THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MATH1020
Exercise 14
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Conics can be classified by computing the discriminant,

$$B^2 - 4AC$$

of the general quadratic equation,

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

We will not be studying any conics with the Bxy term, so we will always assume that

$$B = 0.$$

If

$$B \neq 0,$$

then the conic is rotated such that its major (and minor) axis is no longer parallel to one of the coordinate axes.

1. If

   $$B^2 - 4AC > 0,$$

   then the conic is a hyperbola. This is equivalent to showing that A and C have opposite signs.

2. If

   $$B^2 - 4AC = 0,$$

   then the conic is a parabola. This is equivalent to showing that either A or C is equal to zero.

3. If

   $$B^2 - 4AC < 0,$$

   then the conic is either an ellipse or a circle. This is equivalent to showing that A and C have the same sign. Circles are distinguished from ellipses when

   $$A = C.$$

Exercise 1 Identify each of the following conics:

1. $x^2 + 2y^2 - 4x + 6y - 1 = 0$;
2. $2x^2 + 2y^2 - 4x + 6y - 1 = 0$;
3. $x^2 - 2y^2 - 4x + 6y - 1 = 0$;
4. $x^2 - 4x + 6y - 1 = 0$.

Solution: Using the discriminant,

$$B^2 - 4AC$$

to classify each of the above equations, we have

1. ellipse
2. circle
3. hyperbola
4. parabola