LECTURE 4.2 – 2D KINEMATICS
PROJECTILE MOTION

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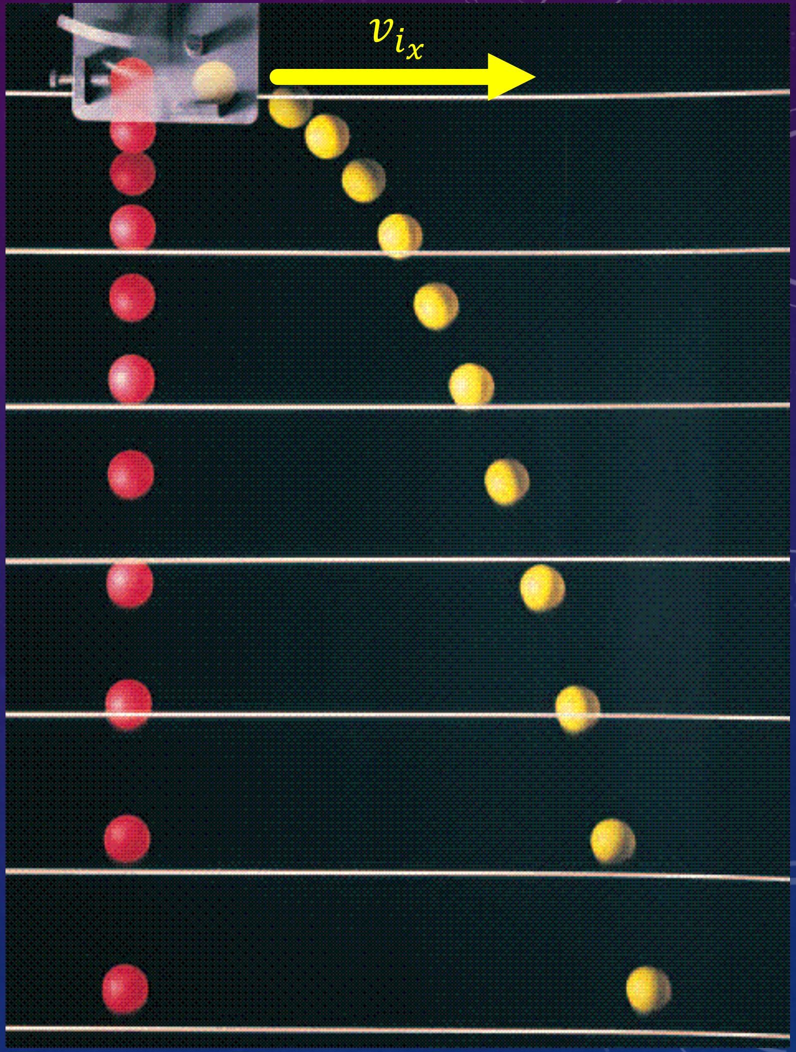
PROJECTILE MOTION

- A projectile is an object that moves in two dimensions under the influence of only gravity.
- Projectile motion is an extension of free-fall motion.
- We can neglect the influence of air resistance if:
    1. The object is *relatively* heavy
    2. The object is moving *relatively* slowly
    3. The object is traveling over a *relatively* short distance
- Projectiles in two dimensions follow a *parabolic trajectory*



The red ball is dropped from rest while the yellow ball has an initial velocity in the x-direction. Still, both balls hit the ground at the same time.

