

THE CHINESE UNIVERSITY OF HONG KONG
 Department of Mathematics
 MATH1020
 Exercise 3
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Exercise 1 Find the limits:

- (a) $\lim_{x \rightarrow 2^-} \frac{1}{x-2};$ (b) $\lim_{x \rightarrow 2^+} \frac{1}{x-2};$
 (c) $\lim_{x \rightarrow 2} \frac{1}{x-2};$ (d) $\lim_{x \rightarrow +\infty} \frac{1}{x-2};$ (e) $\lim_{x \rightarrow -\infty} \frac{1}{x-2}.$

| | | | | | | | | |
|------------------------|-----|------|-------|--------|---------|----------|---------------|---------|
| x | 1.9 | 1.99 | 1.999 | 1.9999 | 1.99999 | 1.999999 | \rightarrow | \dots |
| $f(x) = \frac{1}{x-2}$ | | | | | | | | |

| | | | | | | | | |
|------------------------|---------|--------------|----------|---------|--------|-------|------|-----|
| x | \dots | \leftarrow | 2.000001 | 2.00001 | 2.0001 | 2.001 | 2.01 | 2.1 |
| $f(x) = \frac{1}{x-2}$ | | | | | | | | |

| | | | | | | | | |
|------------------------|---|----|-----|------|-------|--------|---------------|---------|
| x | 1 | 10 | 100 | 1000 | 10000 | 100000 | \rightarrow | \dots |
| $f(x) = \frac{1}{x-2}$ | | | | | | | | |

| | | | | | | | | |
|------------------------|---------|--------------|---------|--------|-------|------|-----|----|
| x | \dots | \leftarrow | -100000 | -10000 | -1000 | -100 | -10 | -1 |
| $f(x) = \frac{1}{x-2}$ | | | | | | | | |

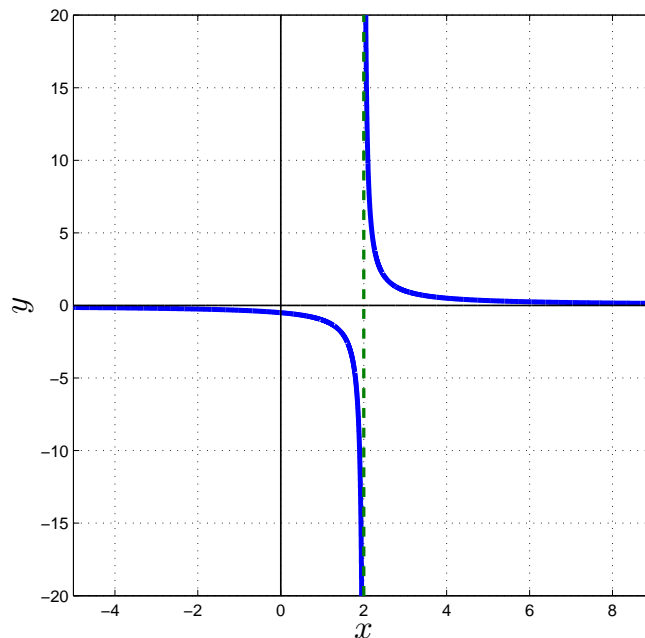


Figure 1: Graph of $y = \frac{1}{x-2}$, where $x \in [-5, 9]$.

Exercise 2 Find the limits:

$$\begin{array}{lll} \text{(a)} \quad \lim_{x \rightarrow -1^-} \frac{3x}{x+1}; & \text{(b)} \quad \lim_{x \rightarrow -1^+} \frac{3x}{x+1}; & \\ \text{(c)} \quad \lim_{x \rightarrow -1} \frac{3x}{x+1}; & \text{(d)} \quad \lim_{x \rightarrow +\infty} \frac{3x}{x+1}; & \text{(e)} \quad \lim_{x \rightarrow -\infty} \frac{3x}{x+1}. \end{array}$$

| | | | | | | | | |
|-------------------------|------|-------|--------|---------|----------|-----------|---------------|---------|
| x | -1.1 | -1.01 | -1.001 | -1.0001 | -1.00001 | -1.000001 | \rightarrow | \dots |
| $f(x) = \frac{3x}{x+1}$ | | | | | | | | |

| | | | | | | | | |
|-------------------------|---------|--------------|-----------|----------|---------|--------|-------|------|
| x | \dots | \leftarrow | -0.999999 | -0.99999 | -0.9999 | -0.999 | -0.99 | -0.9 |
| $f(x) = \frac{3x}{x+1}$ | | | | | | | | |

| | | | | | | | | |
|-------------------------|---|----|-----|------|-------|--------|---------------|---------|
| x | 1 | 10 | 100 | 1000 | 10000 | 100000 | \rightarrow | \dots |
| $f(x) = \frac{3x}{x+1}$ | | | | | | | | |

| | | | | | | | | |
|-------------------------|---------|--------------|---------|--------|-------|------|-----|----|
| x | \dots | \leftarrow | -100000 | -10000 | -1000 | -100 | -10 | -1 |
| $f(x) = \frac{3x}{x+1}$ | | | | | | | | |

