

CSCI 3230

Fundamentals of Artificial Intelligence

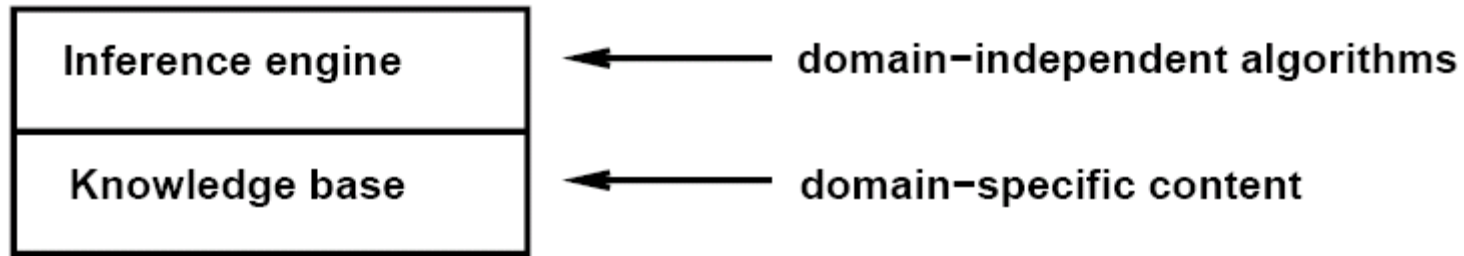
Chapter 7

LOGICAL AGENTS

Outline

- ▶ Knowledge-based agents
- ▶ Wumpus World
- ▶ Logic in general – models and entailment
- ▶ Propositional (Boolean) logic
- ▶ Equivalence, validity, satisfiability
- ▶ Inference rules and theorem proving
 - Forward chaining
 - Backward chaining
 - Resolution

Knowledge bases



Knowledge base = set of **sentences** in a **formal** language

Declarative approach to building an agent (or other system):
Tell it what it needs to know

Then it can infer what to do – answers should follow from the KB

Can describe a knowledge-based agent at 3 levels:

- The **knowledge level** or **epistemological** level is the most abstract; can describe the agent by saying what it knows.
- The **logical level** – at which the knowledge is encoded into sentences.
- The **implementation level** runs on the agent architecture.

A simple knowledge-based agent

function KB-Agent (*percept*) **returns** *an action*

static: *KB*, a knowledge base

t, a counter, initially 0, indicating time

Tell (*KB*, Make-Percept-Sentence (*percept*, *t*))

action \leftarrow Ask (*KB*, Make-Action-Query (*t*))

Tell (*KB*, Make-Action-Sentence (*action* , *t*))

t \leftarrow *t* + 1

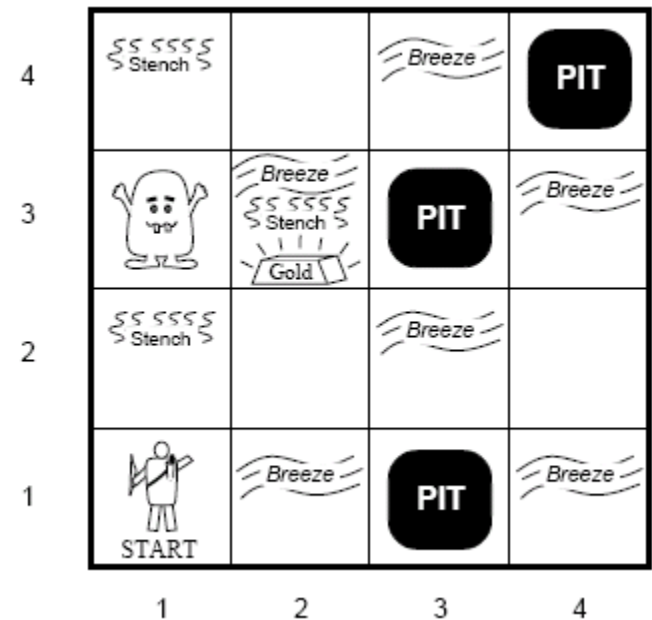
return *action*

The agent must be able to:

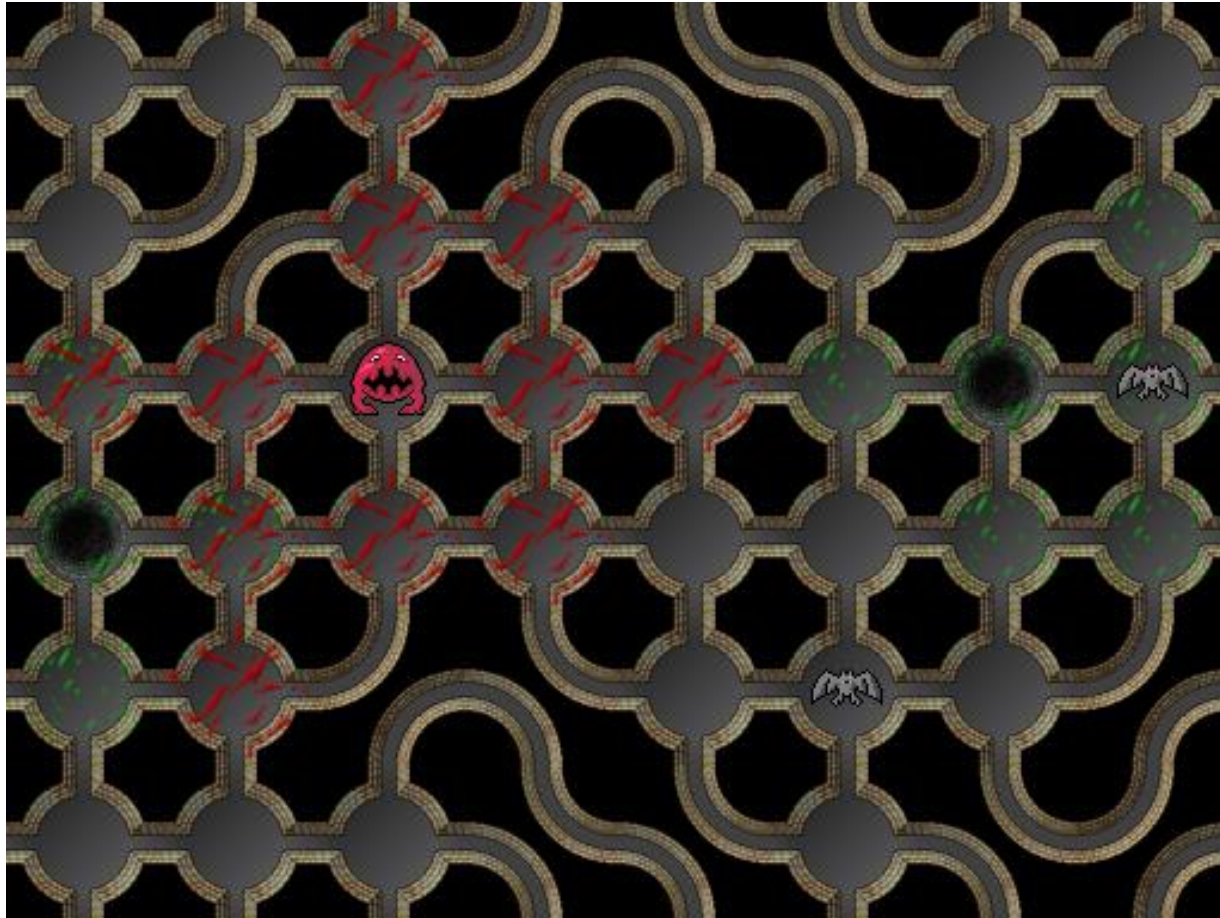
- Represent states, actions, etc.
- Incorporate new percepts
- Update internal representations of the world.
- Deduce hidden properties of the world e.g. diagnose
- Deduce appropriate actions e.g. deduce cure method

