THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MATH1020
Exercise 10
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Exercise 1 A ball is thrown with an initial speed of 30 miles per hour in a direction that makes an angle of 60° with the positive x-axis.

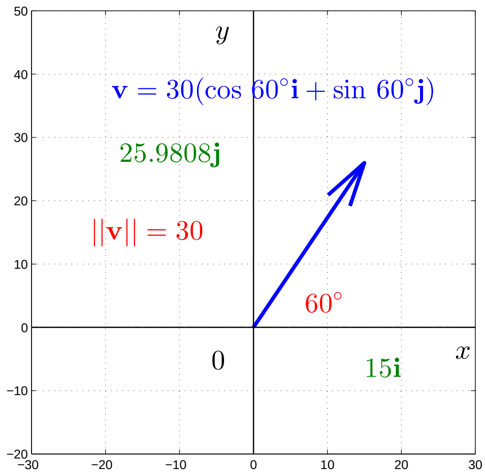
- Express the velocity vector $\mathbf{v}$ in terms of $\mathbf{i}$ and $\mathbf{j}$.
- What is the initial speed in the horizontal direction?
- What is the initial speed in the vertical direction?

Solution:

- The magnitude of $\mathbf{v}$ is $||\mathbf{v}|| = 30$ miles per hour, and the angle between the direction of $\mathbf{v}$ and $\mathbf{i}$, the positive x-axis, is $\alpha = 60°$. We have

$$\begin{aligned} \mathbf{v} &= ||\mathbf{v}||(\cos\alpha\mathbf{i} + \sin\alpha\mathbf{j}) \\ &= 30(\cos\, 60°\mathbf{i} + \sin\, 60°\mathbf{j}) \\ &= 30\left(\frac{1}{2}\mathbf{i} + \frac{\sqrt{3}}{2}\mathbf{j}\right) \\ &= \frac{30}{2}\mathbf{i} + \frac{30\sqrt{3}}{2}\mathbf{j}. \end{aligned}$$

- The initial speed of the ball in the horizontal directions is the horizontal component of $\mathbf{v}$, $\frac{30}{2} = 15$ miles per hour.
- The initial speed in the vertical direction is the vertical component of $\mathbf{v}$, $\frac{30\sqrt{3}}{2} \approx 25.9808$ miles per hour. See Figure 1.



y
$\mathbf{v} = 30(\cos\,60^\circ\mathbf{i} + \sin\,60^\circ\mathbf{j})$
$25.9808\mathbf{j}$
$||\mathbf{v}|| = 30$
60°
0
x
$15\mathbf{i}$
50
40
30
20
10
0
−10
−20
−30
−20
−10
0
10
20
30

Figure 1:

Exercise 2 A box of supplies (in yellow) that weighs 1000 pounds is suspended by two cables attached to the ceiling, as shown in Figure 2, where $\alpha = 30°$ and $\beta = 45°$. What are the tensions in the two cables?