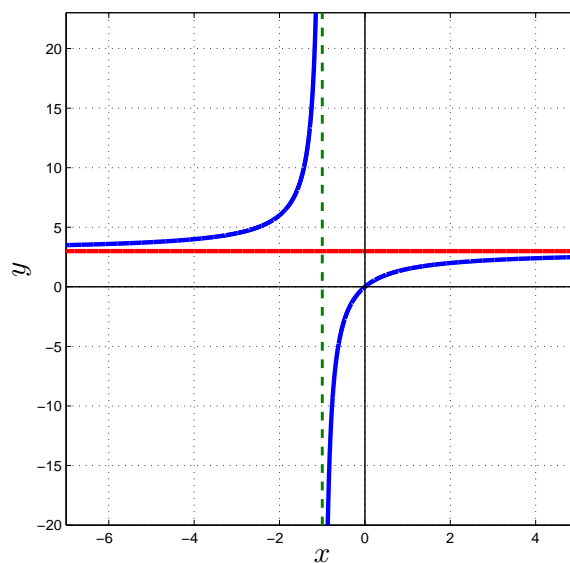


Figure 2: Graph of $y = \frac{3x}{x+1}$, where $x \in [-7, 5]$.

Exercise 3 Find the limits:

$$\begin{array}{lll} \text{(a)} \quad \lim_{x \rightarrow 0^-} \frac{x^2 + 1}{x}; & \text{(b)} \quad \lim_{x \rightarrow 0^+} \frac{x^2 + 1}{x}; & \\ \text{(c)} \quad \lim_{x \rightarrow 0} \frac{x^2 + 1}{x}; & \text{(d)} \quad \lim_{x \rightarrow +\infty} \frac{x^2 + 1}{x}; & \text{(e)} \quad \lim_{x \rightarrow -\infty} \frac{x^2 + 1}{x}. \end{array}$$

| | | | | | | | | |
|----------------------------|------|-------|--------|---------|----------|-----------|---------------|---------|
| x | -0.1 | -0.01 | -0.001 | -0.0001 | -0.00001 | -0.000001 | \rightarrow | \dots |
| $f(x) = \frac{x^2 + 1}{x}$ | | | | | | | | |

| | | | | | | | | |
|----------------------------|---------|--------------|----------|---------|--------|-------|------|-----|
| x | \dots | \leftarrow | 0.000001 | 0.00001 | 0.0001 | 0.001 | 0.01 | 0.1 |
| $f(x) = \frac{x^2 + 1}{x}$ | | | | | | | | |

| | | | | | | | | |
|----------------------------|---|----|-----|------|-------|--------|---------------|---------|
| x | 1 | 10 | 100 | 1000 | 10000 | 100000 | \rightarrow | \dots |
| $f(x) = \frac{x^2 + 1}{x}$ | | | | | | | | |

| | | | | | | | | |
|----------------------------|---------|--------------|---------|--------|-------|------|-----|----|
| x | \dots | \leftarrow | -100000 | -10000 | -1000 | -100 | -10 | -1 |
| $f(x) = \frac{x^2 + 1}{x}$ | | | | | | | | |

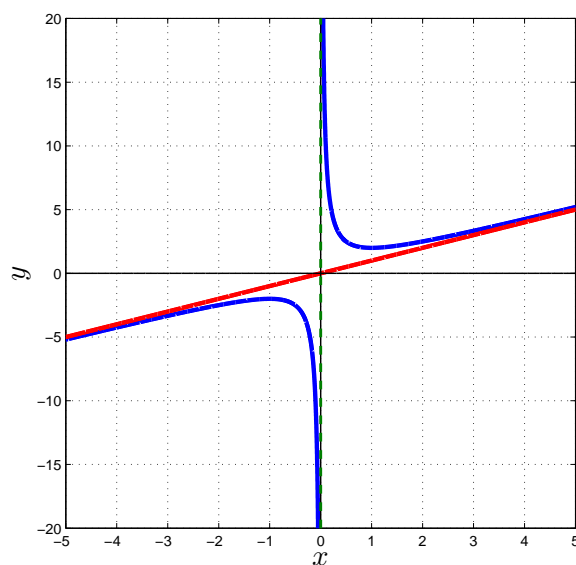


Figure 3: Graph of $y = \frac{x^2 + 1}{x}$, where $x \in [-5, 5]$.