請勿攜去
Not to be taken

香 港 中 文 大 學
The Chinese University of Hong Kong

Course Examinations 2017 – 2018 2nd Term

Course Code & Title : ENGG2430A Probability and Statistics for Engineers

Time allowed : 2 hours 0 minutes

Student I.D. No. : _____ Seat No. : _____

- Answer ALL questions using the answer book provided.
- Full mark is 100.
- This is a closed-book examination.
- A sheet of double-sided handwritten A4 note is allowed.
- Calculators are permitted.
- Write your student ID number and seat number clearly on the front page of your answer book.
- Question papers should be returned after the examination.
- Do not turn over this page until you are instructed to do so.

1. (15 marks) A bag contains 5 red balls and 7 green balls.

   a) (5 marks) Suppose three balls are selected randomly, what is the probability that 2 red balls and 1 green ball are selected?

   b) (10 marks) Suppose three balls are selected from the bag sequentially without replacement, and the first ball is discarded without observing the color. If the second ball is red, what is the probability that the third ball is green?

2. (15 marks) Let X and Y be independent geometric random variables, where X has parameter p and Y has parameter q.

   a) (7 marks) Show that the probability of $P(X = Y) = \frac{pq}{p+q-pq}$.

   b) (8 marks) What is the expectation $E[X|X \leq Y]$?

3. (20 marks) Consider a random variable X with a continuous and strictly increasing CDF $F_X(x)$.

   a) (5 marks) Let $Y = F_X(X)$, find the PDF of Y.

   b) (5marks) Let U be a random variable uniformly distributed in [0, 1] and $V = F_X^{-1}(U)$, find the PDF and CDF of V.

   c) (5 marks) Suppose now you have a computer that can generate values of random variable U that uniformly distributed in [0,1]. State a procedure of how you can generate a random variable X that has a given CDF $F_X(x)$.

   d) (5 marks) Can you describe a procedure to generate two random variables R_1 and R_2 that follow a joint distribution $f_{R_1,R_2}(r_1, r_2)$, assuming that both the marginal CDF and conditional CDF are all strictly increasing (note that this assumption is not critical but only for the ease of explanation)?

4. (20 marks) A scientist plans to perform a series of measurements to find the diameter of a particle in nanometer (nm). He believes that the values obtained from the measurements are independent and identically distributed random variables having a common mean μ (the actual diameter) and a common variance of 4nm. Please estimate the number of measurements he needs to perform so that the average value is accurate to within 0.5nm, with probability at least 0.95, based on

   a) (10 marks) Chebyshev's inequality.

   b) (10 marks) Central limit theorem (Please refer to the appendix for the standard normal distribution table).

5. (20 marks) Suppose the joint probability density function of X and Y is

$$f(x,y) = \begin{cases} k(2-x+y)x & 0 \le x \le 1, 0 \le y \le 1 \\ 0 & otherwise \end{cases}$$

   a) (6 marks) Find the value of constant k.

   b) (6 marks) Find the marginal density function of *X*, i.e., $f_X(x)$.

   c) (8 marks) Find the conditional probability density of *X* given Y = y, i.e., $f_X(x|y)$

6. (10 marks) Let X be a random variable with moment generating function $M_X(s) = e^{18s^2+2s}$.

   a) (4 marks) Find E[X].

   b) (6 marks) Let random variable $Y = \frac{1}{6}(X-2)$. Derive the moment generating function of Y.

END

Appendix

Table of the standard normal distribution values ($z \geq 0$)

