CONTINUOUS RANDOM VARIABLES

MAT 152 – Statistical Methods I

Lecture 1

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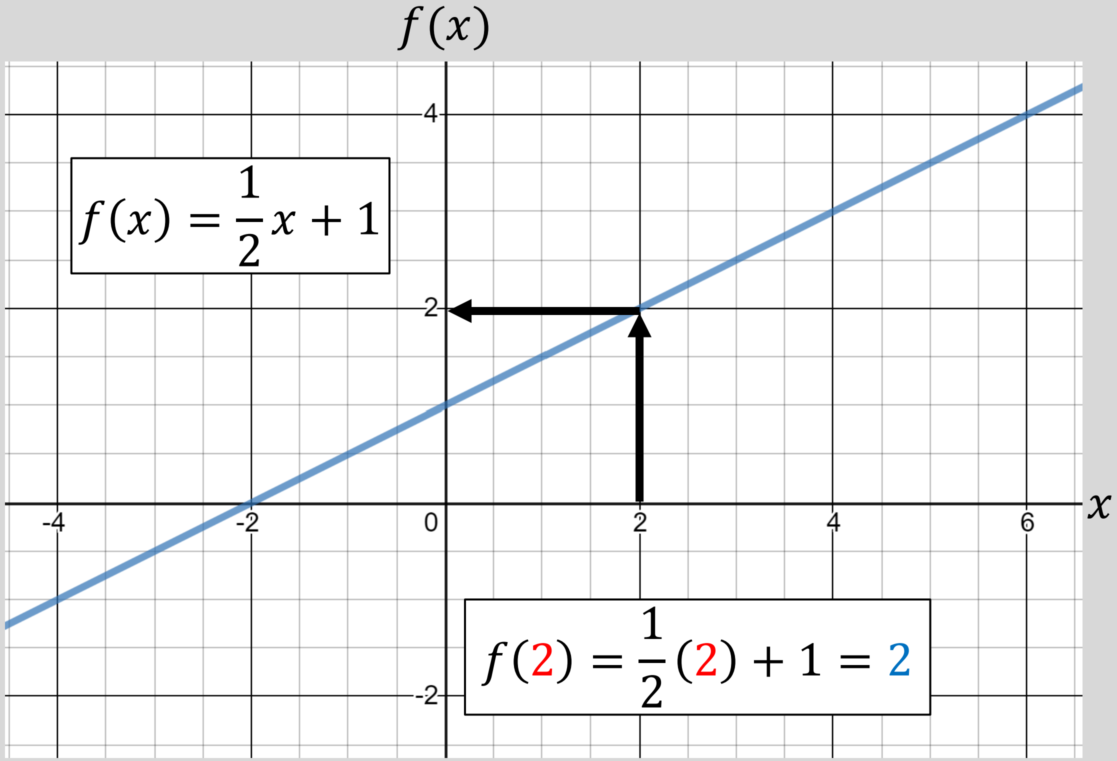
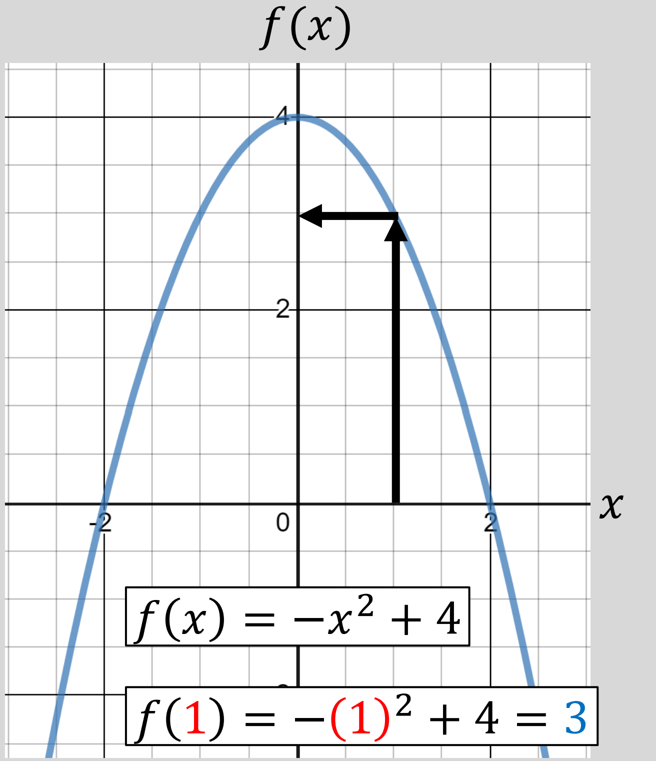