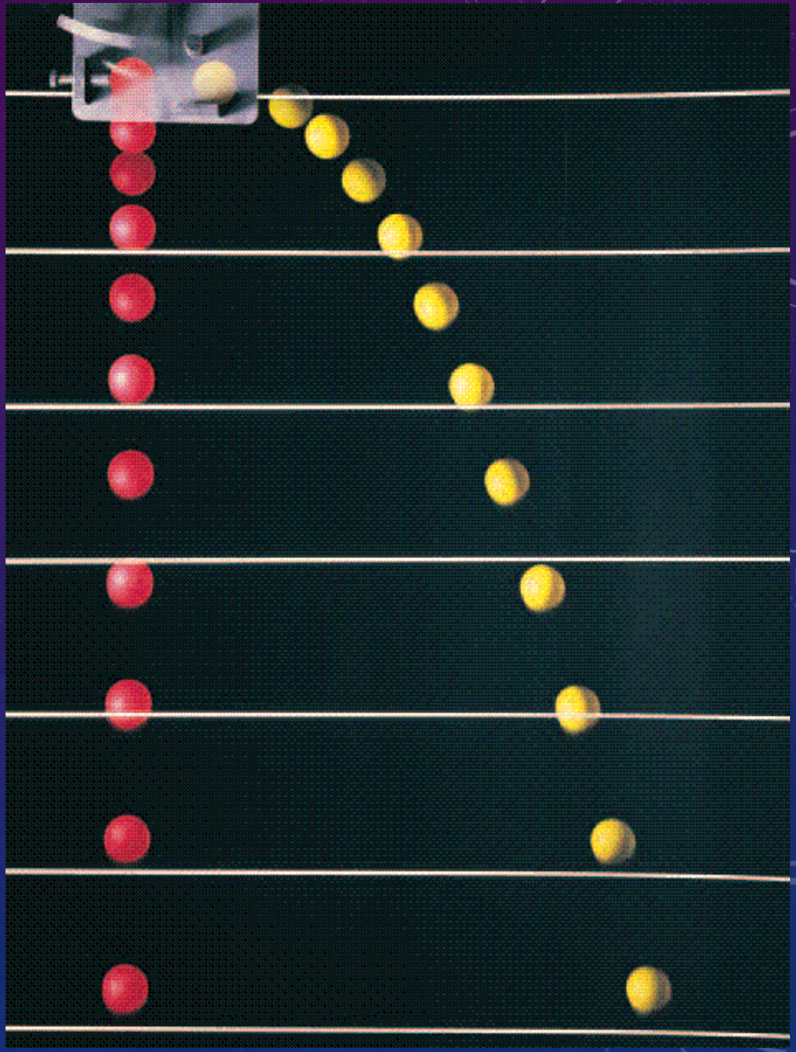
https://www.youtube.com/watch?v=zMF4CD7i3hg&t=60s

DEMO

WHY DOES THIS HAPPEN?

If air resistance is negligible, two objects hit the ground *simultaneously.* They do so because the horizontal and vertical components of projectile motion are independent of each other. The initial horizontal velocity of the second ball has no influence over its vertical motion.

Neither ball has any initial motion in the vertical direction, so both fall a distance, *h*, in the same amount of time.



LET'S MODEL A BALL'S TRAJECTORY WITH PARAMETRIC EQUATIONS!

- What do we know?
  - The initial velocities in the *x* and *y* directions are independent of one another.
  - In free-fall, the motion in the *x* and *y* directions are independent of one another.
  - Gravity (pointing downward) is the only acceleration acting on the ball.

<u>The *x*-direction</u>

$$x(t) = x_i + v_{i_x}\Delta t + \frac{1}{2}a\Delta t^2$$

What about acceleration?

$a_x = 0\frac{m}{s^2}$ (No *a* in the x-direction)

$$x(t) = x_i + v_{i_x}\Delta t$$
$$v_x(t) = v_{i_x}$$

<u>The *y*-direction</u>

$$y(t) = y_i + v_{i_y}\Delta t + \frac{1}{2}a\Delta t^2$$

What about acceleration?

$a_x = -9.8\frac{m}{s^2} = -g$

$$y(t) = y_i + v_{i_y}\Delta t - \frac{1}{2}g\Delta t^2$$
$$v_y(t) = v_{i_y} - g\Delta t$$

$$\vec{r}(t) = \left[x_i + v_{i_x}\Delta t\right]\hat{\imath} + \left[y_i + v_{i_y}\Delta t - \frac{1}{2}g\Delta t^2\right]\hat{\jmath}$$

$$\vec{v}(t) = \left[v_{i_x}\right]\hat{\imath} + \left[v_{i_y} - g\Delta t\right]\hat{\jmath}$$

