# EE 233 Circuit Theory

# Lab 1: RC Circuits
<div align="right">Instructor: Tai-Chang Chen</div>

## Contents

# 1. Introduction

The main purpose of this lab is to teach students methods for characterizing circuit systems, more specifically, an RC circuit system. This lab will also familiarize students with the test bench instruments used in this class by having them use the equipment to analyze some fundamental response trends of step and sinusoidal input functions for an RC circuit.

A circuit system can be pictured as a box with inputs and outputs, and the characteristics of this system can be represented by its input and output signals, e.g. voltage and current. A signal contains three parameters: magnitude, frequency, and phase. Any change of these parameters in the input signal will affect the output signal.

The RC circuit has many interesting characteristics while staying one of the most basic circuit systems. This lab is going to allow students to observe these characteristics and teach them how to analyze the output signals with changes in input magnitude or frequency.

# 2. Precautions

None of the devices used in this set of experiments are particularly static sensitive; nevertheless, you should pay close attention to the circuit connections and the polarity of the power supplies, function generator, and oscilloscope inputs.

# 3. Prelab

## 3.1 The RC Response to a Step Function

1. Write the differential equation for $v_{out}(t)$ in Figure 3.1.1 in terms of $v_{in}(t)$, $R$, and $C$.
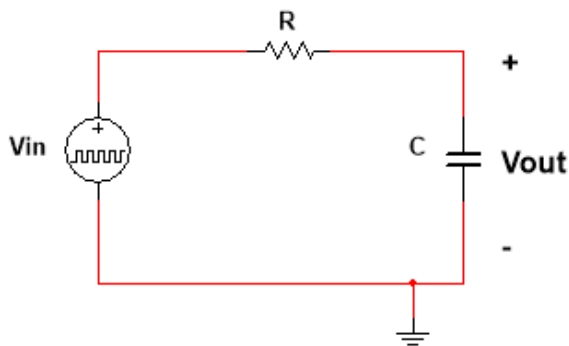


**Figure 3.1.1** An RC circuit

2. The differential equation for $v_{out}(t)$ is the most fundamental equation describing the RC circuit, and can be solved if the input signal $v_{in}(t)$ is given. Now suppose the input signal has been zero for a long time and then is changed to $V_0$, a positive constant, at $t = 0$. The input signal is a step function, which means

$$v_{in}(t) = \begin{cases} 0, when\ t < 0 \\ V_0, when\ t \geq 0 \end{cases}$$

The initial condition for $v_{out}(t)$ is needed in order to solve the differential equation. The output voltage should be zero when $t < 0$, since there is no input before $t = 0$. Thus, the initial condition for $v_{out}(t)$ is $v_{out}(0) = 0$. Now find $v_{out}(t)$ for $t > 0$ in terms of $V_0$, $R$, and $C$.

3. Suppose $V_0 = 5$ V,$R = 10$ kΩ, and $C = 0.01$ μF. Plot both $v_{in}(t)$ and $v_{out}(t)$ from $-1$ ms to 1 ms on the same set of axes.

4. The RC circuit's natural response to the step function is now analyzed, and you have a general idea of what this response looks like by plotting it with the input voltage. Now suppose the input signal has been $V_0$, a positive constant, for a long time before being changed to zero at $t = 0$, which means

$$v_{in}(t) = \begin{cases} V_0, when\ t < 0 \\ 0, when\ t \geq 0 \end{cases}$$

Find the initial condition for $v_{out}(t)$, then find $v_{out}(t)$ for $t > 0$ in terms of $V_0$, $R$, and $C$.

5. Suppose $V_0 = 5$ V, $R = 10$ kΩ, and $C = 0.01$ μF. Plot both $v_{in}(t)$ and $v_{out}(t)$ from $-1$ ms to 1 ms on the same set of axes.

6. If the switch is turned on and off periodically, the input signal becomes a square wave. Suppose the period is $T$ and the duty cycle is 50%. Explain that if half of the period $T/2 \gg RC$ then the output voltage goes to its limit before the switch changes. Find the time constant, rise time, fall time, and delay time in terms of $R$ and $C$. Notice that these timing parameters are independent of the input voltage. Read Reference 5.1 for more information.

