

### **Network Effects**



#### **Network effects**

 Two reasons why individual might imitate the behavior of others

 Informational effect: the behavior of others conveys information about what they know.

 Network effect (direct benefit effect): you incur an explicit benefit when you align behavior with the behavior of others.



examples of network effect









#### **Externalities**

#### Externalities

 situation that the welfare of an individual is affected by the actions of other individuals

#### Positive externalities

- welfare increases when others are joining
- e.g. social networks

#### Negative externality

- welfare decreases when others are joining
- e.g. traffic congestion



### The Economy

- Markets with huge no. of purchasers.
- Individual decisions do not affect the aggregate behavior (no individual effects).
- Each consumer wants at most one unit of the good.
- Each customer has a personal intrinsic interest in obtaining the good that can vary from one consumer to another.





## The Economy Without Network Effects

- With network effects
  - The willingness to pay is determined by
    - intrinsic interest
    - the no. of other people using the good
- Without network effects
  - Each consumer's interest → reservation price

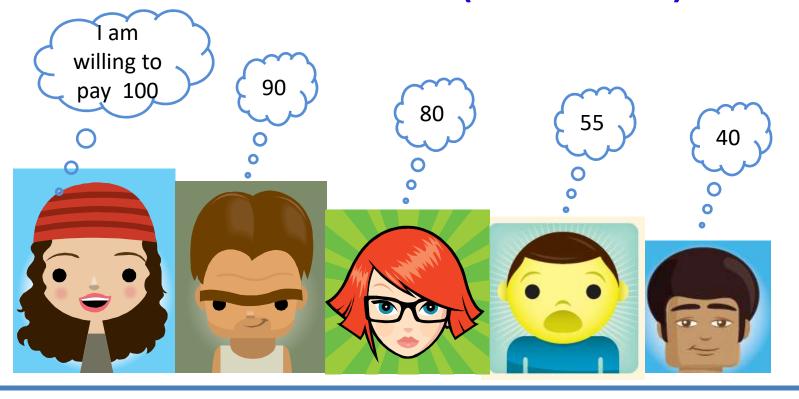








### Customer x $(0 \le x \le 1)$



0.0 0.2 0.4 0.6 0.8 1.0

Reservation price: the max amount one is willing to pay for one unit of the good.



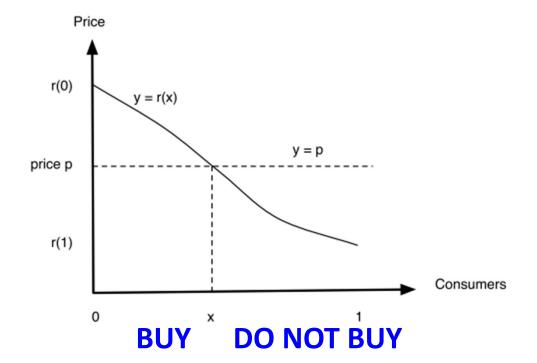
#### Reservation Prices

- If consumer x  $(0 \le x \le 1)$  has a higher reservation price than consumer y, then x < y.
- Let r(x) denote the reservation price of consumer
   x. (with no network effect)
- Assumptions :
  - $r(\cdot)$  is continuous,
  - no two consumers have exactly the same reservation price so the function  $r(\cdot)$  is strictly decreasing as it ranges over the interval from 0 to 1.



### Market price

- p = the market price for a unit of the good
- no units are offered for sale at a price above or below p.
- When  $p \ge r(0)$ , no one will buy the good.
- When p ≤ r(1), everyone will buy the good.
- When r(0) > p > r(1), there exists r(x) = p, customers 0 to x will buy.