EXAMPLE #1

Consider the particle with trajectory:

$$\vec{r}(t) = \left[x_i + v_{i_x}\Delta t\right]\hat{\imath} + \left[y_i + v_{i_y}\Delta t - \frac{1}{2}g\Delta t^2\right]\hat{\jmath}$$

$$(x_i, y_i) = (0\text{m}, 0\text{m})$$

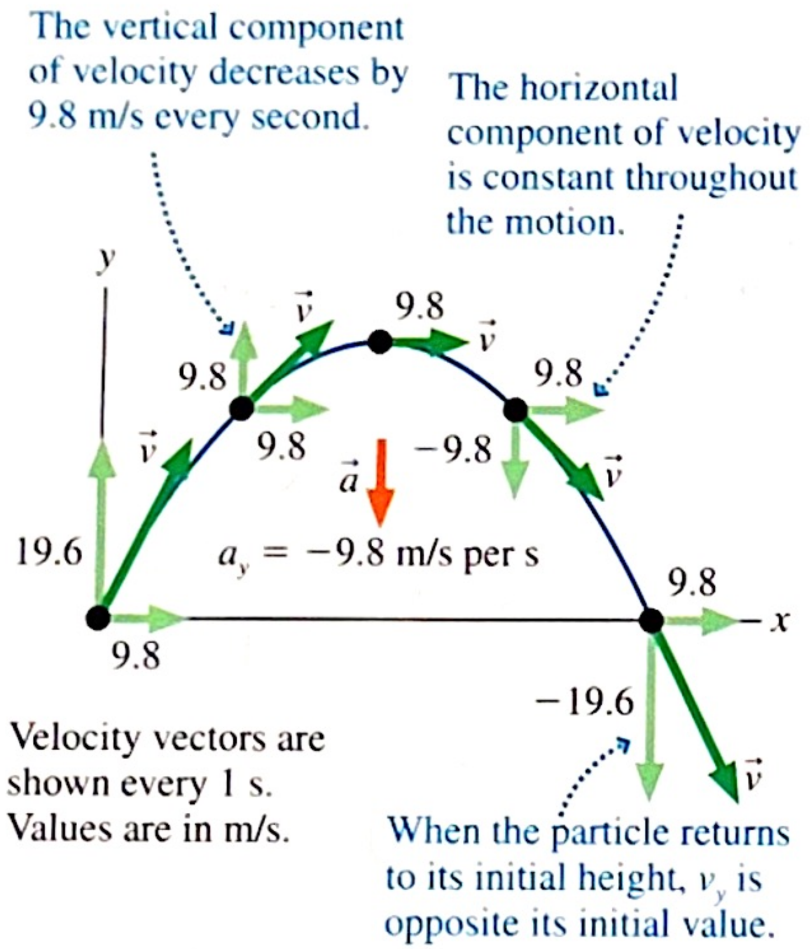
$$v_{i_x} = 9.8\,\frac{\text{m}}{\text{s}};\ v_{i_y} = 19.6\,\frac{\text{m}}{\text{s}}$$

$$a_x = 0\,\frac{\text{m}}{\text{s}^2};\ a_y = -g = -9.8\,\frac{\text{m}}{\text{s}^2}$$

