Report on Pre-Lab 3

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Group Number: 4
Names Of All Teammates: Leqi Ding, Ming Cheng, Wanda Guo, Changhao Bai

1 Inverting Amplifier

1. Assume the current across the negative input is i, in the direction of right. According to the KVL, we can get the following formulas:

$$\begin{cases} v_{in} - v_{out} = i(R1 + R2) \\ v_{in} = iR1 \end{cases}$$

Therefore, we can get the gain $\frac{v_{out}(s)}{V_{in}(s)}$ as follows:

$$\frac{V_{out}(s)}{V_{in}(s)} = -\frac{R2}{R1}$$

Because R1 > 0, R2 > 0, v_{out} and v_{in} are opposite in phase. And if $R_2 > R_1$, v_{out} is larger than v_{in}, implementing the amplification. Therefore, this circuit is called an inverting amplifier.

2.According to the expression of $\frac{v_{out}(s)}{V_{in}(s)}$ in the previous question, $\frac{R_2}{R_1} = 47$. Therefore, we should choose $R_1 = 1\text{k}\Omega$ and $R_2 = 47\text{k}\Omega$, according to the lab kit. The schematic of this circuit with the component values labeled is shown as follows.

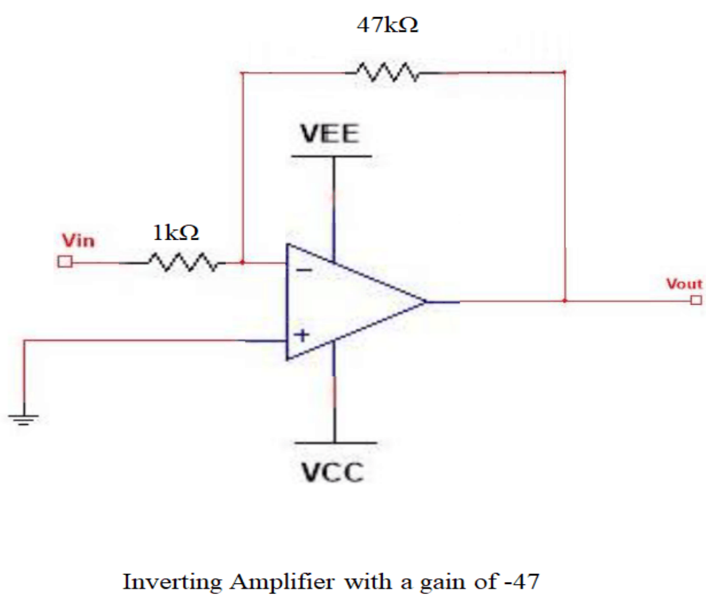

Figure 1: The schematic of inverting amplifier circuit

3.According to the references, we define $H(j\omega) = \frac{V_{out}(j\omega)}{V_{in}(j\omega)}$. Then we do some mathematical process on H(j) to convert its unit to dB.

$$20lg(|H(j\omega)|) = 20lg(\frac{R2}{R1})$$

The Bode Plot is shown as follows.