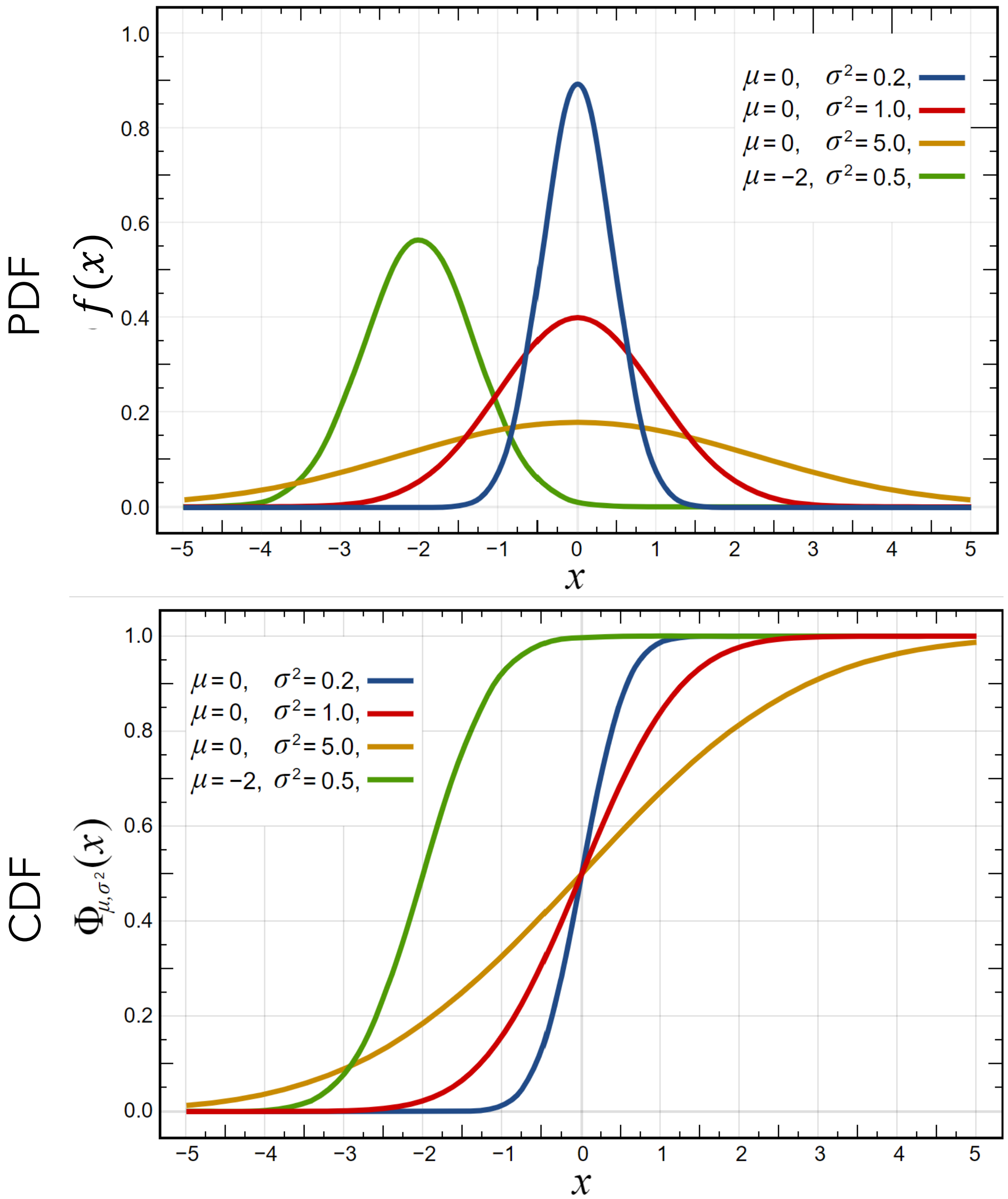
Fall 2020

A Quick Review of Functions

- You will recall that functions, represented as $f(x)$, represent a curve in space.







