Heaps in C

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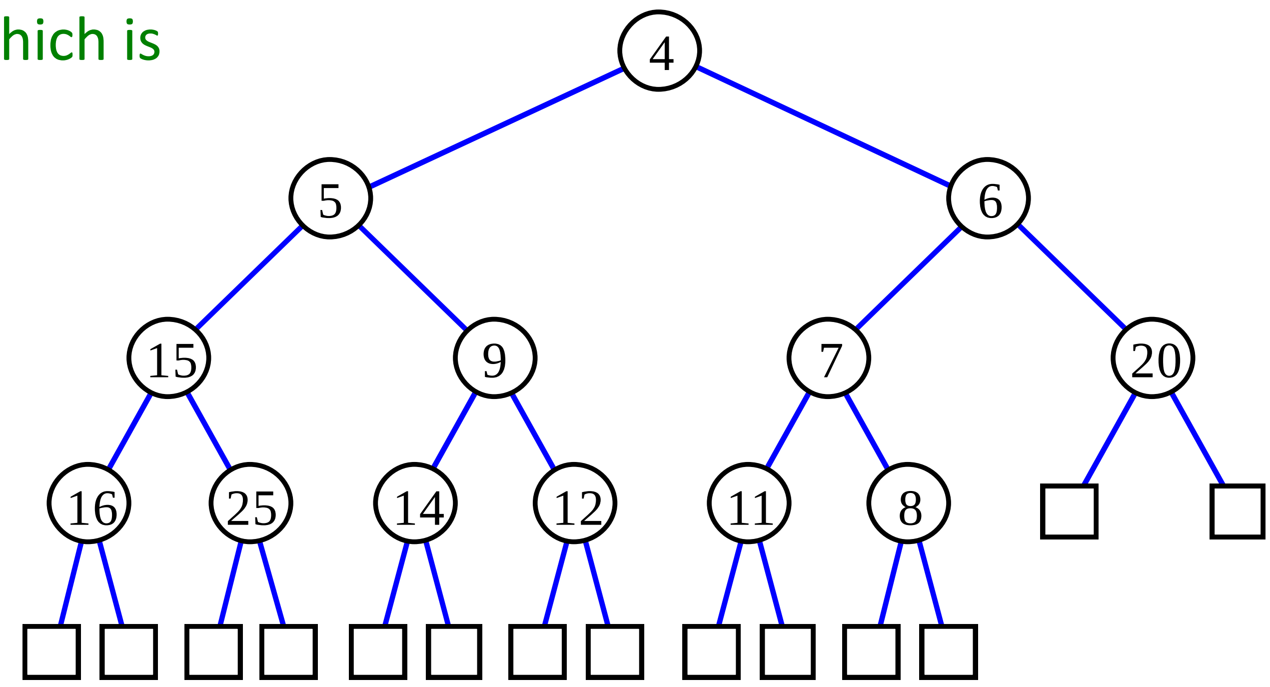
CSCI2100 Data Structures Tutorial 7

Review on Heaps

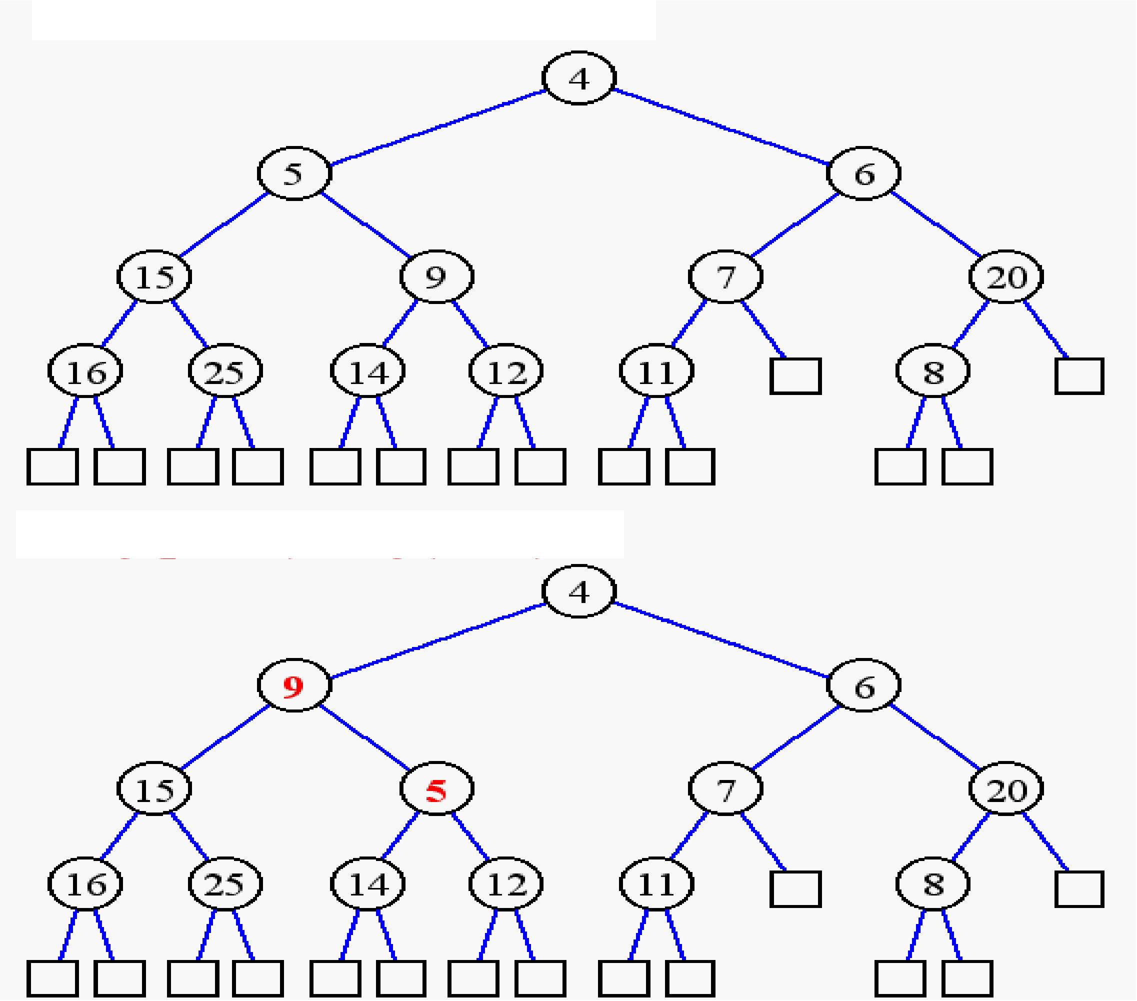
- A *heap* is implemented as a binary tree
- It satisfies two properties:
  - **MinHeap: parent <= child**
  - **[OR MaxHeap: parent >= child]**
  - all levels are full, except the last one, which is left-filled



What are Heaps Useful for?

