Practice questions

1. The PDF of the random variable X is $f(x) = Ce^{-|x|}$, where x ranges over all real numbers. Determine (a) the value of C, (b) the CDF $\mathrm{P}(X \le x)$, and (c) the probability that $|X| \le 1$.

2. The joint PDF of (X, Y) is

$$f_{X,Y}(x,y) = \begin{cases} C(x - 2y + 1)y, & \text{if } 0 \le x \le 2, 0 \le y \le 2, \\ 0, & \text{otherwise.} \end{cases}$$

   Find (a) the value of C and (b) The conditional PDF $f_{Y|X}(y|x)$.

3. Let X be a Uniform$(0, 1)$ random variable. Find the PDF of the random variables (a) $Y = e^X$ and (b) $Z = -2 \log X$.

4. Raindrops hit your head at a rate of 1 per second. What is the PDF of the time at which the second raindrop hits you? How about the third one? (**Hint:** convolution)

5. You draw 10 balls at random among 15 red and 5 blue balls. Let X be the number of red balls drawn.

   (a) What is the expected value of X?

   (b) Write $X = X_1 + X_2 + \cdots + X_{10}$, where X_i indicates if the i-th drawn ball is red. What is the variance of X_i?

   (c) What is the covariance of X_i and X_j $(i \ne j)$?

   (d) What is the variance of X?