

THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MATH1020
Exercise 1
Produced by Jeff Chak-Fu WONG

Exercise 1 Combining graphing procedures

Graph the function $f(x) = \frac{3}{x-2} + 1$. Find the domain and the range of f .

Solution:

It is helpful to write f as $f(x) = \frac{3}{x-2} + 1$. Now we use the following steps to obtain the graph of f .

- | | | |
|--------|---|---|
| Step 1 | $y = \frac{1}{x}$ | Reciprocal function |
| Step 2 | $y = 3\left(\frac{1}{x}\right) = \frac{3}{x}$ | Multiply by 3.
Vertical stretch of the graph
of $y = \frac{1}{x}$ by a factor of 3. |
| Step 3 | $y = \frac{3}{x-2}$ | Replace x by $x - 2$.
Horizontal shift to the right 2 units. |
| Step 4 | $y = \frac{3}{x-2} + 1$ | Add 1.
Vertical shift up 1 unit. |

The domain of $y = \frac{1}{x}$ is $\{x \mid x \neq 0\}$ and its range $\{y \mid y \neq 0\}$.

Because we shifted right 2 units and up 1 unit to obtain f , the domain of f is $\{x \mid x \neq 2\}$ and its range $\{y \mid y \neq 1\}$.

See Figure 1.

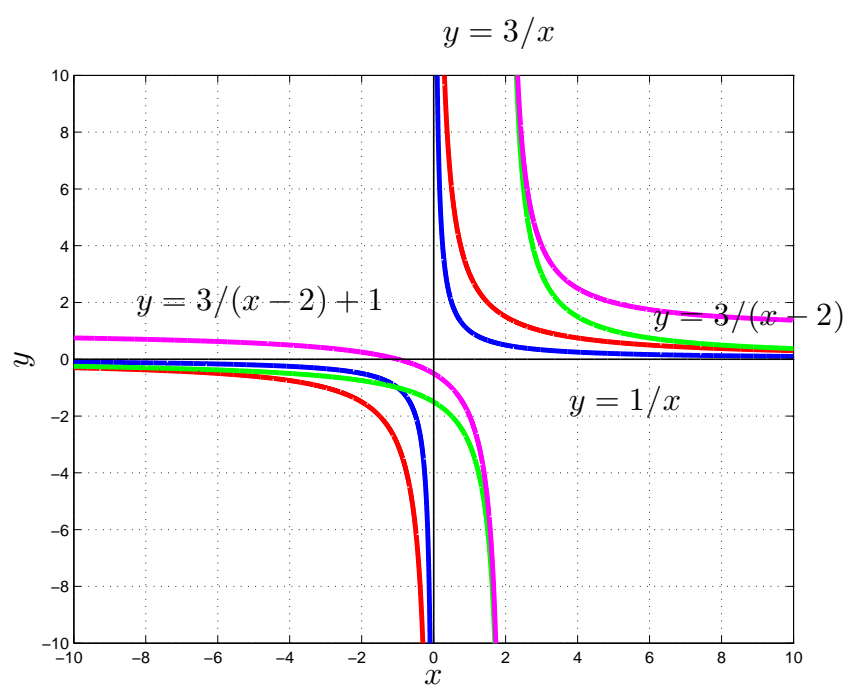


Figure 1:

Exercise 2 Combining graphing procedures

Graph the function $f(x) = \sqrt{1-x} + 2$. Find the domain and the range of f .

Solution:

It is because horizontal shifts require the form $x - h$, we begin by rewriting f as

$$\begin{aligned} f(x) &= \sqrt{1-x} + 2 \\ &= \sqrt{-(x-1)} + 2. \end{aligned}$$

Now use the following steps:

Step 1	$y = \sqrt{x}$	Square root function.
Step 2	$y = \sqrt{-x}$	Replace x by $-x$. Reflect about the y -axis.
Step 3	$y = \sqrt{-(x-1)}$	Replace x by $x-1$. Horizontal shift to the right 1 unit.
Step 4	$y = \sqrt{1-x} + 2$	Add 2. Vertical shift up 2 units.

The domain of $y = \frac{1}{x}$ is $(-\infty, 1]$ and its range $[2, +\infty)$.

