Strategic Games

DEFINITION. A **strategic game** $\langle N, (A_i), (\succsim_i) \rangle$ consists of

- a finite set N (the set of **players**)
- for each player $i \in N$ a nonempty set A_i (the set of **actions** available to player i)
- for each player $i \in N$ a preference relation $\succsim_i$ on $A = \times_{j \in N} A_j$ (the **preference relation** of player i on the set of **action profiles**).

Nash Equilibrium

DEFINITION. A **Nash equilibrium of a strategic game** $\langle N, (A_i), (\succsim_i) \rangle$ is a profile $a^* \in A$ of actions with the property that for every player $i \in N$ we have

$$(a^*_{-i}, a^*_i) \succsim_i (a^*_{-i}, a_i) \text{ for all } a_i \in A_i.$$

Bayesian Games

Consider a situation when the players are not sure about the *state of nature* when playing games.

In different states of nature, the games that are being played are different.

Bayesian Games

$$\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4, \omega_5\}$$

$$A_1 = \{T, B\} \quad A_2 = \{L, R\}$$

States Ω

Players N

Actions (A_i)

ω_1

3,3	0,5
5,0	1,1

ω_2

2,2	6,0
0,6	3,3

ω_3

2,1	0,0
0,0	1,2

ω_4

5,5	1,0
0,1	1,1

ω_5

1,–1	–1,1
–1,1	1,–1

Bayesian Games

$$p_1 = \left\{\omega_1 \mapsto \frac{1}{8}, \omega_2 \mapsto 0, \omega_3 \mapsto \frac{1}{4}, \omega_4 \mapsto \frac{1}{2}, \omega_5 \mapsto \frac{1}{8}\right\},$$
$$p_2 = \left\{\omega_1 \mapsto 0, \omega_2 \mapsto \frac{1}{8}, \omega_3 \mapsto \frac{5}{8}, \omega_4 \mapsto \frac{1}{4}, \omega_5 \mapsto 0\right\}.$$

Prior Beliefs	$^1/_8$	0	$^1/_4$	$^1/_2$	$^1/_8$
(p_1, p_2)	0	$^1/_8$	$^5/_8$	$^1/_4$	0
States Ω	ω_1	ω_2	ω_3	ω_4	ω_5

	ω_1		ω_2		ω_3		ω_4		ω_5	
Players N	3,3	0,5	2,2	6,0	2,1	0,0	5,5	1,0	1,–1	–1,1
Actions (A_i)	5,0	1,1	0,6	3,3	0,0	1,2	0,1	1,1	–1,1	1,–1

Bayesian Games

$$T_1 = \{t_1, t_2, t_3\}$$

$$T_2 = \{t_4, t_5, t_6\}$$

Signals (T_i)	t_1		t_2		t_3
	t_4	t_5			t_6
Prior Beliefs	$^1/_8$	0	$^1/_4$	$^1/_2$	$^1/_8$
(p_1, p_2)	0	$^1/_8$	$^5/_8$	$^1/_4$	0
States Ω	ω_1	ω_2	ω_3	ω_4	ω_5
Players N Actions (A_i)	3,3 0,5 5,0 1,1	2,2 6,0 0,6 3,3	2,1 0,0 0,0 1,2	5,5 1,0 0,1 1,1	1,–1 –1,1 –1,1 1,–1

Bayesian Games

$$\tau_1 : \Omega \to T_1 \qquad \tau_2 : \Omega \to T_2$$

$$\tau_1 = \{\omega_1 \mapsto t_1, \omega_2 \mapsto t_1, \omega_3 \mapsto t_2, \omega_4 \mapsto t_2, \omega_5 \mapsto t_3\}$$

$$\tau_2 = \{\omega_1 \mapsto t_4, \omega_2 \mapsto t_5, \omega_3 \mapsto t_5, \omega_4 \mapsto t_5, \omega_5 \mapsto t_6\}$$

