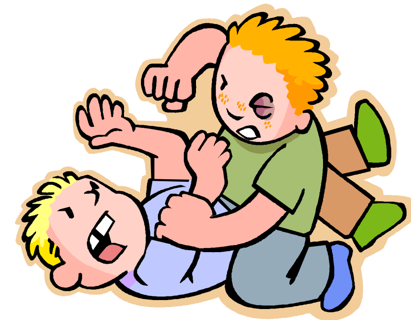
Positive and Negative Relationships

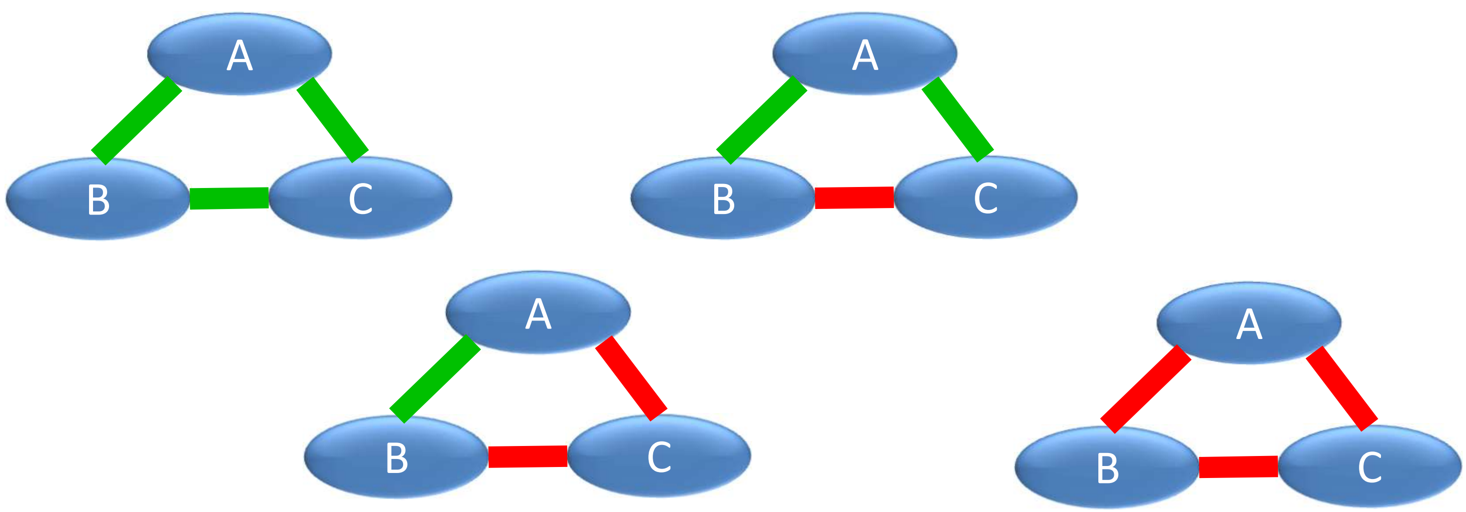

Positive and Negative Relationships



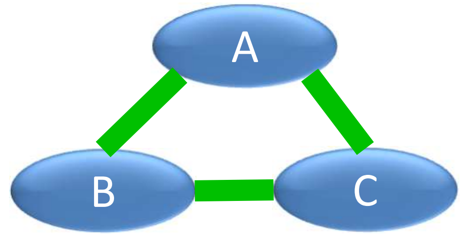
- Some relations are friendly, but others are antagonistic or hostile; interactions between people or groups are regularly beset by controversy, disagreement, and sometimes outright conflict.
- Positive links represent friendship while negative links represent antagonism.
- Structural balance
  – to understand the tension between these two forces.
  – a connection between local and global network properties; local effects → global consequences

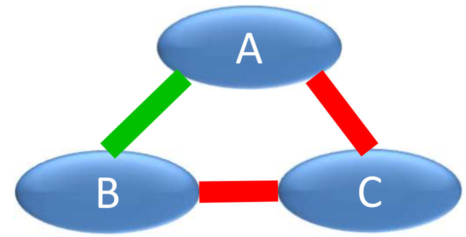
Structure Balance

- A clique or complete graph : graph which an edge connecting each pair of nodes
- Edges are labelled as
  - either friends (+) or enemies (-)
  - no two people are indifferent to one another.
- A set of three people at a time

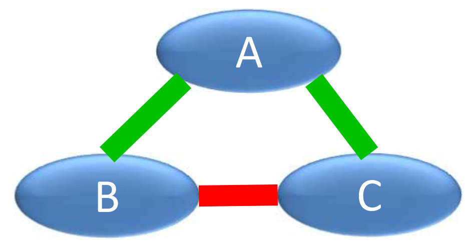


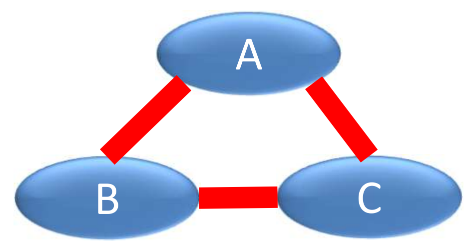
- Balanced Structure
  - Three mutual friends
  - A third mutual enemy





- Unbalanced Structure
  - Two common friends don’t get along with each other
    - A has stress to side with B or C against the other
  - All are enemies
    - Two of them team up against the third





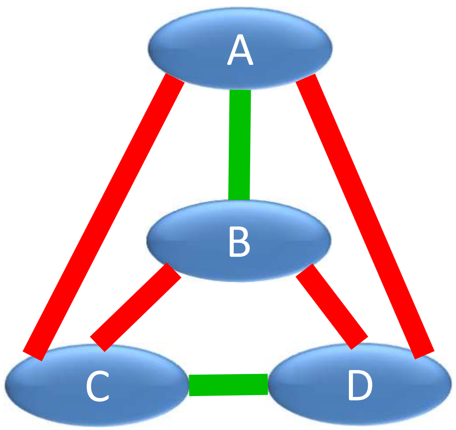
Structural Balance for Networks

A labelled complete graph is structurally balanced if everyone of its triangles is balanced.

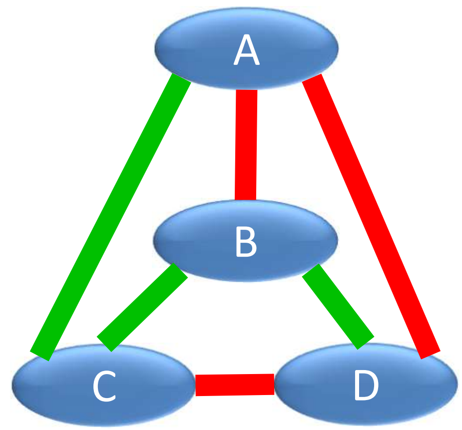
Structural Balance Property:

For every set of three nodes, if we consider the three edges connecting them, either all three of these edges are labeled +, or else exactly one of them is labeled +.

Balanced structure

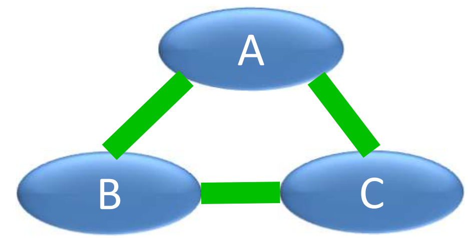


Not Balanced structure



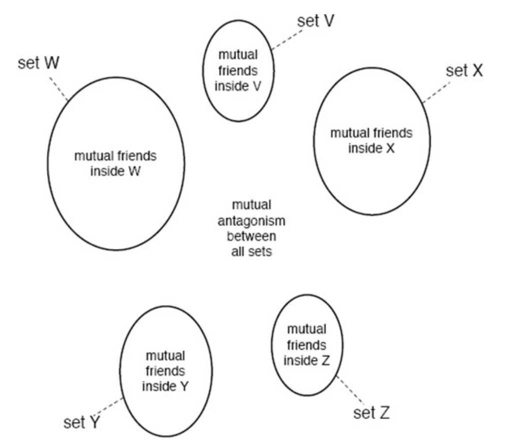
Local views vs global views

- Local view (Structural balanced property)
  - Conditions on each triangle of the network

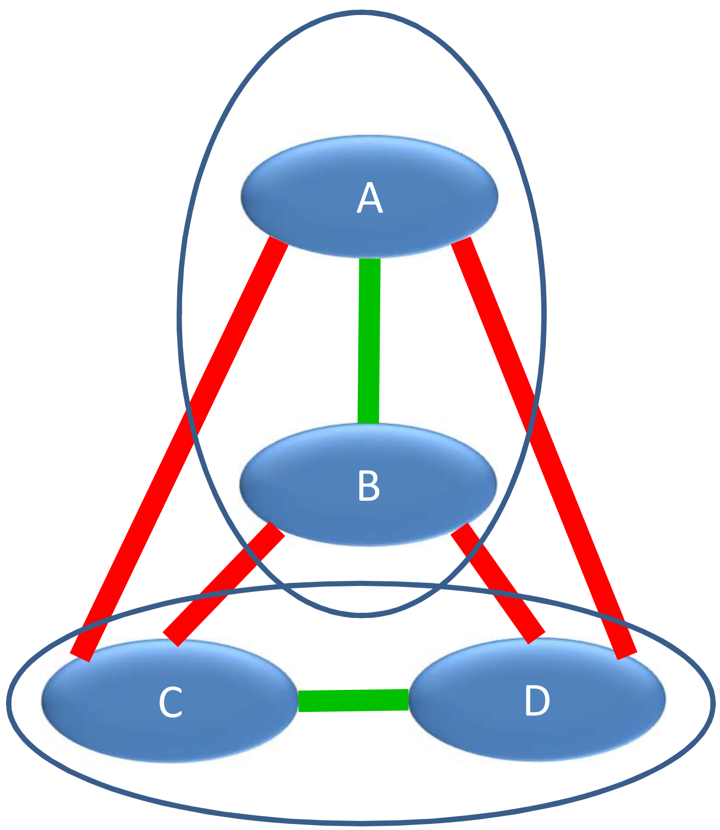


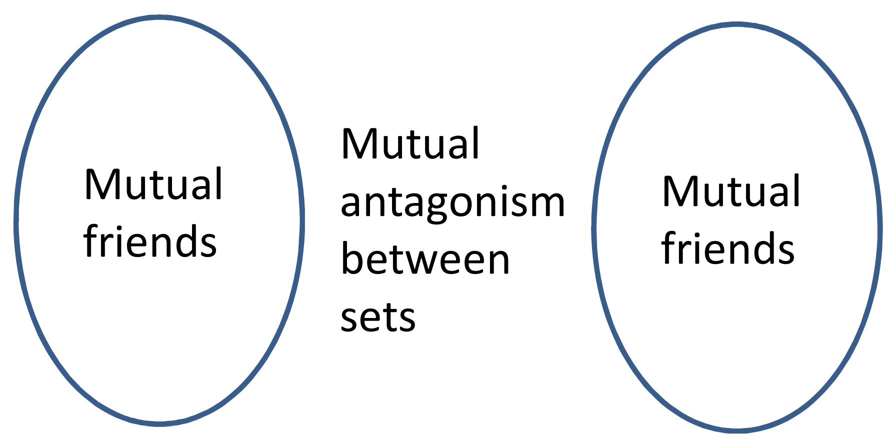
- Global view (balance theorem)
  - Requirement that the world be divided into sets of friends.

