



# Lecture 5.3 – Friction and Drag

DUSTIN ROTEN, PH.D.

WILKES COMMUNITY COLLEGE

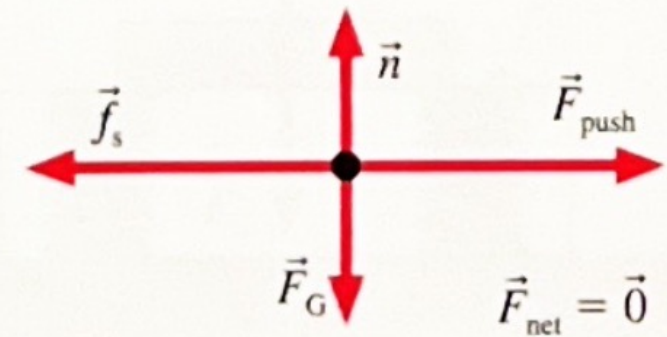
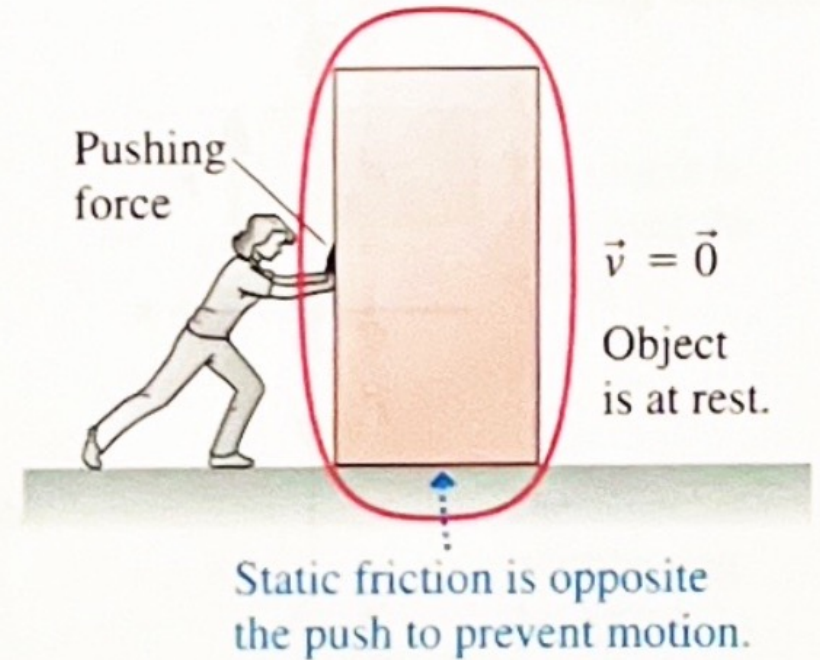
FALL 2023



# Static Friction ( $\vec{f}_s$ )

- ▶ Static friction ( $\vec{f}_s$ ) acts on an object to keep it from slipping.
- ▶ To determine the direction of  $\vec{f}_s$ , decide which way the object would move if there were no friction.  $\vec{f}_s$  points in the *opposite* direction to prevent the motion.
- ▶ The magnitude of  $\vec{f}_s$  depends on how hard an object is “pushed”. As the pushing force increases,  $\vec{f}_s$  will increase in the opposite direction.

**FIGURE 6.11** Static friction keeps an object from slipping.



Free-body diagram

# Static Friction ( $\vec{f}_s$ )

- ▶ If the force acting on an object becomes greater than the maximum possible value of  $\vec{f}_s$ , the object slips and starts to move.
- ▶ This maximum value is called  $\vec{f}_{s,\max}$ .
- ▶ If  $\vec{f}_s < \vec{f}_{s,\max}$  an object remains at rest.
- ▶ If  $\vec{f}_s = \vec{f}_{s,\max}$  the object is at its slipping point.
- ▶ A static friction force of  $\vec{f}_s > \vec{f}_{s,\max}$  is not possible.

The mathematical model of static friction defines it to be proportional to the normal force acting on an object.

$$\vec{f}_{s,\max} = \mu_s \vec{n}$$

$\mu_s$  is the coefficient of static friction. This is a unitless number that depends on the interactions between surfaces.

**FIGURE 6.12** Static friction acts in response to an applied force.



$\vec{F}_{\text{push}}$  is balanced by  $\vec{f}_s$  and the box does not move.