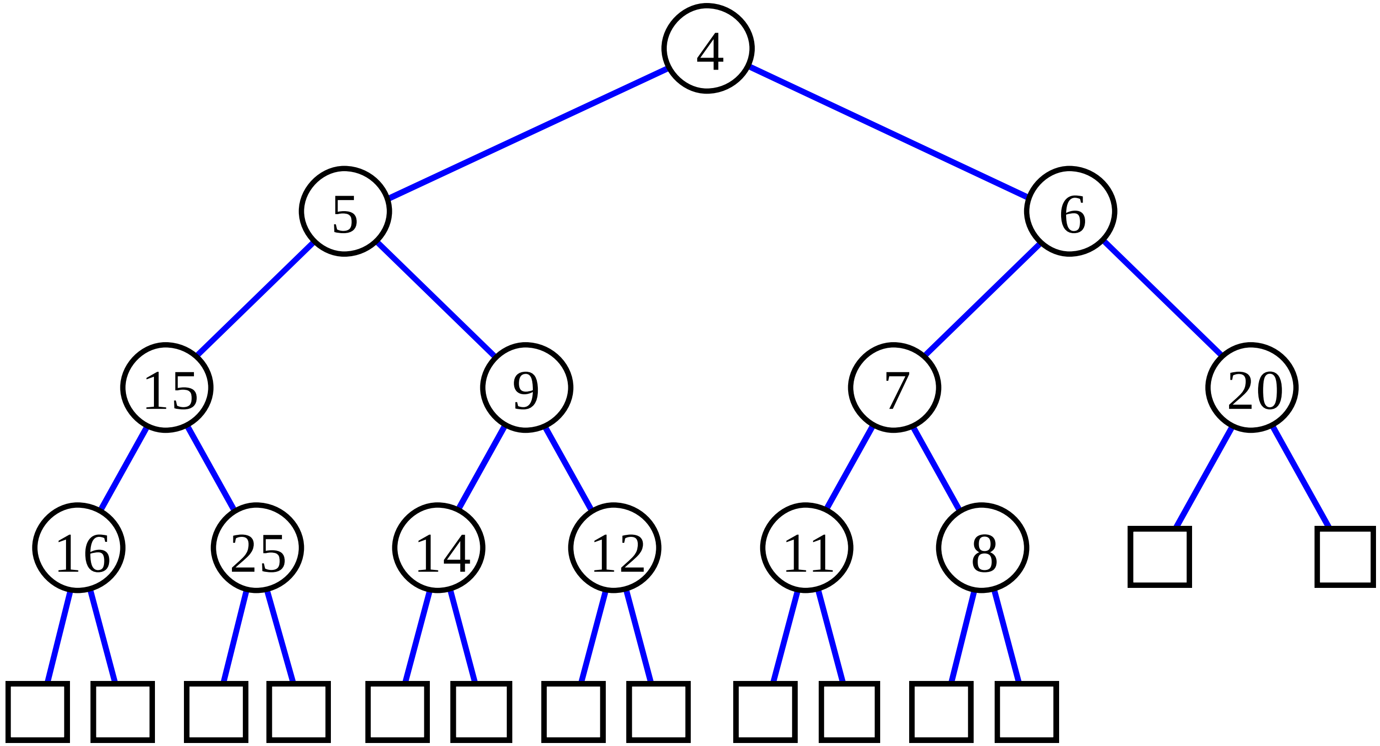
- To implement priority queues
- Priority queue = a queue where all elements have a "priority" associated with them
- Remove in a priority queue removes the element with the smallest priority
- Basic operations:
  - insert
  - removeMin

Heap or Not a Heap?



Heap Properties

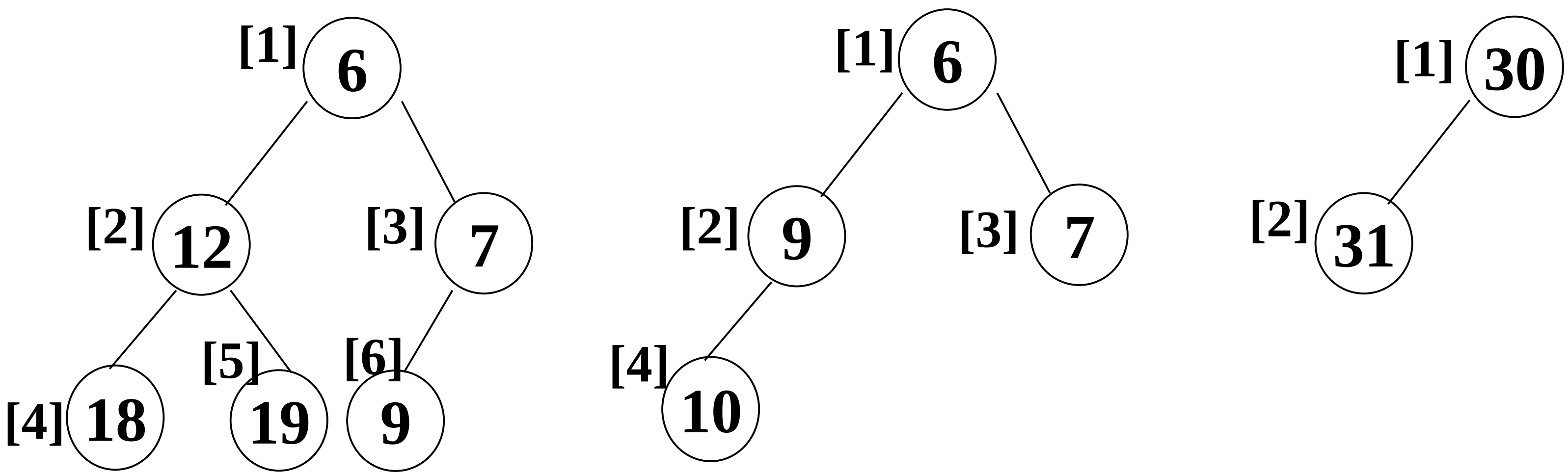
- A heap T storing n keys has height h = $\lfloor log_2 n \rfloor$,
- e.g. 13 keys, height = 3



Heap Implementation

- Using arrays
- Parent = k ; Children = 2k , 2k+1

  (k starts from 1)

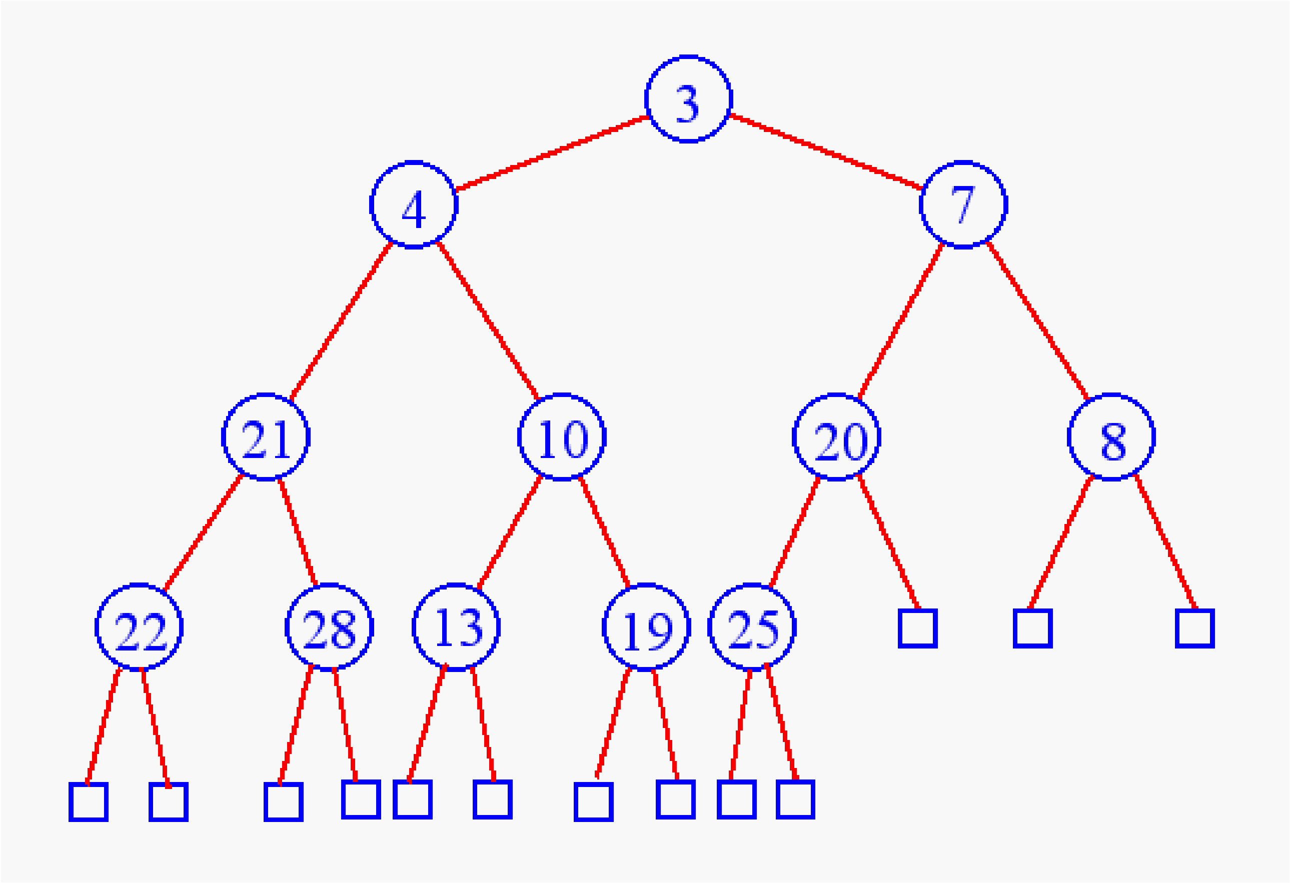


Heap Structure in C

```
struct HeapStruct {
  int capacity;
  int size;
  ElementType *Elements;
};
typedef struct HeapStruct Heap;
```

