THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MATH1020
Exercise 12
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Exercise 1 In Problems (a) - (b),

1. $\mathbf{v} = \mathbf{i} - \mathbf{j}, \quad \mathbf{w} = \mathbf{i} + \mathbf{j}$.
2. $\mathbf{v} = 2\mathbf{i} + \mathbf{j}, \quad \mathbf{w} = \mathbf{i} - 2\mathbf{j}$.

Answer the following questions:

i. Find the dot product $\mathbf{v} \cdot \mathbf{w}$;

ii. Find the angle between $\mathbf{v}$ and $\mathbf{w}$;

iii. State whether the vectors are parallel, orthogonal, or neither.

Solution:

1. $\mathbf{v} \cdot \mathbf{w} = 1 \times 1 + (-1) \times 1 = 0$, the angle between $\mathbf{v}$ and $\mathbf{w}$ is $\theta = \arccos \dfrac{\mathbf{v} \cdot \mathbf{w}}{||\mathbf{v}||||\mathbf{w}||} = \dfrac{\pi}{2}$, and thus the vectors are orthogonal.
2. $\mathbf{v} \cdot \mathbf{w} = 2 \times 1 + 1 \times (-2) = 0$, the angle between $\mathbf{v}$ and $\mathbf{w}$ is $\theta = \arccos \dfrac{\mathbf{v} \cdot \mathbf{w}}{||\mathbf{v}||||\mathbf{w}||} = \dfrac{\pi}{2}$, and thus the vectors are orthogonal.

Exercise 2 In Problems (a) - (b), find each quantity if $\mathbf{v} = 3\mathbf{i} - 5\mathbf{j} + 2\mathbf{k}$ and $\mathbf{w} = -2\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$.

1. $2\mathbf{v} + 3\mathbf{w}$.
2. $\|\mathbf{v} - \mathbf{w}\|$.

Solution:

1. $2\mathbf{v} + 3\mathbf{w} = -\mathbf{j} - 2\mathbf{k}$.
2. $\|\mathbf{v} - \mathbf{w}\| = \|5\mathbf{i} - 8\mathbf{j} + 4\mathbf{w}\| = \sqrt{5^2 + (-8)^2 + 4^2} = \sqrt{105}$.

Exercise 3 In Problems (a) - (b), find the direction angles of each vector. Write each vector in the form of the following question:

$$\mathbf{v} = \parallel \mathbf{v} \parallel [(\cos\,\alpha)\mathbf{i} + (\cos\,\beta)\mathbf{j} + (\cos\,\gamma)\mathbf{k}]\,.$$

1. $\mathbf{v} = 3\mathbf{i} - 6\mathbf{j} - 2\mathbf{k}$.
2. $\mathbf{v} = \mathbf{i} + \mathbf{j} + \mathbf{k}$.

Solution: By the equality,

$$\alpha = \arccos \frac{\mathbf{v}\cdot\mathbf{i}}{\|\mathbf{v}\|}, \quad \beta = \arccos \frac{\mathbf{v}\cdot\mathbf{j}}{\|\mathbf{v}\|}, \quad \gamma = \arccos \frac{\mathbf{v}\cdot\mathbf{k}}{\|\mathbf{v}\|}.$$

So the result is

	α	β	γ
(a)	$\arccos\frac{3}{7}$	$\arccos(-\frac{6}{7})$	$\arccos(-\frac{2}{7})$
(b)	$\arccos\frac{\sqrt{3}}{3}$	$\arccos\frac{\sqrt{3}}{3}$	$\arccos\frac{\sqrt{3}}{3}$

$\square$