As  $\vec{F}_{\text{push}}$  increases,  $\vec{f}_s$  grows . . .



$$f_s = f_{s,\max}$$

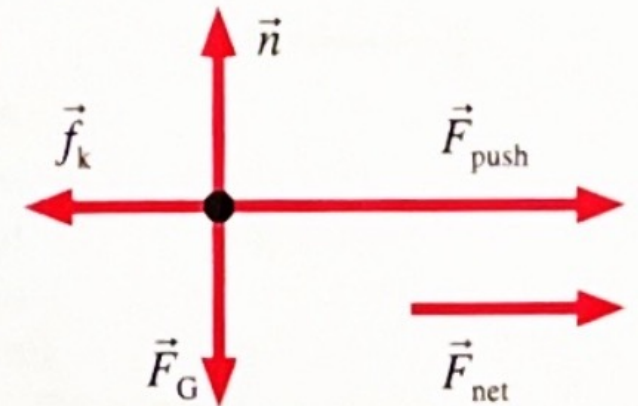
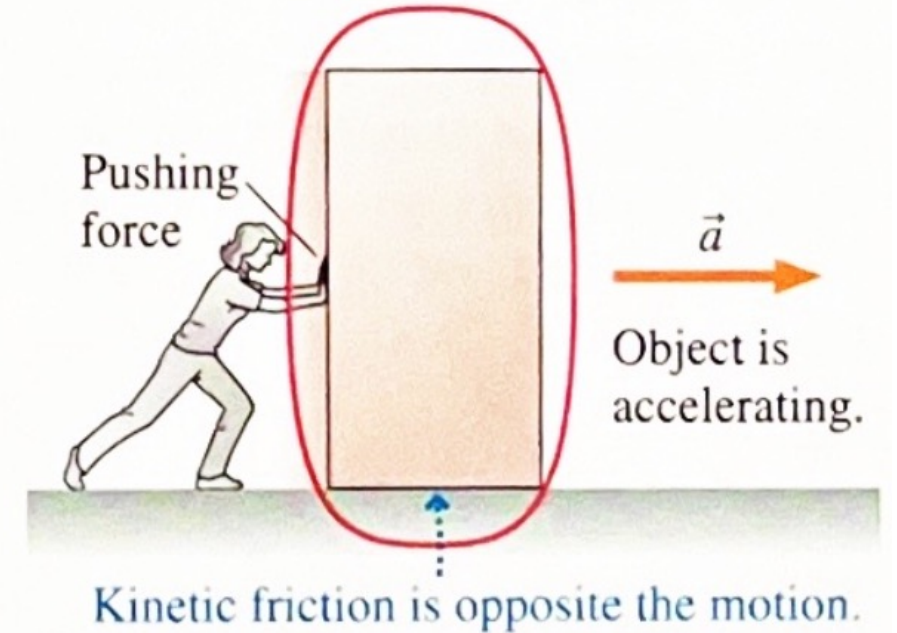
. . . until  $f_s$  reaches  $f_{s,\max}$ . Now, if  $\vec{F}_{\text{push}}$  gets any bigger, the object will start to move.



# Kinetic Friction ( $\vec{f}_k$ )

- ▶ Suppose an object is sitting on the floor at rest. A horizontal force ( $\vec{F}_{\text{push}}$ ) is slowly applied such that its magnitude begins at 0N and increases.
- ▶ Once  $\vec{F}_{\text{push}} = \vec{f}_{s,\text{max}}$ , the object starts sliding across the floor.
- ▶ Even though the influence of static friction was broken, there is still an interaction between the object and the floor while it begins to accelerate. This new friction force is the kinetic friction ( $\vec{f}_k$ ).

