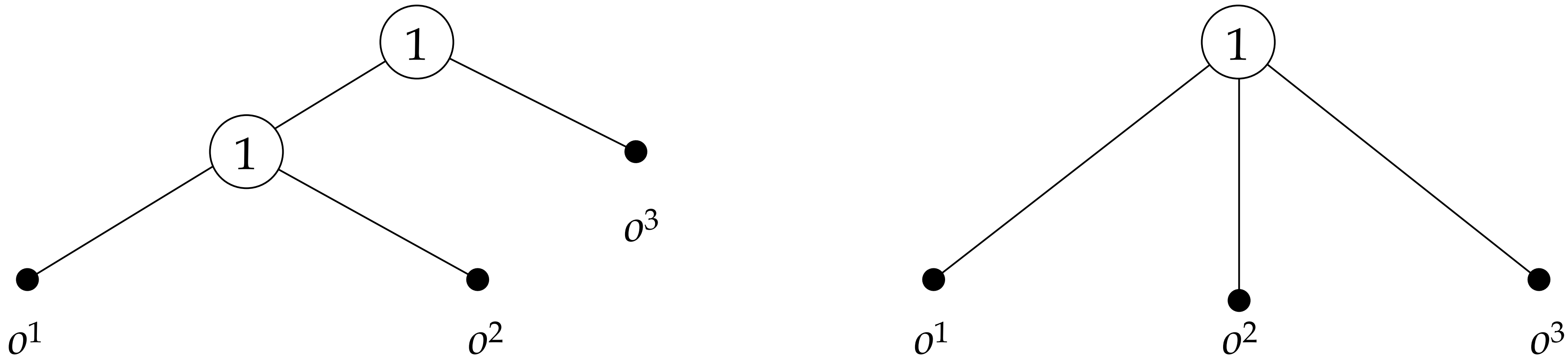
Equivalence of Extensive Games



Intuitively, these two games represent the same situation.

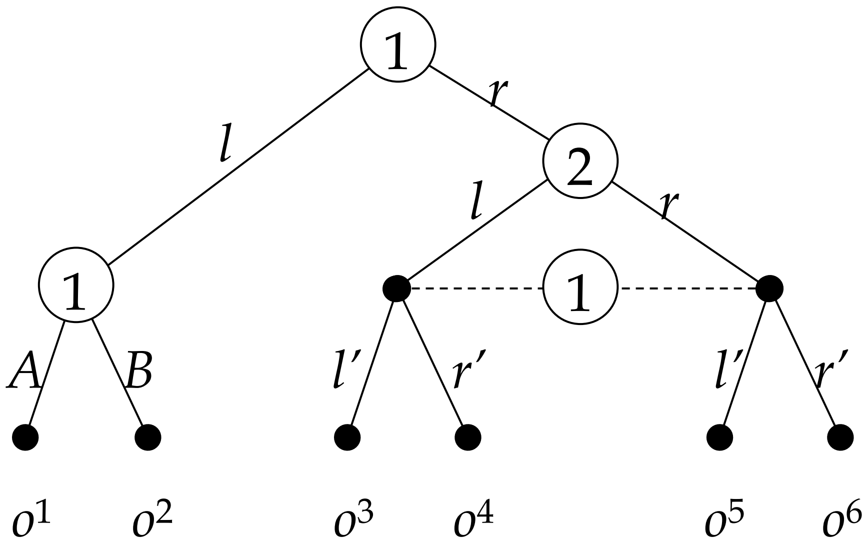
Formally, these two games are different.

Principles for the Equivalence of Extensive Games

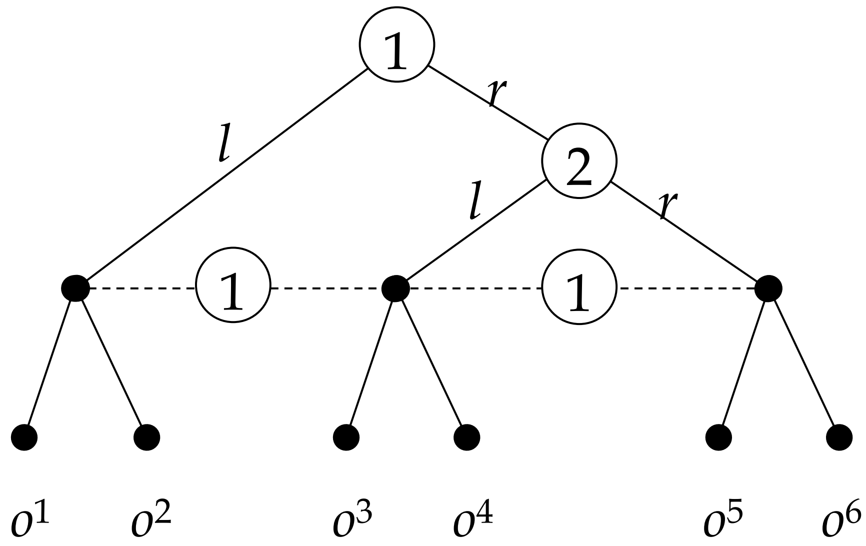
We shall discuss four principles for the equivalence of extensive games.

- *Principle of Inflation-Deflation*
- *Principle of Addition of a Superfluous Move*
- *Principle of Coalescing of Moves*
- *Principle of Interchange of Moves*

Principle of Inflation-Deflation

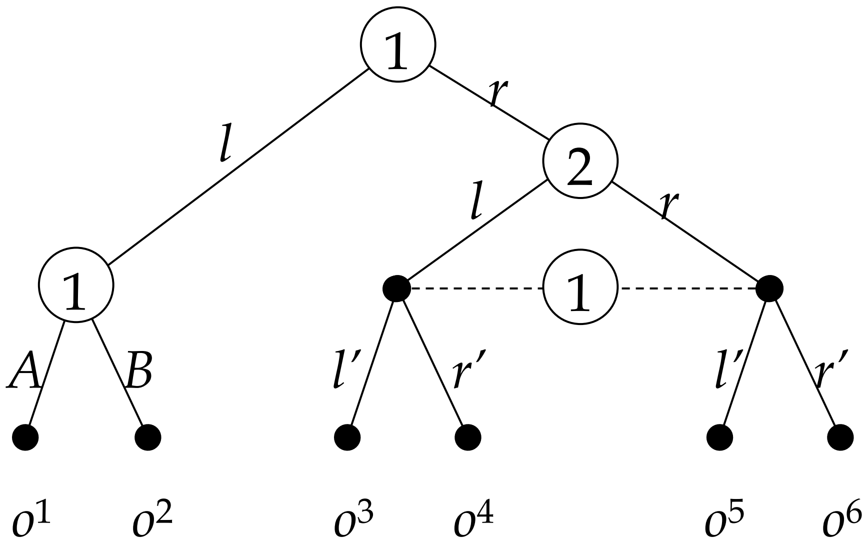


Perfect/ Imperfect recall?

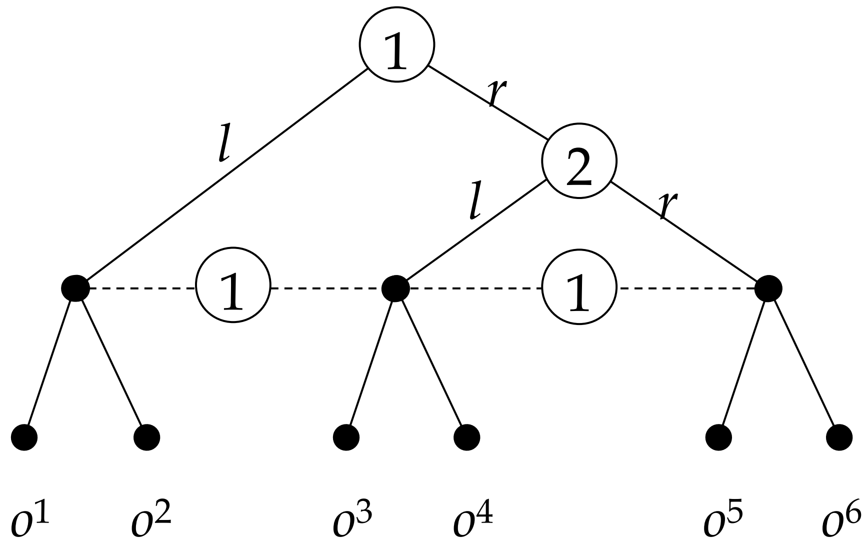


Perfect/ Imperfect recall?

Principle of Inflation-Deflation

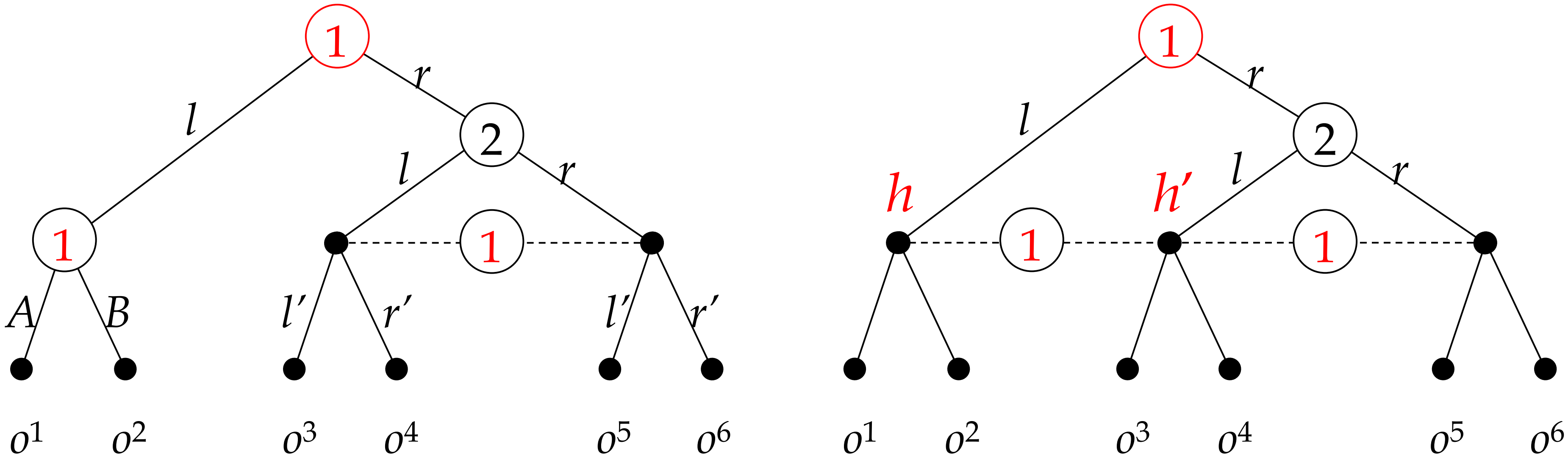


Perfect recall.



Formally, imperfect recall.