Signal	$\tau_1(\omega_1) = \tau_1(\omega_2) = t_1$		$\tau_1(\omega_3) = \tau_1(\omega_4) = t_2$		$\tau_1(\omega_5) = t_3$
Functions (τ_i)	$\tau_2(\omega_1) = t_4$	$\tau_2(\omega_2) = \tau_2(\omega_3) = \tau_2(\omega_4) = t_5$			$\tau_2(\omega_5) = t_6$
Prior Beliefs	$1/8$	0	$1/4$	$1/2$	$1/8$
(p_i)	0	$1/8$	$5/8$	$1/4$	0
States Ω	ω_1	ω_2	ω_3	ω_4	ω_5

Players N
Actions (A_i)

ω_1:

3,3	0,5
5,0	1,1

ω_2:

2,2	6,0
0,6	3,3

ω_3:

2,1	0,0
0,0	1,2

ω_4:

5,5	1,0
0,1	1,1

ω_5:

1,–1	–1,1
–1,1	1,–1

Bayesian Games

Signals (T_i)	t_1		t_2		t_3
	t_4	t_5			t_6
Prior Beliefs	⅛	0	¼	½	⅛
(p_i)	0	⅛	⅝	¼	0
Posterior	0	0	⅓	⅔	0
Beliefs	0	⅛	⅝	¼	0
States Ω	ω_1	ω_2	ω_3	ω_4	ω_5
Players N	3,3 | 0,5	2,2 | 6,0	2,1 | 0,0	5,5 | 1,0	1,–1 | –1,1
Actions (A_i)	5,0 | 1,1	0,6 | 3,3	0,0 | 1,2	0,1 | 1,1	–1,1 | 1,–1

Bayesian Games

States Ω	ω_1		ω_2		ω_3		ω_4		ω_5	
Players N	3,3	0,5	2,2	6,0	2,1	0,0	5,5	1,0	1,–1	–1,1
Actions (A_i)	5,0	1,1	0,6	3,3	0,0	1,2	0,1	1,1	–1,1	1,–1

Player 1's preference is on the set of probability measures over $A \times \Omega = (\times_{j \in N} A_j) \times \Omega$, e.g.,

$$\{ \ ((T,L),\omega_1), \quad ((T,R),\omega_1), \quad \ldots, \quad ((B,R),\omega_5) \ \}$$

$$(\ p_1, \quad p_2, \quad \ldots, \quad p_{20} \ )$$

$$\succsim_1$$

$$(\ p_1', \quad p_2', \quad \ldots, \quad p_{20}' \ )$$

Bayesian Games

DEFINITION. A **Bayesian game** $\langle N, \Omega, (A_i), (T_i), (\tau_i), (p_i), (\succsim_i) \rangle$ consists of

- a finite set N (the set of **players**)
- a finite set Ω (the set of **states**)

and for each player $i \in N$

- a set A_i (the set of **actions** available to player *i*)
- a finite set T_i (the set of **signals** that may be observed by player i) and a function $\tau_i \colon \Omega \to T_i$ (the **signal function** of player i)

- a probability measure p_i on Ω (the **prior belief** of player i) for which $p_i(\tau_i^{-1}(t_i)) > 0$ for all $t_i \in T_i$.
- a preference relation $\succsim_i$ on the set of probability measures over $A \times \Omega$ (the **preference relation** of player i), where $A = \times_{j \in N} A_j$.

