + \frac{4}{12} I(\frac{2}{4}, \frac{2}{4})] = 0 bits$$



Important Points in Learning
– Noise and overfitting

- If many possible hypotheses, not to use freedom to find meaningless "regularity" in the data.
  - This problem is called overfitting.
  - A general phenomenon, occurs even when the target function is not random.
  - Afflicts every kind of learning algorithm, not just decision trees.
  - It'll fit only training data but not testing data.
- Decision tree pruning: Pruning works by preventing recursive splitting on attributes that are not clearly relevant. E.g. ignore attributes with low Inf gains or use χ^2 measure. dependence test
- With pruning --> smaller tree and learning can tolerate more noise in examples.

Ockham's razor; Feature selection; regularized learning

Important Points in Learning
– Noise and overfitting

- **Cross-validation** is another technique that eliminates the dangers of overfitting.
  - It tries to estimate how well the current hypothesis will predict unseen data.
  - Set aside some fraction of the known data, and use it to test the hypothesis induced from the rest of the known data.
  - Do this repeatedly with different subsets of the data, with the results averaged.
    E.g. 10-fold; 5-fold

Important Points in Learning
– Broadening the applicability of decision trees

- **Missing data:** In many domains, not all the attribute values are known for every example: not recorded, or too expensive to obtain. 2 problems:
  - (1) Given a decision tree, how to classify an object with one of the test attributes missing?
  - (2) How to modify the information gain formula when some examples have unknown values for the attribute?
    E.g. guess by statistics; infer by rules; certainty factors
- **Multivalued attributes:** An attribute has too many possible values, its information gain gives an inappropriate indication. (useless)
  E.g. Restaurant Name (Singleton)
- **Continuous-valued attributes:** Attributes such as Height and Weight have a large or infinite set of possible values. ∴ not well-suited for decision-tree learning in raw form.
  - To **discretize** the attribute.
    E.g: Price (continuous) --> $ $$ $$$ (discrete)

Learning General Logical Descriptions
–Hypothesis

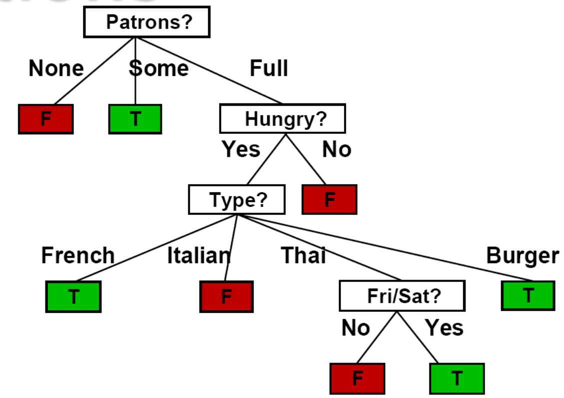
- To learn more general kinds of logical representation
- Inductive learning viewed as searching for a good hypothesis in a large hypothesis space – defined by the representation language used.

Hypothesis

- Start with a (unary) goal predicate Q (e.g. in the restaurant domain, Q is WillWait)
- A candidate definition C_i of the goal for each hypothesis H_i is a logical sentence of the form

$$\forall x\ Q(x) \Leftrightarrow C_i(x)$$

Learning General Logical Descriptions
–Hypothesis



- E.g. the decision tree in Fig 18.8 H_r:
  - $\forall r$ WillWait(r) $\Leftrightarrow$ Patron(r, some)
    $\vee$ (Patrons(r, Full) $\wedge$ Hungry(r) $\wedge$ Type(r, French))
    $\vee$ (Patrons(r, Full) $\wedge$ Hungry(r) $\wedge$ Type(r, Thai) $\wedge$ Fri/Sat(r))
    $\vee$ (Patrons(r, Full) $\wedge$ Hungry(r) $\wedge$ Type(r, Burger))
- **H** denotes the hypothesis space $\{H_1 \ldots H_n\}$. The learning algorithm believes that one of the hypotheses is correct
- Each hypothesis predicts a certain set of examples– i.e. those satisfy its candidate definition – also examples of the goal predicate. This set is called the extension of the predicate.

