Discrete Random Variables

MAT 152 – STATISTICAL METHODS I
LECTURE 3
INSTRUCTOR: DUSTIN ROTEN
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The Poisson Distribution

The Poisson probability distribution gives the probability of a number of events occurring in a **fixed interval** (provided the average rate is known).

$X \sim P_d(\mu)$: "X is a random variable with a Poisson distribution."

For large values of *n* and small values of *p*, $B(n, p) \approx P_d(\mu)$.

$$P(X = k) = \frac{\mu^k e^{-\mu}}{k!}$$

https://en.wikipedia.org/wiki/Sim%C3%A9on_Denis_Poisson

Example

The Zenithal Hourly Rate (ZHR) is the expected number of meteors a single observer would see in an hour of a meteor shower's peak.

Here, the mean ZHR (μ_{ZHR}) for the list of 2020 meteor showers is $\mu_{ZHR} = 54$

Given this average, what is the probability of seeing more than 60 meteors per hour during an event? $P(x > 60)$? (Assuming that the relationship follows a Poisson distribution!)

2020 Major Meteor Showers (Class I)

Shower	**Activity Period**	**Maximum**		**Radiant**		**Velocity**	**r**	**Max.**	**Time**	**Moon**
		Date	**S. L.**	**R.A.**	**Dec.**	**km/s**		**ZHR**		
Quadrantids (QUA)	Dec 22-Jan 17	Jan 04	283.16°	15:21	+49.5°	40.7	2.1	120	0500	08
Lyrids (LYR)	Apr 14-Apr 30	Apr 22	032.3°	18:09	+33.4°	45.5	2.1	18	0400	28
eta Aquarids (ETA)	Apr 17-May 24	May 06	046.2°	22:32	-00.8°	65.7	2.4	60	0400	14
Southern delta Aquarids (SDA)	Jul 21-Aug 23	Jul 29	126.9°	22:42	-16.4°	41.3	3.2	20	0300	09
Perseids (PER)	Jul 17-Sep 01	Aug 12	140.0°	03:13	+58.1°	59.1	2.6	100	0400	24
Orionids (ORI)	Sep 23-Nov 27	Oct 22	208.9°	06:24	+15.7°	66.3	2.5	23	0500	06
Leonids (LEO)	Nov 02-Nov 30	Nov 18	236°	10:15	+21.8°	70.2	2.5	15	0500	02
Geminids (GEM)	Dec 01-Dec 22	Dec 14	262°2	07:33	+32.4°	33.7	2.6	120	0100	00
Ursids (URS)	Dec 19-Dec 24	Dec 21	270°1	14:40	+75.4°	32.9	3.0	10	0500	07

Information and Table Template Courtesy the International Meteor Organization.

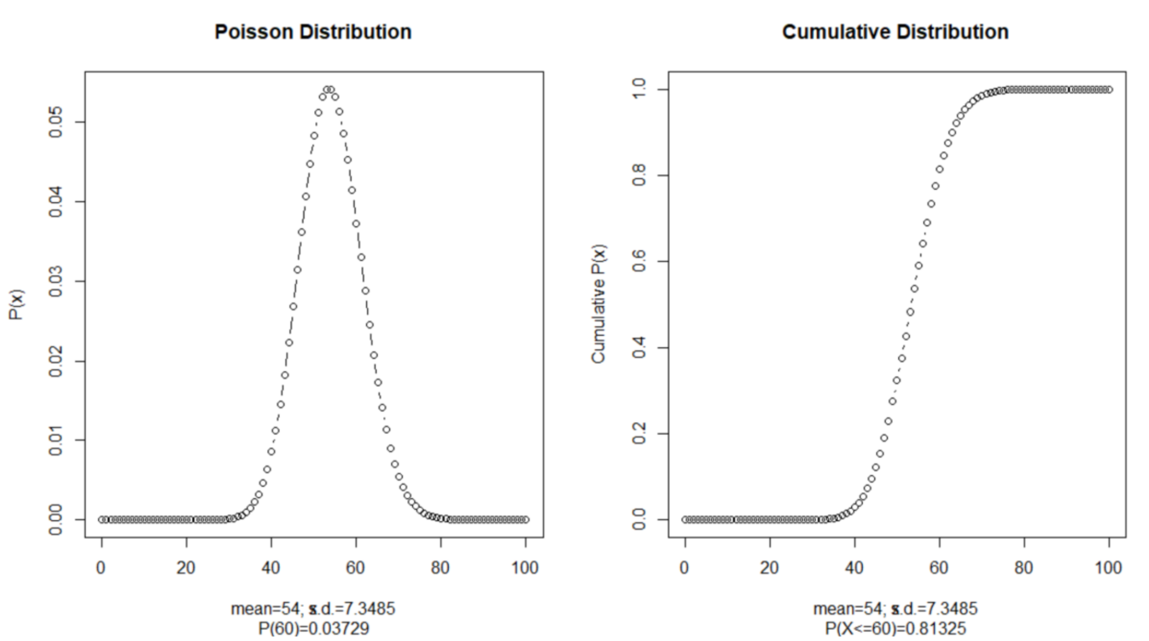
Example (cont.)