**FIGURE 6.13** The kinetic friction force is opposite the direction of motion.





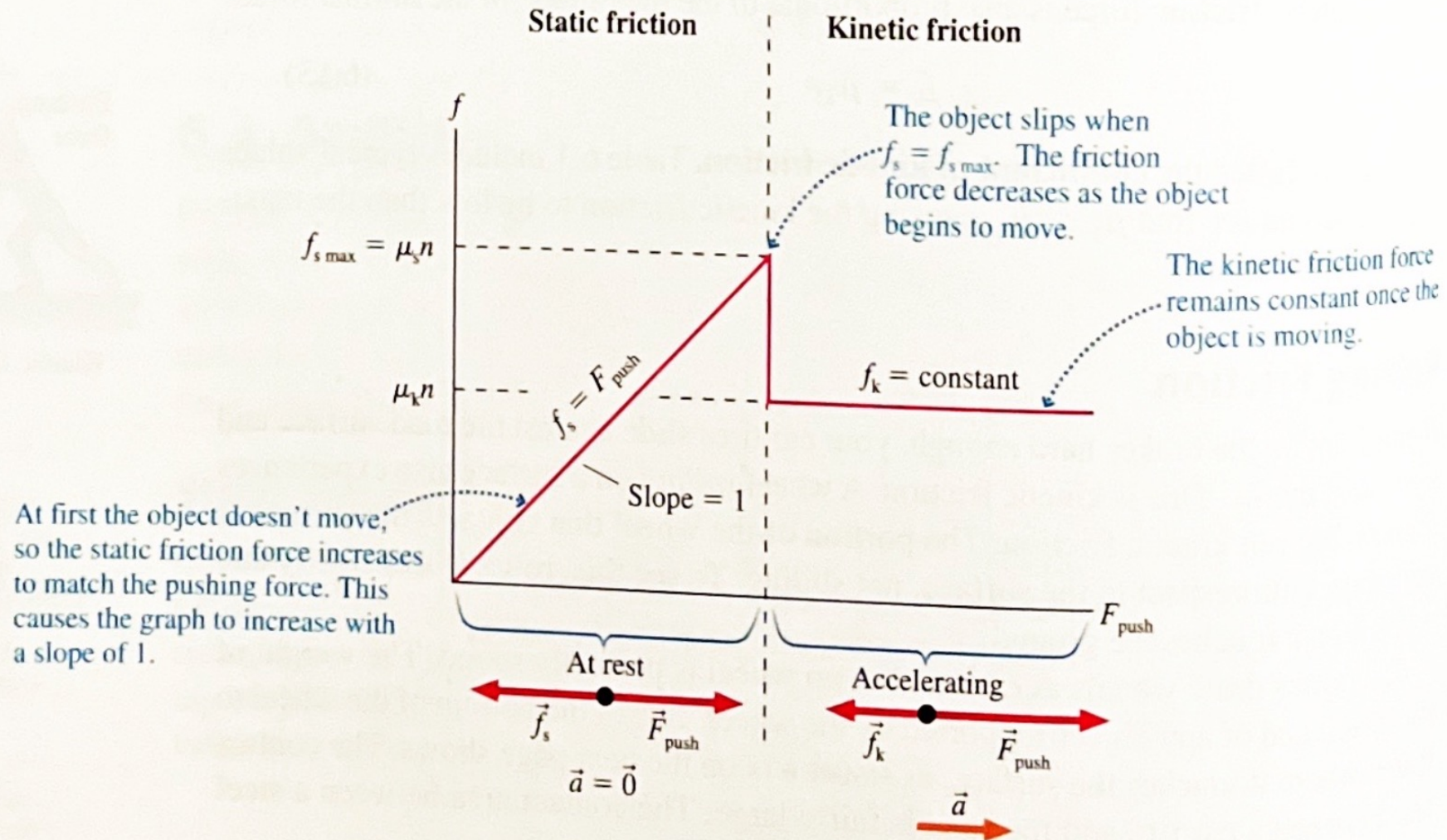
# Kinetic Friction ( $\vec{f}_k$ )

- ▶  $\vec{f}_k$  has a near-constant magnitude.
- ▶ It is always the case that  $\vec{f}_k < \vec{f}_{s,\max}$ . The magnitude of kinetic friction is always smaller than the magnitude of maximum static friction.
- ▶ “It is easier to keep an object moving than it is to get it moving.”
- ▶  $\vec{f}_k$  is always opposite the direction of motion.
- ▶ Like static friction, there is a coefficient of kinetic friction ( $\mu_k$ ) such that:
$$\vec{f}_k = \mu_k \vec{n}$$



# Static vs. Kinetic Friction

FIGURE 6.15 The friction force response to an increasing applied force.