2 hypotheses with difference sets of extensions are inconsistent.

Learning General Logical Descriptions –Example

- For example X_i, The classification should be $Q(X_i)$ for a positive example and $\neg Q(X_i)$ for a negative example.
- An example is false negative (FN) for a hypothesis, if the hypothesis says it should be negative but in fact it is positive.
- An example is false positive (FP) for a hypothesis, if the hypothesis says it should be positive but in fact it is negative.
- For false examples (assuming correct): eliminate the hypothesis or change it to accommodate the false example.

*For medical application: Which is more serious?

Learning General Logical Descriptions
-Current-best-hypothesis search

- To maintain a single hypothesis and to adjust it as new examples arrive in order to maintain consistency.
- For false negative, the extension of the hypothesis must be increased to include the example, called generalization. (Less restrictive)
- For false positive, the extension of the hypothesis must be decreased to exclude the example, called specialization. (More restrictive)
- Recheck after changes for consistence with other examples, if fail, backtrack

Learning General Logical Descriptions
–Current–best–hypothesis search

+: +ve examples; -: -ve examples   ?boundary?

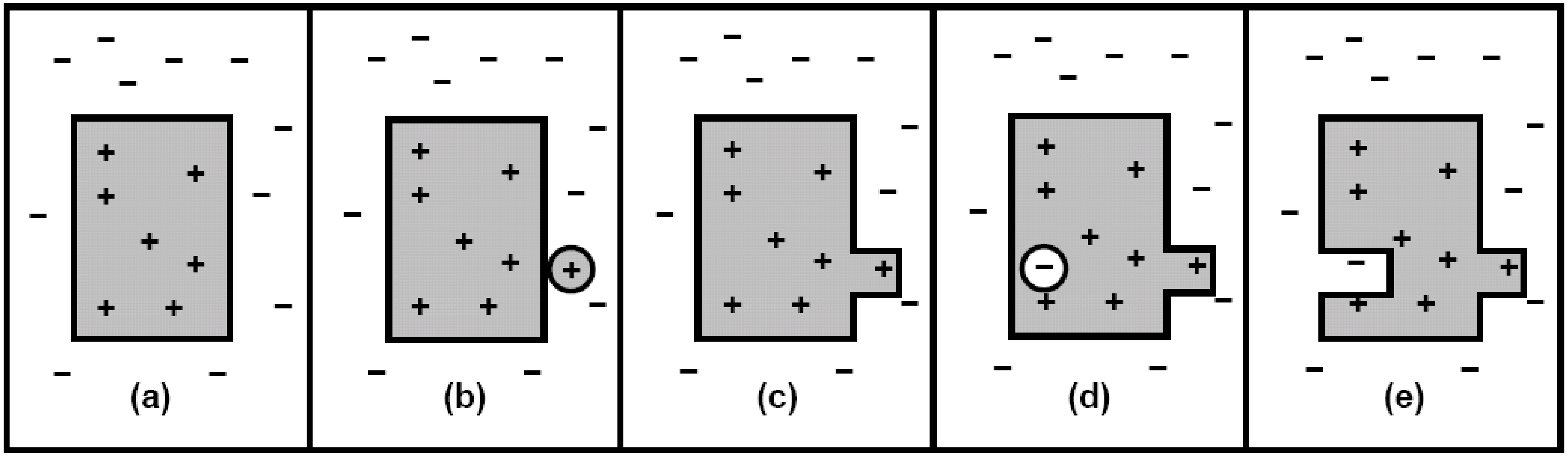


Fig. 18.10 (a) A consistent hypothesis, H (shaded area).
(b) A false negative.
(c) The hypothesis is generalized.
(d) A false positive.
(e) The hypothesis is specialized.