They are equivalent !!



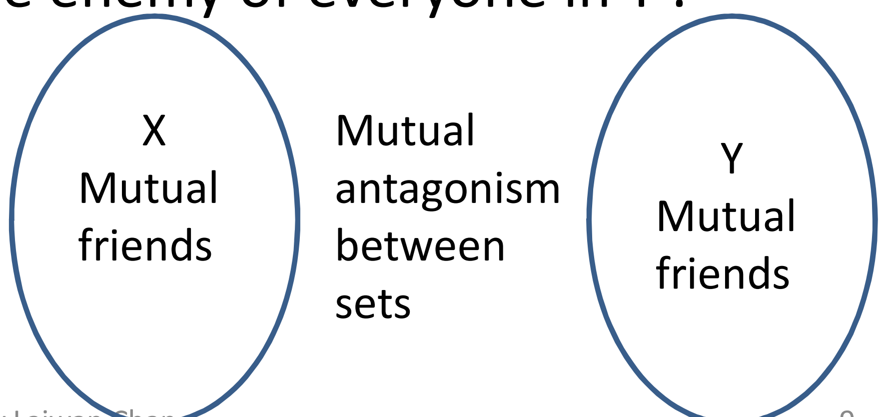
Balanced labeled complete graph





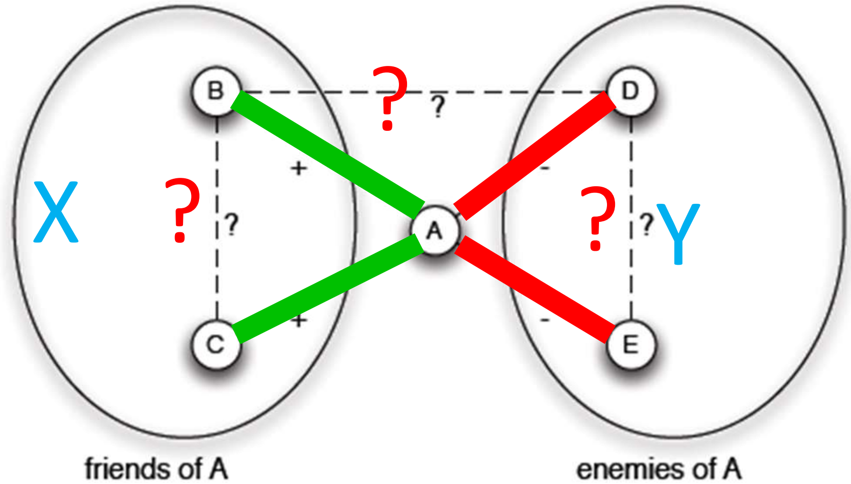
Balance Theorem

- If a labeled complete graph is balanced, then
  - either all pairs of nodes are friends,
  - or else the nodes can be divided into two groups, X and Y ,
    - such that every pair of nodes in X like each other,
    - every pair of nodes in Y like each other,
    - and everyone in X is the enemy of everyone in Y .

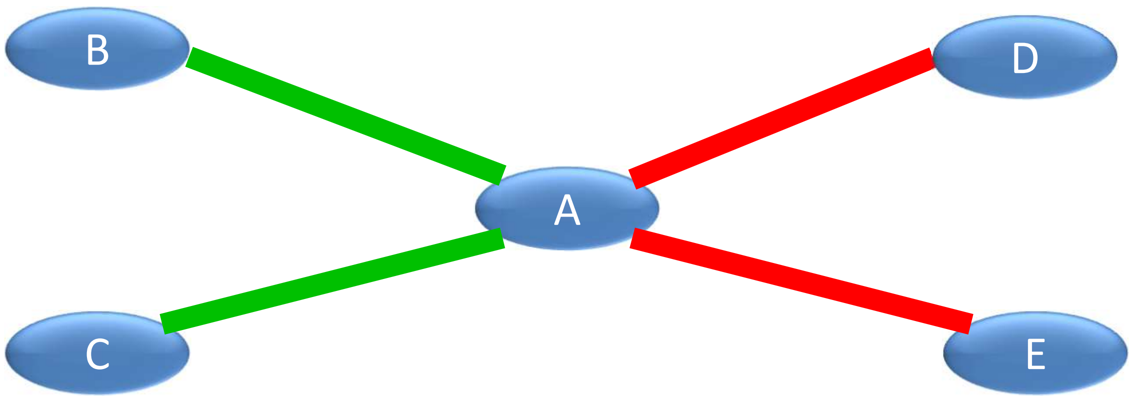


Proving the Balance Theorem

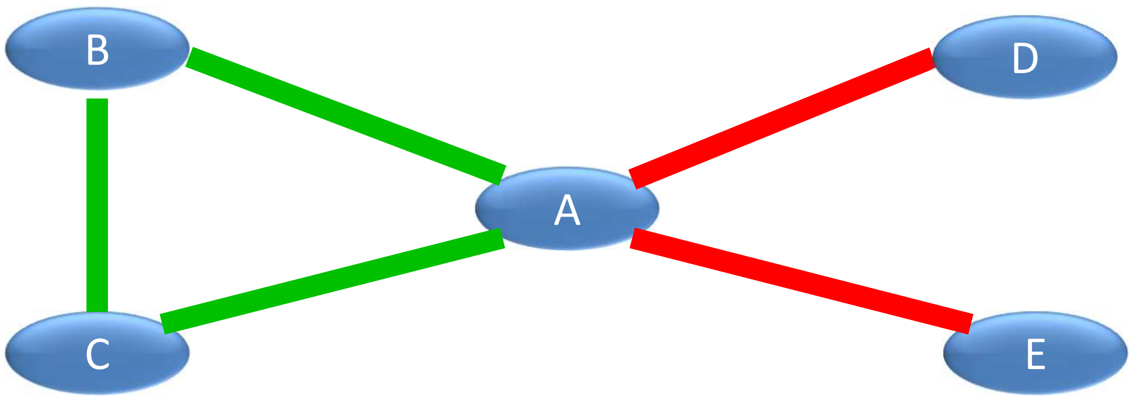
- If all pairs of nodes are friends, the graph is balanced. Otherwise, a balance graph must have at least one negative edge and one positive edge.



- Pick a node A, and let
  – X be the friends of A
  – Y be the enemies of A
- We need to prove
  a) Every two nodes in X are friends.
  b) Every two nodes in Y are friends.
  c) Every node in X is an enemy of every node in Y