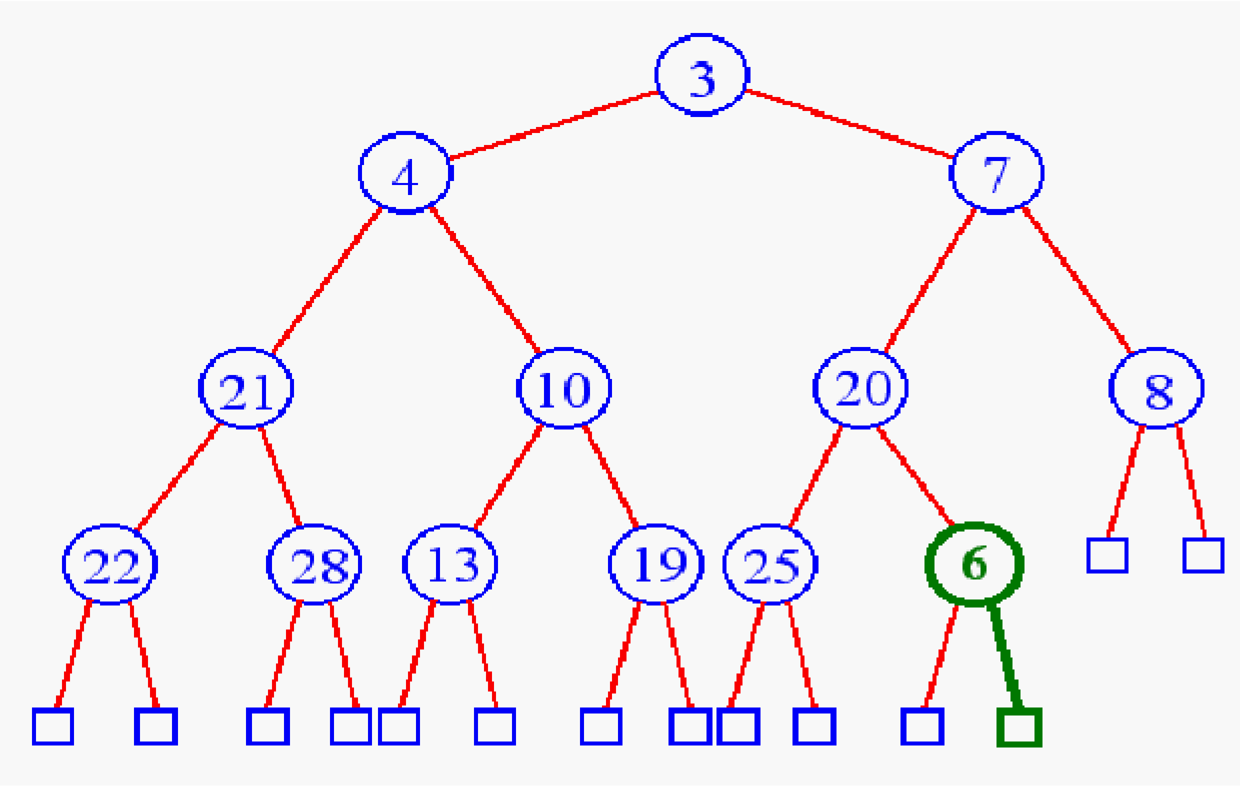
Review on Heap Insertion

- Insert 6



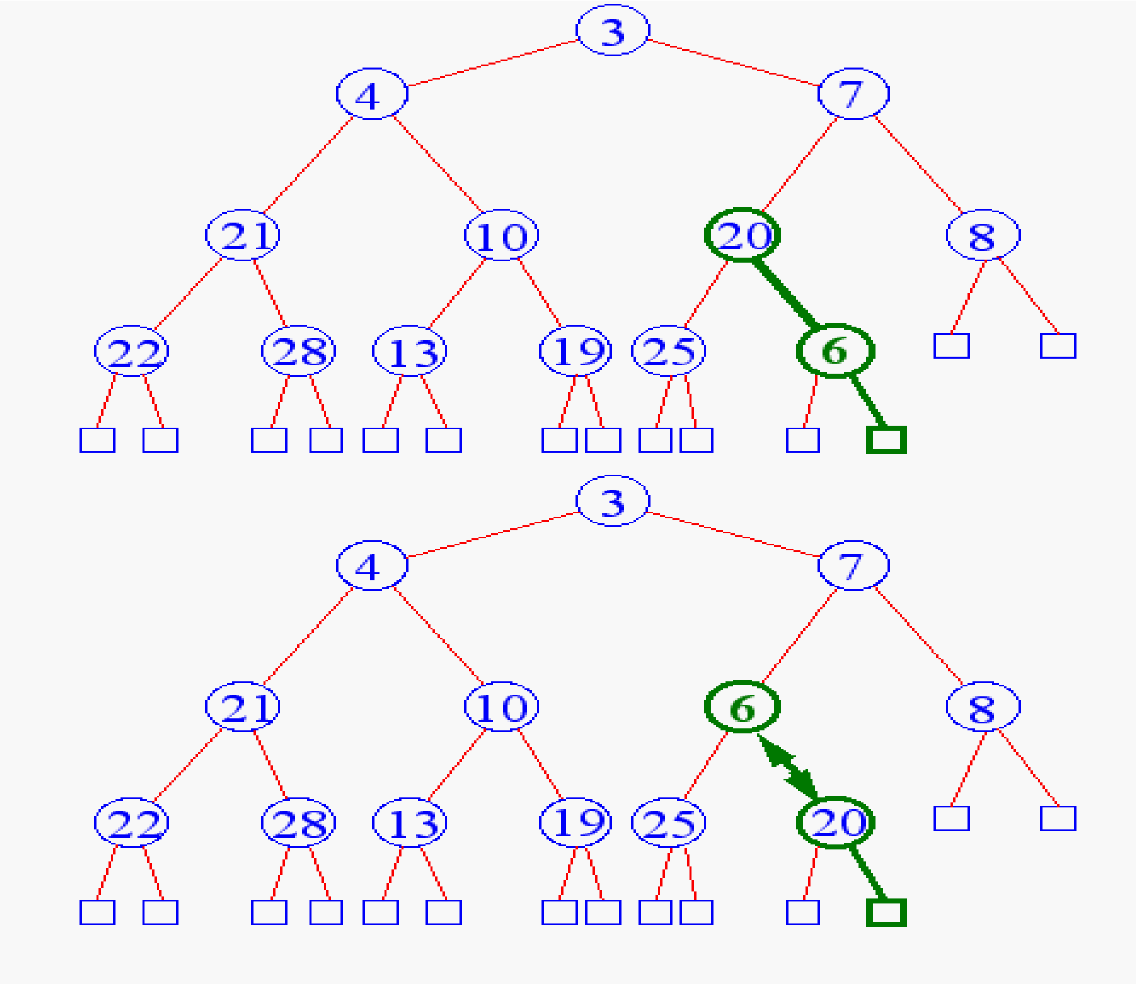
Heap Insertion

- Add key in next available position
- Violate Heap properties

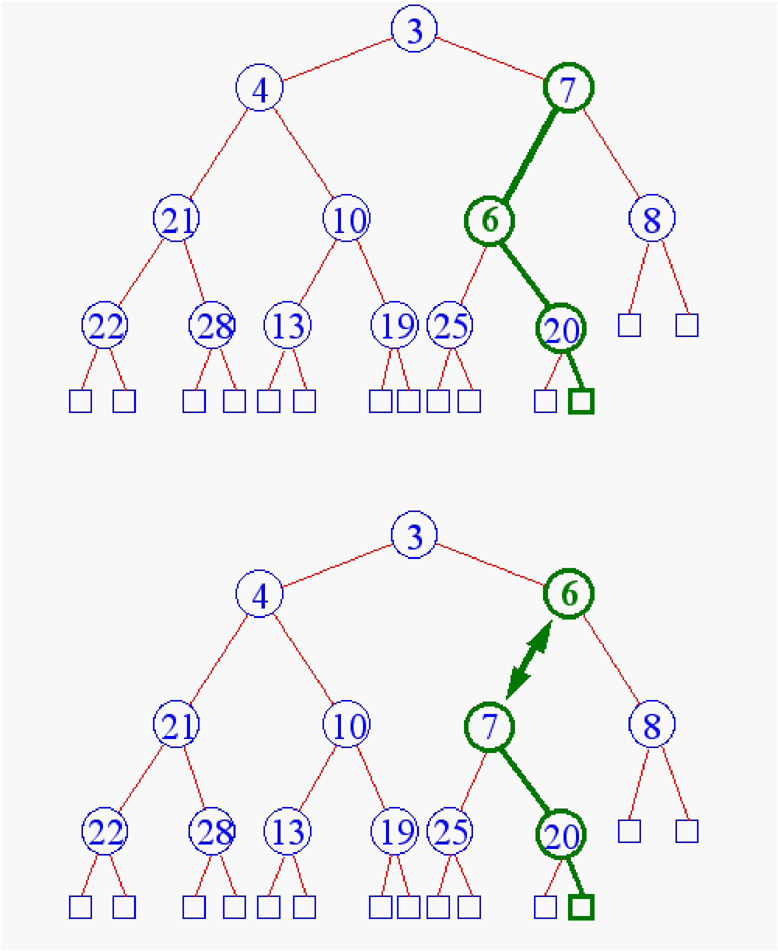


Heap Insertion

- Begin percolate up

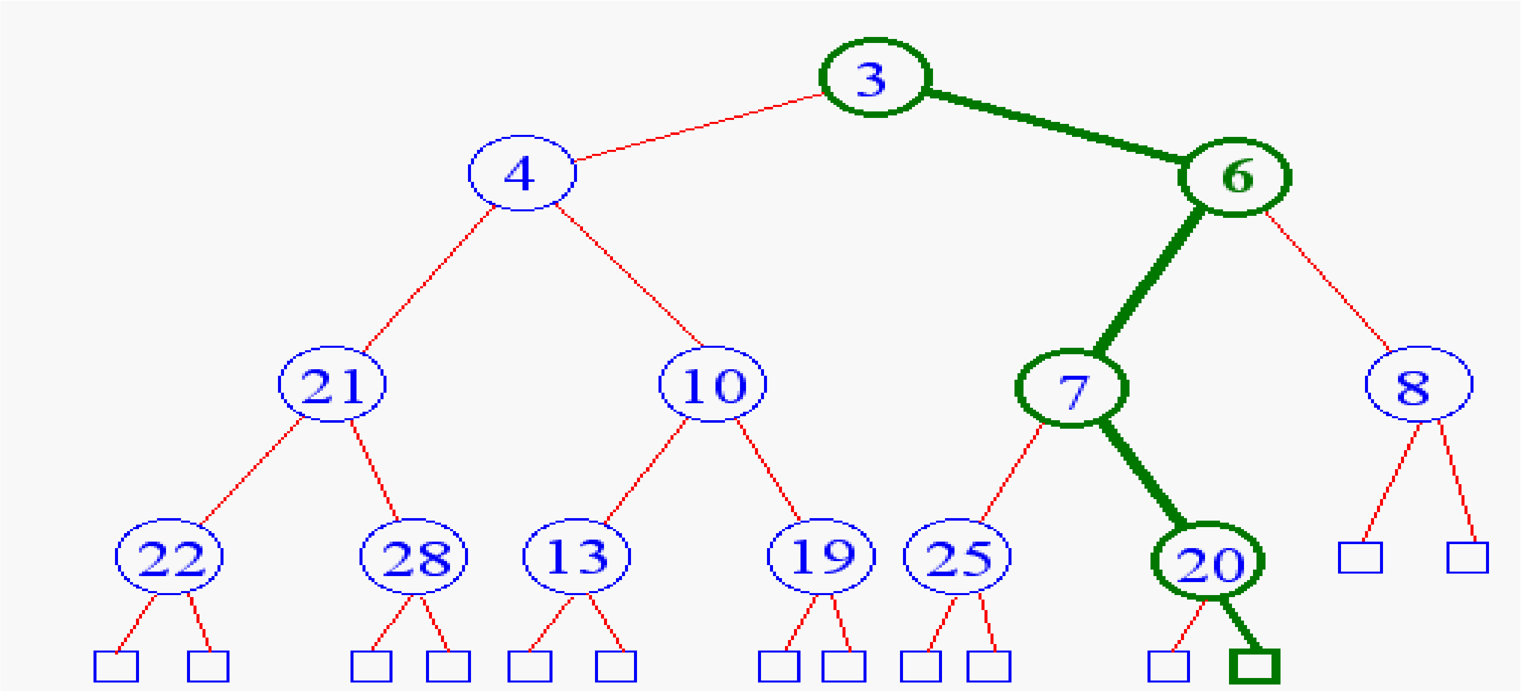


Heap Insertion



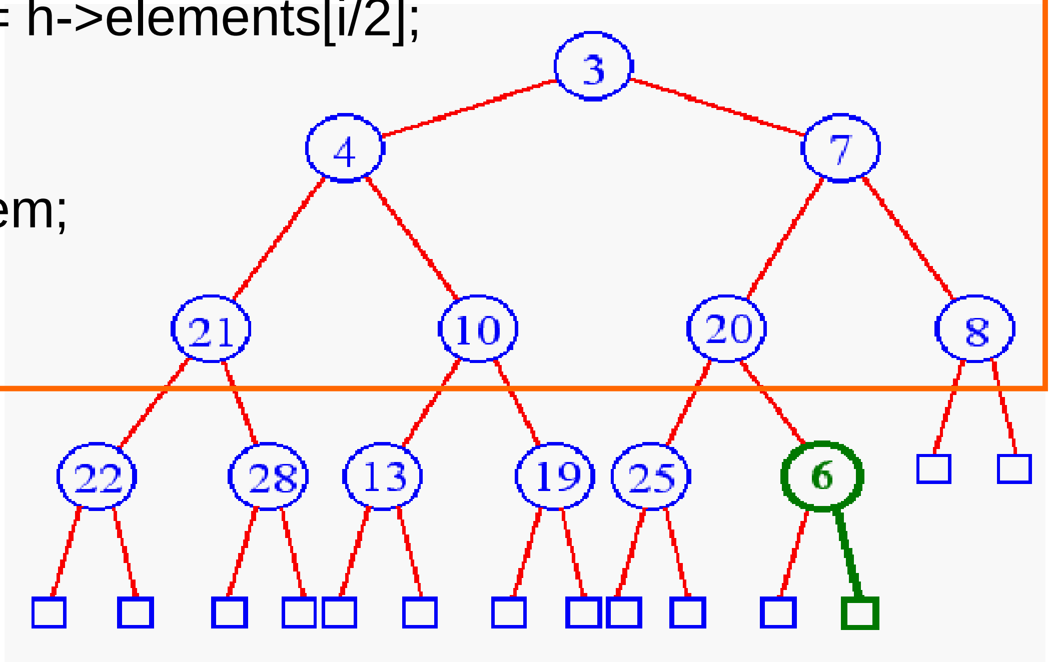
Heap Insertion

- Terminate percolate-up when
  - reach root
  - key child is greater than key parent



Insertion into a Heap $O(\log_2 n)$

```
void insertHeap(Heap *h, ElementType item){
   int i;
   if (HEAP_FULL(h)) {
      printf("The heap is full.\n");
      exit(1);
   }
   i = ++h->size;
   while ( (i!=1) && (item < h->elements[i/2]) ){
      h->elements[i] = h->elements[i/2];
      i /= 2;
   }
   h->elements[i]=item;
}
```



