CSCI 5350 Assignment 3

Due date: 21 December 2020

1. Consider a TU game $\langle N, v \rangle$ where $N = \{1, 2, 3\}$ and v is defined as follows:

   - $v(\{1,2,3\}) = 1$
   - $v(\{1,2\}) = \frac{1}{2}$ $v(\{2,3\}) = \frac{1}{4}$ $v(\{1,3\}) = \frac{1}{8}$
   - $v(\{1\}) = \frac{1}{8}$ $v(\{2\}) = \frac{1}{8}$ $v(\{3\}) = \frac{1}{8}$

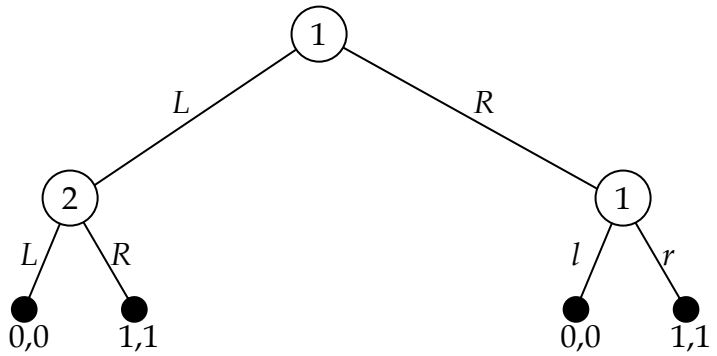
   a) Is this game cohesive? Justify your answer.
   b) Is the payoff profile $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ in the core of the game? Justify your answer.
   c) Is the set $\{(\frac{1}{2}, \frac{1}{2}, 0), (\frac{1}{2}, 0, \frac{1}{2}), (0, \frac{1}{2}, \frac{1}{2})\}$ a stable set of the game? Justify your answer.
   d) Find the Shapley value of this game. Is the Shapley value in the core?

2. Consider a group of six players playing a game together. Player 1, player 2 and player 3 each is given a red card. Player 4 and player 5 each is given a green card. Play 6 is given a white card. There is a rule that players can form groups, such that a group can receive a utility of 1 unit if the group members have one red card and one green card, and, in general, a utility of n if they have n red cards and n green cards. Otherwise, the group will receive a utility of 0 (zero).

   a) The scenario can be formulated as a coalitional game with transferrable payoff $G = \langle N, v \rangle$
      i. Write down N in the game G.
      ii. Write down v in the game G.
      iii. Is the game G cohesive? Justify your answer.
      iv. Give one payoff profile that is in the core, or prove that the core is empty. Justify your answer.
      v. Describe the core (or list all members of the core) of the game G.
      vi. Are there any nonempty stable sets of the game? Give one if there is any, or justify that there is none.
      vii. Calculate the Shapley value of the game.

   (b) The same game can also be formulated as a coalitional game <u>without</u> transferrable payoff.
      i. Write down the set X of all consequences in the NTU game. You may describe or list the contents of X.
      ii. Describe the core (or list all members of the core) of the NTU game.

3. Consider the following game



a) Find all the trembling hand perfect equilibria of this extensive game with perfect information.
b) Show the strategic form of this extensive game with perfect information and find all the trembling hand perfect equilibria of the strategic form of this extensive game with perfect information.
c) Using this extensive game with perfect information, show that the notion of trembling hand perfect equilibrium of an extensive game is not invariant to the coalescing of moves.

— End of Assignment —