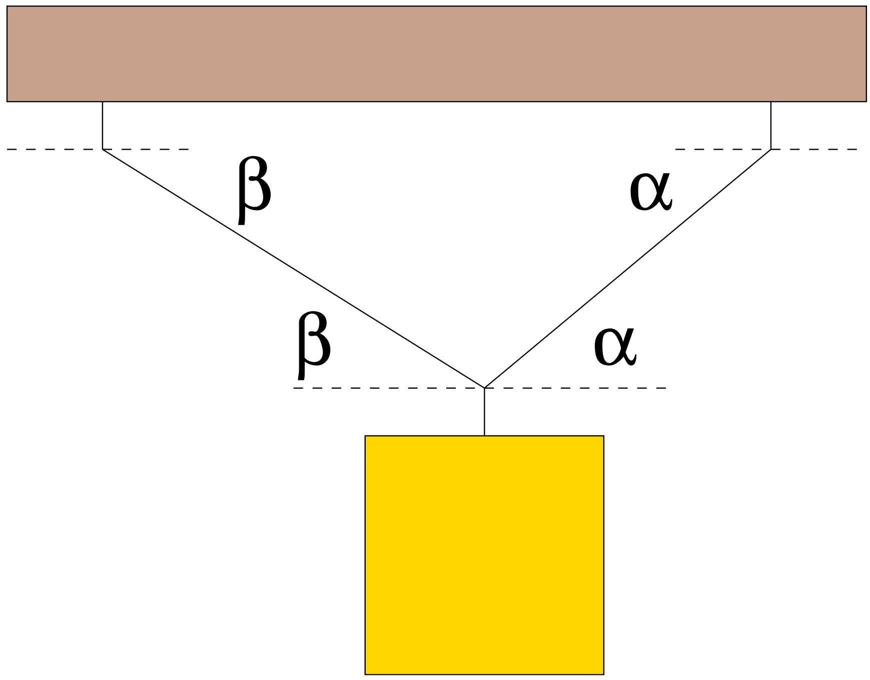
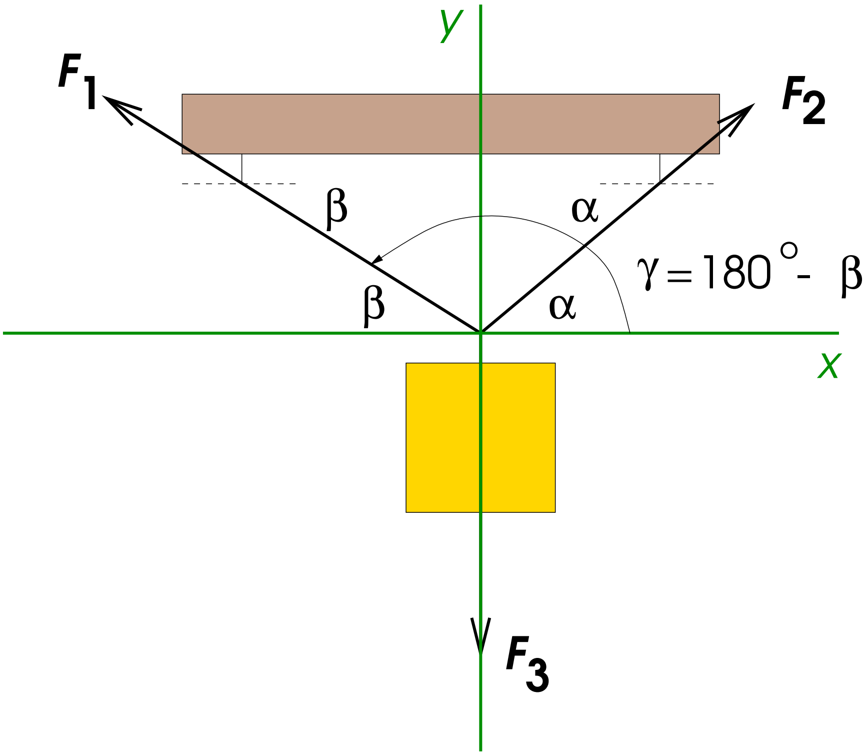


Figure 2:

Solution:

We draw a force diagram using the vectors shown in Figure 3.



[ht]

Figure 3:

Let $\beta = 45°$, $\alpha = 30°$, and $\gamma = 180° - \beta = 180° - 45° = 135°$.

