香 港 中 文 大 學
The Chinese University of Hong Kong

Course Examination 2nd Term, 2011 - 2012

Course Code & Title : CSCI5350 Game Theory in Computer Science

Time allowed : 2 hours .... minutes

Student I.D. No. : .... Seat No. : ....

Answer all three (3) questions.

1. John and Tom go together on an adventure in the Magicland. On their way, they discover four cold coins. They are very happy, so each of them takes 2 gold coins, and they go on their journey. Later on they find a box with a label attached to it, which says:

'Magic Box.

Blessed are the ones who find this box. Put one gold coin in it, say the magic word MALIMALIHUNG, and then you will have one and a half; two, then you will have three; three, then you will have four and a half; four, then you will have six. Only one time can this box be used. Use it wisely.'

So this is a magic box that will magically increase the number of gold coins put into it by 50%, after the magic word *MALIMALIHUNG* is said. Unfortunately this box can only be used once, so no one can use it repeatedly to get really rich.

John and Tom each has two gold coins, so each of them will determine independently the number of their own gold coins (0, 1, or 2 gold coins) to put into the magic box. If John puts n_j gold coins and Tom n_t gold coins in the box, then after saying the magic word, they will find totally $\frac{3}{2}(n_j + n_t)$ gold coins in the magic box. They agree that they will equally share this amount. In other words, at the end John will have $(2-n_j)+\frac{(\frac{3}{2}(n_j+n_t))}{2}$ gold coins, and Tom will have $(2-n_t)+\frac{(\frac{3}{2}(n_j+n_t))}{2}$. That is to say, Tom and John will equally share the gold coins finally found in the magic box, while keeping all the gold coins that they do not put into the magic box.

It is now a decision for John and Tom to be made, that how many gold coins each of them should put inside the magic box. Both John and Tom prefer having more gold coins to less gold coins.

To simply the situation, let's assume that John and Tom have a tool that can magically cut a gold coin into two pieces of any specific sizes. This allows them to share a gold coin with any specific ratio they like.

(a) Model the scenario as a strategic game $G = \langle N,(A_i),(u_i)\rangle$, where $N = \{1,2\}$, with John being Player 1 and Tom being Player 2.
 i. **(2 marks)** Write down (A_i) in the game G.
 ii. **(2 marks)** Write down the set A of outcomes in the game G.
 iii. **(2 marks)** Write down (u_i) in the game G.

(b) **(4 marks)** What are the pure strategy Nash equilibria in the game G? Justify your answer.

(c) **(4 marks)** Are there any completely mixed Nash equilibria in the game G? List all of them if there are any, or show that there is none.

(d) **(2 marks)** Suppose John now consults you, an expert in game theory, how many coins he should put inside the magic box. What advice would you give John? Justify why you think that this is a good advice.

2. Consider again the scenario in Question 1. Having heard your advices, John suggests to Tom that the game should be changed as follows.

Tom will first put 0, 1, or 2 gold coins into the magic box. After knowing how many gold coins Tom has put into the magic box, John will decide whether or not to put some or all of his gold coins into the magic box.

Finally, the magic word will be said, and the magic box will return 1.5 times of the amount of gold coins that Tom and John have been put into it. As in Question 1, Tom and John will equally share the gold coins finally found in the magic box, while keeping all the gold coins that they do not put into the magic box.

Both John and Tom prefer having more gold coins to less gold coins.

(a) Model the new scenario as an extensive game with perfect information $\Gamma = \langle N,H,P,(u_i)\rangle$, where $N = \{1,2\}$, with John being Player 1 and Tom being Player 2.
 i. **(2 marks)** How many pure strategies does John have?
 ii. **(2 marks)** Write down the set Z of terminal histories in the game E.
 iii. **(2 marks)** For each member $h \in Z$, define $u_1(h)$ and $u_2(h)$.
 iv. **(4 marks)** Describe a subgame perfect equilibrium in this game.

