ENGG1410C: LINEAR ALGEBRA AND VECTOR CALCULUS FOR ENGINEERS

2nd Term, 2016-17

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Rm 317, ERB

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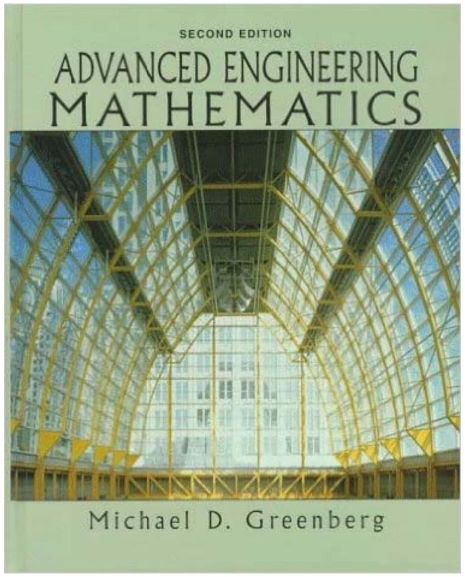
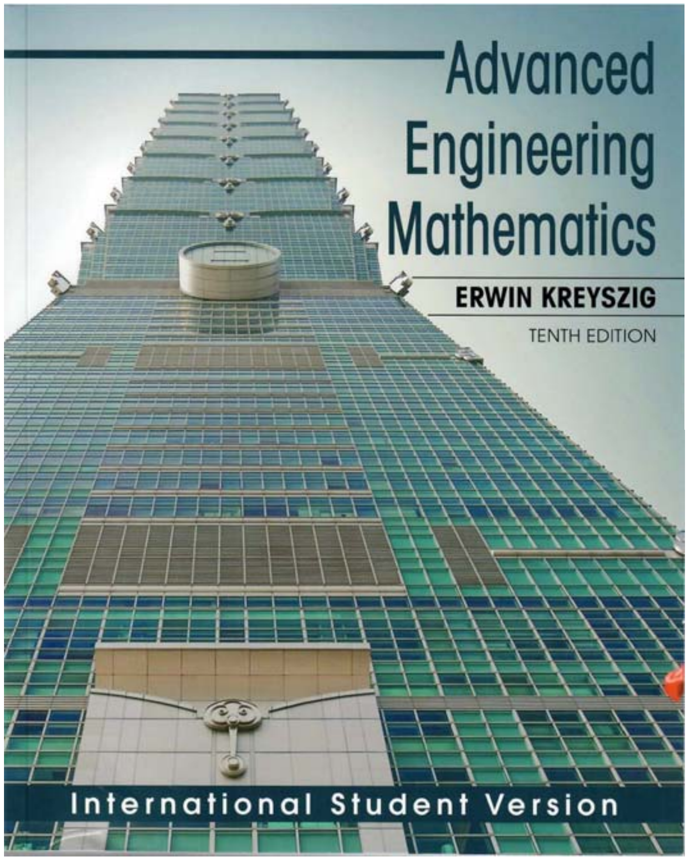
- **Course Objective**
  - To convey the fundamental concepts in *Linear Algebra* and *Vector Calculus* as key mathematical tools for many fields of engineering.
  - To expand upon the mathematical training students acquired in MATH1510 on one-variable calculus and its simple multivariable variations.

- **Textbook**
  - E. Kreyszig, "Advanced Engineering Mathematics," Wiley, 10th Edition, August 2011.
  - Coverage – Part B Linear Algebra. Vector Calculus: Ch. 7-10
  - Supplementary Reading – Michael D. Greenberg, "Advanced Engineering Mathematics," 2nd edition, Prentice Hall, 1988





- **Course Content**
  - <u>Linear Algebra (Chapter 7 and 8)</u>
    - LA.1 Matrices, matrix addition, matrix multiplication, inverses, special matrices
    - LA.2 Vector spaces, basis and dimension, linear independence, rank, determinants
    - LA.3 Linear transformations, projection, orthogonality
    - LA.4 Systems of linear equations, Gaussian elimination, LU decomposition
    - LA.5 Eigenvalues and eigenvectors
  - <u>Vector Calculus (Chapter 9 and 10)</u>
    - VC.1 2-D and 3-D vector space and algebra
    - VC.2 Vector differential calculus. Gradient, divergence, curl
    - VC.3 Vector integral calculus. Green's theorem, Gauss's theorem, Stokes's theorem

- **Lecture/Tutorial**

Lecture	Wednesday Thursday	1:30PM - 2:15PM 2:30PM - 4:15PM	LSK LT2 LSB LT1
Tutorial	Tuesday	4:30PM - 6:15PM	LSB LT1

