Report on Pre-Lab 4

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1 Simple Filter

1. In order to get the transfer function, we assume I_1, I_2, I_3, I_4, I_5 as Figure 1 shows:

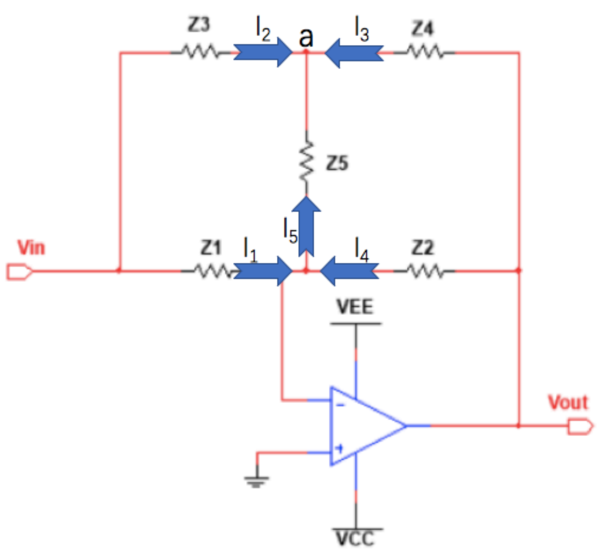

Figure 1: The diagram of simple filter circuit

Because the op-amp is ideal and its positive port is connected to the ground, we can utilize its virtual ground and virtual short circuit characteristics to solve these problems. From these characteristics we find that:

virtual ground:

$$v_+ = v_- = 0$$

virtual short circuit:

$$i_+ = i_- = 0$$

Therefore, we can obtain

$$I_1(s) = \frac{V_{in}(s)}{Z_1}$$

$$I_4(s) = \frac{V_{out}(s)}{Z_2}$$

According to KCL,

$$I_5(s) = I_1(s) + I_4(s) = \frac{V_{in}(s)}{Z_1} + \frac{V_{out}(s)}{Z_2}$$

Therefore, we can obtain the value of voltage in node a:

$$V_a(s) = -\frac{Z_5}{Z_1}V_{in}(s) - \frac{Z_5}{Z_2}V_{out}(s) \tag{1}$$

According to KCL,

$$I_2(s) + I_3(s) + I_5(s) = 0$$

$$\frac{V_{in}(s) - V_a}{Z_3} + \frac{V_{out}(s) - V_a(s)}{Z_4} + \frac{-V_a(s)}{Z_5} = 0$$

Substitute equation 1, we can obtain:

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = -\frac{\frac{1}{Z_3} + \frac{Z_5}{Z_1}(\frac{1}{Z_3} + \frac{1}{Z_4} + \frac{1}{Z_5})}{\frac{1}{Z_4} + \frac{Z_5}{Z_2}(\frac{1}{Z_3} + \frac{1}{Z_4} + \frac{1}{Z_5})} \tag{2}$$

2. When $Z_1 = Z_2 = Z$,

$$H(s) = -\frac{\frac{1}{Z_3} + \frac{Z_5}{Z}(\frac{1}{Z_3} + \frac{1}{Z_4} + \frac{1}{Z_5})}{\frac{1}{Z_4} + \frac{Z_5}{Z}(\frac{1}{Z_3} + \frac{1}{Z_4} + \frac{1}{Z_5})}$$

$$= -[1 + \frac{\frac{1}{Z_3} - \frac{1}{Z_4}}{\frac{1}{Z_4} + \frac{Z_5}{Z}(\frac{1}{Z_3} + \frac{1}{Z_4} + \frac{1}{Z_5})}]$$

To make sure the magnitude of the transfer function is always $H(s)$ =1, the other impedances need to meet the following conditions:

$$Z_3 = Z_4; \quad Z_5 \text{ can take any value.}$$

3. We substitute the specific value of $Z_i(i = 1, 2, 3, 4, 5)$ to equation 2
($Z_1 = Z_2 = 2.4 \times 10^3\Omega, Z_3 = 10^5\Omega, Z_4 = 5 \times 10^4\Omega, Z_5 = \frac{10^8}{j\omega}\Omega$), Then we can obtain:

$$|H(j\omega)| = \frac{\sqrt{1.821 \times 10^{-7} + \frac{1.5625}{\omega^2}}}{\sqrt{1.907 \times 10^{-7} + \frac{1.5625}{\omega^2}}}$$

Then, we plot the gain of the filter from 20 Hz to 20 kHz as Figure 2 shows:

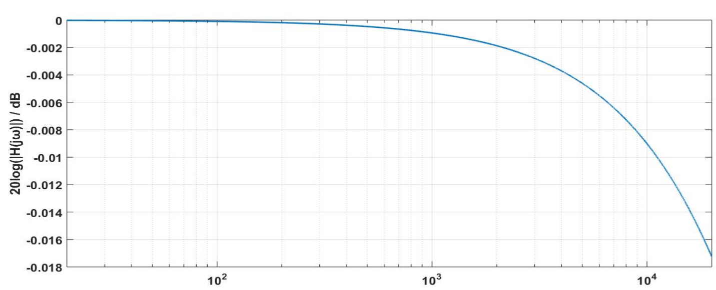

Figure 2: The gain in terms of frequency from 20Hz to 20kHz

According to Figure 2, it proved that the filter is a low-pass filter. Then, we switch R_3 and R_4 and plot the gain of the filter from 20 Hz to 20 kHz again:

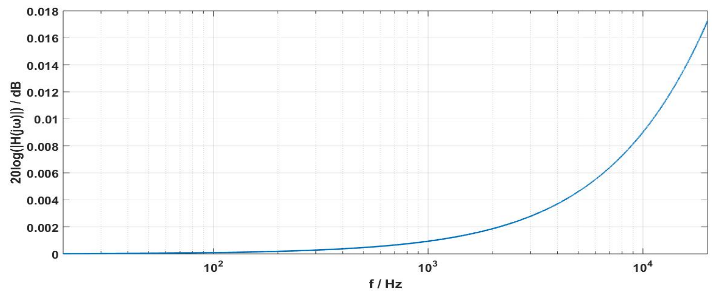

Figure 3: The gain in terms of frequency from 20Hz to 20kHz after switching R_3 and R_4

According to Figure 3, it proved that the filter is a high-pass filter.

4. We assume the new transfer function as H(s)' after switching Z_3 and Z_4, When $Z_1 = Z_2$, we find that

$$H(s)' = \frac{1}{H(s)}$$

Because H(s) is transfer function of a band-pass filter, H(s)' is function of a band-reject filter, which means the new one is a band-reject filter after switching Z_3 and Z_4.

2 Band-pass and Band-reject Filters

1. We can get the expressions below by using wye-delta transformation:

$$Z_1 = (1-\gamma)R_5$$

$$Z_2 = \gamma R_5$$

$$Z_3 = sC_1$$

$$Z_a = \frac{Z_2 Z_3}{Z_1 + Z_2 + Z_3} = \frac{\gamma R_5}{sC_1 R_5 + 1}$$

$$Z_b = \frac{Z_1 Z_3}{Z_1 + Z_2 + Z_3} = \frac{(1-\gamma)R_5}{sC_1 R_5 + 1}$$

$$Z_c = \frac{Z_1 Z_2}{Z_1 + Z_2 + Z_3} = \frac{s\gamma(1-\gamma)R_5^2 C_1}{sC_1 R_5 + 1}$$

In these equations, we assume that the the resistance on the left of the movable terminal is γR_5, then the resistance on the right is $(1-\gamma)R_5$.

2. We utilize the method and equations in item 1. Then we can find the same circuit as 3.1 item 1. Only some coefficients change as the expressions below.

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = -\frac{\frac{1}{Z_3} + \frac{Z_5}{Z_1}(\frac{1}{Z_3} + \frac{1}{Z_4} + \frac{1}{Z_5})}{\frac{1}{Z_4} + \frac{Z_5}{Z_2}(\frac{1}{Z_3} + \frac{1}{Z_4} + \frac{1}{Z_5})}$$

$$Z_a = R_1, Z_b = R_2, Z_c = R_3 + Z_a$$

$$Z_4 = R_4 + Z_b, Z_5 = \frac{1}{sC_2} + Z_c$$

3. As reference 5.1 shows, $f_0 = \frac{\sqrt{2+\frac{R_5}{R_3}}}{20\pi R_5 C_2} = \frac{\sqrt{3}}{\pi 10^5 C_2}$
for

$$f_0 = 500Hz \Rightarrow C_2 = \frac{\sqrt{3}}{2\times\pi\cdot 10^5\cdot 500} = 1.1\times 10^{-8}F$$

$$f_0 = 2.5kHz \Rightarrow C_2 = \frac{\sqrt{3}}{2 \times \pi \cdot 10^5 \cdot 2.5 \times 10^3} = 2.2 \times 10^{-9} F$$

$$f_0 = 10kHz \Rightarrow C_2 = \frac{\sqrt{3}}{2 \times \pi \cdot 10^5 \cdot 10 \times 10^3} = 5.5 \times 10^{-10} F$$

4. The results we obtained in MultiSim are as follows:

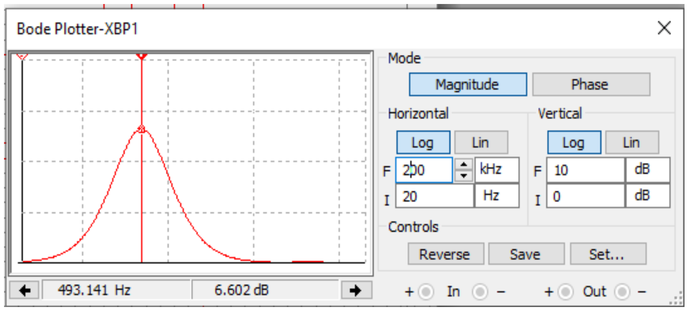

Figure 4: The analysis result of band-pass filter with a center frequency of 500Hz in MultiSim