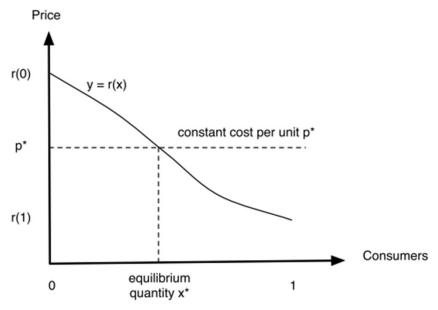


## The Equilibrium Quantity of the Good

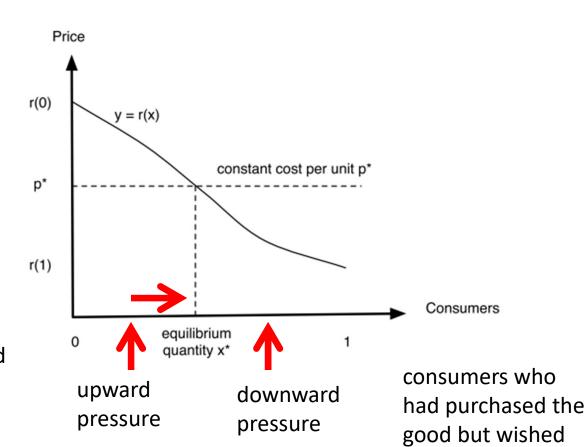
p\*: production cost of one unit of the good
 (the minimum price a producer is willing to accept to sell a good)

x\*: equilibrium quantity of the good

$$r(x^*)=p^*$$







consumers who have not purchased but with an incentive to do so

they had not



### The Economy with Network Effects

- With network effect, r(x) is the intrinsic interest of consumer x in the good. The reservation price is also depending on how many people use/want the product.
  - Assume r(1) = 0 (for convenience)
- When a z fraction of the population is using the good,
   f(z) measures the benefit to each consumer from those who use the good.
  - f(z) is increasing in z
  - f(0) = 0: if no one has purchased the good, no one is willing to pay anything for the good.
- The reservation price of consumer x is equal to r(x)f(z).



- Suppose that the price of the good is p\*, and that consumer x expects a z fraction of the population will use the good.
  - x will purchase provided that  $r(x)f(z) \ge p^*$ .





### Equilibria with Network Effects

- self-fulfilling expectations equilibrium
  - if everyone expects that a z fraction of the population will purchase the product, then this expectation is in turn fulfilled by people's behavior.
  - If everyone expects z = 0,  $r(x)f(0) = 0 < p^*$ , no one buys.
  - If  $p^* > 0$  and 0 < z < 1 form a self-fulfilling expectations equilibrium, then  $p^* = r(z)f(z)$ .