- **Tutors and Office Hours**

Name	Office	Office Extension	Email address	Office Hour
Chen Yu	ERB313	56107426	1155089925@link.cuhk.edu.hk	Mon, 3-5pm
He Changran	ERB411	39438046	hechangran@link.cuhk.edu.hk	Th. 2-4pm
Liang Dong	ERB411	39438046	dliang@mae.cuhk.edu.hk	Tue 2-4pm
Song Chen	AB1, 1st/F		1155088240@link.cuhk.edu.hk	Wed 2-4pm
Wang Dongping	ERB322	39438040	dpwang@mae.cuhk.edu.hk	Fri 3-5pm

- **Assessment Scheme**
  - Homework Assignments (6) 20%
  - Mid-term Exam 30%
  - Final Exam 50%

- **Examination Dates**
  - Midterm March 14, 2017 (using Tutorial period)
  - Final Centralized, TBD

- **Course Learning Outcomes**
  - Competent in understanding the roles and connections between matrices and vectors, linear equation solving, linear algebra and vector calculus
  - Able to formulate solutions to practical applications in engineering and economics using mathematical skills
  - Able to use special matrices such as triangular, diagonal, and orthogonal matrices
  - Able to understand Gauss elimination and Gauss-Jordan method and their relationship with elementary matrices for different types of matrix factorization and decomposition
  - Competent in using vectors and vector space for interpreting matrix rank and the different solutions to linear equations
  - Able to apply methods of vector calculus, including Jacobian, divergence, Green's and Stokes' theorems

- **Course website**

  Logon to at Blackboard for classnotes, homework assignment and course-related announcements:

  https://elearn.cuhk.edu.hk/webapps/login/

- **Attendant Requirement**

  Students are required to attend a least 50% of lectures to ensure a passing grade

- **Student/Faculty Expectations on Teaching & Learning**

A. STUDENT EXPECTATIONS: Students have the right to expect:

1. a positive, respectful, and engaged academic environment inside and outside the classroom;
2. classes offered at regularly scheduled times without undue variations, and to receive before term-end adequate make-ups of canceled classes;
3. to receive a syllabus including an outline of the course objectives, content and schedule, evaluation criteria, and any other requirements;
4. to consult with teacher and course tutors outside of usual classroom times through regularly scheduled office hours;
5. to have reasonable access to University facilities and equipment in order to complete course assignments and/or objectives;
6. to have access to guidelines on University's definition of academic misconduct within any course;
7. to have reasonable access to grading instruments and/or grading criteria for individual assignments, projects, or exams and to review graded material in a timely fashion;
8. to consult with each course's faculty member regarding the petition process for graded coursework.

B. FACULTY EXPECTATIONS: Teachers have the right to expect:

1. a positive, respectful, and engaged academic environment inside and outside the classroom;
2. students to appear for class meetings in a timely fashion;
3. to select qualified course tutors and the right to delegate responsibilities to these individuals;
4. students to appear at office hours or a mutually convenient appointment for official matters of academic concern;
5. full attendance at examination, midterms, presentations, and laboratories, with the exception of approved absences or emergency;
6. students to be prepared for class, appearing with appropriate materials and having completed assigned readings and homework;
7. full engagement within the classroom, including meaningful focus during lectures, raising questions, and class participation (avoid conversation or phone-calls not related to the lecture topic at hand);
8. to cancel class due to emergency situations and to cover missed material during subsequent class meetings;
9. students to act with integrity and honesty.

Full version at **http://www.erg.cuhk.edu.hk/Student-Faculty-Expectations**
and Course website

- **Avoid Plagiarism**
  - Plagiarism is the act of using the work of others as one's own
  - CUHK places very high importance on honesty in academic work submitted by students
  - Offence will lead to disciplinary action including termination of studies
  - All student assignments submitted via VeriGuide for checking
  - Teachers shall report all cases of plagiarism or suspected cheating in examinations to disciplinary committees in Faculty and University level
  - For details on Academic Honesty, please refer to:
    - University Policy
      http://www.cuhk.edu.hk/policy/academichonesty/
    - Faculty of Engineering Guideline
      http://star.erg.cuhk.edu.hk/upload/ENGG_Discipline.pdf

Applications of Linear Algebra

http://aix1.uottawa.ca/~jkhoury/app.htm

- Abstract Thinking
- Chemistry
- Coding theory
- Coupled oscillations
- Cryptography
- Economics
- Elimination Theory
- Games
- Genetics
- Geometry
- Graph theory
- Heat distribution
- Image compression
- Linear Programming
- Markov Chains
- Networks
- Sociology
- The Fibonacci numbers
- Eigenfaces