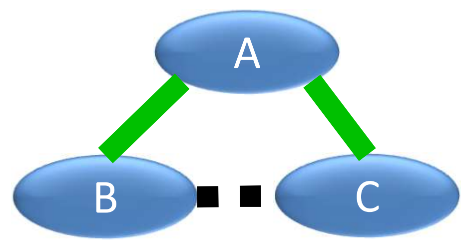

B
D
A
C
E

(a) Every two nodes in X are friends

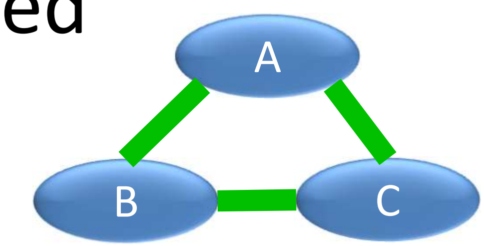


(a) Every two nodes in X are friends

- Let B and C be two nodes in X
- A is friends with both B and C

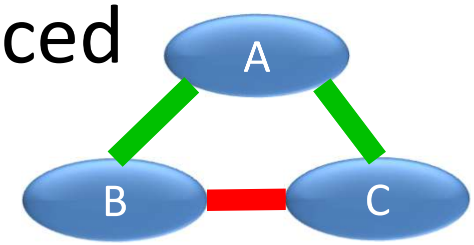


- Balanced

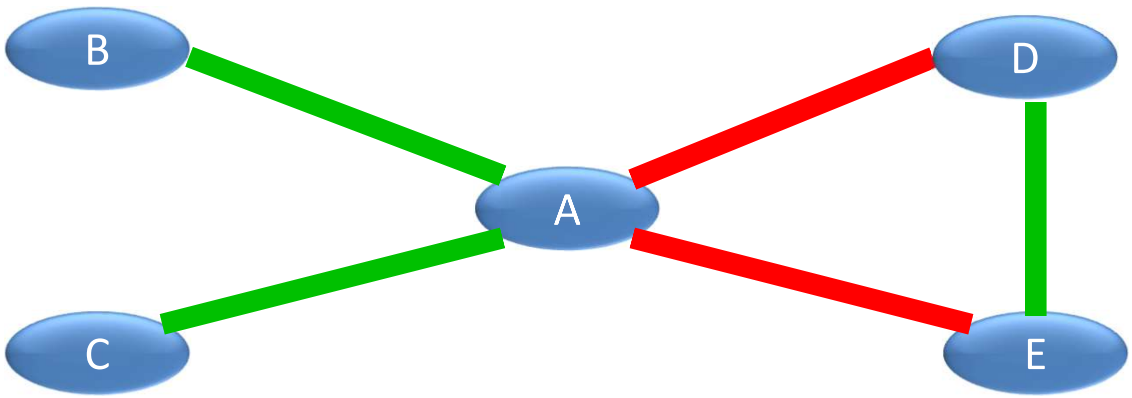


B and C are Friends

- Unbalanced

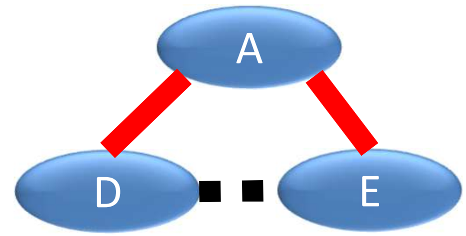


(b) Every two nodes in Y are friends

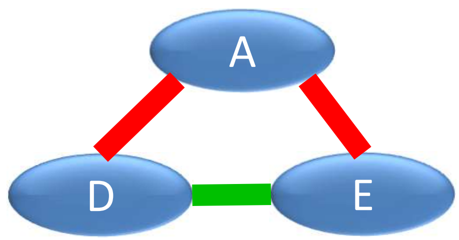


(b) Every two nodes in Y are friends

- Let D and E be two nodes in Y
- A is enemies with both D and E

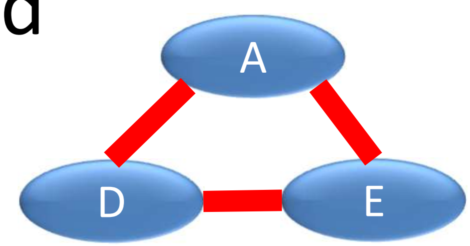


- Balanced

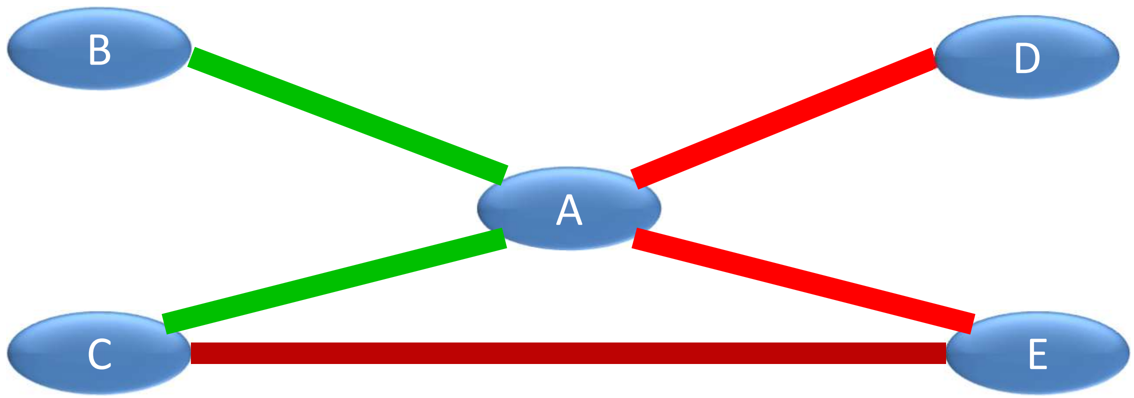


D and E are friends

- Unbalanced

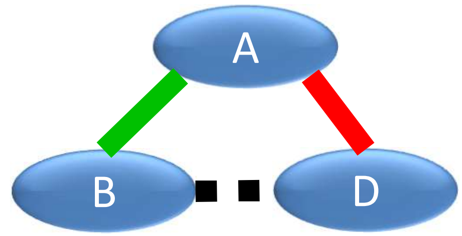


(c) Every node in X is an enemy of every node in Y

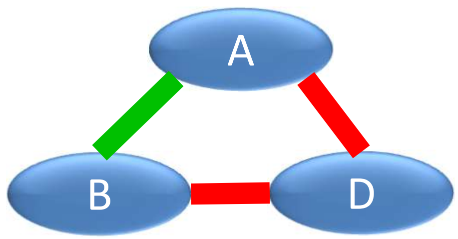


(c) Every node in X is an enemy of every node in Y

- Let B be a node in X
- Let D be a node in Y

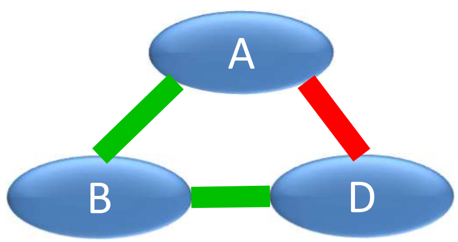


- A is friends with B and enemies with D
- Balanced



B and D are enemies

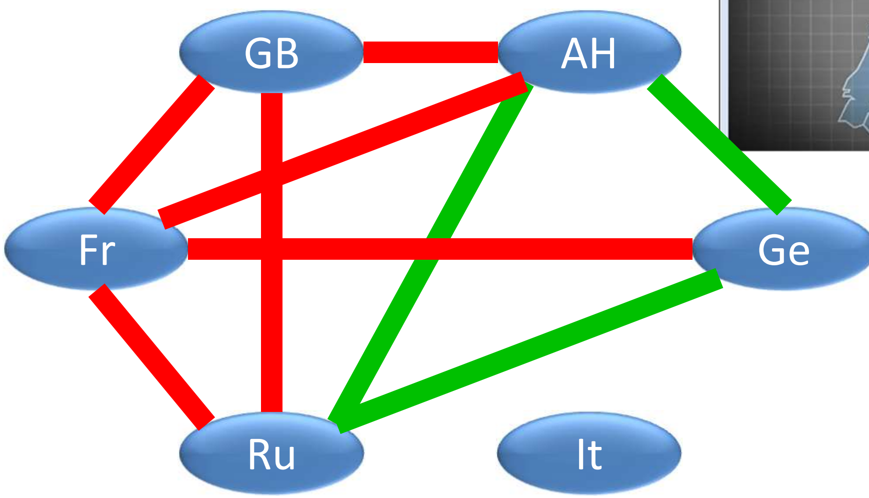
- Unbalanced



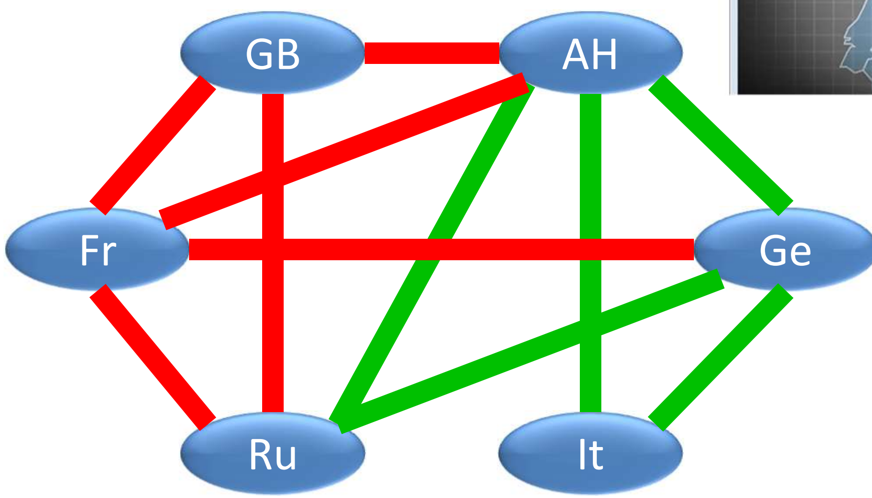
Applications of structural balance

- International relationship (by Antal, Krapivsky, and Redner)
- How the network slides into a balanced labeling and into World War I.
  - GB : Great Britain
  - Fr : France
  - Ru : Russia
  - It : Italy
  - Ge : Germany
  - AH : Austria-Hungary

