CSCI 5350 Assignment 2

Due date: 16 November 2020

1. Amy, Beatrice and Cathy are playing a game with a few marbles and a bowl. There are two steps in the game. First, Cathy will be invited to leave the room so that she will not know how Amy and Beatrice play the game in the first step. After Cathy leaves, Amy (player 1) will decide whether to put a marble into the bowl. If Amy puts a marble into the bowl (action P), then the first step ends. Otherwise, if Amy does not put a marble into the bowl (action N), then Beatrice (player 2) will decide whether to put another marble into the bowl (action P) or not (action N). Then the first step ends. After that, Cathy will be invited to entre the room to play in the second step of the game. Cathy (player 3) will see 0 or 1 marble in the bowl. If there is no marble in the bowl, then Cathy does not need to do anything, but the game ends. However, if there is one marble in the bowl, then Cathy has to guess who puts the marble into the bowl. Cathy can guess that it is Amy who puts the marble into the bowl (action A) or it is Beatrice who puts it into the bowl (action B). After Cathy plays, the game ends, and the scores of the three players are determined by the following rules:

   - If Cathy sees no marble when she plays, then Cathy does not do anything, and each of Amy, Beatrice and Cathy receives a score of 1 (one).

   - If Cathy sees 1 marble when she plays the second step, and she guesses correctly, then Cathy and the person who puts the marble into the bowl each receives a score of 10 (ten), but the person who does not put the marble into the bowl receives a score of 0 (zero).

   - If Cathy sees 1 marble when she plays the second step, but she guesses incorrectly, then Cathy and the person who really puts the marble into the bowl each receives a score of 0 (zero), but the person who does not put the marble into the bowl receives a score of 10 (ten).

   Assume that all players prefer higher scores to lower scores. The situation can be modelled as an extensive game with imperfect information $\langle N, H, P, f_c, (\mathcal{I}_i), (u_i) \rangle$.

   a) Write down all information partitions in this game.

   b) Is this a game with perfect recall? Justify your answer.

   Consider the scenario that both Amy and Beatrice simply randomly decide whether to put a marble into the bowl, that is, the probably to put a marble into the bowl and the probably not to put a marble into the bowl are always ½. On the other hand Cathy believes that the probably that the marble is put into the bowl by Amy is ½, and the probably that the marble is put into the bowl by Beatrice is also ½.

c) Consider the assessment corresponding to this scenario. What behavioural strategy should Cathy play for this assessment to be sequentially rational? Justify your answer.

d) Is this assessment consistent? Justify your answer.

e) Do we have a sequential equilibrium in this scenario? Justify your answer.

f) If your answer to Question 1e) is 'yes,' give another sequential equilibrium of the game, or justify that this is impossible. If your answer to Question 1e) is 'no,' give a sequential equilibrium of the game, or justify that this is impossible.

2. Amy thinks that the game in Question 1 is too complicated, and suggests playing a 'simpler' version of the game with Beatrice and Cathy. The 'simpler' game that Amy suggests is that Amy (player 1) and Beatrice (player 2) will perform the action of putting (action P) or not putting (action N) a marble into the bowl <u>at the same time</u>, and Cathy (player 3) also needs to openly announce what she guesses who puts a marble into the bowl (actions A or B) <u>at the same time</u>. Therefore, the game ends after all players take their actions at the same time. They agree that the rules to determine the scores remain more or less the same, except that if there are 2 marbles in the bowl at the end of the game (because both Amy and Beatrice put marbles into the bowl), then everyone receives 0 (zero) scores. To be specific, the rules are as follows.

- If the outcome is that there is no marble in the bowl, then each of Amy, Beatrice and Cathy receives a score of 1 (one), no matter what Cathy's guess is.

- If the outcome is that there are 2 marbles in the bowl, then each of Amy, Beatrice and Cathy receives a score of 0 (zero), no matter what Cathy's guess is.

- If the outcome is that there is 1 marble in the bowl, and Cathy guesses correctly who puts it into the bowl, then Cathy and the person who puts the marble into the bowl each receives a score of 10 (ten), but the person who does not put the marble into the bowl receives a score of 0 (zero).

- If the outcome is that there is 1 marble in the bowl, but Cathy guesses incorrectly who puts it into the bowl, then Cathy and the person who really puts the marble into the bowl each receives a score of 0 (zero), but the person who does not put the marble into the bowl receives a score of 10 (ten).

a) Are there any pure strategy Nash equilibria in this 'simpler' game? Justify your answer.

b) Consider the hypothetical scenario that Amy, Beatrice and Cathy like this game very much, and play the game repeatedly for infinitely many times.

   i) What are the minmax payoffs of the players?

   ii) Give one feasible and enforceable payoff profile.

   iii) Describe a trigger strategy equilibrium in which the limit-of-means criterion is used to evaluate the utilities of terminal histories.

3. Two players (player 1 and player 2) are arguing over an issue.

- Both players can decide to yield (action *Y*) or fight (action *F*).
- If both players decide to yield, then each will get a utility of 0 (zero).
- If player 1 decides to fight, then he will get a utility of 5 (five) if player 2 yields, or –5 (negative five) otherwise.
- If player 2 decides to fight, then he will get a utility of 3 (three) if player 1 yields, or –5 (negative five) otherwise.
- In any case, if a player decides to yield but his opponent decides to fight, then the player who yields receives a utility of –3 (negative three).

The situation can be modelled as a strategic game $G = \langle N, (A_i), (u_i) \rangle$.

a) List all the pure strategy Nash equilibria in the game *G*.
b) List all the completely mixed strategy Nash equilibria in the game *G*.

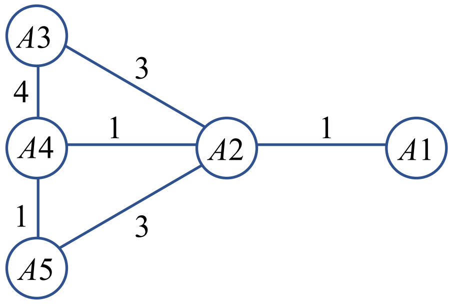
Now consider a limit-of-means infinitely repeated game of *G*.

c) What is the minmax payoff of player 1?
d) What is the minmax payoff of player 2?
e) Consider the payoff profile $(0, -8/3)$ a feasible payoff profile? Justify your answer.

f) Consider the payoff profile $(0, -8/3)$ an enforceable payoff profile? Justify your answer.

g) Describe one Nash equilibrium of the limit-of-means infinitely repeated game of G. What are the utilities these players receive in this Nash equilibrium?

4. There are five people ($A1$, $A2$, $A3$, $A4$, and $A5$) living in different places in a city. The distances between their homes are depicted in the following diagram:



For examples, the length of the shortest path for $A3$ to visit $A1$ is 4, but the length of the shortest path for $A5$ to visit $A1$ is 3. We say that the cost of travelling for $A3$ to go to $A1$'s home is 4, and the cost of travelling for $A5$ to go to $A1$'s home is 3.

One day these people decide that they should hold a party at one of these people's homes. After some discussions, they believe that only the homes of $A2$, $A4$ and $A5$ are big enough. They agree that they would using the Clarke tax mechanism to select the best place among the homes of $A2$, $A4$ and $A5$ as the place for the party. The objective is the minimise the total cost of travelling of all players.

a) What is the outcome of the selection?

b) How much tax each player needs to pay with this outcome?

c) Can any two of the players collude to improve their benefits in this example? Justify your answer.

— End of Assignment —