- Player 1 in the second information set forgets he played *l* or *r*.
- Can he not infer?
- Not <u>really</u> 'imperfect recall…'



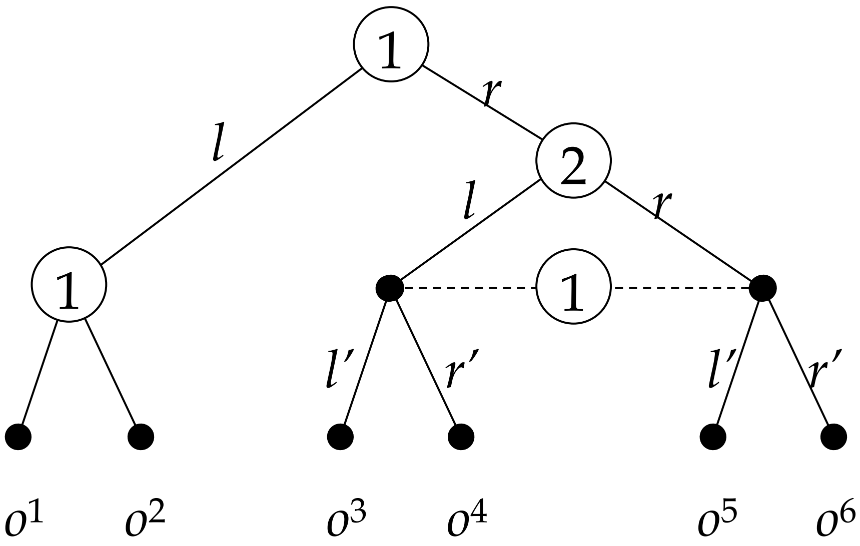
Principle of Inflation-Deflation

The extensive game Γ_2 is equivalent to the extensive game Γ_1 because Γ_1 differs from Γ_2 only in that in the information set $\{l, (r,l), (r,r)\}$ of player 1 in Γ_2, which is a union of information sets $\{l\}$ and $\{(r,l), (r,r)\}$ in Γ_1, with the following property: two histories h and h' (e.g., l and (r,l)) in different members of the union have the same subhistory $\emptyset$, and the player 1′s action at information set $\{\emptyset\}$ is different in h and h'.

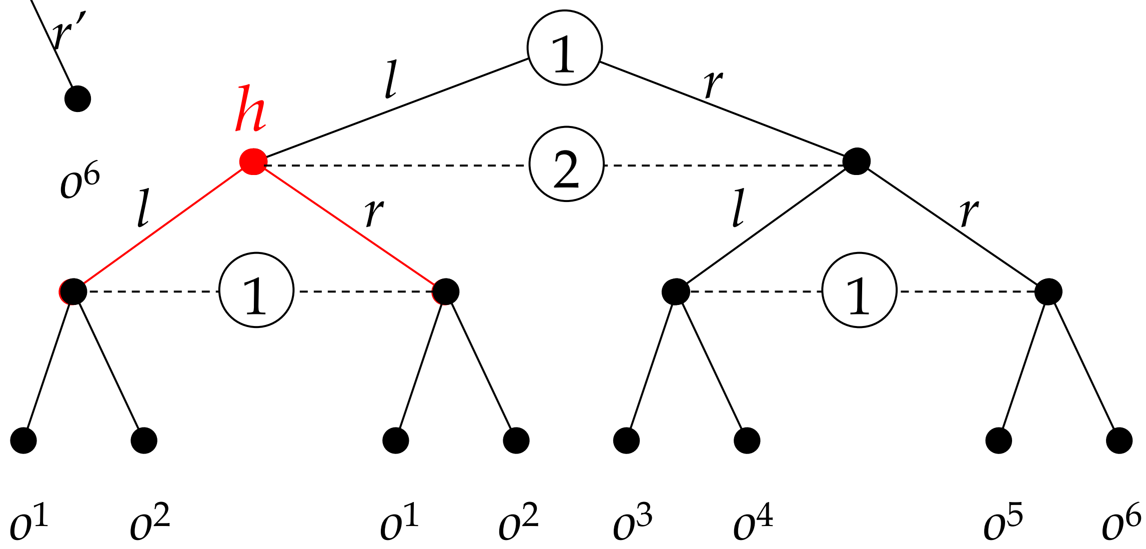
Principle of Inflation-Deflation

Generally, the extensive game Γ is equivalent to the extensive game Γ' if Γ' differs from Γ only in that there is an information set of some player i in Γ that is a union of information sets of player i in Γ' with the following property: any two histories h and h' in different members of the union have subhistories that are in the same information set of player i and player i's action at this information set is different in h and h'.

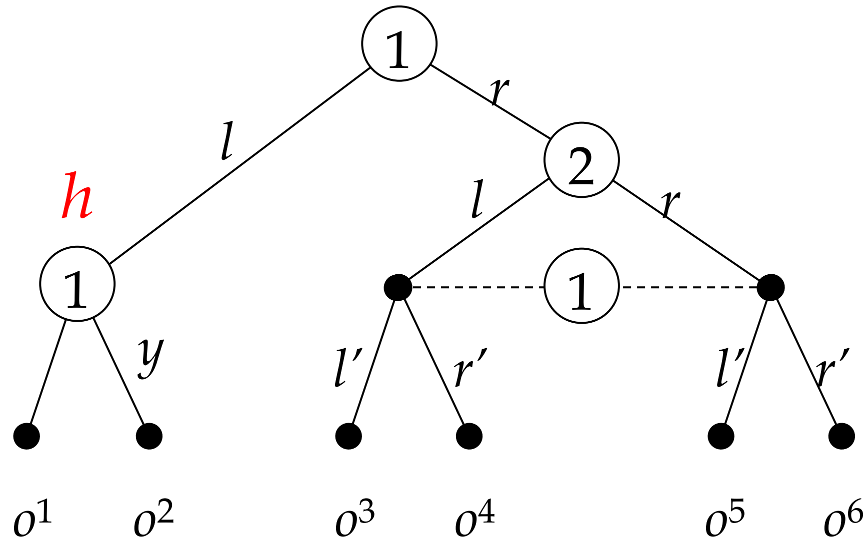
Principle of Addition of a Superfluous Move



At history h, the action of player 2 is irrelevant.



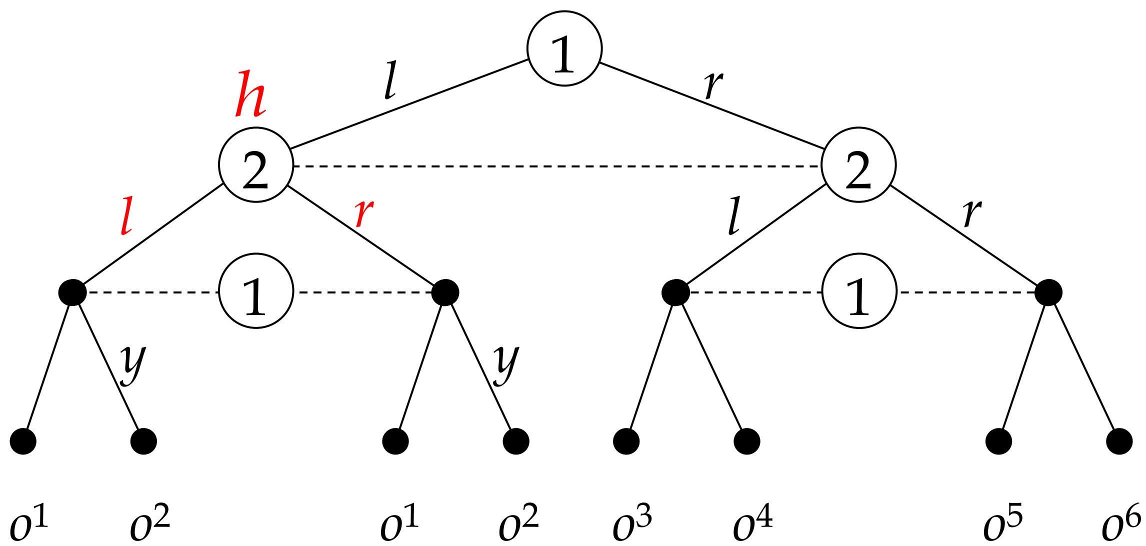
<u>Γ_3 is equivalent to Γ_1.</u>



Superfluous move in Γ_3.

Condition 1 *of* 5.

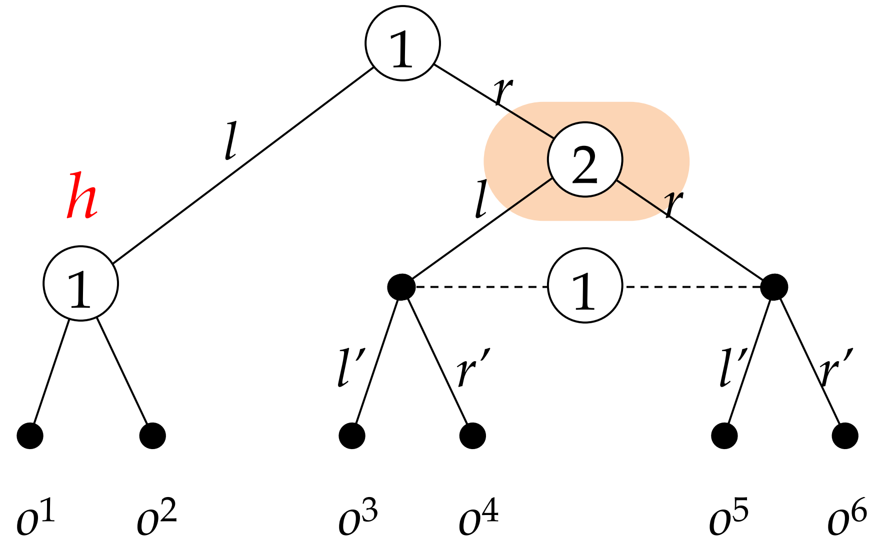
All histories of the form (h, c, h') for $c \in A(h) = \{l, r\}$ in Γ_3 are replaced by the single history (h, h') in Γ_1.

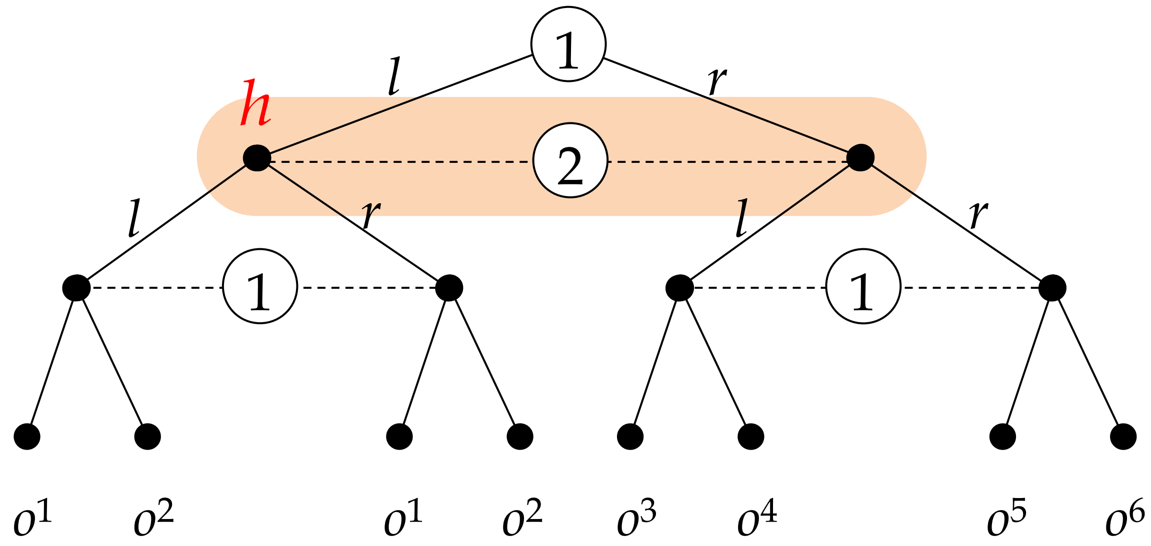


$\underline{\Gamma_3 \text{ is equivalent to } \Gamma_1.}$

Condition 2 *of* 5.