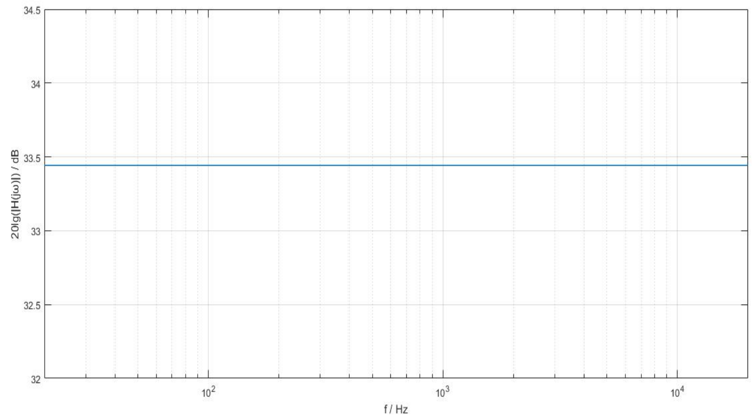

Figure 2: The Bode Plot of inverting amplifier

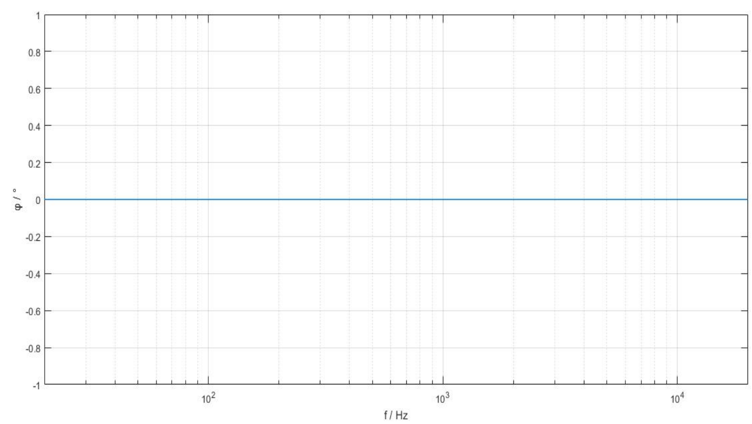

Figure 3: The phase angle of non-inverting amplifier in terms of frequency

4.The Bode Plot is shown as follows. We find that when the frequency is low, the result is the same as the theory value. But when the frequency is high, the value begins to reduce slowly. We think the result from multisim is more reliable, and it warns us the usage of higher frequency in experiment may cause some unpredictable trouble.

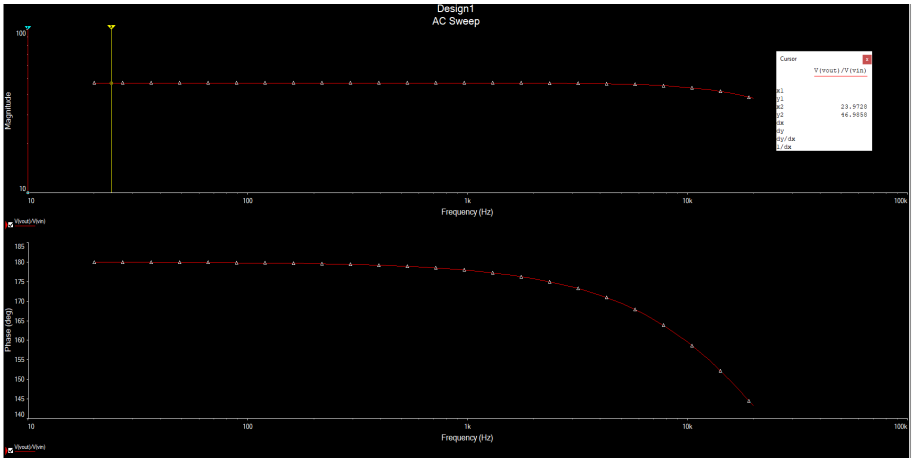

Figure 4: The Bode plot of inverting amplifier generated by MultiSim

5.Because this amplifier is a non-inverting one, we can build the circuit as the following figure shows, where the input signal should be connected to the positive input port and the negative input port should be connected to the ground. The value of the resistors chosen are also labeled in the figure. (According to the lab kit, we should choose two 10kΩ resistors in parallel connection which is connected to the positive input port and one 240kΩ resistor connected in the feedback loop.)

