香 港 中 文 大 學
The Chinese University of Hong Kong

二〇一七至一八年度上學期科目考試

Course Examination 1st Term, 2017-18

科目編號及名稱
Course Code & Title : ENGG 2440A: Discrete Mathematics for Engineers

時間
Time allowed : 2 小時 hours 00 分鐘 minutes

學號
Student I.D. No. :

座號
Seat No. :

Each question is worth 10 points. Explain your solution clearly and concisely. Write in full sentences. Define all terms and notations that you use but were not introduced in class.

1. Let G be a graph with 10 vertices and 9 edges. Is it true that G must be a tree? Justify your answer.

2. Alice places two pebbles at the opposite corners of an 8 by 8 chessboard. At each step, she can

   - put a new pebble in an empty square, if *exactly one* of its neighbors contains a pebble, or
   - remove a pebble from a square, if *at least one* of its neighbors contains a pebble.

   Neighbors are squares that share a common side. Can the board ever have a single pebble on it?

3. The set S_n consists of all length-n strings with symbols $\{A, B, C\}$ in which every B is immediately followed by a C (e.g., BCAC is in S_4 but ACAB is not). Find the value of a for which $f(n)$ is $\Theta(a^n)$.

4. What is the largest integer n for which

$$n \leq 1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \cdots + \frac{1}{\sqrt{9999}}?$$

5. At a party, seven people check in their hats. In how many ways can they be returned so that *exactly one* person receives their own hat? Show your calculations.

6. Show that no matter how you place 17 pieces on an 8 by 8 chessboard, at least two pieces must occupy squares that share a common side or a common corner.

7. Suppose that all arrangements of n plus signs and n minus signs in a row are equally likely. Give a formula for the probability that no two minus signs are adjacent to each other. Specify the relevant sample space and event. Show your calculations.