LECTURE 6.1 – IMPULSE AND MOMENTUM

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CONSIDER THE FOLLOWING:

- A collision is a short-duration interaction between two objects. The particles of a system move together.

- An explosion is the opposite of a collision. The particles of the system move apart from each other after a brief intense interaction.

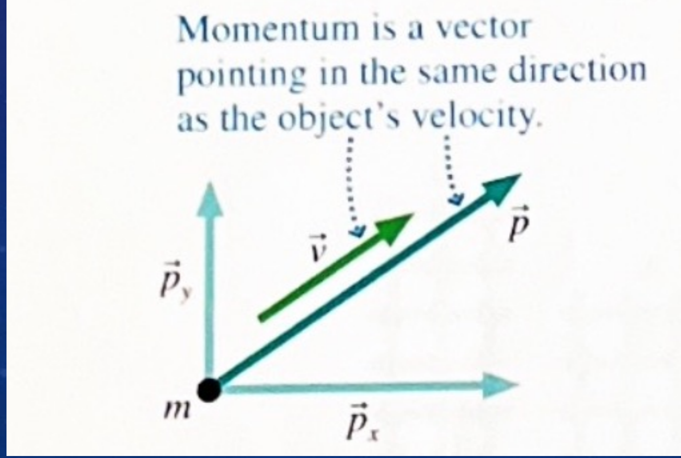
GOAL

- To describe and predict *simple* outcomes of these *complex* interactions.

- To do this, we will consider the *momentum* of the objects *before* and *after* interactions.

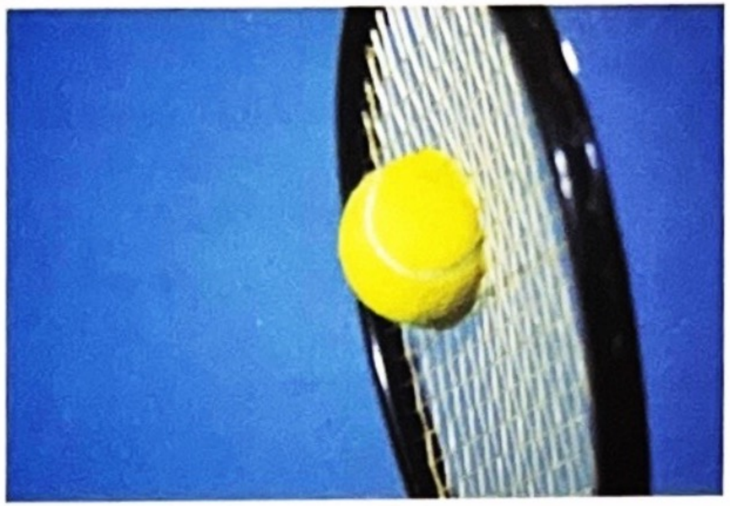
- Momentum ($\vec{p}$) is defined as a particle's mass (m) times velocity ($\vec{v}$): $\vec{p} = m\vec{v}$
- The units of momentum are $\mathrm{kg}\frac{\mathrm{m}}{\mathrm{s}}$.
- "Momentum is a particle's velocity vector scaled by its mass."

FIGURE 9.4 A particle's momentum vector $\vec{p}$ can be decomposed into x- and y-components.



COLLISIONS

- The duration of a collision depends on the materials from which the objects are made.
- Collisions are on the order of milliseconds. (The amount of time that objects are in contact with one another.)
- The harder the objects, the shorter the contact time.
- The molecular bonds (“springs”) making up the objects compress and re-expand during collision.
- This is an action/reaction pair.



A tennis ball collides with a racket. Notice that the right side of the ball is flattened.

FIGURE 9.1 Atomic model of a collision.