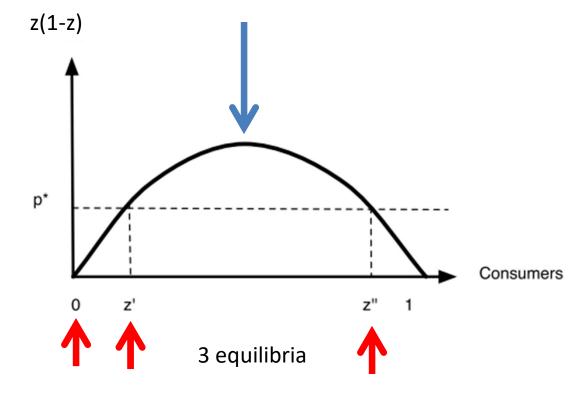
### Example

• 
$$r(x) = 1-x$$

• 
$$f(z) = z$$

• 
$$p^*=r(z)f(z)$$
  
=  $z(1-z)$ 

Max 
$$p^*=1/4$$
 at  $z = 1/2$ 

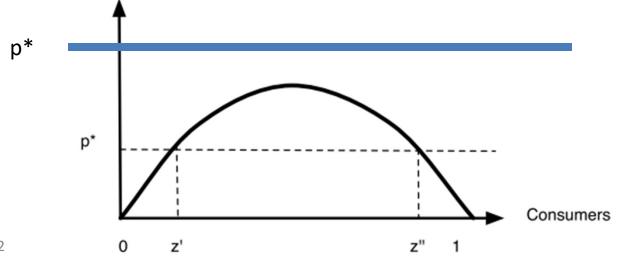




• If  $p^* > \frac{1}{4}$ , no solution to  $p^*=r(z)f(z) = z(1-z)$ 

– only equilibrium at z=0

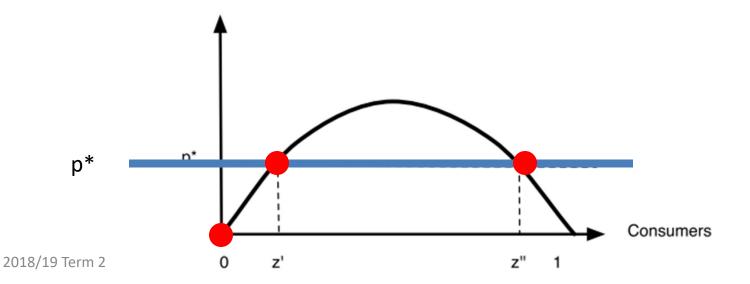
Max  $p^*=1/4$  at z = 1/2



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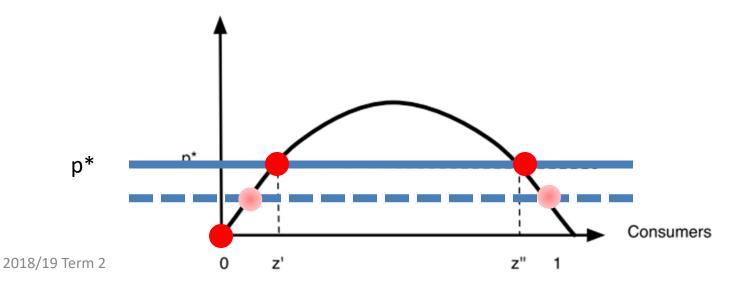


- If p\* > ¼, no solution to p\*=r(z)f(z) = z(1-z)
   only equilibrium at z=0
- If  $0 \le p^* < \frac{1}{4}$ , 3 possible equilibrium, 0, z' and z"
- If p\* drops, z" increases but z' decreases.



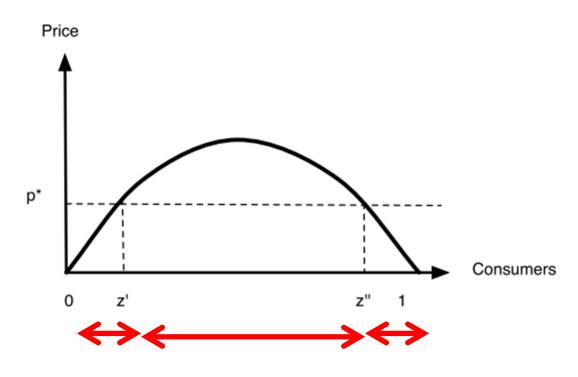


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# Stability, Instability, and Tipping Points

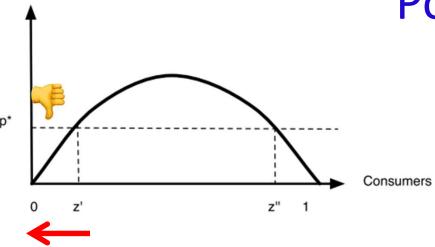


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CSCI4190 by Laiwan Chan



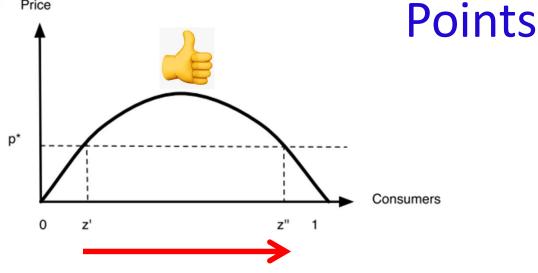
Stability, Instability, and Tipping Points



- 0 < z < z'
- r(z)f(z) < p\*, the purchaser named z (and other purchasers just below z) will value the good at less than p\*, and hence will wish they hadn't bought it.
- "downward pressure" on the consumption of the good : this would push demand downward.