**Hint**: Suppose $T = 10RC$, for example. Find out how close the output voltage is to its limit.

7. Suppose the period of the turning switch is $T = 3$ ms, and that $V_0 = 5$ V, $R = 10$ kΩ, and $C = 0.01$ μF. Plot the input and output voltages $v_{in}(t)$ and $v_{out}(t)$ over one period.

8. Read the supplemental material of Elmore Delay Estimation and calculate the delay time for the circuit in Figure 3.1.1, Figure 3.1.2, and Figure 3.1.3 in terms of $R$ and $C$. Assume $R = R_1 = R_2 = R_3$ and $C = C_1 = C_2 = C_3$.
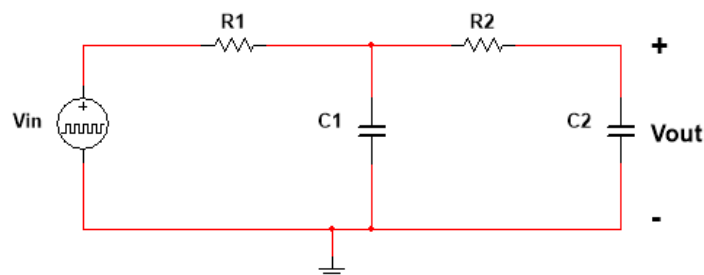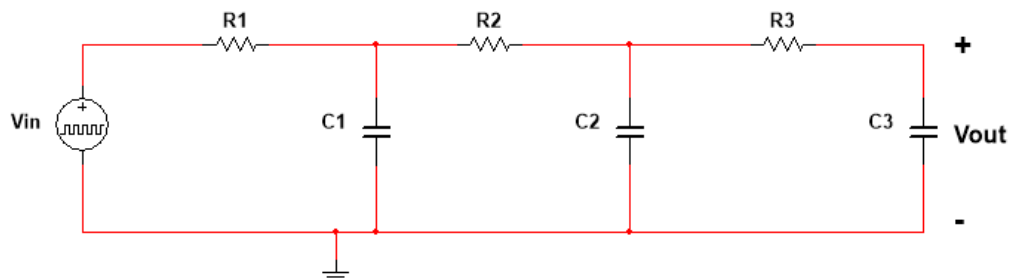


**Figure 3.1.2** A two-stage RC circuit



**Figure 3.1.3** A three-stage RC circuit

### 3.2 The RC Response to a Sinusoidal Function

1. While the input square wave changes the magnitude of the signal, exploration of the RC response to an AC signal can show more interesting characteristics of the RC circuit. Now consider the RC circuit as part of a circuit system, as shown in Figure 3.2.1. The system is the dashed box, and it has two input terminals (input+ and input-) and two output terminals (output+ and output-). The voltage between the two input terminals is the provided input signal, while the voltage between the two output terminals is the output signal. Thus, we can connect input terminals to the voltage source and then calculate the output signal voltage. Suppose we are using a sinusoidal wave as an input signal, this means that $v_{in}(t) = V_0 \cos(\omega t)$, where $\omega$ is the angular frequency of the signal. Write the differential equation for $v_{out}(t)$ in terms of $V_0, \omega, R,$ and $C$.
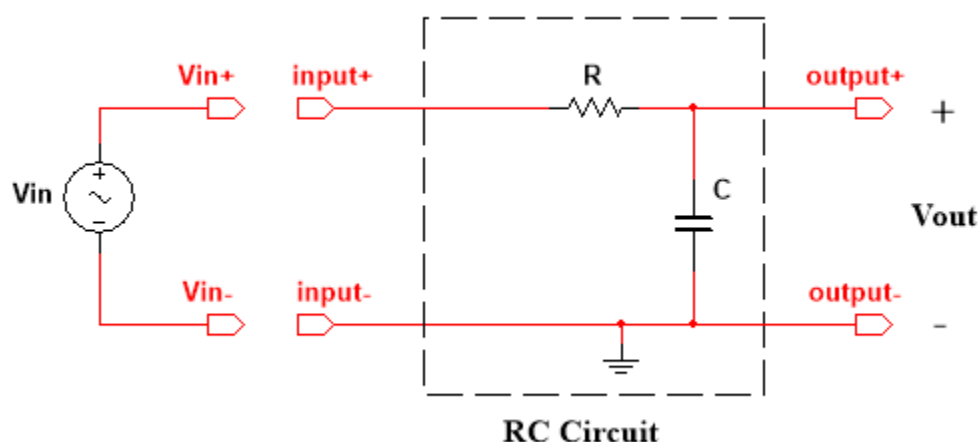