Wumpus World PEAS description

- ▶ Performance measure (Goal: to pick up the gold & return to start)
 - Gold + 1000, death - 1000
 - -1 per step, -10 for using the arrow
- ▶ Environment:
 - Square adjacent to wumpus are **smelly**
 - Square adjacent to pit are **breezy**
 - **Glitter** iff **gold** is in the same square
 - *Shooting* kills wumpus if you are facing it
 - *Shooting* uses up the only arrow
 - *Grabbing* picks up gold if in same square
 - *Releasing* drops the gold in same square
- ▶ Sensors: **Breeze, Glitter, Smell**
- ▶ Actuators:
 - Left turn, Right turn, Forward, Grab, Release, shoot



Demo

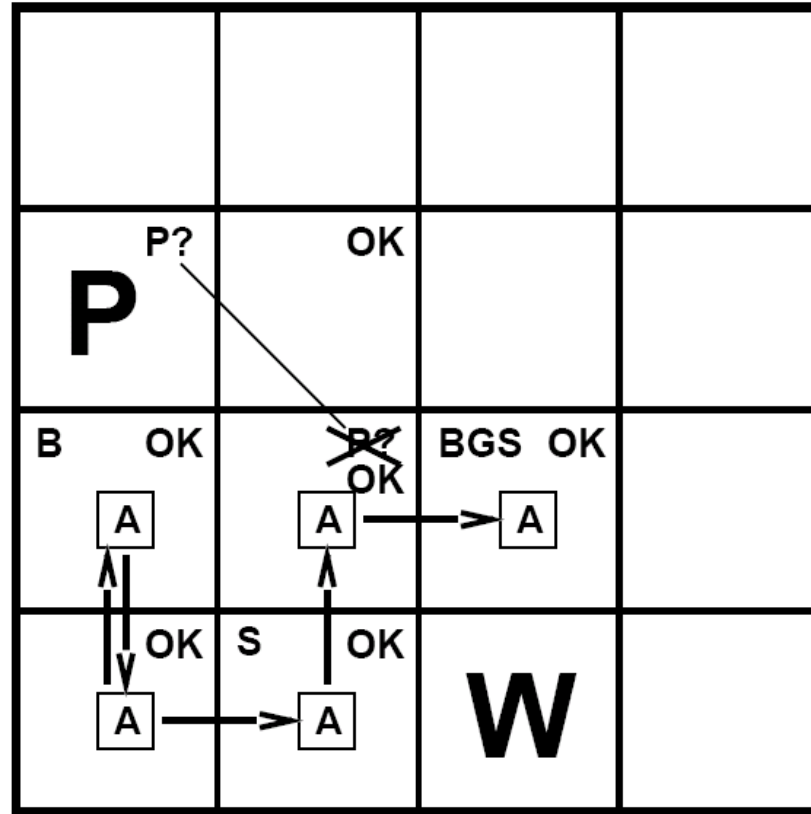


Our goal is to use logic to represent and play it automatically

Wumpus world characterization

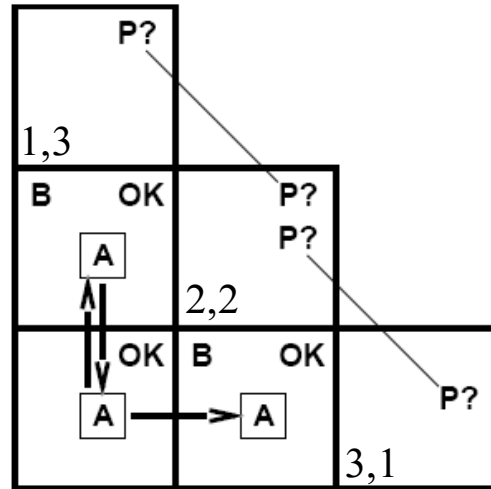
| | |
|---------------|--|
| Observable | No, only local perception |
| Deterministic | Yes, outcomes exactly specified |
| Episodic | No, sequential at the level of actions |
| Static | Yes, wumpus and pits do not move |
| Discrete | Yes |
| Single-agent | Yes, wumpus is essentially a natural feature |

Exploring a wumpus world

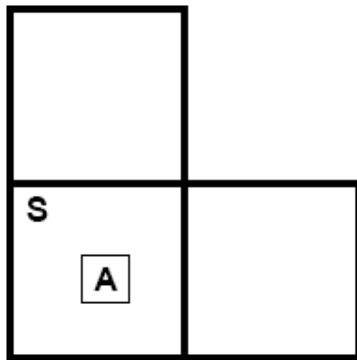


Agent:- Sensor percept -> outside world status -> action

Other tight spots



- ▶ Breeze in (1,2) and (2,1)
 \Rightarrow no safe actions
 - ▶ $B \Rightarrow (P,0) (0,P) (P,P)$ 3 possibilities
- ▶ Assuming pits uniformly distributed, (2,2) has pit w/ prob
 $0.89 = 1 - (1/3) * (1/3)$ vs.
 $2/3 = (0.67) = (1/3) + (1/3)$ for (1,3) and (3,1)



Smell in (1,1) \Rightarrow cannot move
 Can use a strategy of **coercion**:
 shoot straight ahead
 wumpus was there \Rightarrow dead \Rightarrow safe
 wumpus wasn't there \Rightarrow safe

Logic in general

Logics are formal languages for representing information such that conclusions can be drawn ^{knowledge} _{inference}

Syntax 語法 defines the sentences in the language. _{grammar}

Semantics 語義 define the “meaning” of sentences.

i.e. define **truth** of sentences in a world.

E.g. the language of *arithmetic*

- $x + 2 \geq y$ is sentence; $x^2 + y >$ is not a sentence _{syntax error}
- $x + 2 \geq y$ is true iff the number $x + 2$ is not less than the number y
- $x + 2 \geq y$ is true in a world where $x = 7, y = 1$
- $x + 2 \geq y$ is false in a world where $x = 0, y = 6$

Entailment

- ▶ Entailment means that one thing **follows from** another:

$$KB \models \alpha$$

- ▶ Knowledge base KB entails sentence α if and only if (iff) α is true in all worlds where KB is true
- ▶ E.g. the KB containing “HKU won” and “CUHK won” entails “Either HKU won or CUHK won”
- ▶ E.g. $x + y = 4$ entails $4 = x + y$
- ▶ Entailment is a relationship between sentences (i.e. **syntax**) that is based on **semantics**
- ▶ Note: brains process **syntax & semantics** (of some sort) **function** based on grammar. Fuzzy logic?

Models

Logicians typically think in terms of **models**: Formal structured worlds in which truth can be evaluated.

