The Leontief Input-Output Model

Source: http://barnyard.syr.edu/mat183/l32/

This topic introduces an important application of matrix inversion in modern economic theory. The Leontief model describes a simplified view of an economy. Its goal is to predict the proper level of production for each of several types of goods or service. The proper level of production is the one which meets two requirements:

a. There should be enough of each good to meet the demand for it.
b. There should be no "leftovers", i.e, unused goods.

In a real economy there are tens of thousands of different goods and services, but we can often simplify matters by combining goods into categories. For example, let us consider a very simple economy that runs on just 3 different types of output: raw materials, services, and manufacturing. Raw materials include the output of many different industries, agriculture and mining to name two. Services include retailing, advertising, transportation, etc.

Now, the raw materials industry needs some of the output from the other two industries to do its job. For example, it needs trucking to get its goods to market, and it uses some manufactured goods (machines.) The raw materials industry even needs some of its own output to produce its own output -- iron ore to make the steel to build the rails that carry ore from the mines, for example.

Similarly, each of the other two industries requires some amount of output from each of the three to do its job. All of these requirements can be summarized in the form of a table such as the following:

Industry	Raw Materials	Services	Manufacturing
Raw Materials	0.02	0.04	0.04
Services	.05	.03	0.01
Manufacturing	.2	.01	.1

The numbers in the table tell how much output from each industry a given industry requires in order to produce **one dollar** of its own output. For example, to provide $1 worth of service, the service sector requires $.05 worth of raw materials, $.03 worth of services, and $.01 worth of manufactured goods.

The information in the table can be more compactly described by dropping the headings. The result is a 3x3 matrix called the **input-output matrix:**

$$A = \begin{bmatrix} .02 & .04 & .04 \\ .05 & .03 & .01 \\ .2 & .01 & .1 \end{bmatrix}$$

A second important matrix, the **demand matrix,** tells how much (in, say, billions of dollars) of each type of output is demanded by consumers and others outside the economy ("exports".) For example, we might have

$$D = \begin{bmatrix} 400 \\ 200 \\ 600 \end{bmatrix}$$

For example, the 200 in the second position means that $200 billion worth of services are demanded by consumers and exporters.

Finally, let X denote the **production matrix.** It is another column of length 3 that represents the amounts (in billions of dollars of value) produced by each of the three industries. We can't fill in the entries of X yet because we don't yet know the levels of production that will meet the two requirements above. But we can still interpret the meaning of the matrix product AX: it is also a column of length 3 that represents that part of the production which is used **internally,** i.e, by the industries themselves in order to produce their goods. The difference X - AX = (I-A)X then represents how much of the output remains to satisfy the external demand. This demand will be exactly met with no leftover waste provided (I-A)X = D. But we can easily solve this matrix equation for X

$$X = (I - A)^{-1} \times D$$

thereby obtaining a formula for the required levels of production. In our example we obtain

$$\begin{bmatrix} .98 & -.04 & -.04 \\ -.05 & .97 & -.01 \\ -.2 & -.01 & .9 \end{bmatrix}^{-1} \times \begin{bmatrix} 400 \\ 200 \\ 600 \end{bmatrix} = \begin{bmatrix} 449.24 \\ 237.27 \\ 769.13 \end{bmatrix}$$

Thus, the service sector should produce $237.27 billion worth of services, etc, in order for the economy to "balance".

The overall approach is quite flexible in that small sectors of the economy, even individual businesses, can be modelled in a similar way -- we merely have to regard the rest of the economy as being part of the "customers and exports" category.

Practice problem:

Assume there are 5 sectors of an economy:

Sector 1: Auto
Sector 2: Steel
Sector 3: Electricity
Sector 4: Coal
Sector 5: Chemical

For Sector 1, to produce $1 of output it requires the following inputs from each sector.

Sector 1	Sector 2	Sector 3	Sector 4	Sector 5
$0.15	$0.10	$0.05	$0.05	$0.10

Similarly, for other sectors:

Sector 2:

Sector 1	Sector 2	Sector 3	Sector 4	Sector 5
$0.40	$0.20	$0.10	$0.10	$0.10

Sector 3

Sector 1	Sector 2	Sector 3	Sector 4	Sector 5
$0.10	$0.25	$0.20	$0.10	$0.20

Sector 4

Sector 1	Sector 2	Sector 3	Sector 4	Sector 5
$0.10	$0.20	$0.30	$0.15	$0.10

Sector 5

Sector 1	Sector 2	Sector 3	Sector 4	Sector 5
$0.05	$0.10	$0.05	$0.02	$0.05

I. Construct the Input-Output Matrix in R from the data given above. Remember the correct way to use the **matrix(data,rows,columns)** command. For initializing the matrix you will have to use the **c(…)** command on the correct numeric data.
II. We are given a demand vector **D** as follows:

$$\mathbf{D} = \begin{bmatrix} 100 \\ 200 \\ 300 \\ 400 \\ 500 \end{bmatrix}$$

Calculate the production vector **X** to satisfy both internal consumption and demand.