CSCI 5350 Advanced Topics in Game Theory

Discussion Session 3

Game Theory Exercise 1

Consider a 3-player strategic game $G = \langle N, (A_i), (\succsim_i) \rangle$. Each player has three available actions: a_1, a_2 and a_3. That is, $A_1 = A_2 = A_3 = \{a_1, a_2, a_3\}$. Suppose in G, player 1 always receives a payoff of 10 if he plays a_1, or 0 otherwise, regardless of what strategies the other two players play; player 2 always receives a payoff of 5 if he plays a_2, or 0 otherwise, regardless of what strategies the other two players play; player 3 always receives a payoff of 2 if he plays a_3, or 0 otherwise, regardless of what strategies the other two players play.

(a) **(6 marks)** Give the payoff matrix of G.

(b) **(6 marks)** What are the pure strategy Nash equilibria in G?

(c) An outcome ω of a strategic game is said to be *Pareto optimal* if there is no other outcome ω' existing in the same game, such that at least one player i gets a higher payoff in ω' than in ω, while no player gets a lower payoff in ω' than in ω.

   i. **(4 marks)** Identify all Pareto optimal outcomes in the game G.

   ii. **(4 marks)** Consider an arbitrary strategic game, in which A is the set of pure strategy profiles and N the set of players. Prove that pure strategy profiles that maximise the total utilities of all players are always Pareto optimal. (That is, each outcome of the set $\{\omega \in A : \sum_{i \in N} u_i(\omega) \geq \sum_{i \in N} u_i(\omega') \text{ for all } \omega' \in A\}$ is a Pareto optimal outcome.)

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One day John and Mary play a game. Each of them has one red card and one green card, and they play the game by presenting either one of these two cards at the same time. The rule is that if both of them present a red card (action R), then each of them receives a utility of 100; if both of them present a green card (action G), then each of them receives a utility of 200. However, if they present different cards, then John will receive a utility of 100 and Mary will receive a utility of 50 if John presents a green card and Mary presents a red card, or both players receive a utility of 0 (zero) otherwise.

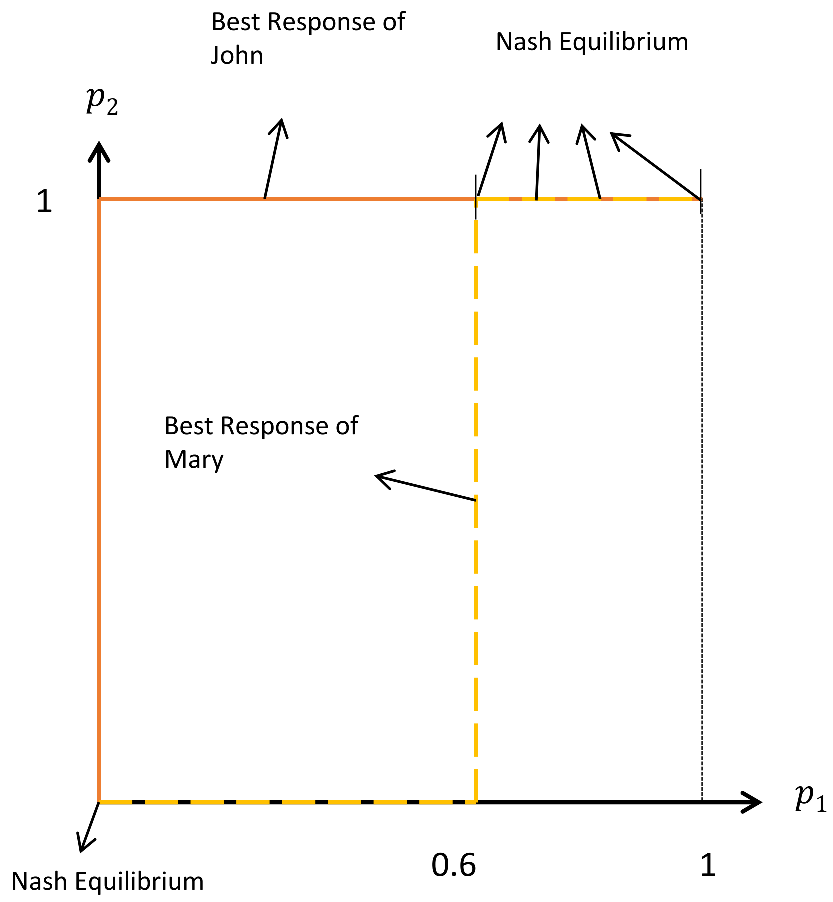
a) Model this game as a strategic game $\langle N,(A_i),(u_i)\rangle$.
   i) **(3 marks)** Write down N, (A_i), and (u_i).
   ii) **(2 marks)** Are there any pure strategy Nash equilibria in the game? Write down all of them if there are any.
   iii) **(5 marks)** Are there any completely mixed strategy Nash equilibria in the game? Write down all of them if there are any.
   iv) **(2 marks)** Which outcome(s) is/are Pareto optimal?
   v) **(2 marks)** Can you conclude what action(s) should John play and what action(s) should Mary play? Explain your answer.

b) Suppose now the game is changed a little bit so that both John and Mary will receive a utility of 50 when John presents a green card and Mary presents a red card.
   i) **(2 marks)** Are there any symmetric equilibria in the changed game?
   ii) **(4 marks)** Give one evolutionary stable solution of the changed game. Justify your answer.

DEFINITION. An strategy profile a^* in a strategic game in which each player has the same set of strategies is a **symmetric Nash equilibrium** if it is a Nash equilibrium and a_i^* is the same for every player i.

From 2009-10 Final Exam Q1

a) iii)


Best Response of
John
Nash Equilibrium
p_2
1
Best Response of
Mary
p_1
Nash Equilibrium
0.6
1

Consider the strategic game G represented by the following payoff matrix:

G:

		Player 2	
		H	D
Player 1	H	1, 1	6, 2
	D	2, 6	3, 3

(a) **(4 marks)** What are the pure strategy Nash equilibria in G?

(b) **(5 marks)** Suppose player 1 plays a mixed strategy α_1 such that $\alpha_1(H) = p$ (and hence $\alpha_1(D) = 1 - p$). What should be player 2's best response to α_1? Express your answer in terms of p.

(c) **(5 marks)** Suppose player 2 plays a mixed strategy α_2 such that $\alpha_2(H) = q$ (and hence $\alpha_2(D) = 1 - q$). What should be player 1's best response to α_2? Express your answer in terms of q.

(d) **(6 marks)** Find <u>one</u> mixed strategy Nash equilibrium (which is not a pure strategy Nash equilibrium) in G.

From 2004-05 Final Exam Q2

End