CSCI 5350
Advanced Topics in Game Theory

Discussion Session 6

Game Theory Exercise 2

Keith and Diana discuss where they should go in the afternoon. After discussion, they agree that Diana should first propose and then Keith would say what he thinks. Suppose Diana is to choose an action between shopping (action S) and seeing a movie (action M). If Diana chooses shopping, and Keith agrees (action Y), then they will both go shopping. If Keith disagrees (action N), then they will go home. On the other hand, if Diana decides to see a movie, and Keith agrees (action Y), and then Keith will decide whether to go to Cinema Aster (action A) or the Sunbeam Theatre (action B), and they will go to whichever cinema Keith decides to go. Finally, if Keith does not agree to see a movie (action N), then they will go home.

The utilities of Keith and Mary in different situation are shown below.

	Going shopping	Going home	Seeing Movie at Cinema Aster	Seeing Movie at the Sunbeam Theatre
Keith's utility	1	0	4	2
Diana ~~Mary~~'s utility	2	0	2	4

a) Model this game as an extensive game with perfect information $\langle N, H, P, (u_i) \rangle$.
   i) **(2 marks)** How many pure strategies does Keith have? List all of them.
   ii) **(5 marks)** Are there any subgame perfect equilibria in the game? Describe all of them.
   iii) **(4 marks)** Are there any strategies that are reducible? For <u>each</u> of strategies that are reducible, explain why it are reducible, and show the corresponding reduced strategy.
   iv) **(5 marks)** Are there any pure strategy Nash equilibria in the reduced strategic form of the game? List all of them.

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		Going shopping	Going home	Seeing Movie at Cinema Aster	Seeing Movie at the Sunbeam Theatre
	Keith's utility	1	0	4	2
Diana	~~Mary~~'s utility	2	0	2	4

b) Diana wants to change the game a bit, and proposes that she will first write her decision on a piece of paper, and Keith needs to decide whether to agree or disagree before knowing what Diana's decision is. In order that the game is 'fairer,' they agree that if Diana proposes to see a movie and Keith agrees, then they will always go to Cinema Aster because it is Keith's favourite cinema.

Now suppose Keith and Diana plays this game once every day for a long time. To simplify the analysis, we model the situation as an infinitely repeated game with perfect information.

i) **(2 marks)** Show the constituent game of the infinitely repeated game with perfect information (you may represent the game using a payoff matrix).

ii) **(2 marks)** What are Keith's and Diana's minmax payoffs in the constituent game?

iii) **(10 marks)** Suppose the limit-of-means criterion is used in the infinitely repeated game. Describe a Nash equilibrium in the infinitely repeated game. You should describe the strategy played by Keith and that played by Mary, and the resulting payoff profile when this Nash equilibrium is played.

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Consider an infinitely repeated game of the game G

	Player 2	
	L	R
T	1, 2	8, 7
B	4, 3	5, 6

Player 1: T, B (rows)

a) **(2 marks)** What are the minmax payoffs of player 1 and player 2?

Consider that the players play the infinitely repeated game of G in such a way that the sequence of outcomes consists of an indefinite repetition of the following cycle of length 3: (T, R), (B, L), (B, R). That is, the sequence of outcomes is (T, R), (B, L), (B, R), (T, R), (B, L), (B, R), (T, R), (B, L), (B, R), ...

b) **(2 marks)** What is the utility profile of the outcome of the infinitely repeated game, if the limit of means criterion is used?

c) **(4 marks)** What is the utility profile of the outcome of the infinitely repeated game, if the discounting criterion is used, and the discount factor is ½?

d) **(2 marks)** Is the utility profile in question b) feasible, if the limit of means criterion is used? Justify your answer.

e) **(4 marks)** Is the utility profile in question b) enforceable, if the limit of means criterion is used? Justify your answer.

f) **(6 marks)** Is it possible to find a trigger strategy equilibrium such that the players play a <u>limit of means</u> infinitely repeated game of G in such a way that its sequence of outcomes consists of an indefinite repetition of the following cycle of length 3: (T, R), (B, L), (B, R)? If the answer is yes, describe the strategy each of the two players use in the trigger strategy equilibrium. Otherwise, explain why it is not possible.

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End