Bayesian Games

$$\langle N, \Omega, (A_i), (T_i), (\tau_i), (p_i), (\succsim_i) \rangle$$

For the game we just considered

- $N = \{1,2\}$.
- $\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4, \omega_5\}$.
- $A_1 = \{T, B\},\ A_2 = \{L, R\}$.
- $T_1 = \{t_1, t_2, t_3\},\ T_2 = \{t_4, t_5, t_6\}$.
- $\tau_1 = \{\omega_1 \mapsto t_1, \omega_2 \mapsto t_1, \omega_3 \mapsto t_2, \omega_4 \mapsto t_2, \omega_5 \mapsto t_3\}$,
  $\tau_2 = \{\omega_1 \mapsto t_4, \omega_2 \mapsto t_5, \omega_3 \mapsto t_5, \omega_4 \mapsto t_5, \omega_5 \mapsto t_6\}$.
- $p_1 = \{\omega_1 \mapsto \frac{1}{8}, \omega_2 \mapsto 0, \omega_3 \mapsto \frac{1}{4}, \omega_4 \mapsto \frac{1}{2}, \omega_5 \mapsto \frac{1}{8}\}$,
  $p_2 = \{\omega_1 \mapsto 0, \omega_2 \mapsto \frac{1}{8}, \omega_3 \mapsto \frac{5}{8}, \omega_4 \mapsto \frac{1}{4}, \omega_5 \mapsto 0\}$.

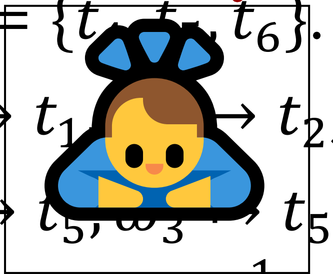
Bayesian Games

$$\langle N, \Omega, (A_i), (T_i), (\tau_i), (p_i), (\succsim_i) \rangle$$

This is not a Bayesian Game

For the game we just considered

- $N = \{1, 2\}$.
- $\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4, \omega_5\}$.
- $A_1 = \{T, B\}$, $A_2 = \{L, R\}$.
- $T_1 = \{t_1, t_2, t_3\}$, $T_2 = \{t_4, t_5, t_6\}$.



- $\tau_1 = \{\omega_1 \mapsto t_1, \omega_2 \mapsto t_1, \omega_3 \mapsto t_2, \omega_4 \mapsto t_2, \omega_5 \mapsto t_3\}$,
  $\tau_2 = \{\omega_1 \mapsto t_4, \omega_2 \mapsto t_5, \omega_3 \mapsto t_5, \omega_4 \mapsto t_5, \omega_5 \mapsto t_6\}$.
- $p_1 = \{\omega_1 \mapsto \frac{1}{8}, \omega_2 \mapsto 0, \omega_3 \mapsto \frac{1}{4}, \omega_4 \mapsto \frac{1}{2}, \omega_5 \mapsto \frac{1}{8}\}$,
  $p_2 = \{\omega_1 \mapsto 0, \omega_2 \mapsto \frac{1}{8}, \omega_3 \mapsto \frac{5}{8}, \omega_4 \mapsto \frac{1}{4}, \omega_5 \mapsto 0\}$.

p p

- $(\succsim_i)$ (the **preference relation** of player i)

States Ω	ω_1		ω_2		ω_3		ω_4		ω_5	
Players N	3,3	0,5	2,2	6,0	2,1	0,0	5,5	1,0	1,–1	–1,1
Actions (A_i)	5,0	1,1	0,6	3,3	0,0	1,2	0,1	1,1	–1,1	1,–1

p_2

For a player, there are 20 probabilities in a single probability measure over $A \times \Omega$.

$$A \times \Omega = \{\underbrace{((T,L),\omega_1)}_{p_1}, \underbrace{((T,R),\omega_1)}_{p_2}, \dots, \underbrace{((B,R),\omega_5)}_{p_{20}}\}$$

For any player, some of these probability measures are better than some others.

Bayesian Games

Example.

		Wife	
		Boxing	*Opera*
Husband	*Boxing*	2, 1	0, 0
	Opera	0, 0	1, 2

What if the husband does not know the wife's exact preference, and the wife does not know the husband's exact preference, either?

