LECTURE 8.1 - WORK AND KINETIC ENERGY

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INITIAL QUESTIONS

- Under what conditions is energy conserved?
- How does a system gain or lose energy?
- How is energy transferred into/out of a system?
- What is "work"?

DEFINITION OF “WORK”

- Energy is transferred into and out of a system by *pushes* and *pulls*. These pushes/pulls do *work*.

- A system can be characterized by two quantities: *kinetic* and *potential* energy.
  - Kinetic energy *(K)* is the energy of motion.
  - Potential energy *(U)* is the energy of position and interactions between objects.

- The sum of the kinetic and potential energies of a system is the *mechanical* energy. $E_{mech} = K + U$.

THERMAL ENERGY

- The microscopic motion of atoms and molecules within an object is a form of energy that is different from the object's mechanical energy. The total energy of the bonds between these atoms is called the system's *thermal energy* (E_{th}).

- Thermal energy is associated with a system's temperature.
  - A higher temperature indicates more microscopic motion (more thermal energy).

- Mechanical energy is transformed into thermal energy through friction.

- The system energy (E_{sys}) is given by $E_{\text{sys}} = E_{\text{mech}} + E_{\text{th}} = (K + U) + E_{\text{th}}$

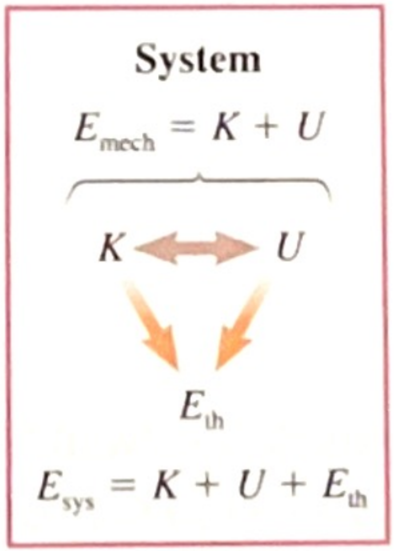
THERMAL ENERGY

- Kinetic and potential energy can be transformed into thermal energy.

- Thermal energy is not (normally) changed into kinetic or potential energy.

- Energy exchanges *within* the system are called *energy transformations*.
  - Energy transformations do not change E_{sys}.

FIGURE 11.1 Energy can be transformed within the system.



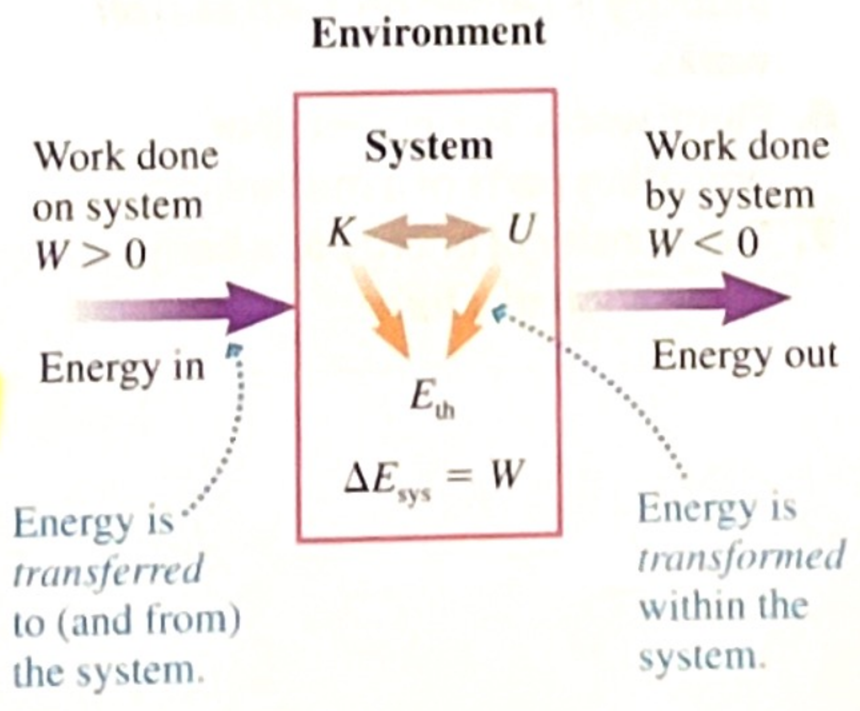
THERMAL ENERGY

- A system is always situated within an *environment*. Unless the system is completely isolated, it has the possibility of exchanging energy with the environment.

