

Problem 1

Q1.

The prior of C_1 is 0.34125, C_2 is 0.33833 and C_3 is 0.32042.

```
total_num = len(input1_train)
pC1 = len(classes[1])/total_num
pC2 = len(classes[2])/total_num
pC3 = len(classes[3])/total_num
pCi = [pC1, pC2, pC3]
print("pCi value: ",pCi)
```

pCi value: [0.34125, 0.3383333333333333, 0.3204166666666667]

Q2.

$$p(x=1|C1) = p_1^x \cdot (1 - p_1)^{1-x}$$

$$= p_1$$

$$= 0.34432$$

$$p(x=1|C2) = p_2^x \cdot (1 - p_2)^{1-x}$$

$$= p_2$$

$$= 0.61207$$

$$p(x=1|C3) = p_3^x \cdot (1 - p_3)^{1-x}$$

$$= p_3$$

$$= 0.40182$$

```
p1 = (classes[1]['feature_value']==1).sum()/len(classes[1])
p2 = (classes[2]['feature_value']==1).sum()/len(classes[2])
p3 = (classes[3]['feature_value']==1).sum()/len(classes[3])
pi = [p1, p2, p3]
print("pi value: ",pi)
```

pi value: [0.3443223443223443, 0.6120689655172413, 0.40182054616384916]

Q3.

My discriminant function is $\log p_i^x + \log(1 - p_i)^{1-x} + \log P(C_i)$ for prior $P(C_i)$ and the log likelihood $\log p_i^x + \log(1 - p_i)^{1-x}$.

```
def confusion_matrix():
    #pirj = predicted result i and actual result j
    plr1, plr2, plr3, p2r1, p2r2, p2r3, p3r1, p3r2, p3r3 = 0,0,0,0,0,0,0,0,0
    for i in range(len(y_test)):
        if(y_test.iloc[i]==1):
            if(result[i]==1): plr1+=1
            elif(result[i]==2): p2r1+=1
            else: p3r1+=1
        elif(y_test.iloc[i]==2):
            if(result[i]==1): plr2+=1
            elif(result[i]==2): p2r2+=1
            else: p3r2+=1
        else:
            if(result[i]==1): plr3+=1
            elif(result[i]==2): p2r3+=1
            else: p3r3+=1
    print('\t\t\033[96mpredict\033[0;0m\t')
    print('\t\t\033[96m1\t2\t3\033[0;0m')
    print('\t\033[96m1\033[0;0m\t'+'\033[1m'+str(plr1)+'\t'+str(p2r1)+'\t'+str(p3r1)+'\033[0;0m')
    print('\033[96mactual\t2\033[0;0m\t'+'\033[1m'+str(plr2)+'\t'+str(p2r2)+'\t'+str(p3r2)+'\033[0;0m')
    print('\t\033[96m3\033[0;0m\t'+'\033[1m'+str(plr3)+'\t'+str(p2r3)+'\t'+str(p3r3)+'\033[0;0m')
    return "\033[96m-----confusion_matrix-----"
print(confusion_matrix())
print("acc: ",(y_test==result).sum()/len(y_test))
```

		predict		
		1	2	3
actual	1	135	70	0
	2	70	108	0
	3	148	69	0

-----confusion_matrix-----
acc: 0.405

Confusion matrix is as above and along the row is the number of instances in predicted class i while along the column is the the number of instances in actual class i for $1 \leq i \leq 3$.

Q4. accuracy = 40.5%

As precision = TP/TP+FP,

$$\text{precision for class 1} = 135 / (135+70+148) \\ = 0.38244$$

$$\text{precision for class 2} = 108 / (70+108+69) \\ = 0.43725$$

$$\text{precision for class 3} = 0 / 0 \\ = \text{undetermined} / 0$$

As recall = TP/TP+FN,

$$\text{recall for class 1} = 135 / (135+70+0) \\ = 0.65854$$

$$\text{recall for class 2} = 108 / (70+108+0) \\ = 0.60674$$

$$\text{recall for class 3} = 0 / 148+69+0 \\ = 0$$

As F1 Score = $2 * (\text{Recall} * \text{Precision}) / (\text{Recall} + \text{Precision})$,

$$\text{F1 score for class 1} = 2 * (0.65854 * 0.38244) / (0.65854 + 0.38244) \\ = 0.48387$$

$$\text{F1 score for class 2} = 2 * (0.60674 * 0.43725) / (0.60674 + 0.43725) \\ = 0.50824$$

$$\text{F1 score for class 3} = \text{undetermined} / 0$$

Problem2

Q1.

