Matrix Review

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Outline:

- Matrix Review

Matrix review

- $\boldsymbol{A}_{r\times c} = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1c} \\ a_{21} & a_{22} & \dots & a_{2c} \\ \vdots & \vdots & \vdots & \vdots \\ a_{r1} & a_{r2} & \dots & a_{rc} \end{bmatrix}$
- Vectors (one column) and scalars (one row and one column) are special types of matrix.

Matrix review

- Square matrices (r=c) and diagonal matrices (only elements are a_{ij} where $i = j$ and 0 elsewhere)
- Symmetric matrix has $a_{ij} = a_{ji}$ for all i, j

Matrix review

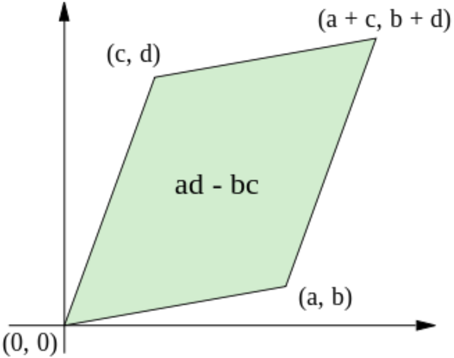
- Trace of a matrix trace($\boldsymbol{A}$) $= \sum_{i=1}^{c} a_{ii}$, sum of diagonals
- Transpose of $\boldsymbol{A}$: $\boldsymbol{A}'_{r\times c}$ flip matrix on diagonal
- $\boldsymbol{A}_{r\times c} + \boldsymbol{B}_{r\times c} = \{a_{ij} + b_{ij}\}_{r\times c}$
- Five types of multiplication: elementwise, matrix, scalar, Kronecker, horizontal direct product
- Most common is matrix multiplication which must have conformable dimenstions
- $\boldsymbol{A}_{r\times c} * \boldsymbol{B}_{r\times c}$ does not work (non-conformable)
- $\boldsymbol{A}_{r\times c} * \boldsymbol{B}_{c\times r}$ does (conformable)
- $\begin{bmatrix} 1 & 2 \\ 1 & 3 \\ 1 & 5 \end{bmatrix} * \begin{bmatrix} 1 & 2 & 2 \\ 1 & 3 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 8 & 10 \\ 4 & 11 & 14 \\ 6 & 17 & 22 \end{bmatrix}$

Matrix review

- Commutative law works for addition
- Distributive laws: Remember that $\boldsymbol{A} * \boldsymbol{B} \neq \boldsymbol{B} * \boldsymbol{A}$ except in special cases
- $(\boldsymbol{A} * \boldsymbol{B})' = \boldsymbol{B}' * \boldsymbol{A}'$
- Determinant: $|\boldsymbol{A}|$, only for square matrices
- $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bd$

Matrix review

- Determinant is a index of the variability in a matrix
- Larger magnitude implies larger variability
- Determinant visualization



Matrix review

- Rank, the number of linearly independent columns in a matrix (typically the number of columns)
- A less than full rank matrix, where rank is less than number of columns, is called singular
- A full rank matrix is called nonsingular
- The inverse of a matrix, $\boldsymbol{A}^{-1}$, exists for all nonsingular square matrices such that $\boldsymbol{A} * \boldsymbol{A}^{-1} = \boldsymbol{I}$
- For singular square matrices, the generalized inverse ($\boldsymbol{A}^{+}$) exists but is not unique
- Eigenvalues, any number λ that is the solution to the equation $|\boldsymbol{A} - \lambda \boldsymbol{I}|$

Matrix Review: Statistical quantities

- Sum of Squares
- The matrix $\boldsymbol{A} * \boldsymbol{A}'$ has elements $\sum a_{ij} * a_{rs}$ with diagonal elements $\sum a_{ij}^2$
- Covariance matrix
- $\boldsymbol{S} = \boldsymbol{D}'\boldsymbol{D}\frac{1}{N-1}$ where $\boldsymbol{D}_{r \times c}$ is $\boldsymbol{A}_{r \times c}$ subtracting the column mean from each corresponding column entree
- OR $\boldsymbol{D}_{r \times c} = (\boldsymbol{I}_{r \times r} - \frac{1}{c}\mathbf{1}_{r \times r})\boldsymbol{A}_{r \times c}$

Matrix Review

- Can use matrices to represent a series of linear equations, e.g.

$$\begin{aligned} 5x + 7y &= -11 \\ 8x + 4y &= 4 \\ 5x + 5y &= -5 \end{aligned}$$

- $\boldsymbol{A}_{2\times 2} = \begin{bmatrix} 5 & 7 \\ 8 & 4 \\ 5 & 5 \end{bmatrix}$, $\boldsymbol{b}_{2\times 1} = \begin{bmatrix} x \\ y \end{bmatrix}$, $\boldsymbol{c}_{2\times 1} = \begin{bmatrix} -11 \\ 4 \\ -5 \end{bmatrix}$.
- Thus, the series of equations can be written as

$$\boldsymbol{A} * \boldsymbol{b} = \boldsymbol{c}$$

Questions?