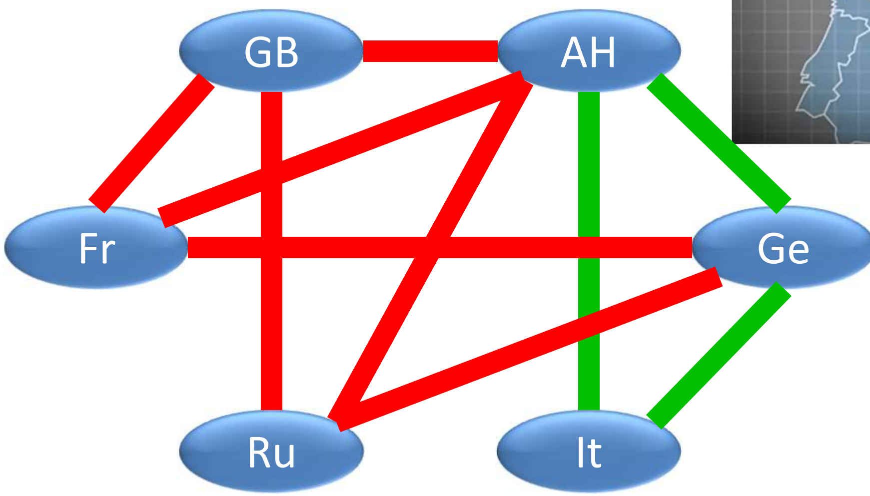
Three Emperors' League 1872-81



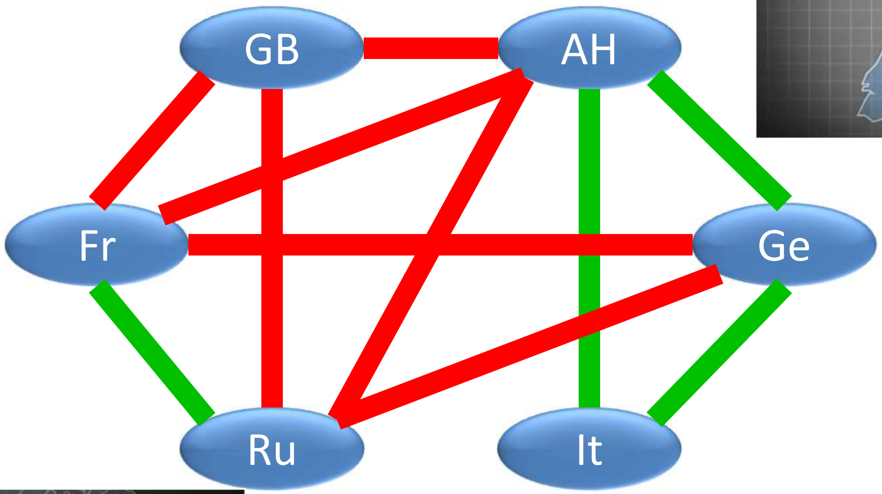
Triple Alliance 1882



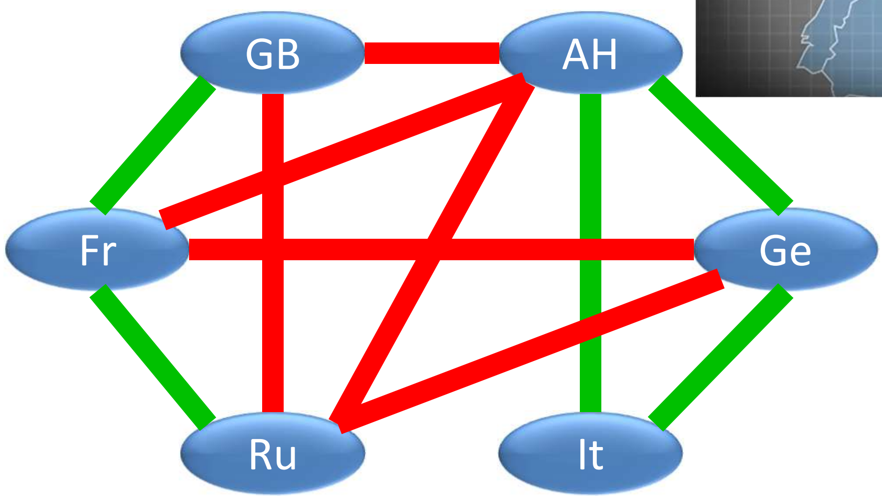
German-Russian Lapse 1890



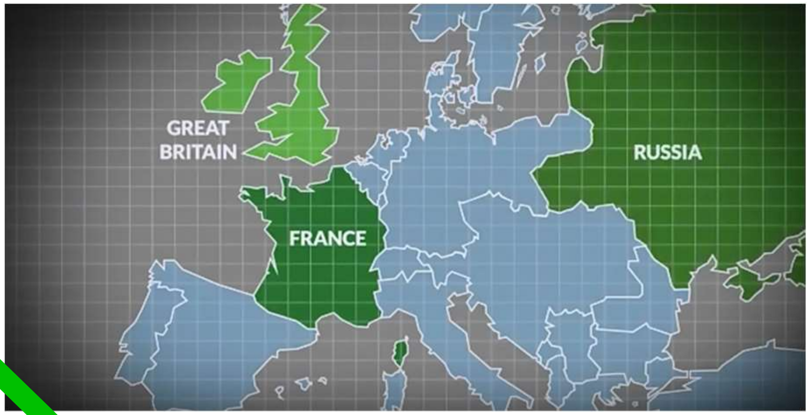
French-Russian Alliance 1891–94

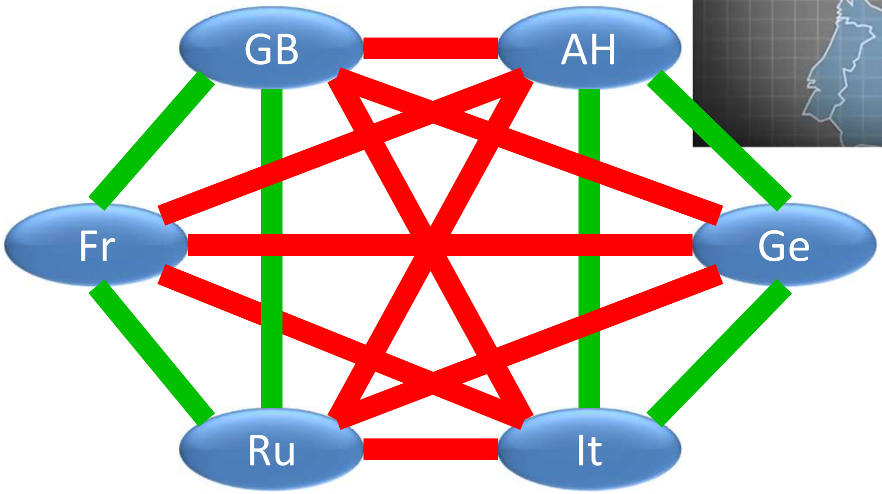


Entente Cordiale 1904



British Russian Alliance 1907





Trust and Distrust

- Consumer review sites or on-line rating sites that users can express trust/distrust dichotomy in on-line ratings.
- Directed Graph or Undirected graph
  - When A expresses trust or distrust of B, we don't know what B thinks of A

- Transferability
  - A trusts B, B trusts C, does A trust C ?
  - A distrusts B, B distrusts C, does A trust or distrust C ?

A weaker form of structural balance

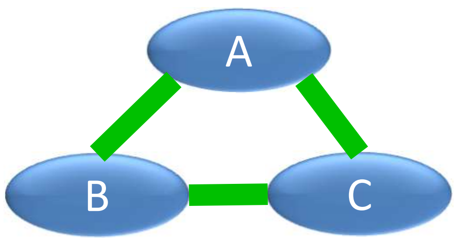
- Structural Balance Property:

  For every set of three nodes, if we consider the three edges connecting them, either all three of these edges are labeled +, or else exactly one of them is labeled +.

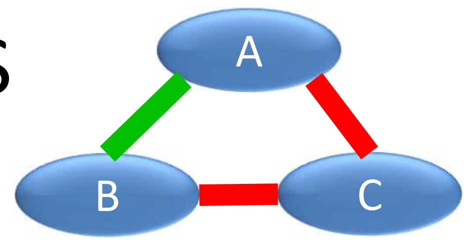
- Weak Structural Balance Property: There is no set of three nodes such that the edges among them consist of exactly two positive edges and one negative edge.

- Structural Balance Property:

YES

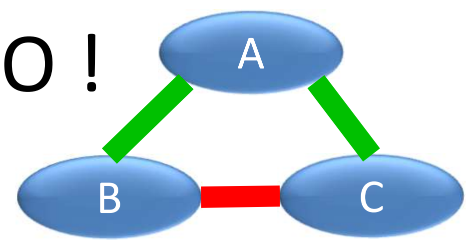


YES

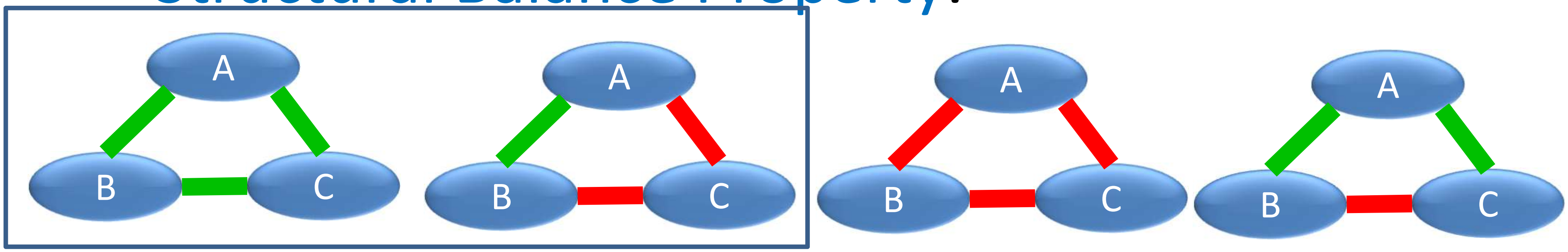


- Weak Structural Balance Property:

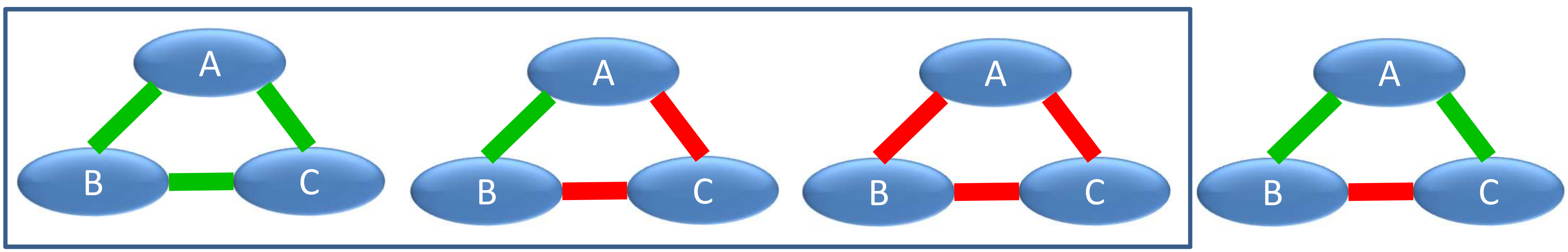
NO !



- Structural Balance Property:

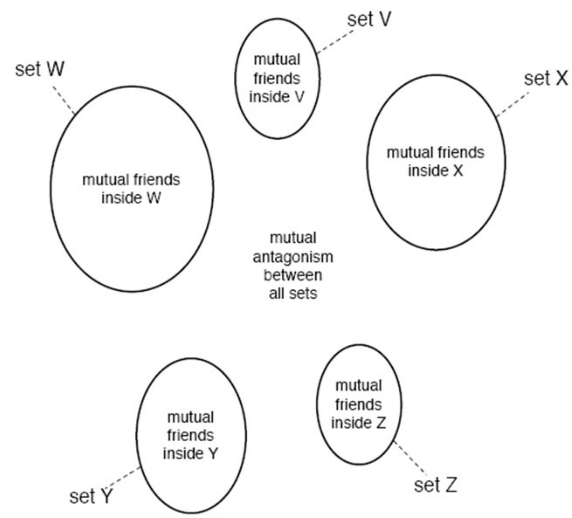


- Weak Structural Balance Property:

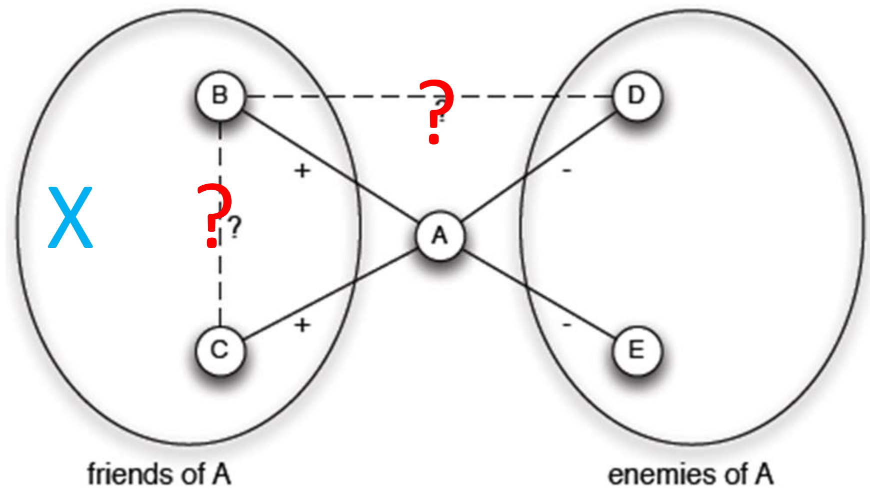


Characterization of Weakly Balanced Networks

- If a labeled complete graph is weakly balanced, then its nodes can be divided into groups in such a way that every two nodes belonging to the same group are friends, and every two nodes belonging to different groups are enemies.



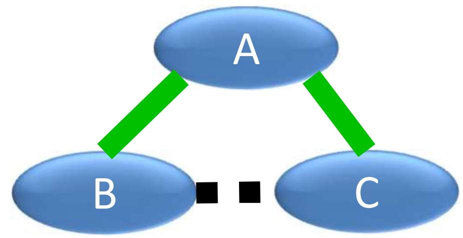
Proving the characterization



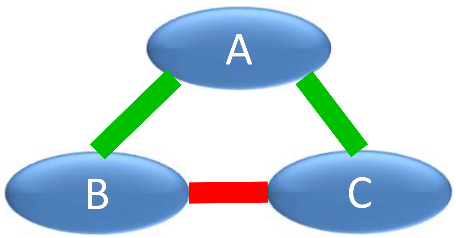
- Pick a node A, and let
  – X be the friends of A
- We need to prove
  – All of A's friends are friends with each other
  – A and all his friends are enemies with everyone else.