The prior of C_1 is 0.34208, C_2 is 0.32792 and C_3 is 0.33.

```
#Calc P(Ci)
pC1 = len(classes[1])/len(input2_train)
pC2 = len(classes[2])/len(input2_train)
pC3 = len(classes[3])/len(input2_train)
pCi = [pC1, pC2, pC3]
print("pCi value: ",pCi)
```

```
pCi value: [0.34208333333333335, 0.32791666666666667, 0.33]
```

Q2.

$m_1 = -0.02506$; $m_2 = 3.04947$; $m_3 = -2.85995$;

```
#calc mean_i
mean1 = classes[1]['feature_value'].mean()
mean2 = classes[2]['feature_value'].mean()
mean3 = classes[3]['feature_value'].mean()
mean_list = [mean1, mean2, mean3]
print("mean value: ", mean_list)
```

```
mean value: [-0.025056430115166065, 3.0494696695961974, -2.859953974797454]
```

$\sigma_1^2 = 1.06049$; $\sigma_2^2 = 3.92569$; $\sigma_3^2 = 9.70644$;

```
#calc sd (not using std as it calc (n/n-1)*s^2)
sd1 = np.sqrt(((classes[1]['feature_value']-mean1)**2).sum()/len(classes[1]))
sd2 = np.sqrt(((classes[2]['feature_value']-mean2)**2).sum()/len(classes[2]))
sd3 = np.sqrt(((classes[3]['feature_value']-mean3)**2).sum()/len(classes[3]))
sd_list = [sd1, sd2, sd3]
print("variance value: ", list(map(lambda x: x**2, sd_list)))
```

```
variance value: [1.0604933047619924, 3.9256875493805583, 9.706441924363155]
```

Q3.

My discriminant function is $\log p_i^x + \log(1 - p_i)^{1-x} + \log P(C_i)$ for prior $P(C_i)$ and the log likelihood $\log p_i^x + \log(1 - p_i)^{1-x}$.

```
def confusion_matrix():
    #pirj = predicted result i and actual result j
    plr1, plr2, plr3, p2r1, p2r2, p2r3, p3r1, p3r2, p3r3 = 0,0,0,0,0,0,0,0,0
    for i in range(len(y2_test)):
        if(y2_test.iloc[i]==1):
            if(result[i]==1): plr1+=1
            elif(result[i]==2): p2r1+=1
            else: p3r1+=1
        elif(y2_test.iloc[i]==2):
            if(result[i]==1): plr2+=1
            elif(result[i]==2): p2r2+=1
            else: p3r2+=1
        else:
            if(result[i]==1): plr3+=1
            elif(result[i]==2): p2r3+=1
            else: p3r3+=1
    print('\t\t\t\033[96mpredict\033[0;0m\t')
    print('\t\t\t\033[96m1\t2\t3\033[0;0m')
    print('\t\t\033[96m1\033[0;0m\t'+'\033[1m'+str(plr1)+'\t'+str(p2r1)+'\t'+str(p3r1)+'\033[0;0m')
    print('\033[96mactual\t2\033[0;0m\t'+'\033[1m'+str(plr2)+'\t'+str(p2r2)+'\t'+str(p3r2)+'\033[0;0m')
    print('\t\t\033[96m3\033[0;0m\t'+'\033[1m'+str(plr3)+'\t'+str(p2r3)+'\t'+str(p3r3)+'\033[0;0m')
    return "\033[96m-----confusion_matrix-----"
print(confusion_matrix())
print("acc: ",(y2_test==result).sum()/len(y2_test))
```

		predict		
		1	2	3
actual	1	194	11	15
	2	42	162	2
	3	37	13	124

-----confusion_matrix-----
acc: 0.8

Confusion matrix is as above and along the row is the number of instances in predicted class i while along the column is the the number of instances in actual class i for $1 \leq i \leq 3$.