We say m is **a model** of a sentence α if α is true in m

Model: a world state represented
by a logic sentence (with truth values)
– a Truth Table row

$M(\alpha)$ is the set of all models of α

Then $KB \models \alpha$ iff $M(KB) \subseteq M(\alpha)$

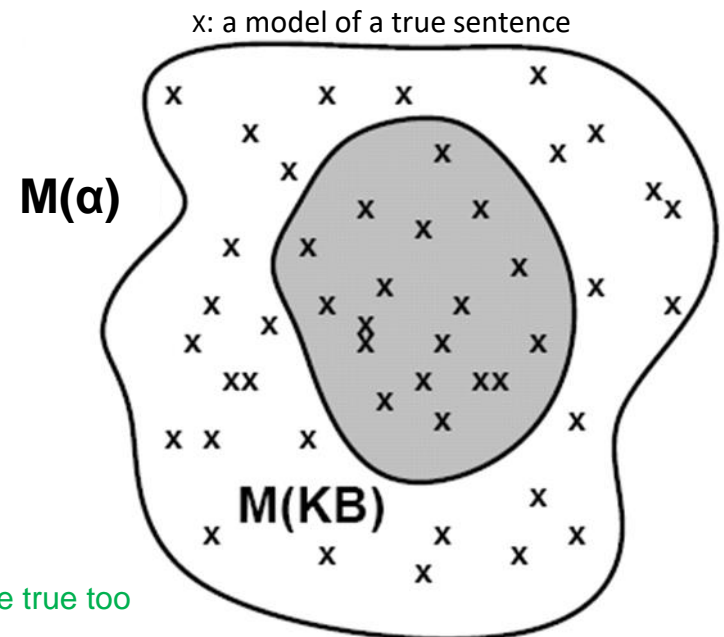
E.g. World: CUHK, HKU won or not?

4 possible models (TT, TF, FT, FF)

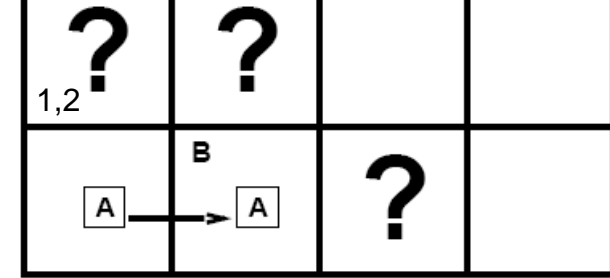
KB = CUHK won and HKU won (TT)

α = CUHK won $M(\alpha) = (TT, TF)$

$M(KB) \subseteq M(\alpha) \Rightarrow$ whenever KB is T, α must be true too



Models Examples



3 Models of KB

| $B_{1,1}$ | $B_{2,1}$ | $P_{1,1}$ | $P_{1,2}$ | $P_{2,1}$ | $P_{2,2}$ | $P_{3,1}$ | KB | α_1 |
|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-------------|-------------|
| false | false | false | false | false | false | false | false | true |
| false | false | false | false | false | false | true | false | true |
| ... | ... | ... | ... | ... | ... | ... | ... | ... |
| false | true | false | false | false | false | false | false | true |
| false | true | false | false | false | false | true | <u>true</u> | <u>true</u> |
| false | true | false | false | false | true | false | <u>true</u> | <u>true</u> |
| false | true | false | false | false | true | true | <u>true</u> | <u>true</u> |
| false | true | false | false | true | false | false | false | true |
| ... | ... | ... | ... | ... | ... | ... | ... | ... |
| true | true | true | true | true | true | true | false | false |

KB is true when all the 5 sentences in KB is true

$$\alpha_1 = \neg P_{1,2}?$$

KB $\models \alpha_1$?

iff $M(KB) \subseteq M(\alpha_1)$

$$\neg P_{1,1}; \neg B_{1,1}; B_{2,1};$$

$$B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1});$$

$$B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})$$

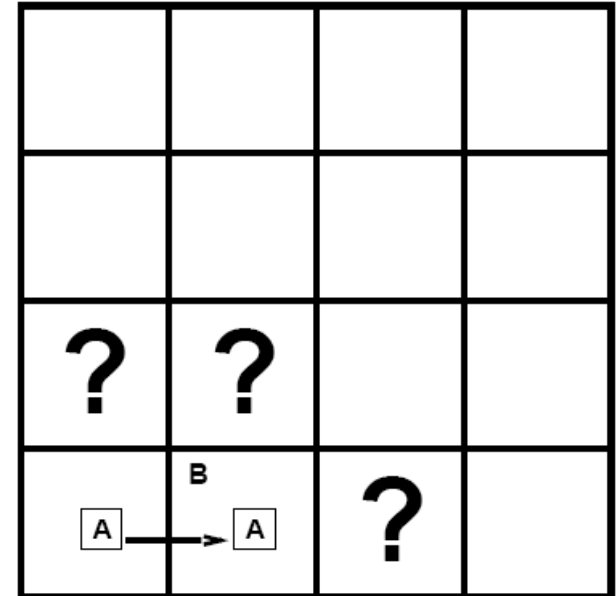
Inference by Truth Table Enumeration

Entailment in the wumpus world

Situation after detecting nothing in [1,1]
moving right, breeze in [2,1]

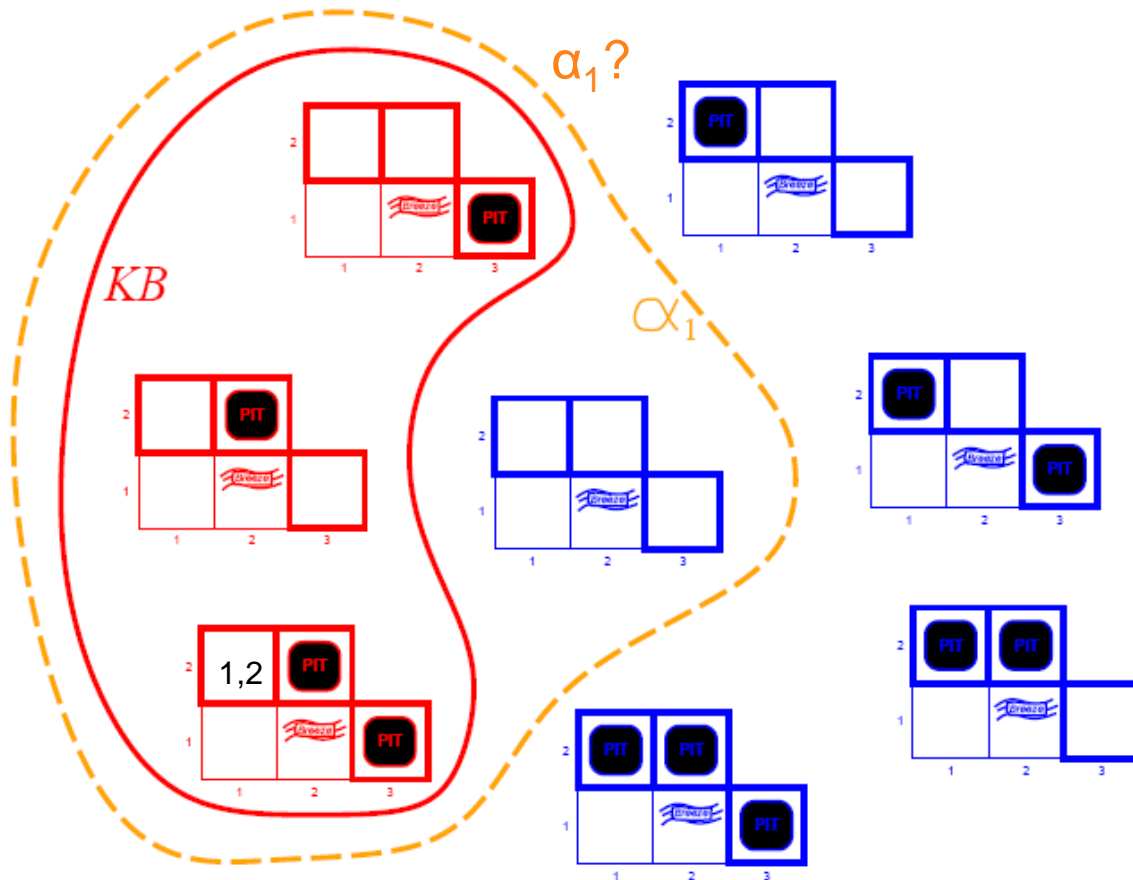
Consider possible models for ?s (3 Sqs)
assuming only **pits**

