香 港 中 文 大 學
The Chinese University of Hong Kong

Course Examination 1st Term, 2020 – 2021

Course Code & Title : CSCI5350 Advanced Topics in Game Theory

Student I.D. No. : .... Seat No. : ....

This is a take-home examination.

- **Candidate should work on the paper he/ she receives at his/ her own CUHK email address (i.e., xxxxxxxxxx@link.cuhk.edu.hk), and submit the answers to Blackboard by <u>17:30 on 16 December 2020</u>.**

- **Candidate should write down his/ her student ID on the first page of the answers.**

- **Candidate should not communicate with other people (including other candidates) after the examination starts, until 17:30 on 16 December 2020.**

- **Candidate should include all his/ her answers <u>in one single file</u>. The file should be in PDF or JPG format.**

<u>Answer ALL Questions. The full mark of the paper is 140.</u>

1. Consider the Hawk-Dove game $G = \langle N, (A_i), (u_i) \rangle$ shown below, where $0 < c \leq 5$.

Player 2

		H	D
Player 1	H	$5-c, 5-c$	$10,0$
	D	$0,10$	$5,5$

(a) **(5 marks)** Is there a completely mixed Nash equilibrium of the game? Justify your answer and discuss whether and how the answer depends on the value of c.

(b) **(5 marks)** Is there a pure strategy Nash equilibrium of the game? Justify your answer and discuss whether and how the answer depends on the value of c.

(c) **(10 marks)** Consider a limit of means infinitely repeated game of the above Hawk-Dove game G with $c = 3$. Is it possible that there is a Nash equilibrium, the payoff profile of which is $(4,4)$? Justify your answer if this is not possible, or describe such a Nash equilibrium if it is possible.

2. Consider a group of six players playing a game together. Players 1 and 2 each is given a red card. Players 3, 4, 5 and 6 each is given a green card. There is a rule that players can form groups, such that a group receives a utility of 1 if the group members have one (1) red card and two (2) green cards, and, in general, an integer utility of n if they have n red cards and $2n$ green cards. Otherwise, the group will receive a utility of 0 (zero).

   The scenario can be formulated as a coalitional game with transferrable payoff $G = \langle N, v \rangle$.

   (a) **(5 marks)** Is the game G cohesive? Justify your answer.
   (b) **(10 marks)** What is the core of the game? Justify your answer.
   (c) **(10 marks)** Are there any nonempty stable sets of the game? Give one if there is any, or justify that there is none.
   (d) **(5 marks)** Calculate the Shapley value of the game.

3. One day three friends Amy, Betty and Catherine discuss to decide where they shall go for dinner together. After some discussions, they have two options left: they shall either have dinner at Pizza House, or Queenie's Kitchen. They agree that they shall go to the restaurant that the majority of them want to go. Moreover, they shall use the following procedure to decide which restaurant they shall go.

   Step 1. Amy (Player 1) will say whether she wants to eat at Pizza House (action *P*), or Queenie's Kitchen (action *Q*). She can even decide to cancel the dinner (action *S*).
   Step 2. If the dinner is not cancelled, Betty (Player 2) will then say whether she wants to eat at Pizza House (action *P*), or Queenie's Kitchen (action *Q*). Note that Betty (Player 2) cannot cancel the dinner.
   Step 3. If both Amy (Player 1) and Betty (Player 2) choose the same restaurant, then all three people will go to that restaurant, which is either Pizza House or Queenie's Kitchen. Otherwise, Catherine (Player 3) will say whether she wants to eat at Pizza House (action *P*), or Queenie's Kitchen (action *Q*). Obviously, since Amy (Player 1) and Betty (Player 2) choose different restaurants, Catherine (Player 3) will make the final decision. Note that Catherine (Player 3) cannot cancel the dinner.

   In fact, these three people have their individual preferences, which are represented by the utilities each of them will receive for different outcomes:

Player	Pizza House	Queenie's Kitchen	Dinner cancelled
Amy	1	2	0
Betty	1	1	0
Catherine	2	1	0

   The scenario can be formulated as an extensive game $\langle N, H, P, (\succsim_i) \rangle$ with perfect information.

   (a) **(4 marks)** List all the pure strategies available to Betty (Player 2) and Catherine (Player 3).
   (b) **(6 marks)** Describe all pure strategy subgame perfect equilibria of the game.

(c) **(5 marks)** Are there any subgame perfect equilibria in which at least one player plays a completely mixed strategy? Justify your answer and show one if there are.

(d) **(5 marks)** Is there a Nash equilibrium in which Amy decides to cancel the dinner?

(e) **(10 marks)** If your answer to Question (d) above is 'no,' justify your answer. Otherwise, discuss whether it is trembling hand perfect.