If the information set that contains the history h in Γ_3 is not a singleton then h is excluded from the information set in Γ_1.

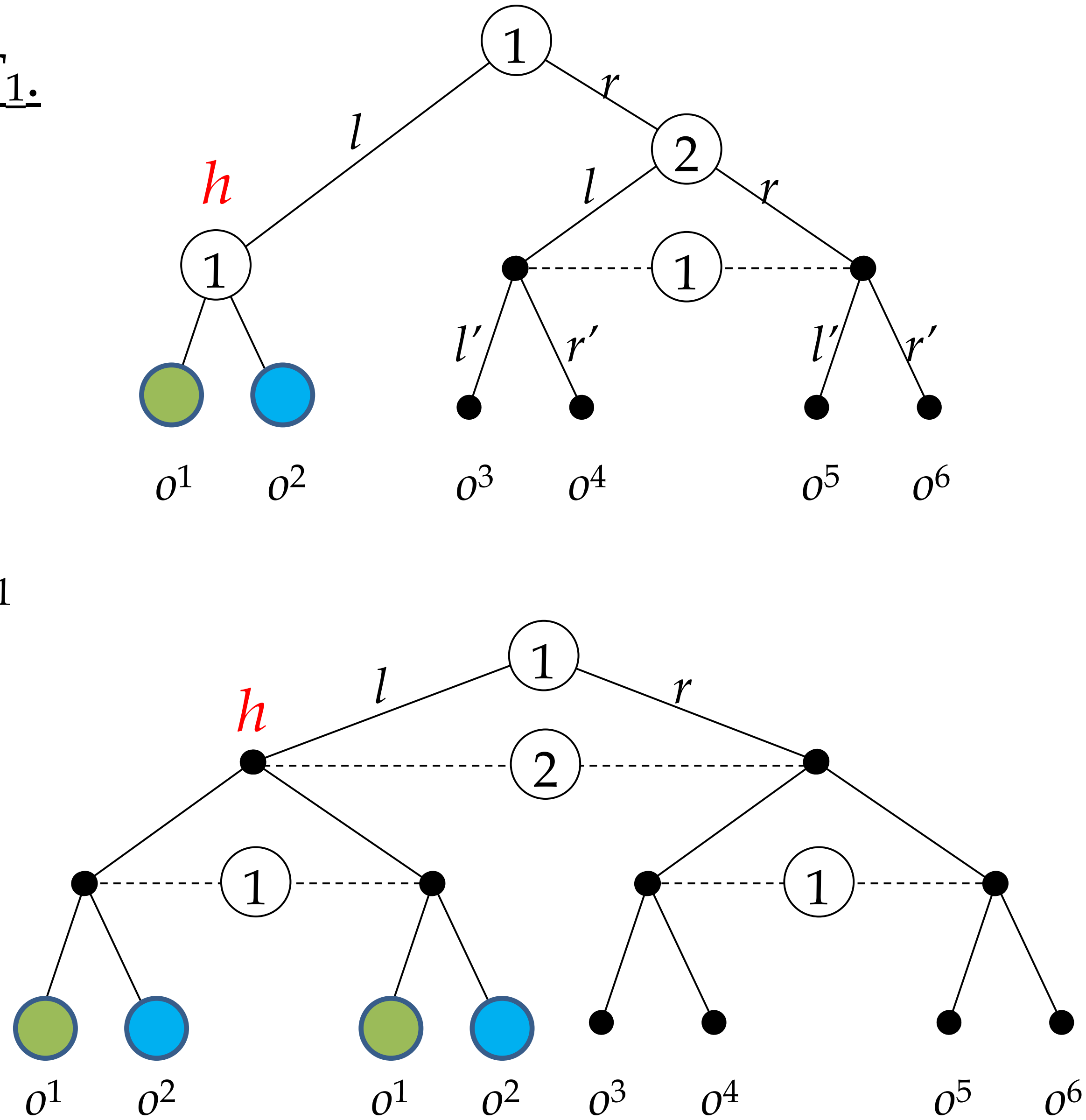




$\underline{\Gamma_3 \text{ is equivalent to } \Gamma_1.}$

Condition 3 *of* 5.

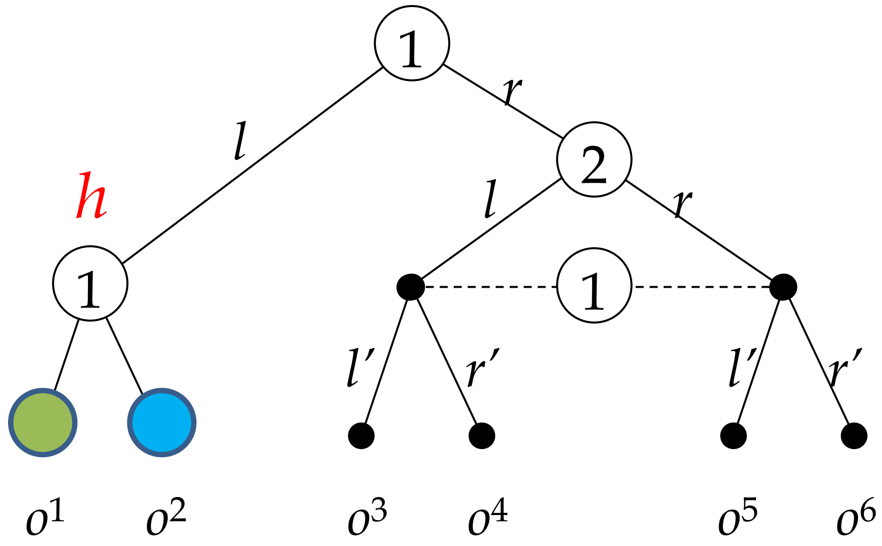
The player who is assigned to the history (h, h') in Γ_1 is the one who is assigned to (h, a, h') in Γ_3.

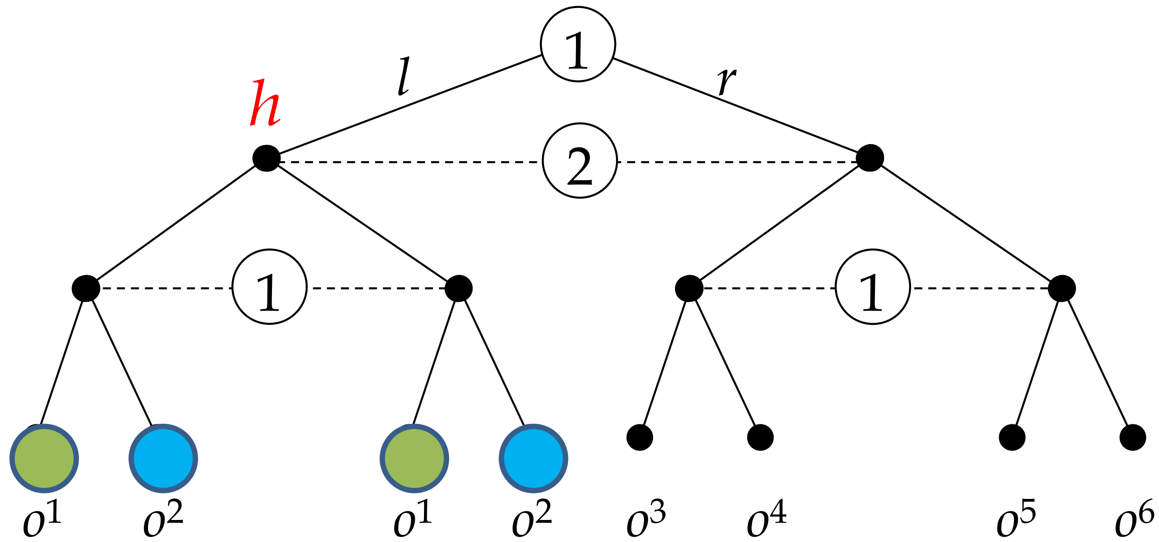


Γ$_3$ is equivalent to Γ$_1$.

Condition 4 *of* 5.

(h, h') and (h, h'') are in the same information set of Γ$_1$ if and only if (h, a, h') and (h, a, h'') are in the same information set of Γ$_3$.

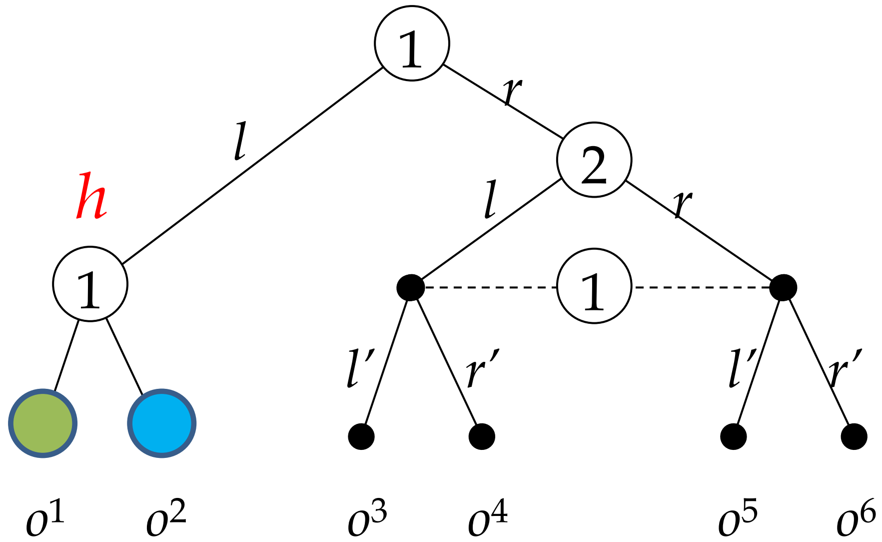


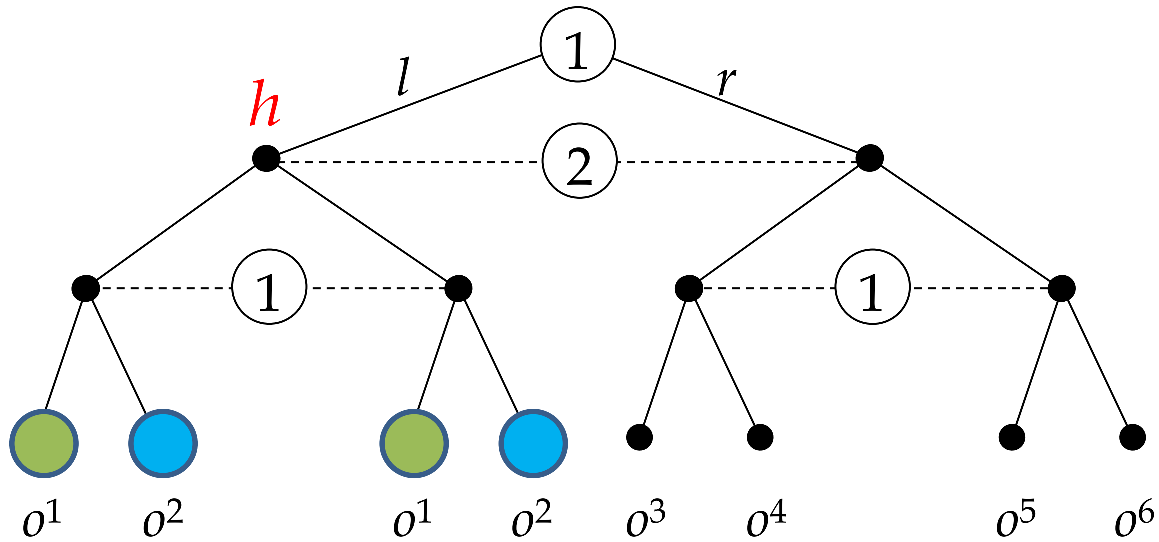


$\underline{\Gamma_3 \text{ is equivalent to } \Gamma_1.}$

Condition 5 *of* 5.