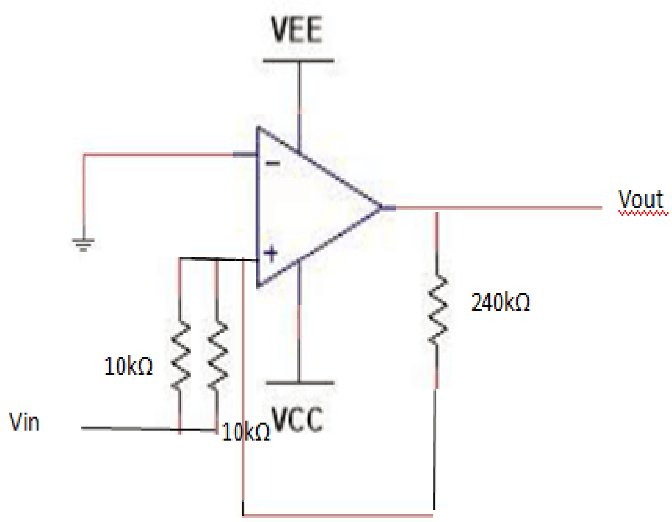

Figure 5: The schematic of non-inverting amplifier circuit

6.The Bode Plot of non-inverting amplifier generated by MultiSim is shown as follows.

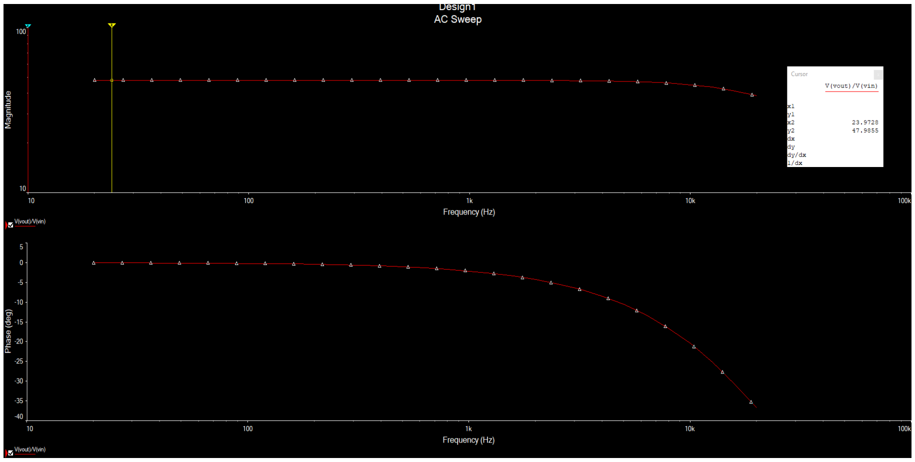

Figure 6: The Bode Plot of non-inverting amplifier generated by MultiSim

2 Integrator

1. Because the op-amp is ideal and its non-inverting port is connected to the ground, we can utilize its virtual ground and virtual short circuit characteristics to solve these problems. From these characteristics we find that:

virtual ground:

$$v_+ = v_- = 0 \tag{1}$$

virtual short circuit:

$$i_+ = i_- = 0. \tag{2}$$

Then according to Kirchhoff Law, we can get the equation below:

$$i_{R1} = i_C \tag{3}$$

$$\frac{V_{in}(t) - V_-}{R_1} = C\frac{d(V_- - V_{out}(t))}{dt}. \tag{4}$$

After utilizing equations 1, 2 and3 and executing Laplace Transform to equation 4, we get the equation below:

$$\frac{V_{in}(s)}{R_1} = -C[sV_{out}(s) - V_{out}(0^-)] \tag{5}$$

Simplify this equation, then we find that

$$V_{out}(s) = -\frac{V_{in}(s)}{sCR_1} + \frac{V_{out}(0^-)}{s} \tag{6}$$

Then we execute Inverse Laplace Transform to this equation, and then derive the equation in time domain:

$$V_{out}(t) = -\frac{1}{CR_1}\int_{0^-}^{t} V_{in}(t)\mathrm{d}t + V_{out}(0^-) \tag{7}$$

In this equation, we assume that t is not less than zero to adapt it to dimension of the unilateral Laplace Transform. As we can easily notice, the output voltage has positive relationship with the integral of the input voltage. This phenomenon verifies that it is an integrator.