Since our current tools won't allow us to calculate $P(x > 60)$ directly, the complement can be used.

$$P(x > 60) = 1 - P(x \leq 60) = 1 - 0.81325 = 0.18675$$

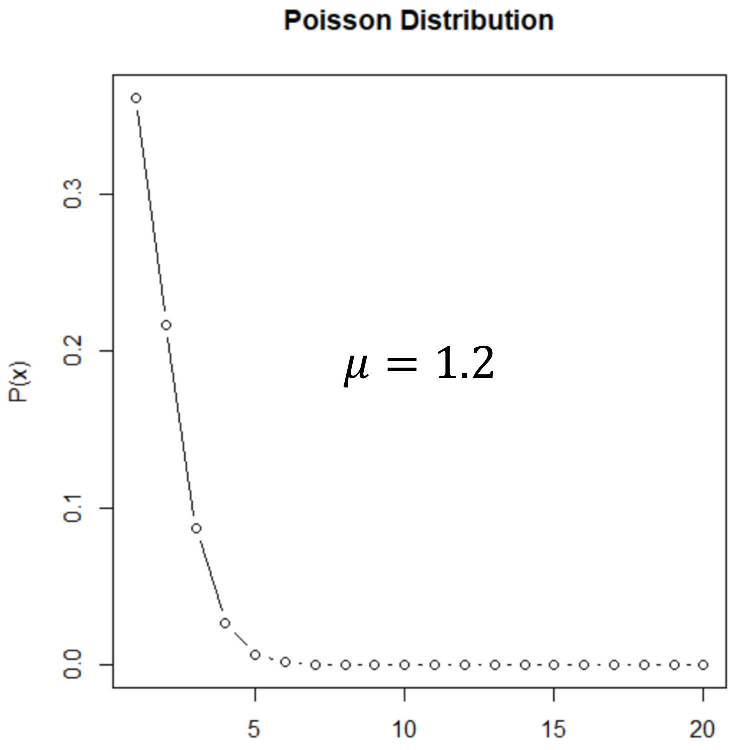
What is the probability of a ZHR of 60?