Continuous Probability Distributions

- The graph of a continuous probability distribution is a curve. It is called the **probability density function (pdf)** and is notated as $f(x)$.
- The area under the curve is the probability and is given by the **cumulative density function (cdf)**.
- Outcomes are measured, not counted.
- The entire area under $f(x)$ is equal to 1.
- Probability is found for intervals, not individual x values.

Example

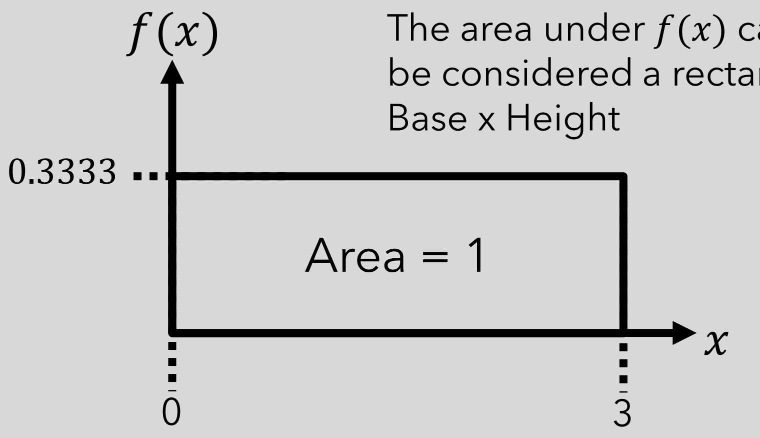
Suppose the amount of time a customer must wait at a supermarket's checkout counter is between 0 and 3 minutes. All times between these values are equally likely. Use the probability density function, $f(x)$, to determine the probability of a customer waiting 1.25 to 1.5 minutes.

$P(1.25 < x < 1.5) = ?$

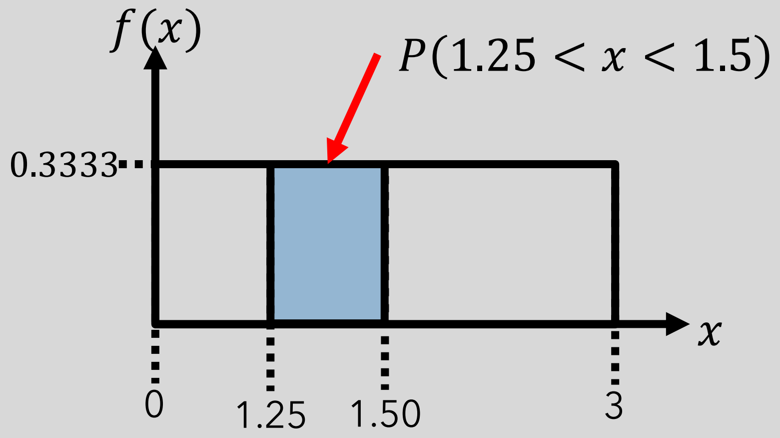
Note the area under the pdf.
$\text{Area} = \text{Base} \times \text{Height} = 1$
$\text{Area} = (3 - 0) \cdot 0.3333 \approx 1$

$P(1.25 < x < 1.5) = (1.50 - 1.25) \cdot 0.3333 \approx 0.08333$

Note that, for continuous distributions:
$P(1.25 < x < 1.5) = P(1.25 \leq x \leq 1.5)$



The area under $f(x)$ can be considered a rectangle: Base x Height



Example

Consider the following pdf.

$$f(x) = \begin{cases} \frac{1}{5}x & 0 \le x \le 2 \\ \frac{2}{3}\left(1 - \frac{1}{5}x\right) & 2 < x \le 5 \end{cases}$$

Determine $P(x = 2)$.

