Information cascades

Information cascades

- People are connected by a network and one would influence the other's behavior and decision.
  - products they buy
  - political positions they support
  - activities they pursue
  - technologies they use
  - opinions they hold
- Why such influence occurs ?
- Networks serve to aggregate individual behavior and produce population-wide outcomes

Information Cascade (Herding)

- When you wonder which restaurant to go in a new area, you ….
- When you want to buy a new mobile phone, you ….

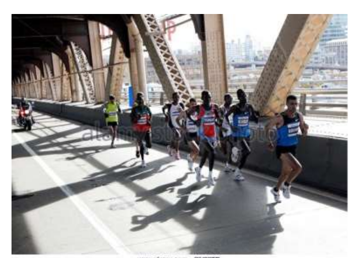
- When you arrive at a restaurant A recommended to you, you see no one is eating there.
- But restaurant B next door is nearly full.
- Choose between
  - Private information
  - Sequences or multiple independent but imperfect information

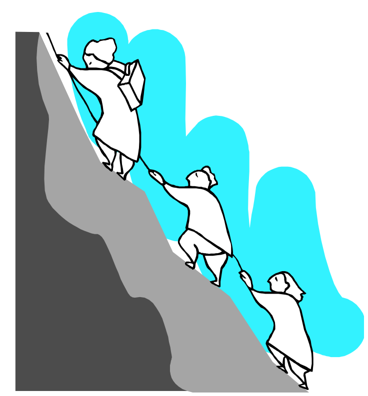
An experiment

- one person stood on a street corner and stare up into the sky
  - few passersby stopped
- five people were staring up
  - more passersby stopped
- fifteen people were staring up
  - 45% passersby stopped

- Information cascade : a social force for conformity

- Reasons why individual might imitate the behavior of others
  - Informational effect :
    - The behavior of others conveys information about what they know
    - Peer pressure
    - Imitate what others (e.g. idols) are doing
  - Network effect (direct benefit effect) : you incur an explicit benefit when you align behavior with the behavior of others.
    - Join the chat group when many of your friends have already joined.





- examples of direct-benefit effects

Telegram

Informational effect and direct benefit effect

- Many decisions exhibit both effects
  - social networks
  - enrolling in a course that many of your friends did
- In some cases, the two effects are in conflict
  - long queue in front of a popular restaurant

A game to guess the urn

- Two urns

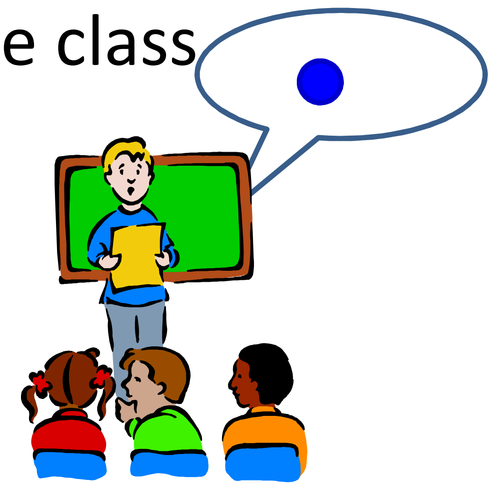
50% 50%

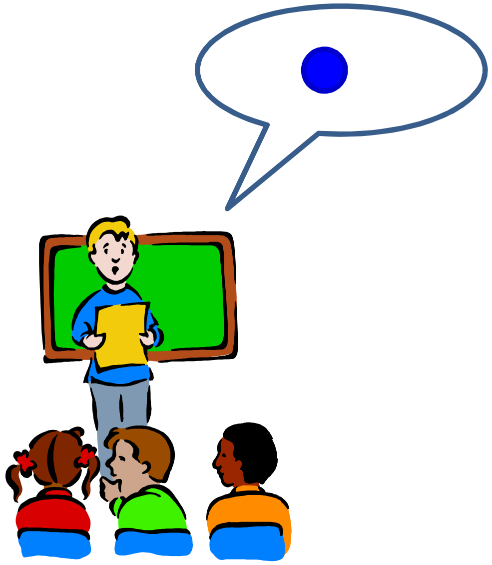
majority – red urn

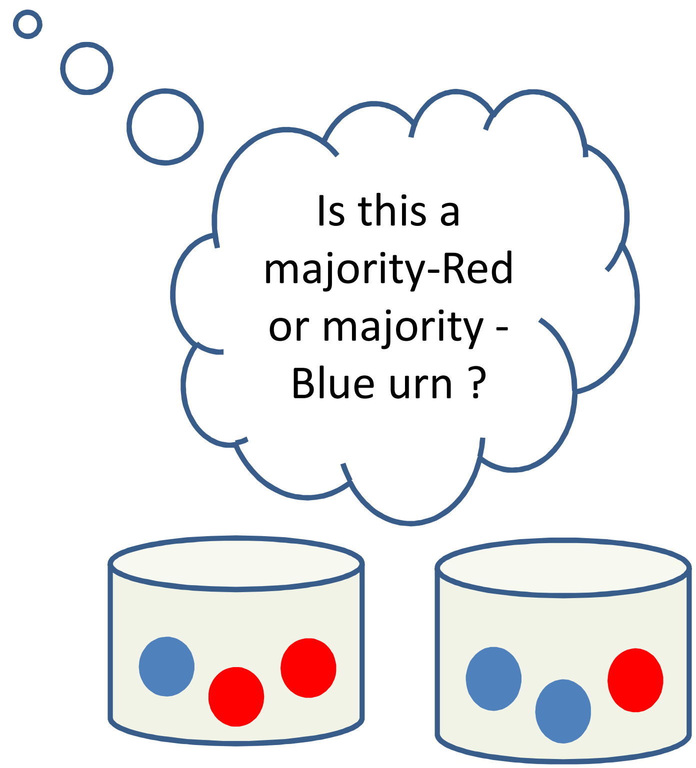
majority – blue urn

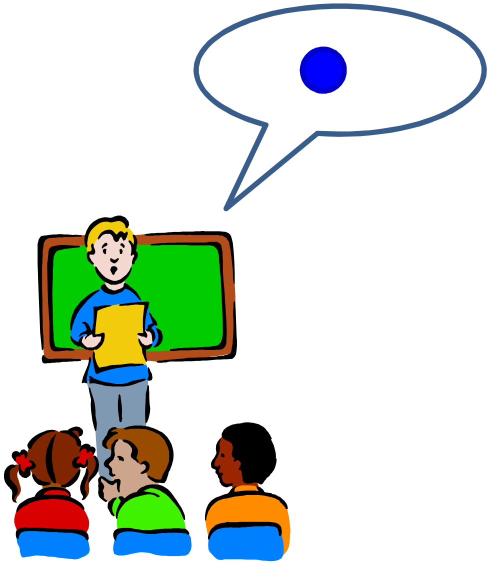
A game to guess the urn

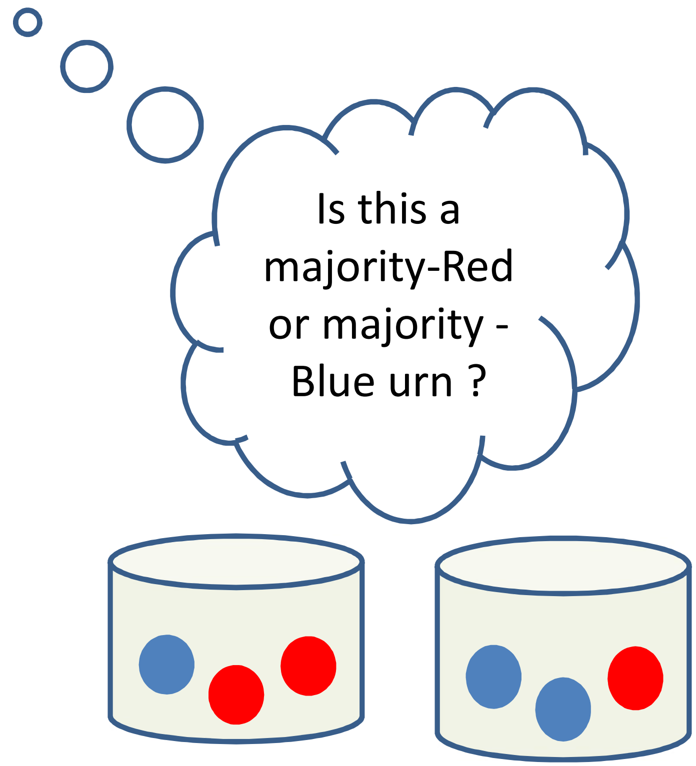
- Each student
  - comes forward and draw a marble
  - looks at the color by himself only
  - puts the marble back to the urn
  - announces his guess of the urn to the class






Is this a majority-Red or majority - Blue urn ?

