Lecture 2.2
1D Kinematics – Acceleration

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Consider the following race…

- Suppose the driver of a Volkswagen Beetle wanted to race the driver of a Porsche where both drivers must achieve $30\frac{m}{s}$ (roughly 67mph).
- Obviously, both are capable of reaching this speed. So, *how can their performances be compared?*
- Although we all know how this will turn out, what will the results of the race tell us about each car's *acceleration*?

Consider the following race…

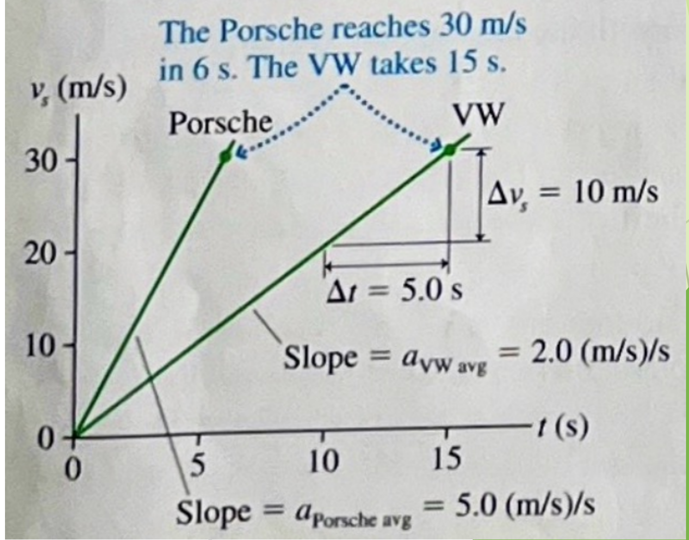
- Here are velocities of each car in 0.1s increments. What do you notice?

TABLE 2.1 Velocities of a Porsche and a Volkswagen Beetle

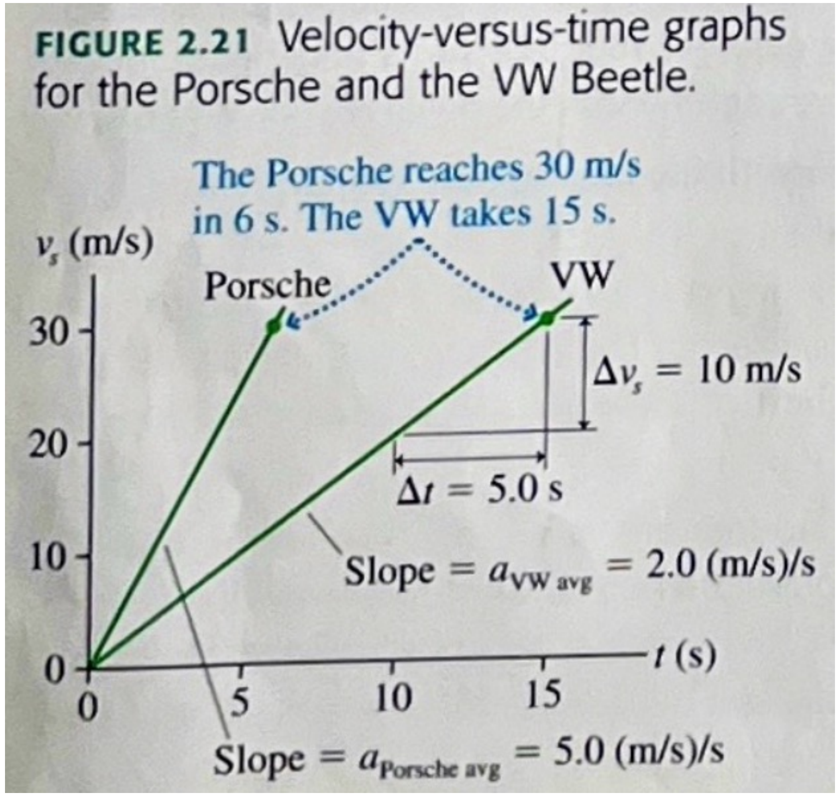
t(s)	$v_{Porsche}$ (m/s)	v_{VW} (m/s)
0.0	0.0	0.0
0.1	0.5	0.2
0.2	1.0	0.4
0.3	1.5	0.6
0.4	2.0	0.8
⋮	⋮	⋮

- Over the same time interval, the velocity of the Porsche increases more than the VW's. This is indicated by the steeper slope on the v vs. t graph.

FIGURE 2.21 Velocity-versus-time graphs for the Porsche and the VW Beetle.