2. The Bode Plot is shown as follows. We can see that the input wave form is a sine wave, and after transformed by the integrator, the output wave form is a cosine wave. Therefore, the output wave is leading the input wave. (And we meet a little distortion.)

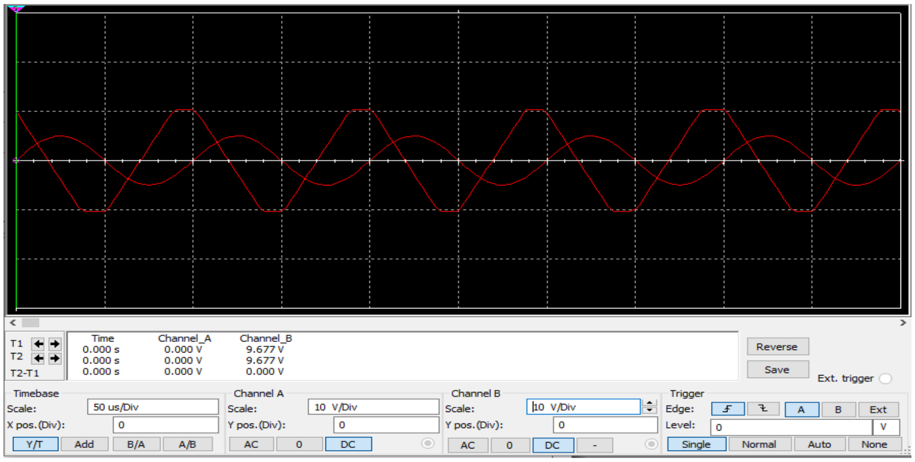

Figure 7: The plot of the input and output waveforms with MultiSim

3. First, we set the value of Vcc and Vee into ±12V. The waveforms of input and output are shown in Figure 8

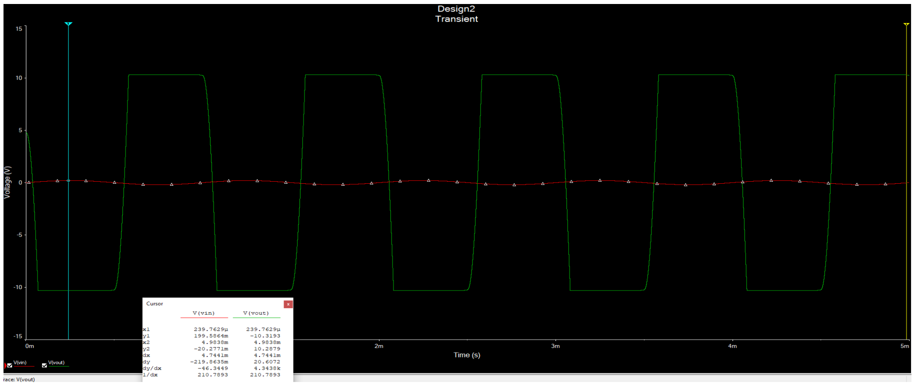

Figure 8: The plot of the input and output waveforms with default power supply

The maximum and minimum voltage of the output signal are 10V and -10V. And the reason to the distortion is the value of Vcc and Vee are low, which makes the quiescent operating point low, and cause the cut-off distortion and the saturation distortion.

After we change Vcc and Vee into 100V and -100V, the distortion was gone. And the maximum and minimum voltage of the output signal are 59.5V and -97V. The waveforms of input and output are shown in Figure 9

