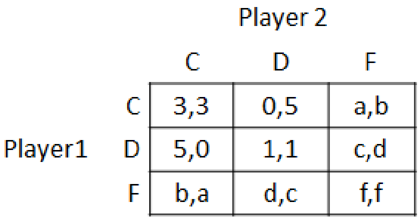
CSCI5350 Assignment1 Suggested Solution

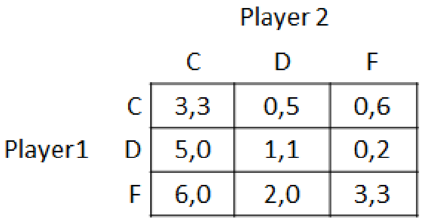
1. (25pts)
   (a) $N = \{1, 2\}$
   (b) $A_1 = A_2 = \{C, D\}$
   (c) The revised game G' is shown below:



		Player 2		
		C	D	F
Player1	C	3,3	0,5	a,b
	D	5,0	1,1	c,d
	F	b,a	d,c	f,f

   Number of outcomes in $G' = 9$

   (d) Yes, consider $b = 6, d = 2, a = c = 0, f = 3$, we have

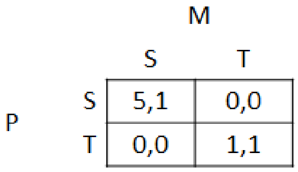


		Player 2		
		C	D	F
Player1	C	3,3	0,5	0,6
	D	5,0	1,1	0,2
	F	6,0	2,0	3,3

   $NE = (F, F)$

   (e) The general conditions are $b > 5, d > 1, f > a, f > c$
   (f) Correlated equilibrium $= \langle(\Omega, \pi), (P_i), (\sigma_i)\rangle$
   $\Omega = \{x\}$
   $\pi(x) = 1$
   $P_1 = P_2 = \{\{x\}\}$
   $\sigma_1 = \sigma_2 = \{x \mapsto F\}$

2. (25pts)
   (a) i. $N = \{1, 2\}$, $A_1 = A_2 = \{S, T\}$, where S represents Spasso restaurant and T represents Tin Tak Heen restaurant.



		M	
		S	T
P	S	5,1	0,0
	T	0,0	1,1

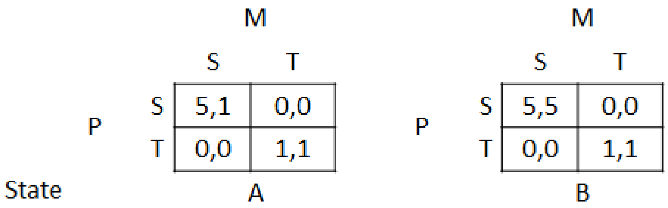
   ii. $NE = (S, S), (T, T)$

iii. Let the mixed strategies of player 1 and 2 be $\alpha_1 = (p, 1-p)$ and $\alpha_2 = (q, 1-q)$.
Consider the expected payoff for player 1,
$U_1 = 5pq + (1-p)(1-q)$
$\quad = 1 - q + (6q-1)p$
To maximize his utility, player 1 choose
$$\begin{cases} p = 0 & \text{if } q < \frac{1}{6} \\ p = 1 & \text{if } q > \frac{1}{6} \end{cases}$$
Consider the expected payoff for player 2,
$U_2 = pq + (1-p)(1-q)$
$\quad = 1 - p + (2p-1)q$
To maximize his utility, player 2 choose
$$\begin{cases} q = 0 & \text{if } p < \frac{1}{2} \\ q = 1 & \text{if } p > \frac{1}{2} \end{cases}$$
Combining both cases, we have completely mixed $NE = ((\frac{1}{2}, \frac{1}{2}), (\frac{1}{6}, \frac{5}{6}))$.

(b) Correlated equilibrium $= \langle(\Omega, \pi), (P_i), (\sigma_i)\rangle$
$\Omega = \{x_{SS}, x_{ST}, x_{TS}, x_{TT}\}$
$\pi(x_{SS}) = \frac{1}{12}, \pi(x_{ST}) = \frac{5}{12}, \pi(x_{TS}) = \frac{1}{12}, \pi(x_{TT}) = \frac{5}{12}$
$P_1 = \{\{x_{SS}, x_{ST}\}, \{x_{TS}, x_{TT}\}\}$
$P_2 = \{\{x_{SS}, x_{TS}\}, \{x_{ST}, x_{TT}\}\}$
$\sigma_1 = \{x_{SS} \mapsto S, x_{ST} \mapsto S, x_{TS} \mapsto T, x_{TT} \mapsto T\}$
$\sigma_2 = \{x_{SS} \mapsto S, x_{ST} \mapsto T, x_{TS} \mapsto S, x_{TT} \mapsto T\}$

(c) No. The concept of ESS is only defined under the context of symmetric games.

