Lecture 2.1
1D Kinematics – Uniform Motion

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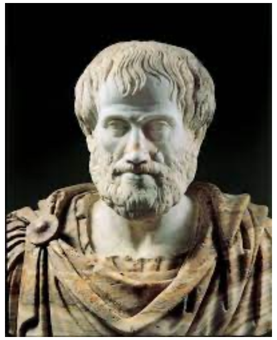
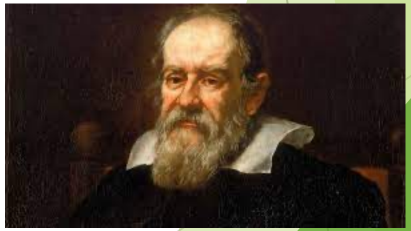
History

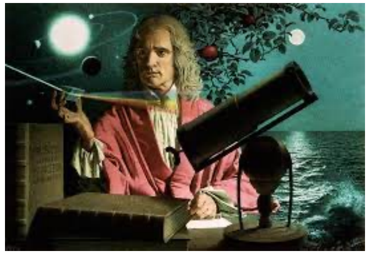
- The motion of the stars and planet across the night sky has intrigued many throughout history.



  - Aristotle
    - Proposed a geocentric model of the solar system



  - Galileo
    - Astronomical observations supported a heliocentric model of the solar system
  - Newton
    - Invented Calculus



Motion

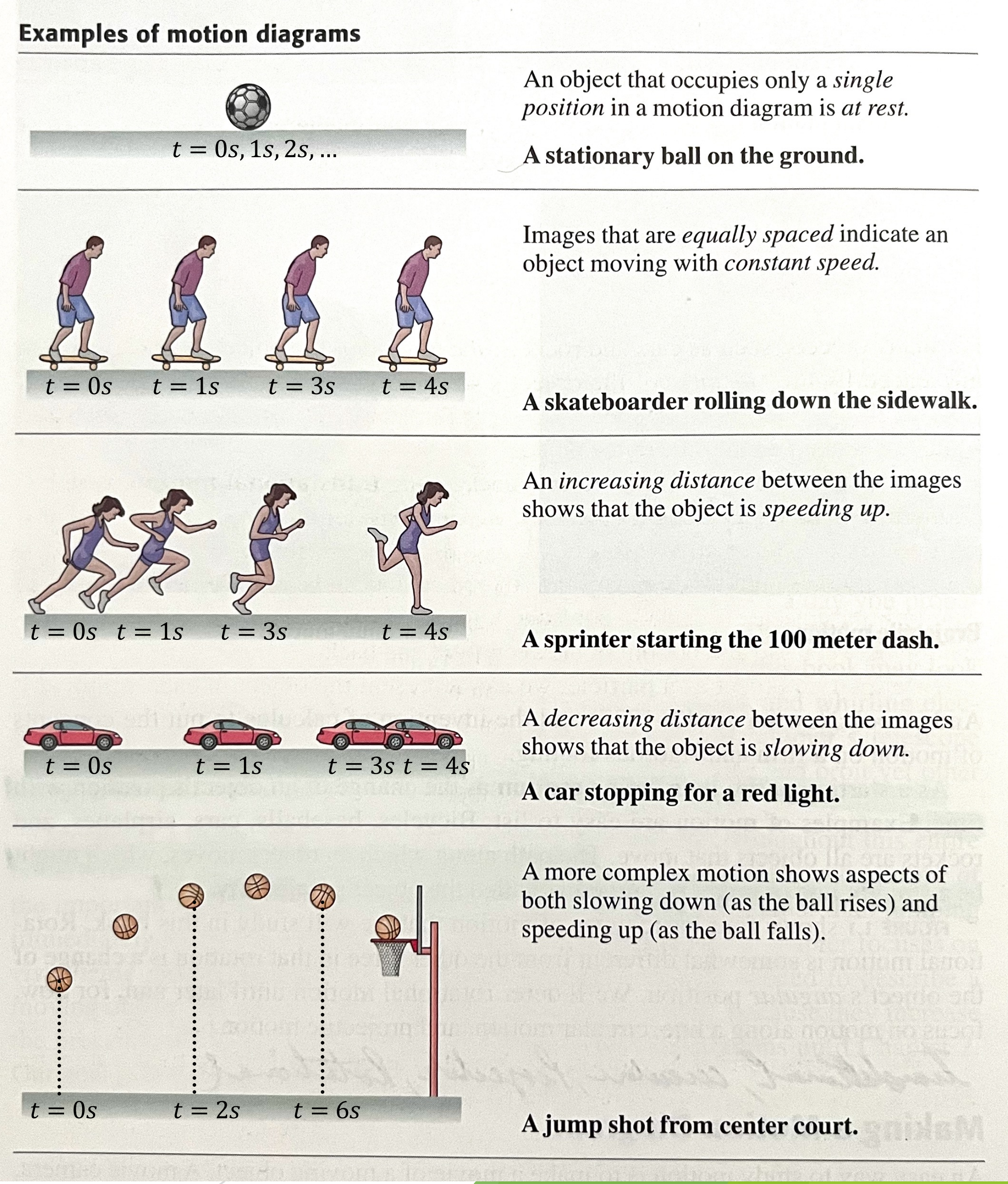
- A fundamental definition of motion is the change of an object's position with time. The path along which it moves is called the object's trajectory.