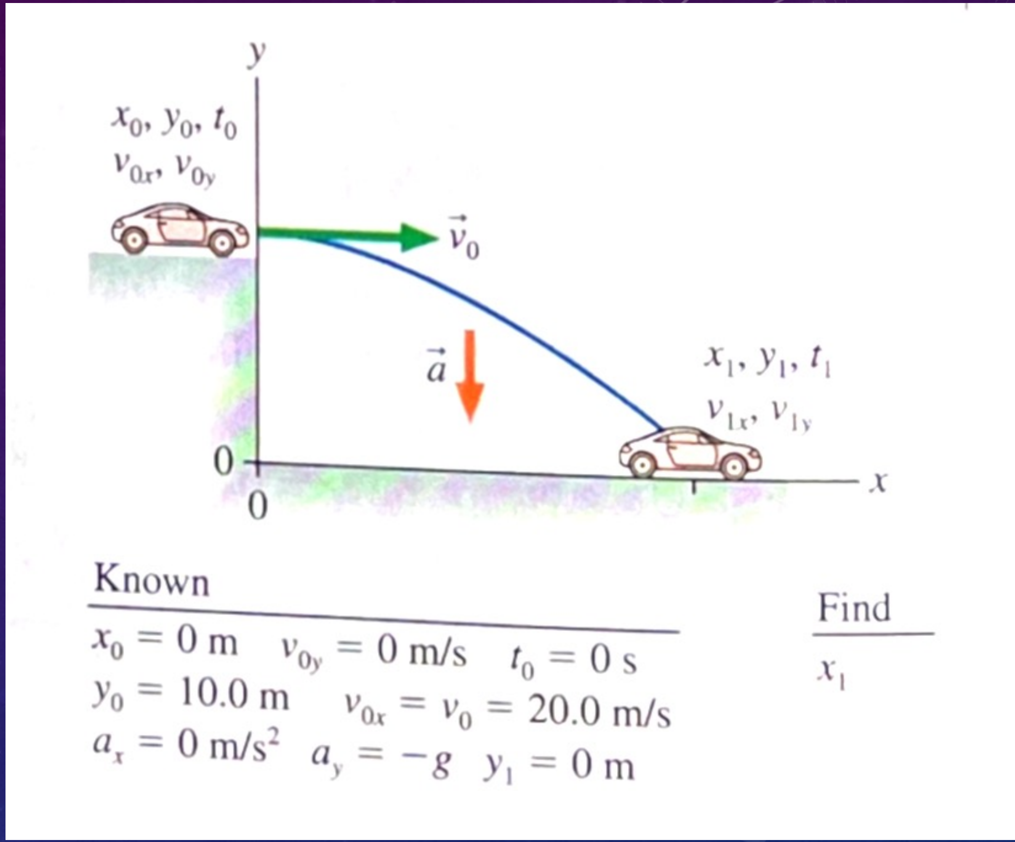
$$\vec{r}(t) = \left[\left(9.8\,\frac{\text{m}}{s}\right)\Delta t\right]\hat{\imath} + \left[\left(19.6\,\frac{m}{s}\right)\Delta t - \frac{1}{2}\left(9.8\,\frac{\text{m}}{\text{s}^2}\right)\Delta t^2\right]\hat{\jmath}$$

Notice that $9.8\,\frac{m}{s}$ is subtracted from the *vertical* velocity each second. The horizontal velocity stays the same.

FIGURE 4.17 The velocity and acceleration vectors of a projectile moving along a parabolic trajectory.



EXAMPLE #2



A stunt man drives a car off a 10.0-m-high cliff at a speed of $20.0\frac{\text{m}}{\text{s}}$. How far does the car land from the base of the cliff?

$$x_f = x_i + v_{i_x}\Delta t = 0\text{m} + \left(20.0\frac{\text{m}}{\text{s}}\right)(t_f - 0\text{s}) = \left(20.0\frac{\text{m}}{\text{s}}\right)t_f$$

$$y_f = y_i + v_{i_y}\Delta t + \frac{1}{2}a\Delta t^2 = 10\text{m} + \left(0\frac{\text{m}}{\text{s}}\right)(t_f - 0s) + \frac{1}{2}\left(-9.8\frac{\text{m}}{s^2}\right)(t_f - 0s)^2$$

We have enough information to calculate the amount of time taken for the y-direction motion to be completed.

$$0\text{m} = (10m) - \frac{1}{2}\left(9.8\frac{\text{m}}{s^2}\right)t_f^2$$

$$t = 1.43\text{s}$$

The time required for the *y*-direction motion to happen is the same amount of time required for the *x*-direction.

$$x_f = \left(20.0\frac{\text{m}}{\text{s}}\right)\cdot(1.43s) = 28.6\text{m}$$

NO TIME GIVEN

In the previous example, the time-of-flight was not known. We were able to determine the TOF from the motion in the y-direction then use this in the x-direction.