All of A's friends are friends with each other

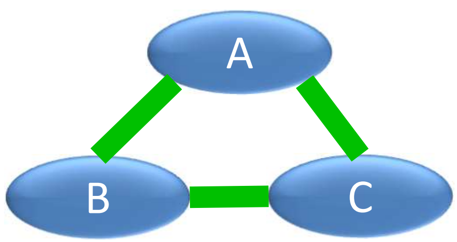
- Let B and C be two nodes who are friends of A
- A is friends with both B and C



- If



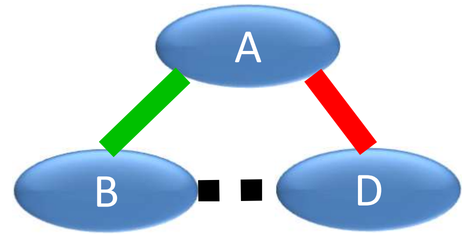
Violet the weak structural balanced



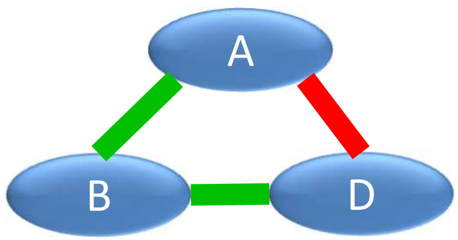
B and C are friends

A and all his friends are enemies with everyone else

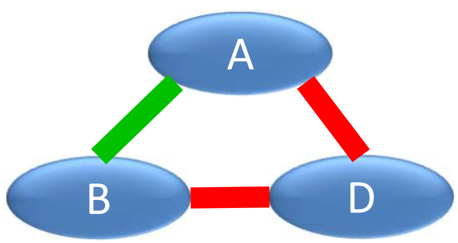
- Let B be a node in X
- Let D be a node outside X



- A is friends with B and enemies with D
- If

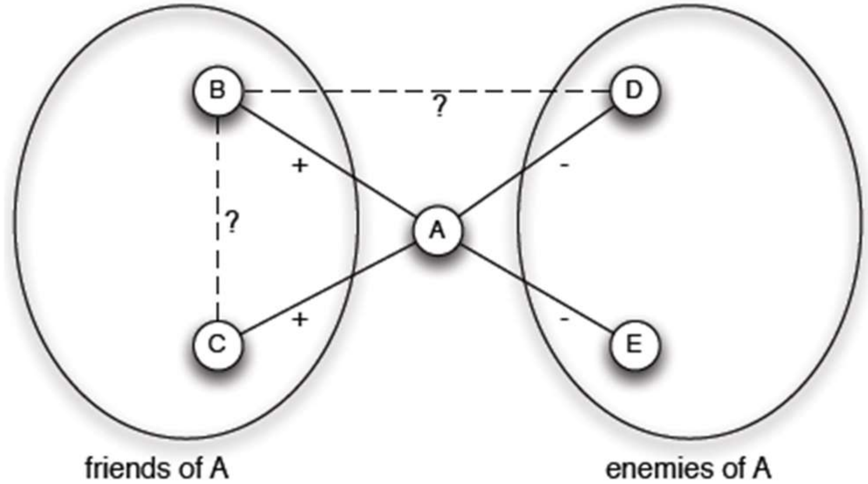


violet the weak structural balanced

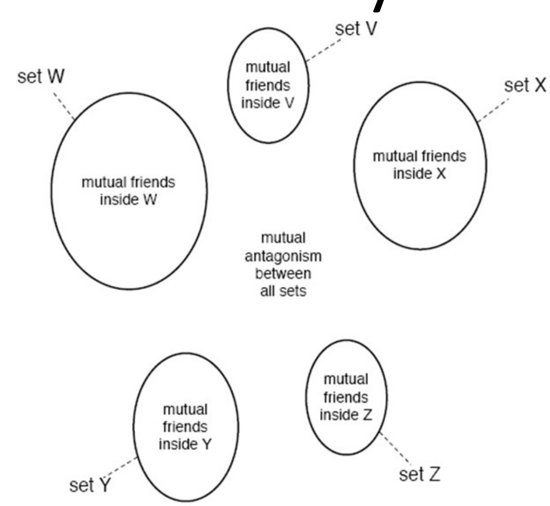


B and D are enemies

Proving the characterization



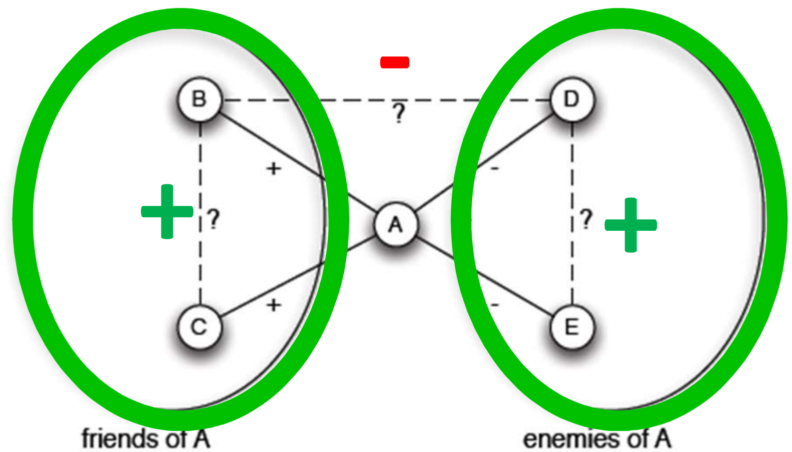
- Pick a node A, and let
  - X be the friends of A
- We have proved
  - All of A's friends are friends with each other
  - A and all his friends are enemies with everyone else.
- Remove the set X and A. Proceed to the subsequent groups in the graph.



A weaker form of structural balance

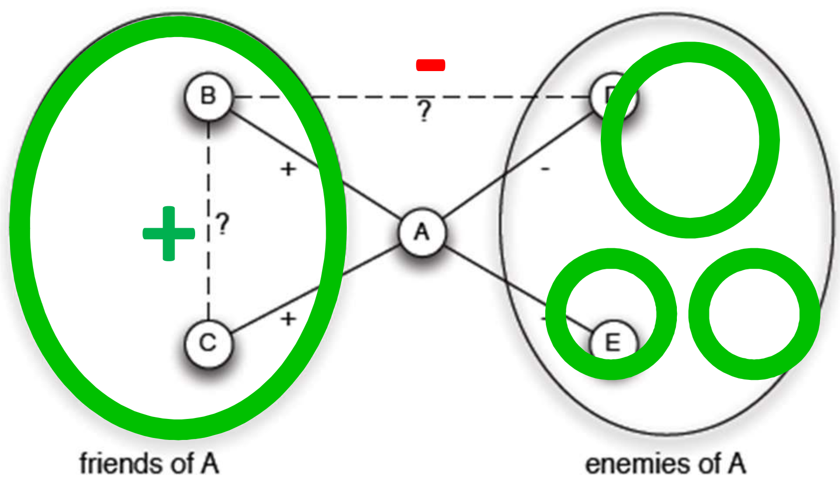
- Structural Balance

  Property:



- Weak Structural Balance

  Property:

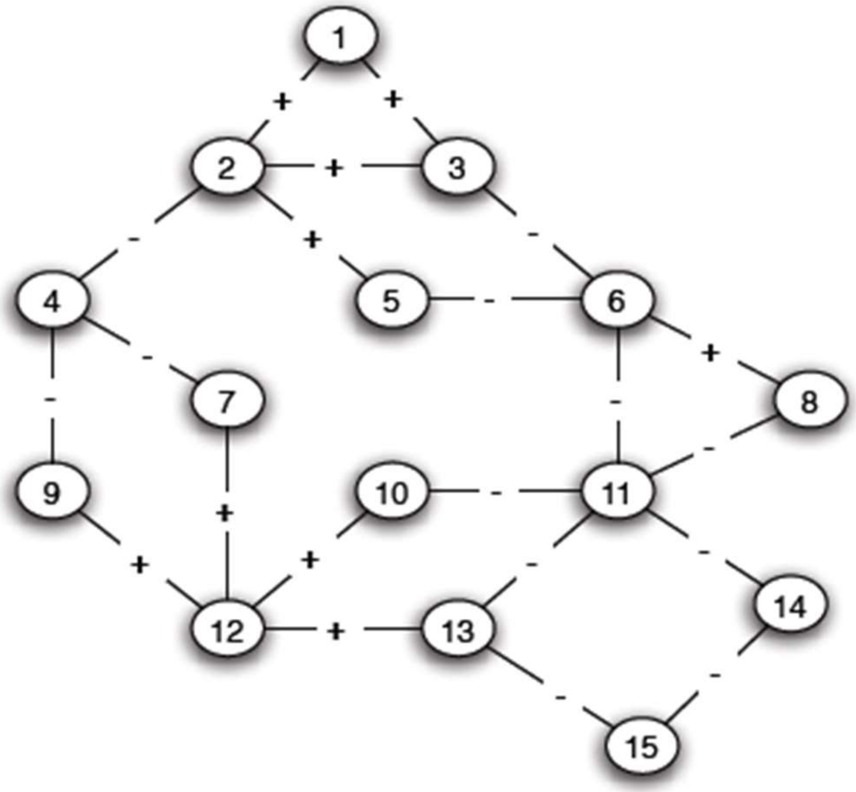


Generalizing the Definition of Structural Balance

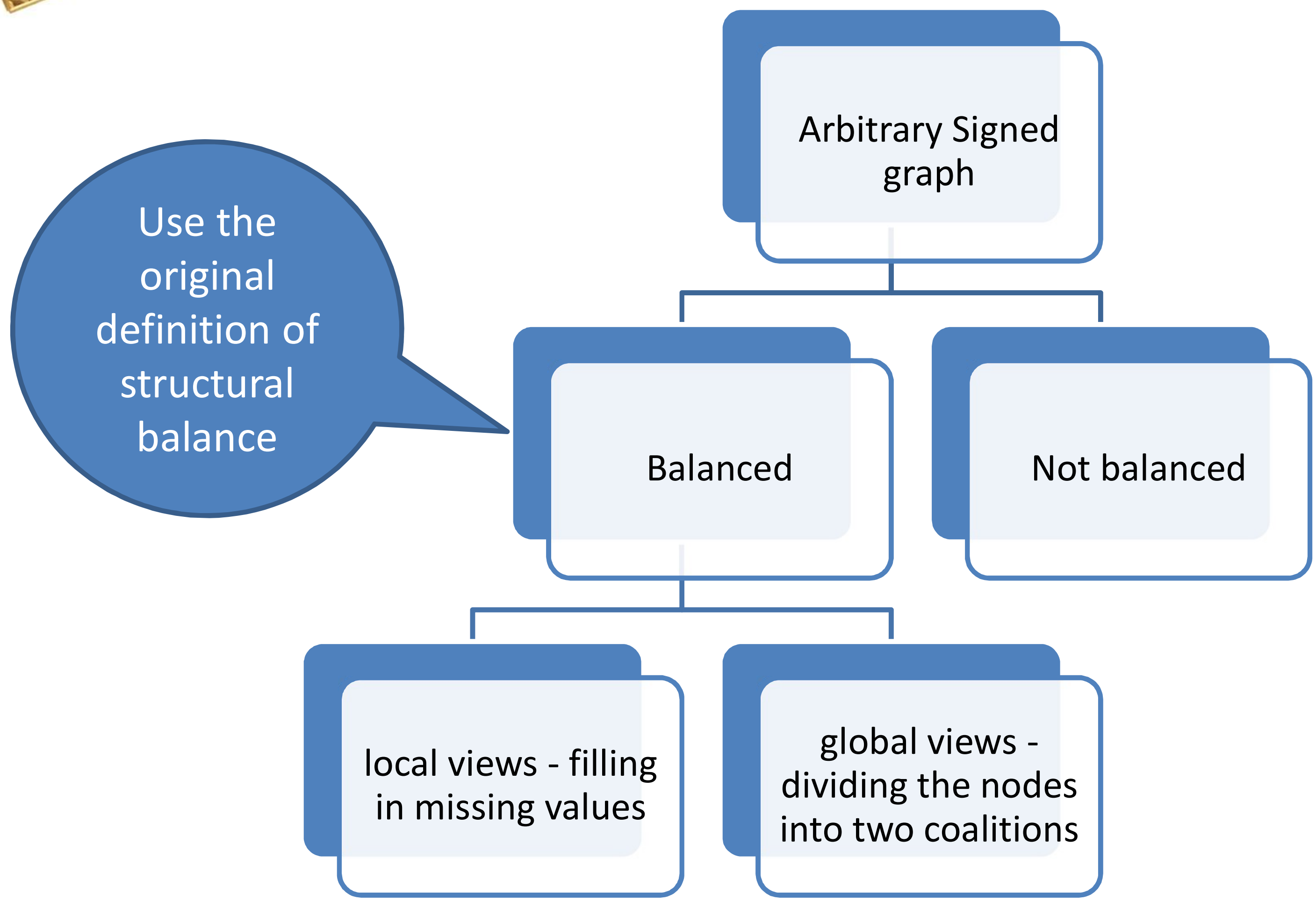
- Assumptions we have made
  1. Complete graphs
     - Each person must be either friend or enemy to others
     - What if some are *unknown* ?
  2. The balanced theorem
     - Apply to all triangles in the graph
     - Can we relax this ?
       – What if *most* triangles are balanced ?