- There are four basic types of motion:
  - Translational
  - Projectile
  - Circular
  - Rotational

Motion Diagram

- A simple way to study motion is with a motion diagram. This tool shows an object’s position at equally spaced time intervals.

- Some translational diagrams can be simplified as particle models
  - Objects are treated as single points. This point has no size, shape, or distinguishing features. This only works if the object’s shape doesn’t affect its motion. (For example, a rotating gear cannot be represented as a point particle.)



Position and Time

- Position
  - Formal position measurements can be made by laying a coordinate grid over a motion diagram. The origin of a coordinate system is arbitrary and can be placed anywhere.
- Time
  - Time is also a coordinate system. An arbitrary point in the motion of an object can be set as $t = 0s$. (This is the “origin” of the time coordinate.)
  - Motion occurring before $t = 0s$ is represented as a negative time.
- It is important to note that multiple observers can have different position and time origins, but the physics (motion of a particle) will be described the same way in all reference frames.

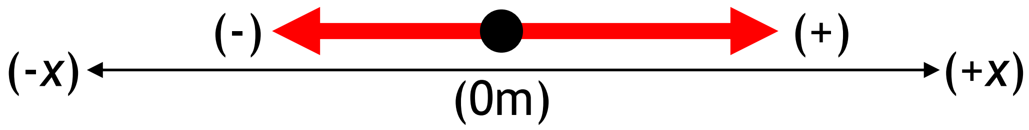
1D Vectors

- Position is a vector quantity. This means that is has a magnitude and a direction. (A consequence of adding a coordinate system!)

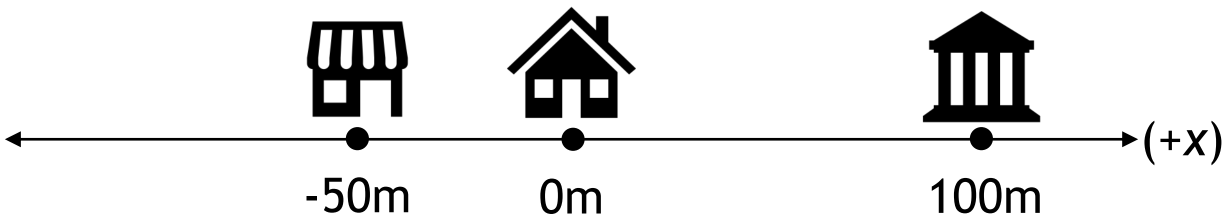
- Scalar quantities (such as temperature) do not have an associated direction.

- Position vectors are indicated as $\vec{r}$ and are measured from an origin.