3 Boolean choices \Rightarrow 8 possible models



Wumpus models

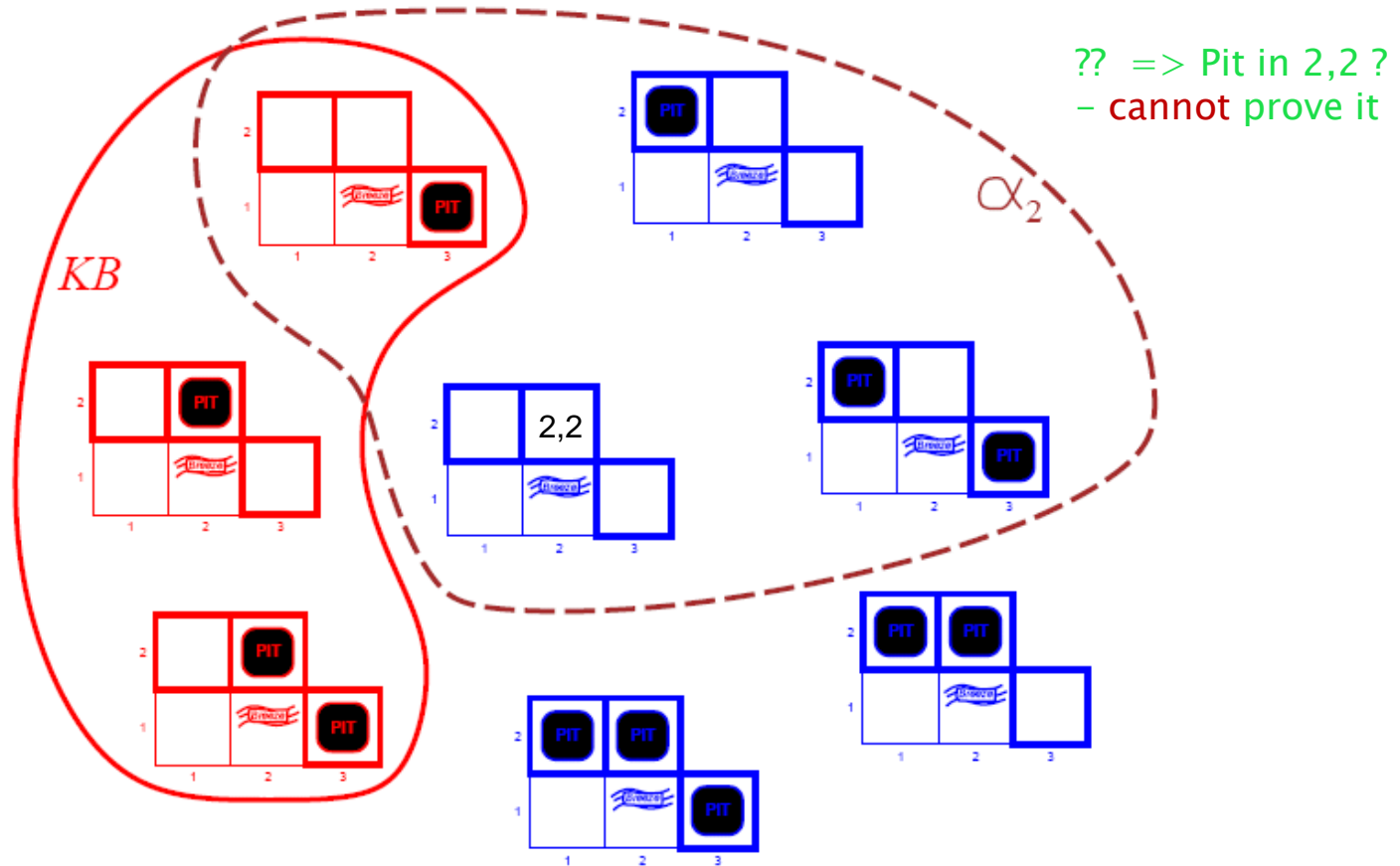
8 possible models



KB = wumpus-world rules + observations

α_1 = "[1,2] is safe", $KB \models \alpha_1$, proved by model checking enumeration

Wumpus models



KB = wumpus-world rules + observations

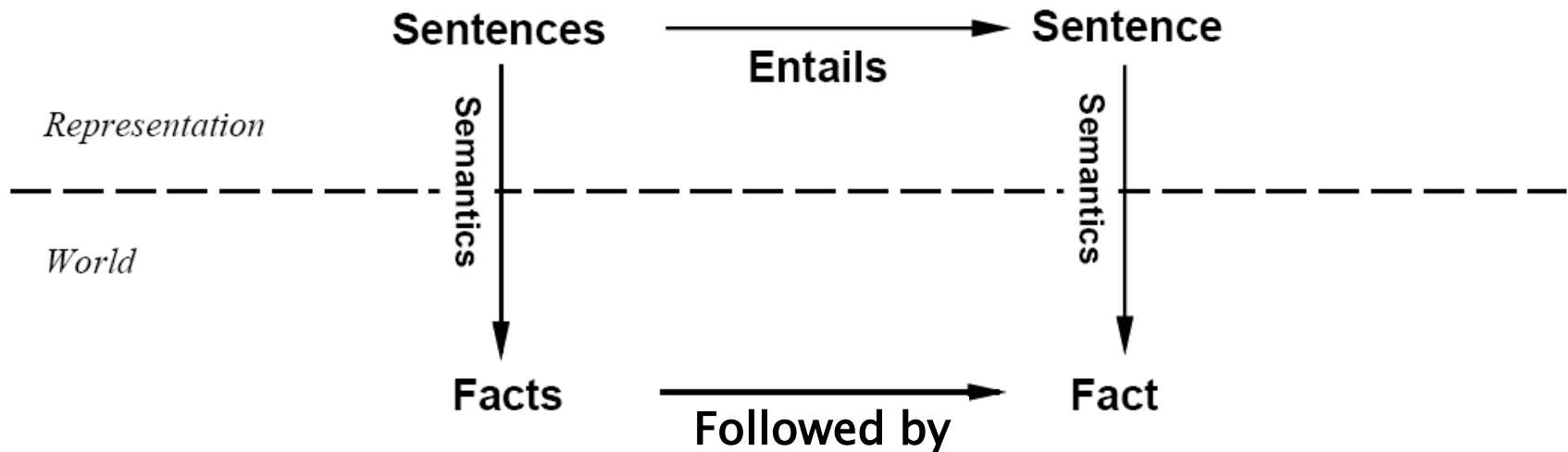
α_2 = "[2,2] is safe", $KB \not\models \alpha_2$??

Logic

- ▶ The **syntax** of a language describes the possible configurations of legal sentences.
- ▶ The **semantics** determines the facts in the world to which the sentences refer.

A **logic language** defines the syntax and semantics precisely. From the syntax and semantics, we can derive an **inference** mechanism for an agent to use the language.