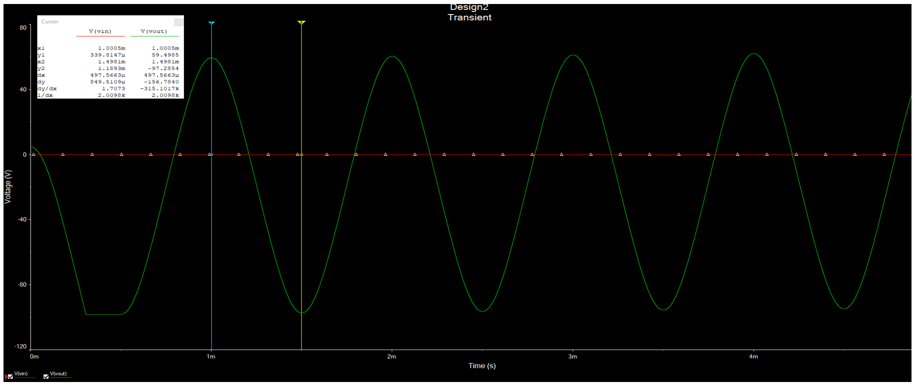

Figure 9: The plot of the input and output waveforms with higher power supply

4. The Bode Plot is shown as follows.

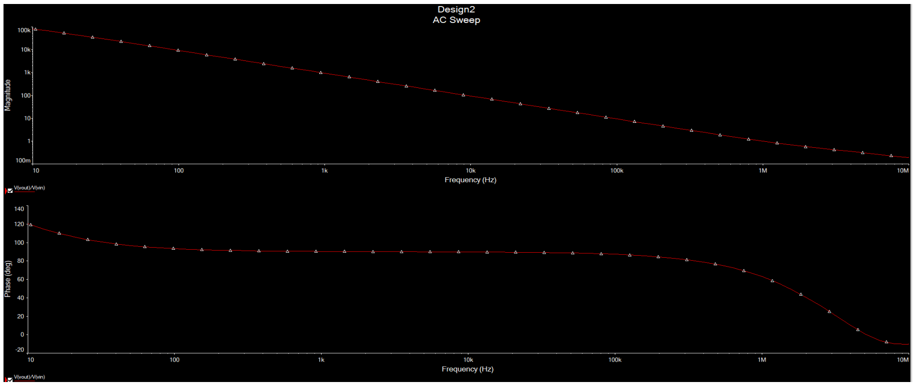

Figure 10: The Bode plot of Integrator generated by MultiSim

5. Using the approach similar to item 1 in 3.2, we also use virtual ground and virtual short circuit characteristics. Then we get the equation below: virtual ground:

$$v_+ = v_- = 0 \tag{8}$$

virtual short circuit:

$$i_+ = i_- = 0 \tag{9}$$

Then we utilize the Kirchhoff Law and Ohm Law. We find that

$$\frac{V_{in}(t) - V_-}{R_1} = \frac{V_- - V_{out}(t)}{R_2} + C\frac{d[V_- - V_{out}(t)]}{dt} \tag{10}$$

Then we solve this equation utilizing Laplace Transform and get the expression in frequency domain:

$$\frac{V_{in}(s)}{R_1} = -\frac{V_{out}(s)}{R_2} - C[sV_{out}(s) - V_{out}(0^-)] \tag{11}$$

$$V_{out}(s) = -\frac{\frac{V_{in}(s)}{CR_1} - V_{out}(o^-)}{s + \frac{1}{CR_2}} \tag{12}$$

Finally, we get the equation in time domain by using Inverse Laplace Transform:

$$V_{out}(t) = -\frac{1}{CR_1}e^{-\frac{1}{CR_2}t}\int_{0^-}^{t} V_{in}(t)\mathrm{d}t + e^{-\frac{1}{CR_2}t}V_{out}(0^-) \tag{13}$$

Similarly, we can notify the form of the equation and verify that the circuit is an integrator with a feedback resistor.

