

Exercises: Eigenvalues, Eigenvectors, and Similarity Transformation

Problem 1. Find all the eigenvalues and eigenvectors of $\mathbf{A} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$.

Problem 2. Let \mathbf{A} be an $n \times n$ square matrix. Prove: \mathbf{A} and \mathbf{A}^T have exactly the same eigenvalues.

Problem 3. Let \mathbf{A} be an $n \times n$ square matrix. Prove: \mathbf{A}^{-1} exists if and only if 0 is not an eigenvalue of \mathbf{A} .

Problem 4. Let \mathbf{A} be an $n \times n$ square matrix such that \mathbf{A}^{-1} exists. Prove: if λ is an eigenvalue of \mathbf{A} , then $1/\lambda$ is an eigenvalue of \mathbf{A}^{-1} .

Problem 5. Diagonalize the following matrix:

$$\mathbf{A} = \begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix}$$

Problem 6. Consider again the matrix \mathbf{A} in Problem 5. Calculate \mathbf{A}^t for any integer $t \geq 1$.

Problem 7. Diagonalize the matrix \mathbf{A} in Problem 1.