The players' preferences are modified accordingly



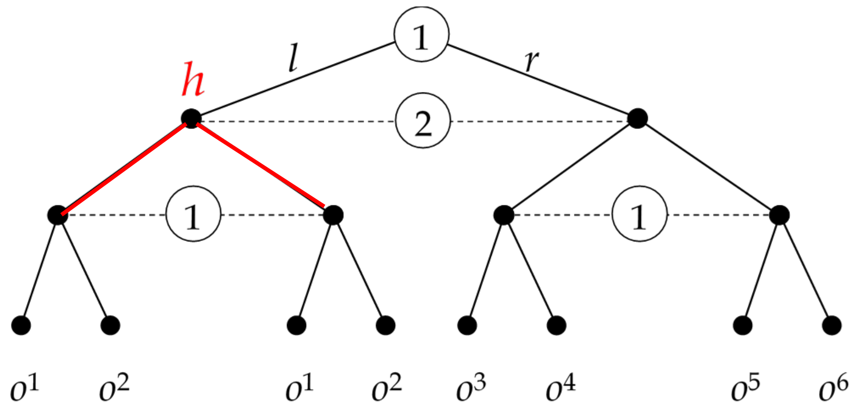


Consider the game Γ_3 with superfluous moves.

Consider $h = l,\ P(h) = 2$ and $a \in A(h) = \{l, r\}$.

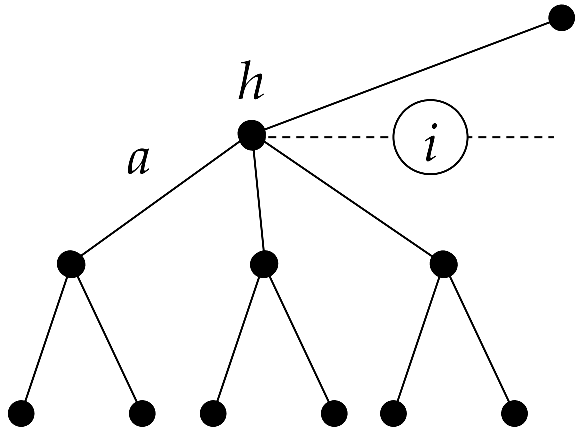
For any h' that follows (h, a) and for any $b \in A(h)$.

- $(h, a, h') \in H$ iff $(h, b, h') \in H$, and $(h, a, h') \in Z$ iff $(h, b, h') \in Z$
- If $(h, a, h') \in Z$ and $(h, b, h') \in Z$, then $(h, a, h') \sim_i (h, b, h')$ for all $i \in N$.
- If both (h, a, h') and (h, b, h') are nonterminal, they are in the same information set.



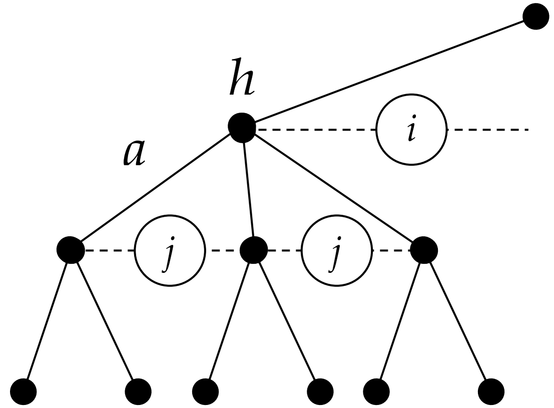
Principle of Addition of a Superfluous Move

Generally, let Γ be an extensive game, let $P(h) = i$, and let $a \in A(h)$.



Suppose that for any sequence h' of actions (including the empty sequence $\emptyset$) that follows the history (h, a) and for any $b \in A(h)$ we have

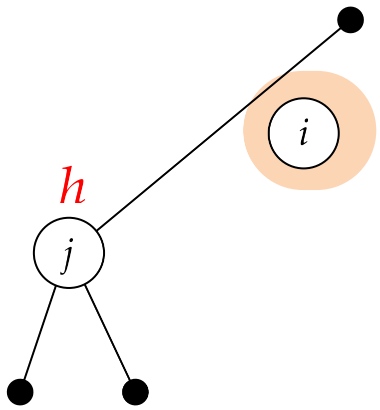
- $(h, a, h') \in H$ iff $(h, b, h') \in H$, and (h, a, h') is terminal iff (h, b, h') is terminal
- if both (h, a, h') and (h, b, h') are terminal, then $(h, a, h') \sim_i (h, b, h')$ for all $i \in N$.
- If both (h, a, h') and (h, b, h') are nonterminal, then they are in the same information set.

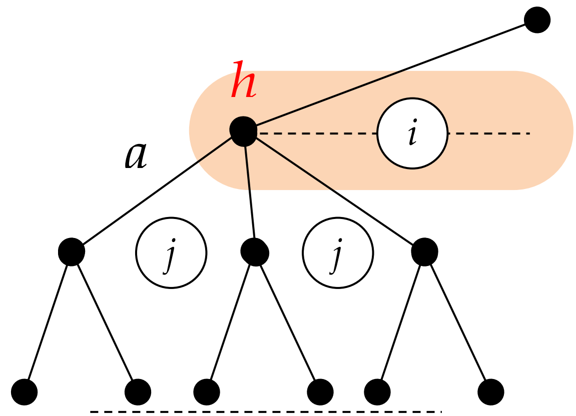


Then Γ is equivalent to the game Γ' that differs from Γ only in that

(i) all histories of the form (h, c, h') for $c \in A(h)$ are replaced by the single history (h, h'),

(ii) if the information set I_i that contains the history h in Γ is not a singleton then h is excluded from I_i in Γ',

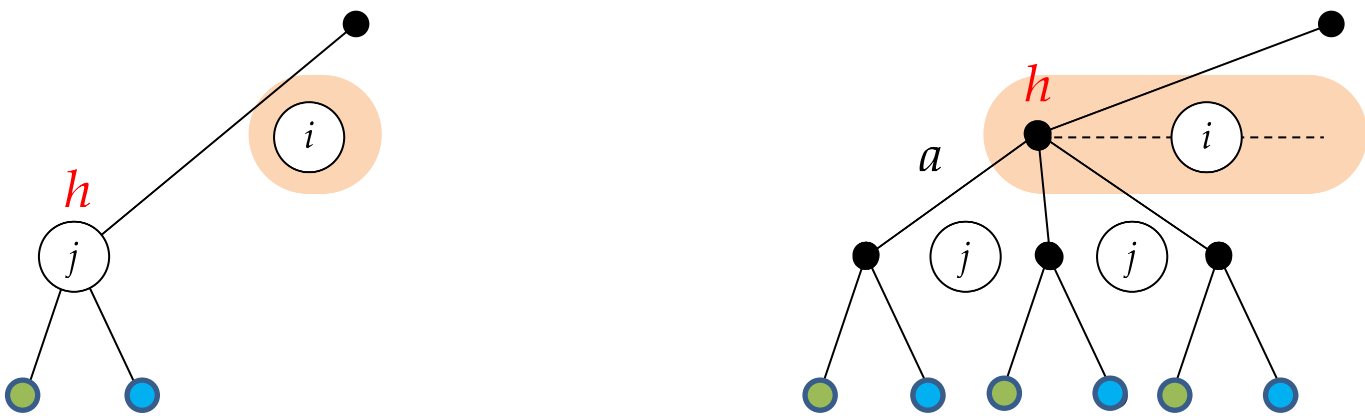




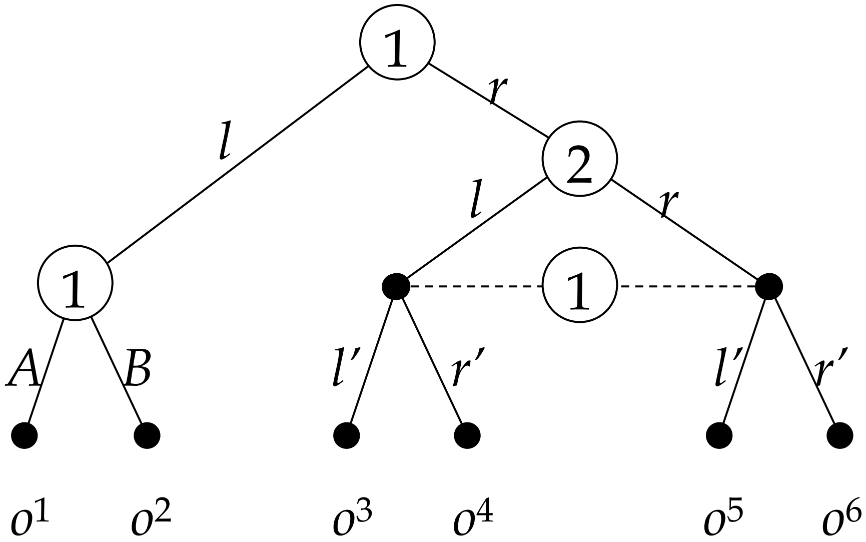
(iii) the player who is assigned to the history (h, h') in Γ' is the one who is assigned to (h, a, h') in Γ,

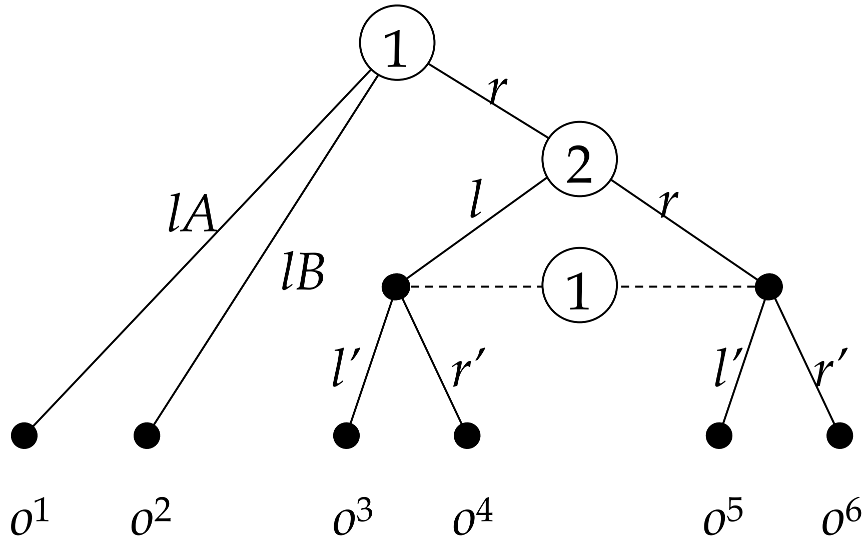
(iv) (h, h') and (h, h'') are in the same information set of Γ' if and only if (h, a, h') and (h, a, h'') are in the same information set of Γ, and

(v) the players' preferences are modified accordingly.