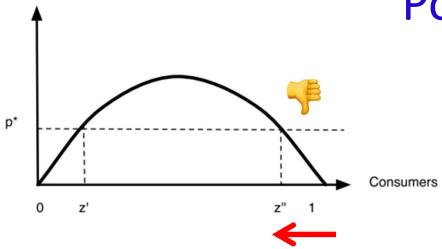
Stability, Instability, and Tipping



- z' < z < z"
- r(z)f(z) > p\*, consumers with names slightly above z have not purchased the good but will wish they had.
- "upward pressure" on the consumption of the good: this would drive demand upward

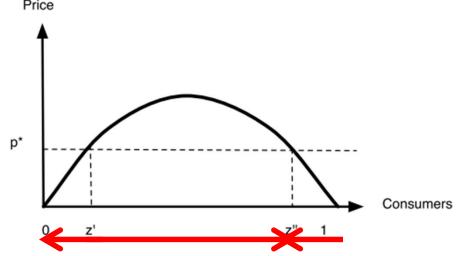


Stability, Instability, and Tipping Points



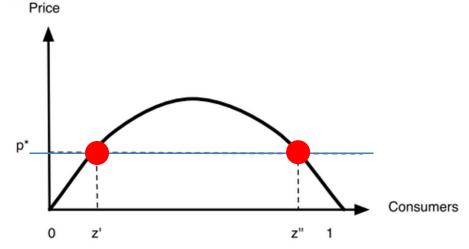
- z'' < z < 1
- r(z)f(z) < p\*, the purchaser z and other just below will wish they hadn't bought the good.
- "downward pressure" on the consumption of the good: this would push demand downward.





- z" has a strong stability property
- z' is a critical point (or a tipping point).
- If z > z', upward pressure push to z"
- If z < z', downward pressure push to 0</li>





- If we lower p\*,
  - z' moves left, i.e. a lower critical point
  - z" moves right, larger z"
    - free trials for products
    - low introductory prices



### A Dynamic View of the Market

- If everyone believes a z fraction of the population will use the product, x will buy if  $r(x)f(z) \ge p^*$ .
- Let  $\hat{z}$  be the fraction of population who buy the product, where  $r(\hat{z})f(z)=p^*$  or  $\hat{z}=r^{-1}\left(\frac{p^*}{f(z)}\right)$
- Define  $g(z) = \hat{z}$  as the function that gives the outcomes as a function of z.



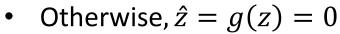
### Example

- If r(x) = 1 x and f(z) = z
- $r^{-1}(x) = 1 x$

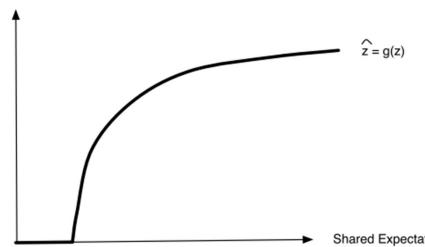
• 
$$\hat{z} = g(z) = r^{-1} \left( \frac{p^*}{f(z)} \right) = 1 - \frac{p^*}{z}$$

• 
$$p^* \le r(0)f(z) \rightarrow p^* \le z$$

• If 
$$z \ge p^*$$
,  $\hat{z} = g(z) = r^{-1} \left( \frac{p^*}{f(z)} \right) = 1 - \frac{p^*}{z}$ 