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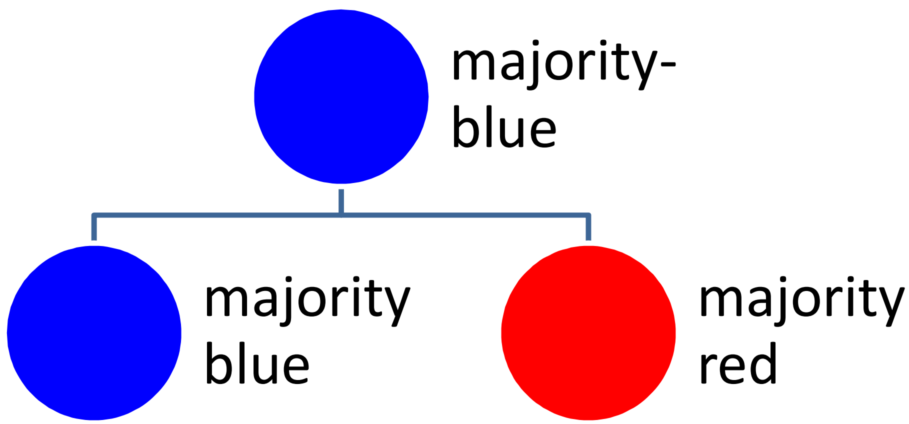
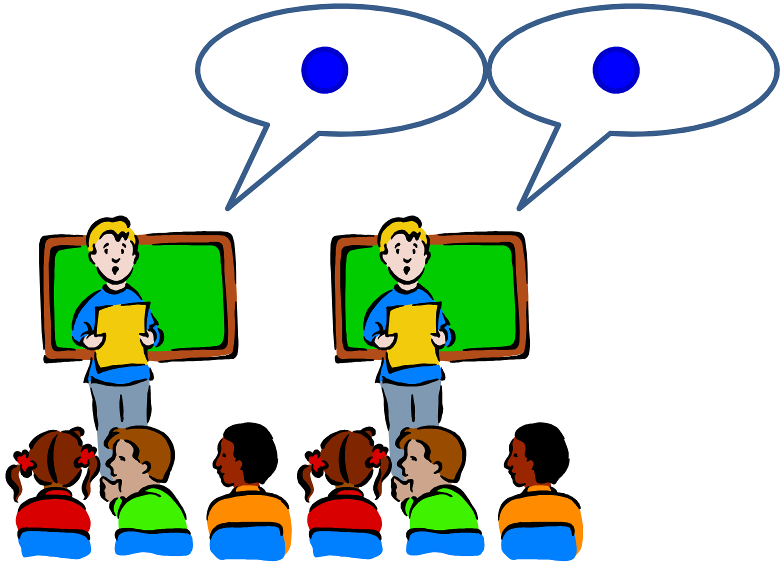
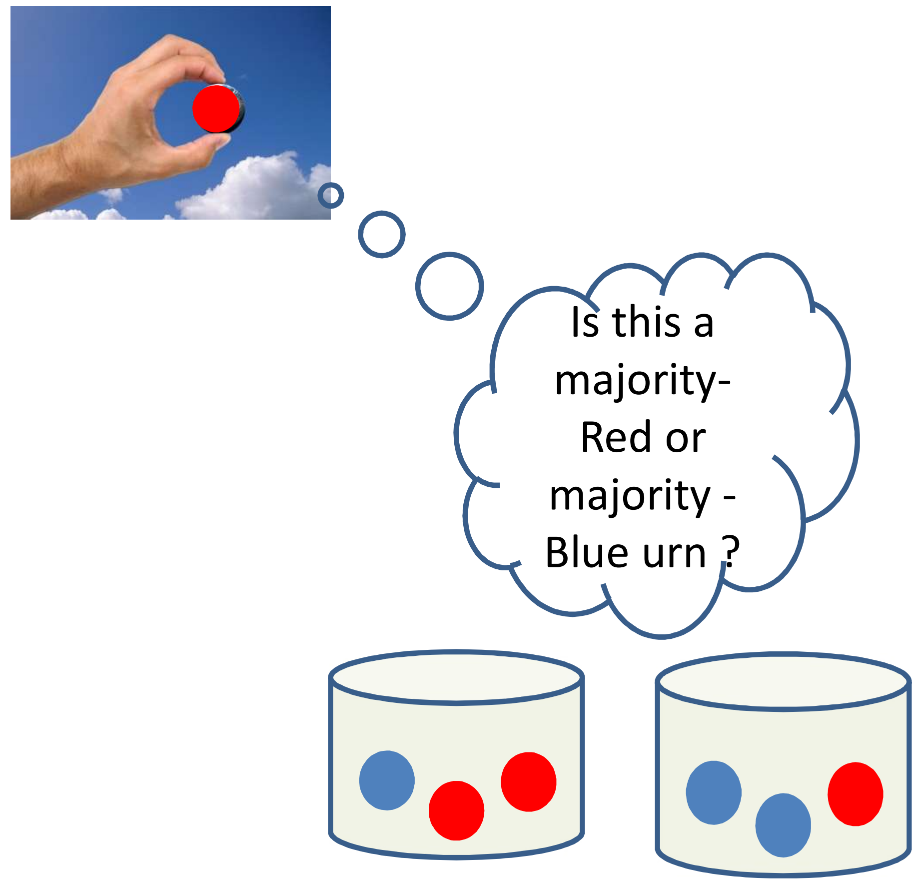
or majority -
Blue urn ?


majority-
blue
majority
blue
majority
red

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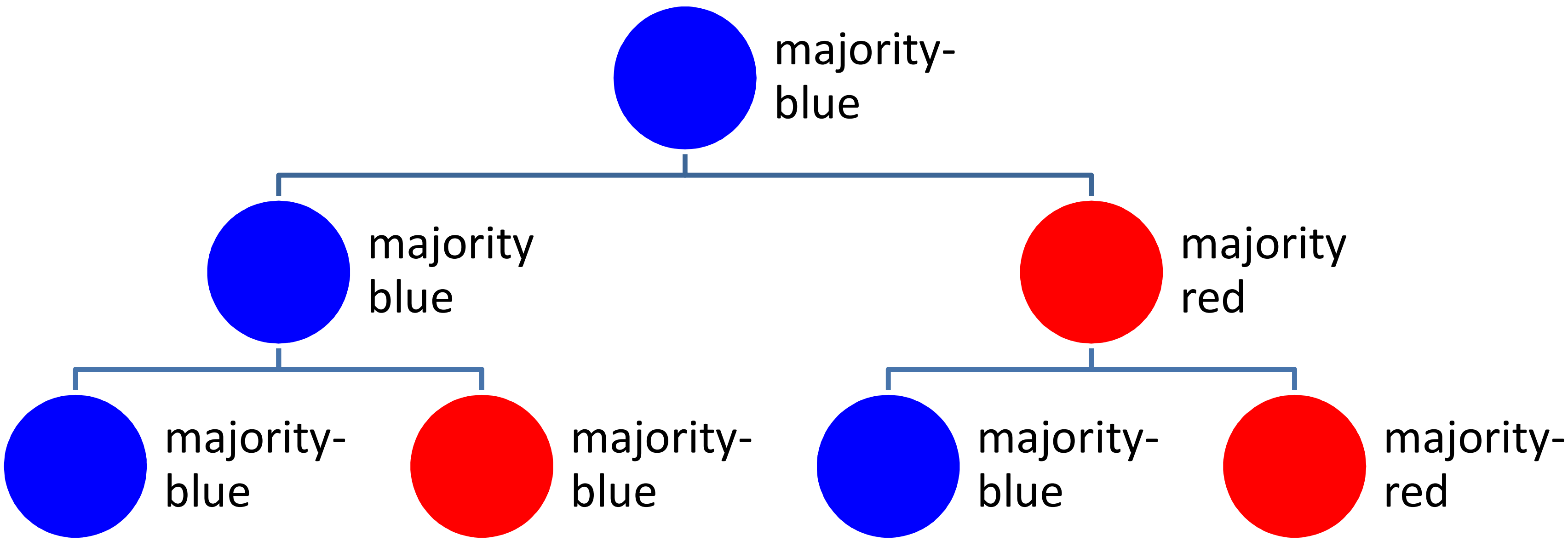
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majority-
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majority-
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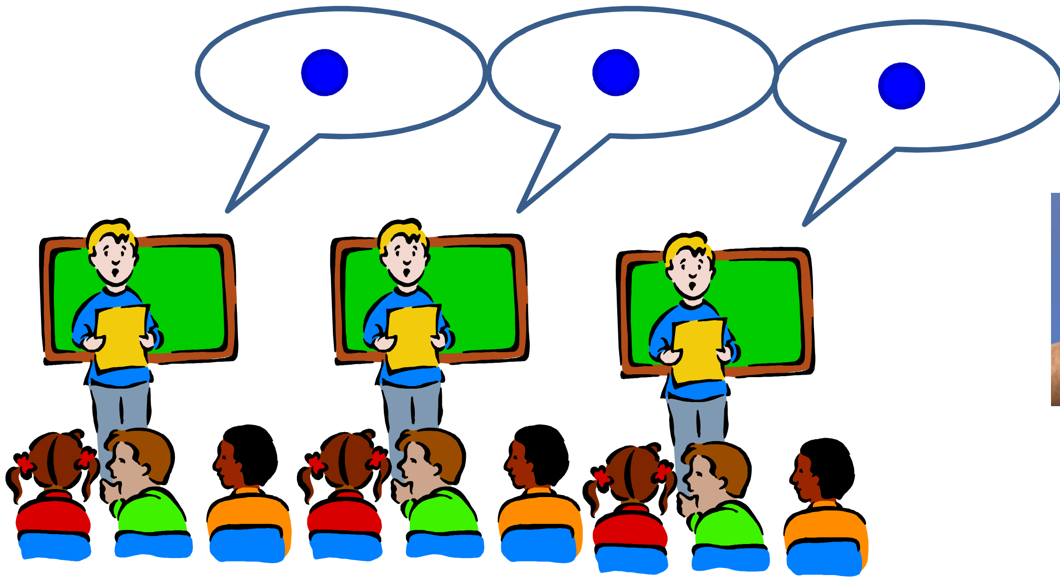
majority-
red