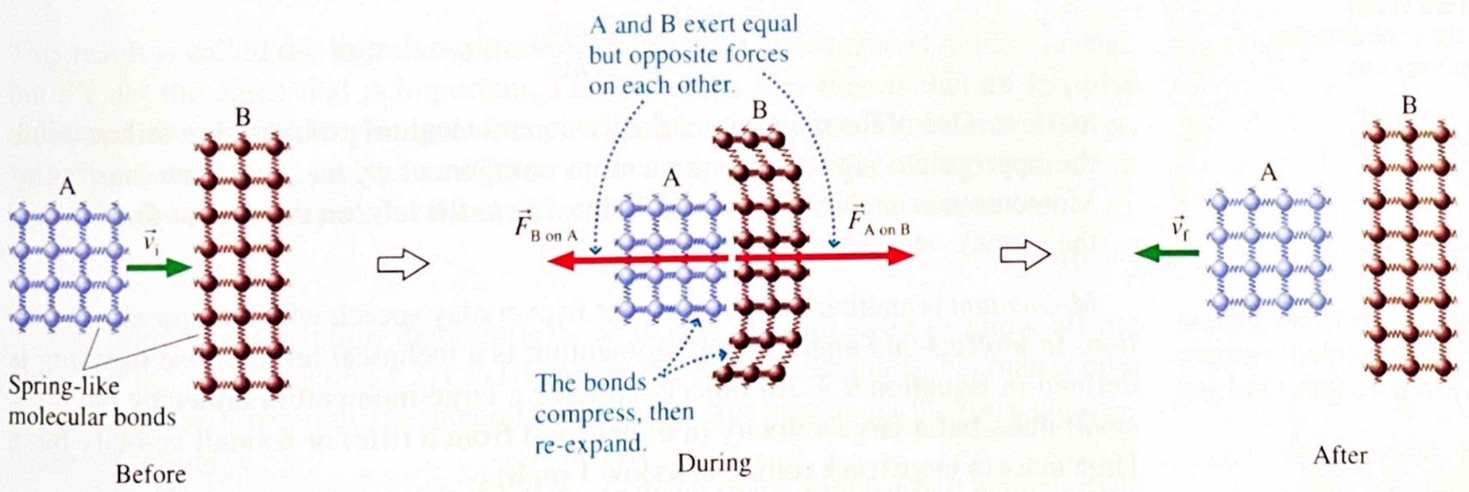


FIGURE 9.1 Atomic model of a collision.

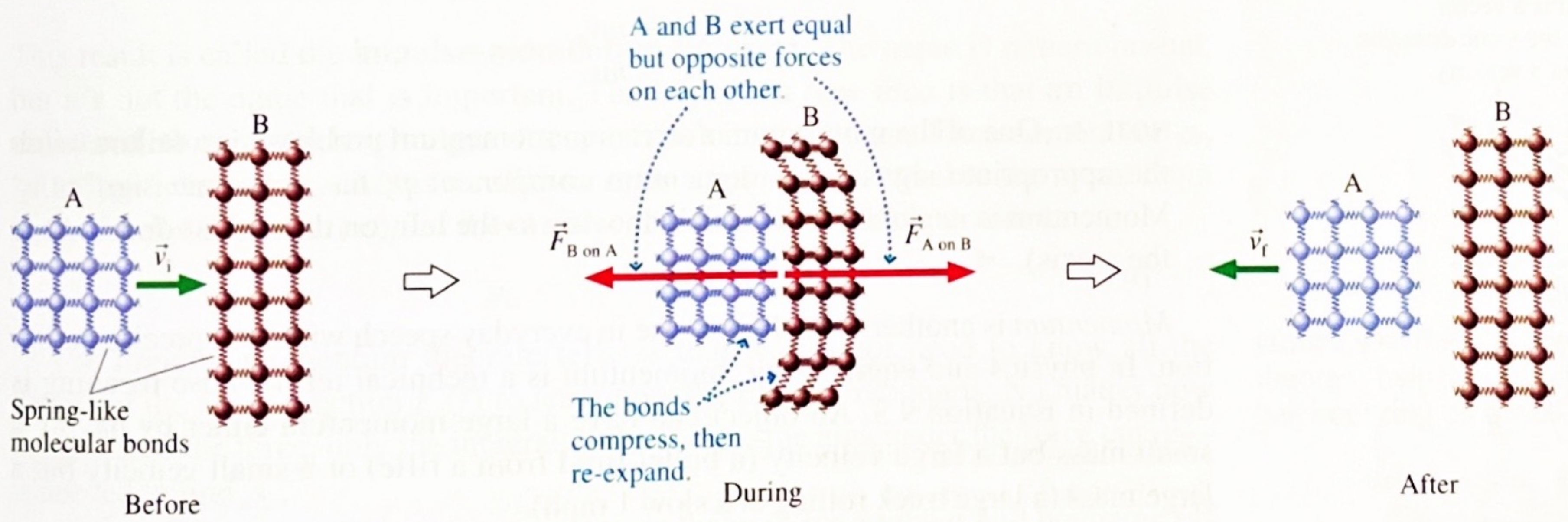
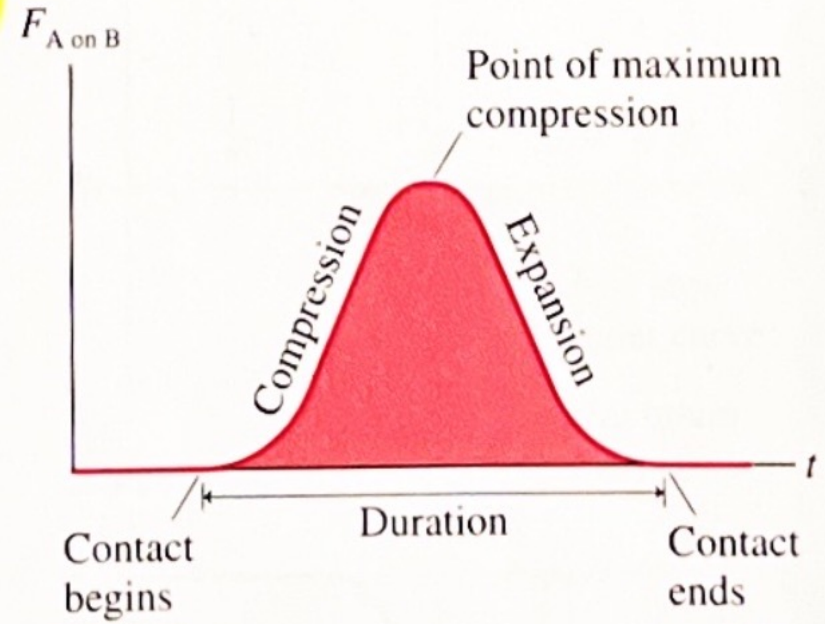


