CSCI 5350 Advanced Topics in Game Theory

Discussion Session 12

Game Theory Exercise 4

Suppose there are five agents in a market with transferable payoff. Agents 1 and 2 each has two red tickets; agents 3, 4 and 5 each has two green tickets. The rule of the game says that a coalition can obtain $1 in exchange of one red ticket *and* one green ticket.

(a) Find the coalitional game with transferable payoff that is associated with this market.

 i. **(2 marks)** What is the set N of players? and

 ii. **(5 marks)** What is the worth of each of the coalitions?

(b) **(5 marks)** Find one feasible payoff profile that is in the core. Justify your answer.

From 2004-05 Final Exam Q5

Three players are considering forming a team to work on a project. If they all contribute to the project, then they can sell the final product at the price of \$100. However, if only any two of them work on the project, then they can still produce a product, but the price of the product depends on who work on it. If Players 1 and 2 work together, then they can make a product that sells at \$60. For Players 1 and 3, their product can sell at \$70. Finally, if Players 2 and 3 work together to build a product, then the product can sell at a price as high as \$90. On the other hand, for each of these Players, none can make a product (hence he receives no money) if he does not work with others. The situation can be modelled as a coalitional game with transferable payoff $G = \langle N, v \rangle$.

(a) **(1 mark)** Write down N in the game G.
(b) **(2 marks)** Write down v in the game G.
(c) **(2 marks)** Is the game G cohesive? Justify your answer.
(d) **(5 marks)** Give one payoff profile that is in the core. Justify your answer.
(e) **(4 marks)** Describe the core (or list all members of the core) of the game G.
(f) **(6 marks)** Describe one stable set. Justify your answer. What is the 'standard of behaviour' in your answer?

From 2008-09 Final Exam Q3

Two players agree to share a piece of cake using the following procedure. First, player 1 cuts the cake into two pieces. Then player 2 chooses one of these two pieces for himself (so that player 1 gets the one that player 2 does not choose). Each player prefers more of the cake to less. For simplicity, assume that the whole piece of cake is homogeneous (so that you do not need to care who gets the cherry, who gets more chocolate, and so on).

(a) **(6 marks)** Formulate this situation as an extensive game with perfect information. You may present your solution using a game tree.

(b) **(6 marks)** Find all subgame perfect equilibria of the game.

(c) Now suppose after player 1 cuts the cake, there is a probability of $3/8$ that the smaller piece is stolen by someone else, so that only the larger piece is left. (If the two pieces are of the same size, then arbitrary one is stolen.) If the smaller piece is *not* stolen, then the game goes on normally, and player 2 chooses one of these two pieces for himself (so that player 1 gets the one that player 2 does not choose). However, if the smaller piece is really stolen, player 1 will cut the remaining piece into two, and immediately asks player 2 to choose, so that no cake will be stolen again.

i. **(6 marks)** Formulate the whole situation as an extensive game with perfect information and chance moves. You may present your solution using a game tree.

ii. **(4 marks)** Find one Nash equilibrium of the game.

From 2004-05 Final Exam Q3

End