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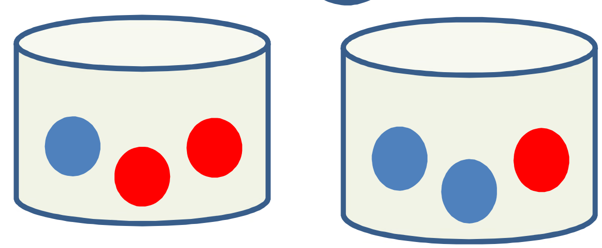

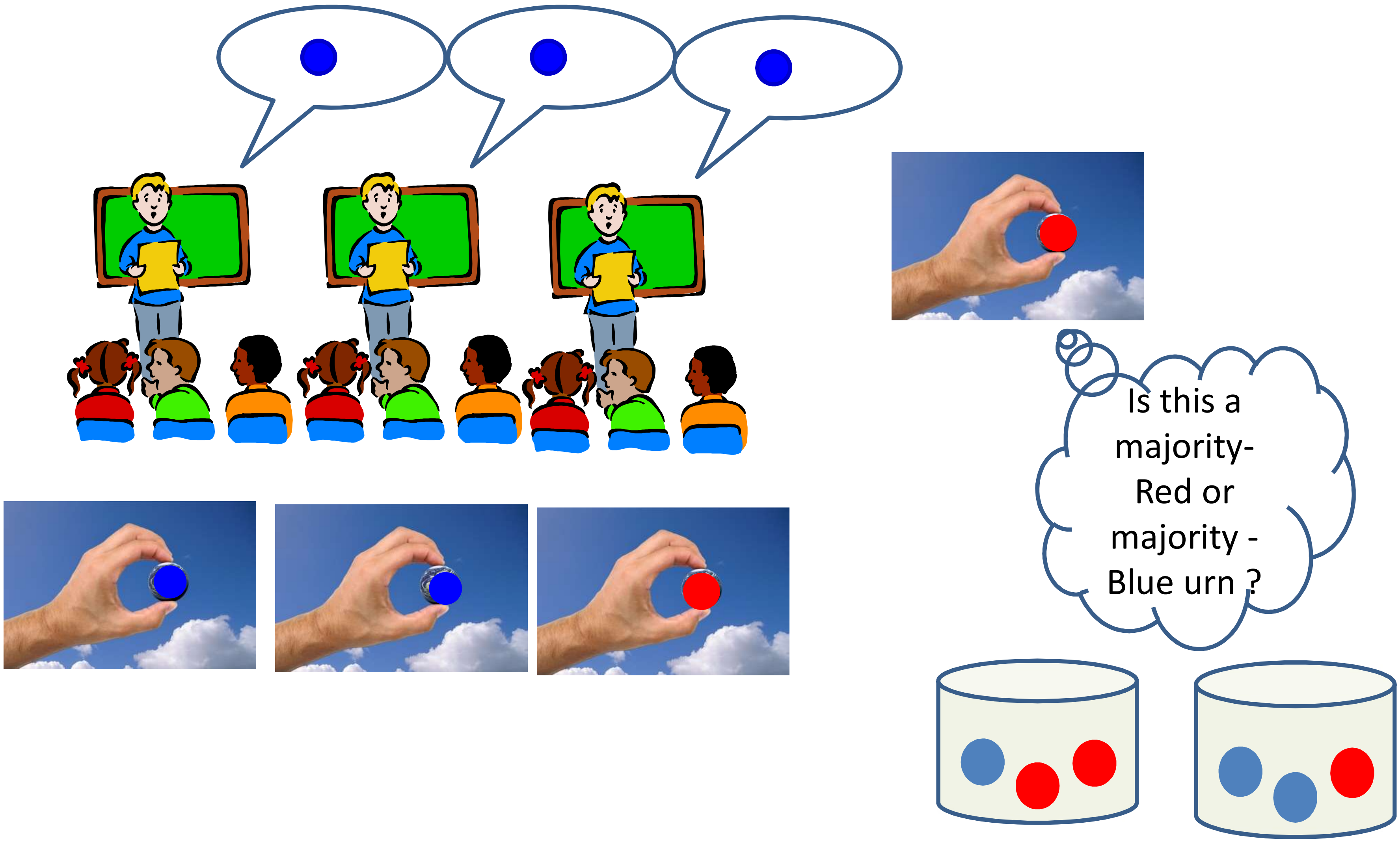
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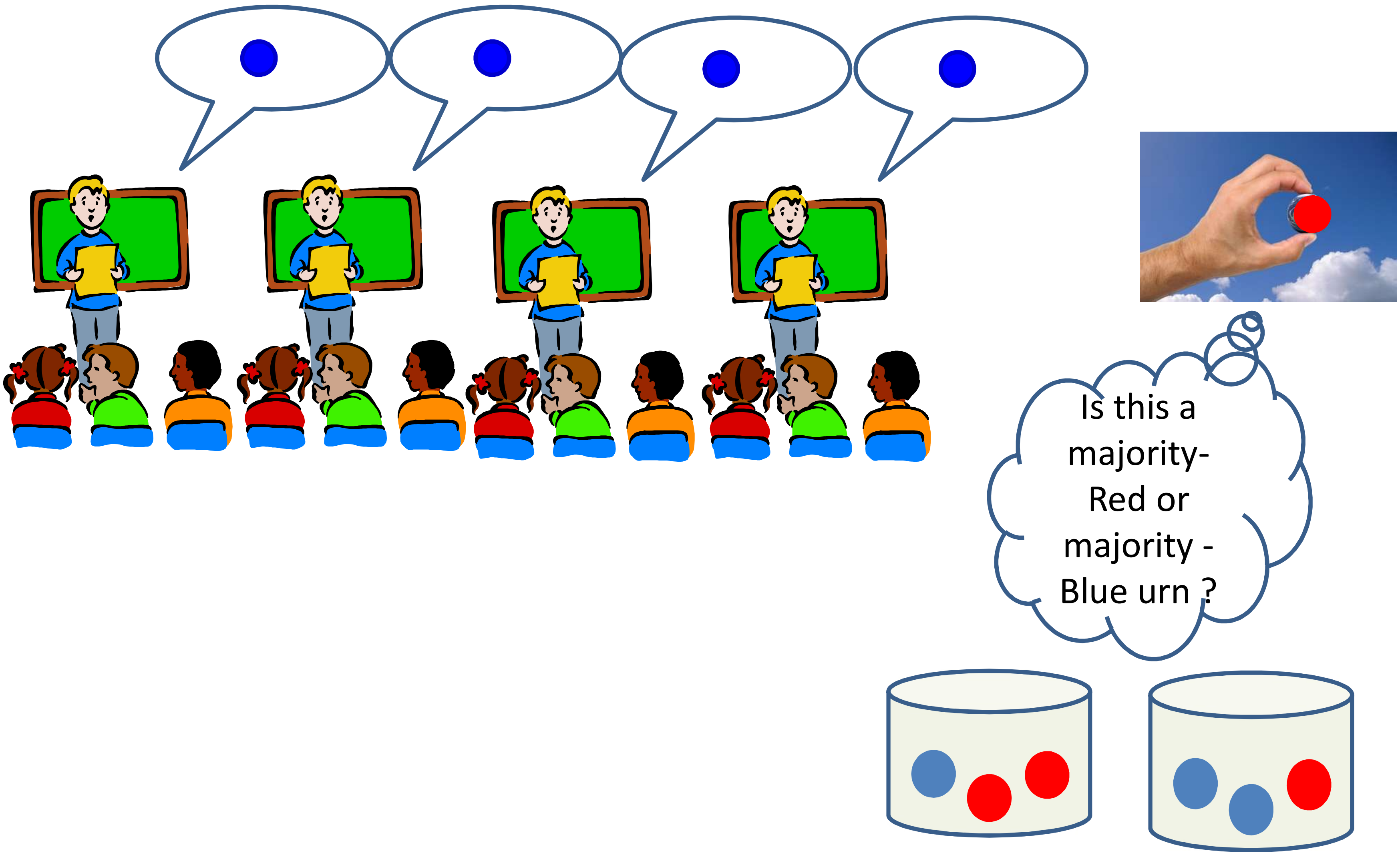
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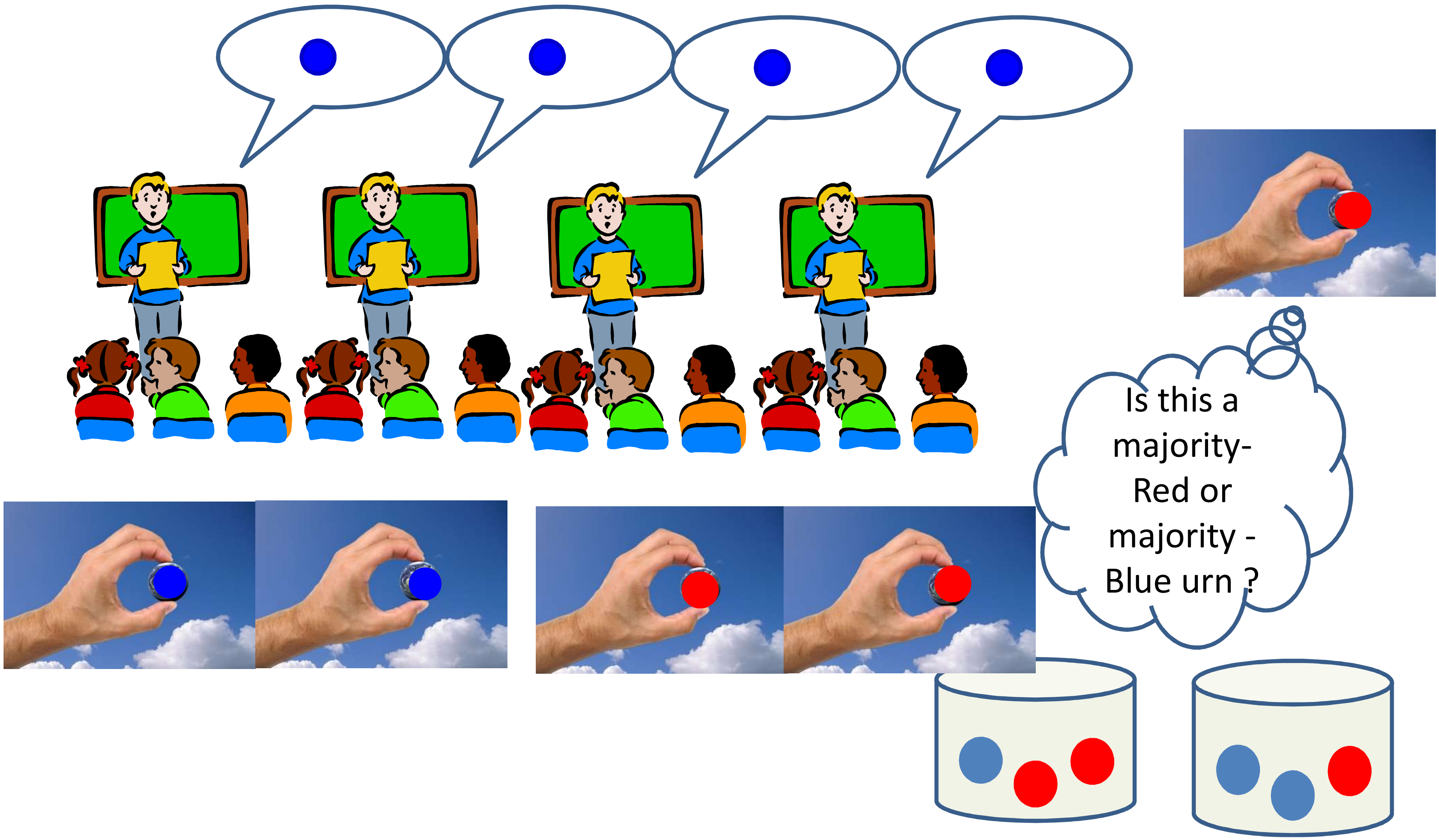