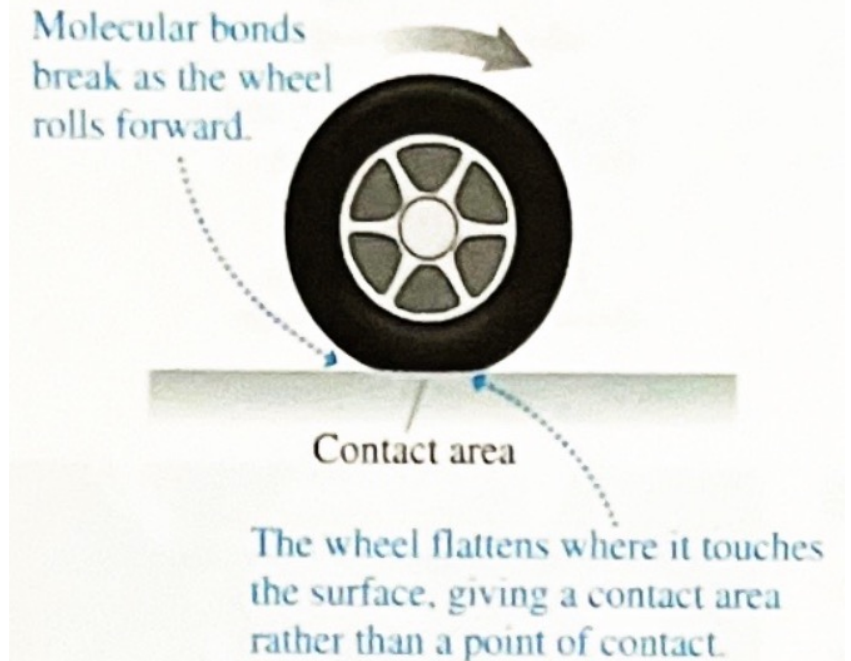


# Rolling Friction ( $\vec{f}_r$ )

- ▶ The portion of the wheel that contacts the surface is stationary with respect to the surface, not sliding.
- ▶ Molecular bonds are quickly established where the wheel presses against the surface. These bonds must be broken as the wheel rolls forward.
- ▶ The effort needed to break these bonds is called rolling friction.
- ▶ The coefficients of rolling friction ( $\mu_r$ ) are generally less than kinetic friction.