Consider the following race…



An object has *uniformly accelerated motion* if and only if it's acceleration is constant and unchanging. Its v vs. t plot is a straight line and $\vec{a}_{avg}$ is the slope.

We see from the graph that the Porsche achieved a velocity of 30m/s in 6s while the VW took 15s.

What does that tell us about about the *rate of change of their velocities?*

$$\frac{\Delta \mathrm{v_{Porsche}}}{\Delta t} = \frac{v_f - v_i}{t_f - t_i} = \frac{30\frac{\mathrm{m}}{\mathrm{s}} - 0\frac{\mathrm{m}}{\mathrm{s}}}{6\mathrm{s} - 0\mathrm{s}} = \frac{30\frac{\mathrm{m}}{\mathrm{s}}}{6\mathrm{s}} = 5\frac{\mathrm{m}}{\mathrm{s}^2}$$

$$\frac{\Delta \mathrm{v_{VW}}}{\Delta t} = \frac{v_f - v_i}{t_f - t_i} = \frac{30\frac{\mathrm{m}}{\mathrm{s}} - 0\frac{\mathrm{m}}{\mathrm{s}}}{15\mathrm{s} - 0\mathrm{s}} = \frac{30\frac{\mathrm{m}}{\mathrm{s}}}{15\mathrm{s}} = 2\frac{\mathrm{m}}{\mathrm{s}^2}$$

This is each car's *change in velocity per second*. The units are "meters per second per second" or "meters per second squared".

The Porsche adds 5m/s to its velocity each second while the VW adds 2m/s to its velocity.

Another Kinematic Equation

We can leverage our previous work with *constant average velocity* to derive another useful kinematic equation!

$$\vec{a}_{avg} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i} = \frac{v_f - v_i}{\Delta t}$$

$$v_f = v_i + \vec{a}_{avg}\Delta t$$

$$v_f = v_i + a\Delta t$$

This is just the slope equation!

$$v_f = v_i + a\Delta t$$

$$y = b + mx$$

Instantaneous Acceleration

An *instantaneous acceleration* is defined in a similar way to *instantaneous velocity.*

The instantaneous acceleration, $\vec{a}(t)$, at a specific instant in time *t* is the slope of the line that is tangent to the v vs. t curve at time t.

$$\vec{a}(t) = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$

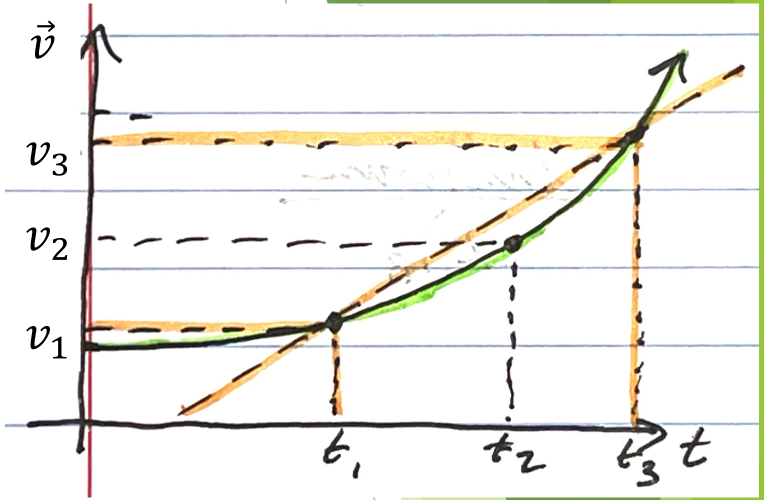
The instantaneous acceleration is the derivative of velocity.

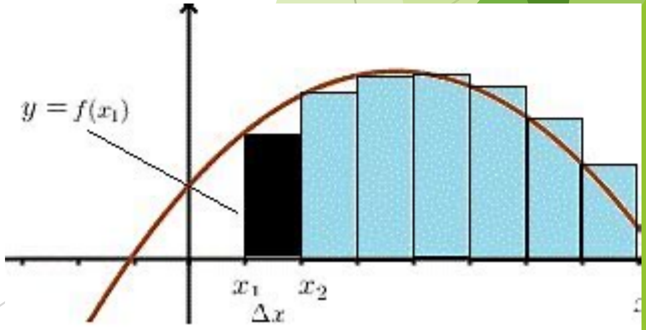
From the previous kinematic equation:

$$\vec{v}(t) = v_i + \lim_{\Delta t \to 0} \sum_{k=1}^{n} \left(\vec{a}_{\text{avg}}\right)_k \Delta t_k = v_i + \int_{t_i}^{t_f} \vec{a}(t)dt$$