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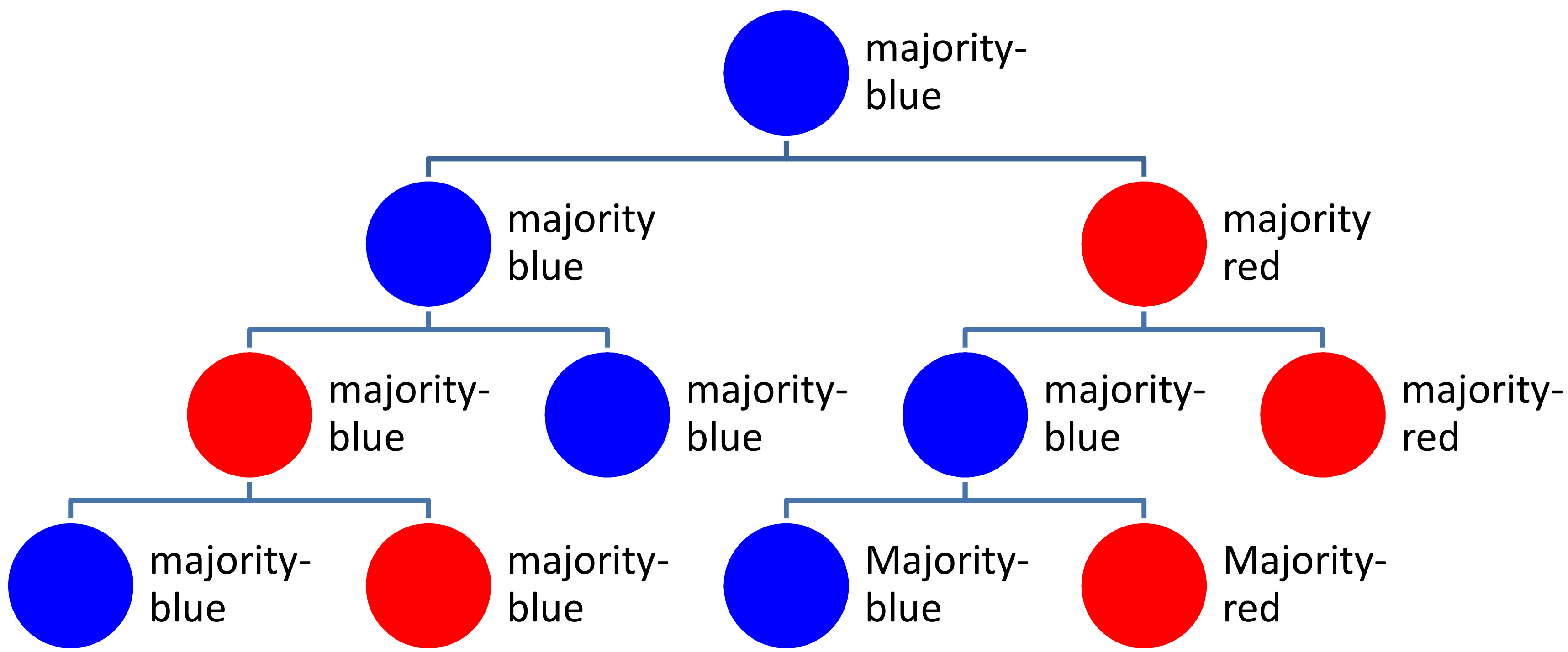
majority-
blue
majority
blue
majority
red
majority-
blue
majority-
blue
majority-
blue
majority-
red
majority-
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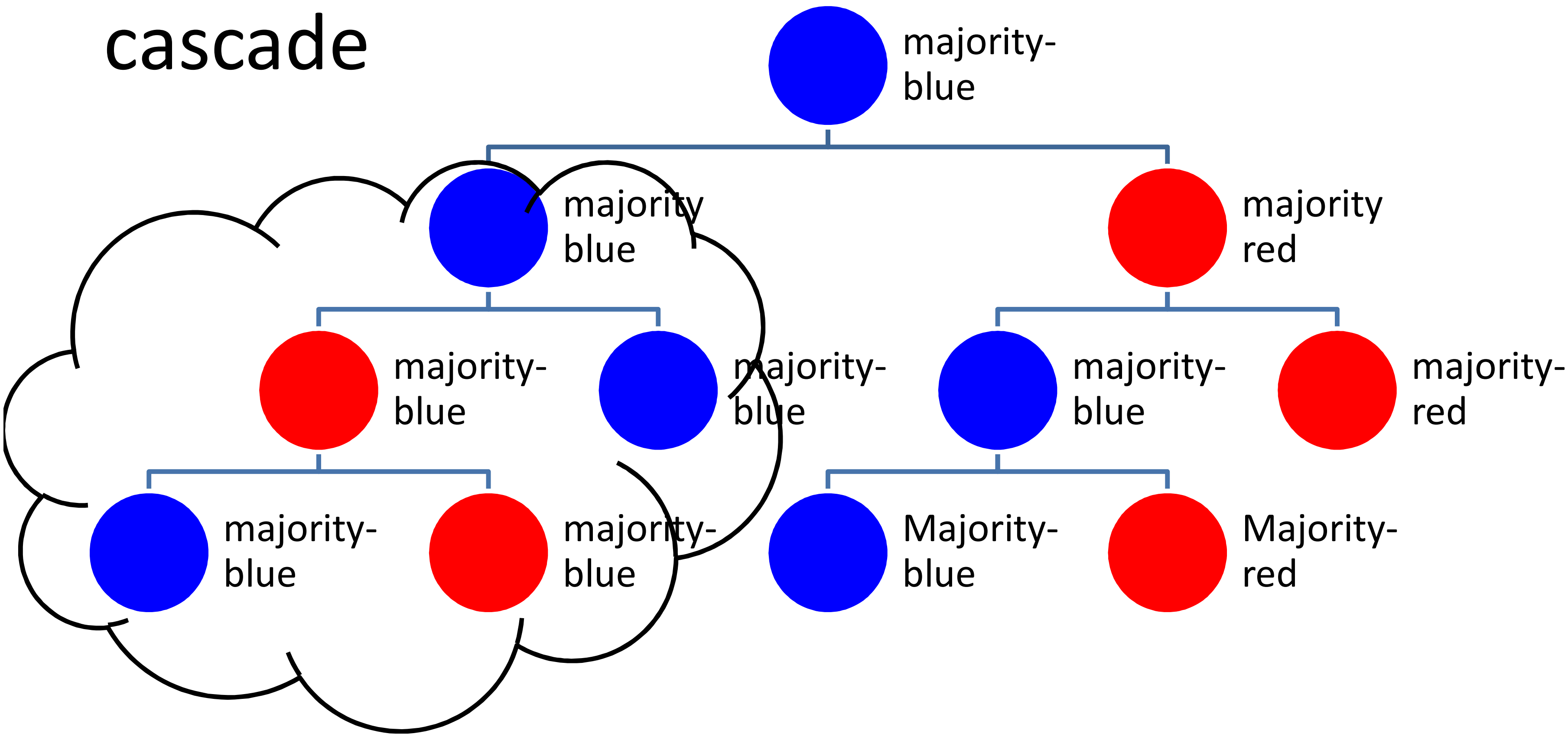
majority-
blue
Majority-
blue
Majority-
red

