# Report on Pre-Lab 1

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**3.1.1**

$$RC\frac{dV_{out}}{dt} + V_{out} = V_{in}$$

**3.1.2**

$$V_{out}(t) = V_0(1 - e^{-\frac{t}{RC}})$$

**3.1.3**

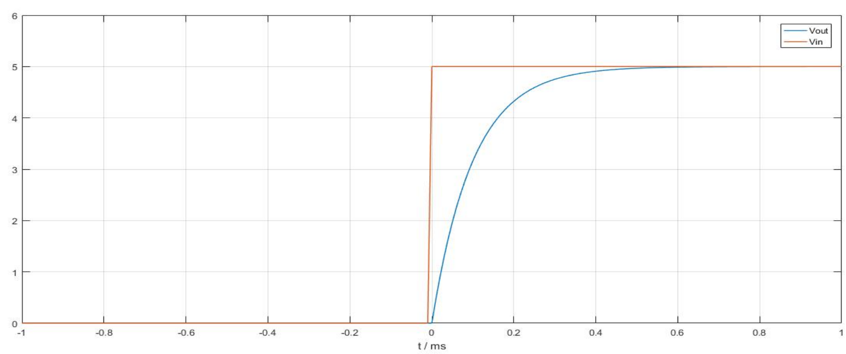


Figure 1: $V_{in} and V_{out}$

**3.1.4**

$$V_{out}(t) = V_0 e^{-\frac{t}{RC}}$$

**3.1.5**

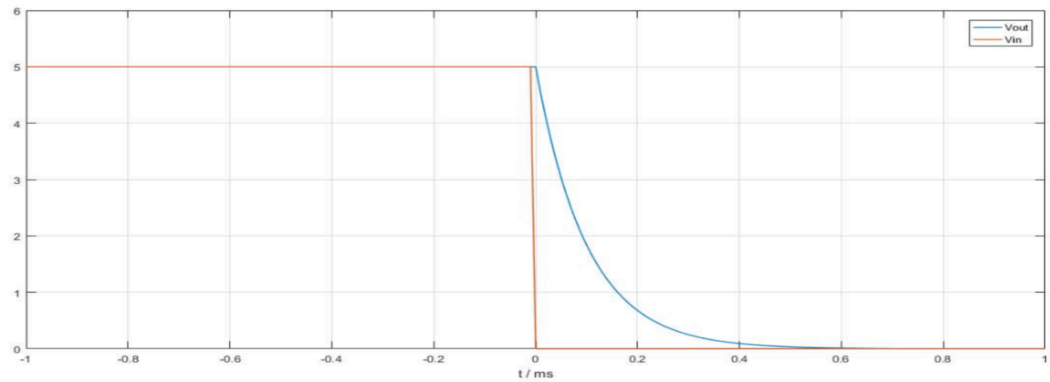


Figure 2: $V_{in} and V_{out}$

**3.1.6**

$$V_{out} = \begin{cases} V_0(1 - e^{-\frac{t}{RC}}) & 0 < t < 5RC \\ V_0 e^{-\frac{t-5RC}{RC}} & 5RC \leq t < 10RC \end{cases}$$

rise time: 10% - 90%

$$0.9V_0 = V_0(1 - e^{-\frac{t}{RC}})$$
$$t = 2.30RC$$

$$0.1V_0 = V_0(1 - e^{-\frac{t}{RC}})$$
$$t = 0.105RC$$

then,

$$t_r = 2.195RC$$

fall time: 90% - 10%

$$0.9V_0 = V_0 e^{-\frac{t-5RC}{RC}}$$
$$t = 5.105RC$$

$$0.1V_0 = V_0 e^{-\frac{t-5RC}{RC}}$$
$$t = 7.30RC$$

then,

$$t_f = 2.195RC$$

then,

$$t_c = 5.40RC$$

Delay time:

$$0.5V_0 = V_0(1 - e^{-\frac{t}{RC}})$$
$$t = 0.69RC$$

$$0.5V_0 = V_0 e^{-\frac{t-5RC}{RC}}$$
$$t = 5.69RC$$

then,

$$t_d = 0.69RC$$

**3.1.7**

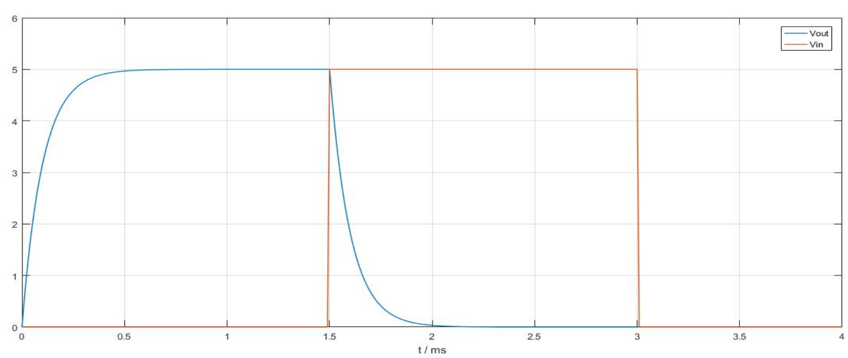


Figure 3: $V_{in} and V_{out}$

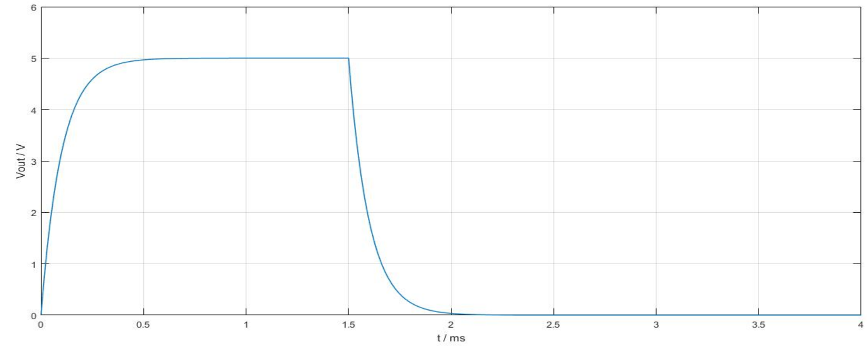


Figure 4: $V_{out}$

**3.1.8**

According to the supplemental material,

1. for Figure 3.1.1, the delay time can be described as

$$\tau = RC$$

for Figure 3.1.2 and 3.1.3, the delay time can be described as

$$\tau = \sum_{n=1}^{N}(R_n \sum_{m=n}^{N} C_m)$$

2. substitute $N = 2$, we can figure out that

$$\tau = 3RC$$

3. substitute $N = 3$, we can figure out that

$$\tau = 6RC$$

**3.2.1**

the differential equation of the circuit is:

$$RC\frac{dV_{out}}{dt} + V_{out} = V_{in}$$

for the given $V_{in}(t) = V_0 cos(\omega t)$, we can figure out that

$$V_{out}(t) = \frac{V_0}{1 + R^2C^2\omega^2}[cos(\omega t) + RC\omega sin(\omega t)]$$

$$where \quad sin(\psi) = \frac{1}{\sqrt{1 + R^2C^2\omega^2}}, cos(\psi) = \frac{RC\omega}{\sqrt{1 + R^2C^2\omega^2}}$$