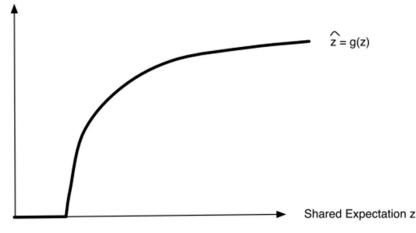


Outcome

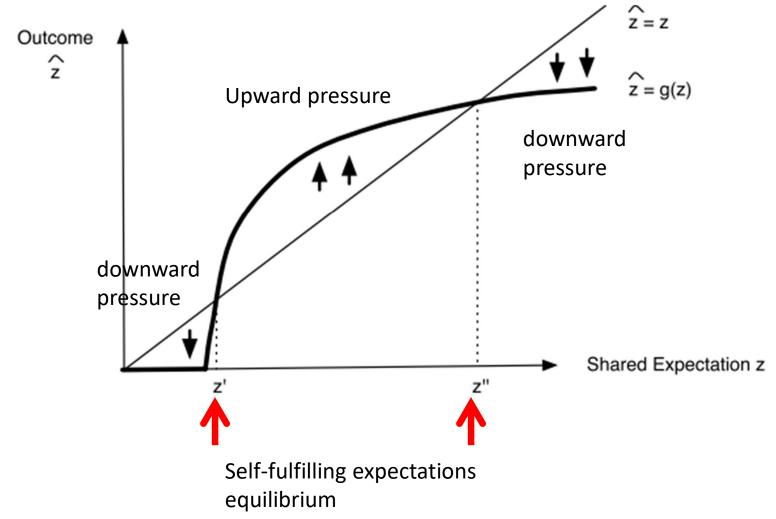


## A Dynamic View of the Market

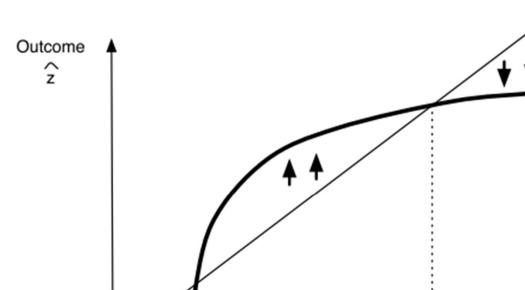
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- Let  $\hat{z}$  be the fraction of population who buy the
- product, where  $r(\hat{z})f(z)=p^*$  or  $\hat{z}=r^{-1}\left(\frac{p^*}{f(z)}\right)$  When  $p^* \leq r(0)f(z)$ , let  $\hat{z}=g(z)=r^{-1}\left(\frac{p^*}{f(z)}\right)$
- Otherwise  $\hat{z} = g(z) = 0$  outcome





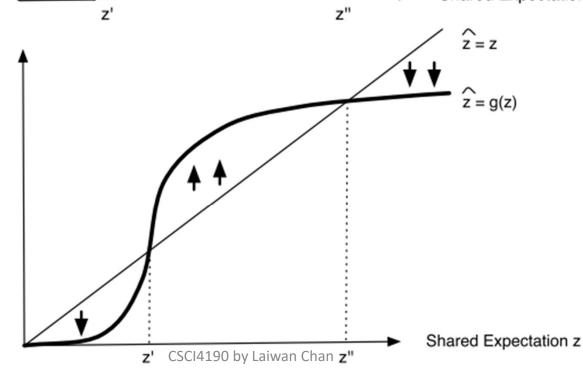






Outcome

More general case for the relation between the expected number of users and the actual number of purchsers



