Practice questions

1. X is a Geometric(Θ) random variable, where Θ itself is a random variable with PDF $f_\Theta(\theta) = 2\theta$ where $0 \leq \theta \leq 1$. What are the MAP (Maximum a Posteriori) estimator and ML (Maximum Likelihood) estimates for Θ?

2. Jason has two 4-sided dice in a bag. Die A has sides 1, 2, 3, 4 and die B has sides 2, 2, 3, 3. Jason picks one of the dice randomly, rolls it twice, and reports the sum S of the rolls. Your task is to guess which die Jason rolled based on the value of S.

   (a) For which values of S would you guess that Bob rolled die A?

   (b) If you guess like in part (a), what is the probability that your guess is wrong?

3. A food processing company packages honey in glass jars. The volume of honey in a random jar is a Normal($\mu, 5$) millilitre random variable for an unknown value of μ. The government wants to verify that μ is at least 100 millilitres.

   (a) The government proposes the following test: Choose a random jar and verify that the jar has at least t millilitres of honey. Which value of t should be chosen so that a complying company passes the test with probability at least 95%?

   (b) The ACME company jars contain Normal($95, 5$) millilitres of honey. What is the probability that ACME passes the test?

4. A random variable X is Normal($1, 1$) with probability p and Normal($-1, 1$) with probability $1 - p$, where the parameter p is unknown.

   (a) What is the PDF of X?

   (b) What is the maximum likelihood estimate of p given that $X = x$?

   (c) Let X_1 and X_2 be independent samples of X. What is the maximum likelihood estimate of p given that $X_1 = x_1$ and $X_2 = x_2$?

5. Coin A has probability of heads 40%. Coin B has probability of tails 40%. One of these coins is tossed is n times. How large does n need to be so that you can identify the coin with probability about 99%? (**Hint:** Use a normal approximation, or write a computer program.)