Learning General Logical Descriptions
-Current-best-hypothesis search

```
function Current-Best-Learning(examples) returns a hypothesis
    H ← any hypothesis consistent with the first example in examples
    for each remaining example, e in examples do
        if e is false positive for H then
            H ← choose a specialization of H consistent with examples   //all examples
        else if e is false negative for H then
            H ← choose a generalization of H consistent with examples
        if no consistent specialization/generalization can be found then fail
    end
    return H
```

Fig 18.11 The current-best-hypothesis learning algorithm.
It searches for a consistent hypothesis and backtracks when no consistent specialization/generalization can be found.

Learning General Logical Descriptions
-Current-best-hypothesis search

- Generalization & specialization are logical relationships between hypotheses. If hypothesis H_1, with definition C_1, is a generalization of H_2, with definition C_2, then we must have:

  $$\forall x\ C_2(x) \Rightarrow C_1(x)$$
  (generalization)

  To generalize H_2, find a definition C_1 that is logically implied by C_2.
- E.g. if $C_2(x)$ is alternate(x) $\wedge$ Patrons(x, Some), then possible generalization: $C_1(x) \equiv$ Patrons(x, Some), called dropping conditions, or add disjunctive (OR) conditions. (?)
- Specialization: add extra conditions to its candidate definition or remove disjuncts (OR) from a disjunctive definition (?)

Learning General Logical Descriptions
–Some examples from the restaurant example in Fig. 18.5

- Example X_1 is positive. Alternate(X_1) is true, so let us assume an initial hypothesis

$$H_1 : \forall x \text{ WillWait}(x) \Leftrightarrow \text{Alternate}(x)$$

- X_2 is negative. H_1 predicts it to be positive, so it is a false positive. Therefore, need to specialize H_1. Add an extra condition to rule out X_2. One possibility is

$$H_2 : \forall x \text{ WillWait}(x) \Leftrightarrow \text{Alternate}(x) \wedge \text{Patrons}(x, \text{Some})$$

Example	Attributes										Goal
	Alt	*Bar*	*Fri*	*Hun*	*Pat*	*Price*	*Rain*	*Res*	*Type*	*Est*	*WillWait*
X_1	*Yes*	*No*	*No*	*Yes*	*Some*	*\$\$\$*	*No*	*Yes*	*French*	*0–10*	*Yes*
X_2	*Yes*	*No*	*No*	*Yes*	*Full*	*\$*	*No*	*No*	*Thai*	*30–60*	*No*
X_3	*No*	*Yes*	*No*	*No*	*Some*	*\$*	*No*	*No*	*Burger*	*0–10*	*Yes*
X_4	*Yes*	*No*	*Yes*	*Yes*	*Full*	*\$*	*No*	*No*	*Thai*	*10–30*	*Yes*
X_5	*Yes*	*No*	*Yes*	*No*	*Full*	*\$\$\$*	*No*	*Yes*	*French*	*>60*	*No*
X_6	*No*	*Yes*	*No*	*Yes*	*Some*	*\$\$*	*Yes*	*Yes*	*Italian*	*0–10*	*Yes*
X_7	*No*	*Yes*	*No*	*No*	*None*	*\$*	*Yes*	*No*	*Burger*	*0–10*	*No*
X_8	*No*	*No*	*No*	*Yes*	*Some*	*\$\$*	*Yes*	*Yes*	*Thai*	*0–10*	*Yes*
X_9	*No*	*Yes*	*Yes*	*No*	*Full*	*\$*	*Yes*	*No*	*Burger*	*>60*	*No*
X_{10}	*Yes*	*Yes*	*Yes*	*Yes*	*Full*	*\$\$\$*	*No*	*Yes*	*Italian*	*10–30*	*No*
X_{11}	*No*	*No*	*No*	*No*	*None*	*\$*	*No*	*No*	*Thai*	*0–10*	*No*
X_{12}	*Yes*	*Yes*	*Yes*	*Yes*	*Full*	*\$*	*No*	*No*	*Burger*	*30–60*	*Yes*

(Why not Bar...Hun ?)