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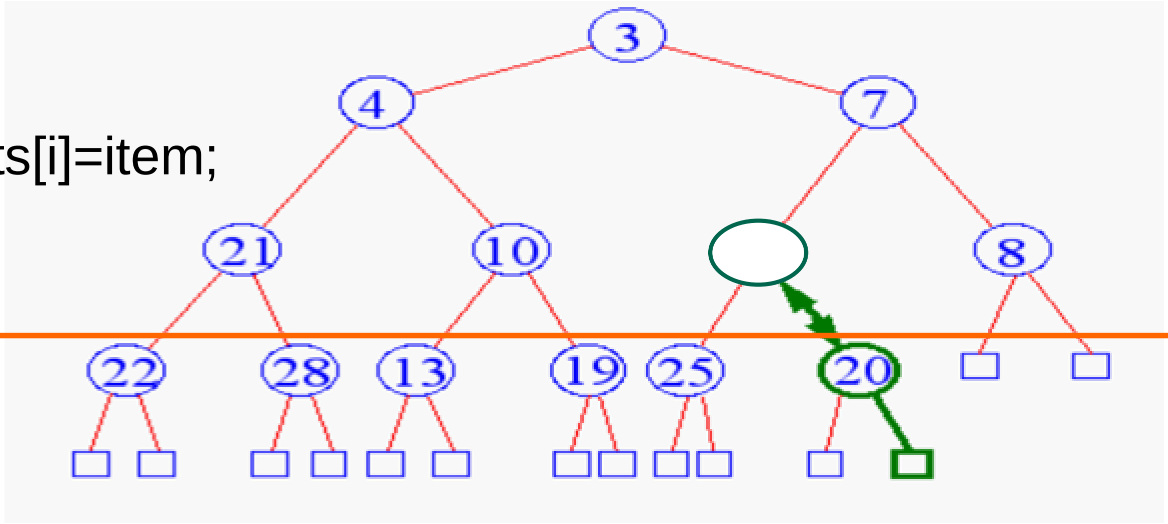
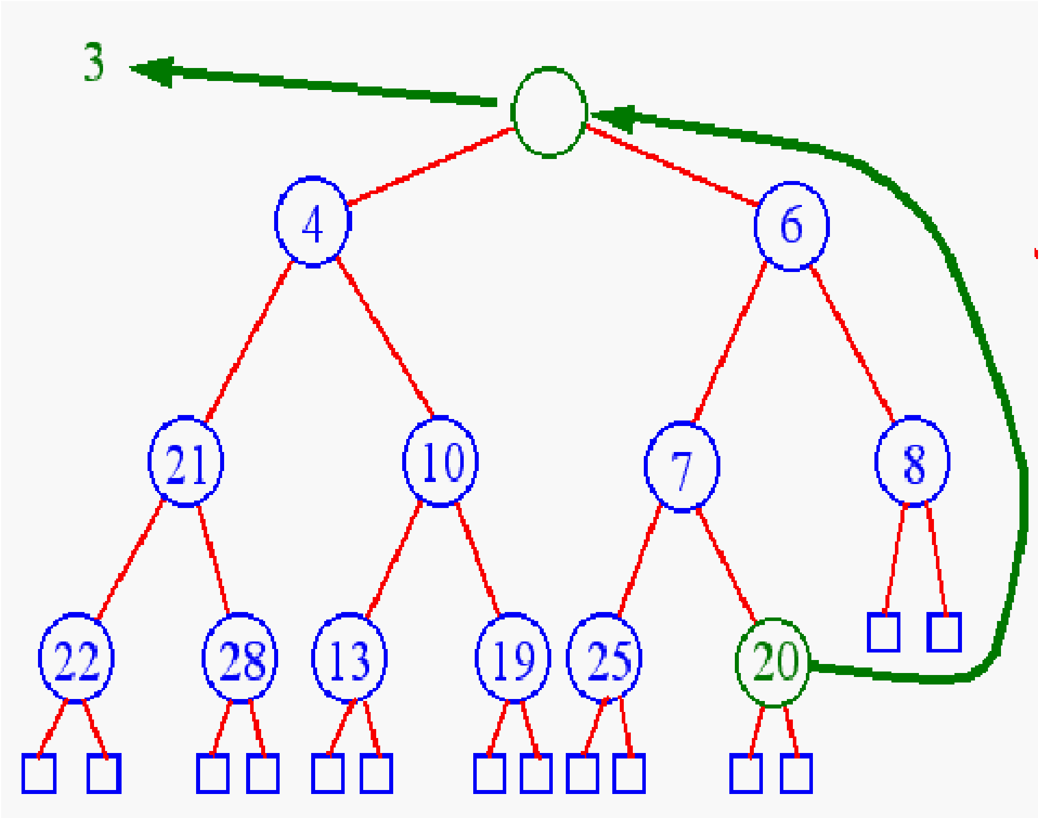


Heap Removal

- Remove element

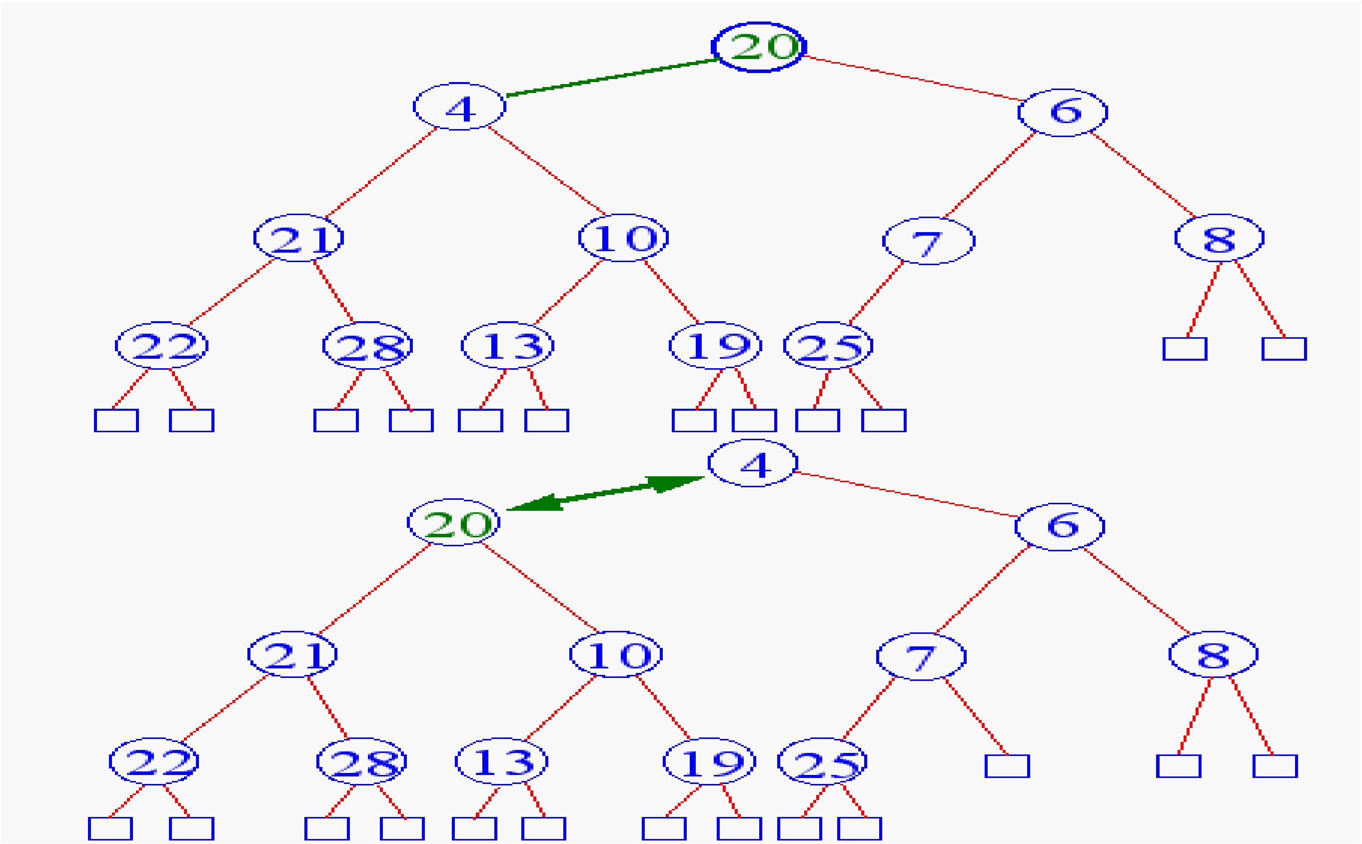
from priority queues?

removeMin( )