$$\vec{f}_r = \mu_r \vec{n}$$

**FIGURE 6.14** Rolling friction is due to the contact area between a wheel and the surface.

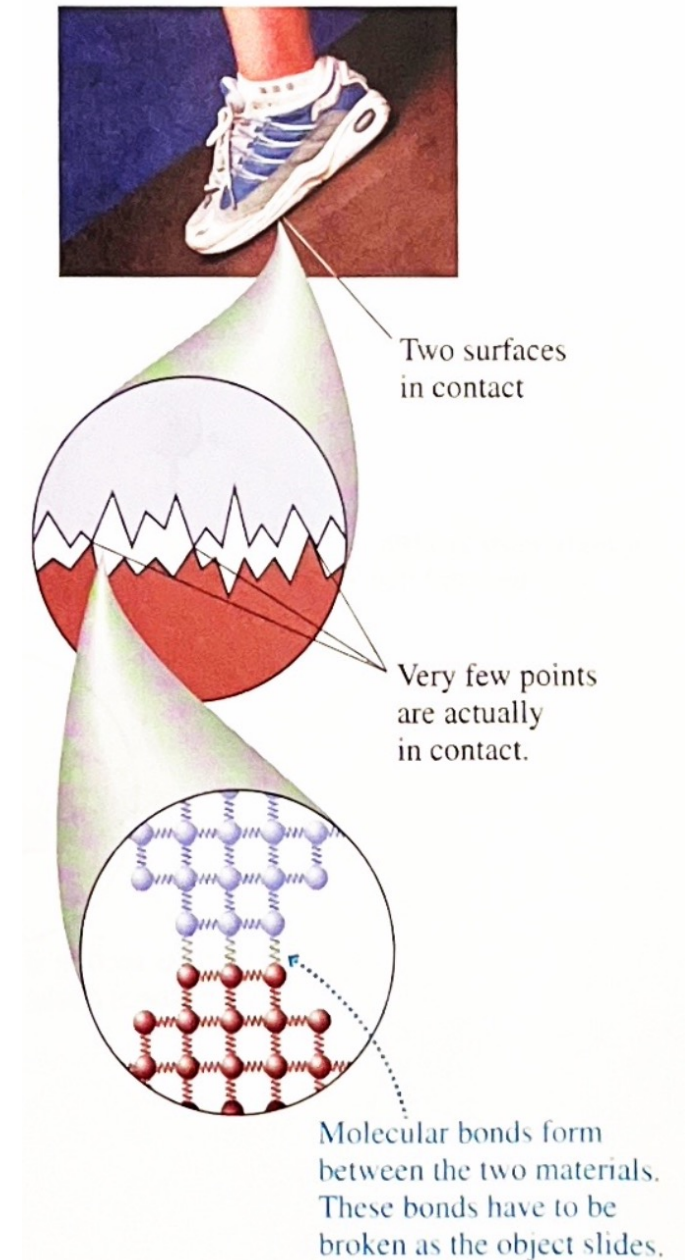




# The Microscopic View of Friction

- ▶ When two surfaces touch, the microscopic “high points” of each surface interact with each other.
- ▶ The amount of contact depends on how hard these surfaces are forced into one another. (This is why  $\vec{f} \propto \vec{n}$ )
- ▶ At the point of contact, molecular bonds are formed between the atoms of the two surfaces. (static friction)
- ▶ As two surfaces move past one another, there are still forces between atoms, but molecular bonds do not form. (kinetic friction)
- ▶ Wear and tear of such surfaces are called *abrasions*.

FIGURE 6.19 An atomic-level view of friction.



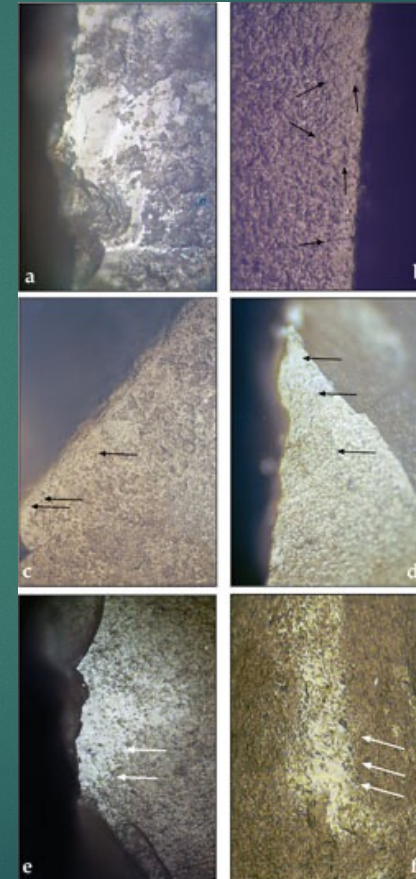
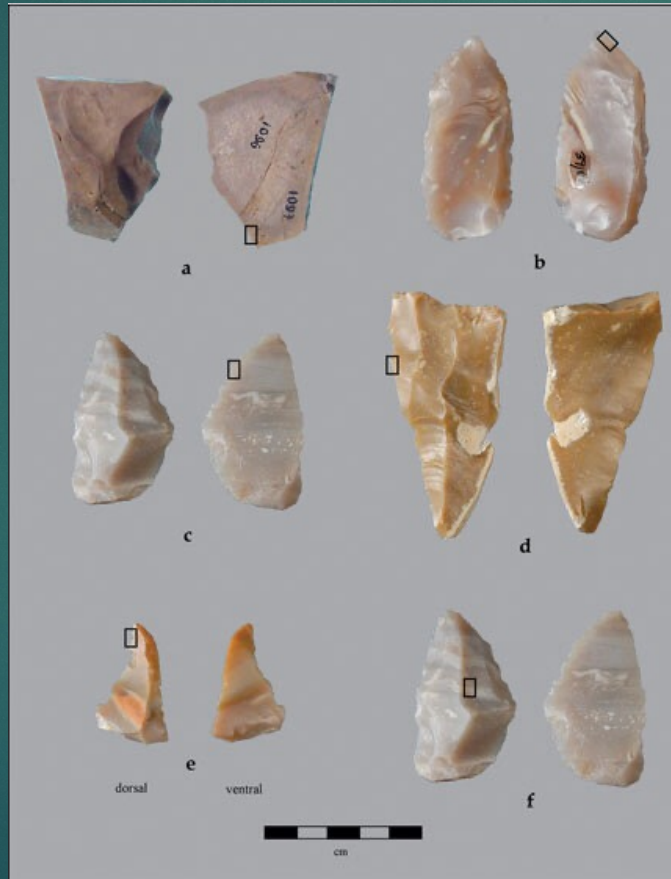


# Tribology

## Atomic Force Microscopy (AFM)

Tribology – The study of friction

Dr. Tonya Coffey at Appalachian State University used an AFM to examine the wear and tear on ancient stone tools.