Q4.

Accuracy = 80%

As precision = $TP / (TP + FP)$,

$$\text{precision for class 1} = 194 / (194 + 42 + 27) = 0.73764$$

$$\text{precision for class 2} = 162 / (11 + 162 + 13) = 0.87097$$

$$\text{precision for class 3} = 124 / (15 + 2 + 124) = 0.87943$$

As recall = $TP / (TP + FN)$,

$$\text{recall for class 1} = 194 / (194 + 11 + 15) = 0.88182$$

$$\text{recall for class 2} = 162 / (42 + 162 + 2) = 0.78641$$

$$\text{recall for class 3} = 124 / (37 + 13 + 124) = 0.71264$$

As F1 Score = $2 * (\text{Recall} * \text{Precision}) / (\text{Recall} + \text{Precision})$,

$$\text{F1 score for class 1} = 2 * (0.88182 * 0.73764) / (0.88182 + 0.73764) = 0.80331$$

$$\begin{aligned} \text{F1 score for class 2} &= 2 * (0.78641 * 0.87097) / (0.78641 + 0.87097) \\ &= 0.82653 \end{aligned}$$

$$\begin{aligned} \text{F1 score for class 3} &= 2 * (0.71264 * 0.87943) / (0.71264 + 0.87943) \\ &= 0.78730 \end{aligned}$$

Appendix:

```
#visualize but the class size are too similar and only single feature with 2 possible value
base = len(classes[1]) + len(classes[2]) + len(classes[3])
plt.scatter(classes[1]['feature_value'], [1]*len(classes[1]), color='r', \
            s=100, alpha=0.02)
plt.scatter(classes[2]['feature_value'], [1]*len(classes[2]), color='b', \
            s=100, alpha=0.02)
plt.scatter(classes[3]['feature_value'], [1]*len(classes[3]), color='g', \
            s=100, alpha=0.02)
plt.legend(["class 1", "class 2", "class 3"])
plt.show()
```

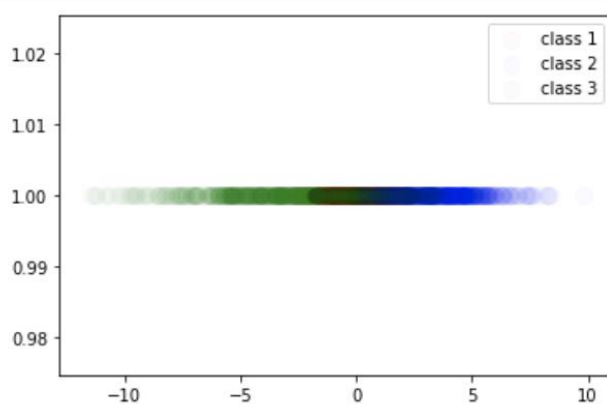


Fig.1. Visualization of Feature Distribution

```
#separate layers
base = len(classes[1]) + len(classes[2]) + len(classes[3])
plt.scatter(classes[1]['feature_value'], classes[1]['class'], color='r', \
            s=len(classes[1])/(base)*100)
plt.scatter(classes[2]['feature_value'], classes[2]['class'], color='b', \
            s=len(classes[2])/(base)*100)
plt.scatter(classes[3]['feature_value'], classes[3]['class'], color='g', \
            s=len(classes[3])/(base)*100)
plt.legend(["class 1", "class 2", "class 3"])
plt.show()
```