Logic



- ▶ The combination between sentences and facts is provided by the **semantics** of the language.
- ▶ The property of one fact **following** from some other facts is mirrored by the property of one sentence being **entailed** by some other sentences.
- ▶ Logical **inference** generates new sentences that are entailed by existing sentences. ⇔ reasoning in real world

Inference

$KB \vdash_i \alpha \Rightarrow$ sentence α can be ^{inferred} **derived** from KB by procedure i

Consequences of KB are a haystack; α is a needle.

Entailment = needle in haystack; inference = finding it

Soundness: i is sound if
whenever $KB \vdash_i \alpha$, it is also true that $KB \models \alpha$

Completeness: i is complete if
whenever $KB \models \alpha$, it is also true that $KB \vdash_i \alpha$

Preview: first-order logic is expressive enough to say almost anything of interest with a sound and complete inference procedure.

That is, the procedure will answer any question whose answer entailed by the KB .

Propositional logic: Syntax

- ▶ Propositional logic is the simplest logic – illustrates basic ideas
- ▶ The proposition symbol P_1 , P_2 etc... are sentences
- ▶ If S is a sentences, $\neg S$ is a sentence (**negation**)
- ▶ If S_1 and S_2 are sentences, $S_1 \wedge S_2$ is a sentence (**conjunction**)
- ▶ If S_1 and S_2 are sentences, $S_1 \vee S_2$ is a sentence (**disjunction**)
- ▶ If S_1 and S_2 are sentences, $S_1 \Rightarrow S_2$ is a sentence (**implication**)
- ▶ If S_1 and S_2 are sentences, $S_1 \Leftrightarrow S_2$ is a sentence (**biconditional**)
- ▶ More complicated sentences can be made using above rules recursively

Many rule-based Expert systems!

Propositional logic: Semantics

Each model specifies true/ false for each proposition symbol

E.g. ^{Pit in 1,2} $P_{1,2}$ $P_{2,2}$ $P_{3,1}$
 true true false

(With these symbols, 8 possible models, can be enumerated automatically.)

Rules for evaluating truth with respect to a model m :

| | | | |
|---------------------------------------|-----------------------------------|-------------------------------|--|
| $\neg S$ is true iff | S is false | | |
| $S_1 \wedge S_2$ is true iff | S_1 is true and | S_2 is true | |
| $S_1 \vee S_2$ is true iff | S_1 is true or | S_2 is true | |
| $S_1 \Rightarrow S_2$ is true iff | S_1 is false or | S_2 is true | |
| i.e., is false iff | S_1 is true and | S_2 is false | |
| $S_1 \Leftrightarrow S_2$ is true iff | $S_1 \Rightarrow S_2$ is true and | $S_2 \Rightarrow S_1$ is true | |

Simple **recursive** process evaluates an arbitrary sentence, e.g.

$\neg P_{1,2} \wedge (P_{2,2} \vee P_{3,1}) = \text{true} \wedge (\text{false} \vee \text{true}) = \text{true} \wedge \text{true} = \text{true}$

Truth tables for connectives

| P | Q | $\neg P$ | $P \wedge Q$ | $P \vee Q$ | $P \Rightarrow Q$ | $P \Leftrightarrow Q$ |
|-------|-------|----------|--------------|------------|-------------------|-----------------------|
| false | false | true | false | false | true | true |
| false | true | true | false | true | true | false |
| true | false | false | false | true | false | false |
| true | true | false | true | true | true | true |

rule base systems

$F \Rightarrow F$??

SARS \Rightarrow Die

No SARS \Rightarrow Won't die X

Wumpus world sentences

KB:

Let $P_{i,j}$ be true if there is a pit in $[i, j]$

Let $B_{i,j}$ be true if there is a breeze in $[i, j]$

$\neg P_{1,1}$

$\neg B_{1,1}$

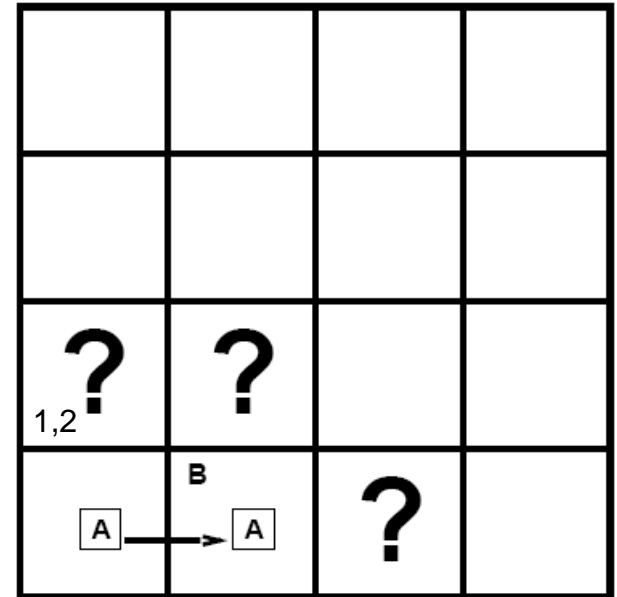
$B_{2,1}$

“Pits cause breeze in adjacent squares”

$B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$

$B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})$

“A square is breezy **iff** there is an adjacent pit”



Question: $\alpha_1 = \neg P_{1,2}$; $KB \models \alpha_1$?

Truth tables for inference (Inference by model enumeration)

| | $B_{1,1}$ | $B_{2,1}$ | $P_{1,1}$ | $P_{1,2}$ | $P_{2,1}$ | $P_{2,2}$ | $P_{3,1}$ | KB | α_1 |
|----------------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-------------|-------------|
| | false | false | false | false | false | false | false | false | true |
| | false | false | false | false | false | false | true | false | true |
| | ... | ... | ... | ... | ... | ... | ... | ... | ... |
| | false | true | false | false | false | false | false | false | true |
| 3 Models of KB | false | true | false | false | false | false | true | <u>true</u> | <u>true</u> |
| | false | true | false | false | false | true | false | <u>true</u> | <u>true</u> |
| | false | true | false | false | false | true | true | <u>true</u> | <u>true</u> |
| | false | true | false | false | true | false | false | false | true |
| | ... | ... | ... | ... | ... | ... | ... | ... | ... |
| | true | true | true | true | true | true | true | false | false |

KB is true when all the 5 sentences in KB is true

$$\alpha_1 = \neg P_{1,2}$$

KB $\models \alpha_1$ why?

iff $M(KB) \subseteq M(\alpha)$

$\neg P_{1,1}; \neg B_{1,1}; B_{2,1};$
 $B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1});$
 $B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})$

How to
program
it??

Goto p.32

Inference by enumeration

Build the complete truth table by recursive calls forming a depth-first tree(?)