FIGURE 9.2 The rapidly changing magnitude of the force during a collision.



IMPULSE

- A large force exerted during a small well-defined interval of time is called an *impulsive force*.
- Now, a force that changes over time must be considered: $F(t)$
- With Newton's Second Law: $\sum_i \vec{F}_i = m\vec{a} = F(t)$

NEWTON'S SECOND LAW

Newton's Second Law can be written in terms of momentum:

$$\vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt} = \frac{d}{dt}(m\vec{v}) = \frac{d\vec{p}}{dt}$$

Also from Newton's Second Law:

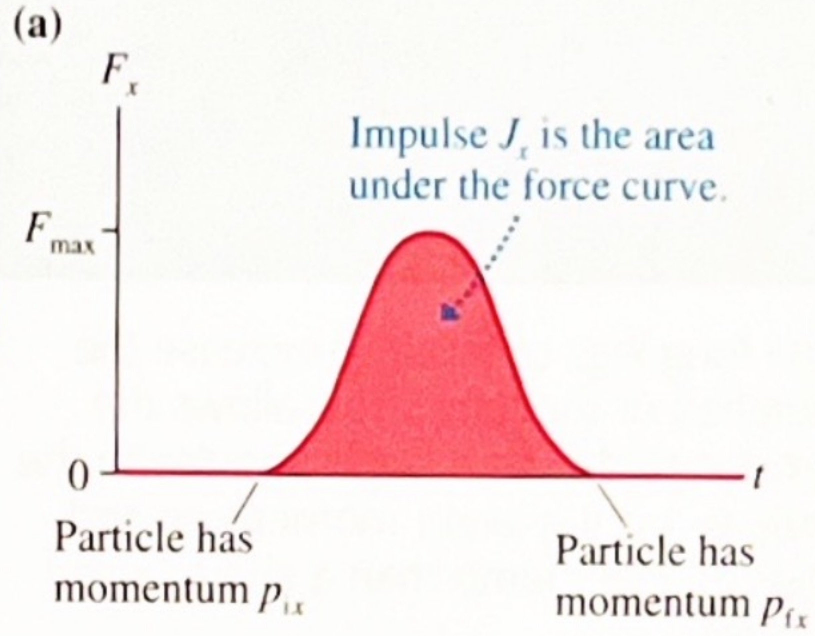
$$mdv = F(t)dt$$

$$m\int_{v_i}^{v_f} dv = mv_f - mv_i = \Delta p = \int_{t_i}^{t_f} F(t)\,dt$$

This time integral of the force (change in momentum) is the impulse (J).

$$J = \int_{t_i}^{t_f} F(t)\,dt$$

FIGURE 9.5 Looking at the impulse graphically.