RC Circuit

**Figure 3.2.1** An RC circuit in a systematic view

2. This differential equation is the fundamental equation describing the RC circuit system. The solution for the steady-state output voltage is

$$v_{out}(t) = \frac{V_0}{1 + R^2C^2\omega^2}[\cos(\omega t) + RC\omega \sin(\omega t)]$$

This solution shows that $v_{out}$ is a function of signal frequency $f$ and time $t$. Suppose $V_0 = 1$ V, $f = 1$ kHz, $R = 10$ k$\Omega$, and $C = 0.01$ µF. Plot $v_{out}(t)$ from 0 to 5 ms. Is the output signal a sinusoidal function? If so, what is the period $T$ and the magnitude $|v_{out}(t)|$?

**Hint:** The relation between angular frequency and signal frequency is $\omega = 2\pi f$.

3. Now consider the solution

$$v_{out}(t) = \frac{V_0}{1 + R^2C^2\omega^2}[\cos(\omega t) + RC\omega \sin(\omega t)].$$

Is the output signal a sinusoidal function? If so, is the output frequency the same as the input signal's frequency? Find the magnitude $|v_{out}(t)|$ for the output voltage.

**Hint**: $\cos(\omega t) + RC\omega \sin(\omega t) = \sqrt{1 + R^2C^2\omega^2}[\sin(\omega t)\cos(\phi) + \cos(\omega t)\sin(\phi)]$

$= \sqrt{1 + R^2C^2\omega^2}\sin(\omega t + \phi)$, where $\sin(\phi) = \frac{1}{\sqrt{1+R^2C^2\omega^2}}$ and $\cos(\varphi) = \frac{RC\omega}{\sqrt{1+R^2C^2\omega^2}}$.

4. Suppose $V_0 = 1$ V, $R = 10$ k$\Omega$, and $C = 0.01$ µF. Plot the magnitude of the output voltage, $|v_{out}(f)|$, versus frequency. The x-axis (frequency) should be plotted on a log ($\log_{10}$) scale from 1 Hz to 1 MHz.

5. Comment on the output signal's characteristics at a very low frequency, e.g. 1 Hz, and at a very high frequency, e.g. 1 MHz. Read Reference 5.3 for more information.

6. Now consider another RC system in Figure 3.2.2, in which the output voltage is over the resistor rather than the capacitor. Find the magnitude $|v_{out}(t)|$ for the output voltage. Suppose $V_0 = 1$ V, $R = 10$ kΩ, and $C = 0.01$ μF. Plot the function $|v_{out}(t)|$ versus frequency. The x-axis (frequency) should be plotted on a log scale from 1 Hz to 1 MHz.

**Hint:** The voltage over resistor is $v_R = v_{in}(t) - v_C(t)$. In addition, $RC\omega \cos(\omega t) - \sin(\omega t) = \sqrt{1 + R^2C^2\omega^2}[\cos(\omega t)\cos(\phi) - \sin(\omega t)\sin(\phi)] = \sqrt{1 + R^2C^2\omega^2}\cos(\omega t + \phi)$, where $\sin(\phi) = \frac{1}{\sqrt{1+R^2C^2\omega^2}}$ and $\cos(\phi) = \frac{RC\omega}{\sqrt{1+R^2C^2\omega^2}}$.



