LECTURE 6.3 – MOMENTUM IN TWO DIMENSIONS

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CONSERVATION IN TWO DIMENSIONS

In cases where momentum is conserved, $\vec{P}_i = \vec{P}_f$.

Momentum is conserved in the x- and y-directions

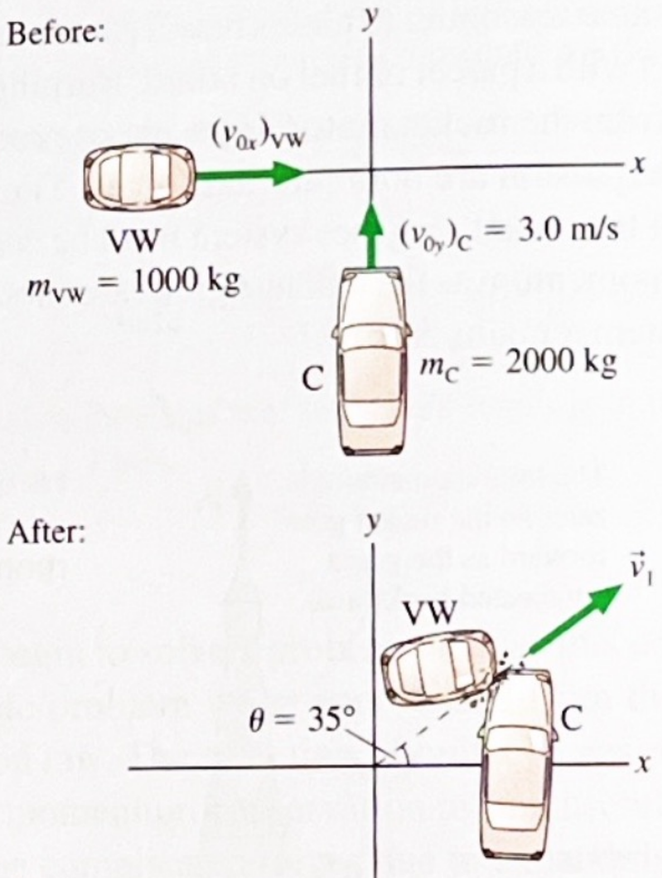
$$(p_{i,x})_1 + \cdots + (p_{i,x})_N = (p_{f,x})_1 + \cdots + (p_{f,x})_N$$
$$(p_{i,y})_1 + \cdots + (p_{i,y})_N = (p_{f,y})_1 + \cdots + (p_{f,y})_N$$

INELASTIC COLLISION (EXAMPLE #1)

EXAMPLE 9.9 Momentum in a 2D car crash

The 2000 kg Cadillac and the 1000 kg Volkswagen of Example 9.6 meet again the following week, just after leaving the auto body shop where they had been repaired. The stoplight has just turned green, and the Cadillac, heading north, drives forward into the intersection. The Volkswagen, traveling east, fails to stop. The Volkswagen crashes into the left front fender of the Cadillac, then the cars stick together and slide to a halt. Officer Tom, responding to the accident, sees that the skid marks go 35° northeast from the point of impact. The Cadillac driver, who keeps a close eye on the speedometer, reports that he was traveling at 3.0 m/s when the accident occurred. How fast was the Volkswagen going just before the impact?

MODEL This is an inelastic collision. The total momentum of the Volkswagen + Cadillac system is conserved.



- x-direction: $m_C v_{C,x,i} + m_V v_{V,x,i} = (m_C + m_V) v_{x,f}$
- y-direction: $m_C v_{C,y,i} + m_V v_{V,y,i} = (m_C + m_V) v_{y,f}$

SETTING UP THE EQUATIONS (EXAMPLE #1)

From the x-direction:

$$m_C v_{C,x,i} + m_V v_{V,x,i} = (m_C + m_V) v_{x,f}$$

$$v_{V,x,i} = \frac{(m_C + m_V) v_{x,f} - m_C v_{C,x,i}}{m_V}$$

But what is the combined final velocity!? Let's get it from the y-direction!

$$m_C v_{C,y,i} + m_V v_{V,y,i} = (m_C + m_V) v_{y,f} = (m_C + m_V) v_f \sin\theta$$

$$v_f = \frac{m_C v_{C,y,i} + m_V v_{V,y,i}}{(m_C + m_V)\sin\theta} = \frac{2000\text{kg} \cdot 3.0\,\frac{\text{m}}{\text{s}} + 1000\text{kg} \cdot 0\,\frac{\text{m}}{\text{s}}}{(2000\text{kg} + 1000\text{kg})\sin 35^\circ} = 3.49\,\frac{\text{m}}{\text{s}}$$

Now, use this value in the x-direction calculation:

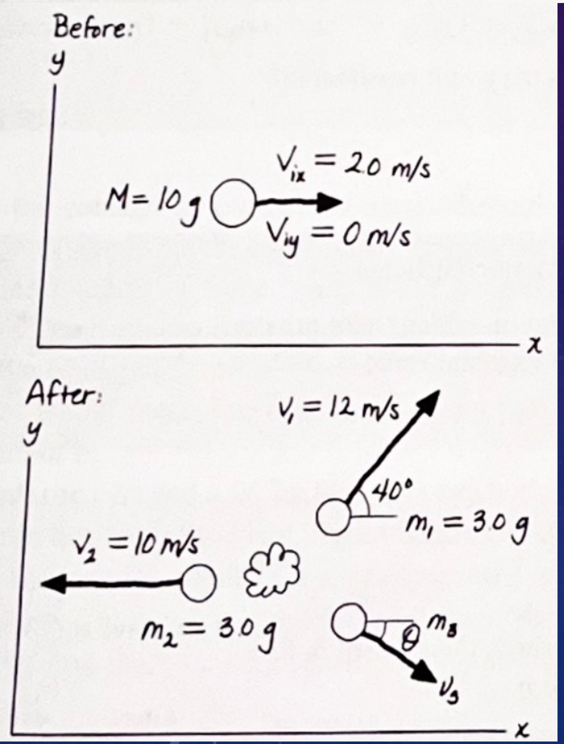
$$v_{V,x,i} = \frac{(m_C + m_V) v_f \cos\theta - m_C v_{C,x,i}}{m_V} = \frac{(2000\text{kg} + 1000\text{kg})\left(3.49\,\frac{\text{m}}{\text{s}}\right)\cos 35^\circ - 2000\text{kg} \cdot 0\,\frac{\text{m}}{\text{s}}}{1000\text{kg}} = 8.6\,\frac{\text{m}}{\text{s}}$$

AN EXPLOSION IN 2D (EXAMPLE #2)

EXAMPLE 9.10 A three-piece explosion

A 10 g projectile is traveling east at 2.0 m/s when it suddenly explodes into three pieces. A 3.0 g fragment is shot due west at 10 m/s while another 3.0 g fragment travels 40° north of east at 12 m/s. What are the speed and direction of the third fragment?

MODEL Although many complex forces are involved in the explosion, they are all internal to the system. There are no external forces, so this is an isolated system and its total momentum is conserved.



x-direction

$$MV_{i,x} = m_1 v_{f,x,1} + m_2 v_{f,x,2} + m_3 v_{f,x,3}$$

$$MV_{i,x} = m_1 v_{f,1} \cos\theta_1 - m_2 v_{f,2} + m_3 v_{f,3} \cos\theta_3$$

y-direction

$$MV_{i,y} = m_1 v_{f,y,1} + m_2 v_{f,y,2} + m_3 v_{f,y,3}$$

$$0\mathrm{kg}\frac{\mathrm{m}}{\mathrm{s}} = m_1 v_{f,1} \sin\theta_1 - m_3 v_{f,3} \sin\theta_3$$

Two equations, two unknowns!

From y-direction:

$$m_1 v_{f,1} \sin\theta_1 = m_3 v_{f,3} \sin\theta_3$$

$$v_{f,3} = \left(\frac{m_1 v_{f,1} \sin\theta_1}{m_3 \sin\theta_3}\right)$$

$$MV_{i,x} = m_1 v_{f,1} \cos\theta_1 - m_2 v_{f,2} + m_3 \left(\frac{m_1 v_{f,1} \sin\theta_1}{m_3 \sin\theta_3}\right) \cos\theta_3$$

SETTING UP THE EQUATIONS (EXAMPLE #2)

$$MV_{i,x} = m_1 v_{f,1} \cos\theta_1 - m_2 v_{f,2} + m_3 \left(\frac{m_1 v_{f,1} \sin\theta_1}{m_3 \sin\theta_3}\right) \cos\theta_3$$

$$MV_{i,x} + m_2 v_{f,2} - m_1 v_{f,1} \cos\theta_1 = \left(\frac{m_1 v_{f,1} \sin\theta_1}{\tan\theta_3}\right)$$

$$\tan\theta_3 = \left(\frac{m_1 v_{f,1} \sin\theta_1}{MV_{i,x} + m_2 v_{f,2} - m_1 v_{f,1} \cos\theta_1}\right)$$

$$\theta_3 = \tan^{-1}\left(\frac{m_1 v_{f,1} \sin\theta_1}{MV_{i,x} + m_2 v_{f,2} - m_1 v_{f,1} \cos\theta_1}\right) = \tan^{-1}\left(\frac{0.003\mathrm{kg} \cdot 12\frac{\mathrm{m}}{\mathrm{s}} \cdot \sin 40^\circ}{0.010\mathrm{kg} \cdot 2.0\frac{\mathrm{m}}{\mathrm{s}} + 0.003\mathrm{kg} \cdot 10\frac{\mathrm{m}}{\mathrm{s}} - 0.003\mathrm{kg} \cdot 12\frac{\mathrm{m}}{\mathrm{s}} \cos 40^\circ}\right) \approx 45.9^\circ$$

SETTING UP THE EQUATIONS (EXAMPLE #2)

FINALLY,

$$v_{f,3} = \left(\frac{m_1 v_{f,1} \sin\theta_1}{m_3 \sin\theta_3}\right) = \left(\frac{0.003\text{kg} \cdot 12\,\frac{\text{m}}{\text{s}} \cdot \sin 40°}{0.004\text{kg} \cdot \sin 45.9°}\right) = 8.1\,\frac{\text{m}}{\text{s}}$$

(Note: $m_3 = M - m_1 \ - m_2$)