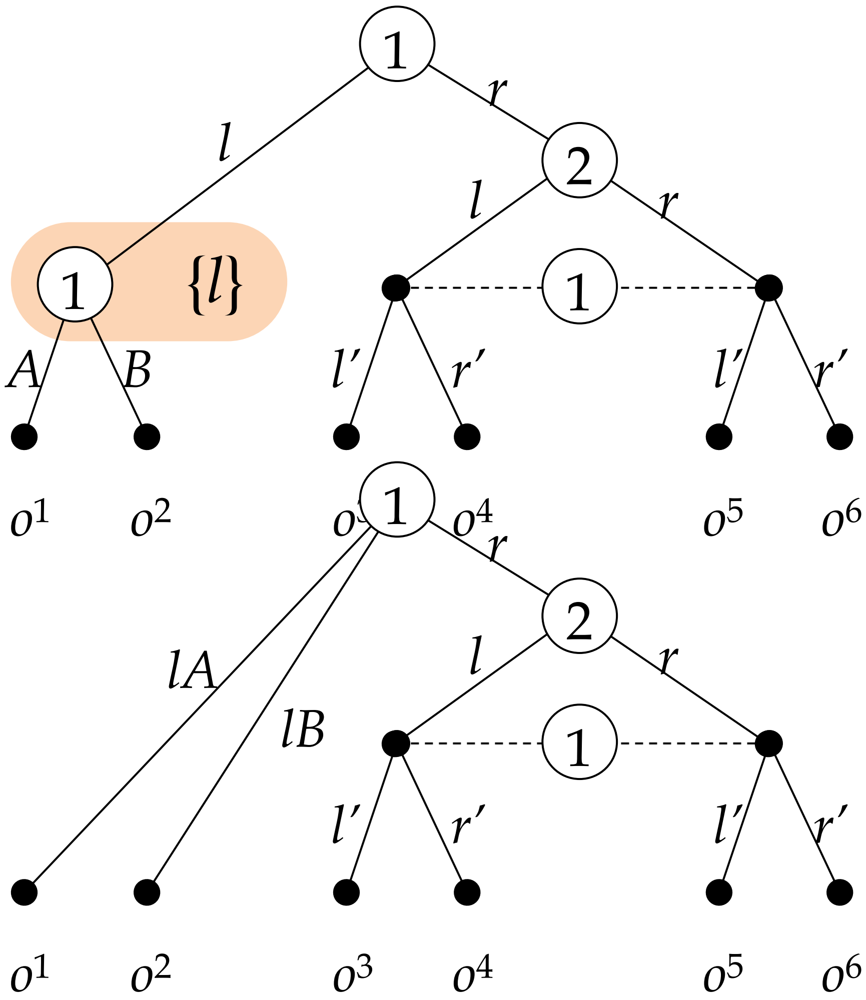
Principle of Coalescing of Moves





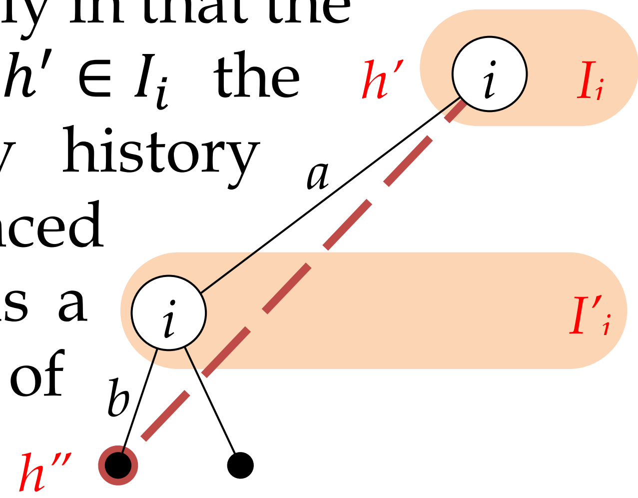
Principle of Coalescing of Moves

Γ_4 differs from Γ_1 only in that the information set $\{l\}$ is deleted, the history (l) is deleted and every history (l, X, h'') is replaced by (lX, h'') where lX is a new action, and the information sets, player function, and players' preferences are changed accordingly.

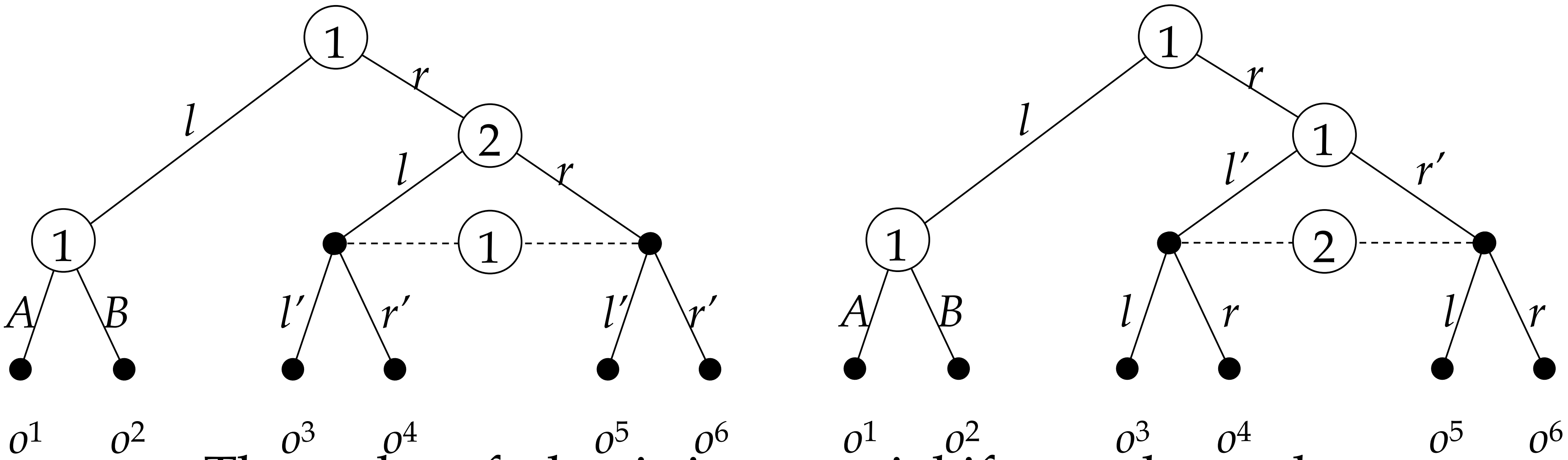


Principle of Coalescing of Moves

Generally, let Γ be an extensive game and let $P(h) = i$, with $h \in I_i$. Let $a \in A(I_i)$ and suppose that $\{(h', a) : h' \in I_i\} = I_i'$ is an information set of player i. Let Γ′ be the game that differs from Γ only in that the information set I_i' is deleted, for all $h' \in I_i$ the history (h', a) is deleted and every history (h', a, b, h'') where $b \in A(h', a)$ is replaced by the history (h', ab, h'') where ab is a new action (that is not a member of $A(h')$), and the information sets, player function, and players' preferences are changed accordingly. Then Γ and Γ′ are equivalent.



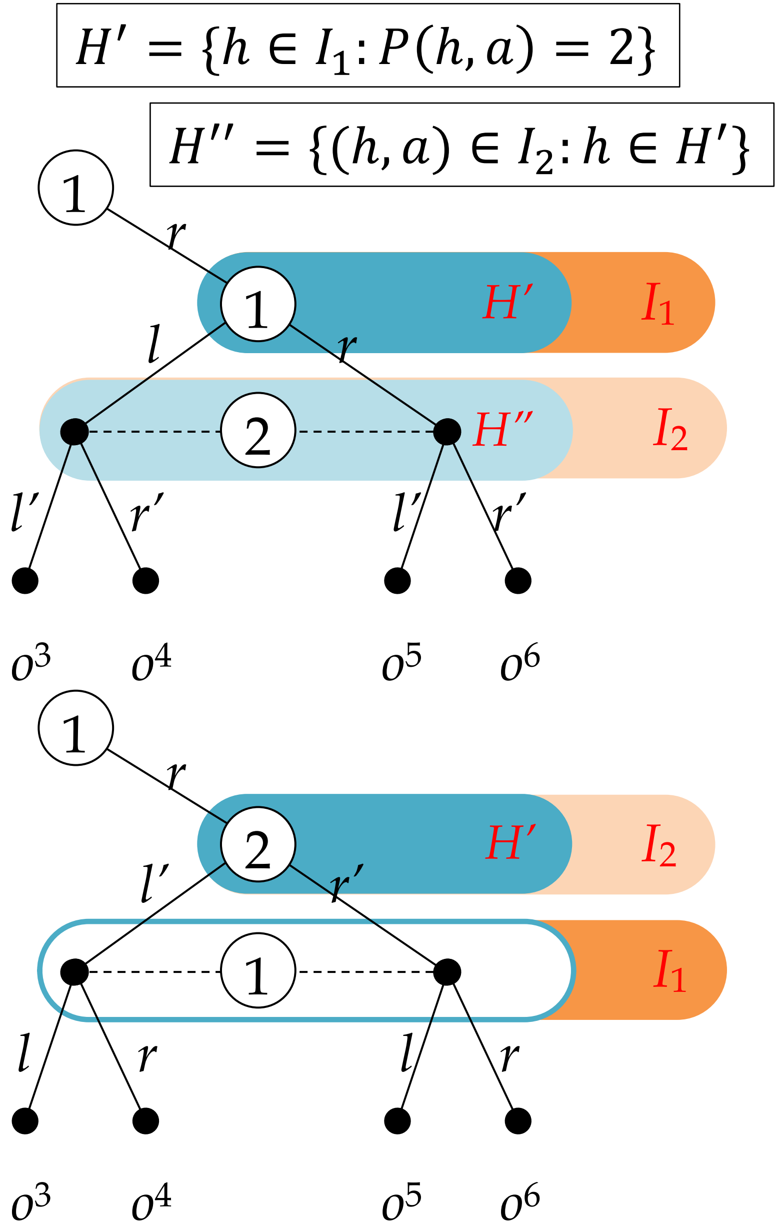
Principle of Interchange of Moves



The order of play is immaterial if one player does not have any information about the other player's action when making his choice.

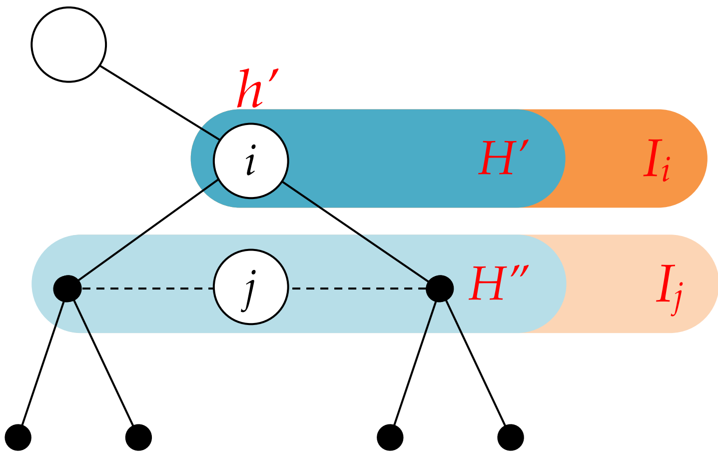
Suppose $(r,a) \in I_2$ for all $a \in A(r)$. Then Γ_1 **is equivalent to** Γ_5 in which

- every history (r,a,b) is replaced by (r,b,a),
- the information set I_1 is replaced by the union of $I_1 \backslash H'$ and all histories of the form (r,b), and
- the information set I_2 is replaced by $(I_2 \backslash H'') \cup H'$.