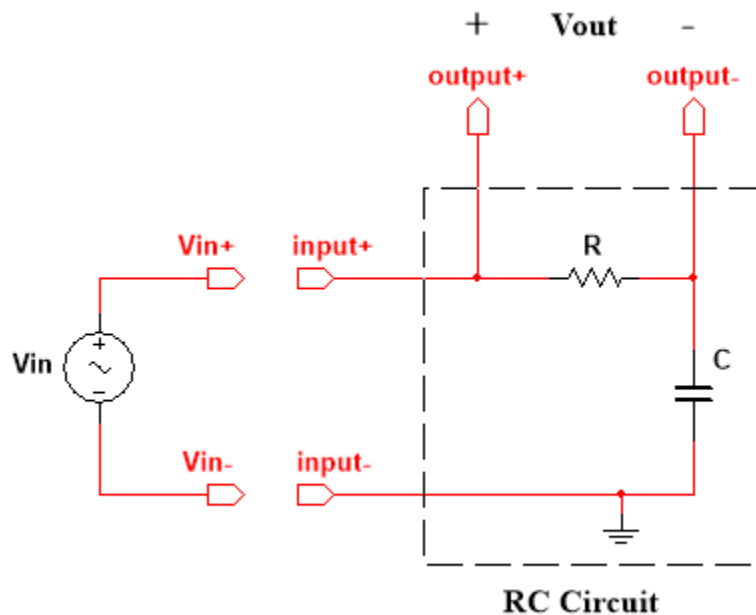**Figure 3.2.2** An RC circuit with the output over the resistor

## 4. Experimental Procedure and Data Analysis

### 4.1 The RC Response to a Step Function

1. Build the circuit in Figure 4.1, using $R = 10$ kΩ and $C = 0.01$ μF. Set the function generator to provide a square wave input as follows:

   a. Period $T \geq 3$ ms (to ensure that $T \gg RC$). This value of $T$ guarantees that the output signal has sufficient time to reach a final value before the next input transition. Record the value of $T$.

   b. Minimum voltage is 0 V and maximum voltage is 5 V. Note that you may need to manually set the offset to achieve this waveform. Use the oscilloscope to display this waveform on Channel 1 to verify that the amplitude is correct. We use this amplitude since it is common in computer systems.
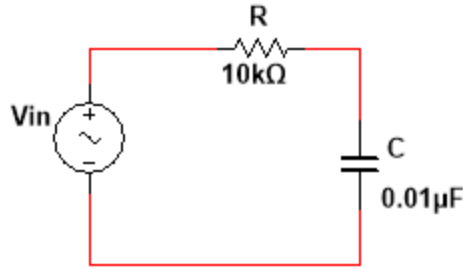
**Figure 4.1** The RC response to a step function

Use Channel 2 of the oscilloscope to display the output voltage over the capacitor. Adjust the time base to display 3 complete cycles of the signals. Get a hardcopy output from the scope display with both the waveforms and the measured values. Turn this hardcopy in as part of your lab report.

**Question-1:** Does the oscilloscope display the same waveform that you plotted in section 3.1 item 7? Explain the similarities or differences.

2. Record the period $T$ of the input signal, and the maximum and minimum values of the output signal. Then use the measurement capability of the oscilloscope to measure the time value of the 10%-point of $v_{out}$, the time value of the 90%-point of $v_{out}$, and the time value of the 50%-point of $v_{out}$.

**Question-2:** Calculate the rise time, fall time, and delay time, then compare them with the theoretical values. Read Reference 5.1 for more information. Explain the likely sources of errors leading to any differences.

3. Clear all the measurements. Use the measurement capability of the oscilloscope to measure the rise time of $v_{out}$, the fall time of $v_{out}$, and the two delay times $t_{PHL}$ and $t_{PLH}$. Read Reference 5.1 for more information.

**Question-3:** Compare the measured data with the theoretical values, as well as the measurements in section 4.1 item 2. Explain the likely sources of errors leading to any differences.

4. The time constant $\tau = RC$ is one of the most important characteristics of $RC$ circuit, and its value can be extracted from measured data.