Depth-first enumeration of all models is sound and complete

```
function TT-Entails? (KB,  $\alpha$ ) returns true or false
  symbols  $\leftarrow$  a list of proposition symbols in KB and  $\alpha$ 
  return TT-Check-All (KB,  $\alpha$ , symbols, [ ] )

function TT-Check-All(KB,  $\alpha$ , symbols, model) returns true or false
  if Empty?(symbols) then // row completed in TT
    if PL-True?(KB, model) then return PL-True?( $\alpha$ , model) //  $T \Rightarrow T/F \Leftrightarrow T/F$ 
    else return true //  $F \Rightarrow T/F \Leftrightarrow T$ 
  else do
    P  $\leftarrow$  First(symbols); rest  $\leftarrow$  Rest(symbols)
    return TT-Check-All(KB,  $\alpha$ , rest, Extend(P, true, model)) and
      TT-Check-All(KB,  $\alpha$ , rest, Extend(P, false, model))
```

$O(2^n)$ for n symbols; problem is co-NP-complete

Extend(*P*, *true*, *model*) returns a new partial model in which *P* has the value *true*

Extend(*P*, *false*, *model*) returns a new partial model in which *P* has the value *false*

S

T

F

Symbols-1 (P)

T F

T F

Symbols-2 (P)

T F T F

T F T F

Symbols-3 (P)

▶ TTT

FTT

▶ TTF

FTF

▶ TFT

FFT

▶ TFF

FFF

function TT-Check-All(*KB*, α , *symbols*, *model*) **returns** *true or false*
if Empty?(*symbols*) **then** // row completed in TT
 if PL-True?(*KB*, *model*) **then** **return** PL-True?(α , *model*) // $T \Rightarrow T/F \Leftrightarrow T/F$
 else **return** *true* // $F \Rightarrow T/F \Leftrightarrow T$
else do
 $P \leftarrow$ First(*symbols*); *rest* \leftarrow Rest(*symbols*)
 return TT-Check-All(*KB*, α , *rest*, Extend(*P*, *true*, *model*)) **and**
 TT-Check-All(*KB*, α , *rest*, Extend(*P*, *false*, *model*))

Logical Equivalence

Two sentences are **logically equivalent** iff true in same models:

$\alpha \equiv \beta$ if and only if $\alpha \models \beta$ and $\beta \models \alpha$

$(\alpha \wedge \beta) \equiv (\beta \wedge \alpha)$ commutativity of \wedge

$(\alpha \vee \beta) \equiv (\beta \vee \alpha)$ commutativity of \vee

$((\alpha \wedge \beta) \wedge \gamma) \equiv (\alpha \wedge (\beta \wedge \gamma))$ associativity of \wedge

$((\alpha \vee \beta) \vee \gamma) \equiv (\alpha \vee (\beta \vee \gamma))$ associativity of \vee

$\neg(\neg \alpha) \equiv \alpha$ double-negation elimination

$(\alpha \Rightarrow \beta) \equiv (\neg \beta \Rightarrow \neg \alpha)$ contraposition

$(\alpha \Rightarrow \beta) \equiv (\neg \alpha \vee \beta)$ implication elimination

$(\alpha \Leftrightarrow \beta) \equiv ((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha))$ biconditional elimination

$\neg(\alpha \wedge \beta) \equiv (\neg \alpha \vee \neg \beta)$ de Morgan

$\neg(\alpha \vee \beta) \equiv (\neg \alpha \wedge \neg \beta)$ de Morgan

$(\alpha \wedge (\beta \vee \gamma)) \equiv ((\alpha \wedge \beta) \vee (\alpha \wedge \gamma))$ distributivity of \wedge over \vee

$(\alpha \vee (\beta \wedge \gamma)) \equiv ((\alpha \vee \beta) \wedge (\alpha \vee \gamma))$ distributivity of \vee over \wedge

[Back to p.40](#)

Validity and satisfiability

A sentence is **valid** if it is true in all models,
e.g. True, $A \vee \neg A$, $A \Rightarrow A$, $(A \wedge \overline{(A \Rightarrow B)}) \Rightarrow B$

Validity is connected to inference via the **Deduction Theorem**:
 $KB \models \alpha$ if and only if $(KB \Rightarrow \alpha)$ is valid

A sentence is **satisfiable** if it is true in **some** models
e.g. $A \vee B$, C

A sentence is **unsatisfiable** if it is true in **no** models [Back to p.41](#)
e.g. $A \wedge \neg A$

Satisfiability is connected to inference via the following:

$KB \models \alpha$ iff $(KB \wedge \neg \alpha)$ is unsatisfiable; $(\neg(\neg KB \vee \alpha) \equiv KB \wedge \neg \alpha)$
 $(\neg(KB \Rightarrow \alpha))$
i.e. prove α by **reductio ad absurdum** (reduction to an absurd thing)
▶ prove by refutation or by contradiction

Proof methods

Proof methods divide into (roughly) two kinds:

- ▶ **Application of inference rules (FC, BC, resolution)**
 - Legitimate (sound) generation of new sentences from old
 - **Proof** = a sequence of inference rule applications. Can use inference rules as operators in a standard search algorithm
 - Typically require translation of sentences into a **normal form**
- ▶ **Model checking**
 - Truth table enumeration (always exponential in n)
 - Improved backtracking, e.g. Davis–Putnam–Logemann–Loveland
 - Heuristic search in model space (sound but incomplete)
E.g. min–conflicts–like hill–climbing algorithms

Forward and backward chaining

Horn Form (restricted)

KB = conjunction of Horn clauses

Horn clause =

- Proposition symbol; or
- (conjunction of symbols) \Rightarrow symbol

E.g. $C \wedge (B \Rightarrow A) \wedge (C \wedge D \Rightarrow B)$

Modus Ponens (for Horn Form: complete for Horn KBs)

$$\frac{\alpha_1, \dots, \alpha_n, \quad \alpha_1 \wedge \dots \wedge \alpha_n \Rightarrow \beta}{\beta} \quad \text{inference rule}$$

Can be used with forward chaining or backward chaining.
Their algorithms are very natural and run in linear time

Forward chaining

Idea: fire any rule whose premises are satisfied in the KB. Add its conclusion to the KB, until query Q is found.

Rules, implications

Facts

Antecedent-consequence

-conclusion

-Head in the program

Premises: elements of Ante.

$$P \Rightarrow Q$$

$$L \wedge M \Rightarrow P$$

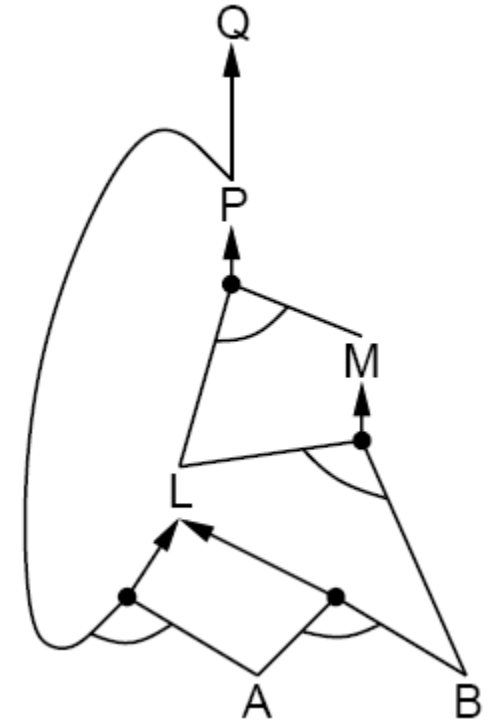
$$B \wedge L \Rightarrow M$$

$$A \wedge P \Rightarrow L$$

$$A \wedge B \Rightarrow L$$

A

B



Where to check Q first??