- An energy exchange between the system and the environment is called an *energy transfer*.

- This mechanical transfer of energy to or from the system is called *work (W)*.
  - $W > 0$ indicates work is done *on* the system.
  - $W < 0$ indicates work is done *by* the system.

FIGURE 11.2 The basic energy model of a system interacting with its environment.



THE BASIC ENERGY MODEL

$$\Delta E_{\text{sys}} = \Delta E_{\text{mech}} + \Delta E_{\text{th}} = \Delta K + \Delta U + \Delta E_{\text{th}} = W$$

Energy transferred in/out of system: $\Delta E_{sys} = W$

Energy transformed within a system: $\Delta E_{sys} = 0$

How much energy does a force transfer?

FORCE AND ENERGY

- How might an applied force do work on a particle?

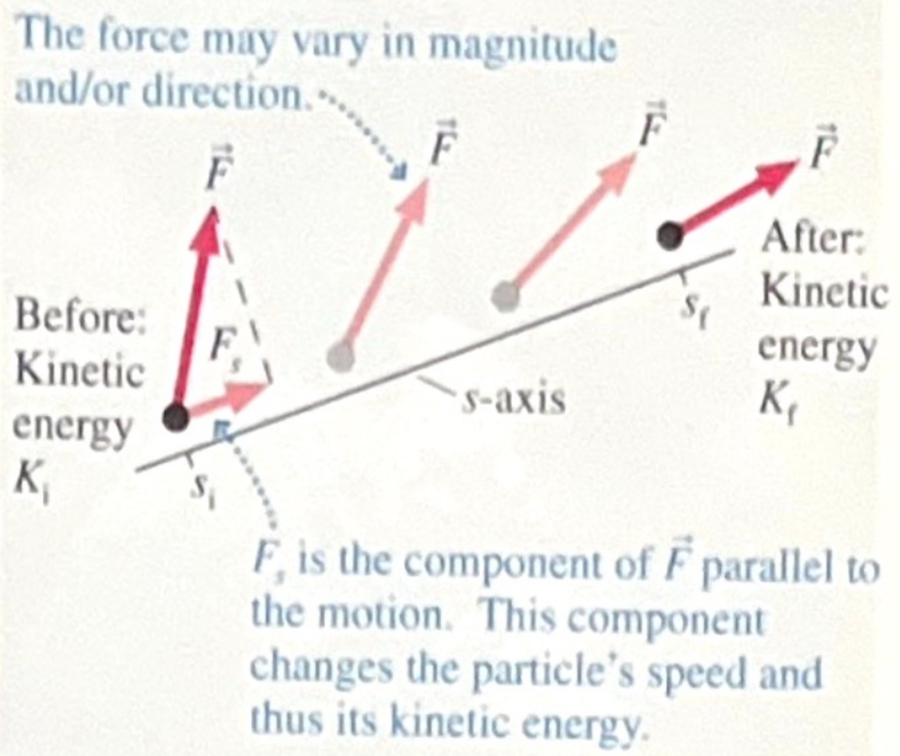
- An applied force causes an acceleration: $\vec{F} = m\vec{a}$. This is a change in velocity.

$$\vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt}$$

- Thus, the component of a force doing work on a particle results in a change of velocity/energy.

$$W = \Delta K$$

FIGURE 11.3 Force $\vec{F}$ does work as the particle moves from s_i to s_f.



FORCE AND ENERGY

- From Newton's Second Law:

$$F_s = ma_s = m\frac{dv_s}{dt}$$

(Chain Rule) $\frac{dv_s}{dt} = \frac{ds}{dt}\frac{dv_s}{ds} = v_s\frac{dv_s}{ds}$

$$F_s = mv_s\frac{dv_s}{ds}$$

$$mv_s dv_s = F_s ds$$

$$\int_{v_i}^{v_f} mv_s\, dv_s = \int_{s_i}^{s_f} F_s\, ds$$

$$\int_{v_i}^{v_f} mv_s\, dv_s = \int_{s_i}^{s_f} F_s\, ds$$

$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \int_{s_i}^{s_f} F_s\, ds$$

$$\mathrm{W} = \Delta K = \int_{s_i}^{s_f} F_s\, ds$$

Work is force × distance so the units are Joules ($N \cdot m = J$). This is the same as energy!