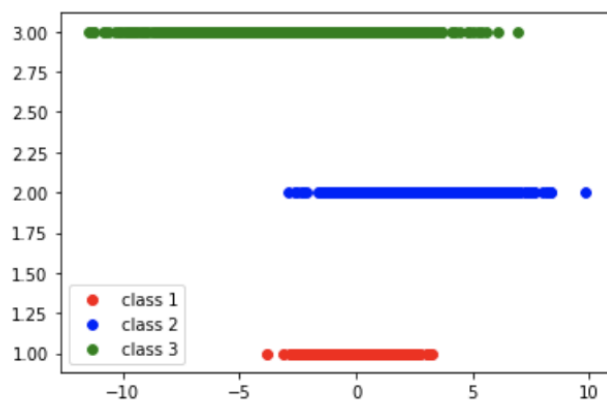


Fig.2. Visualization of Feature Distribution in Separated Layers

```
#visualization of result
#p(x/Ci)
input_pt = np.linspace(-10,10, num = 1000)
plt.plot(input_pt, stats.norm.pdf(input_pt,mean1,sd1)*pCi[0], 'r',\
         input_pt, stats.norm.pdf(input_pt,mean2,sd2)*pCi[1], 'g',\
         input_pt, stats.norm.pdf(input_pt,mean3,sd3)*pCi[2], 'b')
plt.xlim(-10,10)
plt.xlabel('x value')
plt.legend(['class1', 'class2', 'class3'])
plt.title('Approximate p(Ci|x) Distribution')
plt.show()
```

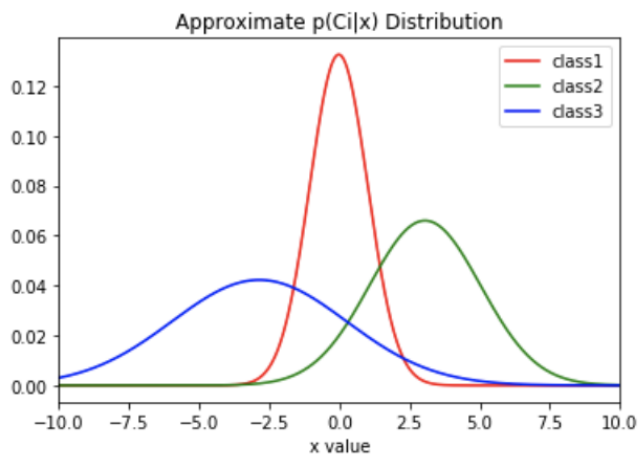


Fig.3. Visualization of Posterior Probability

```
input_pt = np.linspace(-20,20, num = 1000)
plt.plot(input_pt, g(input_pt,1), 'r',\
         input_pt, g(input_pt,2), 'g',\
         input_pt, g(input_pt,3), 'b')
plt.ylim(-20,10)
plt.xlim(-20,20)
plt.xlabel('x value')
plt.legend(['class1', 'class2', 'class3'])
plt.title('Discrimination Function')
plt.show()
```

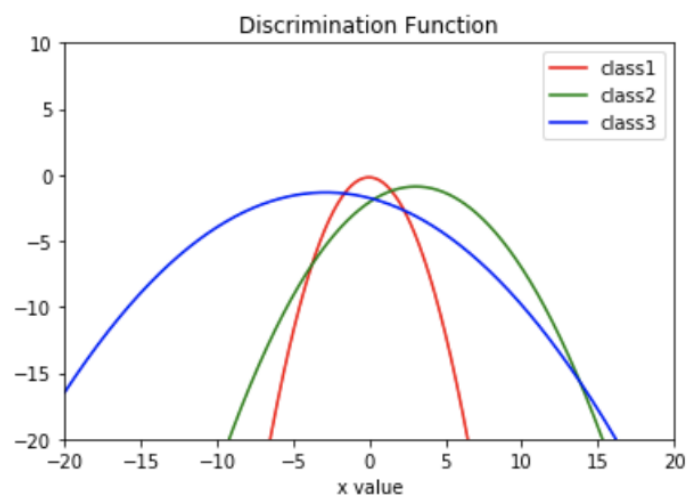


Fig.4. Visualization of Discrimination Function

Problem 3

Q1.

The prior of C_1 is 0.34125, C_2 is 0.33833 and C_3 is 0.32042.

```
pC1 = len(classes[1])/len(input3_train)
pC2 = len(classes[2])/len(input3_train)
pC3 = len(classes[3])/len(input3_train)
pCi = [pC1, pC2, pC3]
print("pCi value: ", pCi)
```

```
pCi value: [0.32916666666666666, 0.3333333333333333, 0.3375]
```

Q2.