THE IMPULSE-MOMENTUM THEOREM ($\Delta p = J$)

From the previous slide:

$$\Delta p = \int_{t_i}^{t_f} F(t)\,dt = J$$

This impulse, added to the *initial momentum* of an object, changes the *final momentum* of an object:

$$p_f = p_i + J$$

This momentum accounting can be represented as a bar chart.

The impulse-momentum theorem applies only during the *brief* interaction between the object. The momenta just before and after the collision is considered. (During this moment, other forces in the environment may be ignored.)

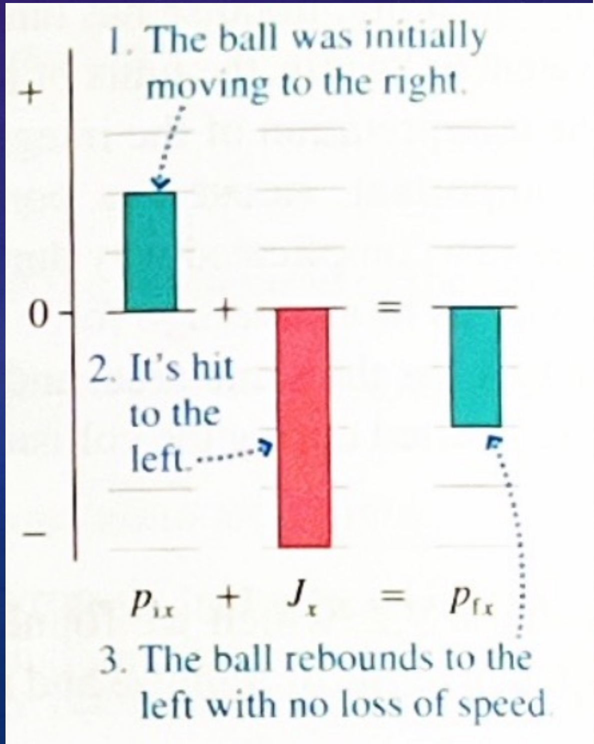
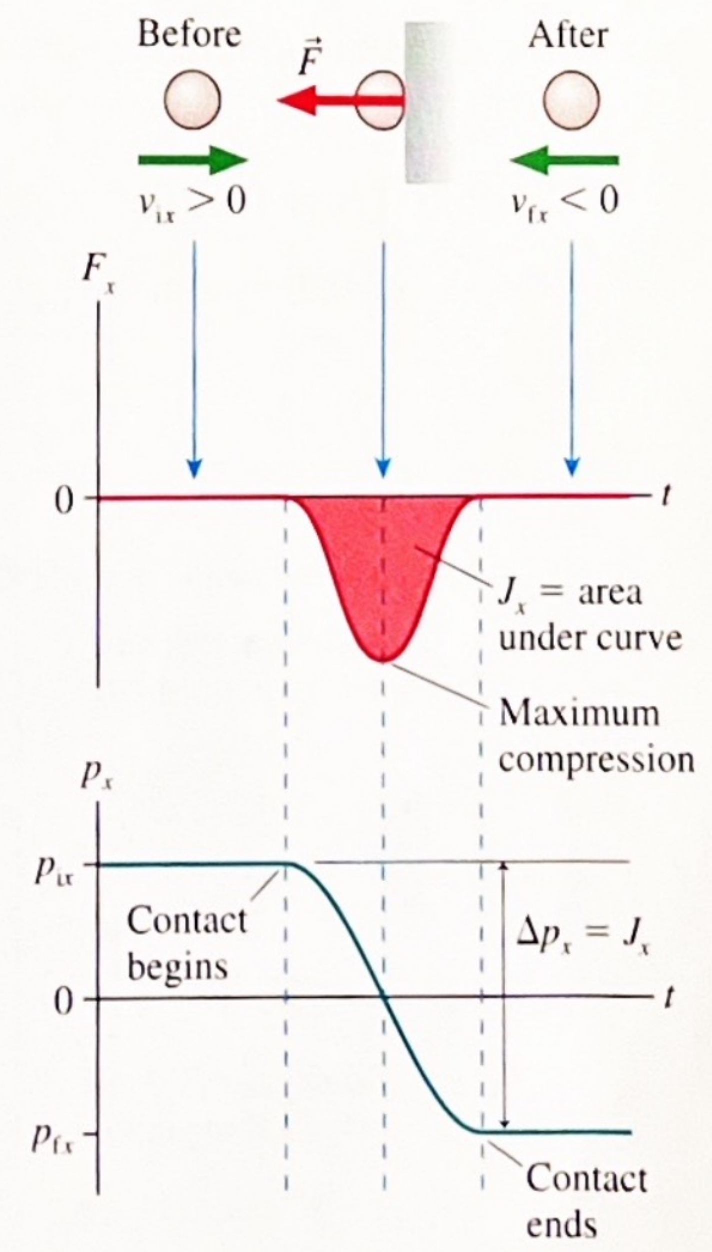