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.50000	0.50399	0.50798	0.51197	0.51595	0.51994	0.52392	0.52790	0.53188	0.53586
0.1	0.53983	0.54380	0.54776	0.55172	0.55567	0.55962	0.56356	0.56749	0.57142	0.57535
0.2	0.57926	0.58317	0.58706	0.59095	0.59483	0.59871	0.60257	0.60642	0.61026	0.61409
0.3	0.61791	0.62172	0.62552	0.62930	0.63307	0.63683	0.64058	0.64431	0.64803	0.65173
0.4	0.65542	0.65910	0.66276	0.66640	0.67003	0.67364	0.67724	0.68082	0.68439	0.68793
0.5	0.69146	0.69497	0.69847	0.70194	0.70540	0.70884	0.71226	0.71566	0.71904	0.72240
0.6	0.72575	0.72907	0.73237	0.73565	0.73891	0.74215	0.74537	0.74857	0.75175	0.75490
0.7	0.75804	0.76115	0.76424	0.76730	0.77035	0.77337	0.77637	0.77935	0.78230	0.78524
0.8	0.78814	0.79103	0.79389	0.79673	0.79955	0.80234	0.80511	0.80785	0.81057	0.81327
0.9	0.81594	0.81859	0.82121	0.82381	0.82639	0.82894	0.83147	0.83398	0.83646	0.83891
1.0	0.84134	0.84375	0.84614	0.84849	0.85083	0.85314	0.85543	0.85769	0.85993	0.86214
1.1	0.86433	0.86650	0.86864	0.87076	0.87286	0.87493	0.87698	0.87900	0.88100	0.88298
1.2	0.88493	0.88686	0.88877	0.89065	0.89251	0.89435	0.89617	0.89796	0.89973	0.90147
1.3	0.90320	0.90490	0.90658	0.90824	0.90988	0.91149	0.91308	0.91466	0.91621	0.91774
1.4	0.91924	0.92073	0.92220	0.92364	0.92507	0.92647	0.92785	0.92922	0.93056	0.93189
1.5	0.93319	0.93448	0.93574	0.93699	0.93822	0.93943	0.94062	0.94179	0.94295	0.94408
1.6	0.94520	0.94630	0.94738	0.94845	0.94950	0.95053	0.95154	0.95254	0.95352	0.95449
1.7	0.95543	0.95637	0.95728	0.95818	0.95907	0.95994	0.96080	0.96164	0.96246	0.96327
1.8	0.96407	0.96485	0.96562	0.96638	0.96712	0.96784	0.96856	0.96926	0.96995	0.97062
1.9	0.97128	0.97193	0.97257	0.97320	0.97381	0.97441	0.97500	0.97558	0.97615	0.97670
2.0	0.97725	0.97778	0.97831	0.97882	0.97932	0.97982	0.98030	0.98077	0.98124	0.98169
2.1	0.98214	0.98257	0.98300	0.98341	0.98382	0.98422	0.98461	0.98500	0.98537	0.98574
2.2	0.98610	0.98645	0.98679	0.98713	0.98745	0.98778	0.98809	0.98840	0.98870	0.98899
2.3	0.98928	0.98956	0.98983	0.99010	0.99036	0.99061	0.99086	0.99111	0.99134	0.99158
2.4	0.99180	0.99202	0.99224	0.99245	0.99266	0.99286	0.99305	0.99324	0.99343	0.99361
2.5	0.99379	0.99396	0.99413	0.99430	0.99446	0.99461	0.99477	0.99492	0.99506	0.99520
2.6	0.99534	0.99547	0.99560	0.99573	0.99585	0.99598	0.99609	0.99621	0.99632	0.99643
2.7	0.99653	0.99664	0.99674	0.99683	0.99693	0.99702	0.99711	0.99720	0.99728	0.99736
2.8	0.99744	0.99752	0.99760	0.99767	0.99774	0.99781	0.99788	0.99795	0.99801	0.99807
2.9	0.99813	0.99819	0.99825	0.99831	0.99836	0.99841	0.99846	0.99851	0.99856	0.99861
3.0	0.99865	0.99869	0.99874	0.99878	0.99882	0.99886	0.99889	0.99893	0.99896	0.99900
3.1	0.99903	0.99906	0.99910	0.99913	0.99916	0.99918	0.99921	0.99924	0.99926	0.99929
3.2	0.99931	0.99934	0.99936	0.99938	0.99940	0.99942	0.99944	0.99946	0.99948	0.99950
3.3	0.99952	0.99953	0.99955	0.99957	0.99958	0.99960	0.99961	0.99962	0.99964	0.99965
3.4	0.99966	0.99968	0.99969	0.99970	0.99971	0.99972	0.99973	0.99974	0.99975	0.99976
3.5	0.99977	0.99978	0.99978	0.99979	0.99980	0.99981	0.99981	0.99982	0.99983	0.99983