 $\hat{z} = z$ 

 $\hat{z} = g(z)$ 

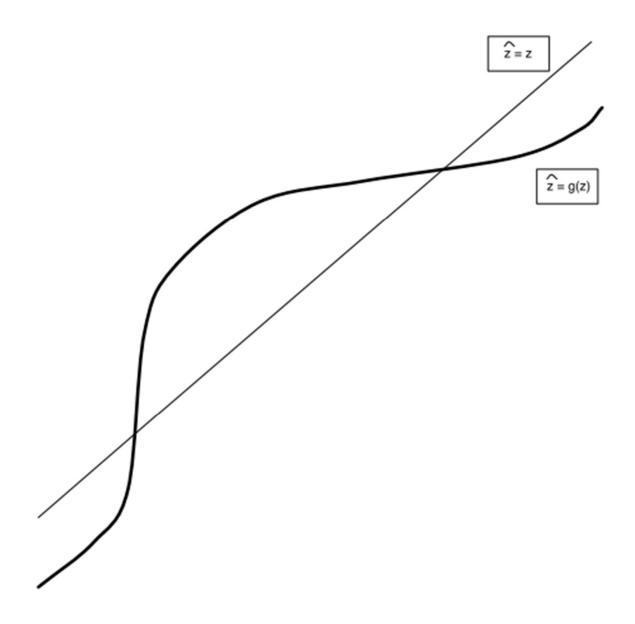
Shared Expectation z



# The dynamic behavior of the population

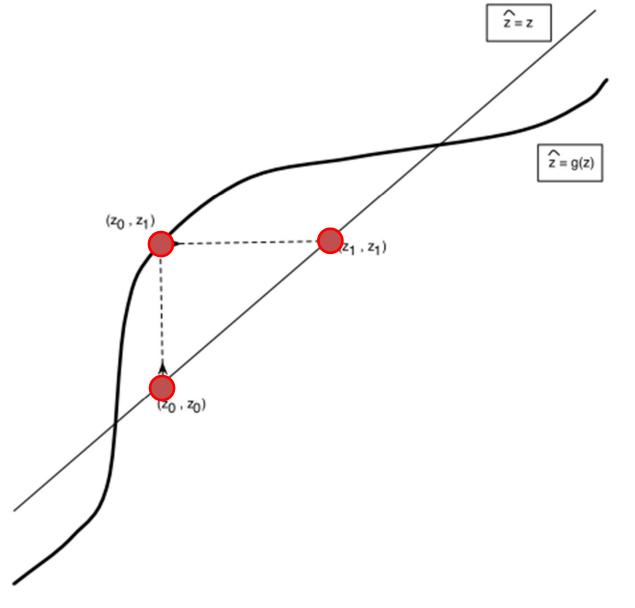
- Participation in a large social media site.
- r(x): intrinsic interest in using the site
- f(z): attractiveness when there are z users
- p\* expenditure of effort required to use the site.
- Time proceeds in a fixed set of periods, t=0,1,2, ...
- z<sub>0</sub>:initial audience size



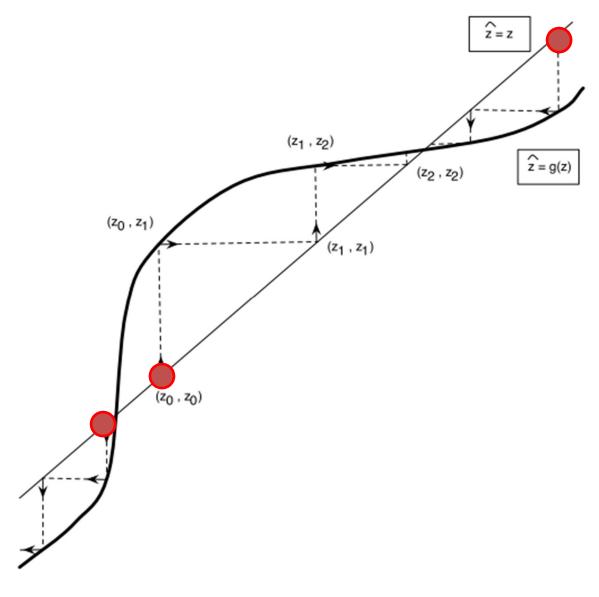




- z<sub>0 :</sub> initial audience
- $z_1 = g(z_0)$
- $z_2 = g(z_1)$









## Marketing a product with network effect

- New piece of software, communication technology, or social media
- Need to get past the tipping point (at  $z_0$ ) by convincing a large initial group to adopt your product before others will be willing to buy it
  - set an initial low, introductory price for the good, perhaps even offering it for free.
    - if it gets over the tipping point then raise the price to overcome the initial losses
  - identify fashion leaders, those whose purchase or use of the good will attract others to use it



### Social Optimality with Network Fffects

- For goods with network effects, the equilibria are typically not social optimal.
- Suppose at an equilibrium with audience size z\*.
- $r(z^*)f(z^*) = p^*$
- Consider consumer z between z\* and z\*+c for c > 0
  - They do not buy, since  $r(z)f(z^*) < p$
  - But if they all did purchase the good, all current purchasers would benefit. The value of the product to each purchaser  $x < z^*$  would increase from  $r(x)f(z^*)$  to  $r(x)f(z^* + c)$ .
  - When this benefit to the existing customers outweighs the loss of customers  $z^* < z < z^*+c$ , not social optimal.
  - Consumers between z\* and z\*+c do not take this into account when they make their decisions.



