CSCI3320 HW 5 Chung Tsz Ting / 1155110208

1) The new covariance matrix will be appending a row and column to the original covariance matrix,

i.e. $\boldsymbol{S_{new}} = \begin{bmatrix} \boldsymbol{S_{old}} & \boldsymbol{s_{old,n+1}} \\ \boldsymbol{s_{old,n+1}}^T & s_{n+1}{}^2 \end{bmatrix}$, where

$\boldsymbol{S_{old}}$ is the previous covariance matrix with shape of nxn and

$\boldsymbol{s_{old,n+1}}$ is a column covariance vector with shape 1xn and

$\boldsymbol{s_{old,n+1}}^T$ is the row covariance vector with shape nx1 and same value as $\boldsymbol{s_{old,n+1}}$ due to the property of covariance matrix and

$s_{n+1}{}^2$ is the variance of the new feature.

Thus,

$$\boldsymbol{S_{new}}^{-\boldsymbol{1}} = \frac{1}{s_{n+1}{}^2 - \boldsymbol{s_{old,n+1}}^T \boldsymbol{S_{old}}^{-\boldsymbol{1}} \boldsymbol{s_{old,n+1}}} \begin{bmatrix} s_{n+1}{}^2 \boldsymbol{S_{old}}^{-\boldsymbol{1}} & -\boldsymbol{S_{old}}^{-\boldsymbol{1}} \boldsymbol{s_{old,n+1}} \\ -\boldsymbol{s_{old,n+1}}^T \boldsymbol{S_{old}}^{-\boldsymbol{1}} & 1 \end{bmatrix}$$