

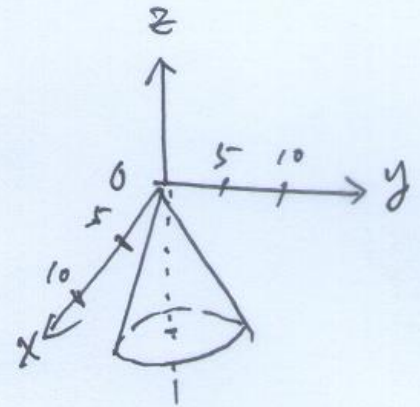
(a) $r + 5z = 0$

Approach 1 Treat r as non-negative

assume
 $r \geq 0$

$$\sqrt{x^2 + y^2} = -5z$$

$$x^2 + y^2 - 25z^2 = 0$$



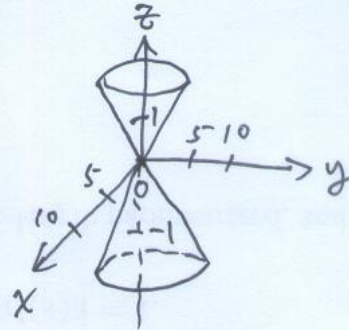
Approach 2 r can be negative (refer to wikipedia definition) - of course r can be ≥ 0 as well

$$r = -5z$$

$$r^2 = 25z^2$$

$$x^2 + y^2 = 25z^2$$

$$x^2 + y^2 - 25z^2 = 0$$



It depends on how you interpret " r " in polar coordinates

When $r \leq 0$, you move from the pole in the **direction opposite to** the given positive angle. (dummies.com).

Grading: Both approaches are accepted, and will be awarded full credits

Example

When $z = 1$, $r = -5$, we plot the point $(r, \theta, z) = (-5, \theta, 1)$, where $\theta \in [0, 2\pi)$, then we obtain a circle ($\because \theta$ can vary from 0 to 2π , revolving a loop) on the horizontal plane $z = 1$.

(b) $\phi = \frac{\pi}{4}$ represents a half-cone and can be described as follows

$$\frac{z}{\sqrt{x^2+y^2+z^2}} = \cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$2z^2 = x^2 + y^2 + z^2$$

$$\boxed{x^2 + y^2 - z^2 = 0}$$

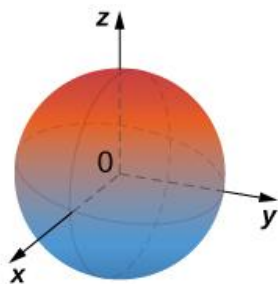
or

For physics people, they will swap the role of “theta” and “phi”.

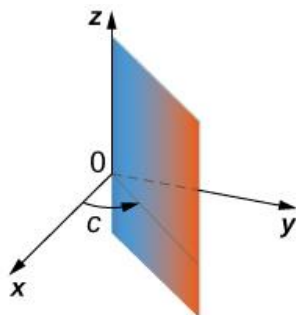
Ref.: https://en.wikipedia.org/wiki/Spherical_coordinate_system

Extra Notes for students:

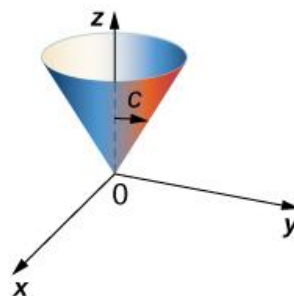
In mathematics notation,



Sphere $\rho = c$ (constant)

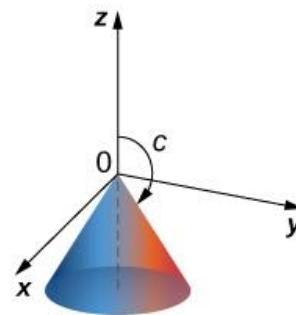


Half plane $\theta = c$ (constant)



$$0 < c < \frac{\pi}{2}$$

Half cone $\phi = c$ (constant)



$$\frac{\pi}{2} < c < \pi$$

Ref.: <https://opentextbc.ca/calculus3openstax/chapter/triple-integrals-in-cylindrical-and-spherical-coordinates/>

For grading, we will accept ALL possible answers, provided that your explanations are sufficient. Don't need to worry! We will cater the needs of different students with different thoughts and interpretations regarding the variables in Question 3.