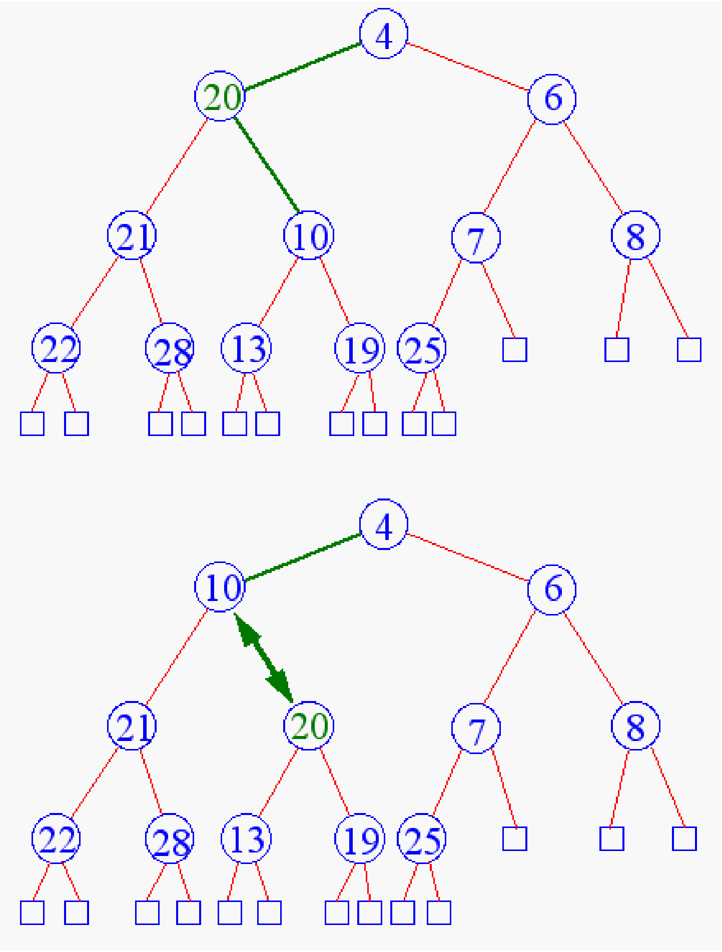


Heap Removal

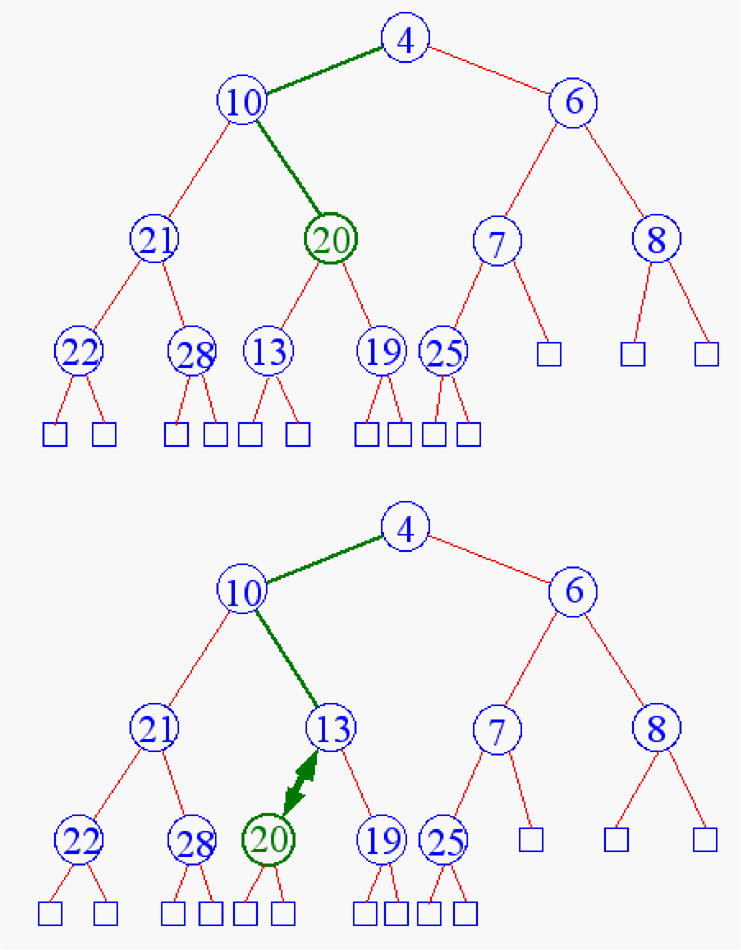
- Begin percolate down



Heap Removal

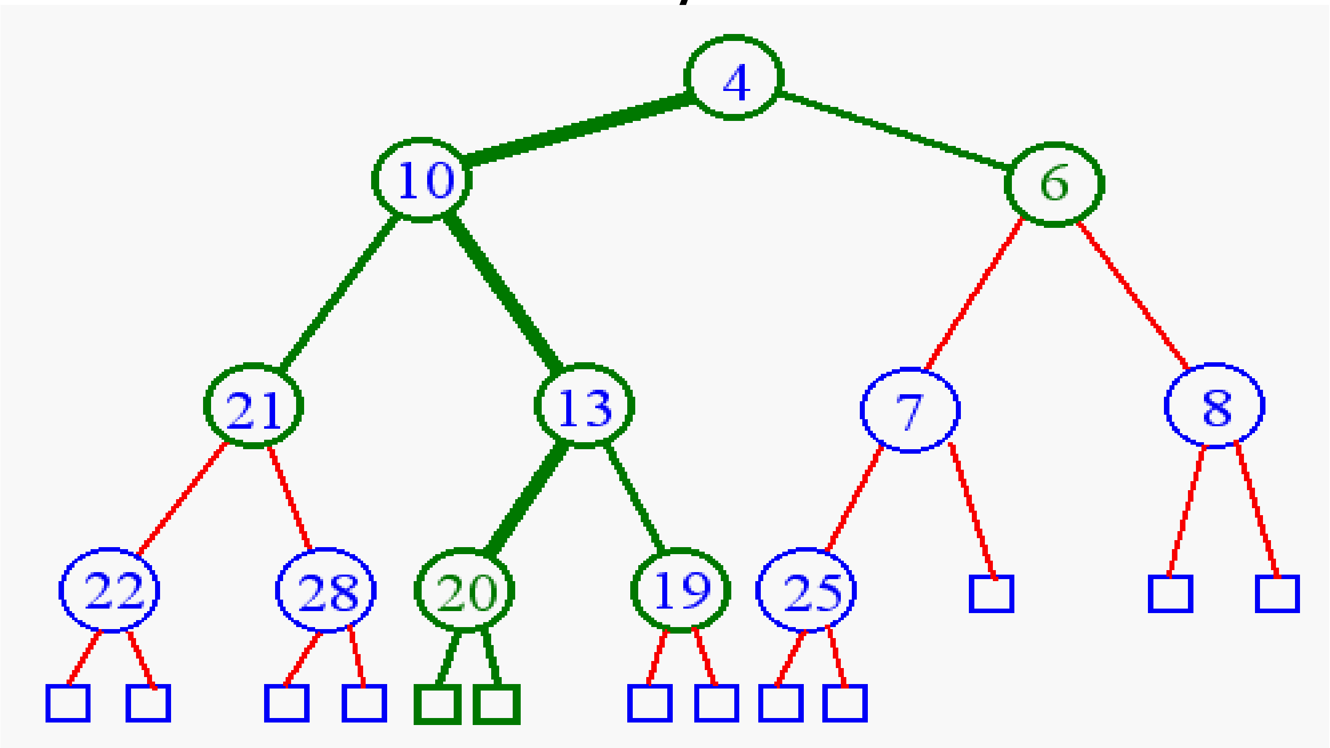


Heap Removal



Heap Removal

- Terminate percolate-down when
  - reach leaf level
  - key parent is smaller than key child