6. We analyze the circuit in frequency domain and derive the equation above:

$$\frac{\overrightarrow{V_{in}}}{R_1} = -\frac{\overrightarrow{V_{out}}}{\frac{R_2\times\frac{1}{j\omega C}}{R_2+\frac{1}{j\omega C}}} \tag{14}$$

Then we can get the expression of the gain and its magnitude:

$$\frac{\overrightarrow{V_{out}}}{\overrightarrow{V_{in}}} = \frac{R_2}{R_1(j\omega CR_2 + 1)} \tag{15}$$

$$|\frac{\overrightarrow{V_{out}}}{\overrightarrow{V_{in}}}| = \frac{R_2}{R_1\sqrt{1+\omega^2C^2R_2^2}} \tag{16}$$

We choose the 1kΩ resistor and 47kΩ resistor for R1 and R2. Then we use MATLAB to plot the Bode plot below:

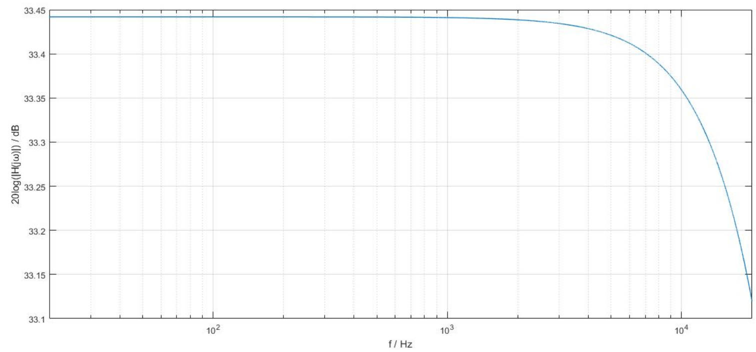

Figure 11: The Bode plot of Integrator

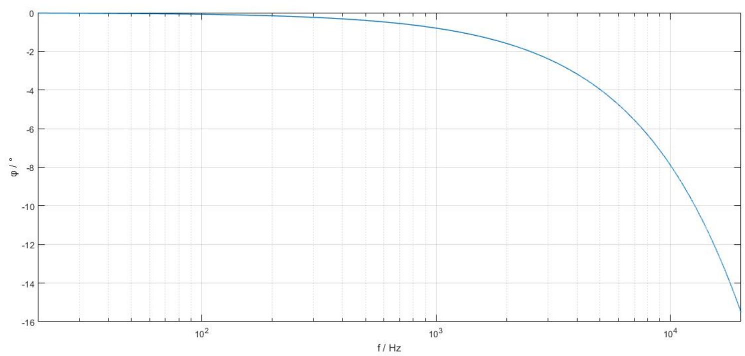

Figure 12: The phase angle of the $\frac{V_{out}(j\omega)}{V_{in}(j\omega)}$

7. When the angle frequency ω approaches zero, according to equation 16, we find that:

$$|\frac{\overrightarrow{V_{out}}}{\overrightarrow{V_{in}}}| \to \frac{R_2}{R_1} \tag{17}$$

It is the same as that of the inverting amplifier. The reason is that when frequency is low enough, the impedance of the capacitor is very high and it can be regarded as infinite value, which means that it can be seen as an open circuit. Hence, the circuit of integrator can be handled as the circuit of inverting amplifier.

8. When ω approaches $\frac{1}{R_2 C}$, we find that

$$|\frac{\overrightarrow{V_{out}}}{\overrightarrow{V_{in}}}| \to \frac{1}{\omega C R_1} \tag{18}$$

This indicates that the complex integrator can be regarded as a simple integrator at high frequency. The reason is that when the frequency becomes very high, the effect of the resistor R2 is really not obvious because the impedance of the capacitor is very small and almost all of the current passes through the capacitor. Then R2 can be handled as an open circuit.

9. The resistor R_2 serves as a feedback resistor. When the circuit works, the bias voltage will charge and discharge the capacitor repeatedly. The resistor helps the capacitor discharge its electric energy. Also the resistor controls the output voltage especially at low frequency and avoid extremely large gain of the circuit.

10.The Bode Plot is shown as follows. If the input signal has a low frequency, the expected gain is 47. If the input signal has a high frequency, the plot is reduced mildly. So we can see this circuit is a low-pass filter. The resistance2 is used as a shunt resistance to avoid the noise to make the output voltage more accurate, makes it a better choice for preamplifier than a simple inverting or non-inverting amplifier.