(B, B)			(B, O)			(O, B)			(O, O)	
2, 2	0, 0		2, 1	0, 0		1, 2	0, 0		1, 1	0, 0
0, 0	1, 1		0, 0	1, 2		0, 0	2, 1		0, 0	2, 2

Signal functions

$\tau_1((B,B)) = \tau_1((B,O)) = B \quad \tau_1((O,B)) = \tau_1((O,O)) = O$

$\tau_2((B,B)) = \tau_2((O,B)) = B \quad \tau_2((B,O)) = \tau_2((O,O)) = O$

(Both players know what he/she prefers.)

(B, B)

2, 2	0, 0
0, 0	1, 1

(B, O)

2, 1	0, 0
0, 0	1, 2

(O, B)

1, 2	0, 0
0, 0	2, 1

(O, O)

1, 1	0, 0
0, 0	2, 2

Husband: I need to decide an action for two cases

- if signal is B; and
- if signal is O.

These two actions are generally independent by *type B of husband* and *type O of husband*.

(B, B)

2, 2	0, 0
0, 0	1, 1

(B, O)

2, 1	0, 0
0, 0	1, 2

(O, B)

1, 2	0, 0
0, 0	2, 1

(O, O)

1, 1	0, 0
0, 0	2, 2

Husband

(type B and type O)

Wife

(type B and type O)

(B, B)

2, 2	0, 0
0, 0	1, 1

$p_1((B,B)) = ½$
$p_2((B,B)) = ¼$

(B, O)

2, 1	0, 0
0, 0	1, 2

$p_1((B,O)) = 0$
$p_2((B,O)) = ¼$

(O, B)

1, 2	0, 0
0, 0	2, 1

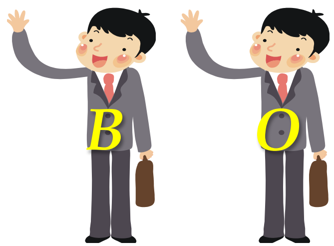
$p_1((O,B)) = ¼$
$p_2((O,B)) = 0$

(O, O)

1, 1	0, 0
0, 0	2, 2

$p_1((O,O)) = ¼$
$p_2((O,O)) = ½$

Question: Is (B, O, B, O) a Nash equilibrium?



(*B*, *B*)

2, 2	0, 0
0, 0	1, 1

$p_1((B,B)) = ½$

(*B*, *O*)

2, 1	0, 0
0, 0	1, 2

$p_1((B,O)) = 0$

(*O*, *B*)

1, 2	0, 0
0, 0	2, 1

$p_1((O,B)) = ¼$

(*O*, *O*)

1, 1	0, 0
0, 0	2, 2

$p_1((O,O)) = ¼$

Question: Is (B, O, B, O) a Nash equilibrium?

For player $(1, B)$, the posterior beliefs are $(1,0,0,0)$.

The 16-probability lottery over $A \times \Omega$ is

$L_1((B,O,B,O),B) = (1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0)$.

The utility of player $(1, B)$ is 2.

(B, B)

2, 2	0, 0
0, 0	1, 1

$p_1((B, B)) = ½$

(B, O)

2, 1	0, 0
0, 0	1, 2

$p_1((B, O)) = 0$

(O, B)

1, 2	0, 0
0, 0	2, 1

$p_1((O, B)) = ¼$

(O, O)

1, 1	0, 0
0, 0	2, 2

$p_1((O, O)) = ¼$

Question: Is (B, O, B, O) a Nash equilibrium?

For player $(1, B)$, the posterior beliefs are $(1,0,0,0)$, so B is the best response in (B, O, B, O).

Note: There are 16 elements in $A \times \Omega$. Probability of $((B, B), (B, B)) \in A \times \Omega$ is 1. Probability of any $(a, \omega) \in A \times \Omega$ is 0 if $(a, \omega) \neq ((B, B), (B, B))$.

(B, B)

2, 2	0, 0
0, 0	1, 1

$p_1((B, B)) = ½$

(B, O)

2, 1	0, 0
0, 0	1, 2

$p_1((B, O)) = 0$

(O, B)

1, 2	0, 0
0, 0	2, 1

$p_1((O, B)) = ¼$

(O, O)

1, 1	0, 0
0, 0	2, 2

$p_1((O, O)) = ¼$

Question: Is (B, O, B, O) a Nash equilibrium?