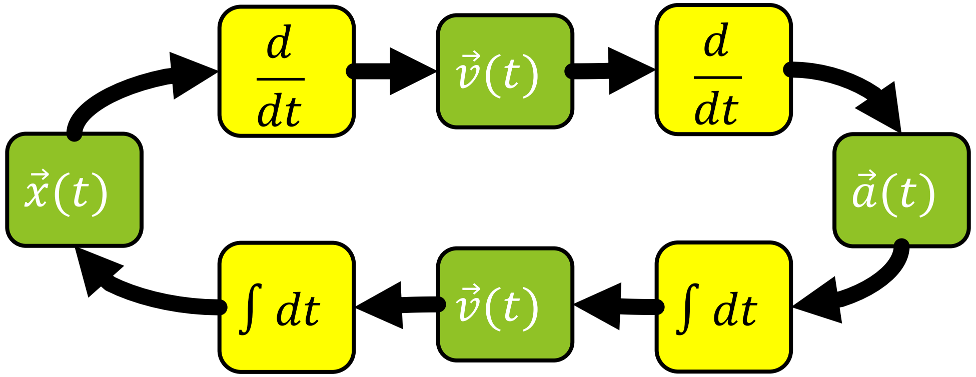
$$\vec{v}(t) = v_i + \int_{t_i}^{t_f} \vec{a}(t)dt$$

This is v_i + the area under the a vs. t graph from t_i to t_f. An integral.





The Calculus of Kinematics



Another Kinematic Equation with Constant Acceleration

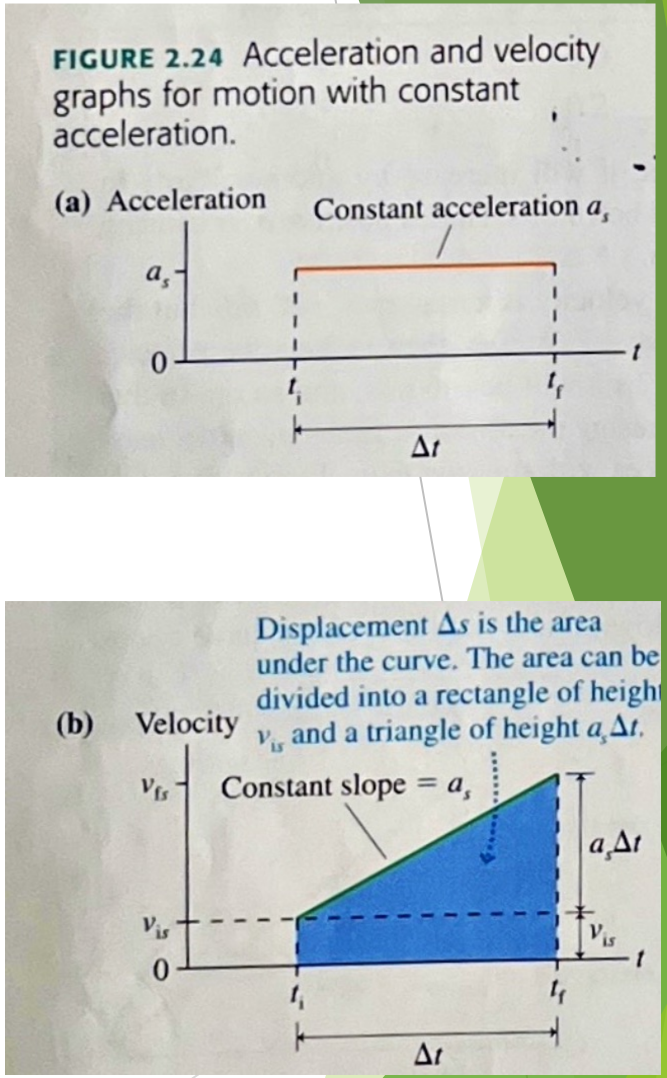
- If acceleration is constant, the graph of $\vec{a}(t)$ is a straight line.
- What does this mean for the graph of $\vec{v}(t)$?
  - It will be linear (constant slope)
- If we apply our definition of position (area under the velocity curve)