- Vectors are represented graphically by arrows. The direction of the arrow represents the direction of the position from the origin. The amount of distance (magnitude) is represented by the length of the arrow.



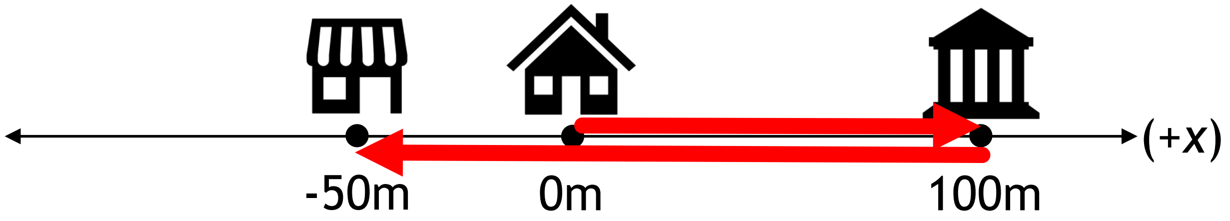
Example



Suppose Sam starts at his house (origin) and walks 100m to the East, arriving at the post office. Shortly after, he walks -150m to the coffee shop to the West of his home.

- The negative sign here tells us that there was a change in direction. (+) is motion to the East and (-) is motion to the West. This gives us information about the associated vectors.

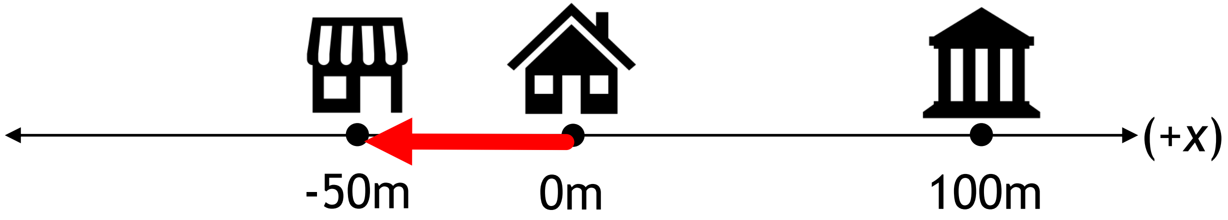
Total Distance vs. Total Displacement



- The total distance traveled is how much distance Sam covered on foot. Here, $100\text{m (East)} + |-150\text{m}|\text{(West)}$ (only the magnitude is needed). 250m is covered.

- What about the displacement? This is the difference between the starting point and ending point regardless of the object's trajectory.
  - This means that an object could travel to the moon and back but, if it returns to the same spot it started from, its displacement would be zero.
  - Displacement vectors are represented as $\Delta\vec{r}$ where the Delta (Δ) represents the difference between the final and initial positions: $\Delta\vec{r} = \vec{r}_f - \vec{r}_i$
    - $\vec{r}_i$ and $\vec{r}_f$ are position vectors and always measured from the origin.

Sam's Displacement

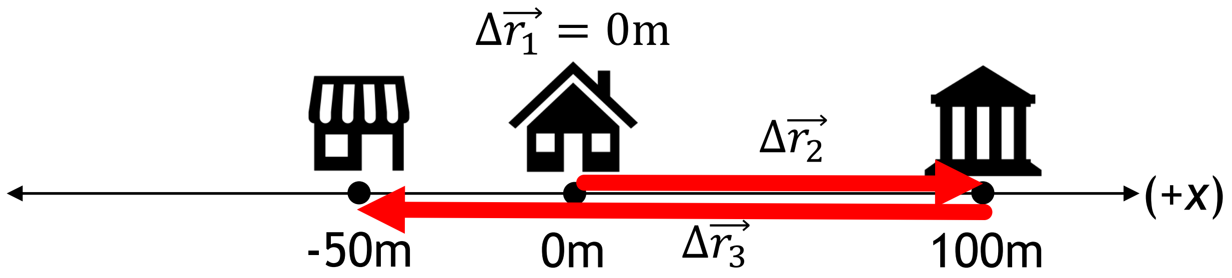


- $\Delta\vec{r} = \vec{r}_f - \vec{r}_i = -50\text{m} - 0\text{m} = -50\text{m}$

  - Since Sam started at the origin (his home), $\vec{r}_i = 0\text{m}$. Since the coffee shop is 50m West of his home (origin), $\vec{r}_f = -50\text{m}$.