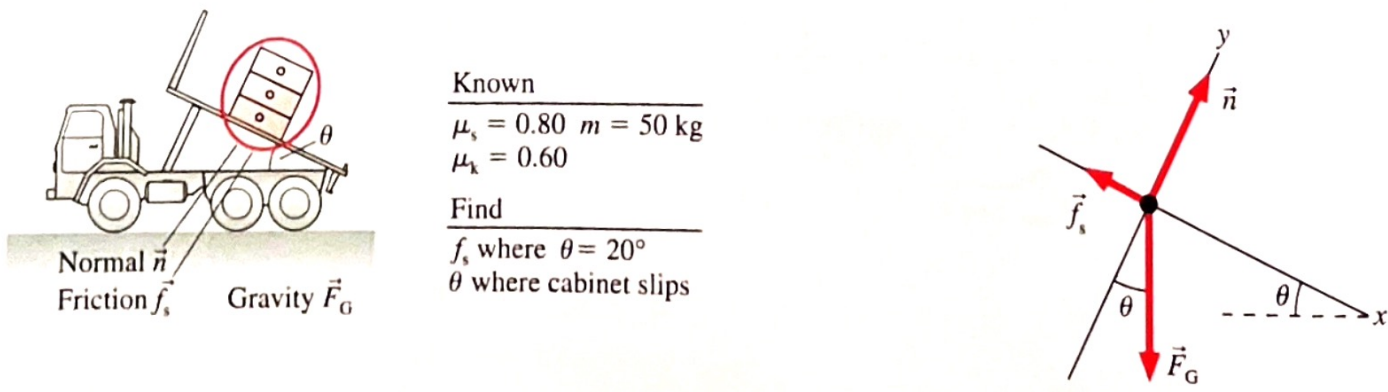
Dr. Tonya Coffey



# Friction Example (#1)

A 50kg steel file cabinet is in the back of a dump truck. The truck's bed (also steel) is slowly tilted. Given that  $\mu_s = 0.80$  and  $\mu_k = 0.60$  for steel-on-steel, at what angle will the file cabinet begin to slide?

FIGURE 6.17 The pictorial representation of a file cabinet in a tilted dump truck.



$$\vec{F} = \sum_i F_i = \vec{F}_G + \vec{f}_{s,\max} + \vec{n} = \vec{0}$$

$$(F_{\text{net}})_x = F_G \sin \theta - f_{s,\max} = mg \sin \theta - \mu_s n = 0 \text{ N}$$

$$(F_{\text{net}})_y = n - F_g \cos \theta = n - mg \cos \theta = 0 \text{ N}$$

Two equations, two unknowns

From the y-direction:

$$n = mg \cos \theta$$

Putting this into the x-direction:

$$mg \sin \theta - \mu_s mg \cos \theta = 0 \text{ N}$$

Divide both sides by  $mg$ :

$$\sin \theta - \mu_s \cos \theta = 0 \text{ N}$$

Rearrange:

$$\mu_s = \frac{\sin \theta}{\cos \theta} = \tan \theta$$

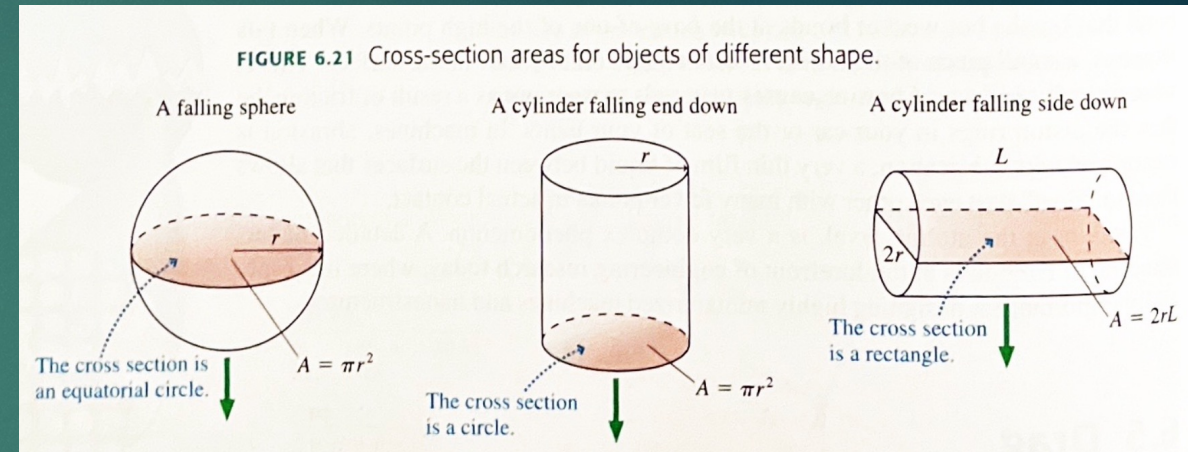
Determine the angle:

$$\theta = \tan^{-1}(\mu_s) = \tan^{-1}(0.8) \approx 39^\circ$$



# Drag Force

- ▶ Any time you move through a fluid (such as water or a gas/atmosphere) you experience drag ( $\vec{D}$ ).
- ▶ Like friction,  $\vec{D}$  is in the opposite direction of  $\vec{v}$ .
- ▶ Like static friction,  $\vec{D}$  increases as  $\vec{v}$  increases.
- ▶ A simple drag model for everyday motion (small, dense objects at slower speeds) is approximated by
$$\vec{D} \approx \frac{1}{4} A v^2$$
- ▶ Like any other force, drag is measured in Newtons



$A$  is the cross-section area of the object

What are the units of the variables?



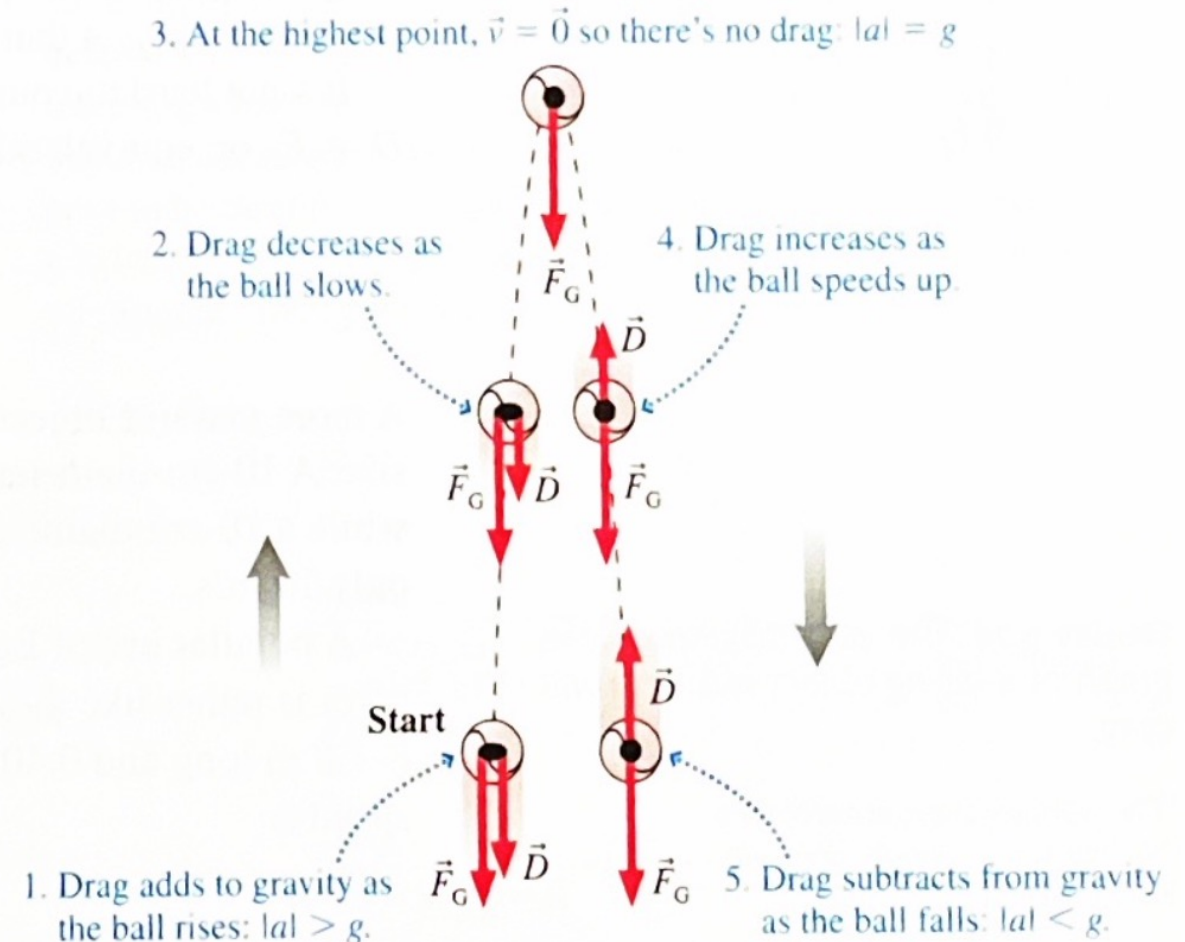
# An Example of Drag

FIGURE 6.23 shows a ball moving up and down vertically. If there were no air resistance, the ball would be in free fall with  $a_{\text{free fall}} = -g$  throughout its flight. Let's see how drag changes this.

Referring to Figure 6.23:

1. The drag force  $\vec{D}$  points down as the ball rises. This *increases* the net force on the ball and causes the ball to slow down *more quickly* than it would in a vacuum. The magnitude of the acceleration, which we'll calculate below, is  $|a| > g$ .
2. The drag force decreases as the ball slows.
3.  $\vec{v} = \vec{0}$  at the highest point in the ball's motion, so there's no drag and the acceleration is simply  $a_{\text{free fall}} = -g$ .
4. The drag force increases as the ball speeds up.
5. The drag force  $\vec{D}$  points up as the ball falls. This *decreases* the net force on the ball and causes the ball to speed up *less quickly* than it would in a vacuum. The magnitude of the acceleration is  $|a| < g$ .

