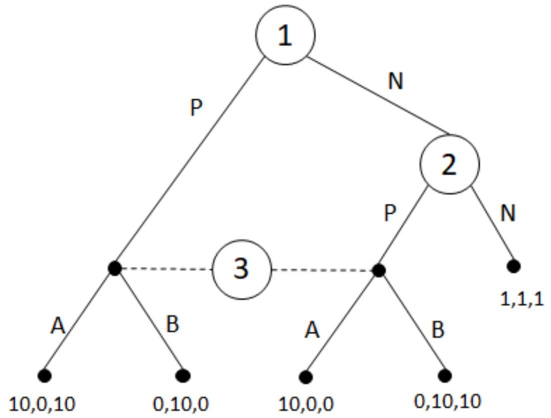
CSCI5350 Assignment2 Suggested Solution

1. (30pts)



   (a) $\mathbb{I}_1 = \{\phi\}$
   $\mathbb{I}_2 = \{N\}$
   $\mathbb{I}_3 = \{\{P, NP\}\}$

   (b) Yes, since $X(P) = X(NP) = \{P, NP\}$

   (c) $\beta_1 = (\phi \mapsto (P(1/2), N(1/2)))$
   $\beta_2 = (N \mapsto (P(1/2), N(1/2)))$
   $\mu = \{\{P, NP\} \mapsto (P(1/2), NP(1/2))\}$
   such strategy does not exist.
   Justification: Cathy can play any strategy $\beta_3 = (\{P, NP\} \mapsto (A(p), B(1-p)))$ for her to be rational. However, Amy and Beatrice using β_1 and β_2 will not be both rational for any p.

   (d) No. μ is not derived from β_1, β_2. $Pr(P|\{P, NP\}) = 2/3, Pr(NP|\{P, NP\}) = 1/3$.

   (e) No. The assessment is not consistent.

   (f) (The answer is not unique)
   assessment $= (\beta, \mu)$
   $\beta_1 = (\phi \mapsto P)$
   $\beta_2 = (N \mapsto P)$
   $\beta_3 = (\{P, NP\} \mapsto A)$
   $\mu = \{\{P, NP\} \mapsto (P(1), NP(0))\}$

2. (20pts)

Cathy: A

		Beatrice P	Beatrice N
Amy	P	0,0,0	10,0,10
	N	10,0,0	1,1,1

Cathy: B

		Beatrice P	Beatrice N
Amy	P	0,0,0	0,10,0
	N	0,10,10	1,1,1

(a) Yes, $NE = (P, N, A), (N, P, B)$

(b) i. $v_i = min_{a_j:j\neq i} max_{a_i} u_i(a_{-i}, a_i)$
$v_1 = v_2 = v_3 = 0$

ii. (The answer is not unique)
payoff profile: $w = (1, 1, 1)$

iii. (The answer is not unique)
trigger strategy:
play (N, N, A) repeatedly, $\gamma = 1, w = (1, 1, 1)$.
if player 1 deviates, other players will play (P,A);
if player 2 deviates, other players will play (P,A);
if player 3 deviates, other players will play (P,P).

3. (30pts)

		P2 Y	P2 F
P1	Y	0,0	-3,3
	F	5,-3	-5,-5

(a) $NE = (Y, F), (F, Y)$

(b) Let the mixed strategies of player 1 and 2 be α_1 and α_2.
We have $\alpha_1 = (2/5, 3/5)$ and $\alpha_1 = (2/7, 5/7)$

(c) $v_1 = -3$

(d) $v_2 = -3$

(e) Yes. Consider playing $(Y, Y), (F, Y), (F, F)$ repeatedly, $\gamma = 3, w = (0, -8/3)$

(f) Yes. Since $(0, -8/3) > (-3, -3)$

(g) (The answer is not unique)
Nash equilibrium payoff profile: $w = (0, -8/3)$ trigger strategy:
play $(Y, Y), (F, Y), (F, F)$ repeatedly, $\gamma = 3$.
if player 1 deviates, player 2 will play F forever;
if player 2 deviates, player 1 will play F forever.

4. (20pts)

(a)

	worth of each outcome			sum for each state without aj			
	A2	A4	A5	A2	A4	A5	tax j
a1	-1	-2	-3	* -6	* -6	-8	0
a2	0	-1	-2	* -7	* -7	-9	0
a3	-3	-4	-5	* -4	* -4	-6	0
a4	-1	0	-1	* -6	-8	-10	0
a5	-2	-1	0	* -5	-7	-11	0
Sum	* -7	-8	-11				

outcome = $A2$

(b) every players pay zero tax

(c) (The answer is not unique)
Yes. Consider a_4, a_5 collude and report the cost to $A2$ be 10.

	worth of each outcome			sum for each state without aj			
	A2	A4	A5	A2	A4	A5	tax j
a1	-1	-2	-3	-23	* -6	-8	0
a2	0	-1	-2	-24	* -7	-9	0
a3	-3	-4	-5	-21	* -4	-6	0
a4	-10	0	-1	-14	* -8	-10	0
a5	-10	-1	0	-14	* -7	-11	0
Sum	-24	* -8	-11				

new outcome = $A4$, and payoff for a_4, a_5 increased.