  - Notice that his trip to the post office is not reflected in his displacement.

Displacement Vectors



- Intermediate displacement vectors can also be calculated. (The “pieces” of Sam’s walk)

$$\Delta\vec{r} = \vec{r}_f - \vec{r}_i = \sum_{k=1}^{n} \Delta\vec{r}_k$$

$$\Delta\vec{r} = \vec{r}_f - \vec{r}_i = \Delta\vec{r}_1 + \Delta\vec{r}_2 + \Delta\vec{r}_3 = 0\text{m} + 100\text{m} + (-150\text{m}) = -50\text{m}$$

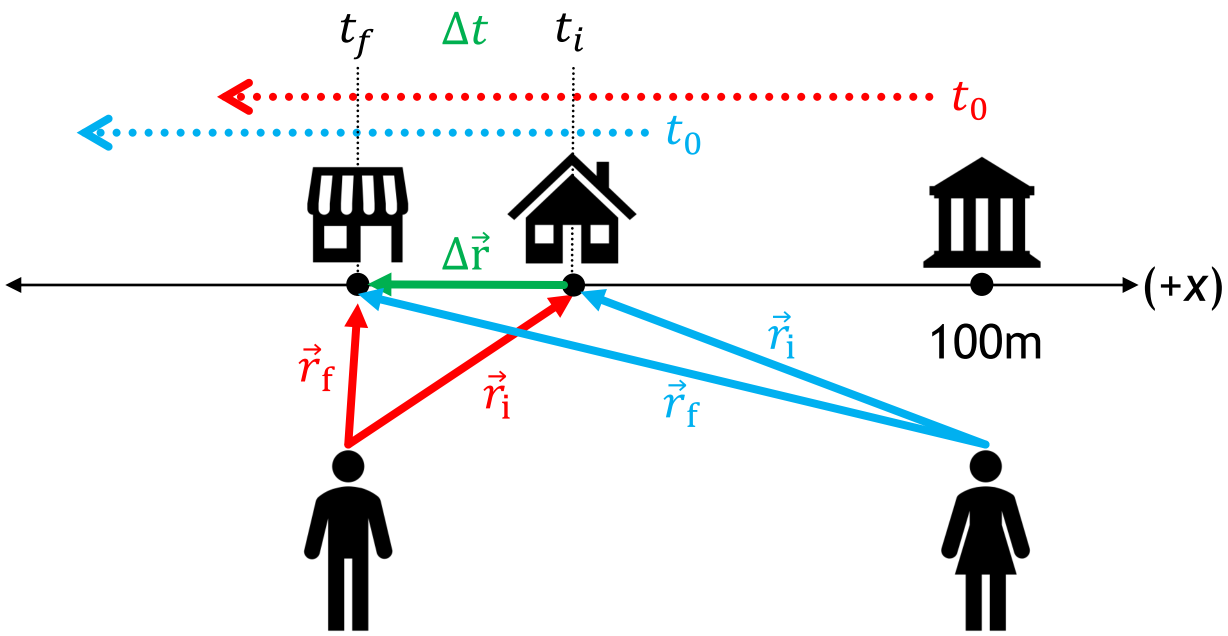
Time

- It is also useful to consider changes in time.

- Two times, t_1 and t_2, may be assigned to two different positions of a particle in motion. The two values of time are arbitrary since they are measured from an origin ($t = 0s$). However, the difference in time, $\Delta t = t_2 - t_1$, is not arbitrary. It represents the amount of time taken for an object to move from $\vec{r}_1$ to $\vec{r}_2$.

Note!