FIGURE 6.23 Drag force on a ball moving vertically.



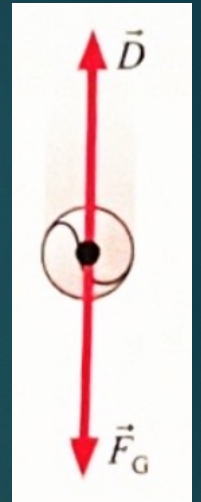


# A Ball in Free-fall

- Consider dropping a ball such that it undergoes free-fall

$$\vec{a} = \frac{\vec{F}_{\text{net}}}{m} = \frac{-mg + D}{m} = -\left(g - \frac{D}{m}\right)$$

- If the ball is light-weight (small/plastic) then  $m$  is small. This means that  $D$  isn't "cut" by much. (see equation above) Therefore,  $g$  is reduced.
- If the ball is heavy (ball bearing) then  $m$  is large. This means that  $D$  is divided by a large number, decreasing its influence. Thus, a heavy object falls at  $\approx -g$ .





# Terminal Speed

- ▶ The drag force increases as an object falls and gains speed. If the object falls for long enough, it will eventually reach its terminal velocity,  $v_t$ .

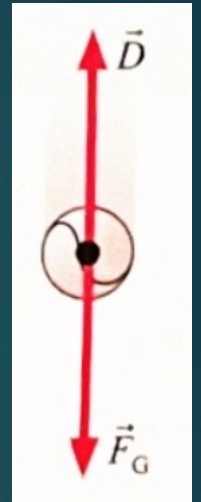
- ▶ This is when the drag and gravitational forces “cancel out”.

$$\vec{F}_{\text{net}} = \vec{D} - \vec{F}_G = 0\text{N}$$

- ▶ At  $v_t$ , an object no longer accelerates ( $\vec{a} = 0 \frac{\text{m}}{\text{s}}$ ).

- ▶ In the case where  $\vec{D} = \vec{F}_G$ :

$$\frac{1}{4}Av_t^2 \approx mg \text{ becomes } v_t = \sqrt{\frac{4mg}{A}}$$





# Summary

- ▶ There's three types of friction
  - ▶ Static friction ( $\vec{f}_s$ )
    - ▶ While an object is at rest, static friction opposes the potential motion.
    - ▶ The maximum value of static friction is  $f_{s,\max} = \mu_s n$
  - ▶ Kinetic friction ( $\vec{f}_k = \mu_k n$ )
    - ▶ Resists the motion of a moving object.
    - ▶ Always less than  $f_{s,\max}$
  - ▶ Rolling friction ( $\vec{f}_r = \mu_r n$ )
- ▶ Like friction, drag ( $\vec{D}$ ) opposes the motion of an object through a fluid.