To measure the time constant $\tau$, use the paired measurement capability of the oscilloscope to measure the voltage and time values at 10 points on the $v_{out}$ waveform during one interval when $v_{out}$ rises or falls with time (pick one interval only). Note that the time values should be referred to time $t = 0$ at the point where the input signal rises from 0 V to 5 V or falls from 5 V to 0 V. Record the 10 measurements.

**Question-4:** Use these 10 measurements to calculate the time constant $\tau$. Compare it with theoretical value $\tau = RC$. Explain the likely sources of errors leading to any differences.

**Hint**: Consider the ratio of $|v_{out} - v_{in}|$ and high voltage $V_0$. It is $R(t) = \frac{|v_{out}(t) - v_{in}|}{|V_0|} = e^{-\frac{t}{\tau}}$, and it can be calculated by measured data. So the function $\ln(R(t))$ is linear according to time, and the slope is $-\frac{1}{\tau}$. Read Reference 5.2 for more information.

5. Build two-stage and three-stage RC circuits and measure time constant $\tau_{two-stage}$ and $\tau_{three-stage}$ using the same method in section 4.1 item 4. Record all your measurements.

**Question-5:** Compare the measured values of the time constant with the theoretical values. Are they the same values? Explain the likely sources of errors leading to any differences.

### 4.2 The RC Response to a Sinusoidal Function

1. Use the circuit in section 4.1 with $R = 10$ kΩ and $C = 0.01$ μF. Set the function generator to provide a sinusoidal wave input as follows:

a. Amplitude of 1 V, which means $V_{pk-pk} = 2$ V

b. Frequency of 1 kHz.

Connect Channel 1 to the input voltage and Channel 2 to the voltage over the capacitor as the output. Display the input and output voltages simultaneously on the oscilloscope in 3 complete cycles. Get the hardcopy of both waveforms and the measured values. Turn this hardcopy in as part of your lab report.

2. Measure the RC response to sinusoidal signals with various frequencies. Keep the input amplitude at 1 V. Sweep the frequency from the starting input frequency of 10 Hz and varying it using a 1-2-5 sequence up to 1 MHz (i.e. set input frequency to 10 Hz, 20 Hz, 50 Hz, 100 Hz, 200 Hz … up to 1 MHz). Record the amplitudes of the output signals.

**Question-5:** Plot the amplitude of the output voltage in terms of frequency. Make sure the frequency is plotted on a log scale. Compare it to what was plotted in section 3.2 item 4.

3. Change the output to the voltage over the resistor. Set the function generator to provide a sinusoidal wave input with $1$ $V$ amplitude. Sweep the frequency starting from $10$ Hz using the 1-2-5 sequence up to 1 MHz. Record the amplitudes of the output signals.

**Question-6:** Plot the amplitude of the output voltage in terms of frequency and compare it to what was plotted in section 3.2 item 6.

## 5. Reference

### 5.1 Step Response and Timing Parameters

The step response of a simple RC circuit, illustrated in Figure 5.1, is an exponential signal with time constant $\tau = RC$. Besides this timing parameter, four other timing parameters are important in describing how fast or how slow an RC circuit responds to a step input. These timing parameters are marked in Figure 5.1, as three voltage levels:

a. The 10%-point is the point at which the output voltage is 10% of the maximum output voltage.

b. The 50%-point is the point at which the output voltage is 50% of the maximum output voltage.