Suppose a group of observers detect an object moving from an initial position ($\vec{r}_i$) to a final position ($\vec{r}_f$) but they all used a different coordinate system. Furthermore, all observers recorded the time of these positions as t_i and t_f but all started their stopwatches at different times before the motion started. Nonetheless, even though $\vec{r}_i$, $\vec{r}_f$, t_i, and t_f are different for each observer, $\Delta\vec{r}$ and Δt are independent of the coordinate systems



Speed vs. Velocity

- A common way to think about the "fastness" or "slowness" of an object is to consider its average speed. This is the distance traveled divided by the time spent traveling.

$$\text{average speed} = \frac{\text{distance traveled}}{\text{time spent traveling}}$$

- The problem here is that there is no directional information. Thus, average speed is a scalar!

- When directional information is required, the average velocity is necessary. For this, we can scale the displacement vector ($\Delta\vec{r}$) by the amount of time required for the displacement to occur (Δt).

$$\vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t}$$

- $\vec{v}_{avg}$ points in the same direction as $\Delta\vec{r}$. Longer velocity vectors indicate faster motion.

Sam Revisited (…again)

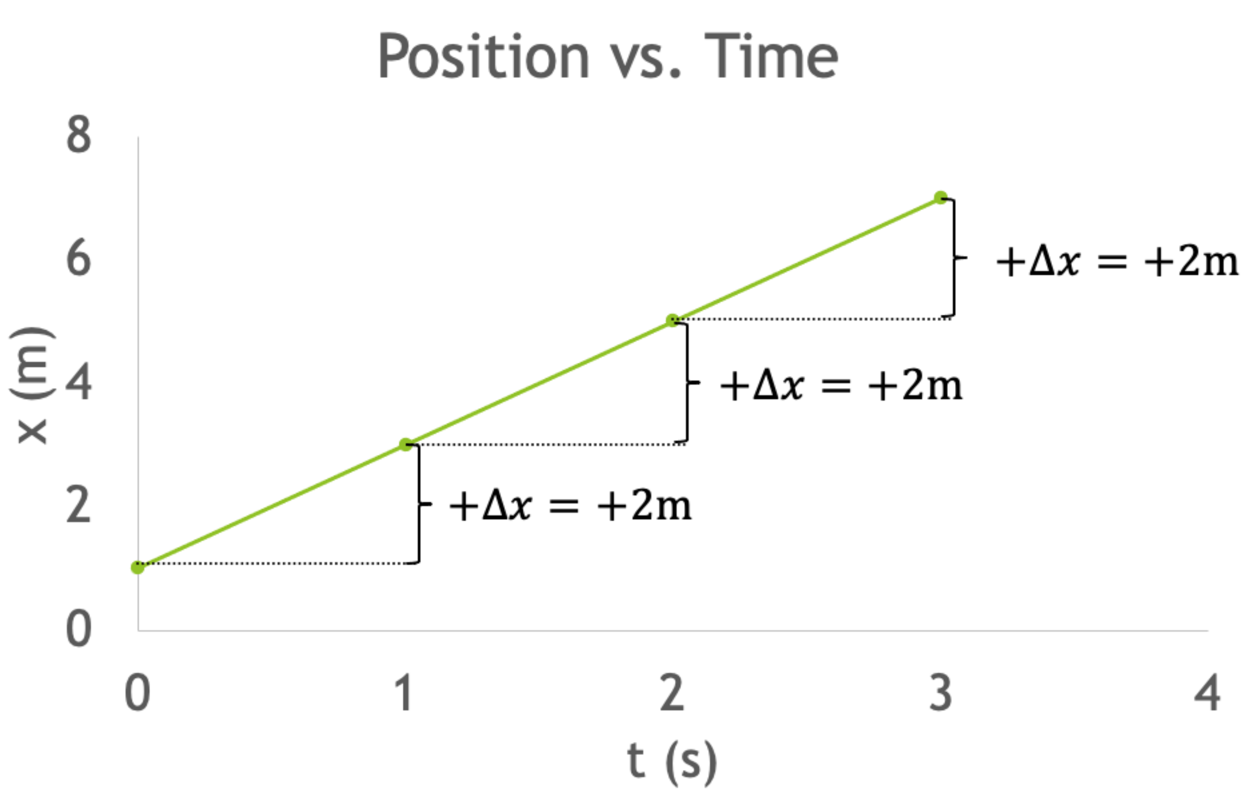
Suppose Sam starts at his home (origin) and walks 100m to the East in 100s to arrive at the post office. He stays here for 3 minutes before heading West -150m to the coffee shop near his home. This segment was completed in 200s. What was his average speed and velocity?