For player $(1, O)$, the posterior beliefs are ________, so O is/ is not the best response in (B, O, B, O).

Note: In lottery $L_1((B, O, B, O), O)$, probabilities of $((O, B), (O, B))$ and $((O, O), (O, O))$ are both ½. Probability of any other $(a, \omega) \in A \times \Omega$ is 0.

(B, B)

2, 2	0, 0
0, 0	1, 1

(B, O)

2, 1	0, 0
0, 0	1, 2

(O, B)

1, 2	0, 0
0, 0	2, 1

(O, O)

1, 1	0, 0
0, 0	2, 2

$p_2((B,B)) = ¼$ $p_2((B,O)) = ¼$ $p_2((O,B)) = 0$ $p_2((O,O)) = ½$

For player $(2, B)$, the posterior beliefs are ________, so $L_2((B,O,B,O),B) =$ _ and B is/ is not the best response in (B,O,B,O).

For player $(2, O)$, … O is/ is not the best response in (B,O,B,O).

(B, B)

2, 2	0, 0
0, 0	1, 1

$p_1((B,B)) = ½$
$p_2((B,B)) = ¼$

(B, O)

2, 1	0, 0
0, 0	1, 2

$p_1((B,O)) = 0$
$p_2((B,O)) = ¼$

(O, B)

1, 2	0, 0
0, 0	2, 1

$p_1((O,B)) = ¼$
$p_2((O,B)) = 0$

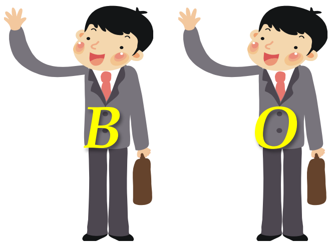
(O, O)

1, 1	0, 0
0, 0	2, 2

$p_1((O,O)) = ¼$
$p_2((O,O)) = ½$

Question: Is (B, O, B, O) a Nash equilibrium?

Answer: Yes, (B, O, B, O) is a Nash equilibrium.



Summary

(B, B)

2, 2	0, 0
0, 0	1, 1

$p_1((B,B)) = ½$

$p_2((B,B)) = ¼$

(B, O)

2, 1	0, 0
0, 0	1, 2

$p_1((B,O)) = 0$

$p_2((B,O)) = ¼$

(O, B)

1, 2	0, 0
0, 0	2, 1

$p_1((O,B)) = ¼$

$p_2((O,B)) = 0$

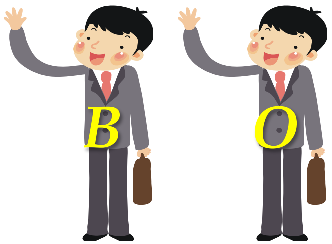
(O, O)

1, 1	0, 0
0, 0	2, 2

$p_1((O,O)) = ¼$

$p_2((O,O)) = ½$

- Four players: $(1, B)$, $(1, O)$, $(2, B)$, and $(2, O)$.
- For players $(1, B)$ and $(1, O)$, actions are A_1.
  For players $(2, B)$ and $(2, O)$, actions are A_2.
- Set of action profiles (outcomes):
  $A_{(1,B)} \times A_{(1,O)} \times A_{(2,B)} \times A_{(2,O)} = \times_{j \in N} (\times_{t_j \in T_j} A_j)$.