FIGURE 9.6 The impulse-momentum theorem helps us understand a rubber ball bouncing off a wall.



SOLVING MOMENTUM PROBLEMS (EXAMPLE #1)

EXAMPLE 9.2 A bouncing ball

A 100 g rubber ball is dropped from a height of 2.00 m onto a hard floor. **FIGURE 9.10** shows the force that the floor exerts on the ball. How high does the ball bounce?

MODEL Model the ball as a particle subjected to an impulsive force while in contact with the floor. Using the impulse approximation, we'll neglect gravity during these 8.00 ms. The fall and subsequent rise are free-fall motion.

VISUALIZE **FIGURE 9.11** is a pictorial representation. Here we have a three-part problem (downward free fall, impulsive collision, upward free fall), so the pictorial motion includes both the before and after of the collision (v_{1y} changing to v_{2y}) and the beginning and end of the free-fall motion. The bar chart shows the momentum change during the brief collision. Note that p is negative for downward motion.

FIGURE 9.10 The force of the floor on a bouncing rubber ball.

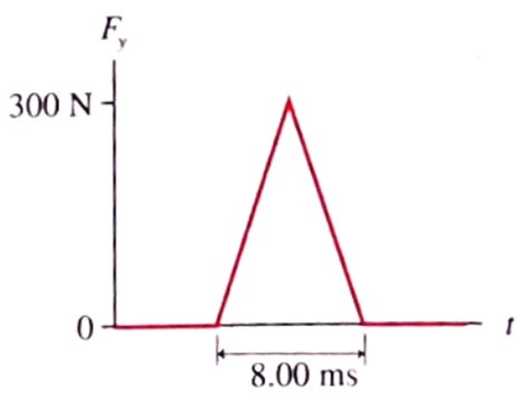
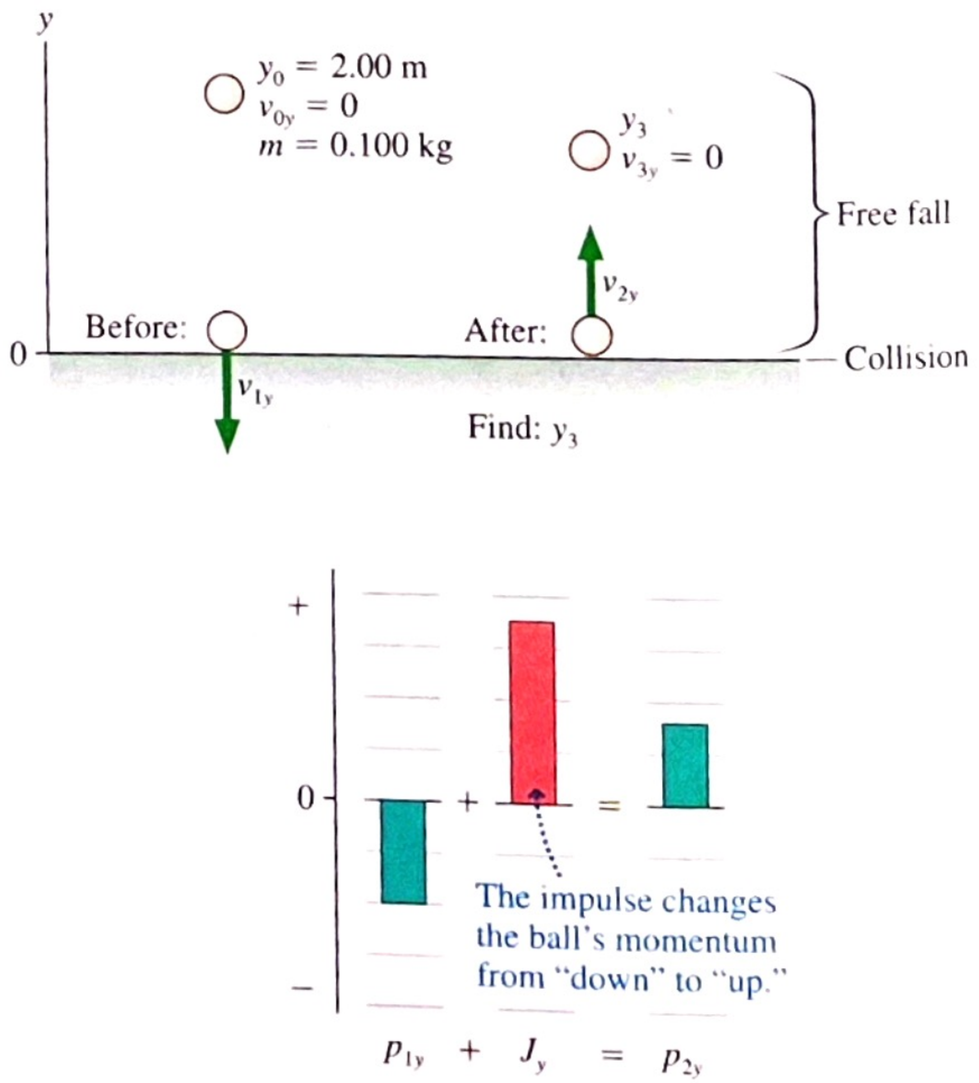


FIGURE 9.11 Pictorial representation of the ball and a momentum bar chart of the collision with the floor.



SOLVING MOMENTUM PROBLEMS (EXAMPLE #1)