Information cascade



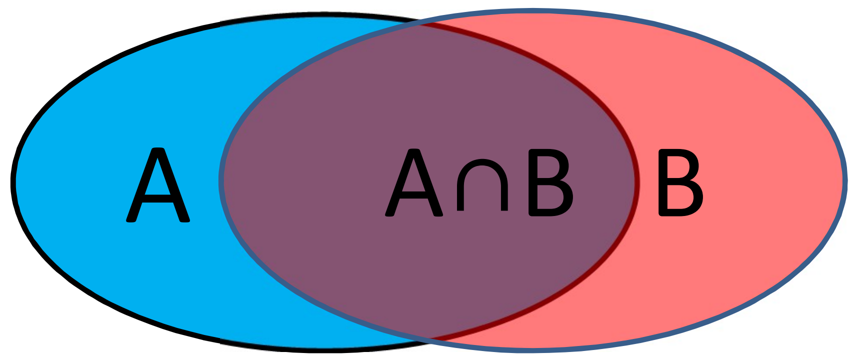
- Majority-red urn
  - $\frac{1}{3}$ chance that the first student draws a blue marble.
  - $\frac{1}{9}$ chance that both first and second student draw a blue marble.
  - All subsequent guesses will be blue.

Bayes rule

- Probability of an event based on given information
  - “What is the probability this is the better restaurant, given the reviews I’ve read and the crowds I see in each one?”
  - “What is the probability this urn is majority-red, given the marble I just drew and the guesses I’ve heard?”

Bayes' Rule

- A : event
- Pr[A] : probability of event A occurring
- Pr[A|B] : conditional probability of A given B
- $\Pr[A|B] = \dfrac{\Pr[A \cap B]}{\Pr[B]}$
- $\Pr[B|A] = \dfrac{\Pr[B \cap A]}{\Pr[A]}$



- $\Pr[A \cap B] = \Pr[B \cap A]$
  $= \Pr[A|B]\Pr[B] = \Pr[B|A]\Pr[A]$

Bayes' Rule

- $\Pr[A|B]\Pr[B] = \Pr[B|A]\Pr[A]$
- $\Pr[A|B] = \frac{\Pr[A]\Pr[B|A]}{\Pr[B]}$
- Pr[A] : prior probability
- Pr[A|B] : posterior probability

Example 1 : taxi cab

- 80% of taxi cabs are black and 20% are yellow

- Given a cab and report its color right = 80%

- If a witness claims he saw a yellow cab, what is the probability that the cab is yellow ?

$P[true=Y]$
$= 0.2$

- 80% of taxi cabs are black and 20% are yellow

- Given a cab and report its color **right** = 80%

$P[report=Y|true=Y] = 0.8$

$P[report=B|true=B] = 0.8$

- If a witness claims he saw a yellow cab, what is the probability that the cab is yellow ?

$P[true=Y|report=Y] = ?$

- Prior probability = $P[true=Y] = 0.2$
- $P[report=Y \mid true = Y] = 0.8$

$$\Pr[A|B] = \frac{\Pr[A]\Pr[B|A]}{\Pr[B]}$$

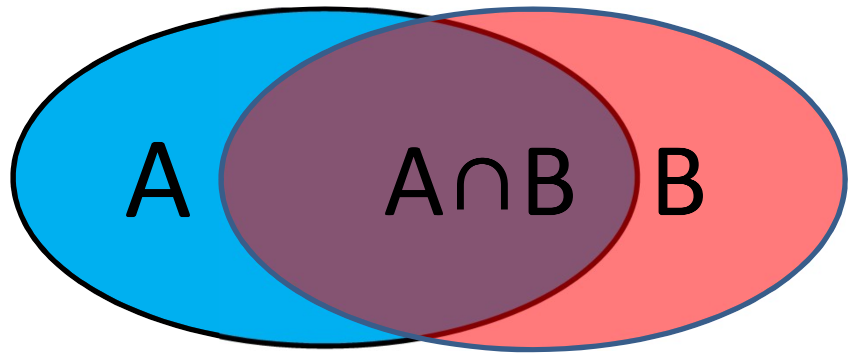
- $P[true = Y \mid report = Y]$

$$= \frac{P[true = Y] \cdot P[report=Y \mid true = Y]}{P[report = Y]}$$

$$= \frac{0.2 \cdot 0.8}{P[report = Y]}$$

$P[report=Y]$
$= ?$

- $\Pr[A|B] = \dfrac{\Pr[A \cap B]}{\Pr[B]}$



- $\Pr[B] = \Pr[B \cap A] + \Pr[B \cap A^c]$
- $\Pr[B \cap A] = \Pr[A]\Pr[B|A]$
- $\Pr[B \cap A^c] = \Pr[A^c]\Pr[B|A^c]$
- $\Pr[B] = \Pr[A]\Pr[B|A] + \Pr[A^c]\Pr[B|A^c]$

- $P[report = Y] = P[true = Y] \cdot P[report = Y \mid true = Y] + P[true = B] \cdot P[report = Y \mid true = B]$

  $= 0.2 \cdot 0.8 + 0.8 \cdot 0.2 = 0.32$

- $P[true = Y \mid report = Y]$

  $= \dfrac{0.2 \cdot 0.8}{P[report = Y]} = \dfrac{0.2 \cdot 0.8}{0.32} = 0.5$

Example 2: spam filtering

- $P[message\ is\ spam\ |subject\ contains\ \text{"check this out"}] = ?$
- $P[spam\ ] = 40\%$
- $P["check\ this\ out"|spam\ ] = 1\%$
- $P["check\ this\ out"|not\ spam\ ] = 0.4\%$
- $P[spam\ |\text{"check this out"}]$

$$= \frac{P[spam] \cdot P["check\ this\ out"|spam]}{P["check\ is\ out\ "]} = \frac{0.4 \cdot 0.01}{0.4 \cdot 0.01 + 0.6 \cdot 0.004}$$

= 0.625

Bayes' Rule in the Herding Experiment : Guess the Urn

- Pr[]=Pr[]=0.5
- Pr[● | ]=Pr[● | ] = $\frac{2}{3}$

- If student A draws ● , what is Pr[| ● **]** ?