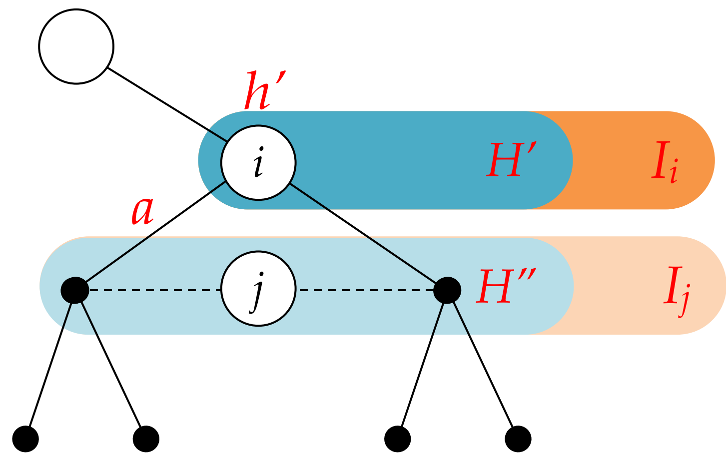


Principle of Interchange of Moves

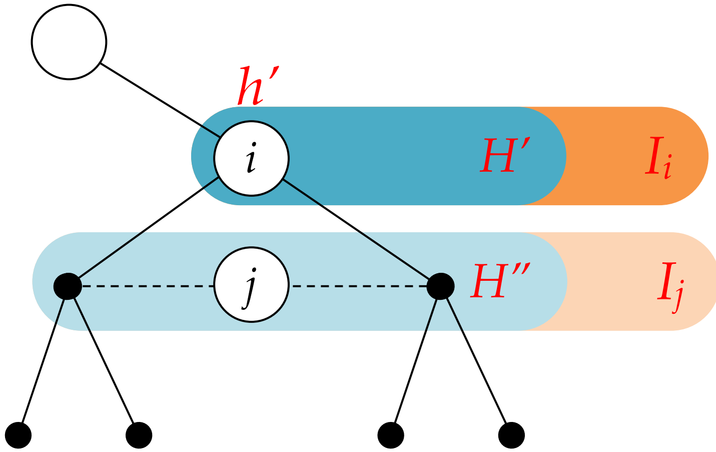
Generally, let Γ be an extensive game and let I_i be an information set of player i. Suppose that for all histories h' in some subset H' of I_i the player who takes an action after i has done so is j, who is not informed of the action that i takes at h'.

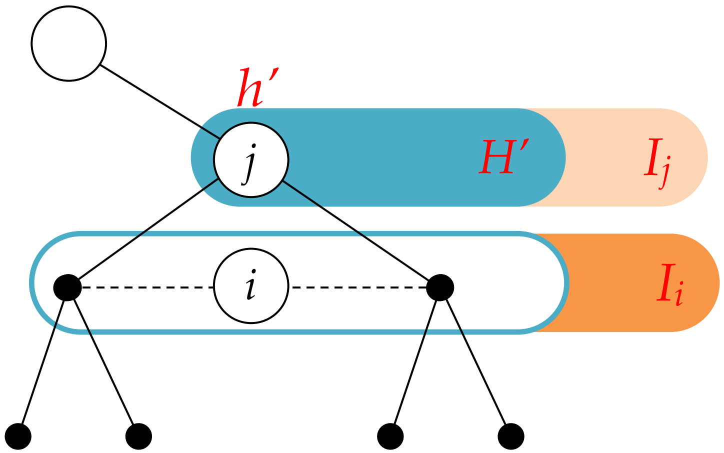


That is, suppose that $(h', a) \in I_j$ for all $h' \in H'$ and all $a \in A(h')$, where I_j is an information set of player j. The information set I_i may contain other histories; let H'' be the subset of I_j consisting of histories of the form (h', a) for some $h' \in H'$.

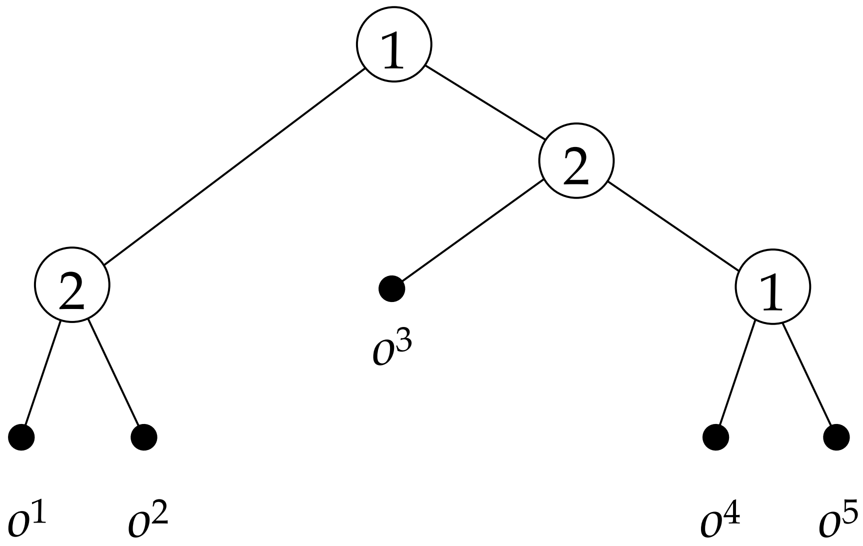


Then Γ is equivalent to the extensive game in which every history of the type (h', a, b) for $h' \in H'$ is replaced by (h', b, a), the information set I_i of player i is replaced by the union of $I_i \backslash H'$ and all histories of the form (h', b) for $h' \in H'$ and $b \in A(h', a)$, and the information set I_j of player j is replaced by $(I_j \backslash H'') \cup H'$.

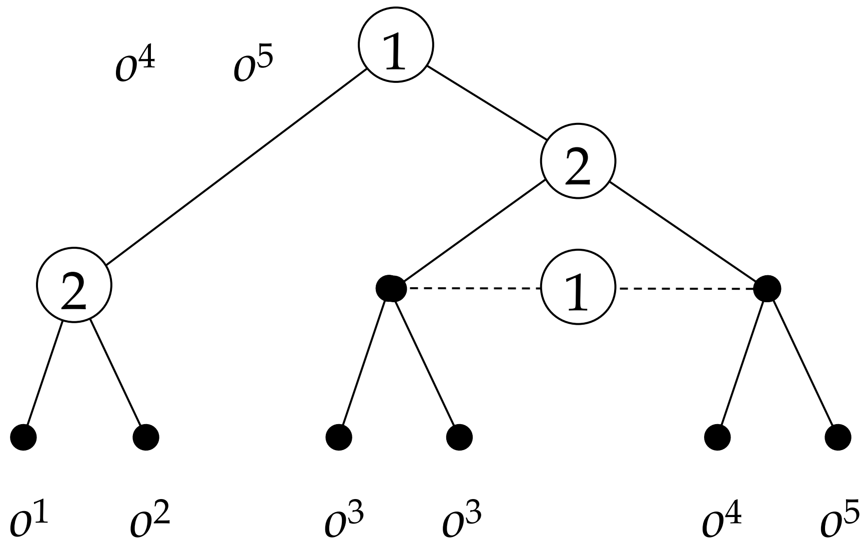




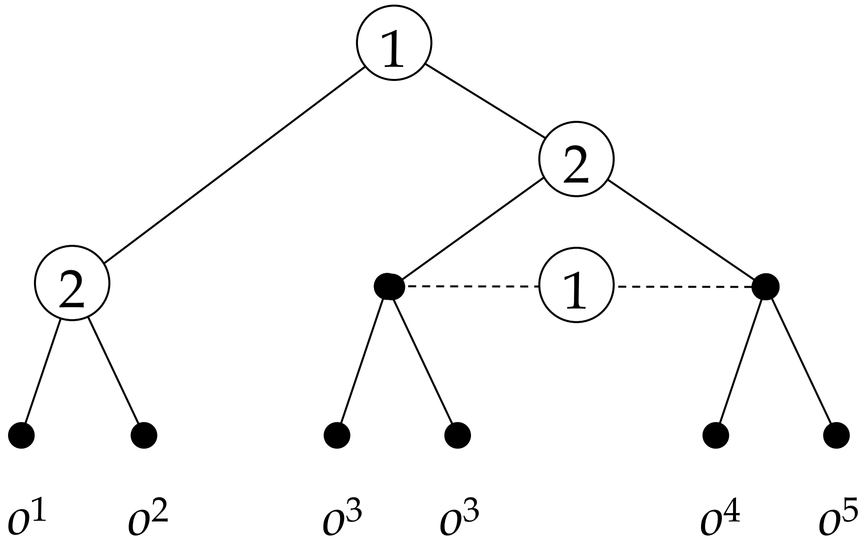
EXAMPLE.

