# Exercises: Line Integral by Coordinate

**Problem 1.** Let $C$ be the curve from point $p = (0, 0)$ to $q = (2, 4)$ on the parabola $y = x^2$. Calculate $\int_C (x^2 - y^2) dx$.

**Problem 2.** Let $\boldsymbol{r}(t) = [t, t^2, t^3]$ and $\boldsymbol{f}(\boldsymbol{r}) = [x - y, y - z, z - x]$. Let $C$ be the curve from the point of $t = 0$ to the point of $t = 1$. Calculate $\int_C \boldsymbol{f}(\boldsymbol{r}) \cdot d\boldsymbol{r}$.

**Problem 3.** Let $\boldsymbol{r}(t) = [x(t), y(t)]$ where $x(t) = \cos(t)$ and $y(t) = \sin(t)$. Let $p$ be the point given by $t = \pi/4$. Calculate $\frac{dx}{ds}$ at $p$.

**Problem 4.** Let $\boldsymbol{r}(t) = [x(t), y(t), z(t)]$. Let $p$ be the point given by $t = t_0$. Prove that $[\frac{dx}{ds}(t_0), \frac{dy}{ds}(t_0), \frac{dz}{ds}(t_0)]$ is a unit tangent vector at $p$.

**Problem 5.** This problem allows you to see the equivalence of line integral by length and line integral by coordinate. Let $\boldsymbol{r}(t) = [x(t), y(t)]$ where $x(t) = \cos(t)$ and $y(t) = \sin(t)$. Convert $\int_C x\,dx + \int_C y^2\,dy$ to line integral by length.