# ENGG1410-E: Short Test 2

**Name:** **Student ID:**

Write all your answers on this sheet, and use the back if necessary.

**Problem 1 (30%).** Let $C$ be the curve $\boldsymbol{r}(t) = [5t, t^2, 3]$ from $t = 0$ to $t = 1$. Calculate $\int_C t\,ds$.

**Answer:**

$$
\begin{aligned}
\int_C t\,ds &= \int_0^1 t\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}\,dt \\
&= \int_0^1 \sqrt{5^2 + (2t)^2 + 0^2}\,dt \\
&= \int_0^1 t\sqrt{4t^2 + 25}\,dt. \\
&= \frac{1}{8}\int_0^1 \sqrt{4t^2 + 25}\,d(4t^2 + 25). \\
&= \frac{1}{12}(4t^2 + 25)^{3/2}\Big|_0^1. \\
&= \frac{1}{12}(29^{3/2} - 25^{3/2}).
\end{aligned}
$$

**Problem 2 (30%).** Let $f(x, y, z) = e^x y + 5z$. Compute the directional derivative of $f(x, y, z)$ in the direction of $[1, 1, 1]$ at point $(1, 2, 3)$.

**Answer:** The unit vector in the direction of $[1, 1, 1]$ is $\boldsymbol{u} = [1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3}]$. Thus, the directional derivative equals

$$
\begin{aligned}
\nabla f(x, y, z) \cdot \boldsymbol{u} &= \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right] \cdot \boldsymbol{u} \\
\nabla f(x, y, z) \cdot \boldsymbol{u} &= [e^x y, e^x, 5] \cdot \boldsymbol{u} \\
&= [2e, e, 5] \cdot [1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3}] \\
&= (3e + 5)/\sqrt{3}.
\end{aligned}
$$

**Problem 3 (40%).** Consider the curve $C$ that is the intersection of the following two faces:

$$
\begin{aligned}
x^2 + y + z^2 &= 5 \\
x + y &= 1.
\end{aligned}
$$

Give a tangent vector of $C$ at point $(0, 1, 2)$.

**Answer:** From the two equations, we have

$$
\begin{aligned}
x^2 - x + z^2 &= 4 \\
\Rightarrow x^2 - x + 1/4 + z^2 &= 17/4 \\
\Rightarrow \left(\frac{x - 1/2}{\sqrt{17}/2}\right)^2 + \left(\frac{z}{\sqrt{17}/2}\right)^2 &= 1
\end{aligned}
$$

Hence we can write $C$ in the parametric form $[x(t), y(t), z(t)]$ where

$$\begin{aligned} x(t) &= \frac{1}{2} + \frac{\sqrt{17}}{2}\cos t \\ y(t) &= 1 - x = \frac{1}{2} - \frac{\sqrt{17}}{2}\cos t \\ z(t) &= \frac{\sqrt{17}}{2}\sin t \end{aligned}$$

Therefore, a tangent vector is $[x'(t), y'(t), z'(t)] = [-\frac{\sqrt{17}}{2}\sin t, \frac{\sqrt{17}}{2}\sin t, \frac{\sqrt{17}}{2}\cos t]$. The point $(0, 1, 2)$ is given by $t$ satisfying $(\sqrt{17}/2)\cos t = -1/2$ and $(\sqrt{17}/2)\sin t = 2$. Therefore, a tangent vector at this point is $[-2, 2, -1/2]$.