We can also do the same thing with our parametric equations!

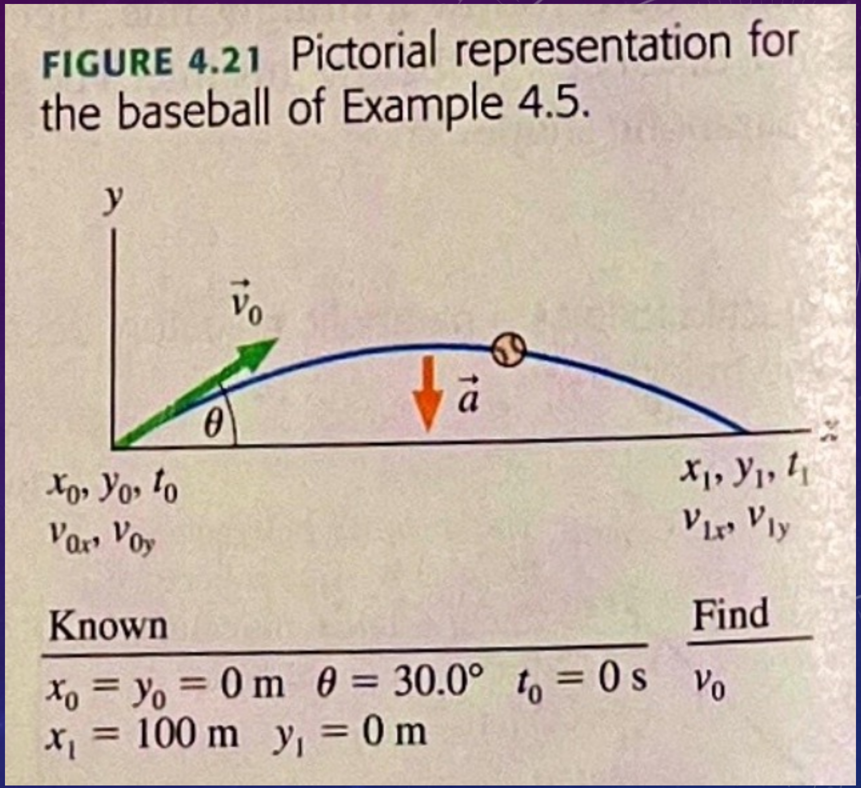
$x = v_{i_x} t$ can be rewritten as $t = \frac{x}{v_{i_x}}$

$$y = y_i + v_{i_y} t - \frac{1}{2} g t^2 = y_i - \frac{1}{2} g \left(\frac{x}{v_{i_x}}\right)^2 = y_i - \frac{1}{2}\left(\frac{g}{v_{i_x}^2}\right) x^2$$

This is $y(x)$. Graphing this is the trajectory!

EXAMPLE #3

A baseball is hit at an angle θ and is caught at the height from which it was hit. If the ball is hit at a $30°$ angle, with what speed must it leave the bat to travel 100m?



$$x_f = v_{0_x} t = \vec{v}_0 \cos(\theta)\, t$$

$$0\text{m} = v_{0_y} t - \frac{1}{2} g t^2 = v_0 \sin(\theta)\, t - \frac{1}{2} g t^2$$

$$\frac{1}{2} g t = v_0 \sin(\theta)$$

The ball has a y-displacement of 0m at $t = 0s$ and $t = \dfrac{2v_0 \sin(\theta)}{g}$.

$$x_f = v_0 \cos(\theta) \left(\frac{2v_0 \sin(\theta)}{g} \right) = \frac{2v_0^2 \sin(\theta)\cos(\theta)}{g} = \frac{v_0^2 \sin(2\theta)}{g}$$

Solving for v_0,

$$v_0 = \sqrt{\frac{g x_f}{\sin(2\theta)}} = \sqrt{\frac{9.8\frac{m}{s^2} \cdot 100m}{\sin(2 \cdot 30°)}} = 33.6\,\frac{\text{m}}{\text{s}}$$

ONE LAST TIDBIT

- A projectile launched at θ or $90^{\circ} - \theta$ will travel the same distance