Deletion from a Heap

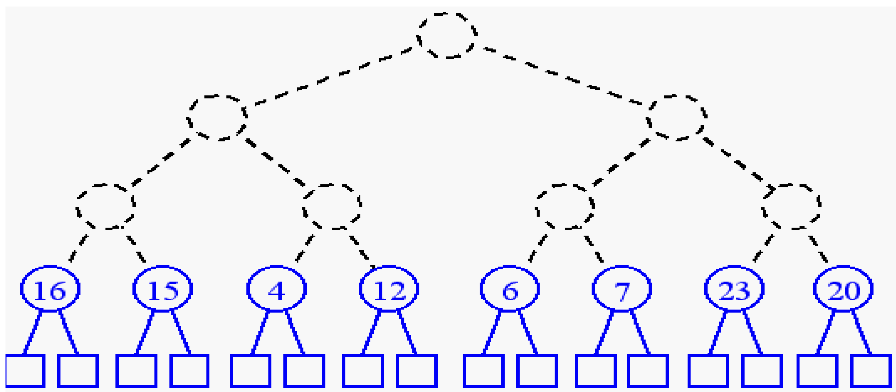
```
ElementType deleteHeap(Heap *h){
   int parent, child;
   ElementType item, temp;
   if (HEAP_EMPTY(h)){
      printf(“The heap is empty\n”);
      exit(1);
   }
   // save value of the minimum element
   item = h->elements[1];
   //use last element in heap to adjust heap
   temp = h->elements[h->size--];
   parent = 1;
   child = 2;
```

Deletion from a Heap (cont'd)

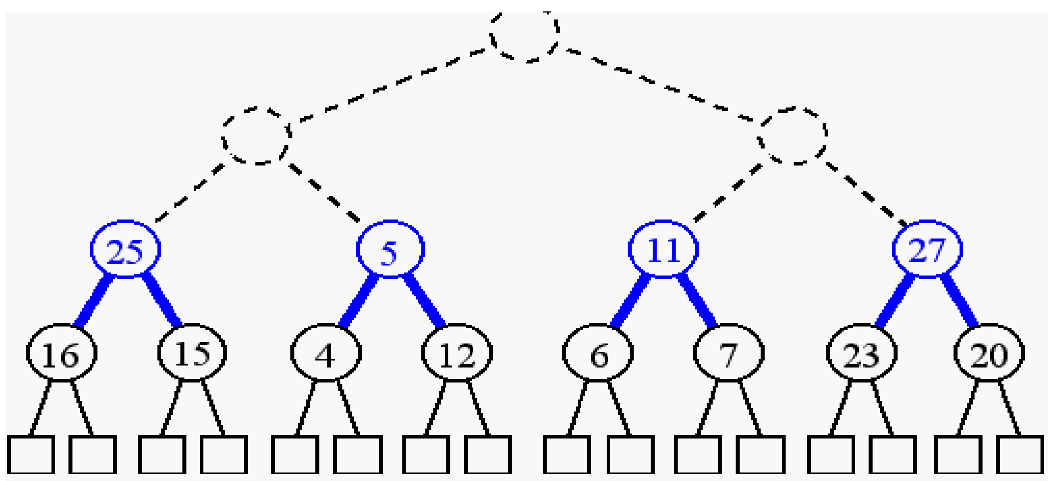
```
    while (child <= h->size){
    // find the smaller child of the current parent
       if ( (child < h->size) &&
       (h->elements[child] > h->elements[child+1] ) )
              child++;
       if (temp<=h->elements[child]) break;
       // move to the next lower level
       h->elements[parent] = h->elements[child];
       parent = child;
       child *=2;
    }
    h->elements[parent] = temp;
    return item;
}
```

Building a Heap level by level

- build (n + 1)/2 trivial one-element heaps

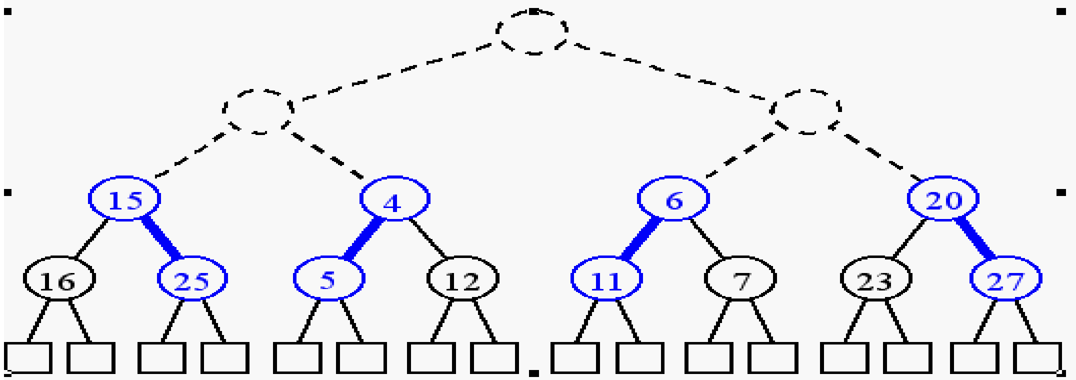


- build three-element heaps on top of them

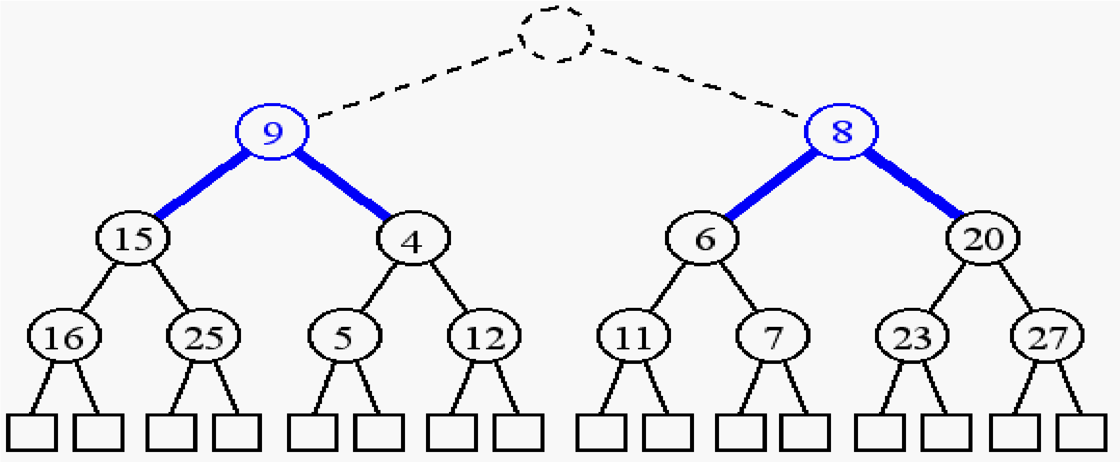


Building a Heap level by level

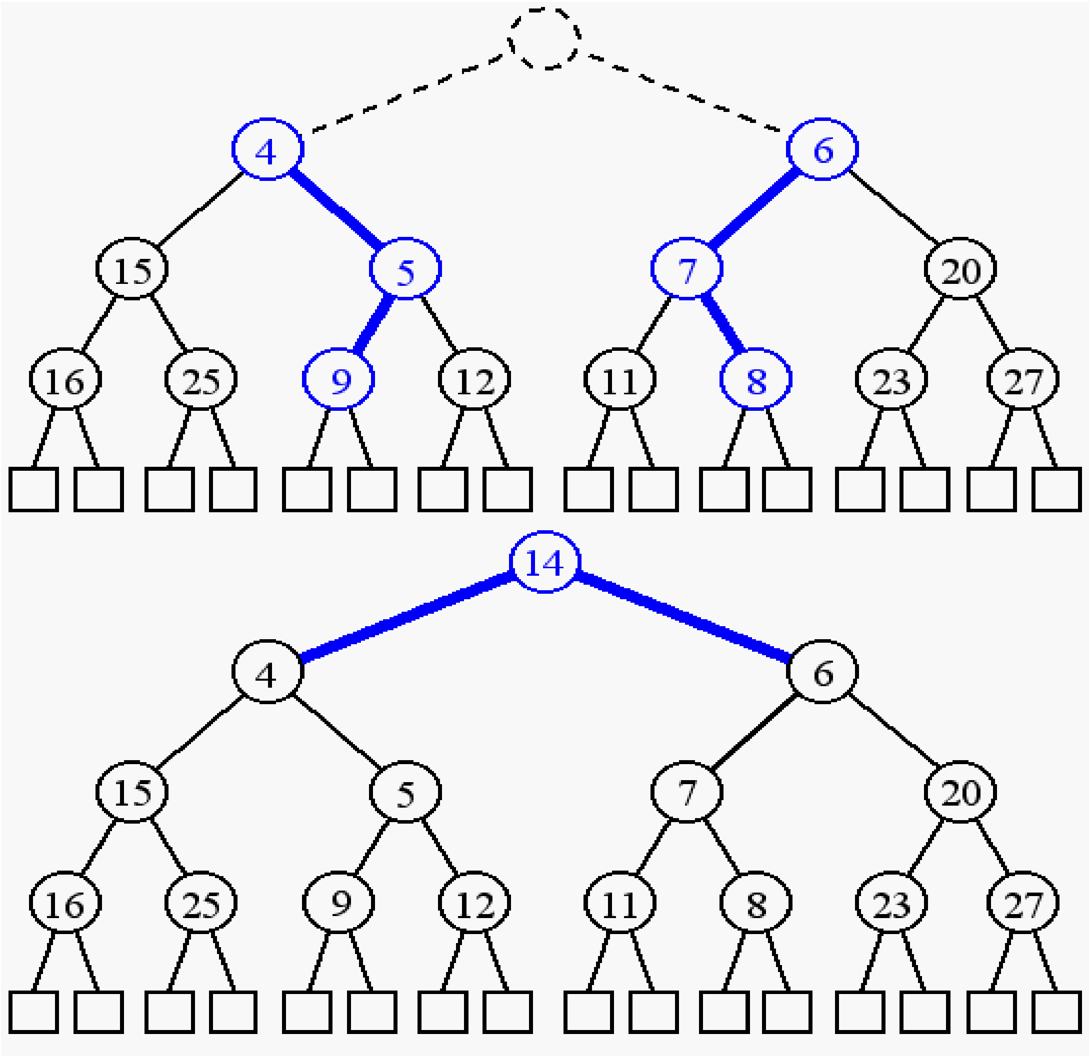
- Percolate-down to preserve the order property



