Lecture Note 14

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MATH1020
General Mathematics

CONIC

The word conic derives from the word cone, which is geometric figure that can be constructed in the following way:

Let a and g be two distinct lines that intersect at a point V. Keep the line a fixed.

Now rotate the line g about a while maintaining the same angle between a and g.

The collection of points swept out (generated) by the line g is called a **(right circular) cone.** See Figure 1.

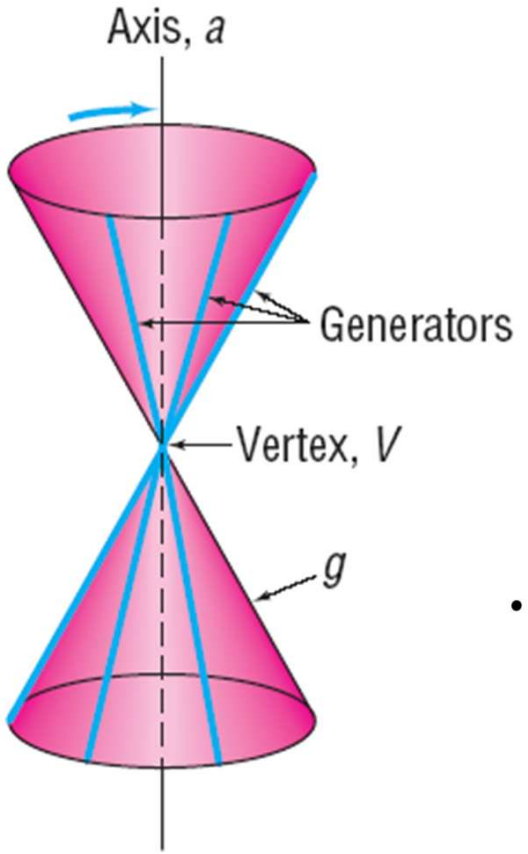
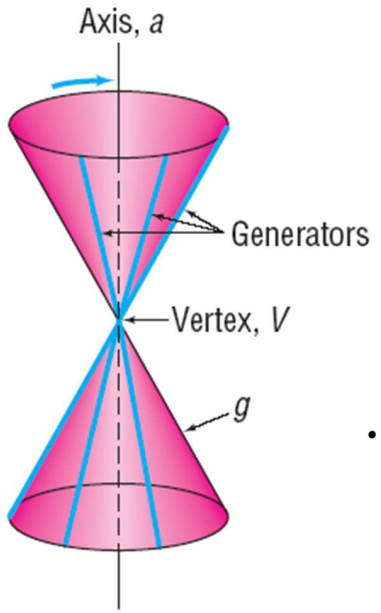


Figure 1:



The fixed line a is called the axis of the cone; the point V is its **vertex;** the lines that pass through V and make the same angle with a as g are **generators** of the cone.

Each generator is a line that lies entirely on the cone. The cone consists of two parts, called **nappes,** that intersect at the vertex.

Conics, an abbreviation for **conics sections,** are curves that from the intersection of a right circular cone and a plane.

The conics we shall arise when the plane does not contain the vertex, as shown in Figure 2 - Figure 5.

- These conics are **circles** when the plane is perpendicular to the axis of the cone and intersects each generator, but intersects only one nappe of the cone, as shown in Figure 2;

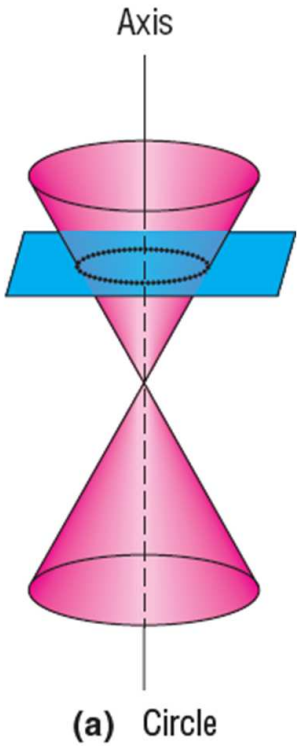


Figure 2:

- **ellipses** when the plane is titled slightly so that it intersects each generator, but intersects only one nappe of the cone, as shown in Figure 3;

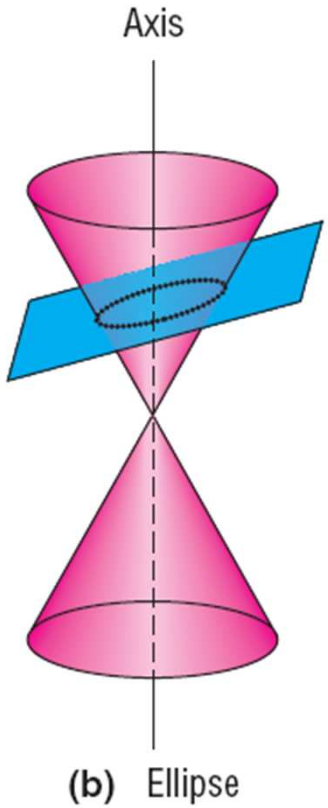


Figure 3:

- **parabolas** when the plane is titled farther so that it is parallel to one (and only one) generator and intersects only one nappe of the cone, as shown in Figure 4;

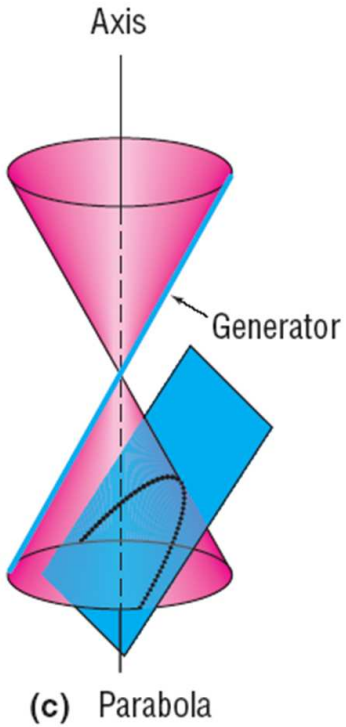


Figure 4:

- and **hyperbolas** when the plane intersects both nappes, as shown in Figure 5.

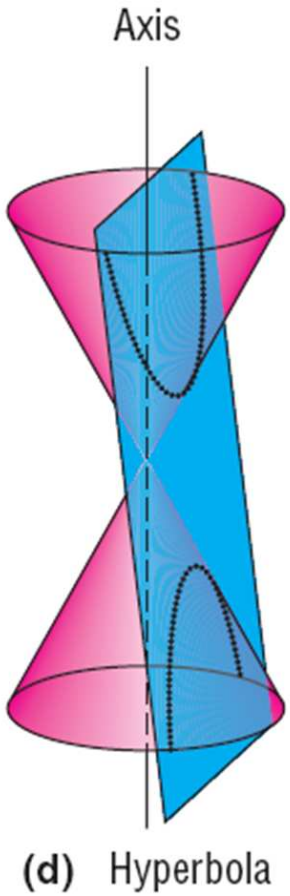


Figure 5:

Definition 1 If the plane does contain the vertex, the intersection of the plane and the cone is a point, a line, or a pair of intersecting lines. These usually called **degenerate conics.**