If student A draws ●, what is Pr[| ●]?

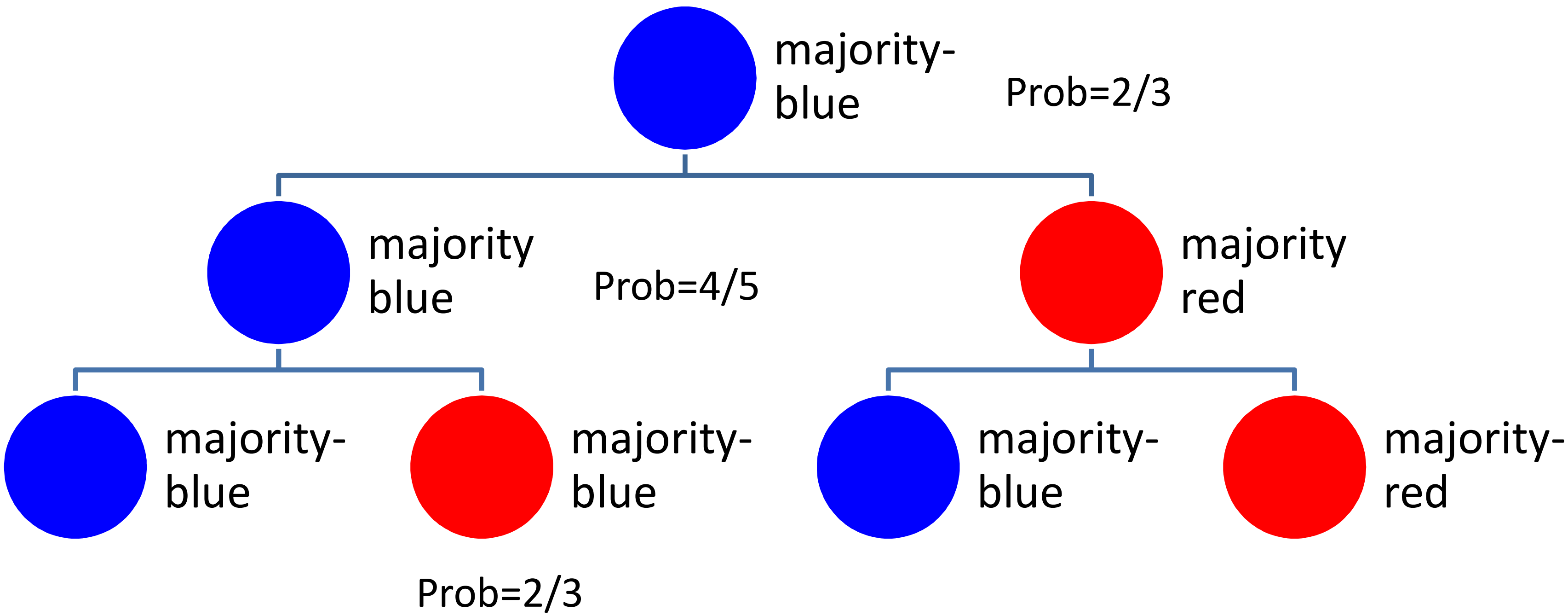
- Pr[ ]=Pr[ ]=0.5
- Pr[● |]=Pr[● |] = $\frac{2}{3}$
- Pr[| ●] = $\frac{\text{Pr[ ]Pr[● |]}}{\text{Pr[●]}}$ $= \frac{0.5 \times \frac{2}{3}}{\frac{1}{2}} = \mathbf{\frac{2}{3}}$
- Pr[●] ={If , Pr[● |]} + {If , Pr[● |]}
  =Pr[ ] Pr[● |]+Pr[ ] Pr[● |]
  =0.5 x $\frac{2}{3}$ + 0.5 x $\frac{1}{3}$ = $\frac{1}{2}$

- If student A behaves rationally, he would guess majority blue as Pr[| ●] =2/3.

what is Pr[💬| 🔵 🔵 🔴] ?

- Pr[💬]=Pr[💬]=0.5
- Pr[🔴 | 💬]=Pr[🔵 | 💬] = $\frac{2}{3}$
- Pr[💬| 🔵 🔵 🔴] = Pr[💬]Pr[⚫ ⚫ ⚫ | 💬] / Pr[⚫ ⚫ ⚫] $= \frac{0.5 \times \frac{4}{27}}{\frac{1}{9}} = \mathbf{\frac{2}{3}}$
- Pr[🔵 🔵 🔴] =Pr[💬] Pr[🔵 🔵 🔴 | 💬]
  +Pr[💬] Pr[🔵 🔵 🔴 | 💬]
  =0.5 x$\frac{2}{3}\frac{2}{3}\frac{1}{3}$ + 0.5 x $\frac{1}{3}\frac{1}{3}\frac{2}{3} = \frac{6}{54} = \frac{1}{9}$

- The third student draws a red marble
  - he knows that the first and second both draw a blue marble
  - Pr[majority blue | blue, blue, red] = 2/3
  - he guesses majority-blue


majority-
blue
Prob=2/3
majority
blue
Prob=4/5
majority
red
majority-
blue
majority-
blue
Prob=2/3
majority-
blue
majority-
red

A Simple General Cascade Model

- States
  - Good (G) with probability p
  - Bad (B) with probability 1-p
- Payoffs
  - 0 if one rejects the option
  - v_g if one accepts a good option
  - v_b if one accepts a bad option
- Signals : a private signal
  - High (H) suggesting that accepting is a good idea
  - Low (L) suggesting that accepting is a bad idea

The recommendation !!

- $\Pr[H|G] = \ q > \frac{1}{2}$, and
- $\Pr[L|G] = 1 - q < \frac{1}{2}$

		States	
		B	G
Signals	L	q	1-q
	H	1-q	q

Restaurant example

- two possible states : restaurant A or B
- accepting : choosing restaurant A
- private information : the review you read of the restaurant A, high if it says A is better than B

Urn example

- two possible states : majority blue or majority red
- accepting : guessing majority blue or red
- private information : the color of the ball draws (e.g. "high" signal if it is blue)
- p = prob[good]=prob[majority blue] = 0.5
- q = P[blue|majority-blue]=2/3

Individual Decisions

- Suppose one receives a high signal
- Expected payoff is

v_g Pr[G|H]+ v_b Pr[B|H]

Pr[G]=p
Pr[H|G]=q

- $\Pr[G|H] = \frac{\Pr[G] \cdot \Pr[H|G]}{\Pr[H]}$

$$= \frac{\Pr[G] \cdot \Pr[H|G]}{\Pr[G] \cdot \Pr[H|G] + \Pr[B] \cdot \Pr[H|B]}$$

$$= \frac{pq}{pq + (1-p)(1-q)} > \frac{pq}{pq + (1-p)q} = p$$

q > 1/2

1-q < q

Multiple signals

- A sequence S of independently generated signals consisting of *a* high signals and *b* low signals, interleaved in some fashion.
  - the posterior probability Pr [G | S] is greater than the prior Pr [G] when $a > b$;
  - the posterior Pr [G | S] is less than the prior Pr [G] when $a < b$; and
  - the two probabilities Pr [G | S] and Pr [G] are equal when $a = b$.