3. (25pts)



M

P	S	T
S	5,1	0,0
T	0,0	1,1

State A

M

P	S	T
S	5,5	0,0
T	0,0	1,1

B

(a) $\Omega = \{A, B\}$

(b) $T_1 = \{t_1\}, T_2 = \{t_2\}$

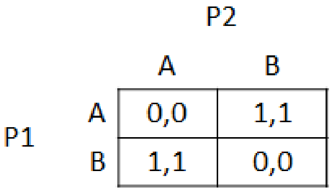
(c) $\tau_1 = \{A \mapsto t_1, B \mapsto t_1\}$
$\tau_2 = \{A \mapsto t_2, B \mapsto t_2\}$

(d) $p_1 = \{A \mapsto \frac{3}{4}, B \mapsto \frac{1}{4}\}$
$p_2 = \{A \mapsto \frac{7}{8}, B \mapsto \frac{1}{8}\}$

(e) $u_1(A, (S, S)) = 5, u_1(A, (S, T)) = 0, u_1(A, (T, S)) = 0, u_1(A, (T, T)) = 1$
$u_1(B, (S, S)) = 5, u_1(B, (S, T)) = 0, u_1(B, (T, S)) = 0, u_1(B, (T, T)) = 1$

(f) Yes. Any reason that makes sense.

4. (10pts)



		P2	
		A	B
P1	A	0,0	1,1
	B	1,1	0,0

NE $= (A, B), (B, A)$

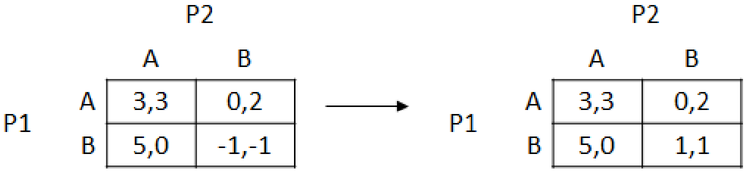
5. (15pts)

   (a) let payoff matrix of player 1 be U, $U' = U + \triangle$, where $\triangle(i, j) \geq 0, \forall i, j$
   $u_1 = max_i min_j U(i, j)$, $u'_1 = max_i min_j (U(i, j) + \triangle(i, j))$
   $U(i, j) \leq U(i, j) + \triangle(i, j)$
   $\Rightarrow min_j U(i, j) \leq min_j (U(i, j) + \triangle(i, j))$
   $\Rightarrow u_1 \leq u'_1$, therefore no equilibrium in in which player 1 is worse off than she was in the equilibrium of G.

   (b) let $v_i = min_j U(i, j)$, suppose action a_j is prohibited
   $u'_1 = max_{i \neq j} v_i \leq max_i v_i = u_1$
   therefore no equilibrium in which player 1 is better off than she was in the equilibrium of G.

   (c) Consider following non-strictly competitive games:

      i. payoff increased

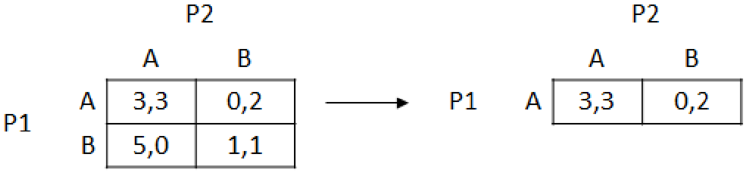


		P2	
		A	B
P1	A	3,3	0,2
	B	5,0	-1,-1

		P2	
		A	B
P1	A	3,3	0,2
	B	5,0	1,1

      NE: $(B, A) \rightarrow (B, B)$
      u_1: $5 \rightarrow 1$

      ii. action prohibited



		P2	
		A	B
P1	A	3,3	0,2
	B	5,0	1,1

		P2	
		A	B
P1	A	3,3	0,2

      NE: $(B, B) \rightarrow (A, A)$
      u_1: $1 \rightarrow 3$