Solution 1

Deadline: April 15

1 Answer 1 (20%)

- **Q1:** (6%) Use the *adjacency matrix* to describe this graph.

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix}.$$

- **Q2:** (6%) List in-degree and out-degree of each node.

$$\mathbf{I}_d = \begin{bmatrix} 0 & 1 & 1 & 3 & 2 & 2 & 2 & 1 & undef/0/1 & 0 \end{bmatrix},$$

$$\mathbf{O}_d = \begin{bmatrix} 3 & 1 & 2 & 1 & 1 & 0 & 2 & 1 & 0 & 2 \end{bmatrix}.$$

- **Q3:** (8%) List all *simple paths* from node A to node F.

$$\{A-B-D-E-F; A-C-E-F; A-C-D-E-F; A-D-E-F;\}$$

2 Answer 2 (20%)

- **Q1:** (5%) Use the *adjacency matrix* to describe this undirected graph.

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix}.$$

- **Q2:** (5%) Compute the cluster coefficient of each node.

$$\mathbf{CLC} = \begin{bmatrix} \frac{2}{3} & 1 & \frac{2}{3} & \frac{1}{2} & \frac{1}{3} & 0 & \frac{1}{6} & 1 & 1 & 1 \end{bmatrix}.$$

- **Q3:** (5%) Find out all bridges and local bridges in this graph

$$\textbf{Bridges} : \{E - F, F - G, G - I\},$$

$$\textbf{Local Bridges} : \{E - F, F - G, G - I\},$$

- **Q4:** (5%) According to the distribution of normal users or malicious users in the graph, measure the homophily of the graph by normal-normal, normal-malicious, malicious-malicious. And figure out if there is evidence of homophily in this graph.

  For fraction of normal user we have $p = 0.7$. For fraction of malicious user we have $q = 0.3$. Then we have:

  $Pr(\text{normal-normal}) = p^2 = 0.49$,

  $Pr(\text{malicious-malicious}) = q^2 = 0.09$,

  $Pr(\text{normal-malicious}) = 2pq = 0.42$.

  Since 5 out of 13 edges are cross node type, and $\frac{5}{13} = 0.38 \approxeq 0.42$. No evidence of homophily.

3 Answer 3 (20%)

- **Q1:** (6%) Is the graph in Figure 3 structurally balanced?

  Yes, it is structurally balanced.

- **Q2:** (7%) Add another node F and build either positive or negative connections with existing five nodes (i.e., A, B, C, D and E), so that the new network satisfies **Structural Balance Property**.

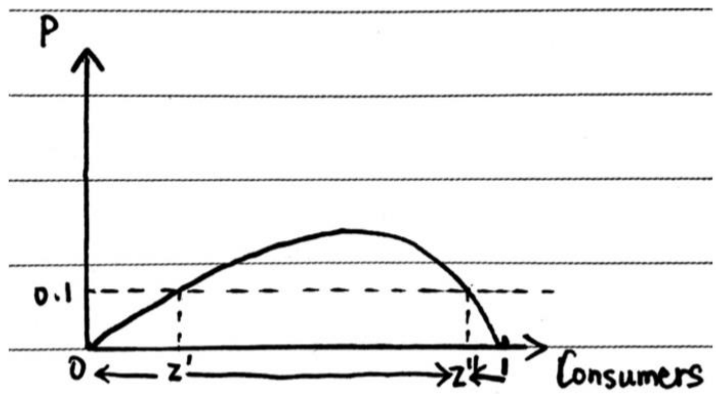
  Build positive connections with nodes B and E. Build negative connections with nodes A, C and D. (There are multiple solutions.)

- **Q3:** (7%) Add another node F and build either positive or negative connections with existing five nodes (i.e., A, B, C, D and E), so that the new network *only* satisfies **Weak Structural Balance Property** but *does not* satisfy **Structural Balance Property**.

  Build negative connections with all existing nodes (i.e., A, B, C, D and E). (There are multiple solutions.)

4 Answer 4 (20%)

- **Q1:** (6%) Suppose p represents the reservation price. Plot the reservation price with the change of fraction of customers z. And find the corresponding fraction of customers when $p = 0.1$.