$$\text{average speed} = \frac{\text{distance traveled}}{\text{time spent traveling}} = \frac{100\text{m} + 0\text{m} + |-150\text{m}|}{100\text{s} + 180\text{s} + 200\text{s}} = \frac{250\text{m}}{480\text{s}} = 0.52\,\frac{\text{m}}{\text{s}}$$

$$\vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\vec{r}_f - \vec{r}_i}{t_f - t_i} = \frac{-50\text{m} - 0\text{m}}{480\text{s} - 0\text{s}} = \frac{-50\text{m}}{480\text{s}} = -0.104\,\frac{\text{m}}{\text{s}}$$

Uniform Motion

- Straight-line motion with equal displacement occurring in equal-time intervals is called uniform motion.
- Consider a plot of the horizontal distance vs. time of an object moving to the right (+x) under uniform motion.
- Because the motion is uniform, the same amount of distance is added for each time interval.



Notes about position vs. time graphs:

- Steeper slopes correspond to faster speeds
    - More speed is being added in less time
- Negative slopes correspond to negative velocities
- The slope is a ratio of intervals, not coordinates ($\frac{\Delta x}{\Delta t}$ not $\frac{x}{t}$)

Our First Kinematic Equation

Suppose we want an equation that will allow us to determine an object's position at some time t provided it is undergoing uniform motion. (This equation is important for space-craft.)

A slight tweak of our average velocity equation allows us to write a new one!

$$\vec{v}_{avg} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i} = \frac{x_f - x_i}{\Delta t}$$

$$x_f = x_i + \vec{v}_{avg}\Delta t$$

$$x_f = x_i + v\Delta t$$

This is just the slope equation!

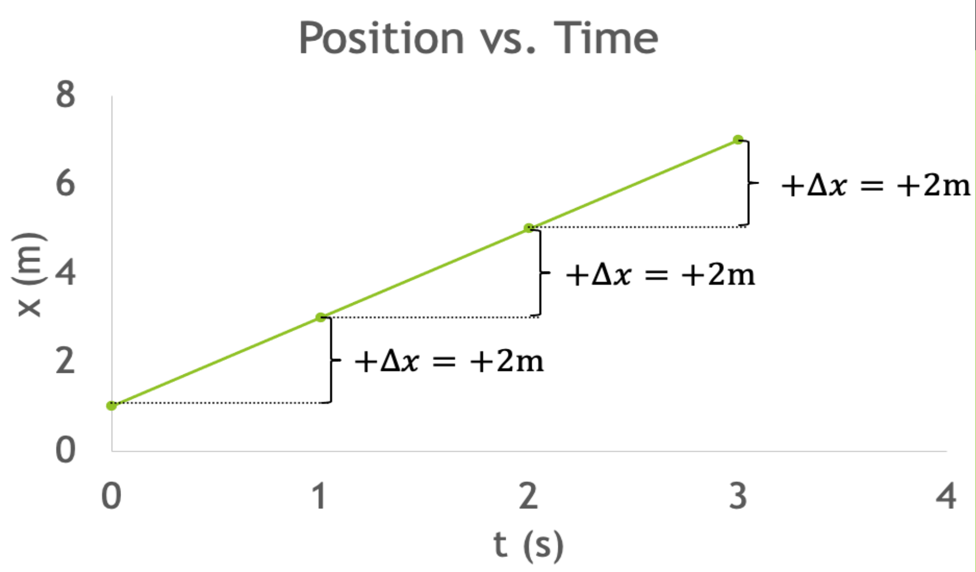
$$x_f = x_i + v\Delta t$$

$$y = b + mx$$

x_i is the initial position (y-intercept, *b*)

v is the velocity (slope, *m*)