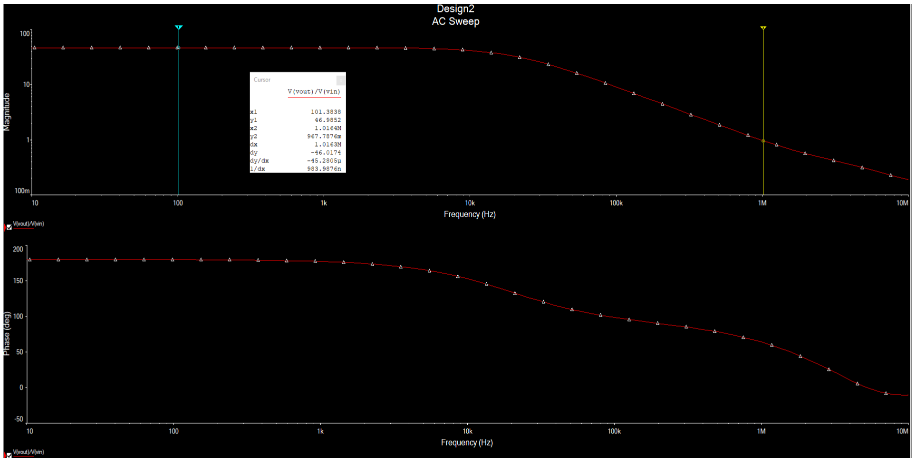

Figure 13: The frequency response of the integrator with shunt resistance

3 Simple Differentiator

1.The characteristic equation of capacitance is:

$$i_c = C\frac{du_c}{dt}$$

Assume the current across the capacitor is i and we can get following formulas by using KVL.

$$\begin{cases} v_{in} = u_c \\ i = C\frac{du_c}{dt} \\ v_{in} - u_c - iR_f = v_{out} \end{cases}$$

So, we can get the expression of V out (s) after applying Laplace Transform to the differential equation above as follows:

$$V_{out}(s) = -sCR_fV_{in}(s)$$

2.After solving the equations in the above question, the time-domain equation is shown as follows:

$$-CR_f\frac{dv_{in}}{dt} = v_{out}$$

According to the equation, the output is equal to the differential of the input. Therefore, this circuit performs the function of a differentiator.

3.According to the references, we define $H(j) = \frac{V_{out}(jw)}{V_{in}(jw)}$. Then we do some mathematical process on H(j) to convert its unit to dB.

$$20lg(|H(j\omega)|) = 20lg(\omega CR_f)$$

The Bode Plot is shown as follows.