#### **Network Effects and Competition**

- If multiple firms develop competing new products, each of which has its own network effects.
- One product will dominate the market, as opposed to a scenario in which both products (or even more than two) flourish.
- The product that first gets over its own tipping point attracts many consumers and this may make the competing product less attractive. Being the first to reach this tipping point is more important than being "best".
- If product A gets over its tipping point, then product B may not be able to survive, unless there are changes to shift the balance after A achieves dominance
  - B improves its product sufficiently and markets it well, and A doesn't respond effectively.



# Mixing Individual Effects with Population-Level Effects

• Previously, we assume the product is useless to consumers when z = 0, i.e f(0) = 0,

$$- e.g. f(z) = z$$

However, the product has some value to the first purchaser

$$-f(0) > 0$$

• E.g.  $f(z) = 1 + az^2$ 



# Mixing Individual Effects with Population-Level Effects

$$f(z) = 1 + az^2$$

• 
$$r(x) = 1 - x$$

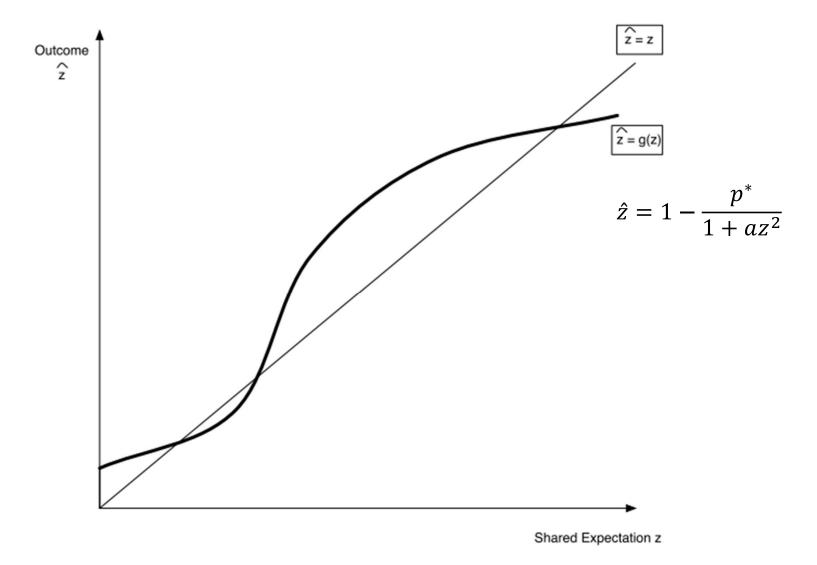
• 
$$r(x)f(z) = (1-x)(1+az^2)$$

• 
$$r(0) = 1, f(z) \ge 1, p^* < 1,$$

so  $p^* \le r(0)f(z)$  is always true

$$\hat{z} = g(z) = r^{-1} \left( \frac{p^*}{f(z)} \right) = 1 - \frac{p^*}{1 + az^2}$$