Multiple signals

- $\Pr[S|G] = q^a(1-q)^b$
- Bayes rule

$$\Pr[G|S] = \frac{\Pr[G]\cdot\Pr[S|G]}{\Pr[G]\cdot\Pr[S|G]+\Pr[B]\cdot\Pr[S|B]}$$

- $\Pr[G|S] = \frac{pq^a(1-q)^b}{p\cdot q^a(1-q)^b+(1-p)\cdot(1-q)^a q^b}$

Compare Pr[G|S] with Pr[G]

$$\Pr[G|S] = \frac{pq^a(1-q)^b}{p\cdot q^a(1-q)^b+(1-p)\cdot \boxed{(1-q)^a q^b}}$$

Compare Pr[G|S] with Pr[G]

$$\Pr[G] = p = \frac{pq^a(1-q)^b}{q^a(1-q)^b} = \frac{pq^a(1-q)^b}{p\cdot q^a(1-q)^b+(1-p)\cdot \boxed{q^a(1-q)^b}}$$

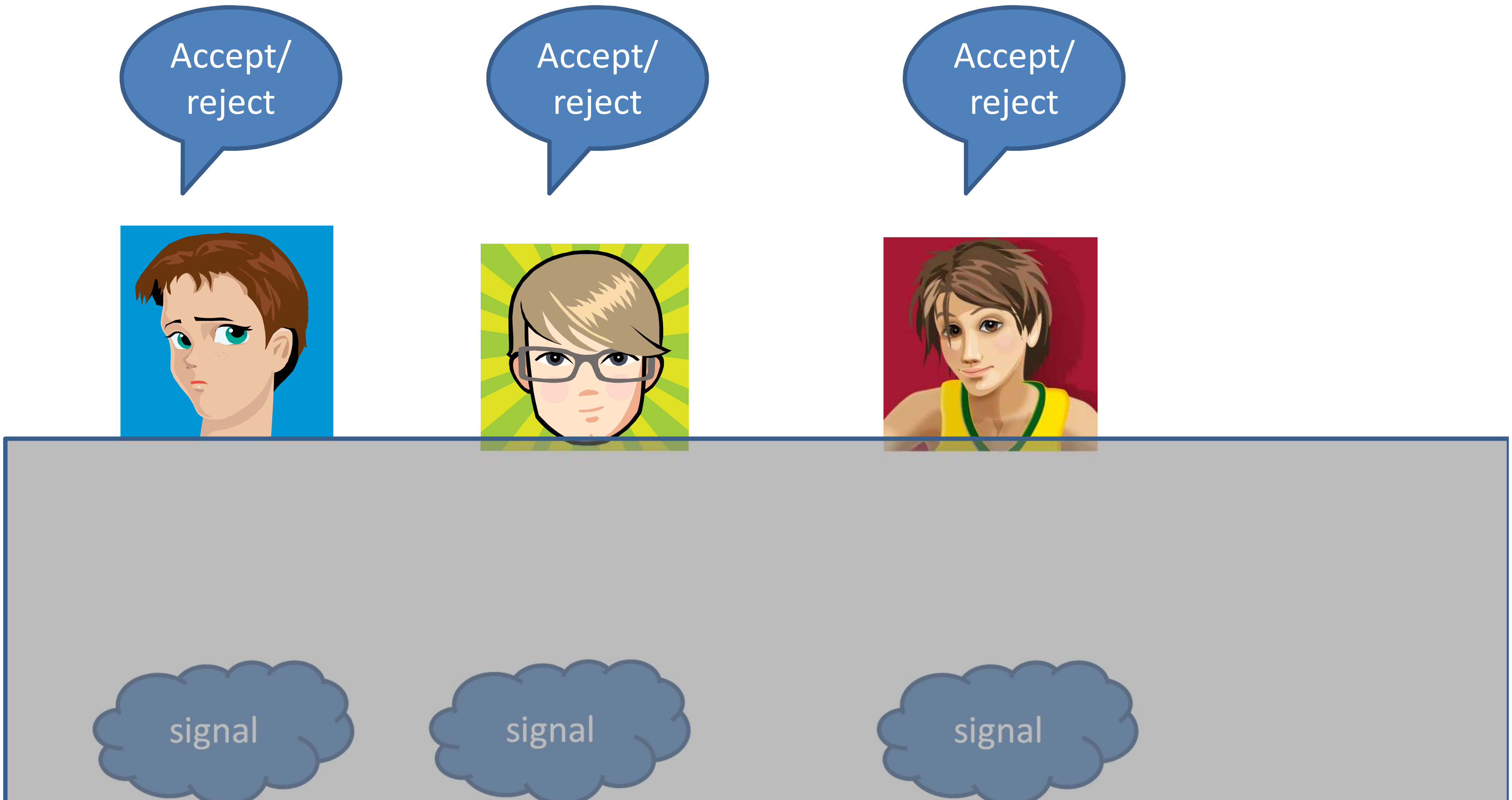
- If a > b, $(1-q)^a q^b < q^a(1-q)^b$, so $\Pr[G|S] > p$
- If a < b, $(1-q)^a q^b > q^a(1-q)^b$, so $\Pr[G|S] < p$
- If a = b, $(1-q)^a q^b = q^a(1-q)^b$, so $\Pr[G|S] = p$

1-q

q

- Cascades can be wrong
  - the wrong choice because of previous people happen to get high signals
- Cascades can be based on very little information
  - Once a cascade starts, people ignore their private information. only the pre-cascade information influences the behavior of the population. This means that if a cascade starts relatively quickly in a large population, most of the private information that is collectively available to the population (in the form of private signals to individuals) is not being used.
- Cascades are fragile
  - Easy to start and easy to stop as well as cascades can be based on relatively little information

