Midterm of ENGG2430A. 9:30-11:00am, Mar 8 2017.

1. (10 points) A 4-sided die is loaded in a way that face i is twice as likely as face $i+1$, for all $i = 1, 2, 3$.
   a) What are the probabilities of the four faces respectively in a roll?
   b) What is the probability that the outcome of a roll is strictly less than 4?

2. (10 points) Suppose that we have three independent random variables X_1, X_2, X_3, with mean $\mathrm{E}(X_i) = i$ and variance $\mathrm{Var}(X_i) = i^2$, for all $i = 1, 2, 3$. What are the mean and variance of $X = X_1 + X_2 + X_3$?

3. (10 points) In a game with n people, someone passes a book to one of the other $n-1$ participants. Thereafter, each recipient passes the book on to one of the other $n-1$ participants, chosen uniformly at random. What is the probability that by the k-th time that the book has been passed, it has not come back to someone who has already received it before?

4. (10 points) Polygraphs are often used by the police to test whether someone is telling the truth. Suppose a person lying fails the polygraph 90% of the time, and one telling the truth fails the polygraph 15% of the time. Assume a general prior probability 80% that a person tells the truth. If a polygraph indicates that a person is lying, what is the probability that he/she is indeed lying?

5. (10 points) On a random day, Alice has some amount X of money in her pocket and Bob has Y in his. Suppose that X and Y are independent random variables with PMFs as follows.

$$p_X(x) = \begin{cases} 0.3 & x = 50 \\ 0.4 & x = 100 \\ 0.3 & x = 200 \\ 0 & otherwise \end{cases}, \qquad p_Y(y) = \begin{cases} 0.5 & y = 50 \\ 0.5 & y = 100 \\ 0 & otherwise \end{cases}$$

   a) What is the joint PMF of (X, Y)?
   b) What is the PMF of $Y - X$?

6. (10 points) Recall that an exponential random variable X has PDF

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x}, & \text{if } x \geq 0 \\ 0, & \text{otherwise} \end{cases}.$$

   Consider an exponential random variable X with mean 4. Suppose that someone draws a sample $\hat{X}$ from the distribution of X and tells us that $\hat{X} \geq 6$.

   What is the probability that $\hat{X} \geq 8$?