CSCI3320 Written Homework 7 Chung Tsz Ting / 1155110208

1)

Proof of (10.4)

$$\text{As we have } \boldsymbol{x} = \boldsymbol{x}_p + r\frac{\boldsymbol{w}}{\|\boldsymbol{w}\|} \text{ and } g(\boldsymbol{x}_p) = \boldsymbol{w}^{\boldsymbol{T}}\boldsymbol{x}_p + w_0 \text{ ,}$$

$$g(\boldsymbol{x}_p) = \boldsymbol{w}^{\boldsymbol{T}}(\boldsymbol{x} - r\frac{\boldsymbol{w}}{\|\boldsymbol{w}\|}) + w_0$$

$$0 = \boldsymbol{w}^{\boldsymbol{T}}(\boldsymbol{x} - r\frac{\boldsymbol{w}}{\|\boldsymbol{w}\|}) + w_0$$

$$0 = \boldsymbol{w}^{\boldsymbol{T}}\boldsymbol{x} - r\frac{\boldsymbol{w}^{\boldsymbol{T}}\boldsymbol{w}}{\|\boldsymbol{w}\|} + w_0$$

$$0 = \boldsymbol{w}^{\boldsymbol{T}}\boldsymbol{x} - r\frac{\|\boldsymbol{w}\|^2}{\|\boldsymbol{w}\|} + w_0$$

$$0 = \boldsymbol{w}^{\boldsymbol{T}}\boldsymbol{x} + w_0 - r\|\boldsymbol{w}\| \quad (10.4.1)$$

$$0 = \boldsymbol{g}(\boldsymbol{x}) - r\|\boldsymbol{w}\|$$

$$r = \frac{\boldsymbol{g}(\boldsymbol{x})}{\|\boldsymbol{w}\|}$$

Proof of (10.5)

$$\text{As proved as above,} \qquad 0 = \boldsymbol{w}^{\boldsymbol{T}}\boldsymbol{x} + w_0 - r\|\boldsymbol{w}\| \quad (10.4.1)$$

$$\text{for } \boldsymbol{x} \text{ be a vector of zeros,}$$

$$0 = \boldsymbol{w}^{\boldsymbol{T}}\boldsymbol{0} + w_0 - r_0\|\boldsymbol{w}\|$$

$$r_0\|\boldsymbol{w}\| = w_0$$

$$r_0 = \frac{w_0}{\|\boldsymbol{w}\|}$$

2)

$$y_i = \frac{\exp(a_i)}{\sum_j \exp(a_j)}$$

$$\frac{dy_i}{da_j} = \frac{d}{da_j}\left(\frac{\exp(a_i)}{\sum_j \exp(a_j)}\right)$$

$$= \frac{\sum_j \exp(a_j)\frac{d\exp(a_i)}{da_j} - \exp(a_i)\frac{d\sum_j \exp(a_j)}{da_j}}{\left(\sum_j \exp(a_j)\right)^2}$$

$$\frac{dy_i}{da_i} = \frac{\sum_j \exp(a_j)\exp(a_i) - \exp(a_i)\exp(a_i)}{\left(\sum_j \exp(a_j)\right)^2} \; for\ i = j$$

$$= \frac{\exp(a_i)\left(\sum_j \exp(a_j) - \exp(a_i)\right)}{\sum_j \exp(a_j) \cdot \sum_j \exp(a_j)}$$

$$= \frac{\exp(a_i)\left(\sum_j \exp(a_j) - \exp(a_i)\right)}{\sum_j \exp(a_j) \cdot \sum_j \exp(a_j)}$$

$$= y_i\ (1 - y_i)$$

$$= y_i\ (1 - y_j)$$

$$\frac{dy_i}{da_j} = \frac{\sum_j \exp(a_j) \cdot 0 - \exp(a_i)\sum_j \exp(a_j)}{\left(\sum_j \exp(a_j)\right)^2} \; for\ i \neq j$$

$$= \frac{-\exp(a_i)\sum_j \exp(a_j)}{\left(\sum_j \exp(a_j)\right)^2}$$

$$= \frac{\exp(a_i)\left(-\sum_j \exp(a_j)\right)}{\sum_j \exp(a_j)\sum_j \exp(a_j)}$$

$$= y_i\ (-y_j)$$

$$= y_i\ (0 - y_j)$$

Thus, we have

$$\frac{dy_i}{da_j} = y_i\ (\delta_{ij} - y_j)$$