Learning General Logical Descriptions
–Some examples from the restaurant example in Fig. 18.5

- X_3 is positive. H_2 predicts it to be negative ⇒ false negative.
  - Therefore, need to generalize H_2.
  - This can be done by dropping the Alternate condition, yielding

    H_3: ∀x WillWait(x) ⇔ Patrons(x, Some)

Example	Attributes										Goal
	Alt	*Bar*	*Fri*	*Hun*	*Pat*	*Price*	*Rain*	*Res*	*Type*	*Est*	*WillWait*
X_1	*Yes*	*No*	*No*	*Yes*	*Some*	*\$\$\$*	*No*	*Yes*	*French*	*0–10*	*Yes*
X_2	*Yes*	*No*	*No*	*Yes*	*Full*	*\$*	*No*	*No*	*Thai*	*30–60*	*No*
X_3	*No*	*Yes*	*No*	*No*	*Some*	*\$*	*No*	*No*	*Burger*	*0–10*	*Yes*
X_4	*Yes*	*No*	*Yes*	*Yes*	*Full*	*\$*	*No*	*No*	*Thai*	*10–30*	*Yes*
X_5	*Yes*	*No*	*Yes*	*No*	*Full*	*\$\$\$*	*No*	*Yes*	*French*	*>60*	*No*
X_6	*No*	*Yes*	*No*	*Yes*	*Some*	*\$\$*	*Yes*	*Yes*	*Italian*	*0–10*	*Yes*
X_7	*No*	*Yes*	*No*	*No*	*None*	*\$*	*Yes*	*No*	*Burger*	*0–10*	*No*
X_8	*No*	*No*	*No*	*Yes*	*Some*	*\$\$*	*Yes*	*Yes*	*Thai*	*0–10*	*Yes*
X_9	*No*	*Yes*	*Yes*	*No*	*Full*	*\$*	*Yes*	*No*	*Burger*	*>60*	*No*
X_{10}	*Yes*	*Yes*	*Yes*	*Yes*	*Full*	*\$\$\$*	*No*	*Yes*	*Italian*	*10–30*	*No*
X_{11}	*No*	*No*	*No*	*No*	*None*	*\$*	*No*	*No*	*Thai*	*0–10*	*No*
X_{12}	*Yes*	*Yes*	*Yes*	*Yes*	*Full*	*\$*	*No*	*No*	*Burger*	*30–60*	*Yes*

- X_4 is posit … negative.
  - Therefo
  - We cann … yield an all-inclusiv
  - One pos

    H_4: ∀x WillWait(x) ⇔ Patrons(x, Some) ∨ (Patrons(x, Full) ∧ Fri/Sat(x))