Forward chaining algorithm

$P \Rightarrow Q$
 $L \wedge M \Rightarrow P$
 $B \wedge L \Rightarrow M$
 $A \wedge P \Rightarrow L$
 $A \wedge B \Rightarrow L$
 A
 B

function PL-FC-Entails? (*KB*, *q*) **return** *true or false* //*q=query*

local variables:

count, a table, indexed by clause, initially the number of premises //*clause=rule*

inferred, a table, indexed by symbol, each entry initially *false*

agenda, a list of symbols, initially the symbols known to be *true* //*fact base*

while *agenda* is not empty **do**

$p \leftarrow \text{Pop}(\text{agenda})$

unless *inferred*[*p*] **do** //check for repeating inferred symbols

inferred[*p*] $\leftarrow \text{true}$

for each Horn clause *c* in whose premise *p* appears **do** //matched premise

decrement *count*[*c*] //*cth rule*

if *count*[*c*] = 0 **then do** //*cth rule fired*

if *Head*[*c*] = *q* **then return** *true*

Push (*Head*[*c*], *agenda*) //Don't push if *inferred*[*Head*[*c*]]

return *false*

Forward chaining algorithm

- ▶ Forward-chaining(FC) for proposition logic(PL)
- ▶ *agenda*: symbols known to be true but not yet “processed”
- ▶ *count*: how many premises of each implication are yet unknown
- ▶ Whenever a new symbol p from the agenda is processed, the count is reduced by 1 for each implication in whose premise p appears.
- ▶ If a count = 0, all the premises of the implication are known, so fire and its conclusion *Head* [c] is added to the agenda
- ▶ Keep track of symbols processed; no need to add an inferred symbol to the agenda if it has been processed previously. This avoids redundant work and prevents infinite loops such as $P \Rightarrow Q$ and $Q \Rightarrow P$. (*inferred*[*Head*[c]])

Forward chaining Example

Agenda

A

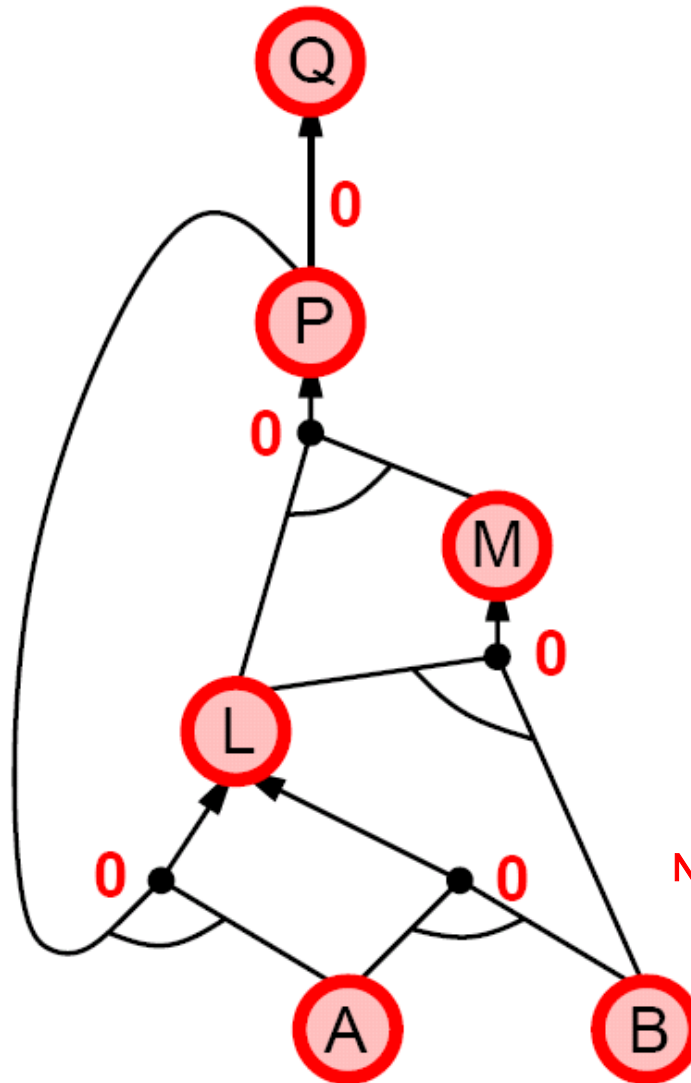
B

L

M

P

Q



$$P \Rightarrow Q$$

$$L \wedge M \Rightarrow P$$

$$B \wedge L \Rightarrow M$$

$$A \wedge P \Rightarrow L$$

$$A \wedge B \Rightarrow L$$

A

B

Nos.: counts of unknown nos.
of premises of rules

Backward chaining

Idea: work backwards from query q :

- ▶ to prove q by BC,
 1. check if q is known already, or
 2. prove by BC all premises of some rule(s) concluding q

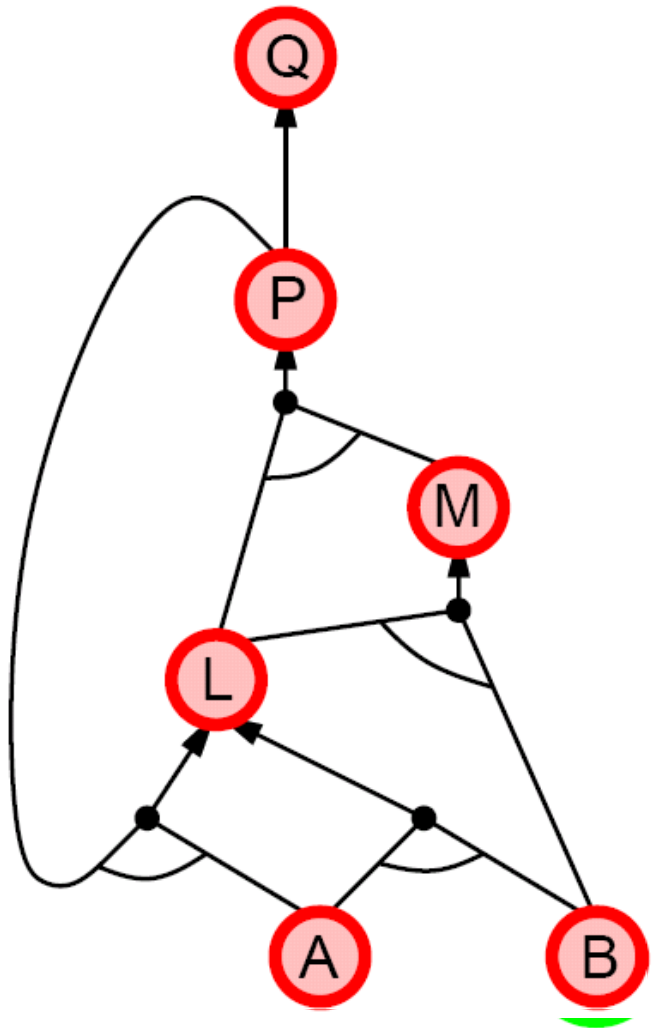
Avoid loops: check if new sub-goal is already on the goal stack

Avoid repeated work: check if new sub-goal

1. has already been proved true or
2. has already failed

Backward chaining example

| <i>Fact</i> | <i>Goal Stack</i> |
|-------------|-------------------|
| <i>A</i> | <i>A</i> |
| <i>B</i> | <i>B</i> |
| <i>L</i> | <i>L</i> |
| <i>M</i> | <i>M</i> |
| <i>P</i> | <i>P</i> |
| <i>Q</i> | <i>Q</i> |



$$P \Rightarrow Q$$

$$L \wedge M \Rightarrow P$$

$$B \wedge L \Rightarrow M$$

$$A \wedge P \Rightarrow L$$

$$A \wedge B \Rightarrow L$$

A

B

Forward vs. backward chaining

Forward chaining:

- FC is **data-driven**, cf. automatic, unconscious processing, e.g. object recognition, routine decisions
- May do lots of works that is irrelevant to the goal

Backward chaining:

- BC is **goal-driven**, appropriate for problem-solving, e.g. Where are my keys? How do I get into a PhD program?
- Complexity of BC can be much less than linear in size of KB

Resolution

Conjunctive Normal Form (CNF–universal)