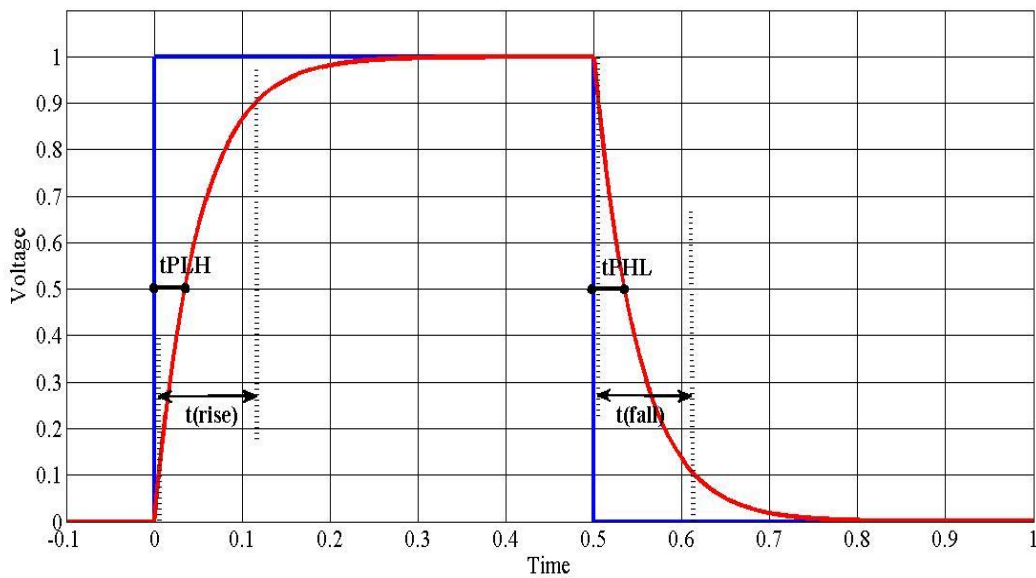c. The 90%-point is the point at which the output voltage is 90% of the maximum output voltage.

**Figure 5.1** Timing parameters of signal waveforms

The three timing parameters are defined as follows:

a. Rise time: the time interval between the 10%-point and the 90%-point of the waveform when the signal makes the transition from low voltage (*L*) to high voltage (H). Notation: $t_r$.

b. Fall time: the time interval between the 90%-point and the 10%-point of the waveform when the signal makes the transition from high voltage (*H*) to low voltage (L). Notation: $t_f$.

c. Delay time (or propagation delay time): the time interval between the 50%-point of the input signal and the 50%-point of the output signal when both signals make a transition. There are two delay times depending on whether the output signal is going from *L* to *H* (delay notation $t_{PLH}$) or from *H* to *L* (delay notation $t_{PHL}$). The subscript P stands for "propagation."

Note that the rise time and the fall time are defined using a single waveform (the output waveform), while the delay time is defined between two waveforms: the input waveform and the corresponding output waveform.

## *5.2 Parameter Extraction via Linear Least-Square-Fit Technique*

The important parameters of $v_{out}(t)$ are the maximum amplitude and the time constant $\tau$. The maximum amplitude is easily measured by using the oscilloscope. Measuring the time constant directly and accurately is more difficult, since the waveform is an exponential function of time. A linear least-square-fit procedure can be used in the lab to extract the time constant from measured voltage and time values as follows.

The equation for $v_{out}(t)$ during the time interval when $v_{out}(t)$ falls with time, which you can write based on what you learned in prerequisite courses, can be manipulated to provide a linear function in terms of the time $t$. The slope of this line is then used to extract the time constant $\tau$.

Alternatively, the equation for $v_{out}(t)$ during the time interval when $v_{out}(t)$ rises with time can also be manipulated to provide a linear function in terms of the time $t$. The slope of this line is then used to extract the time constant $\tau$.

In the lab, you will measure a set of data points $(t, v_{out}(t))$. These values, after the appropriate manipulation as above, can be used to plot a straight line, whose slope is a function of $\tau$. You can use any procedure or a calculator to plot and extract the slop. The slope value will then be used to calculate the time constant $\tau$.

Make sure you understand this procedure and be ready to use it in the lab. Note that the more points you measure, the more accurate the extracted value for $\tau$.

## 5.3 Frequency Response of a Circuit System

An analog circuit system has different responses for sine waves with different frequencies. The magnitude of the output voltage always changes in terms of frequencies if the magnitude of the input sine wave stays the same. Therefore, the frequency response is the quantitative measure to characterize the system.

Since any input signal can be regarded as the sum of a set of sinusoidal waves, the output signal will have different responses to input waves with the set of frequencies. If the circuit has high magnitude for low frequencies, and close to zero magnitude for high frequencies, the high frequencies will be removed by the circuit in the output signal, and vice versa.

The frequency response is one of the main characteristics of the system, and methods of frequency response analysis are going to be carried out in the following labs.