Summary

(B, B)

2, 2	0, 0
0, 0	1, 1

$p_1((B,B)) = ½$

$p_2((B,B)) = ¼$

(B, O)

2, 1	0, 0
0, 0	1, 2

$p_1((B,O)) = 0$

$p_2((B,O)) = ¼$

(O, B)

1, 2	0, 0
0, 0	2, 1

$p_1((O,B)) = ¼$

$p_2((O,B)) = 0$

(O, O)

1, 1	0, 0
0, 0	2, 2

$p_1((O,O)) = ¼$

$p_2((O,O)) = ½$

Consider $\boldsymbol{a}^* = (\boldsymbol{B}, \boldsymbol{O}, \boldsymbol{B}, \boldsymbol{O})$, and player $(\mathbf{1}, \boldsymbol{B})$:

Calculate the probability of $(a, \omega) \in A \times \Omega$:

- For $\omega \in \tau_1^{-1}(B) = \{(B,B),(B,O)\}$ and $a = (a^*(1, \tau_1(\omega)), a^*(2, \tau_2(\omega)))$, probability is $p_1(\omega)/p_1\left(\tau_1^{-1}(B)\right)$.
- Otherwise, probability is 0.

Summary

(B, B)		(B, O)		(O, B)		(O, O)	
2, 2	0, 0	2, 1	0, 0	1, 2	0, 0	1, 1	0, 0
0, 0	1, 1	0, 0	1, 2	0, 0	2, 1	0, 0	2, 2

$p_1((B,B)) = ½$ $p_1((B,O)) = 0$ $p_1((O,B)) = ¼$ $p_1((O,O)) = ¼$

$p_2((B,B)) = ¼$ $p_2((B,O)) = ¼$ $p_2((O,B)) = 0$ $p_2((O,O)) = ½$

Consider $a^* = (B, O, B, O)$**, and player** $(1, B)$**:**

Calculate the probability of $(a, \omega) \in A \times \Omega$:

- First, for $\omega = (B, B) \in \tau_1^{-1}(B)$ and $a = (a^*(1, \tau_1(\omega)), a^*(2, \tau_2(\omega))) = (a^*(1, B), a^*(2, B)) = (B, B)$, probability is $p_1\big((B,B)\big)/p_1\left(\tau_1^{-1}(B)\right)$.
- Otherwise, probability is 0.

Summary

(B, B)

2, 2	0, 0
0, 0	1, 1

$p_1((B,B)) = ½$

$p_2((B,B)) = ¼$

(B, O)

2, 1	0, 0
0, 0	1, 2

$p_1((B,O)) = 0$

$p_2((B,O)) = ¼$

(O, B)

1, 2	0, 0
0, 0	2, 1

$p_1((O,B)) = ¼$

$p_2((O,B)) = 0$

(O, O)

1, 1	0, 0
0, 0	2, 2

$p_1((O,O)) = ¼$

$p_2((O,O)) = ½$

Consider $a^* = (B, O, B, O)$, and player $(1, B)$:

Calculate the probability of $(a, \omega) \in A \times \Omega$:

- Second, for $\omega = (B, O) \in \tau_1^{-1}(B)$ and $a = (a^*(1, \tau_1(\omega)), a^*(2, \tau_2(\omega))) = (a^*(1, B), a^*(2, O)) = (B, O)$, probability is $p_1\big((B,O)\big)/p_1\left(\tau_1^{-1}(B)\right)$.
- Otherwise, probability is 0.

Summary

(B, B)	(B, O)	(O, B)	(O, O)
2, 2 | 0, 0	2, 1 | 0, 0	1, 2 | 0, 0	1, 1 | 0, 0
0, 0 | 1, 1	0, 0 | 1, 2	0, 0 | 2, 1	0, 0 | 2, 2
$p_1((B,B)) = ½$	$p_1((B,O)) = 0$	$p_1((O,B)) = ¼$	$p_1((O,O)) = ¼$
$p_2((B,B)) = ¼$	$p_2((B,O)) = ¼$	$p_2((O,B)) = 0$	$p_2((O,O)) = ½$

Consider $a^* = (B, O, B, O)$**, and player** $(1, B)$**:**

Calculate the probability of $(a, \omega) \in A \times \Omega$:

- In summary, for any $\omega \in \tau_1^{-1}(B) = \{(B,B),(B,O)\}$ and $a = (a^*(1,\tau_1(\omega)), a^*(2,\tau_2(\omega)))$, probability is $p_1(\omega)/p_1\left(\tau_1^{-1}(B)\right)$.
- Otherwise, probability is 0.