Learning (

-Some examp … 18.5

Example	Attributes										Goal
	Alt	*Bar*	*Fri*	*Hun*	*Pat*	*Price*	*Rain*	*Res*	*Type*	*Est*	*WillWait*
X_1	*Yes*	*No*	*No*	*Yes*	*Some*	*$$$*	*No*	*Yes*	*French*	*0–10*	*Yes*
X_2	*Yes*	*No*	*No*	*Yes*	*Full*	*$*	*No*	*No*	*Thai*	*30–60*	*No*
X_3	*No*	*Yes*	*No*	*No*	*Some*	*$*	*No*	*No*	*Burger*	*0–10*	*Yes*
X_4	*Yes*	*No*	*Yes*	*Yes*	*Full*	*$*	*No*	*No*	*Thai*	*10–30*	*Yes*
X_5	*Yes*	*No*	*Yes*	*No*	*Full*	*$$$*	*No*	*Yes*	*French*	*>60*	*No*
X_6	*No*	*Yes*	*No*	*Yes*	*Some*	*$$*	*Yes*	*Yes*	*Italian*	*0–10*	*Yes*
X_7	*No*	*Yes*	*No*	*No*	*None*	*$*	*Yes*	*No*	*Burger*	*0–10*	*No*
X_8	*No*	*No*	*No*	*Yes*	*Some*	*$$*	*Yes*	*Yes*	*Thai*	*0–10*	*Yes*
X_9	*No*	*Yes*	*Yes*	*No*	*Full*	*$*	*Yes*	*No*	*Burger*	*>60*	*No*
X_{10}	*Yes*	*Yes*	*Yes*	*Yes*	*Full*	*$$$*	*No*	*Yes*	*Italian*	*10–30*	*No*
X_{11}	*No*	*No*	*No*	*No*	*None*	*$*	*No*	*No*	*Thai*	*0–10*	*No*
X_{12}	*Yes*	*Yes*	*Yes*	*Yes*	*Full*	*$*	*No*	*No*	*Burger*	*30–60*	*Yes*

- X_3 is positi … negative.
  - Therefor …
  - This can … n, yielding

    H_3: ∀x WillWait(x) ⇔ Patrons(x, Some)

- X_4 is positive, H_3 predicts it to be negative ⇒ false negative.
  - Therefore need to generalize H_3.
  - We cannot drop Patron condition, because it would yield an all-inclusive hypothesis that is inconsistent with X_2.
  - One possibility is to add a disjunct:

    H_4: ∀x WillWait(x) ⇔ Patrons(x, Some) ∨ (Patrons(x, Full) ∧ Fri/Sat(x))

Why Learning Works: Computational Learning Theory

- Answers provided by **computational learning theory**, the intersection of AI and theoretical computer science.
- The underlying principle: any hypothesis that is seriously wrong will almost certainly be "found out" with high probability after a small number of examples, because it will make an incorrect prediction.
- Thus, any hypothesis that is consistent with a sufficiently large set of training examples m is unlikely to be seriously wrong – i.e., it must be Probably Approximately Correct (PAC).
- We'll prove it and find m.
- PAC-learning is a subfield of computational learning theory.

Why Learning Works: Computational Learning Theory
– How many examples are needed?

- Let X be the set of all possible examples.
- Let D be the distribution from which examples are drawn.
- Let H be the set of possible hypotheses.
- Let m be the number of examples in the training set.

Initially, we'll assume that the true function f is a member of **H**.
Define the "**error** of a hypothesis h" w.r.t. the true function f given a distribution D over the examples as the probability P that h is different from f on an example x:

$$error(h) = \mathrm{P}\,(h(\mathrm{x}) \neq f(\mathrm{x}) \mid \mathrm{x} \text{ drawn from D})$$

(given that)

Why Learning Works: Computational Learning Theory
– How many examples are needed?

- A hypothesis h is called **approximately correct** if (probability) $error(h) \leq \epsilon$ (epsilon), where ϵ is a "small (+ve) constant".
- To show that after seeing m examples, with high probability, all consistent hypotheses will be approximately correct.
- Think of an approximately correct hypothesis as being "close" to the true function in hypothesis space – it lies inside what is called the **ϵ-ball** around the true function f.