$p_1 = 0.1$; $p_2 = 0.515$; $p_3 = 0.77160$;

```
#calc pi|
p1 = (classes[1]['feature_value_1']==1).sum()/len(classes[1])
p2 = (classes[2]['feature_value_1']==1).sum()/len(classes[2])
p3 = (classes[3]['feature_value_1']==1).sum()/len(classes[3])
pi = [p1, p2, p3]
print("pi value: ", pi)
```

```
pi value: [0.1, 0.515, 0.7716049382716049]
```

$m_1 = 1.02467$; $m_2 = 4.96953$; $m_3 = 9.88860$;

```
#calc mean_i
mean1 = classes[1]['feature_value_2'].mean()
mean2 = classes[2]['feature_value_2'].mean()
mean3 = classes[3]['feature_value_2'].mean()
mean_list = [mean1, mean2, mean3]
print("mean value: ", mean_list)
```

```
mean value: [1.0246692865939961, 4.969530068401736, 9.888599469860715]
```

$\sigma_1^2 = 0.24928$; $\sigma_2^2 = 6.96917$; $\sigma_3^2 = 20.61892$;

```
#calc sd
sd1 = np.sqrt(((classes[1]['feature_value_2']-mean1)**2).sum()/len(classes[1]))
sd2 = np.sqrt(((classes[2]['feature_value_2']-mean2)**2).sum()/len(classes[2]))
sd3 = np.sqrt(((classes[3]['feature_value_2']-mean3)**2).sum()/len(classes[3]))
sd_list = [sd1, sd2, sd3]
print("variance value: ", list(map(lambda x: x**2, sd_list)))
```

```
variance value: [0.2492844106885291, 6.969167050348478, 20.618919124235582]
```


Q3.

My discriminant function is $\log p_i^x + \log(1 - p_i)^{1-x} + \log P(C_i)$ for prior $P(C_i)$ and the log likelihood $\log p_i^x + \log(1 - p_i)^{1-x}$.

```
def confusion_matrix():
    #pirj = predicted result i and actual result j
    plr1, plr2, plr3, p2r1, p2r2, p2r3, p3r1, p3r2, p3r3 = 0,0,0,0,0,0,0,0,0
    for i in range(len(y3_test)):
        if(y3_test.iloc[i]==1):
            if(result[i]==1): plr1+=1
            elif(result[i]==2): p2r1+=1
            else: p3r1+=1
        elif(y3_test.iloc[i]==2):
            if(result[i]==1): plr2+=1
            elif(result[i]==2): p2r2+=1
            else: p3r2+=1
        else:
            if(result[i]==1): plr3+=1
            elif(result[i]==2): p2r3+=1
            else: p3r3+=1
    print('\t\t\033[96mpredict\033[0;0m\t')
    print('\t\t\033[96m1\t2\t3\033[0;0m')
    print('\t\033[96m1\033[0;0m\t'+'\033[1m'+str(plr1)+'\t'+str(p2r1)+'\t'+str(p3r1)+'\033[0;0m')
    print('\t\033[96m2\033[0;0m\t'+'\033[1m'+str(plr2)+'\t'+str(p2r2)+'\t'+str(p3r2)+'\033[0;0m')
    print('\t\033[96m3\033[0;0m\t'+'\033[1m'+str(plr3)+'\t'+str(p2r3)+'\t'+str(p3r3)+'\033[0;0m')
    return "\033[96m-----confusion_matrix-----"
print(confusion_matrix())
print("acc: ", (y3_test==result).sum()/len(y3_test))
```

```

      predict
      1      2      3
actual 1      204      9      0
      2      15      144      27
      3       4      50      147
-----confusion_matrix-----
acc:  0.825
```

Confusion matrix is as above and along the row is the number of instances in predicted class i while along the column is the the number of instances in actual class i for $1 \leq i \leq 3$.