Structural Balance in Arbitrary (Non-Complete) Networks

- Non-complete graph

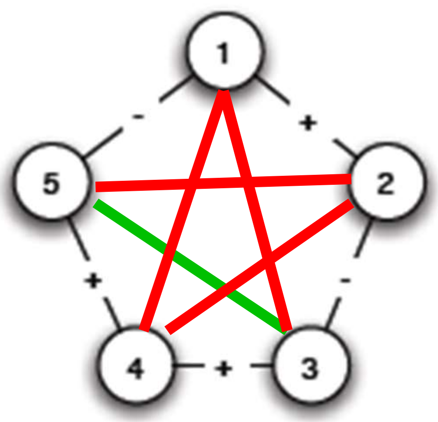


- Edges
  - positive : friendship
  - negative : enmity;
  - absence : two endpoints do not know each other. .


Arbitrary Signed graph
Balanced
Not balanced
local views - filling in missing values
global views - dividing the nodes into two coalitions
Use the original definition of structural balance

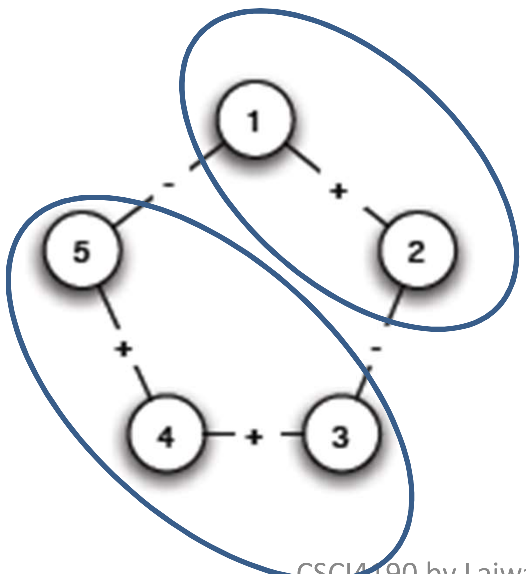
Local view

- Treat non-complete networks as a problem of filling in missing values so as to produce a signed complete graph that is balanced.

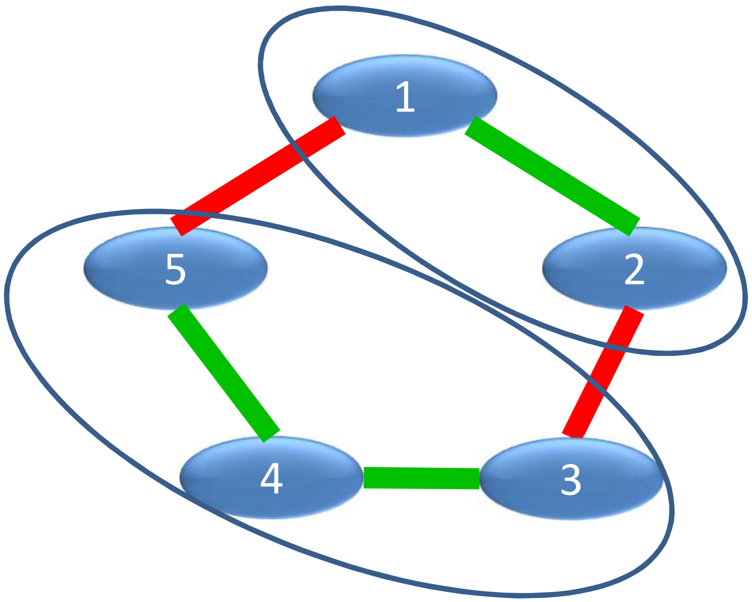


Global view

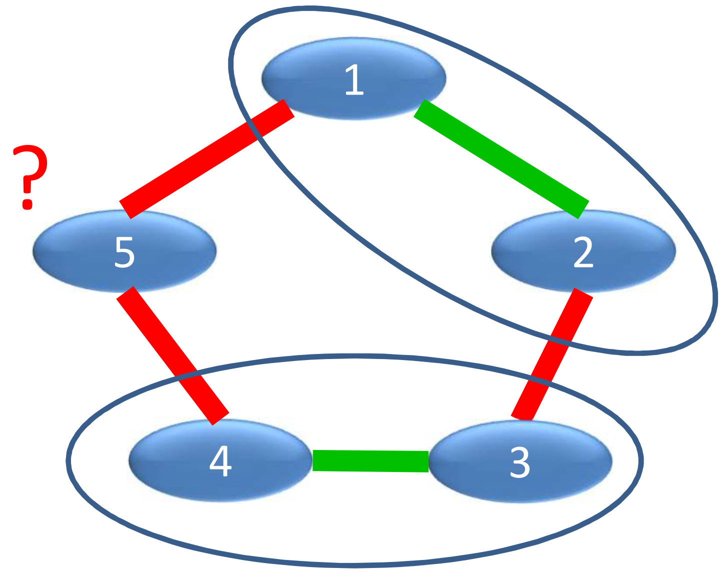
- Divide the nodes into two sets X and Y
  - Positive edge for nodes both inside X
  - Positive edge for nodes both inside Y
  - Negative edge for nodes across X and Y



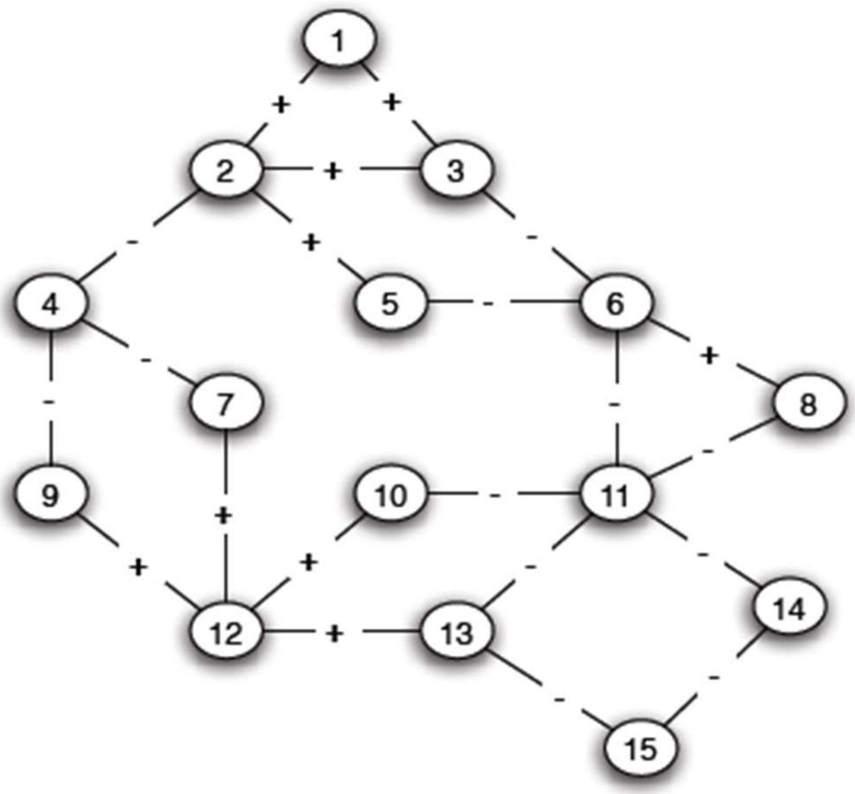
Balanced



Not Balanced

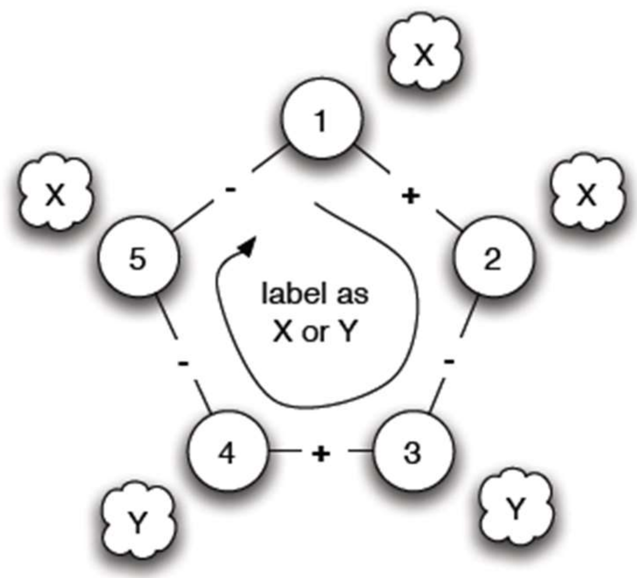


Is this graph balanced ?

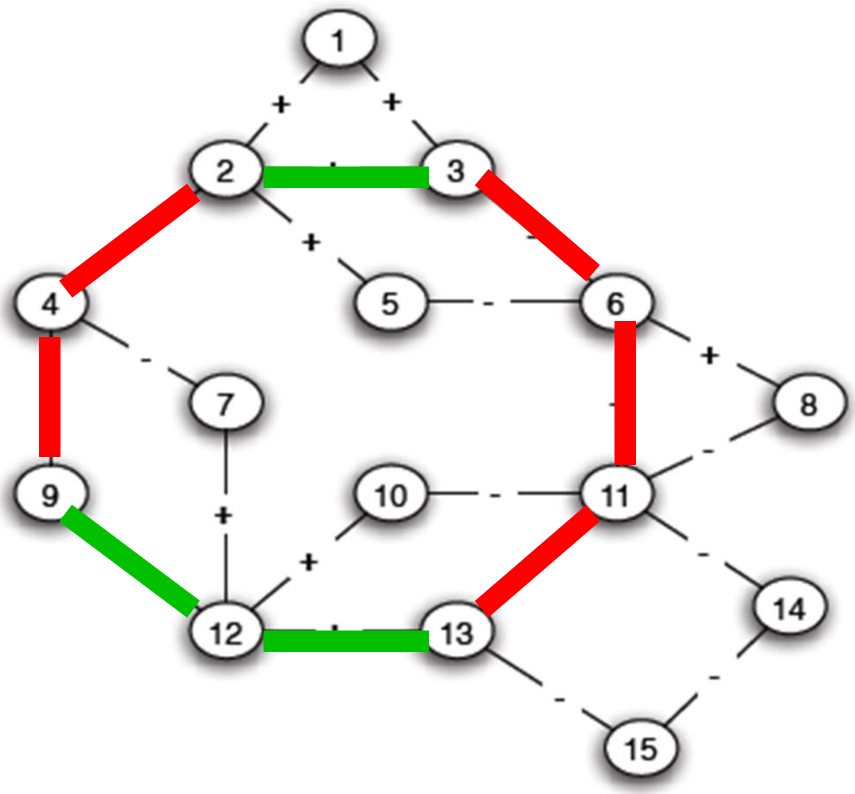


Condition for balanced network

- Claim : A signed graph is balanced if and only if it contains *no cycle with an odd number of negative edges.*

