Exercises: Vector Derivative

Problem 1. Let $f(x) = 3x + 5$. Clearly, $\lim_{x\to 2} f(x) = 11$. Answer the following questions:

- Set $\delta = 1$. By definition of limit, we know that we can find an $\epsilon > 0$, such that for any x satisfying $|x - 2| < \epsilon$, it holds that $|f(x) - 11| < \delta$. Give such an ϵ.
- Repeat the above for $\delta = 0.001$.
- Repeat the above for $\delta = 0.000001$.

Problem 2. Solve the following limits:

- $\lim_{t\to 3} \boldsymbol{f}(t)$, where $\boldsymbol{f}(t) = [5t + 3, \frac{\sin(t-3)}{t-3}]$.
- $\lim_{t\to 0} \boldsymbol{f}(t)$, where $\boldsymbol{f}(t) = [5t^2 + 3t, t^2, \frac{e^t-1}{t}]$.
- $\lim_{t\to 0} \boldsymbol{f}(t)$, where

$$\boldsymbol{f}(t) = \begin{cases} [5t^2 + 3t, t^2, \frac{e^t-1}{t}] & \text{if } t \neq 0 \\ [10, 10, 10] & \text{otherwise} \end{cases}$$

Problem 3. Discuss the continuity of $\boldsymbol{f}(t)$ at $t = 0$.

- $\boldsymbol{f}(t) = [5t^2 + 3t, t^2, \frac{e^t-1}{t}]$.
- $\boldsymbol{f}(t) = [5t^2 + 3t, t^2, \frac{e^t-1}{t}]$ if $t \neq 0$; otherwise, $\boldsymbol{f}(t) = [10, 10, 10]$.
- $\boldsymbol{f}(t) = [5t^2 + 3t, t^2, \frac{e^t-1}{t}]$ if $t \neq 0$; otherwise, $\boldsymbol{f}(t) = [0, 0, 1]$.

Problem 4. Suppose that $\boldsymbol{f}(t) = [\sin(t), \cos(t^3), 5t^2]$. Answer the following questions:

- Give the function $\boldsymbol{f}'(t)$.
- Give the function $\boldsymbol{f}''(t)$ (which is the derivative of $\boldsymbol{f}'(t)$).
- Give the function $\boldsymbol{f}'''(1)$ (where $\boldsymbol{f}'''(t)$ is the derivative of $\boldsymbol{f}''(t)$).

Problem 5. Suppose that $\boldsymbol{f}(t) = [t^2, \sin(t), 2t]$ and $\boldsymbol{g}(t) = 2t\boldsymbol{i} + \frac{1}{\sin(t)}\boldsymbol{j} + 3t^2\boldsymbol{k}$.

- Give the function $h(t) = \boldsymbol{f}(t) \cdot \boldsymbol{g}(t)$.
- Give the function $h'(t)$.
- Give the function $\boldsymbol{f}'(t)$ and $\boldsymbol{g}'(t)$.
- Verify that $h'(t) = \boldsymbol{f}'(t) \cdot \boldsymbol{g}(t) + \boldsymbol{g}'(t) \cdot \boldsymbol{f}(t)$.

Problem 6. Suppose that $\boldsymbol{f}(t) = [t, t^2, 1]$ and $\boldsymbol{g}(t) = [1, t, t^2]$.

- Give the function $\boldsymbol{h}(t) = \boldsymbol{f}(t) \times \boldsymbol{g}(t)$.
- Give the function $\boldsymbol{h}'(t)$.