WORK-KINETIC ENERGY THEOREM

When several forces act on a particle over a distance, $W_{\mathrm{net}} = \sum W_i$.

When one or more forces act on a particle as it is displaced from an initial position to a final position, the net work done on the particle by these forces causes the particle's kinetic energy to change by $\Delta K = W_{\mathrm{net}}$.

A system gains or loses kinetic energy when work transfers energy between the environment and the system.

IMPULSE-MOMENTUM THEOREM
WORK-KINETIC ENERGY THEOREM

I-M Theorem: $\Delta p_s = J_s = \int_{t_i}^{t_f} F_s \, dt$

W-K Theorem: $\Delta K = W = \int_{s_i}^{s_f} F_s \, ds$

A force acting over a time interval (t_i to t_f) and distance (s_i to s_f) changes the momentum *and* kinetic energy of a particle.

An impulse is created AND work is done by a force acting on a particle.

FIGURE 11.4 Impulse and work are both the area under a force graph, but it's very important to know what the horizontal axis is.

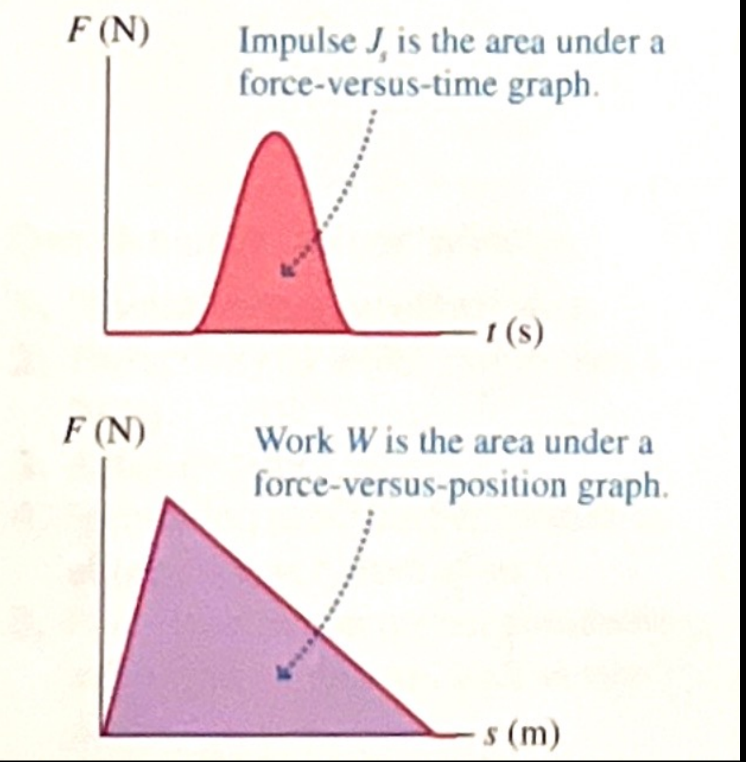
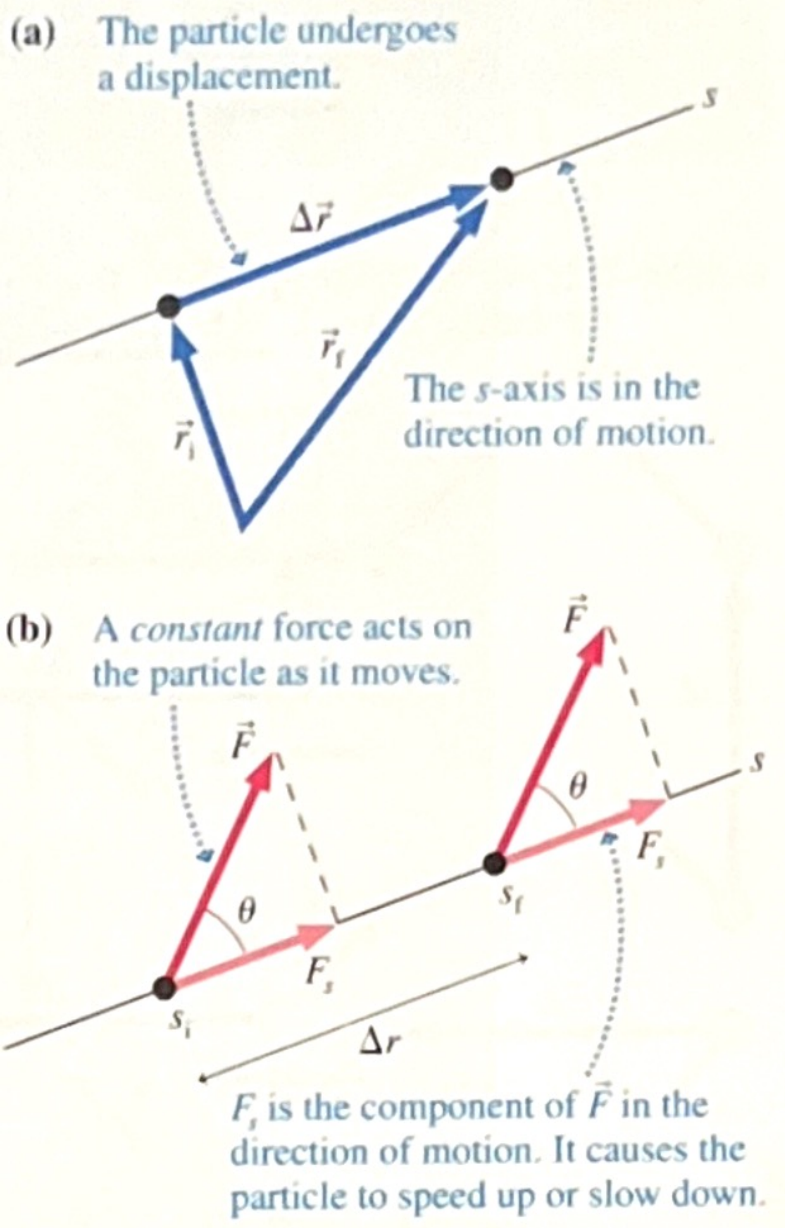


