香 港 中 文 大 學
The Chinese University of Hong Kong

二〇一七至一八年度上學期科目考試
Course Examination 1st Term, 2017-18

科目編號及名稱 Course Code & Title :	ENGG 2440B: Discrete Mathematics for Engineers		
時間 Time allowed :	2	小時 hours	00 分鐘 minutes
學號 Student I.D. No. :		座號 Seat No. :	

SOLVE THE FOLLOWING PROBLEMS.

Problem 1 (15pts). At a party, seven different people check in their hats. How many ways can their hats be returned so that at least one of them receives his/her own hat? Show all your calculations.

Problem 2 (15pts). Show that in a group of $n > 1$ people, there are always two people who have the same number of acquaintances in the group (a person is not considered an acquaintance of himself/herself).

Problem 3 (20pts). Let $n \geq m \geq 1$ be given integers. Give a combinatorial proof of the identity

$$\sum_{k=0}^{m} \binom{n-k}{n-m}\binom{n}{k} = 2^m \binom{n}{m}.$$

State clearly what you are trying to count and how you are counting it.
(*Note: If you cannot find a combinatorial proof of the identity, then you can also try to prove it by other means, such as induction. However, you will only receive a maximum of 15 points for such proofs.*)

Problem 4 (20pts). Let $n \geq 1$ be a given integer. Suppose that all arrangements of n plus signs and n minus signs in a row are equally likely. What is the probability that no two minus signs are adjacent to each other? Show all your calculations. In particular, specify the sample space and the event you are interested in.

Problem 5 (15pts). Let G be a graph with 10 vertices and 9 edges. Is it true that G must be a tree? Justify your answer.

Problem 6 (15pts). Let $G = (V, E)$ be a graph with at least one edge and $a > 0$ be its average degree. Consider the graph G' obtained by removing a vertex with the largest degree from G. Let $a' \geq 0$ be the average degree of G'. Show that $a' < a$.