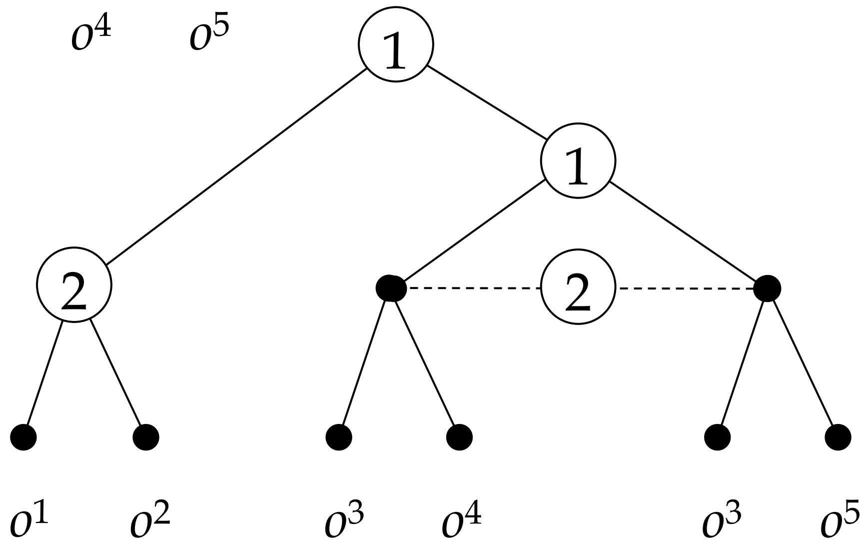


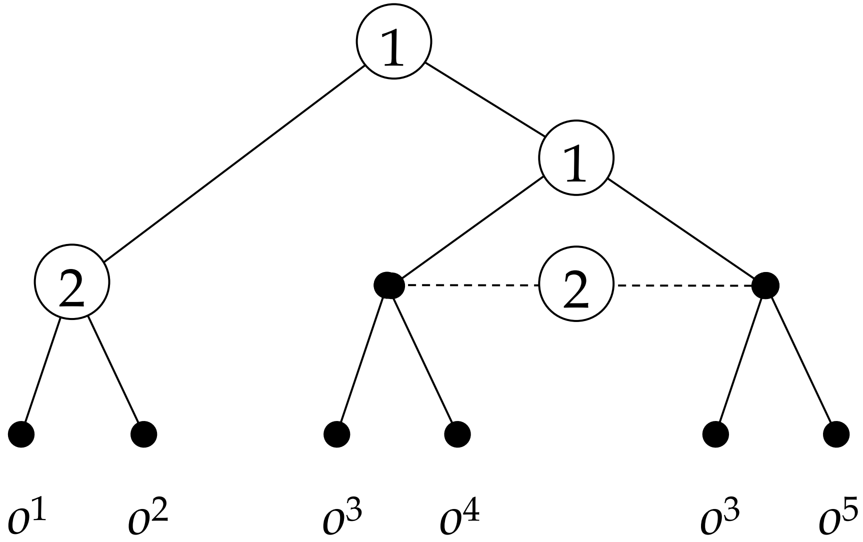
Adding a superfluous move



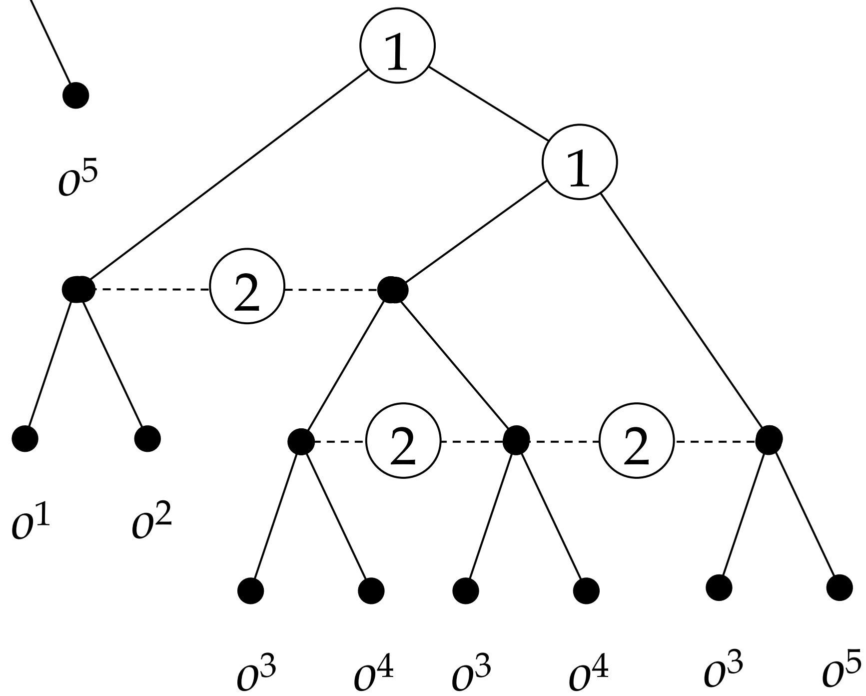
Interchanging moves



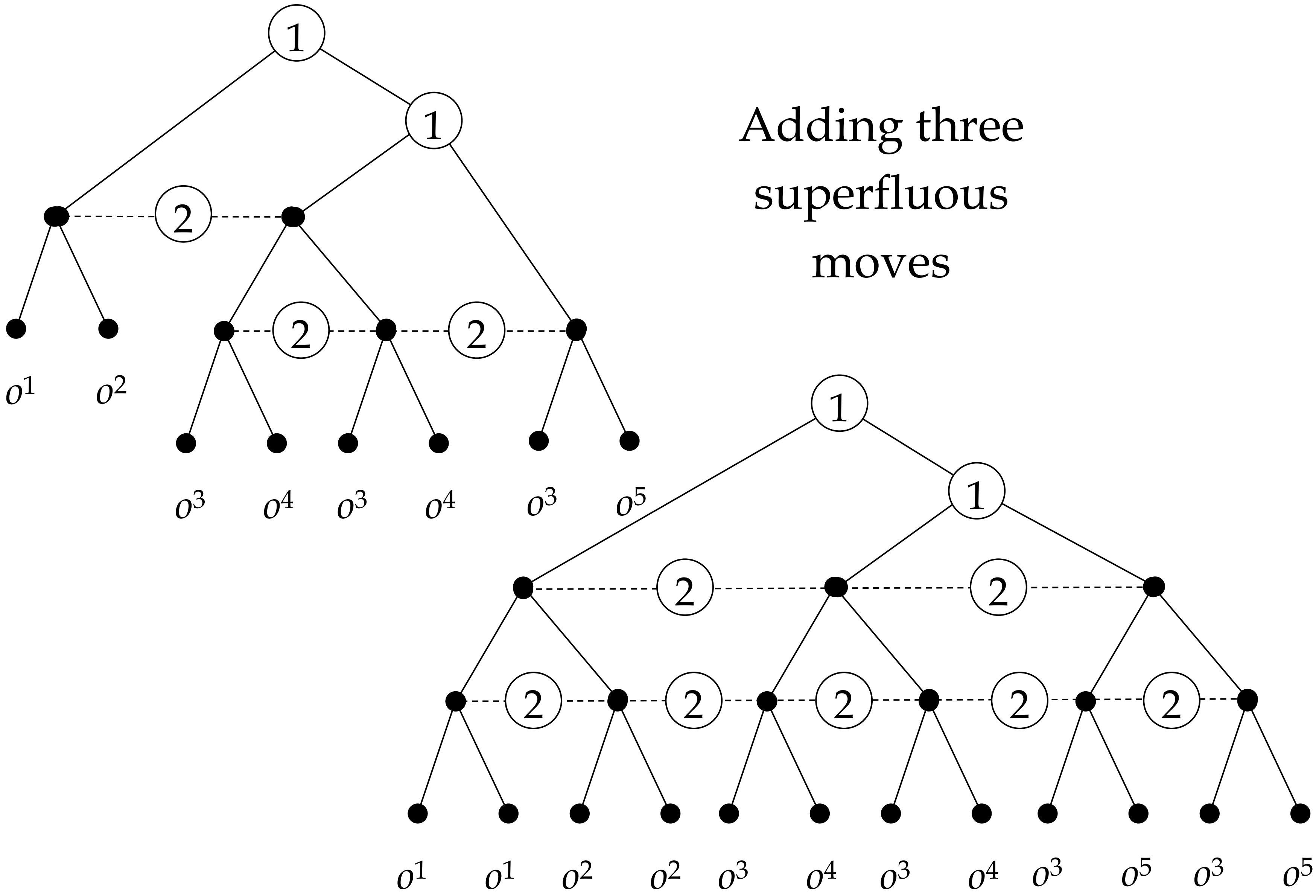


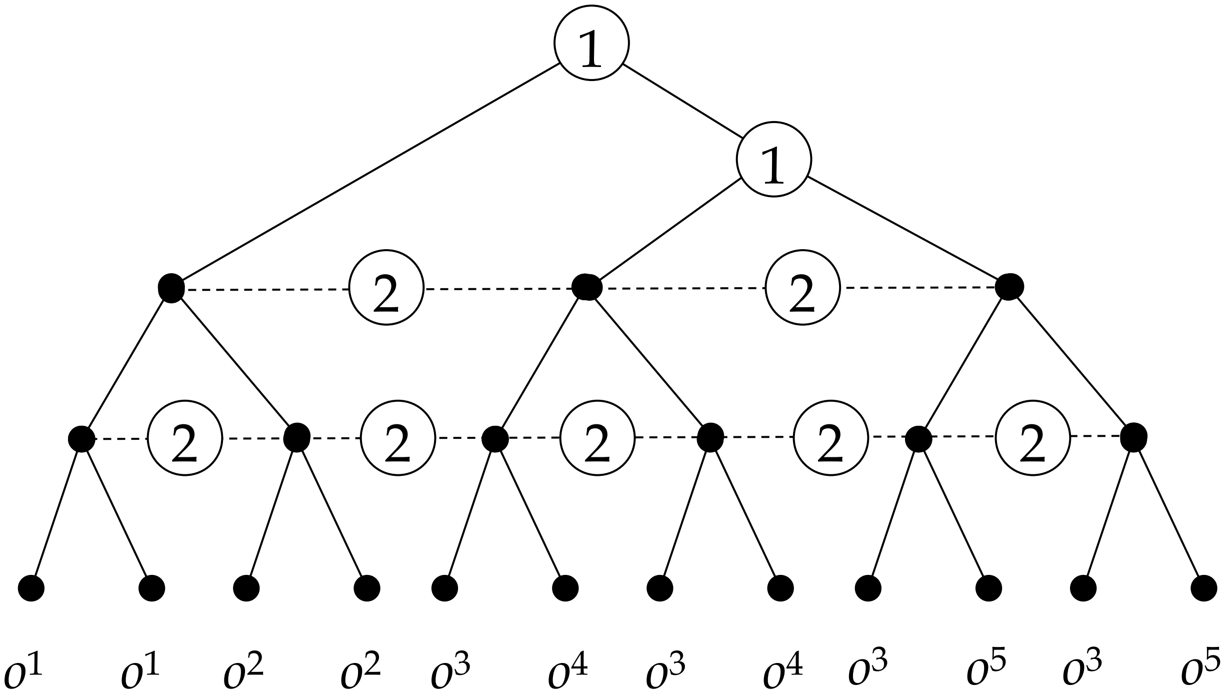


Adding a superfluous move

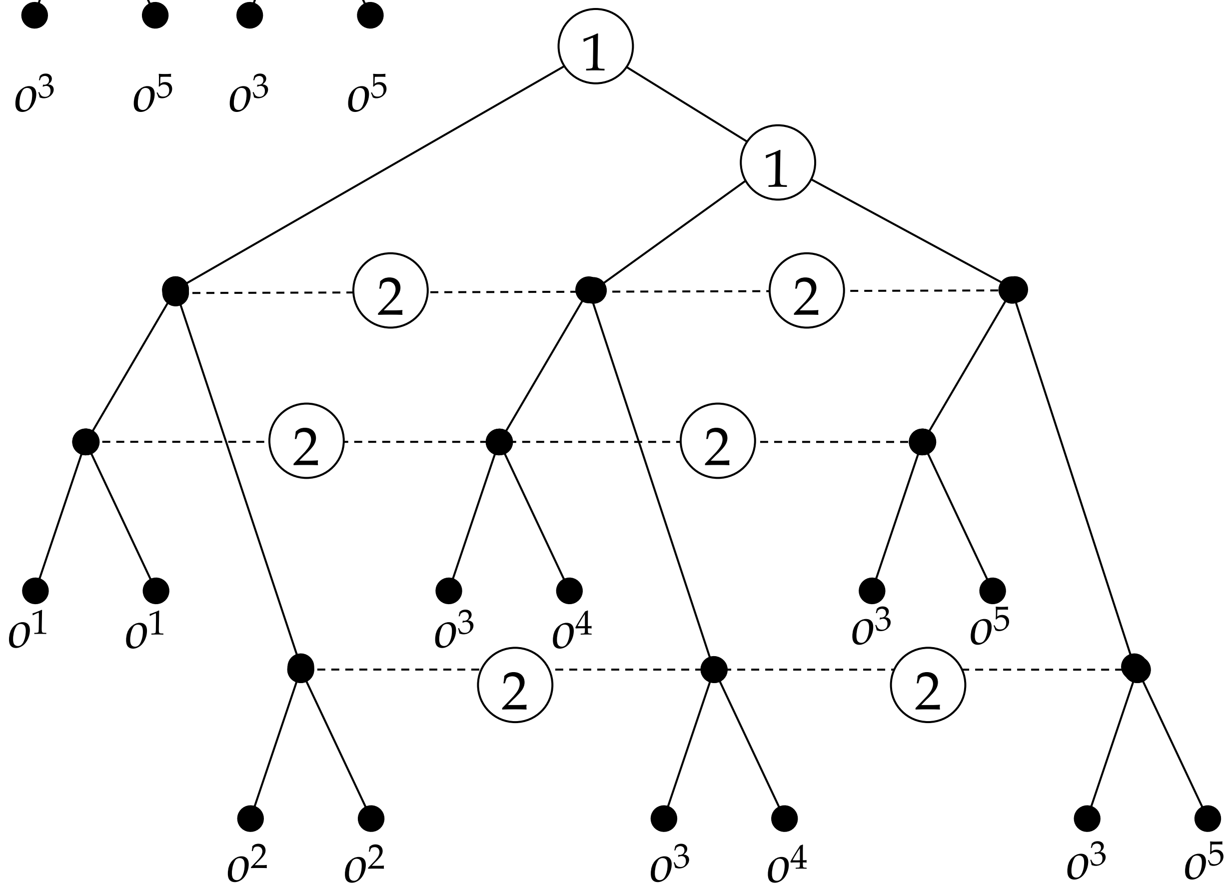


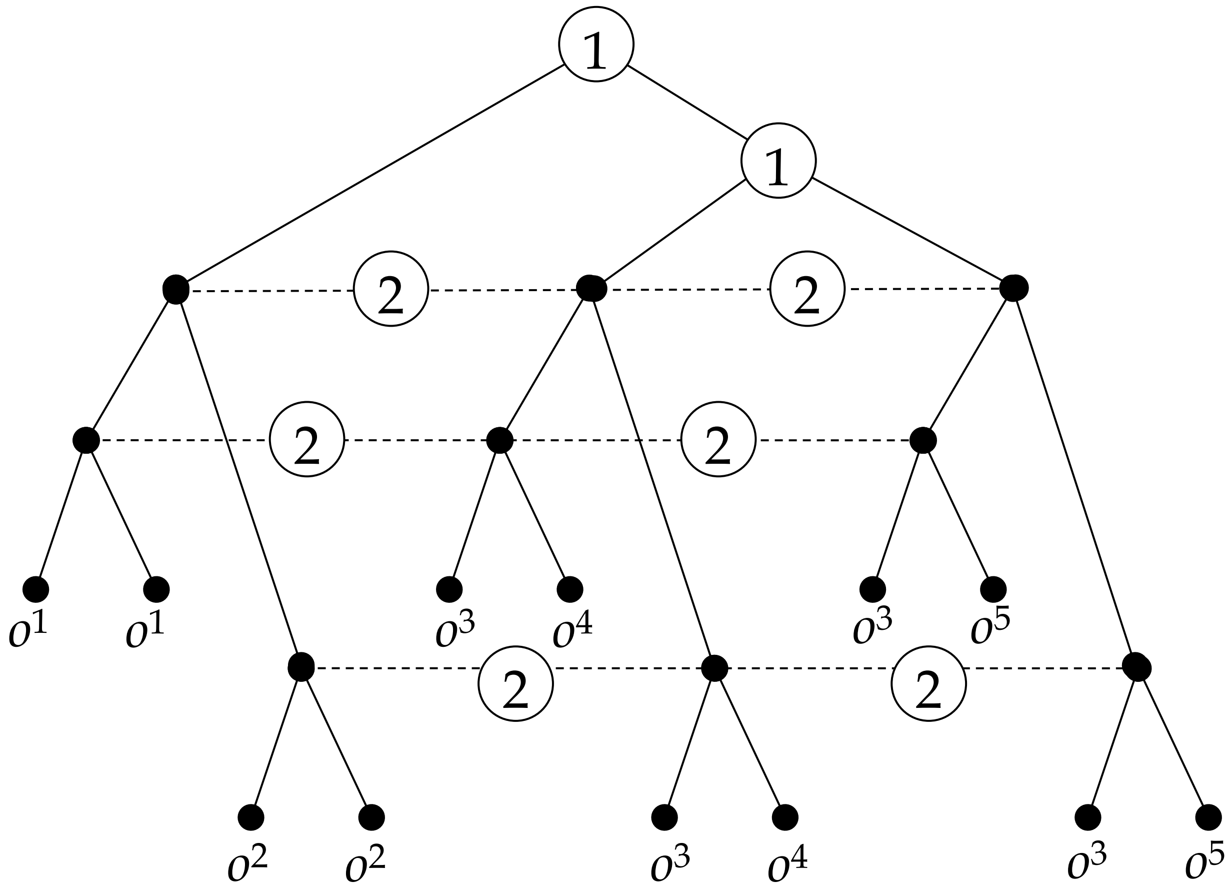
Adding three superfluous moves



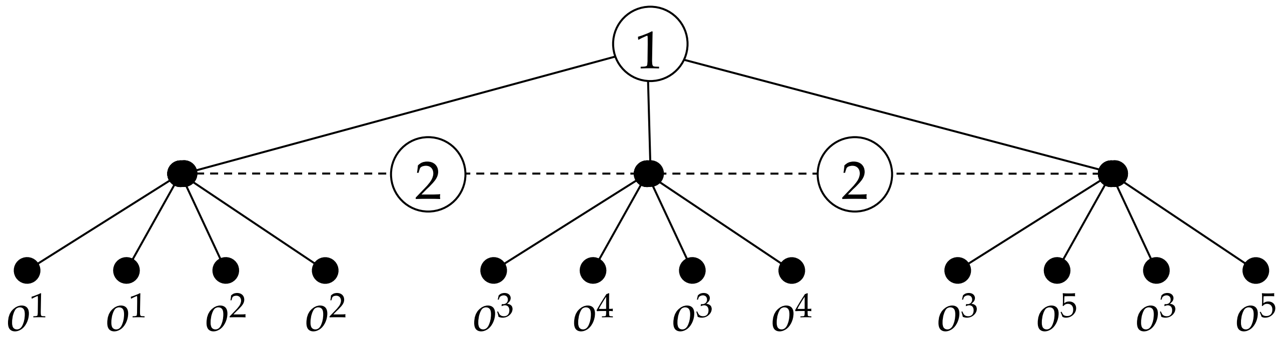


Inflation-deflation





Coalescing moves (six times)



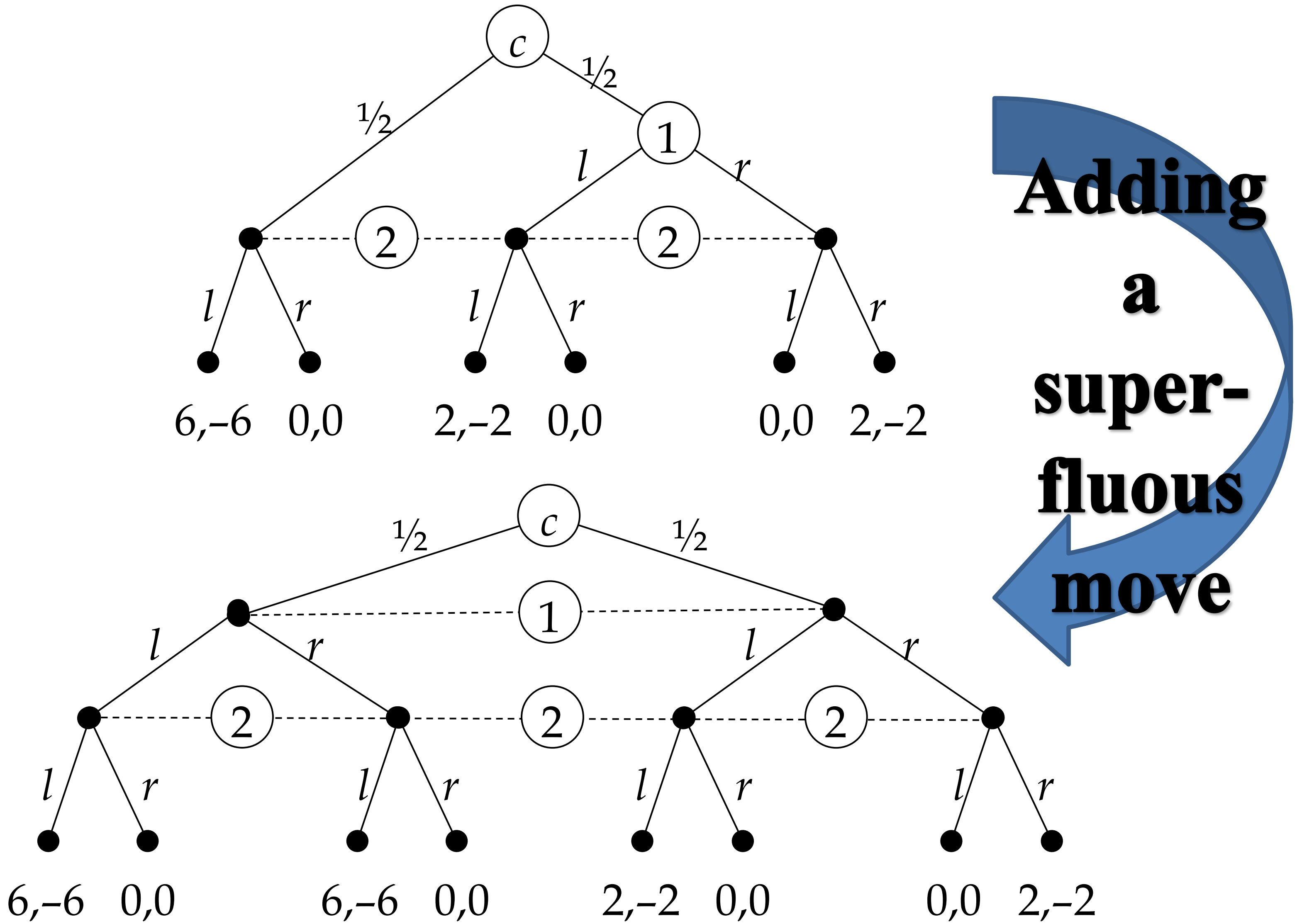
- *Principle of Inflation-Deflation*
- *Principle of Addition of a Superfluous Move*
