ENGG 1130 Multivariable Calculus for Engineers

Assignment 6 (Term 2, 2019-2020)

Assigned Date: 1 Apr 2020 (Wed) 10:00 am

Deadline: **13 Apr 2020 (Mon) 12 noon**

- Show **ALL** your steps and details for each question unless otherwise specified.
- Make a copy of your homework before submitting the original!
- Please submit the **soft copy of your HW 6, TOGETHER WITH THE "DECLARATION FORM" to Blackboard system** on or before the prescribed deadline.
- Feel free to discuss with your friends, but make sure you all present your answers in different manners. **NO** citation (reference) is needed if only discussion takes place.

1. **(25 marks)**

(a) Evaluate $\nabla \cdot \mathbf{F}$ and $\nabla \cdot (\nabla \times \mathbf{F})$ respectively, where $\mathbf{F}(x, y, z) = x^2 z^5\, \mathbf{i} - 2xyz\, \mathbf{j} + xz^3\, \mathbf{k}$.

(b) Evaluate $\nabla \times (\nabla f)$, where $f(x, y, z) = x^3 y^7 e^{z^2 - y^2 + 2x^2}$.

(c) Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is a piecewise smooth curve from $P\left(2, \frac{\pi}{2}, 5\right)$ to $Q\left(1, \frac{\pi}{4}, 3\right)$, then from $Q\left(1, \frac{\pi}{4}, 3\right)$ to $R\left(1, \frac{\pi}{3}, 6\right)$, and $\mathbf{F}(x, y, z) = 8x^3\, \mathbf{i} + z^2 \cos(2y)\, \mathbf{j} + z \sin(2y)\, \mathbf{k}$.

2. **(20 marks)**

(a) Calculate the surface area of the graph of $f(x, y) = x + 20y$ over the region

$$R = \{(x, y) \in \mathbf{R}^2 : 1 \le x \le 4, 2 \le y \le 2x\}$$

in the xy-plane.

(b) Integrate the function $g(x, y, z) = x + y + z$ over the surface that is described as follows:

$$x = 2u - v, \qquad y = v + 2u, \qquad z = v - u$$

Here $u \in [0, 20], v \in [0, 21]$.

3. **(30 marks)**

(a) Consider $\mathbf{F}(x, y) = xy\, \mathbf{i} + xy^2\, \mathbf{j}$.

Without using the approach of curl test, explain whether the vector field $\mathbf{F}$ is conservative.

Show your derivation.

(**Note: NO marks** will be awarded if you use curl test in your answer.)

(b) Consider $\mathbf{F}(x, y) = -\frac{y}{x^2+y^2}\mathbf{i} + \frac{x}{x^2+y^2}\mathbf{j}$, where $(x, y) \neq (0, 0)$.

<u>Without</u> using Green's Theorem, evaluate $\oint_{C'} \mathbf{F} \cdot d\mathbf{r}$. Here $C' = C_1 \cup C_2$, where C_1 is the arc of the circle centered at origin with radius $\sqrt{2}$, from $(1, -1)$ to $(1,1)$, and C_2 is the line segment from $(1,1)$ to $(1, -1)$.

(**Note:** **<u>NO marks</u>** will be awarded if you use Green's Theorem in your answer.)

4. **(25 marks)**

(a) Let E be the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (with $a, b > 0$). Find the area bounded by E by **<u>using Green's Theorem</u>**.

(**Note**: **<u>NO marks</u>** will be awarded if you use other methods apart from Green's Theorem.)

(b) Find the area of one loop of the four-leafed rose described by the equation $r = 3\sin 2\theta$ using **<u>Green's Theorem in polar coordinates</u>**.

(**Note**: **<u>NO marks</u>** will be awarded if you use other methods apart from Green's Theorem.)

Show all your steps and derivations for both (a) and (b).

END OF ASSIGNMENT 6