ENGG 1130 Multivariable Calculus for Engineers

Assignment 5 (Term 2, 2019-2020)

Assigned Date: 15 Mar 2020 (Sunday) 10:00 am

Deadline: **27 Mar 2020 (Friday) 12 noon**

- Show **ALL** your steps and details for each question unless otherwise specified.
- Make a copy of your homework before submitting the original!
- Please submit the **soft copy of your HW 5, TOGETHER WITH THE "DECLARATION FORM" to Blackboard system** on or before the prescribed deadline.
- Feel free to discuss with your friends, but make sure you all present your answers in different manners. **NO** citation (reference) is needed if only discussion takes place.

1. **(15 marks)**

   (a) Given a cone with equation $z^2 = x^2 + y^2$ and a plane with equation $x - 2z = 3$.

   Describe the intersecting curve of the cone and the plane.

   (b) Find the maximum and minimum values of $h(x, y, z) = x^2 + y^2 + z^2$ on the intersecting curve found in (a).

   Also, state clearly the points such that the maximum and minimum values are attained.

2. **(10 marks)** Find the **absolute extrema** of $g(x, y) = (2x - y)^2$ over the triangular region in the xy-plane with vertices (0,1), (1,2) and (2,0), with all boundaries of the triangle included.

   Show **ALL** your steps clearly, and state clearly the points such that the absolute extrema are attained.

3. **(10 marks)**

   We wish to find the region R in the xy-plane such that the value of the double integral

$$\iint_R (2020 - 5x^2 - 5y^2)\,dA$$

   is **maximized**. Find such R with proper explanation, hence evaluate the integral.

4. **(10 marks) Design of a Spherical Souvenir**

   The university is producing a spherical souvenir of radius 30 cm. Evaluate the outer surface area of the souvenir after we drill a hole of radius 3 cm through its centre.

5. **(10 marks)** Suppose $R - r_0 > 0$ and we assume both r_0 and R are positive. We remove a central cylinder of radius r_0 from a sphere of radius R. Show, **with the aid of a triple integral**, that the resulting volume of the remaining portion of sphere is $k[(R + r_0)(R - r_0)]^{\frac{3}{2}}$, where k is a constant. Show the value of the constant k explictly in your final answer.

   (**Hint:** Without loss of generality, we may simply assume that the sphere is centered at the origin.)

6. **(15 marks)** In each of the following questions, **evaluate** $\iint_D f(x,y)dA$ for the given function f and region D.

   Show your **upper limits and lower limits** of all integrals, as well as the **procedures of integration** clearly.

   NOTE: Marks may **NOT** be evenly distributed.

   (a) $f(x,y) = Ax + By$, where A and B are constants.

   D: the triangle formed by the intersection of three lines: $y = 3x, y = \frac{2}{5}x$ and $x - 9y + 26 = 0$.

   (b) $f(x,y) = x\sqrt{y}$

   D: the region above the straight line $x + y = 1$ and inside the unit circle centered at origin.

7. **(15 marks) Re-write the order of integration and Sketching**

   Sketch the region of integration based on the following given integrals. **Label necessary points on your sketch**.

   Then, write down the triple integral with desired order of integration as indicated.

   Show **how you obtain the new upper limits and lower limits** of each integral clearly.

   (**Note:** You cannot evaluate the exact value of these two integrals, since $f(x,y,z)$ is **NOT** given.)

   (a) $\int_0^1 \int_{\sqrt{x}}^1 \int_0^{1-y} f(x,y,z)\, dz\, dy\, dx$

   Desired order of integration : $dy\, dx\, dz$

   (b) $\int_0^2 \int_0^{4-y^2} \int_0^{2-y} f(x,y,z)\, dz\, dx\, dy$

   Desired order of integration : $dy\, dx\, dz$

8. **(15 marks) Mass and Centre of Mass of solid**

   (a) Find the mass of a solid U with density $\delta(x,y,z) = 2020$, where U is bounded by 3 given surfaces:

   $x^2 + y^2 = 2020$, $z = x^2 + y^2$ and the xy-plane.

   (b) We define the first moments with respect to the xy-plane, yz-plane and xz-plane as follows:

   $$M_{z=0} = \iiint_U z\delta(x,y,z)dV \quad ; \quad M_{x=0} = \iiint_U x\delta(x,y,z)dV \quad \text{and} \quad M_{y=0} = \iiint_U y\delta(x,y,z)dV$$

   Then, the coordinates of the center of mass of the solid are given by $(\bar{x}, \bar{y}, \bar{z}) = \left(\frac{M_{x=0}}{M}, \frac{M_{y=0}}{M}, \frac{M_{z=0}}{M}\right)$, where M is the mass of the same solid. Find the centre of mass of the same solid U described in (a).

END OF ASSIGNMENT 5