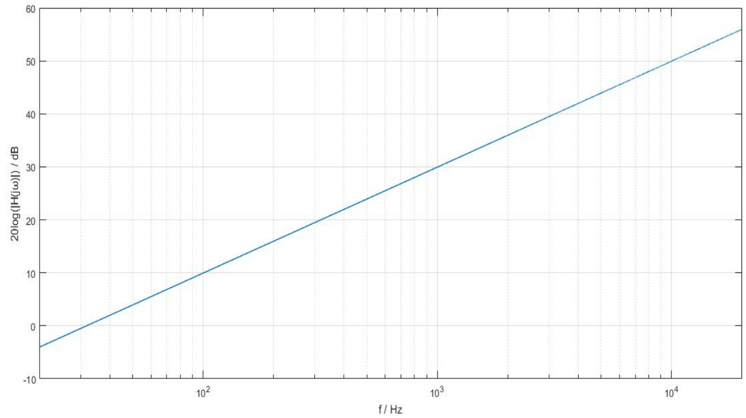

Figure 14: The Bode Plot of simple differentiator

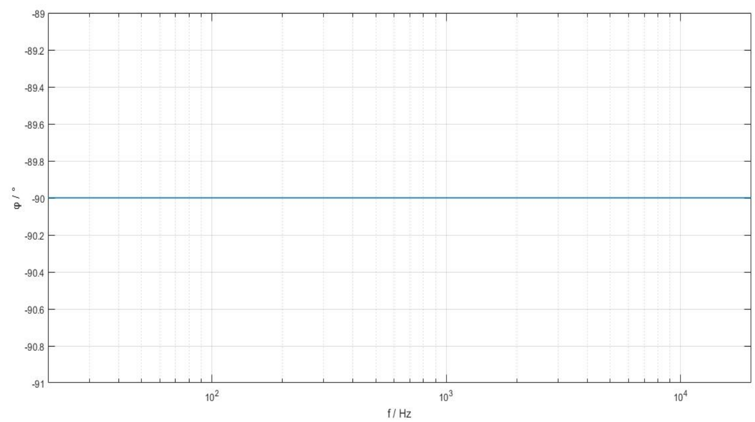

Figure 15: The phase angle of simple differentiator in terms of frequency

4.The Bode Plot of simple differentiator generated by MultiSim is shown as follows.

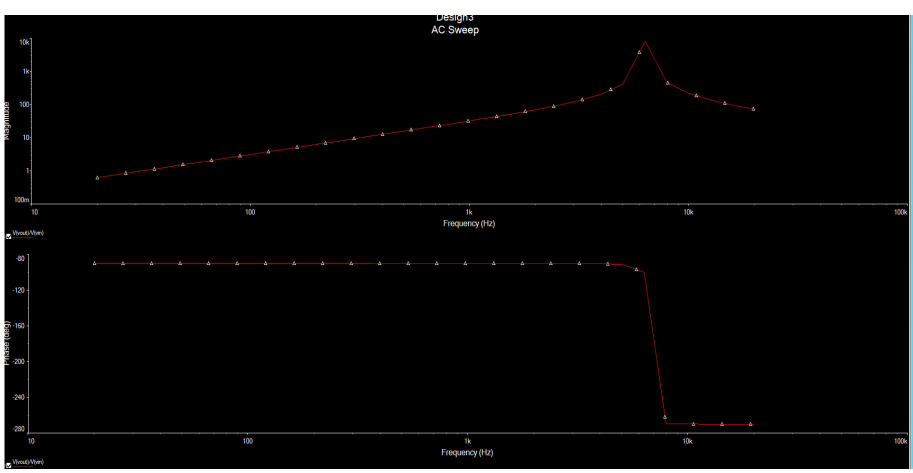

Figure 16: The Bode plot of simple differentiator generated by MultiSim

5.The result is as Figure 17 shows. We can confirm that the circuit we built is a differentiator.

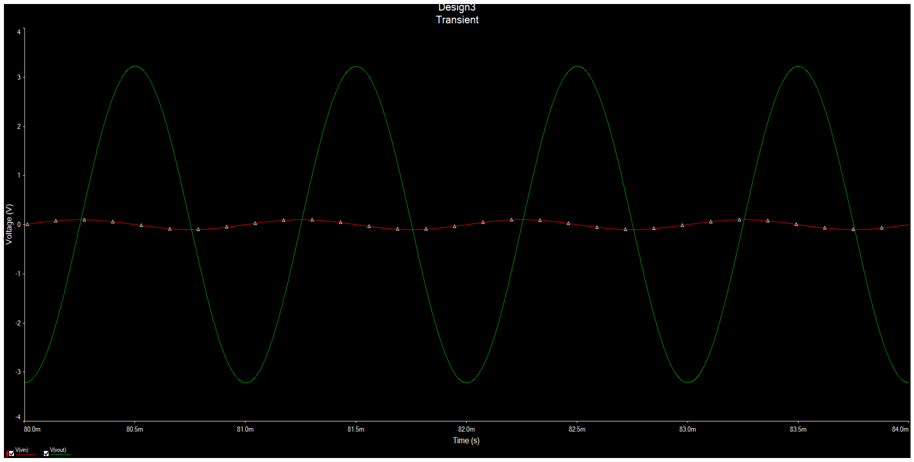

Figure 17: The plot of the input and output waveforms