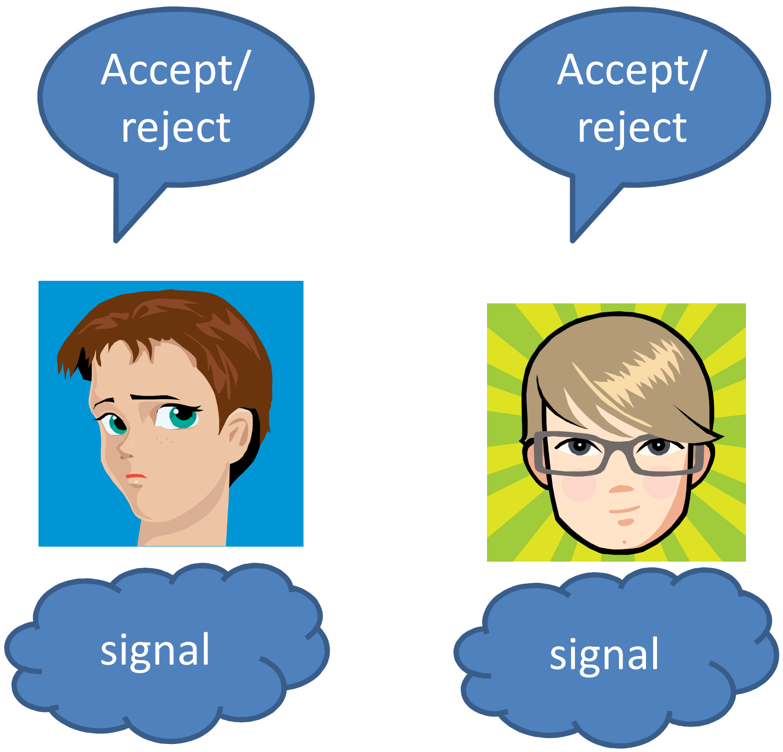
Sequential Decision-Making and Cascades



Sequential Decision-Making and Cascades

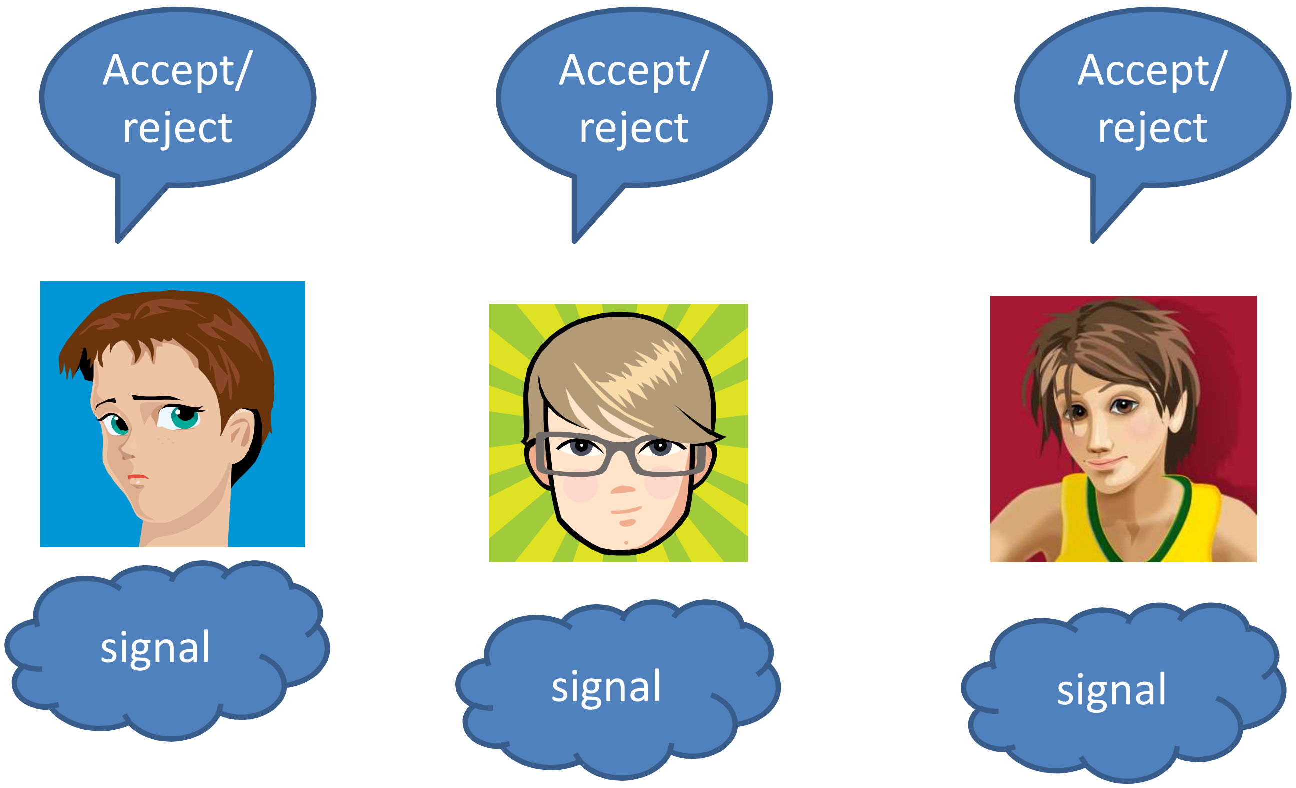
He will follow the private signal

Sequential Decision-Making and Cascades



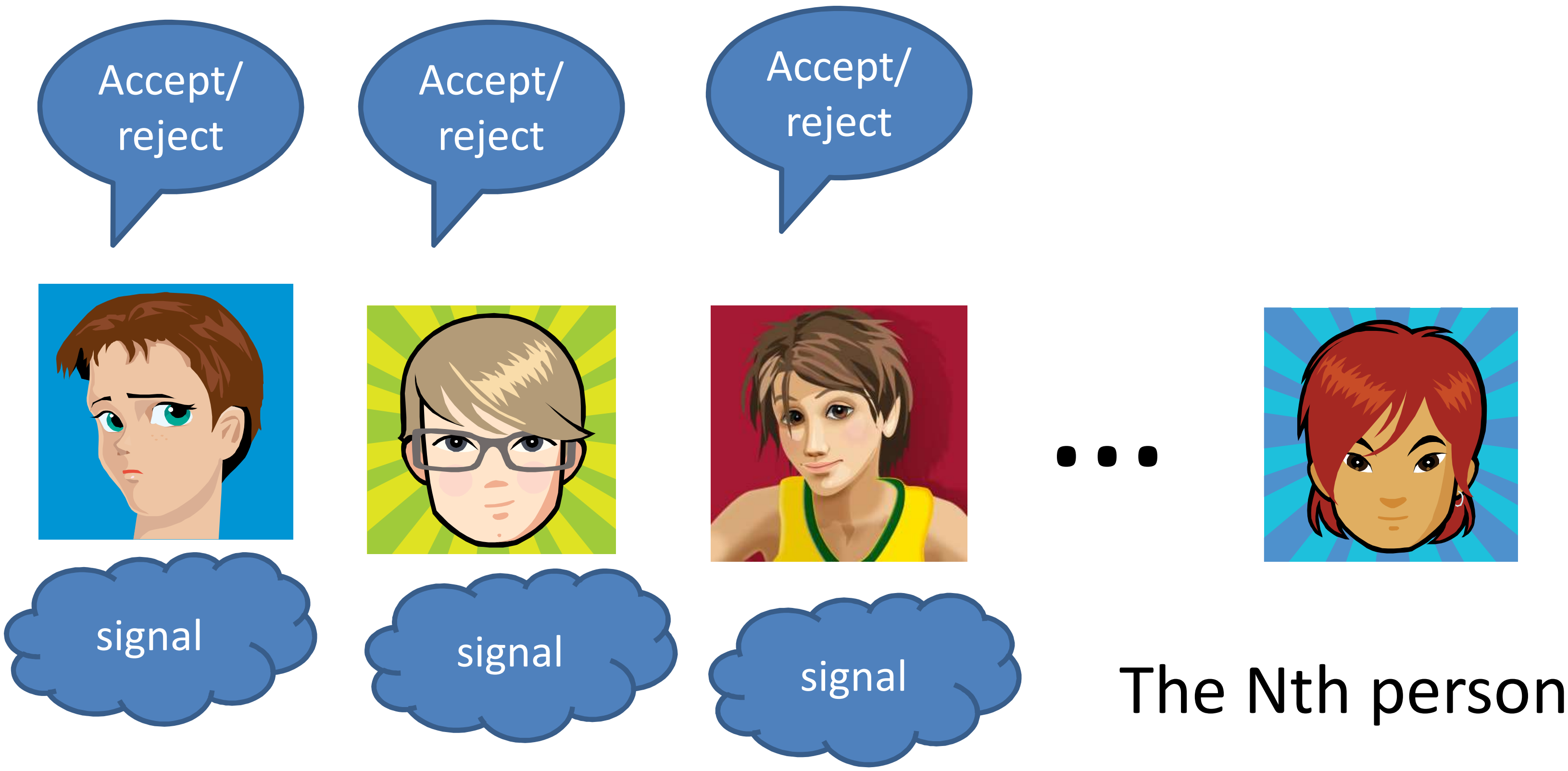
- The first person's decision reveals his private signal
- If the second signal is the same, decision is also the same
- If the signal is different, he will follow his private signal

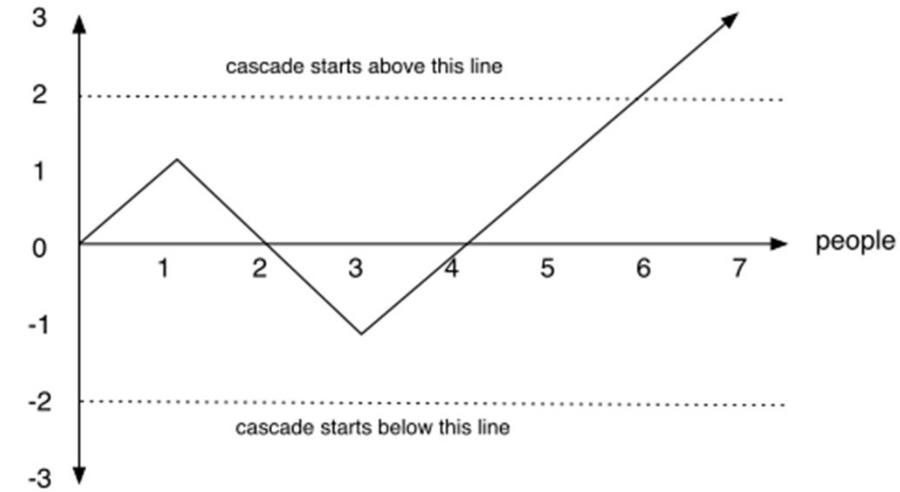
Sequential Decision-Making and Cascades

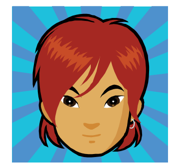


- Follow majority decision
- If the first two people made the same decision, cascade begins

Sequential Decision-Making and Cascades

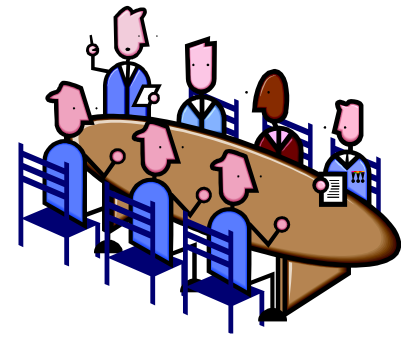




- A= no. of acceptances
- R = no. of rejections
- If A=R,  will follow his signal.
- If |A–R|=1,  will follow his signal.
- If |A-R| ≥ 2,  will follow the majority

Aggregate behavior of people with limited information

- Group decision making , example hiring,
  – go around the table and ask people to express their support for option A or B.
  – who speaks first ?



- Marketing
  – induce an initial set of people to buy new product