Δt is the time coordinate (x-coordinate, *x*)

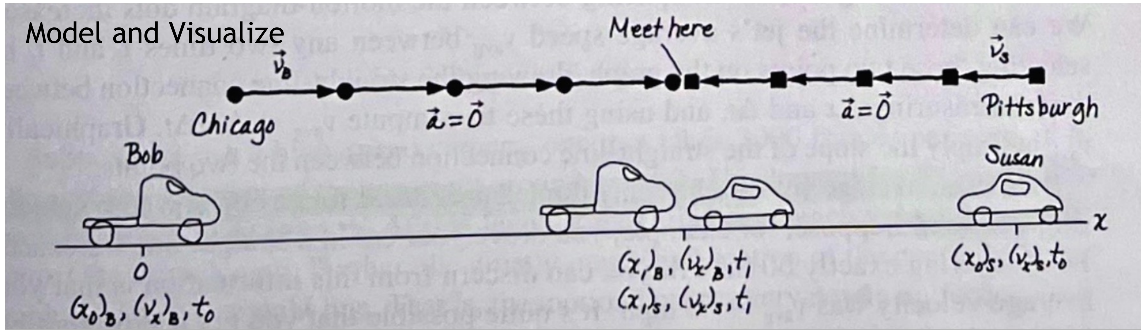


Putting it All Together

EXAMPLE 2.2 Lunch in Cleveland?

Bob leaves home in Chicago at 9:00 A.M. and travels east at a steady 60 mph. Susan, 400 miles to the east in Pittsburgh, leaves at the same time and travels west at a steady 40 mph. Where will they meet for lunch?

Model and Visualize



Known

$(x_0)_B = 0\text{ mi}$ $(v_x)_B = 60\text{ mph}$
$(x_0)_S = 400\text{ mi}$ $(v_x)_S = -40\text{ mph}$
$t_0 = 0\text{ hr}$ t_1 is when $(x_1)_B = (x_1)_S$

Find

$(x_1)_B$

Two equations of motion:

Bob: $(x_1)_B = 0\text{mi} + 60\text{mph} \cdot t_1$

Susan: $(x_1)_S = 400\text{mi} - 40\text{mph} \cdot t_1$

Since they are meeting: $(x_1)_B = (x_1)_S$

$$60\text{mph} \cdot t_1 = 400\text{mi} - 40\text{mph} \cdot t_1$$

Solve for t_1. This is *when* they meet.

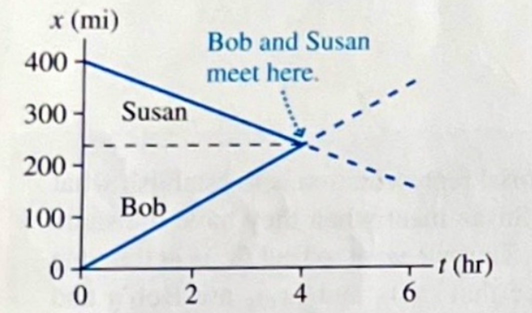
$$t_1 = 4\text{hr}$$

What about *where*?

We can use this time in either equation, but Bob's is easier.