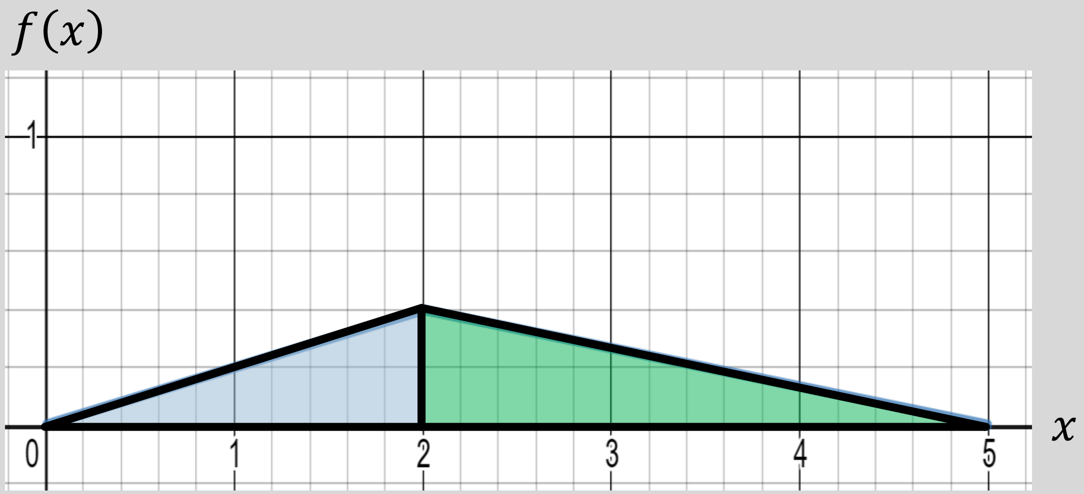
$P(x = 2) = (2 - 2) \cdot (0.4) = 0$

Determine $P(x < 2)$.

$P(x < 2) = \frac{1}{2}bh = \frac{1}{2} \cdot (2 - 0) \cdot (0.4) = 0.4$

Determine $P(x > 2)$.

$1 - P(x < 2) = 0.6$



Area of a triangle $= \frac{1}{2}bh$

"Complicated" Probability Distribution Functions

For more complex probability density functions, calculus is needed to find the area under the curve.

- "Integral calculus" is used to find the area under complicated functions.

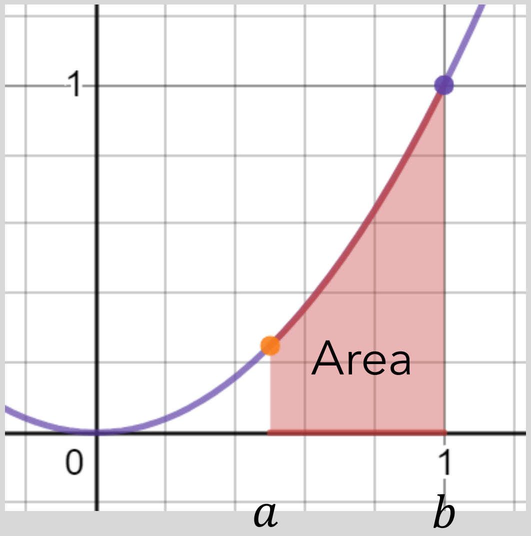
- The integral is used to find areas along segments:

Variable of interest

$$\text{Area} = \int_a^b f(x)\,dx$$

Interval of interest

Function of interest



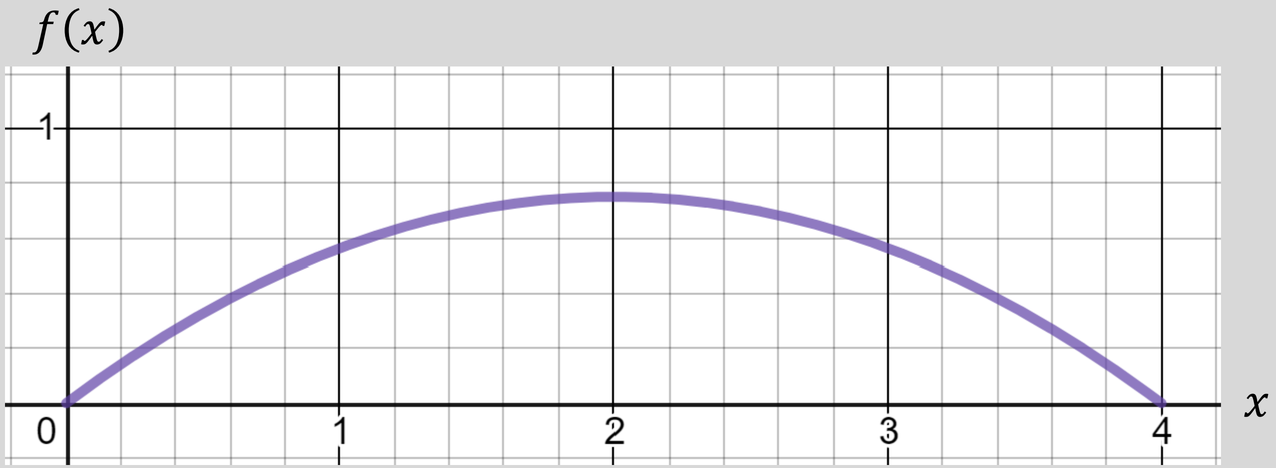
Many integrals can be solved by hand. (However, some cannot!)

Complicated Example

Consider the pdf $f(x) = \frac{3}{32}\left(4 - (x-2)^2\right)$ on the interval $0 \le x \le 4$. (See plot below).

Determine $P(1 < x < 2)$.

$$P(1 < x < 2) = \int_1^2 \frac{3}{32}\left(4 - (x-2)^2\right) dx = \frac{3}{32}\left[\int_1^2 4\, dx - \int_1^2 (x-2)^2\, dx\right] = \frac{3}{32}\left[4[x]_1^2 - \frac{1}{3}[(x-2)^3]_1^2\right] = 0.34375$$



Complicated Example (cont.)

Technology can be used to integrate more complicated probability density functions.

$$\int_1^2 \frac{3}{32}\left(4-(x-2)^2\right)dx$$

Becomes:

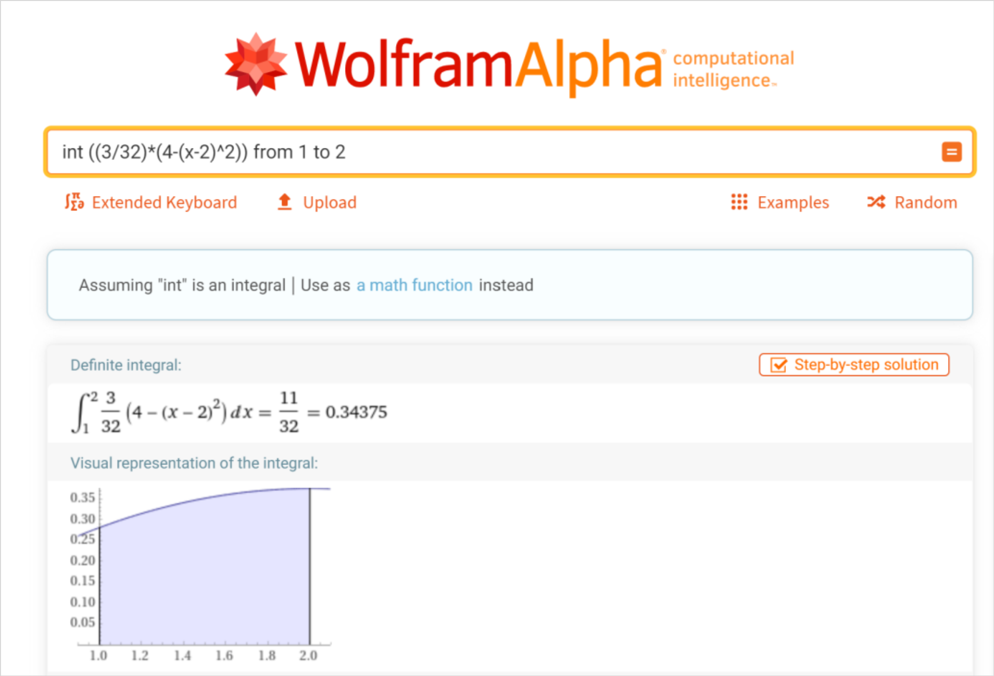
Int (3/32)*(4-(x-2)^2) 1 to 2

Fractions are in parentheses

"*" is used for multiplication

"^" is used for exponents

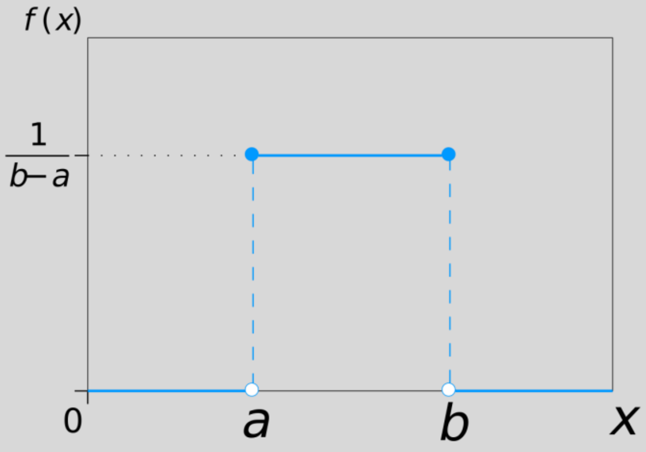
"from 1 to 2" indicates the interval



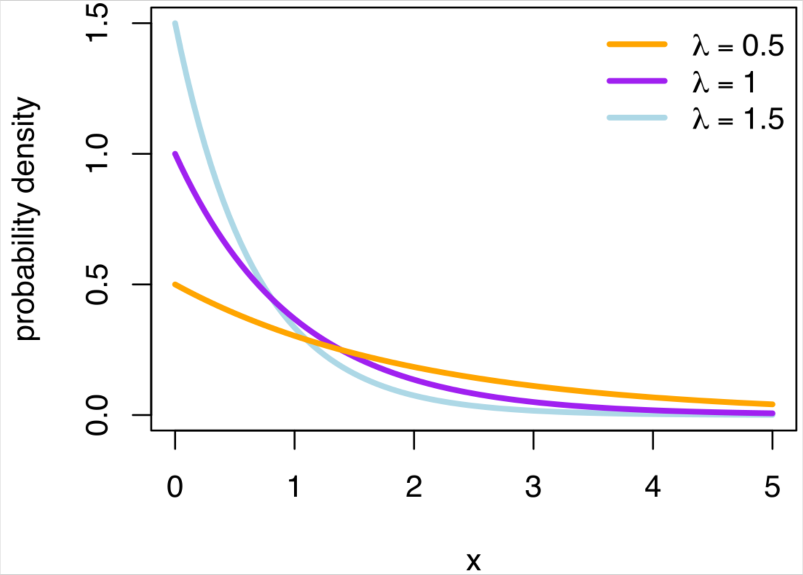
Wolframalpha.com

Common Distributions

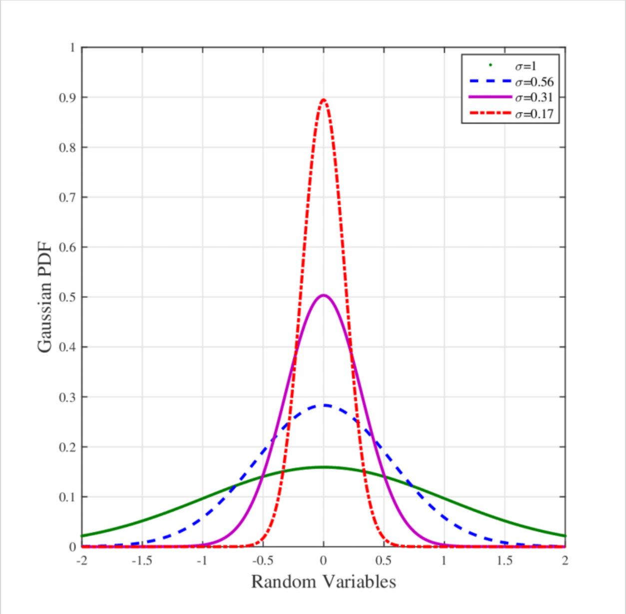