(b) **(10 marks)** Consider a hypothetical situation that John and Tom will need to play the game $\Gamma = \langle N,H,P,(u_i)\rangle$ in 2(a) repeatedly forever, where $N = \{1,2\}$, with John being Player 1 and Tom being Player 2. Describe a Nash equilibrium of this infinitely repeated game.

Tom considers John's suggestion, and thinks that it is unfair, so he proposes to revise the game a little bit, as follows.

Firstly, Tom will tell John whether he will put any gold coins into the magic box. If Tom will NOT put any gold coin into the magic box, then he just tells John. Otherwise, Tom will have to put at least one gold coins into the magic box, but he will not let John know how many gold coin he puts into the magic box. Here we assume that Tom is honest and he will really put one or more gold coins into the magic box.

Secondly, John will decide whether or not to put some or all of his gold coins into the magic box.

Finally, the magic word will be said, and the magic box will return double the amount of gold coins that Tom and John have been put into it. As in question 1, Tom and John will equally share the gold coins finally in the magic box, while keeping all the gold coins that they do not put into the magic box.

Both John and Tom prefer having more gold coins to less gold coins.

(c) Model the new scenario as an extensive game with imperfect information $G = \langle N, H, P, f_c, (\mathcal{I}_i), (u_i') \rangle$, where $N = \{1,2\}$, with John being Player 1 and Tom being Player 2.
  i. **(4 marks)** Write down Player 1's information partition $\mathcal{I}_1$ and Player 2's information partition $\mathcal{I}_2$ in the game G.
  ii. **(4 marks)** Does this game have perfect recall? Justify your answer.

(d) Suppose John believes that, if Tom will put some coins into the magic box, then there is a probability of ⅛ that Tom will put in 2 coins, and a probability of ⅞ that Tom will put in 1 coin.
  i. **(4 marks)** Suggest one pure strategy that John should use if John is rational.
  ii. **(4 marks)** What pure strategy should be Tom's best response if John really uses the strategy in 2(d)i?
  iii. **(6 marks)** If John uses the pure strategy in the answer of 2(d)i, and Tom uses the strategy in the answer of 2(d)ii, do we have a Nash equilibrium? Justify your answer.
  iv. **(6 marks)** Discuss whether there are any behavioural strategies that John should use if John is rational. Describe all of them and justify your answer.
  v. **(5 marks)** Is there any behavioural strategy in the answer of 2(d)iv that is consistent with John's belief (that if Tom will put some coins into the magic box, then there is a probability of ⅛ that Tom will put in 2 coins, and a probability of ⅞ that Tom will put in 1 coin)? Justify your answer.
  vi. **(5 marks)** Give a sequential equilibrium for the game, and justify your answer.

3. Consider a group of six players playing a game together. Player 1, player 2 and player 3 each is given a red card. Player 4 and player 5 each is given a green card. Play 6 is given a white card. There is a rule that players can form groups, such that a group can receive a utility of 1 unit if the group members have one red card and one green card, and, in general, a utility of n if they have n red cards and n green cards. Otherwise, the group will receive a utility of 0 (zero).

(a) The scenario can be formulated as a coalitional game with transferrable payoff $G = \langle N, v \rangle$.

i. **(2 marks)** Write down N in the game G.

ii. **(4 marks)** Write down v in the game G.

iii. **(2 marks)** Is the game G cohesive? Justify your answer.

iv. **(6 marks)** Give one payoff profile that is in the core, or prove that the core is empty. Justify your answer.

v. **(4 marks)** Describe the core (or list all members of the core) of the game G.

(b) The same game can also be formulated as a coalitional game <u>without</u> transferrable payoff $G_1 = \langle N, V, X, (\succsim_i) \rangle$.

i. **(4 mark)** Write down X in the game G_1. Describe or list the contents of X.

ii. **(4 marks)** Describe the core (or list all members of the core) of the game G_1.

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