ENGG 1130 (2019-2020 Term 2) HW 3 Suggested Answer

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① (a) Length $= \int_0^4 \|\underline{r}'(t)\|\,dt$, where $\underline{r}'(t) = \langle 0,0,0\rangle$

$$= \int_0^4 \sqrt{0^2+0^2+0^2}\,dt = \underline{\underline{0}}$$

(b) Length $= \int_0^2 \|\underline{r}'(t)\|\,dt$, where $\underline{r}'(t) = \langle 1,2,3\rangle$

$$= \int_0^2 \sqrt{1^2+2^2+3^2}\,dt$$

$$= \underline{\underline{2\sqrt{14}}}$$

② $\int_0^3 (t\hat{\underline{i}} + t\hat{\underline{j}} + t\hat{\underline{k}})\,dt = \hat{\underline{i}}\int_0^3 t\,dt + \hat{\underline{j}}\int_0^3 t\,dt + \hat{\underline{k}}\int_0^3 t\,dt$

$$= \hat{\underline{i}}\,\tfrac{1}{2}t^2\Big|_0^3 + \hat{\underline{j}}\,\tfrac{1}{2}t^2\Big|_0^3 + \hat{\underline{k}}\,\tfrac{1}{2}t^2\Big|_0^3$$

$$= \tfrac{9}{2}\hat{\underline{i}} + \tfrac{9}{2}\hat{\underline{j}} + \tfrac{9}{2}\hat{\underline{k}} \,//$$

③ $\underline{r}'(t) = \langle 1,1,1\rangle$

$\underline{r}(t) = \int \underline{r}'(t)\,dt = \langle t+C_1, t+C_2, t+C_3\rangle$ (*), where C_1, C_2, C_3 are constants

Now, $\underline{r}(0) = \langle 1,3,10\rangle \Rightarrow$ Sub $t=0$ to (*), $\underline{r}(0) = \langle C_1, C_2, C_3\rangle = \langle 1,3,10\rangle$

$\Rightarrow \underline{r}(t) = \langle t+1, t+3, t+10\rangle \,//$

④ $\underline{r}(t) = \langle t, t, t \rangle,\ t > 0$

$\underline{r}'(t) = \langle 1, 1, 1 \rangle$ ⟶ velocity vector

$\text{Speed} = \|\underline{r}'(t)\| = \sqrt{1^2+1^2+1^2} = \sqrt{3}$

$\text{Acceleration} = \|\underline{r}''(t)\| = \|\langle 0, 0, 0 \rangle\| = \underline{\underline{0}}$

⑤ (a) $f(x,y)$ is only well-defined iff $1 - x^2 - y^2 \geq 0$

i.e. $x^2 + y^2 \leq 1$

When $x^2+y^2 = 1$, $f(x,y) = \sqrt{1-1} = 0$

But $x^2+y^2 \geq 0 \Rightarrow$ when $x^2+y^2 = 0$, $f(x,y) = \sqrt{1-0} = 1$

$\therefore$ Domain: $\{(x,y) \in \mathbb{R}^2 \mid 0 \leq x^2+y^2 \leq 1\}$ Range $= [0, 1]$

⑤(b) $g(x,y) = \ln(x^2+y^2-1)$ is only well-defined iff $x^2+y^2-1 > 0$

i.e. $x^2+y^2 > 1$

Domain $= \{(x,y) \in \mathbb{R}^2 \mid x^2+y^2 > 1\}$

whenever $x^2+y^2 > 1$, $g(x,y)$ will be mapped to a real value

i.e. range $= \underline{\mathbb{R}}$ (or $(-\infty, +\infty)$) / the $\underline{\underline{\text{set}}}$ of real nos

⑥ (a) Since $1+x+y$ is cont on an open disk centered at $(1,2)$

$\Rightarrow \lim\limits_{(x,y)\to(1,2)} (1+x+y) = 1+1+2 = \underline{\underline{4}}$

(b) Since $\tan\left(\frac{x}{y}\right)$ is continuous on an open disk centered at $(\pi,4)$

$\Rightarrow \lim\limits_{(x,y)\to(\pi,4)} \tan\left(\frac{x}{y}\right) = \tan\left(\frac{\pi}{4}\right) = \underline{\underline{1}}$

(c) For $\lim\limits_{(x,y)\to(0,0)} \frac{y(x-y)}{x+y^2}$

<u>Path 1</u>: Take $y=mx$, then $\frac{y(x-y)}{x^2+y^2} = \frac{mx^2(1-m)}{x+m^2x^2} = \frac{mx(1-m)}{m^2x+1}$ — $(*)$

<u>Path 2</u>: Along $x=0$, then $\lim\limits_{(x,y)\to(0,0)} \frac{y(x-y)}{x+y^2} = \lim\limits_{y\to 0} \frac{-y^2}{y^2} = -1$

For path 1, suppose we take $m=3$, the expression $(*)$ becomes $\frac{-6x}{9x+1}$

$\lim\limits_{x\to 0} \frac{-6x}{9x+1} = 0 \neq -1$

$\therefore$ Limit does <u>NOT</u> exist

In particular, if we take $y=0$, limit $= \frac{0}{x} = 0 \neq -1$, so limit does **NOT** exist

<u>Alternatively</u>, we use polar coordinates.

$\lim\limits_{(x,y)\to(0,0)} \frac{y(x-y)}{x+y^2}$

$= \lim\limits_{r\to 0^+} \frac{r\sin\theta(r\cos\theta - r\sin\theta)}{r\cos\theta + r^2\sin^2\theta}$

$= \lim\limits_{r\to 0^+} \frac{r^2(\sin\theta\cos\theta - \sin^2\theta)}{r(\cos\theta + r\sin\theta)}$

Take different choice of θ, then obtain diff limits

(7) Let $f(x,y)=\sqrt{x-y-3}-x$

Let L be the level curve of f that passes through $(3,-1)$

f is well-defined when $x-y-3\geq 0 \Rightarrow y\leq x-3$.

Consider $f(3,-1)=-2$, so we let $(x,y)\in\mathbb{R}^2$ such that

$f(x,y)=-2 \Rightarrow \sqrt{x-y-3}=x-2$

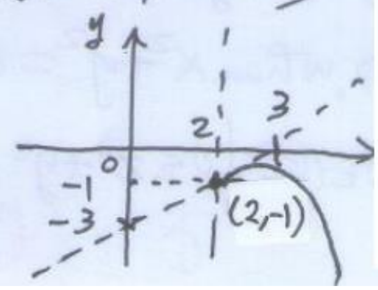
i.e. $x-2\geq 0$ and $x-y-3=(x-2)^2$

i.e. $x\geq 2$ and $y=-x^2+5x-7$

$\therefore L=\{(x,y)\in\mathbb{R}^2 \mid y=-x^2+5x-7,\ y\leq x-3,\ x\geq 2\}$

$\because -x^2+5x-7=-(x-2)^2+x-3\leq x-3$

$\therefore L=\{(x,y)\in\mathbb{R}^2 \mid y=-x^2+5x-7,\ x\geq 2\}$ //



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