Application to Economics

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In 1949, Wassily W

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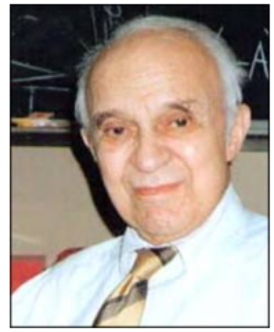
Application to Leontief input-output model



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Introduction In order to understand and be able to manipulate the economy of a country or a region, one needs to come up with a certain model based on the various sectors of this economy. The Leontief model is an attempt in this direction. Based on the assumption that each industry in the economy has two types of demands: external demand (from outside the system) and internal demand (demand placed on one industry by another in the same system), the Leontief model represents the economy as a system of linear equations. The Leontief model was invented in the 30's by Professor Wassily Leontief (picture above) who developed an economic model of the United States economy by dividing it into 500 economic sectors. On October 18, 1973, Professor Leontief was awarded the Nobel Prize in economy for his effort.

1) **The Leontief closed Model** Consider an economy consisting of n interdependent industries (or sectors) $S_1,\ldots,S_n$. That means that each industry consumes some of the goods produced by the other industries, including itself (for example, a power-generating plant uses some of its own power for production). We say that such an economy is **closed** if it satisfies its own needs; that is, no goods leave or enter the system. Let m_{ij} be the number of units produced by industry S_i and necessary to produce one unit of industry S_j. If p_k is the production level of industry S_k, then $m_{ij}\,p_j$ represents the number of units produced by industry S_i and consumed by industry S_j. Then the total number of units produced by industry S_i is given by:

$$p_1 m_{i1} + p_2 m_{i2} + \ldots + p_n m_{in}.$$

In order to have a balanced economy, the total production of each industry must be equal to its total consumption. This gives the linear system:

$$\begin{array}{ccccccccc} m_{11}p_1 & + & m_{12}p_2 & + & \cdots & + & m_{1n}p_n & = & p_1 \\ m_{21}p_1 & + & m_{22}p_2 & + & \cdots & + & m_{2n}p_n & = & p_2 \\ \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ m_{n1}p_1 & + & m_{n2}p_2 & + & \cdots & + & m_{nn}p_n & = & p_n \end{array}$$

If

$$A = \begin{bmatrix} m_{11} & m_{12} & \cdots & m_{1n} \\ m_{21} & m_{22} & \cdots & m_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ m_{n1} & m_{n2} & \cdots & m_{nn} \end{bmatrix}$$

Application to Sociology

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Application to sociology

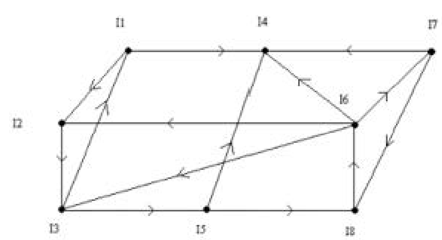
aix1.uottawa.ca/~jkhoury/socio.htm

Google

Application to sociology

<u>Introduction</u> Sociologists interested in various kinds of communications in a group of individuals often use graphs to represent and analyze relations inside the group. For terminology and some results about graph theory that we will use here, check the application of linear algebra to graph theory. The idea is to associate a vertex to each individual in the group, and if individual A influences or dominates individual B, we draw a directed edge from A to B. Note that the obtained graph can have at most one directed edge between two distinct vertices.

<u>Example</u> Consider a group of eight individuals $I_1, \ldots, I_8$. The following digraph represents the dominance relationship among the individuals of the group:



The adjacency matrix of this graph is:

$$M = \begin{array}{c} \\ I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \\ I_7 \\ I_8 \end{array} \begin{array}{c} \begin{array}{cccccccc} I_1 & I_2 & I_3 & I_4 & I_5 & I_6 & I_7 & I_8 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

The row with most 1's in the above matrix corresponds to the most influential individual in the group; in our case it is I_6. In the above graph, walks of length 1 (one edge) correspond to direct influence in the group, whereas walks of greater length correspond to indirect influence. For instance I_3 directly influences I_5 and I_5 directly influences I_4, therefore I_3 indirectly influences I_4.

Now, squaring M gives

$$M^2 = \begin{array}{c} \\ I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \\ I_7 \\ I_8 \end{array} \begin{array}{c} \begin{array}{cccccccc} I_1 & I_2 & I_3 & I_4 & I_5 & I_6 & I_7 & I_8 \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

One can see that individual I_8 has 2-stage influence on half of the group, although he has only one direct influence on I_6.

Applications of Vector Calculus

- Vector field and concepts (gradient, curl, divergence terms and the related theorems) apply to many physical phenomena and related engineering applications

Example: surface area of the Charles Kao Center?