SOLVE Velocity v_{1y}, the ball's velocity *immediately* before the collision, is found using free-fall kinematics with $\Delta y = -2.0$ m:

$$v_{1y}^2 = v_{0y}^2 - 2g\Delta y = 0 - 2g\Delta y$$

$$v_{1y} = \sqrt{-2g\Delta y} = \sqrt{-2(9.80 \text{ m/s}^2)(-2.00 \text{ m})} = -6.26 \text{ m/s}$$

We've chosen the negative root because the ball is moving in the negative y-direction.

The impulse-momentum theorem is $p_{2y} = p_{1y} + J_y$. The initial momentum, just before the collision, is $p_{1y} = mv_{1y} = -0.626$ kg m/s. The force of the floor is upward, so J_y is positive. From Figure 9.10, the impulse J_y is

$$J_y = \text{area under the force curve} = \frac{1}{2} \times (300 \text{ N}) \times (0.0080 \text{ s})$$

$$= 1.200 \text{ N s}$$

Thus

$$p_{2y} = p_{1y} + J_y = (-0.626 \text{ kg m/s}) + 1.200 \text{ N s} = 0.574 \text{ kg m/s}$$

and the post-collision velocity is

$$v_{2y} = \frac{p_{2y}}{m} = \frac{0.574 \text{ kg m/s}}{0.100 \text{ kg}} = 5.74 \text{ m/s}$$

The rebound speed is less than the impact speed, as expected. Finally a second use of free-fall kinematics yields

$$v_{3y}^2 = 0 = v_{2y}^2 - 2g\Delta y = v_{2y}^2 - 2gy_3$$

$$y_3 = \frac{v_{2y}^2}{2g} = \frac{(5.74 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)} = 1.68 \text{ m}$$

The ball bounces back to a height of 1.68 m.

ASSESS The ball bounces back to less than its initial height, which is realistic.