$$x_f = x_i + (\text{bottom rectangle}) + (\text{top triangle})$$

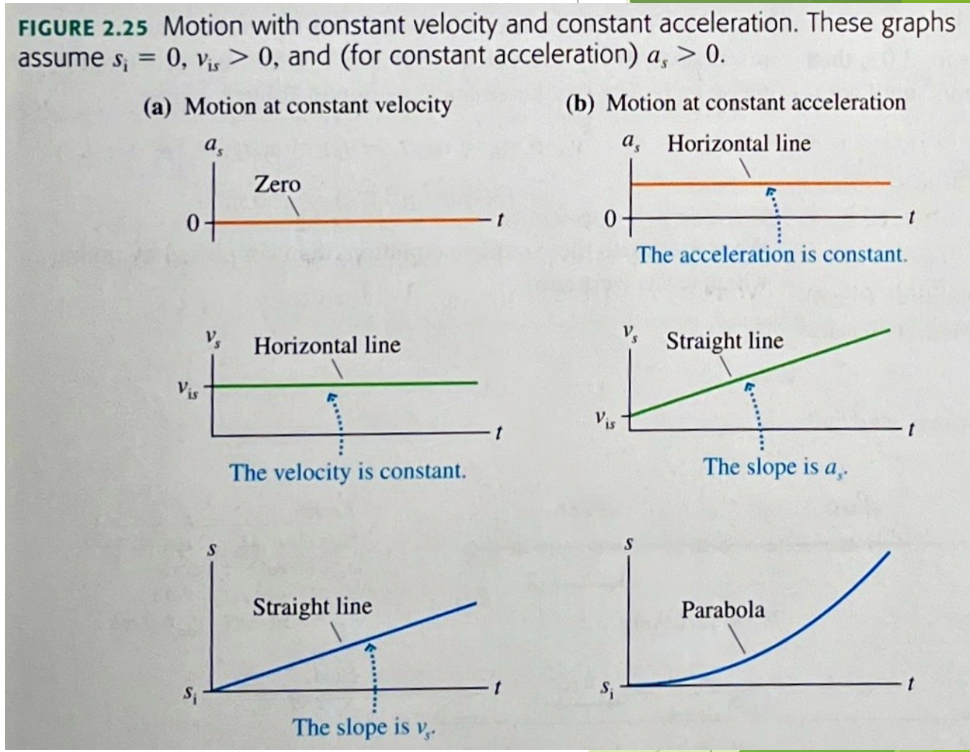
$$x_f = x_i + (\text{length} \times \text{width}) + \left(\frac{1}{2}\text{bh}\right)$$

$$x_f = x_i + v_i \Delta t + \frac{1}{2}(\Delta t)(a\Delta t)$$

$$x_f = x_i + v_i \Delta t + \frac{1}{2}a(\Delta t)^2$$



The Kinematic Equations of Motion with Constant Acceleration



(1) $v_f = v_i + a\Delta t$

(2) $x_f = x_i + v_i\Delta t + \frac{1}{2}a(\Delta t)^2$

(3) $v_f^2 = v_i^2 + 2a\Delta x$

To obtain (3), solve (1) for Δt then "plug it in" to (2)

Objects in Free Fall

The Leaning Tower of Pisa leans at an angle of 4 degrees and is 55.86m tall. If a ball were dropped from the top of the tower, at what speed would it hit the ground?

$$v_f^2 = v_i^2 + 2a\Delta y$$

$$v_f = \sqrt{v_i^2 + 2a\Delta y}$$

$v_i = 0\frac{\mathrm{m}}{\mathrm{s}}$ since the object starts at rest.