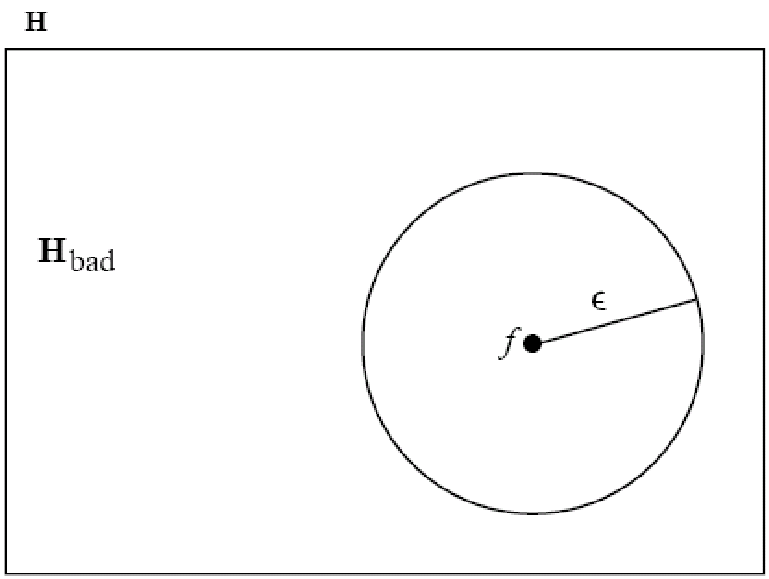
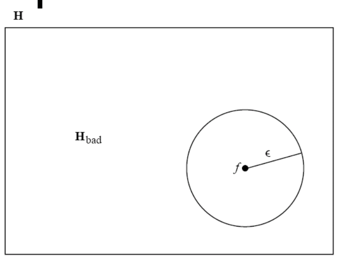


Fig 18.15 Schematic diagram of hypothesis space, showing the ε-ball around the true function f

Why Learning Works: Computational Learning Theory
– How many examples are needed?

- We can calculate the probability that a "seriously wrong" hypothesis $h_b \in H_{bad}$ is consistent with the first m examples as follows: Probability of h_b disagrees with an example:
  - $error(h_b) > \epsilon$. $(error(h) \leq \epsilon)$
  - Thus, the probability that h_b agrees with any given example is (at most) $\leq (1-\epsilon)$. $(> (1-\epsilon)$ for h)
  - The bound for m examples is



$$P(h_b \text{ agrees with m examples}) \leq (1-\epsilon)^m$$

- For $\mathbf{H_{bad}}$ to contain a consistent hypothesis, at least one of the hypotheses in $\mathbf{H_{bad}}$ must be consistent. The probability of this occurring is bounded by the sum of the individual probabilities:

$$P(\mathbf{H_{bad}} \text{ contains a consistent hypothesis}) \leq |\mathbf{H_{bad}}|(1-\epsilon)^m \leq |\mathbf{H}|(1-\epsilon)^m$$

where |H| is the total hypothesis space; |*|: size;

  - e.g. Boolean Function of n attributes: $|H| = 2^{2^n}$

Why Learning Works: Computational Learning Theory
– How many examples are needed?

- We would like to reduce the probability of this event to below some small number δ:

$$|H|(1-\epsilon)^m \leq \delta$$

- We can achieve this if we allow the algorithm to see

$$m \geq \frac{1}{\varepsilon}\left(\ln\frac{1}{\delta} + \ln|H|\right)$$

- Examples needed: (trend only)
  - –Thus, if a hypothesis is consistent with m *examples*, then with probability at least $1-\delta$, it has error at most ϵ. In other words, it is probably approximately correct (*PAC*).
  - –The number, **m**, of required examples, as a function of ϵ, δ, $|H|$, is called the **sample complexity** of the hypothesis space.
  - –E.g. If $|H| = 2^{2^n}$ for Boolean functions, sample complexity, m, grows with 2^n. n: # of attributes
  - –Smaller ϵ, δ and larger H apace $\Rightarrow$ higher m required

Sample Complexity grows with:
$|H| = 2^{2^n}$ for Boolean functions

Consider n=2: 2^{2^2} = 16 possible binary functions (**outputs**)

n = 2 attributes 2^4 = 16 possible functions defined by 16 different outputs

x_1	x_2	h_1	h_2	h_3	h_4	h_5	h_6	h_7	h_8	h_9	h_{10}	h_{11}	h_{12}	h_{13}	h_{14}	h_{15}	h_{16}
0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
0	1	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
1	0	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
1	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1

$2^n=2^2=4$