A Short Note for the Midterm: Divergence

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As you probably have noticed, the teaching of this course has not been following the textbook completely. We have been re-organizing the topics in an attempt to make everything sound more natural. For example, we deferred the discussion of "curve length" until line integral by length (while the textbook discusses it without having formally touched upon line integrals). As another example, we deferred the discussion of "curl" until the time it was needed (i.e., path independence).

In a few weeks, you will need to take the midterm exam of this course. The exam uses the same paper for all the sessions of this course. The scope of the exam covers a concept that we have planned to discuss later: *divergence*. This short note aims to give you a formal introduction to this concept, so that you are not disadvantaged in the exam. The concept is in fact fairly simple:

Definition 1. *Let* $\boldsymbol{f}(x, y, z)$ *be a vector function in* $\mathbb{R}^3$*:* $\boldsymbol{f}(x, y, z) = [v_1(x, y, z), v_2(x, y, z), v_3(x, y, z)]$*. The* **divergence** *of* $\boldsymbol{f}$*, denoted as* div $\boldsymbol{f}$*, is a real value defined as*

$$\text{div } \boldsymbol{f} = \frac{\partial v_1}{\partial x} + \frac{\partial v_2}{\partial y} + \frac{\partial v_3}{\partial z}.$$

Note that the divergence is a scalar function. That is, given a point $p = (x, y, z)$, the divergence of $\boldsymbol{f}$ at p is a real value div $\boldsymbol{f}(x, y, z)$.

Example. Suppose that $\boldsymbol{f}(x, y, z) = [y \sin x, e^y z^2, xz^3]$. Then,

$$\begin{aligned} \textit{div } \boldsymbol{f} &= \frac{\partial v_1}{\partial x} + \frac{\partial v_2}{\partial y} + \frac{\partial v_3}{\partial z}. \\ &= y \cos x + e^y z^2 + 3xz^2. \end{aligned}$$

The divergence of $\boldsymbol{f}$ at $p = (1, 2, 3)$ equals $2 \cos(1) + 9e^2 + 27$. $\square$