

ENGG2430A, Spring 2017, Homework 3

Due at 5pm, Mar 28.

You can put your answer sheets to the Box 10B on the 10th floor of Ho Sin-hang Engineering Building (HSH).

1. (10 points) Suppose that X is nonnegative continuous random variable. Prove that $E[X] = \int_0^\infty P(X > x)dx$.
2. (10 points) Consider a triangle and a point chosen within the triangle according to the uniform probability law. Let X be the distance from the point to the base of the triangle. Given the height of the triangle, find the CDF and the PDF of X .
3. (10 points) Please write down the PDF of a normal random variable with mean μ and standard deviation σ .
4. (10 points) A point is chosen at random (according to a uniform PDF) within a semicircle of the form $\{(x, y): x^2 + y^2 < r^2, y > 0\}$, for some given $r > 0$.
 - a) Find the joint PDF of the coordinates X and Y of the chosen point.
 - b) Find the marginal PDF of Y and use it to find $E[Y]$.
5. (10 points) Let X_1, \dots, X_n be independent random variables. We know that variance has linearity for independent random variables, namely

$$\text{Var}[X_1 + X_2 + \dots + X_n] = \text{Var}[X_1] + \text{Var}[X_2] + \dots + \text{Var}[X_n]$$

How about the variance of the *product*? Can you express

$$\text{Var}[X_1 X_2 \dots X_n]$$

in terms of $\text{Var}[X_i]$ and $E[X_i^2]$?