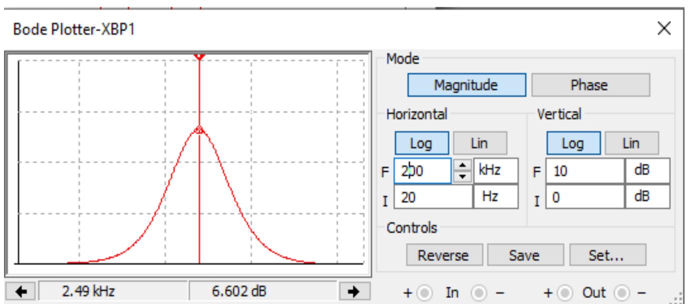

Figure 5: The analysis result of band-pass filter with a center frequency of 2.5kHz in MultiSim

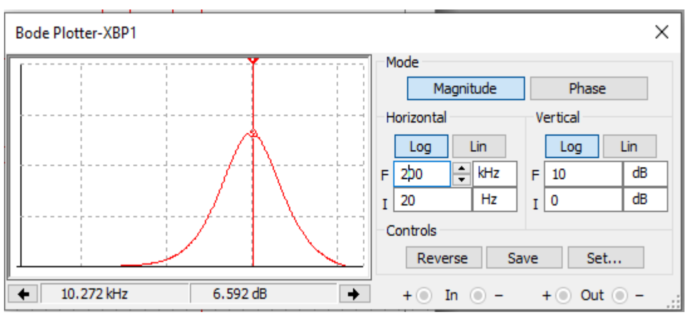

Figure 6: The analysis result of band-pass filter with a center frequency of 10kHz in MultiSim

The data that we obtained in MultiSim is as follows:

Table 1: The result of simulation in MultiSim

	filter 1	filter 2	filter 3
center frequency/Hz	493.141	2.49k	10.272k
cutoff frequency/Hz	259.766	1.268k	5.058k
	936.181	4.728k	18.853k
gain/dB	6.602	6.602	6.592

3 Y-Δ Transform

After reading the hint, we know that two circuits in figure 3 have the same external characteristics. That means, if a test voltage is connected to port N1 and N3, or N2 and N3, or N1 and N2, the current flow through the certain two ports in two circuits is the same. Hence, two circuits should have the same equivalent resistance between any two ports of N1, N2 and N3. So we get three equations listed below:

$$R_{N_1N_2Y} = R_{N_1N_2\Delta}$$

$$R_{N_2N_3Y} = R_{N_2N_3\Delta}$$

$$R_{N_1N_3Y} = R_{N_1N_3\Delta}$$

Then we expand them, and get the equations below:

$$Z_1 + Z_2 = \frac{Z_c(Z_a + Z_b)}{Z_a + Z_b + Z_c}$$

$$Z_2 + Z_3 = \frac{Z_a(Z_b + Z_c)}{Z_a + Z_b + Z_c}$$

$$Z_1 + Z_3 = \frac{Z_b(Z_a + Z_c)}{Z_a + Z_b + Z_c}$$

Then we get the expressions of Z_1 , Z_2 *and* Z_3 below:

$$Z_1 = \frac{Z_bZ_c}{Z_a + Z_b + Z_c}$$

$$Z_2 = \frac{Z_aZ_c}{Z_a + Z_b + Z_c}$$

$$Z_3 = \frac{Z_aZ_b}{Z_a + Z_b + Z_c}$$

These are the equations in item 2.

Then we utilize these three equations to derive the equations in item 1. We get the equations below:

$$\frac{Z_a}{Z_b} = \frac{Z_2}{Z_1}$$

$$\frac{Z_c}{Z_b} = \frac{Z_2}{Z_3}$$

Then we substitute these two equations into the three equations in item 2. We get the equation below:

$$\frac{\frac{Z_2^2}{Z_1Z_3}Z_b}{(1 + \frac{Z_2}{Z_1} + \frac{Z_2}{Z_3})} = Z_2$$

$$Z_b = \frac{Z_1Z_3 + Z_1Z_2 + Z_2Z_3}{Z_2}$$

Similarly, we can also get:

$$Z_a = \frac{Z_1Z_3 + Z_1Z_2 + Z_2Z_3}{Z_1}$$

$$Z_c = \frac{Z_1Z_3 + Z_1Z_2 + Z_2Z_3}{Z_3}$$

Thus we get the equations in item 1.