Exercises: Vector Basics

Problem 1. For each of the following directed segments, give the vector of which the directed segment is an instantiation:

1. $\overrightarrow{(1,2),(2,3)}$
2. $\overrightarrow{(10,20),(11,21)}$
3. $\overrightarrow{(1,-2),(2,3)}$
4. $\overrightarrow{(1,-2,0),(2,3,10)}$

Problem 2. Give the default instantiations and the norms of the following vectors:

1. $[1,2]$
2. $[1,2,3]$
3. $[1,-2,3]$

Problem 3. Give the results of $\boldsymbol{a}+\boldsymbol{b}$ and $\boldsymbol{a}-\boldsymbol{b}$ for each of the following:

1. $\boldsymbol{a}=[1,2], \boldsymbol{b}=[2,5]$
2. $\boldsymbol{a}=[1,2,3], \boldsymbol{b}=[2,5,-7]$
3. $\boldsymbol{a}=10\boldsymbol{i}-209\boldsymbol{j}+32\boldsymbol{k}, \boldsymbol{b}=[2,5,-7]$

Problem 4. Give the results of $c\boldsymbol{a}$ for each of the following:

1. $\boldsymbol{a}=[1,2], c=5$
2. $\boldsymbol{a}=[1,2,3], c=-5$
3. $\boldsymbol{a}=10\boldsymbol{i}-209\boldsymbol{j}+32\boldsymbol{k}, c=10$

Problem 5. Indicate whether $\boldsymbol{a}$ and $\boldsymbol{b}$ have the same directions in each of the following cases:

1. $\boldsymbol{a}=[1,1], \boldsymbol{b}=[2,2]$
2. $\boldsymbol{a}=[1,2,3], \boldsymbol{b}=[20,40,60]$
3. $\boldsymbol{a}=[1,2,3], \boldsymbol{b}=[2,-4,6]$

Problem 6. Let $\boldsymbol{a}$ and $\boldsymbol{b}$ be 2d vectors such that $\boldsymbol{a}+\boldsymbol{b}=[3,5]$, and $\boldsymbol{a}-\boldsymbol{b}=[4,6]$. What are $\boldsymbol{a}$ and $\boldsymbol{b}$?

Problem 7. Let $\boldsymbol{a}$ be a vector and c a scalar. Prove: $|c\boldsymbol{a}|=|c||\boldsymbol{a}|$.

Problem 8. Let A, B, C, D be 4 points in $\mathbb{R}^d$. Suppose that $\overrightarrow{A,B}$, $\overrightarrow{B,C}$, and $\overrightarrow{C,D}$ are instantiations of $\boldsymbol{a}$, $\boldsymbol{b}$, and $\boldsymbol{c}$, respectively; see Figure 1. Prove that $\overrightarrow{A,D}$ is an instantiation of $\boldsymbol{a}+\boldsymbol{b}+\boldsymbol{c}$.

Problem 9. Let A, B, C, D be 4 points in $\mathbb{R}^d$. Suppose that $\overrightarrow{A,B}$, $\overrightarrow{C,B}$, and $\overrightarrow{C,D}$ are instantiations of $\boldsymbol{a}$, $\boldsymbol{b}$, and $\boldsymbol{c}$, respectively; see Figure 2. Give the vector of which $\overrightarrow{A,D}$ is an instantiation.

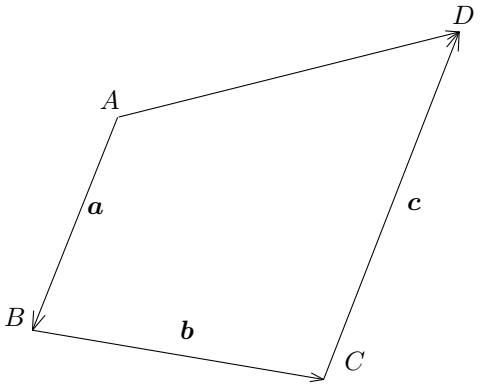

Figure 1: Problem 8

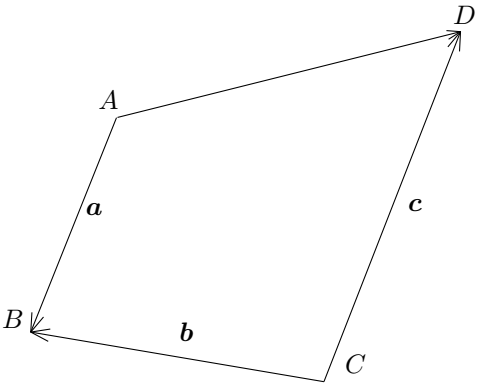

Figure 2: Problem 9