CONSERVATION OF MOMENTUM

From Newton's Second Law (in terms of momentum):

$$\vec{F} = m\vec{a} = \frac{d\vec{p}}{dt}$$

What does Newton's Third Law tell us about momentum exchanges between two objects?

$$\frac{d\vec{p}_1}{dt} = F_{1,2} \qquad \frac{d\vec{p}_2}{dt} = -F_{2,1}$$

What if we wanted to know the change in the *total* momentum of the system?

$$\frac{d}{dt}(\vec{p}_1 + \vec{p}_2) = \frac{d\vec{p}_1}{dt} + \frac{d\vec{p}_2}{dt} = \vec{F}_{1,2} + \left(-\vec{F}_{2,1}\right) = 0$$

$$\int d(\vec{p}_1 + \vec{p}_2) = \vec{p}_1 + \vec{p}_2 = \int 0\, dt = 0 \cdot t + C$$

The sum of the total momentum *over the time of the interaction* is constant!!!

So, the total momentum BEFORE and AFTER a collision is the same: $p_{1,i} + p_{2,i} = p_{1,f} + p_{2,f}$

GENERALIZING THE CONSERVATION OF MOMENTUM

Consider a system of several particles. The total momentum of the system is defined such that:

$$\vec{P} = \vec{p}_1 + \cdots + \vec{p}_N = \sum_{k=1}^{N} \vec{p}_k$$

How does the total momentum of the system change over time?

$$\frac{d\vec{P}}{dt} = \frac{d}{dt}\left(\sum_{k=1}^{N} \vec{p}_k\right) = \sum_{k=1}^{N} \frac{d\vec{p}_k}{dt} = \sum_{k=1}^{N} \vec{F}_k$$

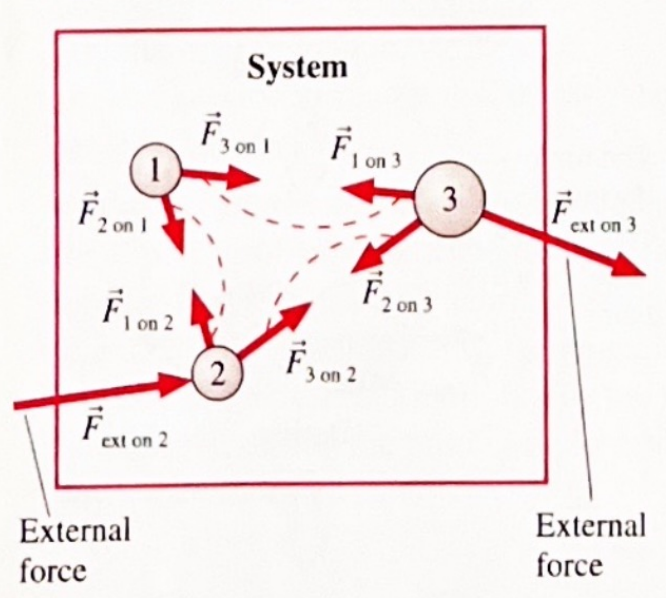
What is a *single* $\vec{F}_k$? This is sum of all forces acting on particle k. So, all the interactions with all the other particles AND any external forces.

$$\vec{F}_k = \sum_{j\neq k}^{N} \vec{F}_{j \text{ on } k} + \vec{F}_{\text{ext on } k}$$

This is all the forces on ONE particle. Now, we must add up the forces in the system:

$$\frac{d\vec{P}}{dt} = \sum_{k=1}^{N}\left(\sum_{j\neq k}^{N} \vec{F}_{j \text{ on } k} + \vec{F}_{\text{ext on } k}\right) = \sum_{k=1}^{N}\sum_{j\neq k}^{N} \vec{F}_{j \text{ on } k} + \sum_{k=1}^{N} \vec{F}_{\text{ext on } k}$$

FIGURE 9.14 A system of particles.



$j \neq k$ is used to denote that particles can't interact with themselves.

FINISHING UP

Remembering that the sum of all interaction forces is zero:

$$\frac{d\vec{P}}{dt} = \sum_{k=1}^{N}\sum_{j\neq k}^{N}\vec{F}_{j\text{ on }k} + \sum_{k=1}^{N}\vec{F}_{\text{ext on }k} = \sum_{k=1}^{N}\vec{F}_{\text{ext on }k} = \vec{F}_{\text{net}}$$

The rate of change of momentum is equal to the net force applied to the system.