$a = -9.8\frac{\mathrm{m}}{\mathrm{s}^2}$

$\Delta y = y_f - y_i = ???$

$55.86\mathrm{m} \cdot \sin(86°) \approx 55.72\mathrm{m} = \Delta y$

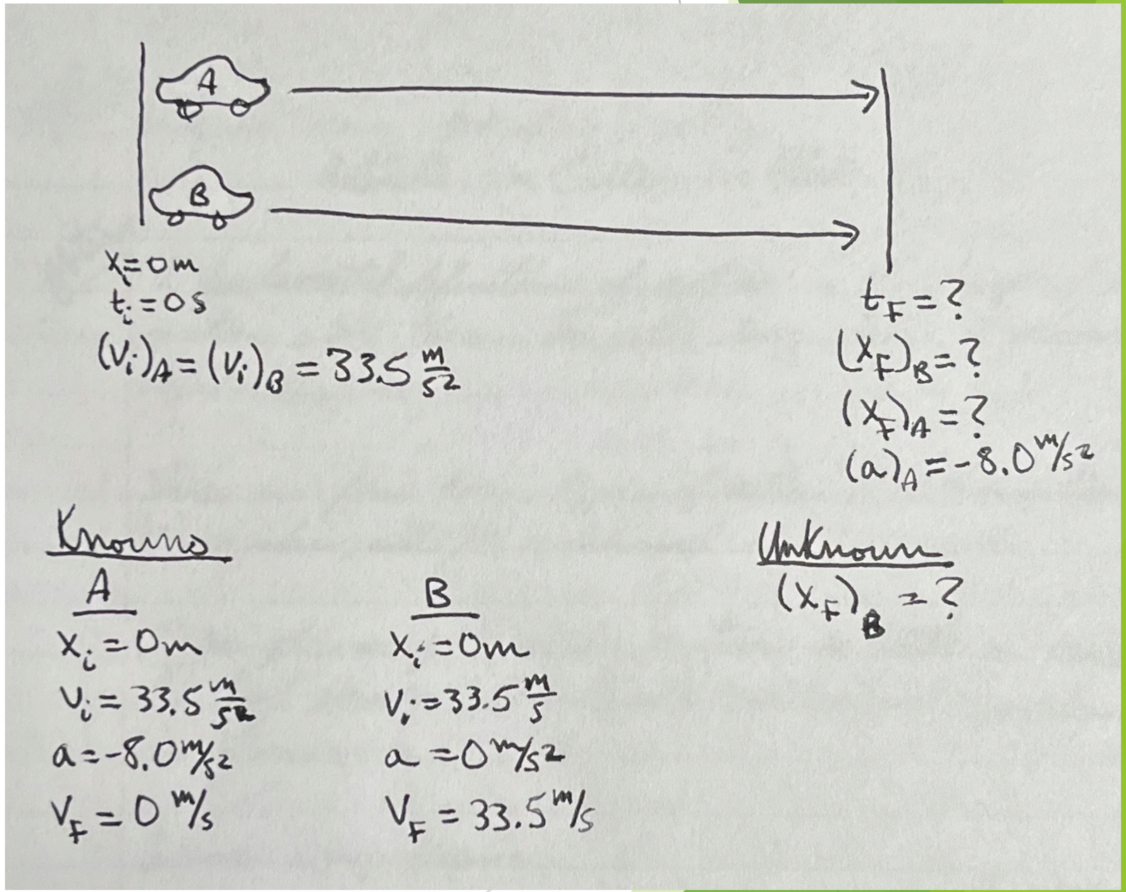
$\Delta y = y_f - y_i = 0\mathrm{m} - 55.72\mathrm{m}$

$$v_f = \sqrt{\left(0\frac{\mathrm{m}}{\mathrm{s}}\right)^2 + 2\left(-9.8\frac{\mathrm{m}}{\mathrm{s}^2}\right)(-55.72\mathrm{m})} = \sqrt{2\left(-9.8\frac{\mathrm{m}}{\mathrm{s}^2}\right)(-55.72\mathrm{m})} \approx 33\frac{\mathrm{m}}{\mathrm{s}}$$

Example

Two cars are driving side-by-side in adjacent lanes on the freeway. Both are cruising at 75mph $(33.5\,\frac{\mathrm{m}}{\mathrm{s}})$ A pick-up truck ahead of one car loses an object off the back. The car following the truck (Car A) skids to a halt, *decelerating* at a rate of $8.0\,\frac{\mathrm{m}}{\mathrm{s}^2}$. The other car (Car B) continues driving at a constant speed. How far away from the origin is Car B when Car A fully stops?

First, we must model and visualize the situation.



Using Equations

We need to get more information from Car A. Specifically, the time taken to stop.

$$v_f = v_i + a\Delta t$$

$$\Delta t = \frac{v_f - v_i}{a}$$

$$\Delta t = \frac{0\,\frac{\mathrm{m}}{\mathrm{s}} - 33.5\,\frac{\mathrm{m}}{\mathrm{s}}}{-8.0\,\frac{\mathrm{m}}{\mathrm{s}^2}} = 4.1875\mathrm{s}$$

This is the same amount of time that Car B will continue traveling. $t_f = 4.1875\mathrm{s}$

$$x_f = x_i + v\Delta t$$

$$x_f = 0\mathrm{m} + \left(33.5\,\frac{\mathrm{m}}{\mathrm{s}}\right) \cdot (4.1875\mathrm{s}) \approx 140\mathrm{m}$$

Assess:

140m seems reasonable!

Summary

Kinematic equations under constant acceleration:

$$x_f = x_i + v\Delta t$$

$$x_f = x_i + v_i t + \frac{1}{2} a t^2$$

$$v_f = v_i + a\Delta t$$

$$v_f^2 = v_i^2 + 2a\Delta x$$