Q4. accuracy = 82.5%

As precision = TP/TP+FP,

$$\text{precision for class 1} = 204 / (204 + 15 + 4) = 0.91480$$

$$\text{precision for class 2} = 144 / (9 + 144 + 50) = 0.70936$$

$$\text{precision for class 3} = 147 / (0 + 27 + 147) = 0.84483$$

As recall = TP/TP+FN,

$$\text{recall for class 1} = 204 / (204 + 9 + 0) = 0.95775$$

$$\text{recall for class 2} = 144 / (15 + 144 + 27) = 0.77419$$

$$\text{recall for class 3} = 147 / (4 + 50 + 147) = 0.73134$$

As F1 Score = $2 * (\text{Recall} * \text{Precision}) / (\text{Recall} + \text{Precision})$,

$$\text{F1 score for class 1} = 2 * (0.95775 * 0.91480) / (0.95775 + 0.91480) = 0.93578$$

$$\begin{aligned} \text{F1 score for class 2} &= 2 * (0.77419 * 0.70936) / (0.77419 + 0.70936) \\ &= 0.74036 \end{aligned}$$

$$\begin{aligned} \text{F1 score for class 3} &= 2 * (0.73134 * 0.84483) / (0.73134 + 0.84483) \\ &= 0.78400 \end{aligned}$$

Appendix:

```
#visualize
from mpl_toolkits.mplot3d import Axes3D
base = len(classes[1]) + len(classes[2]) + len(classes[3])

#           s=len(classes[2])/(base)*100
plt.scatter(classes[1]['feature_value_1'], classes[1]['feature_value_2'],\
            color='r', alpha=0.02, s=100)
plt.scatter(classes[2]['feature_value_1'], classes[2]['feature_value_2'],\
            color='b', alpha=0.02, s=100)
plt.scatter(classes[3]['feature_value_1'], classes[3]['feature_value_2'],\
            color='g', alpha=0.02, s=100)

plt.legend(["class 1", "class 2", "class 3"])
plt.show()
```

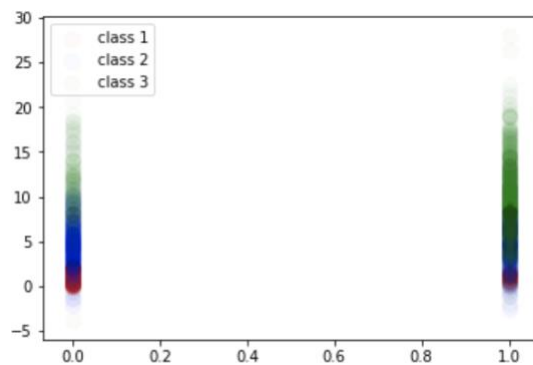


Fig.5. Visualization of Feature Distribution

```
#visualize but the class size are too similar and only single feature with 2 possible value
from mpl_toolkits.mplot3d import Axes3D
base = len(classes[1]) + len(classes[2]) + len(classes[3])
fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')
ax.scatter(classes[1]['feature_value_1'], classes[1]['feature_value_2'],\
          classes[1]['class'], color='r', alpha=0.3)
ax.scatter(classes[2]['feature_value_1'], classes[2]['feature_value_2'],\
          classes[2]['class'], color='b')
ax.scatter(classes[3]['feature_value_1'], classes[3]['feature_value_2'],\
          classes[3]['class'], color='g')
ax.legend(["class 1", "class 2", "class 3"])
ax.set_xlabel('X Label')
ax.set_ylabel('Y Label')
ax.set_zlabel('Z Label')
plt.show()
```

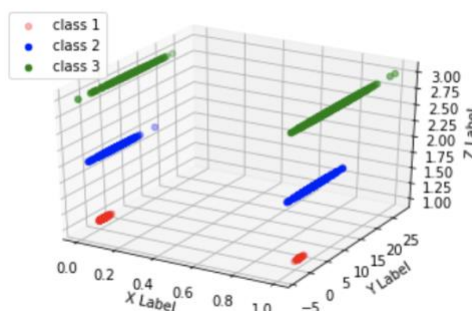


Fig.6. Visualization of Feature Distribution in Separated Layers