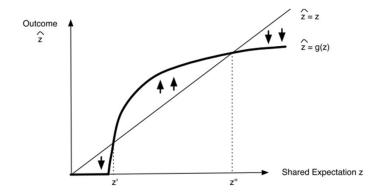


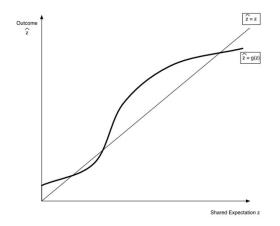




When f(0)=0,
 z = 0 is a
 stable
 equilibrium

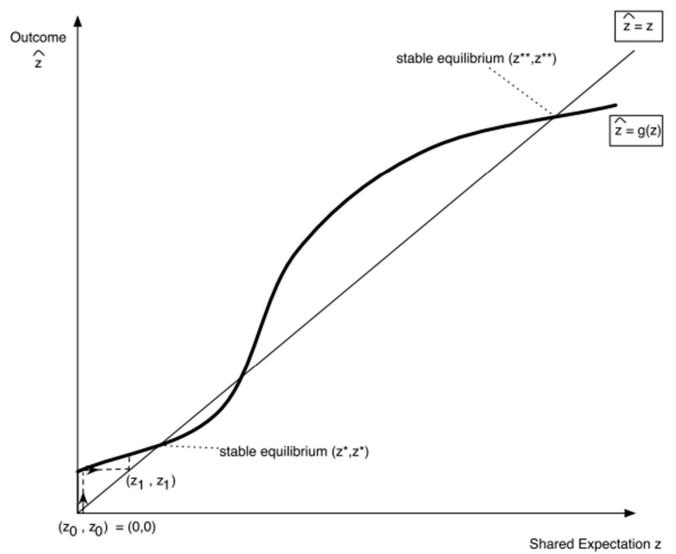
When f(0) > 0,
 it is not





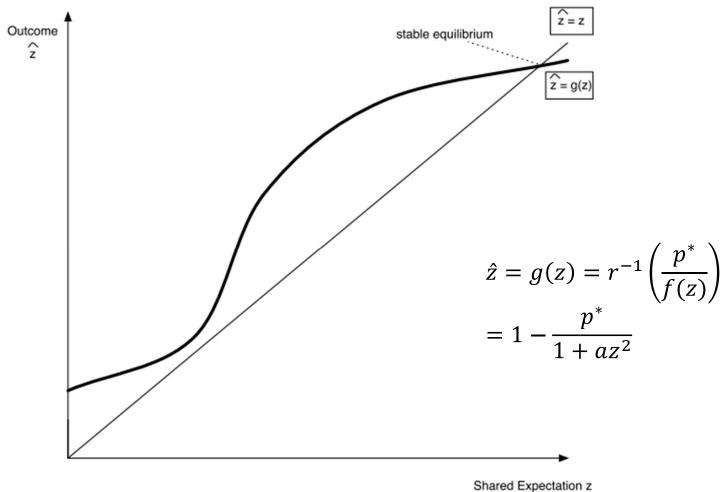


$$r(x)f(z) = (1 - x)(1 + az^2)$$

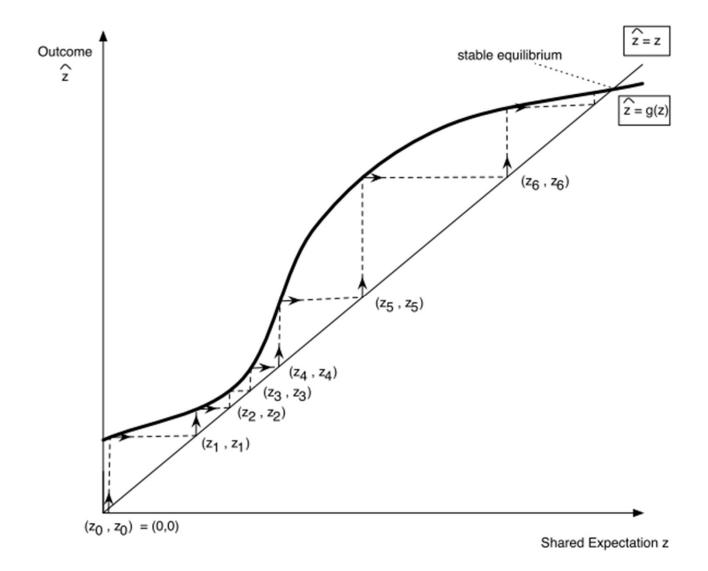




#### If p\* is reduced slightly









- small changes in market conditions can have strong, discontinuous effects on the outcome.
  - Equilibrium at  $z^*$  Equilibrium at  $z^{**}$