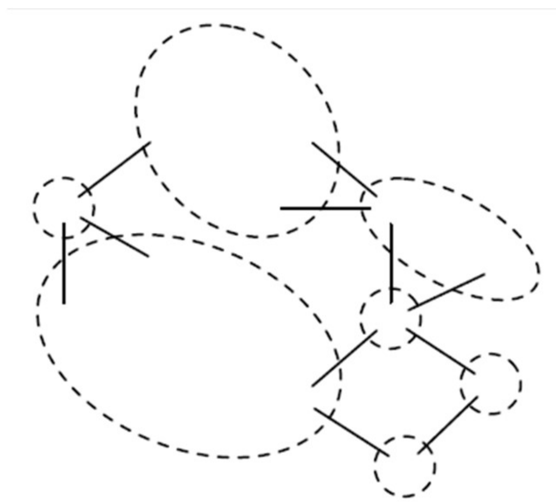


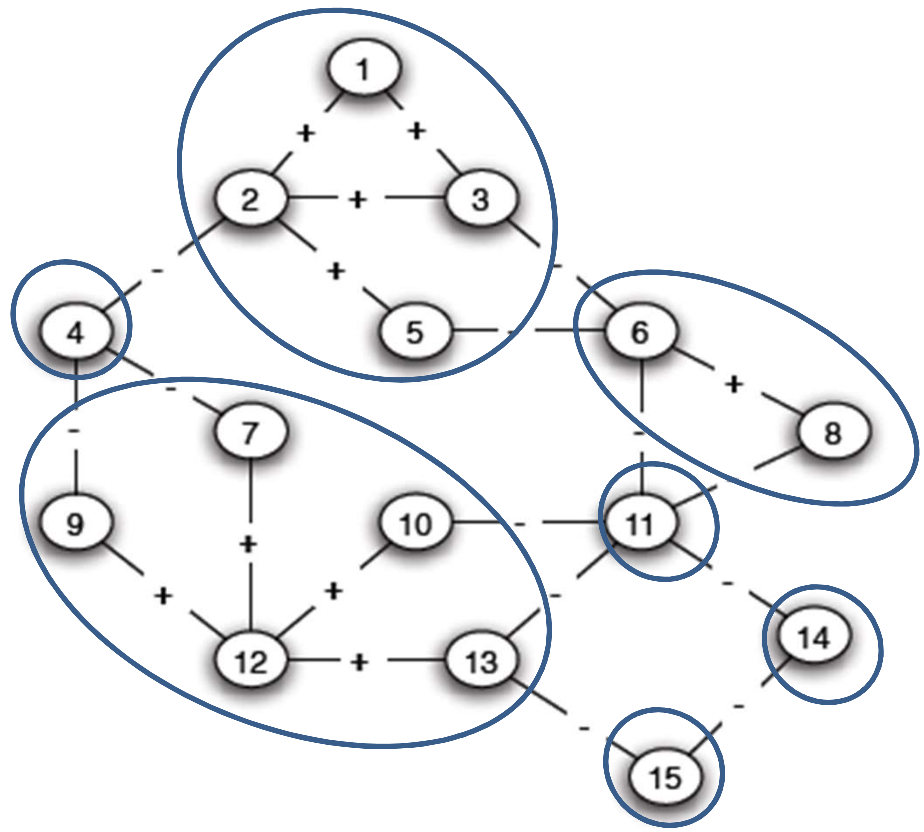
- The graph contains a cycle with an odd number of negative edges. This implies the graph is not balanced.



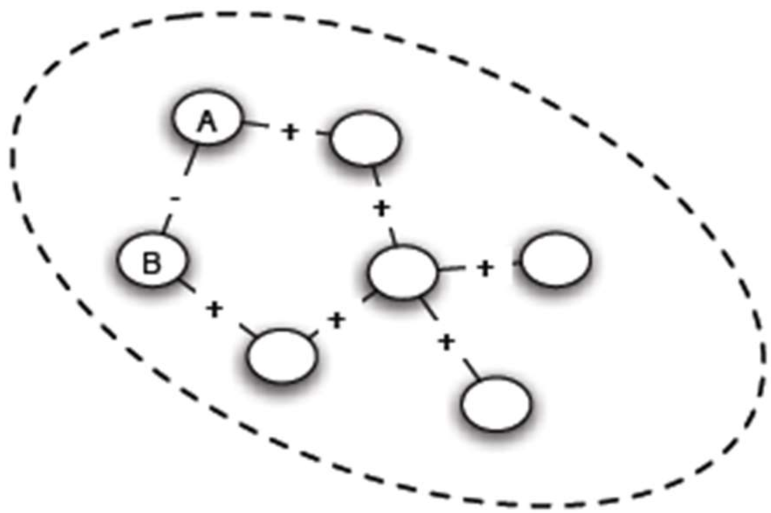
Searching for balanced division

- Step 1 : find the connected components using positive edges. Declare each component to be a <u>supernode</u>.



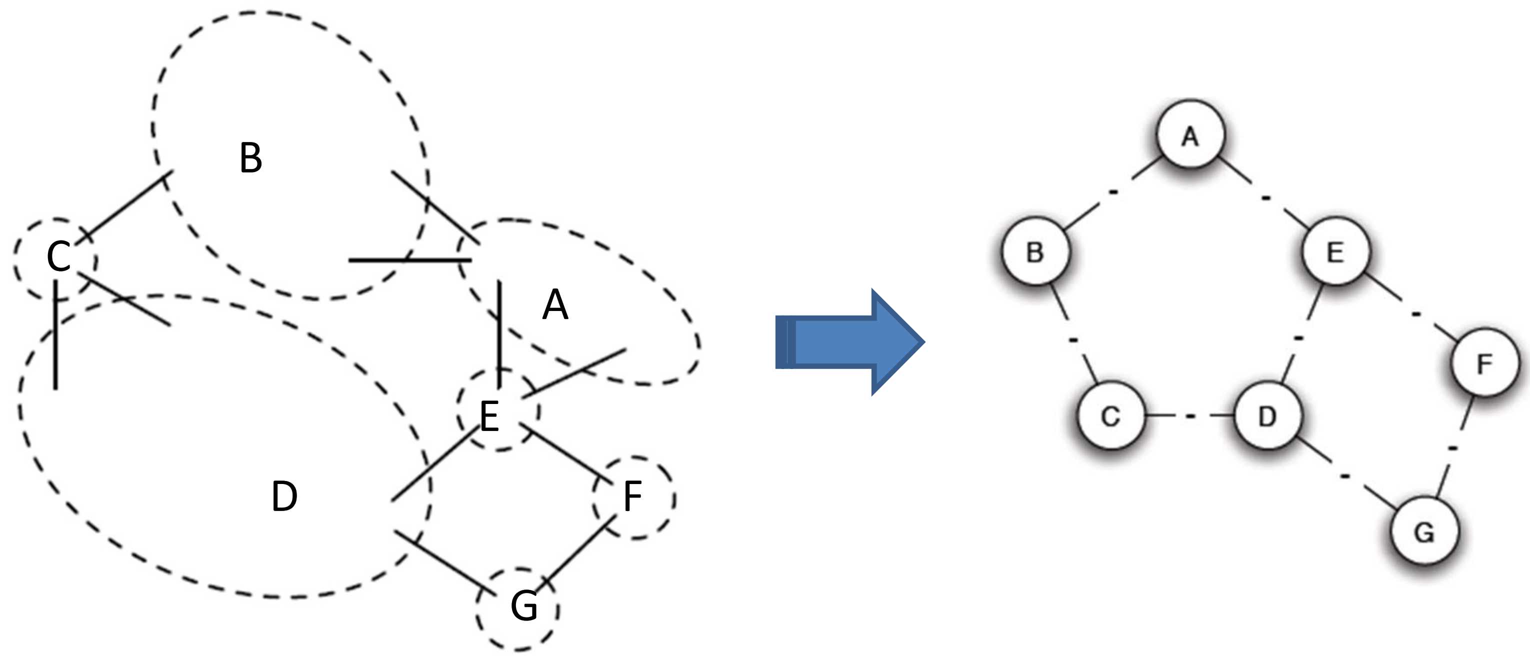


- If any supernode contains a negative edge, there is a cycle with an odd number of negative edges.



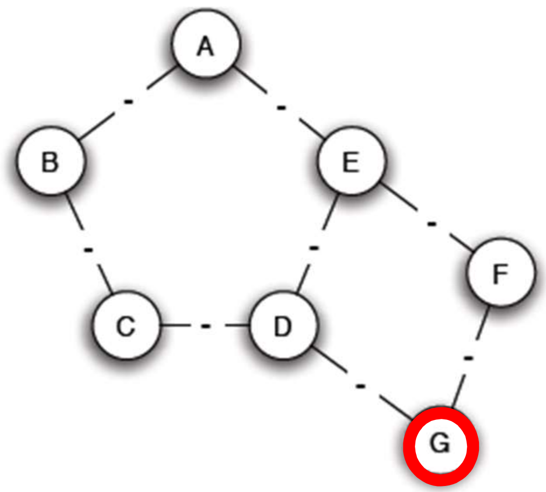
Searching for balanced division

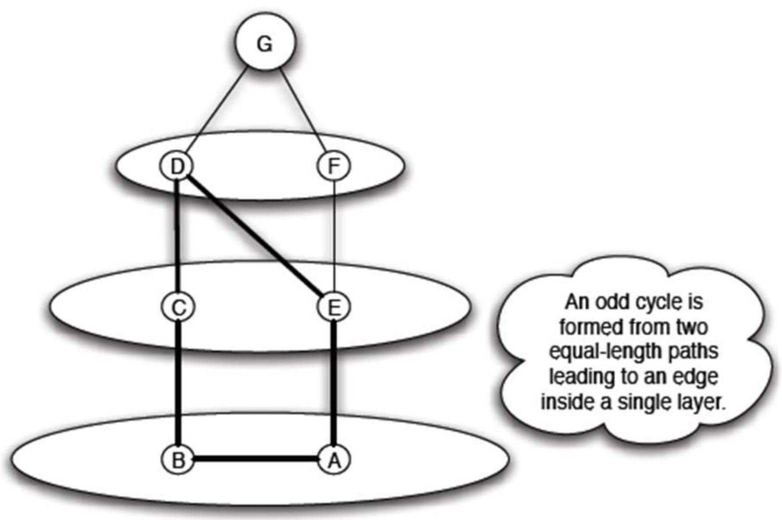
- Step 2 :
  - the supernodes form the reduced graph (negative edges only)
  - breadth first search of the reduced graph



