Practice questions

1. An "early" person will arrive late by some number of minutes that is uniformly distributed from 0 to 10. A "late" person will be late for some number of minutes that is uniformly distributed from 10 to 20. We will be observing the arrival time of Bob, and we have no idea whether he is a late person or not (say he is a late person with probability 0.5).

   - Let θ be the indicator random variable that is 1 if Bob is a late person and 0 otherwise. Write down the prior PMF $f_\Theta(\theta)$ for θ.
   - Let X be the random variable giving how late Bob arrives. Write down the conditional PDF $f_{X|\Theta}(x|\theta)$ of how late Bob is in terms of parameter θ.
   - We observe Bob arrive 8 minutes late. What is the posterior PMF on θ and the CDF of Bob's arrival time for future events? Will observing how late Bob is in the future tell us anything more about how late he typically is?

2. Walter moves to a small town with 5000 valid telephone numbers. He has no idea what his number is so he dials one of them uniformly at random and hears a "busy tone", meaning that the line is in use. The probability that a phone is busy at any given moment is just 0.01, so Walter concludes that he guessed his number correctly. What is the probability that his conclusion is correct?

3. You are running a Brexit poll where people indicate Yes/No for their approval. Show that it should suffice to poll a quarter-million people to obtain both confidence and sampling errors 1%. Your boss is convinced that Brexit has less than 20% public support. Impress him by arguing that, if he is correct, it suffices to poll just 160000 people to obtain the same confidence and sampling errors.

4. Alice is studying some radioactive process where the number of hours $X \geq 0$ until some particle decays is given by the PDF $f_X(x) = \theta \cdot e^{-\theta x}$. Alice doesn't know what θ is, but assumes a prior PDF uniform over $[1, 2]$.

   (a) Write down the prior PDF for θ and the conditional PDF for X given θ.

   Alice runs her experiment and the particle decays after 17 hours. To estimate the probability that the particle decays in fewer than 10 hours, she chooses the θ that is most likely based on her experiment and assumes that this is the correct PDF.

   (b) What value of θ will she choose and what estimate will she then derive for the probability that the particle decays in at most 10 hours?

5. Consider a biased coin where the probability of heads, Θ, is distributed over $[0, 1]$ according to the PDF

$$f_\Theta(\theta) = 2 - 4\left|\frac{1}{2} - \theta\right|$$

   Find the MAP estimate of Θ, assuming that n independent coin tosses resulted in k heads and $n - k$ tails.