- *Principle of Coalescing of Moves*
- *Principle of Interchange of Moves*

The four principles that we consider all preserve the reduced strategic form of the game: if one extensive game is equivalent to another according to the principles then the reduced strategic forms of the two games are the same.

Framing Effects

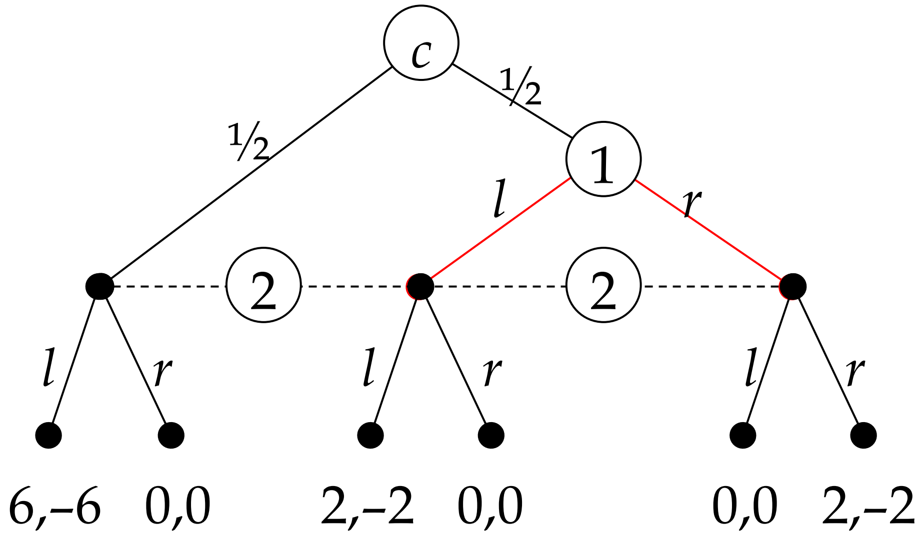
Psychologists find that even minor variations in the framing of a problem may dramatically affect the participants' behaviour (even if the games are equivalent).

Framing Effects



Q: What happens if player 1 maxminimises?

A: (½,½) and (0,1).



	l	r
l	4, −4	0, 0
r	3, −3	1, −1