Conjunction of “disjunctions of literals” (clauses)

E.g. $(A \vee \neg B) \wedge (B \vee \neg C \vee \neg D)$

Cf. Horn Form (clauses)

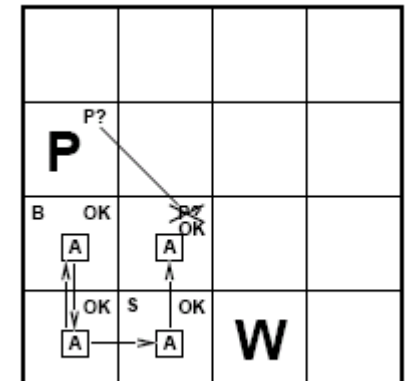
Resolution inference rule (for CNF):
complete for propositional logic

$$\frac{l_1 \vee \dots \vee l_k, \quad m_1 \vee \dots \vee m_n}{l_1 \vee \dots \vee l_{i-1} \vee l_{i+1} \vee \dots \vee l_k \vee m_1 \vee \dots \vee m_{j-1} \vee m_{j+1} \vee \dots \vee m_n}$$

where l_i and m_j are **complementary** literals. E.g.

$$\frac{P_{1,3} \vee P_{2,2}, \quad \neg P_{2,2}}{P_{1,3}}$$

Resolution is sound and complete for
propositional logic



Conversion to CNF

$$B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$$

Goto equivalent rules

1. Eliminate \Leftrightarrow , replacing $\alpha \Leftrightarrow \beta$ with $(\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)$

$$B_{1,1} \Rightarrow (P_{1,2} \vee P_{2,1}) \wedge (P_{1,2} \vee P_{2,1}) \Rightarrow B_{1,1}$$

2. Eliminate \Rightarrow , replacing $\alpha \Rightarrow \beta$ with $\neg \alpha \vee \beta$.

$$(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge (\neg (P_{1,2} \vee P_{2,1}) \vee B_{1,1})$$

3. Move \neg inwards using de Morgan's rules and double-negation:

$$(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge ((\neg P_{1,2} \wedge \neg P_{2,1}) \vee B_{1,1})$$

4. Apply distributivity law (\vee over \wedge) and flatten:

$$(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge (\neg P_{1,2} \vee B_{1,1}) \wedge (\neg P_{2,1} \vee B_{1,1})$$

Conjunctive Normal Form

Resolution algorithm

To proof KB entails α $\neg(KB \Rightarrow \alpha) \equiv KB \wedge \neg \alpha$

Proof by **contradiction**, i.e. show $KB \wedge \neg \alpha$ unsatisfiable [Goto p.28](#)

function PL-Resolution (KB, α) **returns** *true or false*

clauses \leftarrow the set of clauses in the CNF representation of $KB \wedge \neg \alpha$

new $\leftarrow \{\}$

loop do

for each C_i, C_j in *clauses* **do**

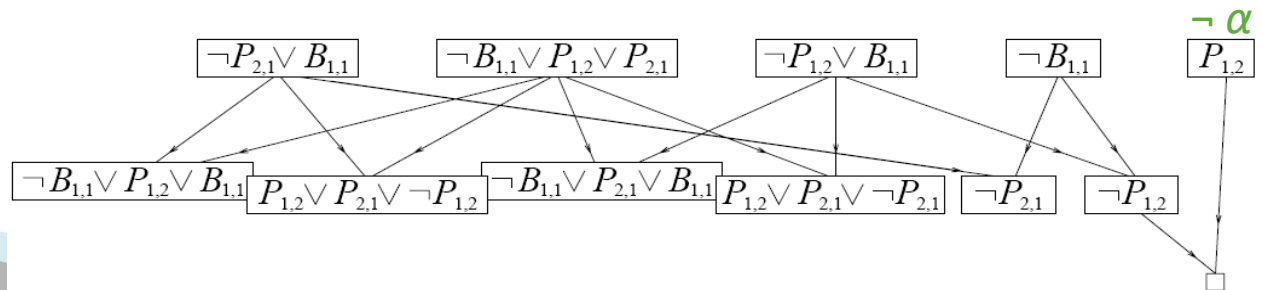
resolvents \leftarrow PL-Resolve (C_i, C_j)

if *resolvents* contains the empty clause **then return** *true*

new $\leftarrow new \cup resolvents$

if $new \subseteq clauses$ **then return** *false* //no new resolvents generated \Rightarrow proof failed

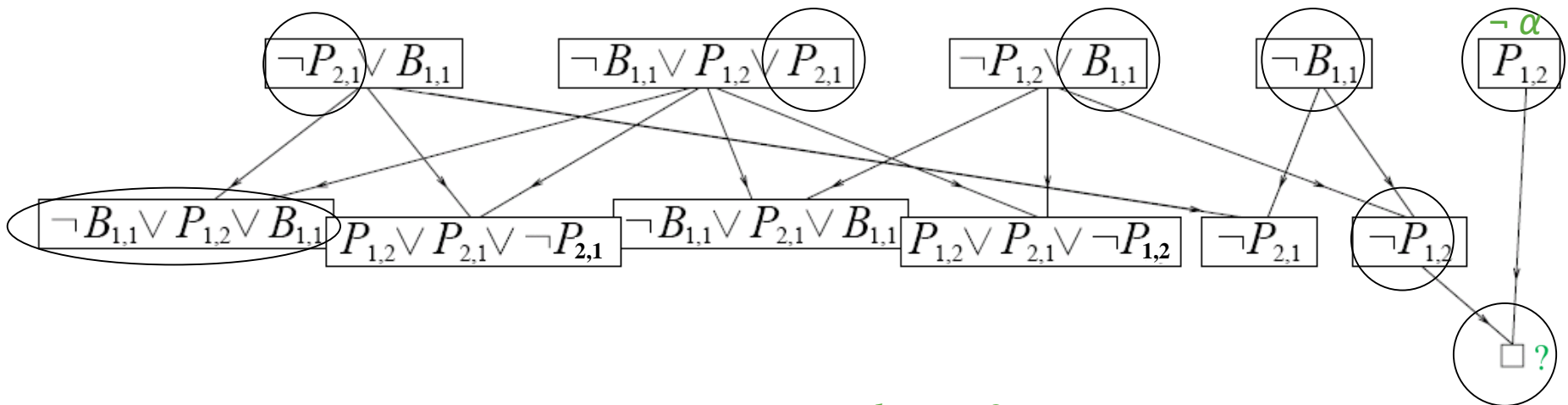
clauses $\leftarrow clauses \cup new$



Resolution example

No breeze in 1,1. Prove NO Pit in 1,2.

$$KB = (B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})) \wedge \neg B_{1,1} \quad \alpha = \neg P_{1,2}$$



new \subseteq clauses ?

Summary

- ▶ Logical agents apply **inference** to a **knowledge base** to derive new information and make decisions
- ▶ Basic concepts of logic:
 - **Syntax**: formal structure of **sentences**
 - **Semantics**: **truth** of sentences with respect to **models**
 - **Entailment**: necessary truth of one sentence given another
 - **Inference**: deriving sentences from other sentences
 - **Soundness**: derivations produce only entailed sentences
 - **Completeness**: derivations can produce all entailed sentences
- ▶ Wumpus world requires the ability to represent partial and negated information, reason by cases, etc.
- ▶ Forward, backward chaining are linear-time, complete for Horn clauses
- ▶ Resolution is complete for propositional logic
- ▶ Propositional logic lacks expressive power