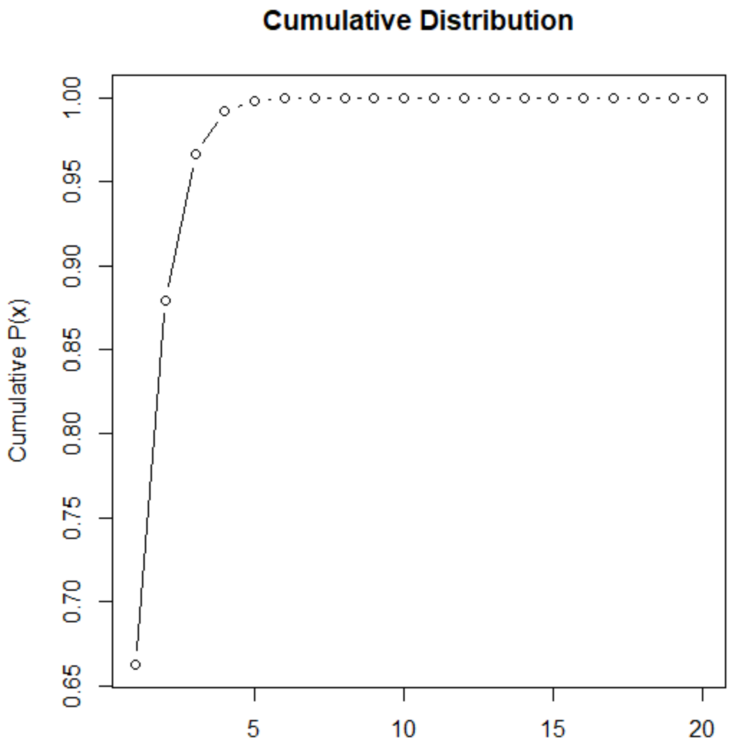
$$P(x = 60) = 0.03729$$



More Notes on the Poisson Distribution

- Poisson distributions may look like binomial distributions.
- However, for small values of μ, they can look much different.
- variance: $\sigma^2 = \mu$
- standard deviation: $\sigma = \sqrt{\mu}$





Example

Suppose the mean annual death rate in a large intensive care unit (ICU) of a hospital is 134 deaths per year. What is the probability of 4 deaths occurring in a week?

X = the number of deaths per week.

The interval must be converted: $\left(\frac{1}{52}\right) \cdot 134 \approx 2.577$ deaths per week.

Also known: $\sigma^2 = 2.577$, $\sigma = \sqrt{2.577} = 1.6053$

Assuming $X \sim P_d(\mu \approx 2.577)$, find $P(4)$.

$P(4) = 0.13966$

FYI, $P(5) = 0.07198$

A Review of Discrete Probability Distributions

Discrete random variables deal with countable events.

As the number of experiments increases, the calculated probabilities get closer to the theoretical probability.

We discussed three discrete probability distributions:

1. Binomial distribution
2. Geometric distribution
3. Poisson distribution

Binomial Distribution Review

Three characteristics:

1. Fixed number of trials.
2. The trials are independent.
3. Two possible outcomes: success and failure.

The binomial distribution answers the question: "Out of *x* number of trials, what is the probability of a certain number of successes?"

$X \sim B(n, p)$

- $n =$ number of trials
- $p =$ probability of a success

Mean outcome: $\mu = np$

Variance: $\sigma^2 = npq$ $\quad (q = 1 - p)$

Standard Deviation: $\sigma = \sqrt{npq}$

$$\boxed{P(X = k) = \binom{n}{k} p^k q^{n-k}}$$

Geometric Distribution Review

Three characteristics:

1. One or more Bernoulli trials. (Can be infinite)
2. The trials are independent.
3. Two possible outcomes: success and failure.

The geometric distribution answers the question: "What is the probability of a certain number of failures before a success?"

$X \sim G(p)$

- $p = \text{probability of a success}$

Mean outcome: $\mu = \frac{1}{p}$

Variance: $\sigma^2 = \left(\frac{1}{p}\right)\left(\frac{1}{p} - 1\right)$

Standard Deviation: $\sigma = \sqrt{\left(\frac{1}{p}\right)\left(\frac{1}{p} - 1\right)}$

$$\boxed{P(X = k) = q^{k-1}p}$$

Poisson Distribution Review

Three characteristics:

1. Provides the probability of events occurring in a fixed interval.
2. The events are independent.
3. The Poisson distribution approximates the binomial distribution in certain cases (small *p*, large *n*).

The Poisson distribution answers the question: "What is the probability of a certain number of events within an interval?"

$X \sim P_d(\mu)$

- $\mu = \text{mean for the interval}$

Variance: $\sigma^2 = \mu$

Standard Deviation: $\sigma = \sqrt{\mu}$

$$\boxed{P(X = k) = \frac{\mu^k e^{-\mu}}{k!}}$$