  $p(z) = \frac{z(1-z^2)}{2}$
  $z' = 0.209$ and $z'' = 0.879$.

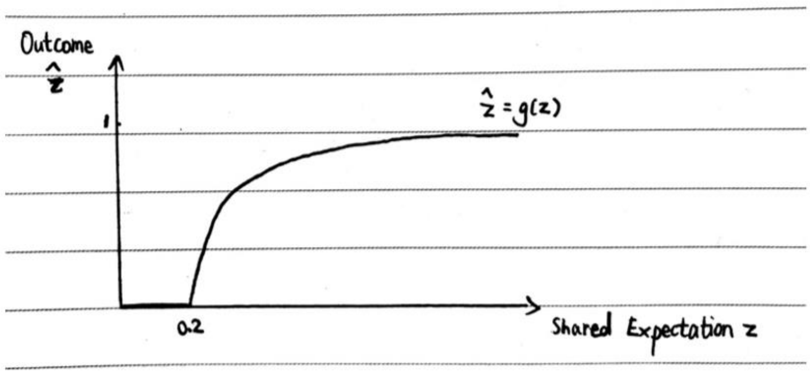
- **Q2:** (7%) Highlight the downward and upward pressure regions for the reservation price plot. And explain why there is downward and upward pressure in these regions.

  1) $0 < z < z'$ $r(z)f(z) < p^*$, the purchaser named z (and other purchasers just below z) will value the good at less than p^*, and hence will wish they had not bought it. – downward pressure on the consumption of the good.

  2) $z' < z < z''$ $r(z)f(z) > p^*$, consumers with names slightly above z have not purchased the good but will wish they had. – upward pressure on the consumption of the good.

  3) $z'' < z < 1$ $r(z)f(z) < p^*$, the purchaser z and other just below will wish they had not bought the good. – downward pressure on the consumption of the good.

- **Q3:** (7%) Now consider the dynamic case of economy. Set the current price $p^* = 0.1$. Please plot the fraction of population who buy the product versus the expected fraction of population who will use the product.



$r(x) = 1 - x^2$ and $f(z) = \frac{z}{2}$

$r^{-1}(x) = \sqrt{1-x}$

$\hat{z} = g(z) = r^{-1}(\frac{p^*}{f(z)}) = \sqrt{1 - \frac{2p^*}{z}}$

$p^* \leq r(0)f(z) \rightarrow p^* \leq \frac{z}{2}$

If $z \geq 2p^* = 0.2$, $\hat{z} = g(z) = r^{-1}(\frac{p^*}{f(z)}) = \sqrt{1 - \frac{2p^*}{z}} = \sqrt{1 - \frac{0.2}{z}}$

Otherwise, $\hat{z} = g(z) = 0$

5 Answer 5 (20%)

- **Q1:** (7%) What is the probability that the first person to decide will choose **Accept**? What is the probability that this person will choose **Reject**?

  The probability that the first person chooses **Accept**, is equal to the probability she sees a high signal. Since the true state is **Good**, this probability is $3/4$. Similarly, the probability of **Reject** is $1/4$.

- **Q2:** (6%) What is the probability of observing each of the four possible pairs of choices by the first two people: $(A, A), (A, R), (R, A)$ and (R, R)? [A pair of choices such as (A, R) means that the first person chose **Accept** and the second person chose **Reject**.]

  Note that the decision of the second player depends only on her own signal. So we can write:

  $Pr(A, A) = 3/4 * 3/4 = 9/16$

  $Pr(A, R) = 3/4 * 1/4 = 3/16$

  $Pr(R, A) = 1/4 * 3/4 = 3/16$

  $Pr(R, R) = 1/4 * 1/4 = 1/16$

- **Q3:** (7%) What is the probability of an **Accept** or a **Reject** cascade emerging with the decision by the third person to choose? Explain why a cascade emerges with this probability.

  A cascade happens when the first and second person both choose the same decision. So the probability of a cascade is

  $Pr(A, A) + Pr(R, R) = 10/16$