**3.2.2**

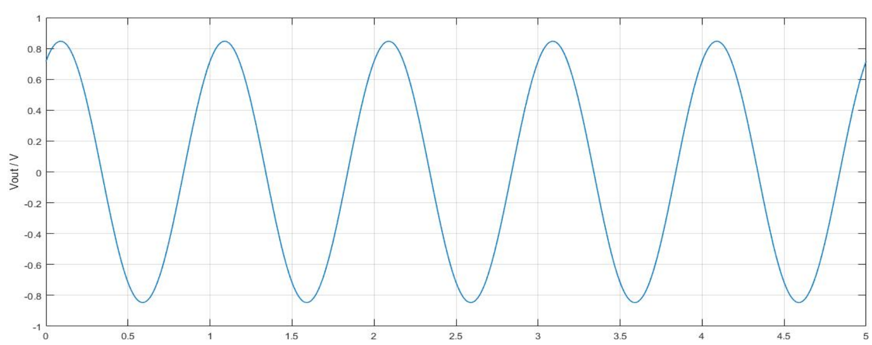


Figure 5: The output signal

As

$$cos(\omega t) + RCsin(\omega t) = \sqrt{1 + R^2C^2\omega^2}[sin(\omega t)cos(\psi) + cos(\omega t)sin(\psi)] = \sqrt{1 + R^2C^2\omega^2}sin(\omega t + \psi)$$

$$where \quad sin(\psi) = \frac{1}{\sqrt{1 + R^2C^2\omega^2}}, cos(\psi) = \frac{RC\omega}{\sqrt{1 + R^2C^2\omega^2}}$$

we find that the output signal is a sinusoidal function.

Substitute $V_0, f, R, C$, we can find that the output signal frequency T and the magnitude is

$$T = \frac{1}{f} = 1ms$$

$$|V_{out}| = 0.845V$$

**3.2.3**

Similar to problem 3.2.2, the output signal is still a sinusoidal function and its frequency is the same as the input signal frequency. The magnitude of output signal is

$$|V_{out}| = \frac{V_0}{\sqrt{1 + R^2C^2\omega^2}}$$

**3.2.4**

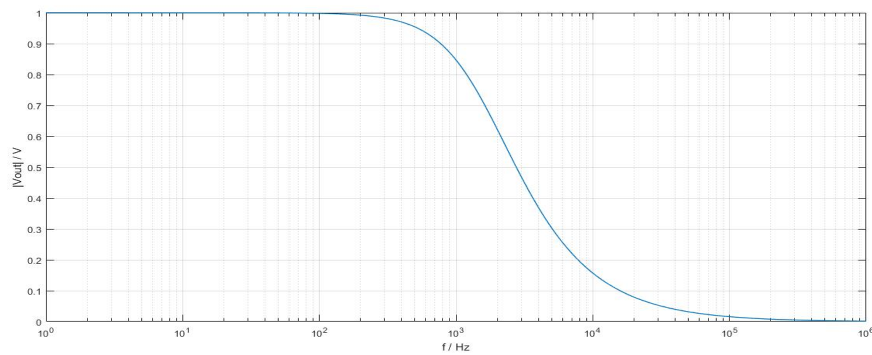


Figure 6: $V_{out}$

**3.2.5**

When the signal frequency is low, the magnitude of the output voltage is close to infinity and the period of the signal is long. When the frequency is high, the magnitude correspondingly becomes close to zero and the period is short.

**3.2.6**

As $V_R(t) = V_{in}(t) - V_C(t)$,we do the similar work to problem 3.2.2. Substitute the given data, we can find that the magnitude is 0.531V.

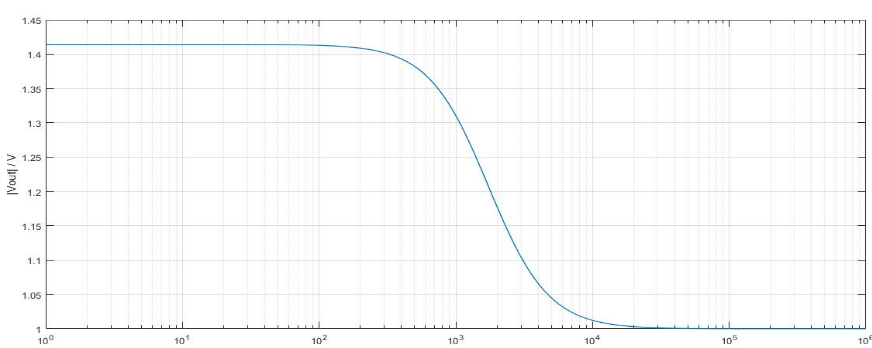


Figure 7: $V_{out}$