Summary

(B, B)	
2, 2	0, 0
0, 0	1, 1

$p_1((B,B)) = ½$

$p_2((B,B)) = ¼$

(B, O)	
2, 1	0, 0
0, 0	1, 2

$p_1((B,O)) = 0$

$p_2((B,O)) = ¼$

(O, B)	
1, 2	0, 0
0, 0	2, 1

$p_1((O,B)) = ¼$

$p_2((O,B)) = 0$

(O, O)	
1, 1	0, 0
0, 0	2, 2

$p_1((O,O)) = ¼$

$p_2((O,O)) = ½$

Consider $a^* \in \times_{j\in N} (\times_{t_j \in T_j} A_j)$ **and player** (i, t_i)**:**

Calculate the probability of $(a, \omega) \in A \times \Omega$:

- In generally, for any $\omega \in \tau_i^{-1}(t_i)$ and $a = (a^*(j, \tau_j(\omega)))_{j\in N}$, probability is $p_i(\omega)/p_i\left(\tau_i^{-1}(t_i)\right)$.
- Otherwise, probability is 0.

> This lottery calculated by (i, t_i) over $A \times \Omega$ is denoted $L_i(a^*, t_i)$.
> The utility of player i (as type t_i) is then calculated accordingly.

Summary

(B, B)		(B, O)		(O, B)		(O, O)	
2, 2	0, 0	2, 1	0, 0	1, 2	0, 0	1, 1	0, 0
0, 0	1, 1	0, 0	1, 2	0, 0	2, 1	0, 0	2, 2

$p_1((B,B)) = ½$ $p_1((B,O)) = 0$ $p_1((O,B)) = ¼$ $p_1((O,O)) = ¼$

$p_2((B,B)) = ¼$ $p_2((B,O)) = ¼$ $p_2((O,B)) = 0$ $p_2((O,O)) = ½$

Consider $a^* = \times_{j \in N} (\times_{t_j \in T_j} A_j)$ and

$b^* = \times_{j \in N} (\times_{t_j \in T_j} A_j)$,

if

$L_i(a^*, t_i) \succsim_i L_i(b^*, t_i)$,

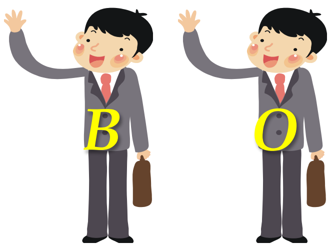
then

$a^* \succsim^*_{(i,t_i)} b^*$, *and vice versa.*

Nash Equilibrium of Bayesian Games

DEFINITION. A **Nash equilibrium of a Bayesian game** $\langle N, \Omega, (A_i), (T_i), (\tau_i), (p_i), (\succsim_i)\rangle$ is a Nash equilibrium of the strategic game defined as follows.

- The set of players is the set of all pairs (i, t_i) for $i \in N$ and $t_i \in T_i$.



- The set of actions of each player (i, t_i) is A_i.

- The preference ordering $\succsim^*_{(i,t_i)}$ of each player (i, t_i) is defined by

$$a^* \succsim^*_{(i,t_i)} b^* \text{ if and only if } L_i(a^*, t_i) \succsim_i L_i(b^*, t_i),$$

where $L_i(a^*, t_i)$ is the lottery over $A \times \Omega$ that assigns probability $p_i(\omega)/p_i\left(\tau_i^{-1}(t_i)\right)$ to $((a^*(j, \tau_j(\omega)))_{j \in N}, \omega)$ if $\omega \in \tau_i^{-1}(t_i)$, 0 otherwise.

Nash Equilibrium of Bayesian Games

In a Nash equilibrium of Bayesian game, each player chooses the best action available to him given the signal that he receives and his belief about the state and the other players' actions that he deduced from the signal.