# ENGG2430A, Spring 2017, Homework 1

## Due at 5pm, Feb 15.

You can put your answer sheets to the Box 10B on the 10th floor of Ho Sin-hang Engineering Building (HSH).

1. (10 points) Prove the set identity $(A \cap B)^c = (A^c \cap B) \cup (A^c \cap B^c) \cup (A \cap B^c)$.

2. (10 points) A six-sided die is loaded in a way that each even face is twice as likely as each odd face. All even faces are equally likely, as are all odd faces. Construct a probabilistic model for a single roll of this die and find the probability that the outcome is less than 4.

3. (10 points) A batch of one hundred items is inspected by testing four randomly selected items. If one of the four is defective, the batch is rejected. What is the probability that the batch is accepted if it contains five defectives?

4. (10 points) Alice and Bob have $2n + 1$ coins, each coin with probability of heads equal to $1/2$. Bob tosses $n + 1$ coins, while Alice tosses the remaining $n$ coins. Assuming independent coin tosses, show that the probability that after all coins have been tossed, Bob will have gotten more heads than Alice is $1/2$.

5. (10 points) A power utility can supply electricity to a city from $n$ different power plants. Power plant $i$ fails with probability $p_i$, independent of the others. Suppose that two power plants are sufficient and necessary to keep the city from a black-out. Find the probability that the city will experience a black-out.