

Exercises: Matrix Rank

Problem 1. Calculate the rank of the following matrix:

$$\begin{bmatrix} 0 & 16 & 8 & 4 \\ 2 & 4 & 8 & 16 \\ 16 & 8 & 4 & 2 \\ 4 & 8 & 16 & 2 \end{bmatrix}$$

Problem 2. Calculate the rank of the following matrix:

$$\begin{bmatrix} 4 & -6 & 0 \\ -6 & 0 & 1 \\ 0 & 9 & -1 \\ 0 & 1 & 4 \end{bmatrix}$$

Problem 3. Judge whether the following vectors are linearly independent.

$$[3, 0, 1, 2]$$

$$[6, 1, 0, 0]$$

$$[12, 1, 2, 4]$$

$$[6, 0, 2, 4]$$

$$[9, 0, 1, 2]$$

If they are not, find the largest number of linearly independent vectors among them.

Problem 4. Prove: if \mathbf{A} is not square, then either the row vectors or the column vectors are linearly dependent.

Problem 5. Let S be an arbitrary set of vectors in \mathbb{R}^3 . Prove that there are at most 3 linearly independent vectors in S .

Problem 6 (Hard). Prove: $\text{rank}(\mathbf{AB}) \leq \text{rank} \mathbf{A}$.

Problem 7 (Very Hard). Prove: $\text{rank}(\mathbf{A} + \mathbf{B}) \leq \text{rank} \mathbf{A} + \text{rank} \mathbf{B}$.