CSCI 5350 Advanced Topics in Game Theory

Discussion Session 9

Game Theory Exercise 3

The Lying Game is a strategic game in which Player 1 decides whether to tell the truth or to tell a lie, while Player 2 determines whether or not to believe what Player 1 says. If Player 1 tells the truth (action T), and Player 2 believes (action B), then both get the same utility of 5. However, if Player 1 tells the truth, but Player 2 does not believe (action N), then both get the same utility of 0. On the other hand, if Player 1 tells a lie (action L) but Player 2 believes, then Player 1 has a utility of 10, while Player 2 has a utility of –5. Finally, if Player 1 tells a lie and Player 2 does not believe, then Player 1 has a utility of –10, while Player 2 has a utility of 0.

(a) Consider a limit of means infinitely repeated game of the Lying Game.

i. **(2 marks)** Is the payoff profile (4,0) a feasible payoff profile? Justify your answer.

ii. **(2 marks)** What is the minmax payoff of Player 1?

iii. **(2 marks)** What is the minmax payoff of Player 2?

iv. **(2 marks)** Is the payoff profile (4,0) an enforceable payoff profile? Justify your answer.

v. **(10 marks)** Describe a trigger strategy equilibrium of the limit of means infinitely repeated game of the Lying Game. What is the outcome of the trigger strategy equilibrium you describe?

From 2008-09 Final Exam Q2

(b) The two players now agree to play a variant of the Lying Game, as follows. First, Player 1 decides whether to continue (action C) or to stop (action S). If Player 1 decides to stop, then the game ends and the payoff profile is (0,0). Otherwise, if Player 1 decides to continue, then Player 2 will decide whether he will believe or will not believe what Player 1 is going to say, before Player 1 really says it. Player 2 writes his decision on a piece of paper without letting Player 1 knows what he writes. Without knowing Player 2's decision, Player 1 now tells the truth or a lie. The final payoff profiles are defined in exactly the same way as before: If Player 1 tells the truth, and Player 2 believes, then both get the same utility of 5. However, if Player 1 tells the truth, but Player 2 does not believe, then both get the same utility of 0. If Player 1 tells a lie but Player 2 believes, then Player 1 has a utility of 10, while Player 2 has a utility of -5. If Player 1 tells a lie and Player 2 does not believe, then Player 1 has a utility of –10, while Player 2 has a utility of 0.

(c) Model the new scenario as an extensive game with imperfect information $G = \langle N, H, P, f_c, (\mathcal{I}_i), (u_i) \rangle$.

i. **(1 mark)** Write down H in the game G.
ii. **(2 marks)** Write down P in the game G.
iii. **(4 marks)** Write down Player 1's information partition $\mathcal{I}_1$ and Player 2's information partition $\mathcal{I}_2$ in the game G.
iv. **(4 marks)** Does this game have perfect recall? Justify your answer.

(d) Suppose when Player needs to decide whether to play T or L, he has a belief μ that the probability that Player 2 has just played B is ¾, the probability that Player 2 has just played N is ¼.

i. **(4 marks)** What should be Player 1's best response behavioural strategy β_1?
ii. **(5 marks)** Describe a consistent assessment that contains β_1 and μ. Justify that this is a consistent assessment.
iii. **(4 marks)** Is the assessment in question 2(d)ii sequentially rational? Justify your answer.
iv. **(3 marks)** Is the assessment in question 2(d)ii a sequential equilibrium? Justify your answer.

From 2008-09 Final Exam Q2

Consider the following extensive game $\langle N, H, P, f_c, (\mathcal{I}_i), (u_i) \rangle$.

$N = \{1,2\}$
$H = \{\varnothing, a, (a,a), (a,b), b, (b,u), (b,v), (b,u,a), (b,u,b), (b,v,a), (b,v,b), (b,v,b,m), (b,v,b,n)\}$
$P(\varnothing) = P(a) = P(b,u) = P(b,v) = 1$
$P(b) = P(b,v,b) = 2$
f_c is undefined
$\mathcal{I}_1 = \{\{\varnothing\}, \{a\}, \{(b,u),(b,v)\}\}$
$\mathcal{I}_2 = \{\{b\}, \{(b,v,b)\}\}$

$u_1(a,a) = 8,$	$u_1(a,b) = 9,$	$u_1(b,u,a) = 1,$	$u_1(b,u,b) = 2,$
$u_1(b,v,a) = 4,$	$u_1(b,v,b,m) = 3,$	$u_1(b,v,b,n) = 4$	
$u_2(a,a) = 0,$	$u_2(a,b) = 0,$	$u_2(b,u,a) = 1,$	$u_2(b,u,b) = 2,$
$u_2(b,v,a) = 3,$	$u_2(b,v,b,m) = 2,$	$u_2(b,v,b,n) = 1$	

a) **(2 marks)** How many pure strategies does player 1 have in this game? List all of them.
b) **(2 marks)** How many pure strategies does player 2 have in this game? List all of them.
c) **(2 marks)** Is this a game with perfect recall? Justify your answer.

Consider a belief system μ such that $\mu(\{(b,u),(b,v)\})((b,u)) = \frac{2}{3}$.

Assume that player **1** plays behavioural strategy β_1 and player 2 plays behavioural strategy β_2, such that $\beta_1(\varnothing)(a) = \beta_1(a)(a) = \beta_1(b,u)(a) = \frac{1}{2}$, and $\beta_2(b)(u) = \beta_2(b,v,b)(m) = 1$.

d) **(5 marks)** Let $\beta = (\beta_1, \beta_2)$. What is $O(\beta, \mu \mid \{(b,u),(b,v)\})$?
e) **(3 marks)** Is the assessment (β, μ) consistent? Justify your answer.
f) **(3 marks)** Is the assessment (β, μ) sequentially rational? Justify your answer.
g) **(3 marks)** Is the assessment (β, μ) a sequential equilibrium? Justify your answer.

From 2013-14 Final Exam Q4

End