FIGURE 11.5 Work being done by a *constant* force as a particle moves through displacement $\Delta\vec{r}$.



CALCULATING WORK

Consider a force, $\vec{F}$, acting on a particle with constant strength and constant direction.

The particle moves through displacement, $\Delta\vec{r}$.

The force vector ($\vec{F}$) makes an angle (θ) with respect to the displacement ($\Delta\vec{r}$) , so the component of the force vector along the direction of motion is $F_{\parallel} = F\cos\theta$.

This is the component of the force that is responsible for changing the particle's velocity (ΔK).

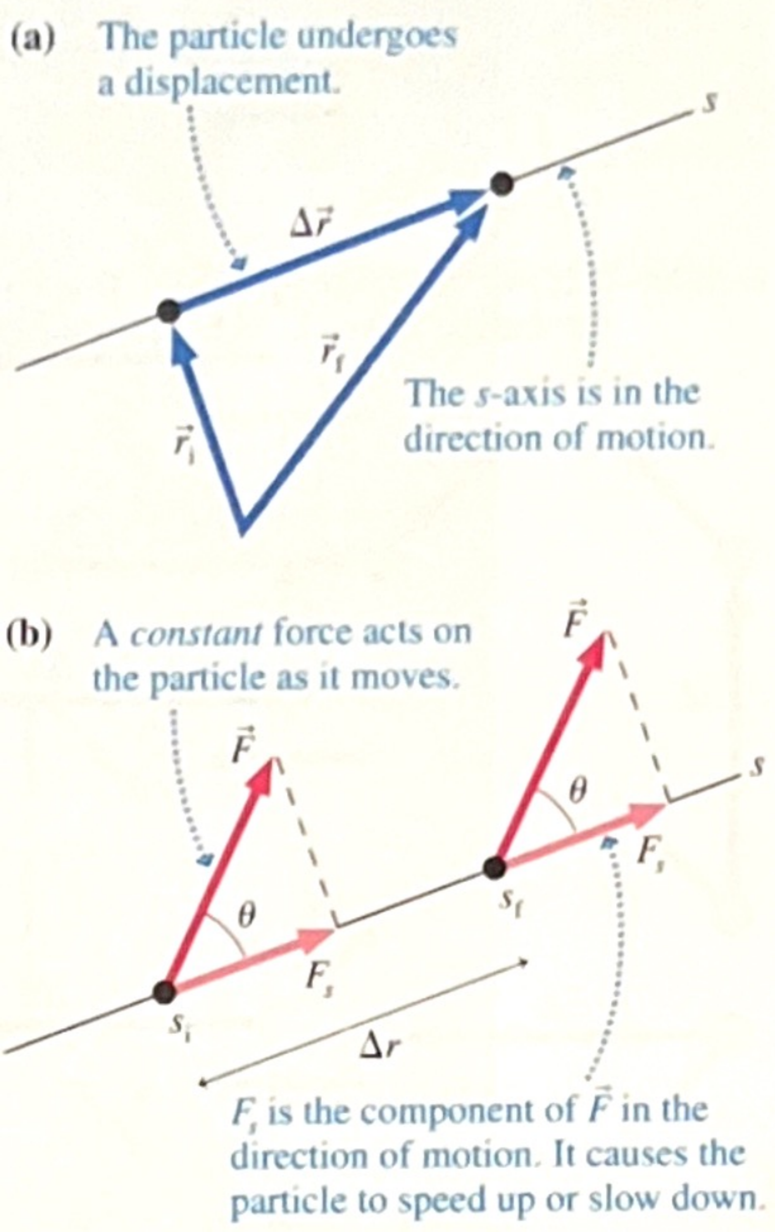