(f) **(4 marks)** Amy finds it very difficult for her to make a decision in Step 1, and comes to you, a game theory expert, for help. What advices would you give her?

After careful consideration, Amy thinks that the game is not fair, and proposes to change the procedures. Specifically, she proposes that in Step 1, Amy (Player 1) should not openly announce her preference, unless she decides to cancel the dinner. Likewise, Betty (Player 2) should not openly announce her preference in Step 2, either. Instead, Amy should write her choice on a piece of paper, and Betty should also write her choice on another piece of paper. Their choices are kept secret until after Catherine (Player 3) makes her choice. These people will then go to the restaurant chosen by the majority of them, unless Amy decides to cancel the dinner in Step 1.

The new scenario can be formulated as an extensive game $\langle N, H, P, f_c, (\mathcal{I}_i), (\succsim_i)\rangle$ with imperfect information.

(g) **(2 marks)** Show all the information set(s) at which Amy is the player.

(h) **(2 marks)** Show all the information set(s) at which Betty is the player.

(i) **(2 marks)** Show all the information set(s) at which Catherine is the player.

(j) **(6 marks)** Is this new extensive game a game with perfect recall? Justify your answer.

Consider an assessment $\langle \beta, \mu \rangle$, in which Amy does not choose action *S*, but she chooses action *P* with probably ¼ and action *Q* with probably ¾; Betty chooses actions *P* and *Q* with the same probably ½ whenever she needs to make decisions. We assume that Catherine is very wise and she always makes the choice that is the best for her.

(k) **(6 marks)** Find a belief system μ so that the assessment $\langle \beta, \mu \rangle$ is consistent.

(l) **(8 marks)** Is the assessment $\langle \beta, \mu \rangle$ in Question (k) sequentially rational? Justify your answer. Hence, or otherwise, find a sequential equilibrium.

4. Consider a print server with one faster laser printer of lower printing quality, and one slower laser printer of higher printing quality. The print server works in a batch mode: within each episode of 10 minutes, it accepts print jobs from different computers, and sends all the collected print jobs to the printers at the end of the 10-minute episode. After that the next episode starts: the print jobs sent to the printers are printed, while the print server receives new print jobs, which are sent to the printers at the end of the episode.

   A print job p represents its preference to be printed using the faster (lower-quality) laser printer and the slower (higher-quality) laser printer by specifying four values. First, the owner process of the print job receives the utility $u_f(p)$ if the print job p is printed using the faster laser printer, or the utility $u_s(p)$ if the print job p is printed using the slower laser printer. It is always true that $u_s(p) > u_f(p) \geq 0$. Second, the time needed for the print job p to be printed on the faster (lower-quality) laser printer is $\tau_f(p)$ and the time needed for the print job p to be printed on the slower (higher-quality) laser printer $\tau_s(p)$. It is always true that $\tau_s(p) > \tau_f(p) \geq 0$.

Now consider an episode in which the following print jobs are collected, which will be sent to the printers:

Print Job	u_s	u_f	τ_s	τ_f
1	20	2	3	2
2	1	0	5	4
3	10	5	2	1
4	5	3	3	1
5	10	5	7	5
6	10	2	4	3

For example, the owner process of print job 1 will receive a utility of 10 if print job 1 is sent to the slower (higher-quality) laser printer, or a utility of 5 if print job 1 is sent to the faster (lower-quality) laser printer. It takes the slower printer 2 minutes to finish the print job, but only 1 minute for the faster printer to finish it.

A *print schedule* $S = \langle P_s, P_f \rangle$ is a partition of the set P of all print jobs into two disjoint sets P_s and P_f, such that $P_s \cup P_f = P$ and $P_s \cap P_f = \emptyset$. (In the episode being considered, $P = \{1,2,3,4,5,6\}$.) A *feasible print schedule* is one in which $\sum_{p \in P_s} \tau_s(p) \leq 10$ and $\sum_{p \in P_f} \tau_f(p) \leq 10$, because all print jobs must be completed before the next episode starts. Let $\mathbb{S}$ denote the set of all feasible print schedules.

(a) **(10 marks)** Show what $\mathbb{S}$ is for the episode being considered.

(b) **(10 marks)** The print server uses the Clarke Tax method to find the best feasible print schedule $S \in \mathbb{S}$ that maximises the total utility to be received by the owner processes of the print jobs. What is the best feasible print schedule $S \in \mathbb{S}$ found for the episode being considered? What are the amounts of tax to be paid by the owner processes of each of the print jobs?

(c) **(10 marks)** Suppose the owner process of a print job p can strategically declare $\hat{u}_s(p)$ and $\hat{u}_f(p)$ as the values of $u_s(p)$ and $u_f(p)$. Is it possible that two owner processes of print jobs beneficially collude by declaring untrue values in the above case? Elaborate your answer.

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