The Uniform Distribution

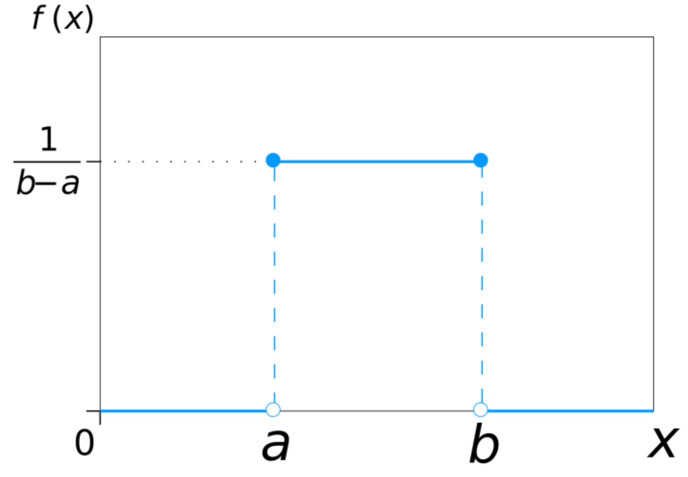


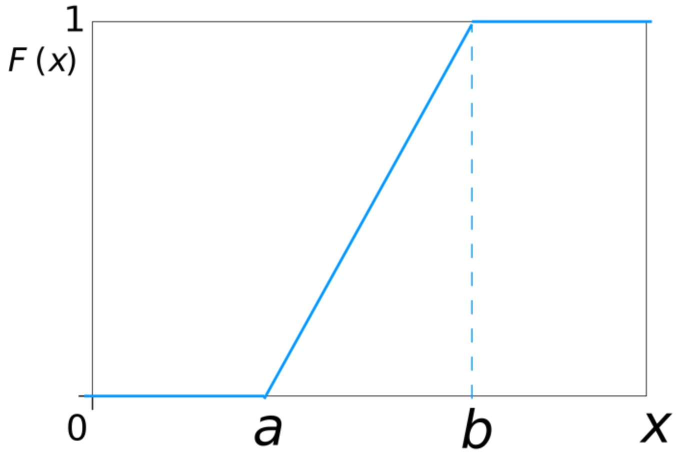
The Exponential Distribution



The Normal Distribution







The Uniform Distribution

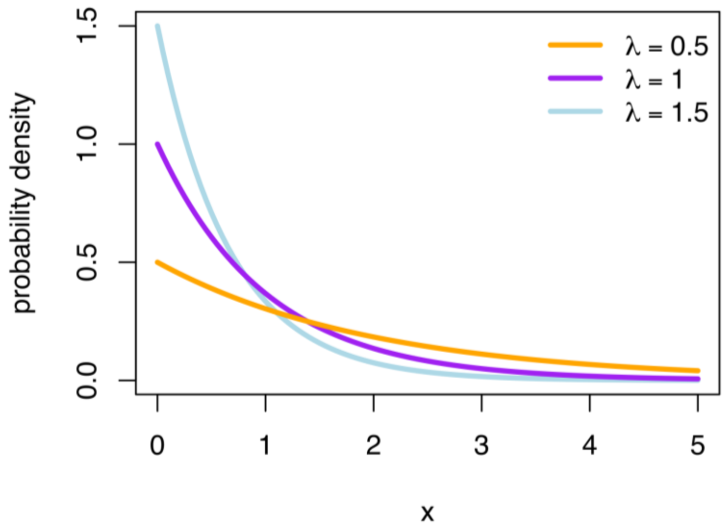
$$f(x) = \begin{cases} \dfrac{1}{b-a} & a \le x \le b \\ 0 & \text{else} \end{cases}$$