The tensions in the cables are the magnitudes $||\mathbf{F_1}||$ and $||\mathbf{F_1}||$ of he force vectors $\mathbf{F_1}$ and $\mathbf{F_2}$. The magnitude of the force vector $\mathbf{F_3}$ equals 1000 pounds, the weight of the positive use Equation (8). Remember that α is the angle between the vector and the positive x-axis. We have

$$\mathbf{F_1} = ||\mathbf{F_1}||(\cos 135°\mathbf{i} + \sin 135°\mathbf{j}) = ||\mathbf{F_1}|| \left(\frac{-\sqrt{2}}{2}\mathbf{i} + \frac{\sqrt{2}}{2}\mathbf{j} \right) = -\frac{\sqrt{2}}{2}||\mathbf{F_1}||\mathbf{i} + \frac{\sqrt{2}}{2}||\mathbf{F_1}||\mathbf{j};$$

$$\mathbf{F_2} = ||\mathbf{F_1}||(\cos 30°\mathbf{i} + \sin 30°\mathbf{j}) = ||\mathbf{F_2}|| \left(\frac{\sqrt{3}}{2}\mathbf{i} + \frac{1}{2}\mathbf{j} \right) = \frac{\sqrt{3}}{2}||\mathbf{F_2}||\mathbf{i} + \frac{1}{2}||\mathbf{F_2}||\mathbf{j};$$

$$\mathbf{F_3} = -1000\mathbf{j}.$$

For static equilibrium, the sum of the force vectors must equal zero. We have

$$\begin{aligned}\mathbf{F_1} + \mathbf{F_2} + \mathbf{F_3} &= -\frac{\sqrt{2}}{2}||\mathbf{F_1}||\mathbf{i} + \frac{\sqrt{2}}{2}||\mathbf{F_1}||\mathbf{j} + \frac{\sqrt{3}}{2}||\mathbf{F_2}||\mathbf{i} + \frac{1}{2}||\mathbf{F_2}||\mathbf{j} - 1000\mathbf{j} \\ &= \mathbf{0}.\end{aligned}$$

The $\mathbf{i}$ component and $\mathbf{j}$ will each equal zero. This results in the two equations

$$\begin{array}{lll} \mathbf{i}: & -\frac{\sqrt{2}}{2}||\mathbf{F_1}|| + \frac{\sqrt{3}}{2}||\mathbf{F_2}|| = 0 & \mathbf{(9)} \\ \mathbf{j}: & \frac{\sqrt{2}}{2}||\mathbf{F_1}|| + \frac{1}{2}||\mathbf{F_2}|| - 1000 = 0 & \mathbf{(10)} \end{array}$$

We solve Equation (9) for $||\mathbf{F_2}||$ and obtain

$$||\mathbf{F_2}|| = \frac{\sqrt{2}}{\sqrt{3}}||\mathbf{F_1}|| \qquad \mathbf{(11)}$$

Substituting into Equation (10) and solving for $||\mathbf{F_1}||$, we obtain

$$\begin{aligned}
\frac{\sqrt{2}}{2}||\mathbf{F_1}|| + \frac{1}{2}||\mathbf{F_2}|| + \frac{\sqrt{2}}{2} - 1000 &= 0 \\
\left(\frac{\sqrt{2}}{2} + \frac{1}{2}\frac{\sqrt{2}}{\sqrt{3}}\right)||\mathbf{F_1}|| - 1000 &= 0 \\
\frac{\sqrt{2}}{2}\left(1 + \frac{\sqrt{2}}{\sqrt{3}}\right)||\mathbf{F_1}|| - 1000 &= 0 \\
||\mathbf{F_1}|| = \frac{1000}{\left(\frac{\sqrt{2}}{2}\left(1 + \frac{\sqrt{2}}{\sqrt{3}}\right)\right)} &\approx 778.5391 \text{ pounds.}
\end{aligned}$$

Substituting this value into Equation (11) yields $||\mathbf{F_2}||$.

$$||\mathbf{F_2}|| = \frac{\sqrt{2}}{\sqrt{3}}||\mathbf{F_1}|| = \frac{\sqrt{2}}{\sqrt{3}}(778.5391) \approx 635.6745 \text{ pounds.}$$

The left cable has tension of approximately 778.5 pounds and the right cable has tension of approximately 635.7 pounds.