Exercises: Path Independence of Line Integral

Problem 1. Calculate $\int_C d\boldsymbol{r} = \int_C dx + \int_C dy$ where C is a smooth curve from point $p = (1, 2)$ to $q = (3, 4)$.

Problem 2. Calculate $\int_C 2xy\,dx + \int_C x^2\,dy$ where C is a smooth curve from point $p = (1, 2)$ to $q = (3, 4)$.

Problem 3. Calculate $\int_C yz\,dx + \int_C xz\,dy + \int_C xy\,dz$ where C is a smooth curve from point $p = (1, 2, 3)$ to $q = (3, 4, 5)$.

Problem 4. Calculate $\int_C yz\,dx + \int_C xz\,dy + \int_C xy\,dz$ where C is the curve given by $\boldsymbol{r}(t) = [\cos(t), \sin(t), 1]$ with $t \in [0, 2\pi]$.

Problem 5. Suppose that $\int_C f_1(x, y, z)dx + \int_C f_2(x, y, z)\,dy + \int_C f_3(x, y, z)\,dz$ equals 0 for *any* closed curve C. Prove that the integral is path independent.