Exercises: Planar-Region Projection, Surface Areas, and Surface Integral by Area

Problem 1. Let g be a region (bounded by a continuous curve) in the plane $x + y + z = 1$. Let g_{xy} be the projection of g onto the xy-plane. If we know that the area of g is 1, what is the area of g_{xy}.

Problem 2. Consider the surface $S : z = x^2 + y^2$ with $0 \le z \le 1$. Compute the area of S.

Problem 3. Consider the surface S in a parametric form $\boldsymbol{r}(u, v) = [x(u, v), y(u, v), z(u, v)]$ where

$$\begin{aligned} x(u,v) &= u+v \\ y(u,v) &= u-v \\ z(u,v) &= uv \end{aligned}$$

with (u, v) in the disc $u^2 + v^2 \le 1$. Compute the area of S.

Problem 4. Let S be the surface $x + y + z = 1$ with $x \in [0, 1]$, $y \in [0, 1]$, and $z \in [0, 1]$. Compute $\iint_S x\, dA$.

Problem 5. Let S be the surface $\boldsymbol{r}(u, v) = [x(u, v), y(u, v), z(u, v)]$ where $x(u, v) = u, y(u, v) = v, z(u, v) = u^3$ with $u \in [0, 1]$ and $v \in [-2, 2]$. Compute $\iint_S (1 + 9xz)^{1/2}\, dA$.

Problem 6. Define $\boldsymbol{f}(x, y, z) = [-x^2, y^2, 0]$. Let S be the surface $\boldsymbol{r}(u, v) = [x(u, v), y(u, v), z(u, v)]$ where $x(u, v) = u, y(u, v) = v, z(u, v) = 3u - 2v$ with $0 \le u \le 1$ and $0 \le v \le 1$. Calculate $\iint_S \boldsymbol{f} \cdot \boldsymbol{n}\, dA$.