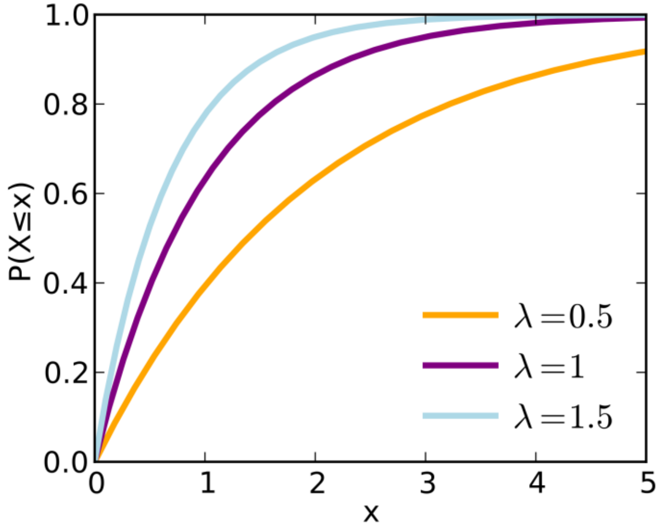
Symmetric distribution

All outcomes are equally likely

Random number generation

Example: location of a raindrop on a patio during a rainstorm.





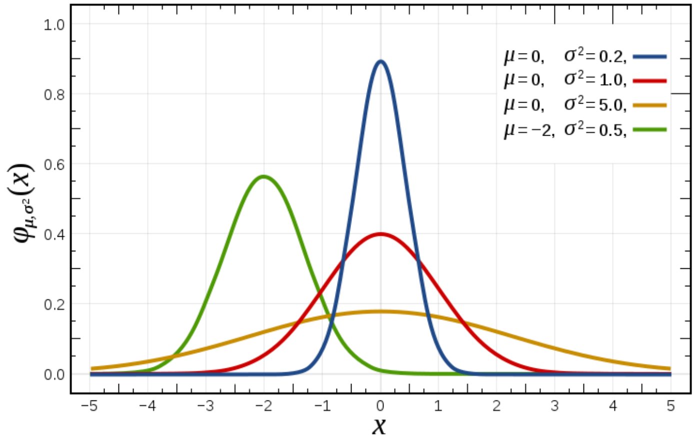
The Exponential Distribution

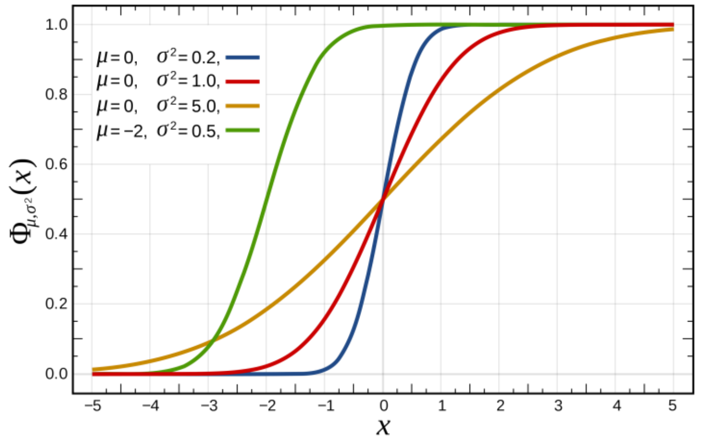
$$f(x) = \frac{1}{\mu} e^{-\frac{1}{\mu}x} \text{ for } x \geq 0$$

Continuous version of the geometric distribution

Inter-arrival time of events in a Poisson process

Example: the number of days ahead of time that airline passengers purchase their tickets.





The Normal Distribution

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

Values centered about a mean.

Heights, grades, etc.