HOWEVER, in an isolated system, $\frac{d\vec{P}}{dt} = \vec{F}_{\text{net}} = 0\text{N}$. (No external forces! The total momentum doesn't change.)

The Law of Conservation of Momentum – The total momentum ($\vec{P}$) of an isolated system is a constant. Interactions within the system do not change the system's total momentum. (Action/reaction pairs)

Thus, $P_f = P_i$. "What you start with is what you end with!"

SOLVING MOMENTUM PROBLEMS

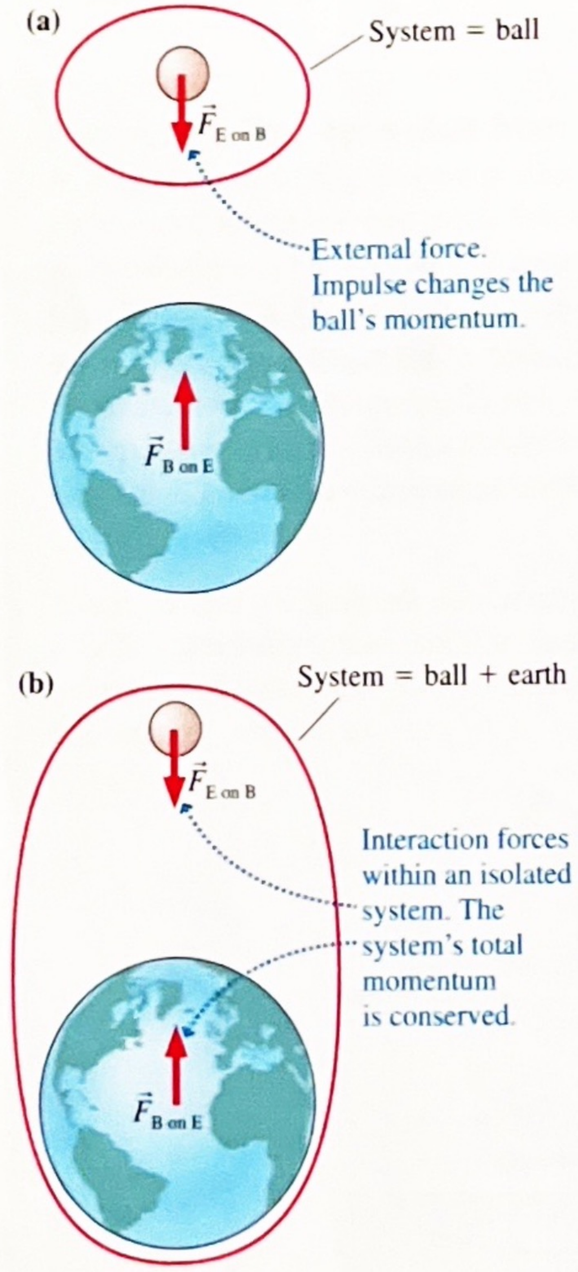
- When solving momentum problems, select a system such that the total momentum is conserved.

- If we only consider a ball being dropped and bouncing off the floor, the momentum is NOT conserved. The momentum BEFORE (-) the collision is not equal to the momentum AFTER (+) the collision since the ball is moving in the opposite direction.

- HOWEVER, if the system is defined such that $\vec{P} = \vec{p}_{\text{ball}} + \vec{p}_{\text{Earth}}$ then the total momentum is conserved.

- The earth does, indeed, have a momentum equal and opposite to that of the ball, but the earth is so massive that it needs only an infinitesimal velocity to match the ball's momentum.

FIGURE 9.17 Whether or not momentum is conserved as a ball falls to earth depends on your choice of the system.



SUMMARY

This impulse, added to the *initial momentum* of an object, changes the *final momentum* of an object: $p_f = p_i + J$

The impulse-momentum theorem applies only during the *brief* interaction between the object.

HOWEVER, in an isolated system, $\frac{d\vec{P}}{dt} = \vec{F}_{\text{net}} = 0\text{N}$. (No external forces! The total momentum doesn't change.)

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