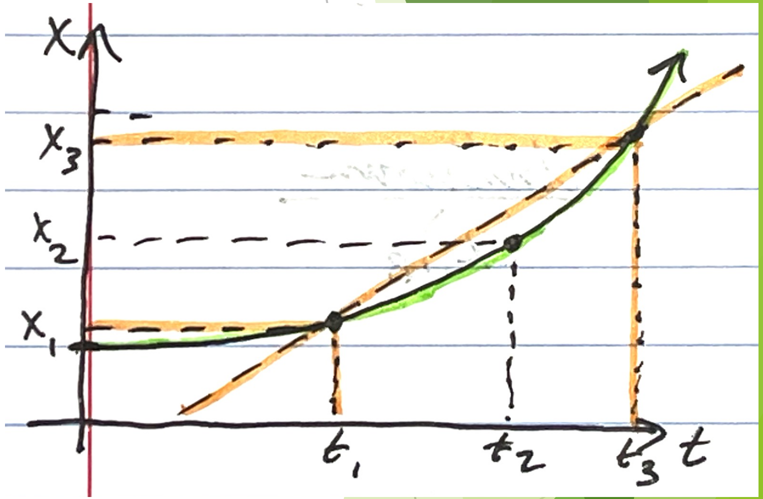
$$(x_1)_B = 60\text{mph} \cdot 4\text{hr} = 240\text{mi}$$

FIGURE 2.5 Position-versus-time graphs for Bob and Susan.



Instantaneous Velocity

- Average velocity is only so useful when an object's velocity is changing. In this situation, determining the instantaneous velocity will be more helpful.
- This is the speed and direction at a particular instance in time.



Suppose we wanted to know the velocity of a particle precisely at t_2. Since it is not undergoing uniform motion (green plot), determining this value may be difficult.

We *could* use apply the definition of average velocity and use the positions at t_1 and t_3 to estimate the slope. However, this will give us the slope of the diagonal orange line. This would only be an approximation.

We need to make the time interval smaller and get it *really close* to t_2. This will provide a more accurate estimate.

$$\vec{v}(t) = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{ds}{dt}$$

This becomes the derivative. Instantaneous velocity is the slope of the line tangent to the x vs. t graph at some time *t*.

Instantaneous Velocity

Conversely, if we know information about an object’s velocity but not its position, we can “work backwards” to get this information.

If we knew the initial position, we could add all the changes in position up to get the final position. However, we only know velocity over time.

$$x_f = x_i + \sum_{k=1}^{n} \Delta x_k$$

But wait! If $\vec{v}_{avg} = \frac{\Delta x}{\Delta t}$, then we could rewrite the equation above using *v* and *t* instead!

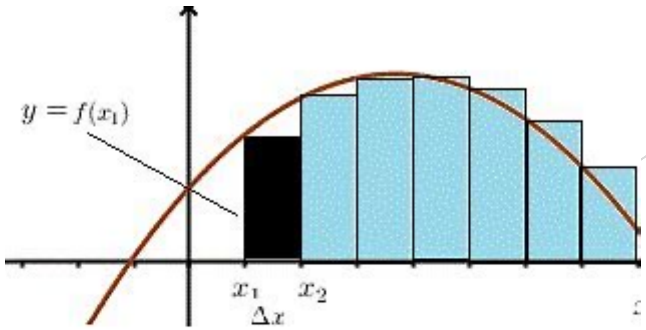
$$x_f = x_i + \sum_{k=1}^{n} \Delta x_k = x_i + \sum_{k=1}^{n} \left(\vec{v}_{\text{avg}}\right)_k \Delta t_k$$

We can take lots of average velocities over short periods of time and add them up! If we make our time intervals really small, we will get a good approximation of position information.

Now,

$$x(t) = x_i + \lim_{\Delta t \to 0} \sum_{k=1}^{n} \left(\vec{v}_{\text{avg}}\right)_k \Delta t_k = x_i + \int_{t_i}^{t_f} v(t)dt$$

$$x(t) = x_i + \int_{t_i}^{t_f} v(t)dt$$



This is $x_i +$ the area under the v vs. t graph from t_i to t_f. An integral.

Summary

- As you work on building intuition about motion, you'll need to move back and forth between four different representations:
    1. The motion diagram
    2. The position vs. time graph
    3. The velocity vs. time graph
    4. Word descriptions

- You should be able to:
    - Sketch motion diagrams
    - Sketch position graphs
    - Sketch velocity graphs
    - Interpret these graphs