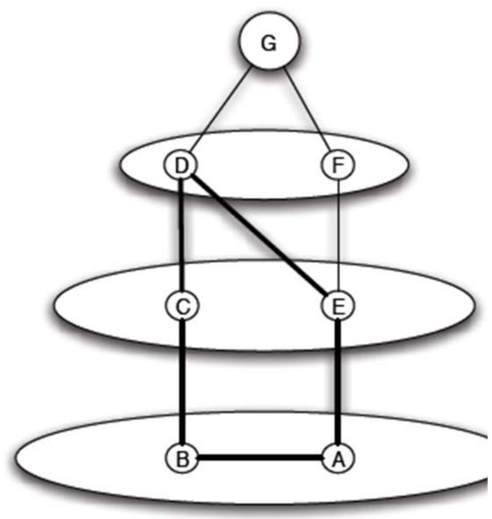
Breadth-first search of the reduced graph

- edges connect two nodes
  - in adjacent layers
    - if all edges are of this type, nodes in alternate layers form a set
  - in the same layer
    - a cycle with odd number of nodes

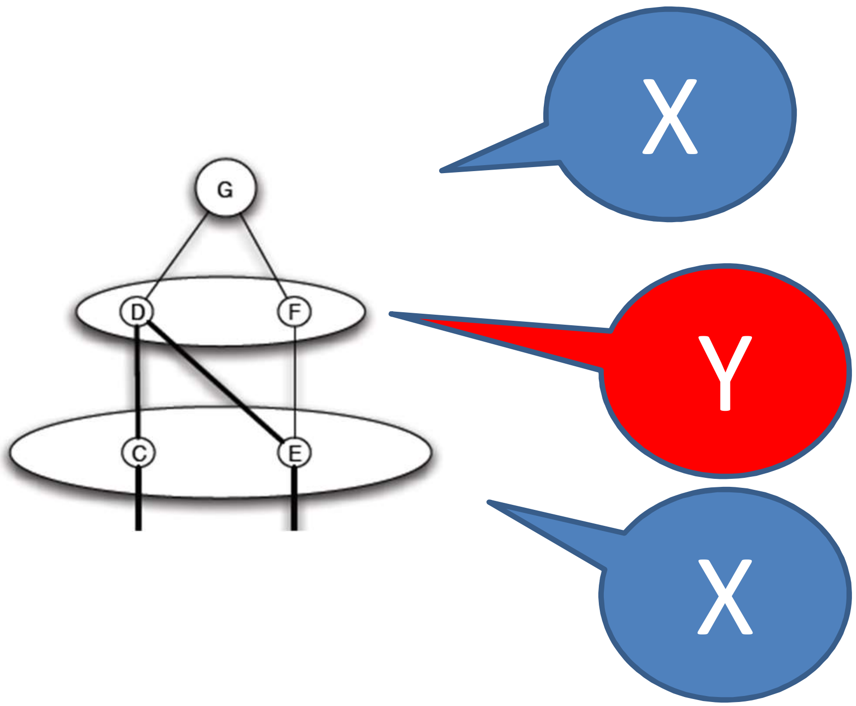




- Each edge connects two nodes
  - in adjacent layers
  - in the same layer
- Edges cannot jump over successive layers



- Each edge connects two nodes
  - in adjacent layers

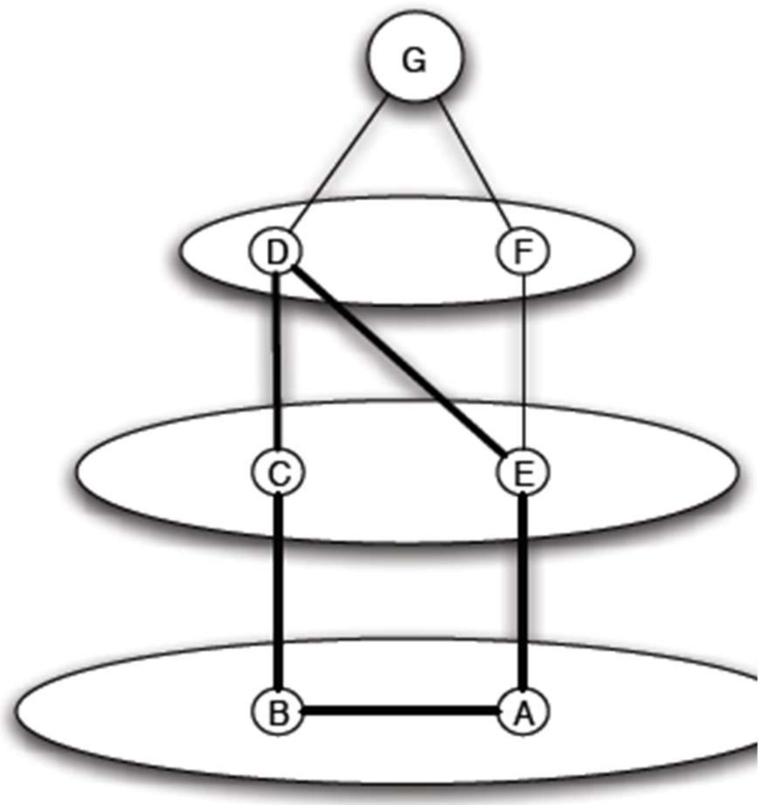


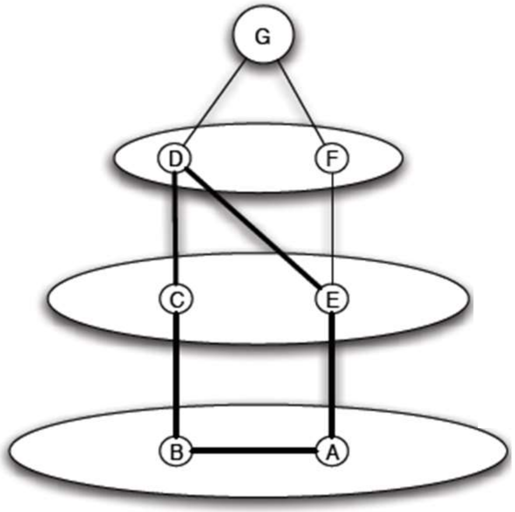
- Each edge connects two nodes
  - in the same layer
- node A and B
- Last node common to them is D

  path length(AD) =k

  path length(BD)= k

  cycle(ABD) = 2k+1




G
D
F
C
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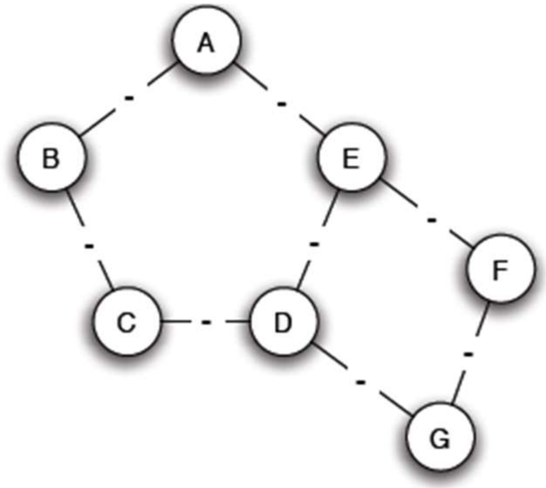
A


A
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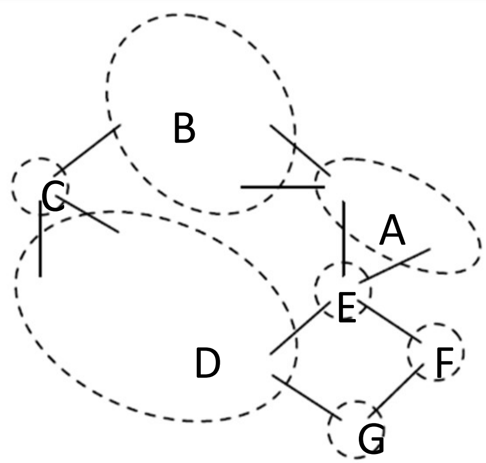
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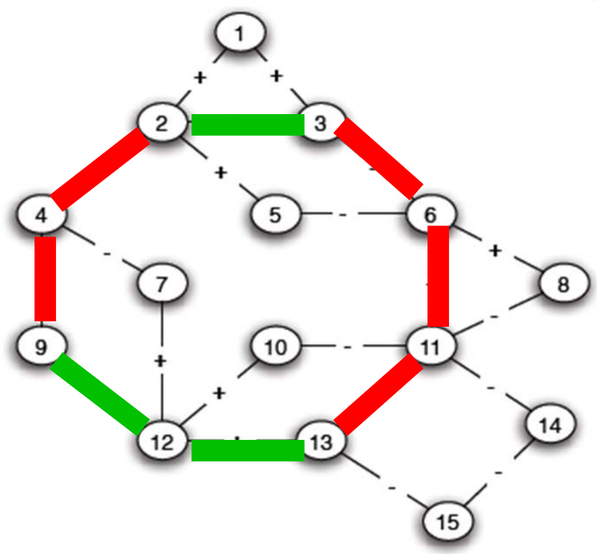
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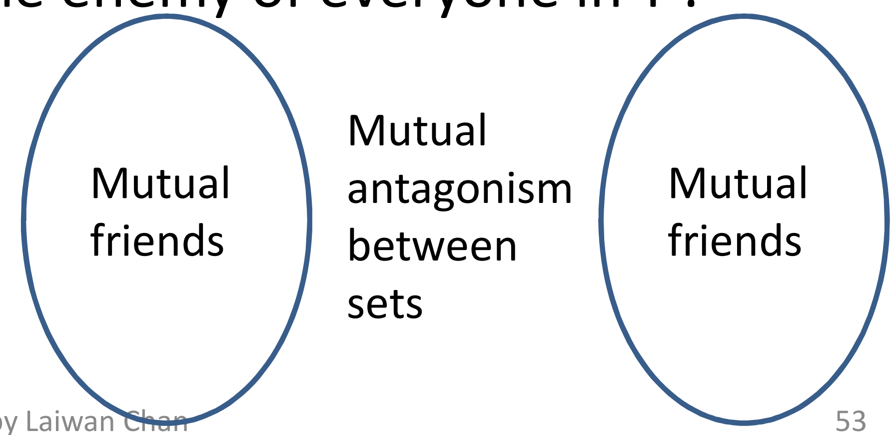
15

Generalizing the Definition of Structural Balance

- Assumptions we have made
  - Complete graphs
    - Each person must be either friend or enemy to others
    - What if some are *unknown* ?
  - The balanced theorem
    - Apply to all triangles in the graph
    - What if *most* triangles are balanced ?

Balance Theorem

- If a labeled complete graph is balanced, then
  - either all pairs of nodes are friends,
  - or else the nodes can be divided into two groups, X and Y ,
    - such that every pair of nodes in X like each other,
    - every pair of nodes in Y like each other,
    - and everyone in X is the enemy of everyone in Y .



Approximately Balance Theorem

- Let ε be any number such that $0 \leq \varepsilon < \frac{1}{8}$, and define $\delta = \sqrt[3]{\varepsilon}$. If at least $1 - \varepsilon$ of all triangles in a labeled complete graph are balanced, then either
  - there is a set consisting of at least $1 - \delta$ of the nodes in which at least $1 - \delta$ of all pairs are friends, or else
  - the nodes can be divided into two groups, X and Y , such that
    - at least $1 - \delta$ of the pairs in X like each other,
    - at least $1 - \delta$ of the pairs in Y like each other, and
    - at least $1 - \delta$ of the pairs with one end in X and the other end in Y are enemies.