ENGG 1130 Multivariable Calculus for Engineers

Assignment 3 (Term 2, 2019-2020)

Assigned Date: 16 Feb 2020 (Sunday) 10:00 am

Deadline: **28 Feb 2020 (Friday) 12 noon**

- Show **ALL** your steps and details for each question unless otherwise specified.
- Make a copy of your homework before submitting the original!
- Please submit the **soft copy of your HW 3, TOGETHER WITH THE "DECLARATION FORM" to Blackboard system** on or before the prescribed deadline.
- Feel free to discuss with your friends, but make sure you all present your answers in different manners. **NO** citation (reference) is needed if only discussion takes place.

Notation: $\langle a, b, c \rangle$ represents the vector $a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$.

Full score: 100

1. **(20 marks)** Find the length of the following curve with specified interval.

   (a) $\mathbf{r}(t) = \langle 1, 1, 1 \rangle$ over the interval $0 \leq t \leq 4$.

   (b) $\mathbf{r}(t) = \langle t, 2t, 3t \rangle$ over the interval $0 \leq t \leq 2$.

2. **(10 marks)** Evaluate the following definite integral.

$$\int_0^3 (t\mathbf{i} + t\mathbf{j} + t\mathbf{k})\,dt$$

3. **(10 marks)** Find $\mathbf{r}(t)$ that satisfies the following equation, as well as the initial condition.

$$\mathbf{r}'(t) = \langle 1, 1, 1 \rangle, \ \ \mathbf{r}(0) = \langle 1,3,10 \rangle$$

4. **(15 marks)** Given that the position vector of a particle moving through space is given by

$$\mathbf{r}(t) = \langle t, t, t \rangle, t > 0$$

   Find the velocity vector, speed and acceleration respectively.

5. **(20 marks)** Find the domain and the range of the following function. Show your steps.

   (a) $f(x, y) = \sqrt{1 - x^2 - y^2}$

   (b) $g(x, y) = \ln(x^2 + y^2 - 1)$

6. **(15 marks)** Evaluate the following limits if they exist. Show **ALL** your explanation clearly.

   If limit does **NOT** exist, **explain clearly why**.

   Note: Marks are **NOT** evenly distributed. You **DON'T need** to show epsilon-delta arguments.

   (a) $\displaystyle\lim_{(x,y)\to(1,2)}(1 + x + y)$

   (b) $\displaystyle\lim_{(x,y)\to(\pi,4)}\tan\left(\frac{x}{y}\right)$

   (c) $\displaystyle\lim_{(x,y)\to(0,0)}\frac{y(x-y)}{x+y^2}$

7. **(10 marks)** Let L be the level curve of $f(x,y) = \sqrt{x - y - 3} - x$ that passes through the point $(3, -1)$.

   Find L and sketch it with <u>clear labels</u>.

   (**Hint:** Think about the domain of $f(x,y)$.)

END OF ASSIGNMENT 3