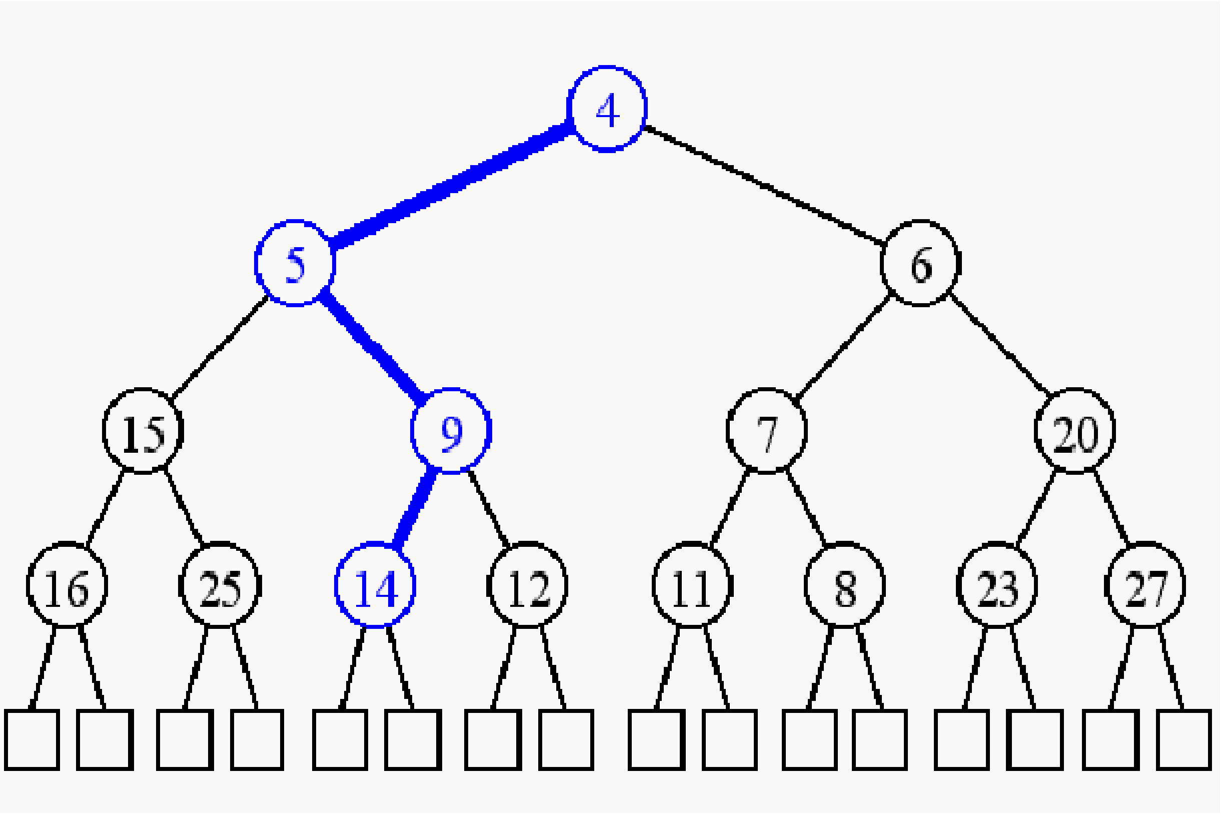
- Now form seven-element heaps



Building a Heap level by level



Building a Heap level by level



- Time complexity: O(n log n)
- We will introduce a more efficient algorithm

A faster algorithm to build Heap

- Step 1:

Insert the keys into the tree in any order

- Step2:

For i = $\lfloor n/2 \rfloor$ down to 1:

  – PercolateDown(i)

- Time complexity: O(n)
- Complete proof on the supplementary file

Heap Sorting

- Step 1: Build a heap
- Step 2: removeMin( )

Appendix: A quick start tutorial for GDB

```c
/* test.c */
/* Sample program to debug. */

#include <stdio.h>
#include <stdlib.h>

int main (int argc, char **argv)
{
    if (argc != 3)
        return 1;
    int a = atoi (argv[1]);
    int b = atoi (argv[2]);
    int c = a + b;
    printf ("%d\n", c);
    return 0;
}
```

A quick start tutorial for GDB

- Compile with the -g option:
  - gcc -g -o test test.c

- Load the executable, which now contain the debugging symbols, into gdb:
  - gdb test

A quick start tutorial for GDB

- Now you should find yourself at the gdb prompt. There you can issue commands to gdb.
- Say you like to place a breakpoint at line 11 and step through the execution, printing the values of the local variables - the following commands sequences will help you do this:

A quick start tutorial for GDB

```
(gdb) break test.c:11
Breakpoint 1 at 0x401329: file test.c, line 11.
(gdb) set args 10 20
(gdb) run
Starting program: c:\Documents and Settings\VMathew\Desktop/test.exe 10 20
[New thread 3824.0x8e8]

Breakpoint 1, main (argc=3, argv=0x3d5a90) at test.c:11
(gdb) n
(gdb) print a
$1 = 10
(gdb) n
(gdb) print b
$2 = 20
(gdb) n
(gdb) print c
$3 = 30
(gdb) c
Continuing.
30

Program exited normally.
(gdb)
```

Commands all you need to start:

```
break file:lineno - sets a breakpoint in the file at lineno.
set args - sets the command line arguments.
run - executes the debugged program with the given command line arguments.
next (n) and step (s) - step program and step program until it
                        reaches a different source line, respectively.
print - prints a local variable
bt -  print backtrace of all stack frames
c - continue execution.
```

- Type help at the (gdb) prompt to get a list and description of all valid commands.

Further GDB guides

- Peter's GDB tutorial http://dirac.org/linux/gdb/
- Tutorial on using the GDB debugger (Video) http://www.youtube.com/watch?v=k-zAgbDq5pk

ADT for Min Heap

objects: n >= 0 elements organized in a binary tree so that the value in each node is at least as large as those in its children

method:

Heap Create(MAX_SIZE)::= create an empty heap that can hold a maximum of max_size elements

Boolean HeapFull(heap)::= if (heap->size== heap->capacity) return TRUE else return FALSE

ADT for Min Heap (cont')

method:

Heap Insert(heap, item)::= if (!HeapFull(heap)) insert item into heap and return the resulting heap else return error

Boolean HeapEmpty(heap)::= if (heap->size>0) return FALSE else return TRUE

Element Delete(heap)::= if (!HeapEmpty(heap)) return one instance of the smallest element in the heap and remove it from the heap else return error