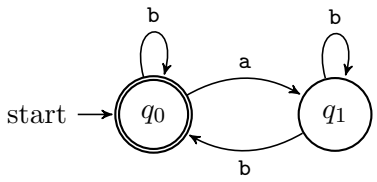
Problem 1 (30 points)

Consider the following NFA M:



(a) (10 points) Convert M to a DFA.

(b) (10 points) Run the DFA minimization algorithm on the resulting DFA. Prove that every pair of states is distinguishable.

(c) (10 points) Write a regular expression for the language of M.

Problem 2 (20 points)

Let L be the language $\{\mathtt{a}^n\mathtt{b}^n \mid n \geqslant 0\}$ over the alphabet $\Sigma = \{\mathtt{a}, \mathtt{b}\}$.

(a) (10 points) Write a context-free grammar for L. Your CFG should be unambiguous and in Chomsky Normal Form.

(b) (10 points) Using the CFG from part (a), apply the Cocke–Younger–Kasami algorithm on input `aabb`. Draw the parse tree derived by the algorithm.

Problem 3 (30 points)

Consider the following context-free grammar G:

$$S \to SS \mid \mathtt{a}\mathtt{a}S\mathtt{b}\mathtt{b} \mid \varepsilon$$

(a) (10 points) For each of these strings, say if it is in the language of G. Justify your answer.

`aaaabbbbaabb`:

`aaab`:

`abaaaabbbb`:

(b) (10 points) Show that G is ambiguous.

(c) (10 points) Draw a pushdown automaton for the language of G. Specify all the states, transitions, and start/final states.

Problem 4 (20 points)

For each of the following languages, say whether it is (i) regular, (ii) context-free but not regular, or (iii) not context-free. Justify your answer by describing a DFA, NFA, regular expression, PDA, CFG, and/or giving a proof via the pumping lemma or pairwise distinguishable strings.

(a) (10 points) $L_1 = \{\mathtt{a}^i\mathtt{b}^j\mathtt{c}^k \mid j = i + k \text{ and } i, j, k \geqslant 0\}$

Please circle: **regular** **context-free but not regular** **not context-free**

(b) (10 points) $L_2 = \{w \in \{\mathtt{a}, \mathtt{b}\}^* \mid \text{every block of } \mathtt{a}\text{'s in } w \text{ contains exactly three } \mathtt{a}\text{'s}\}$

Please circle: **regular** **context-free but not regular** **not context-free**