FIGURE 11.5 Work being done by a *constant* force as a particle moves through displacement $\Delta\vec{r}$.

CALCULATING WORK

$$W = \int_{s_i}^{s_f} F_s\, ds = \int_{s_i}^{s_f} F \cos\theta\, ds$$

$$F\cos\theta \int_{s_i}^{s_f} ds = F\cos\theta\,(s_f - s_i) = F(\Delta r)\cos\theta$$

According to the basic energy model, work can either be positive or negative to indicate energy transfer into or out of the system.

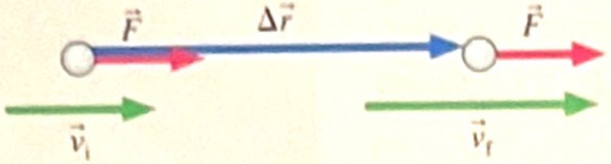
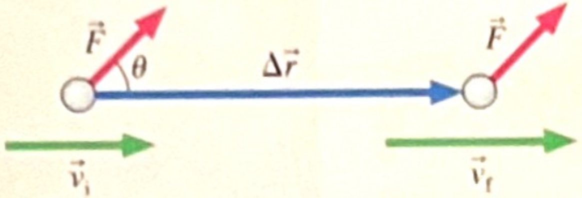
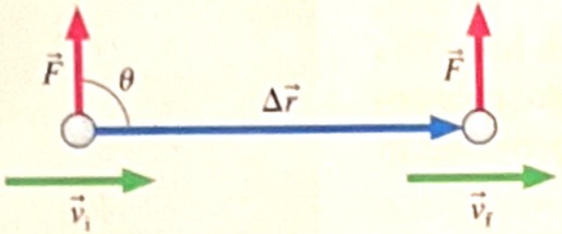
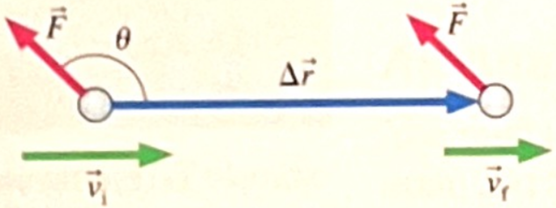
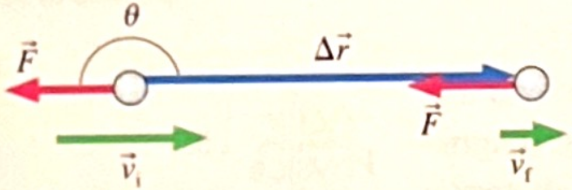
F and Δr are always positive, so the sign is determined by the angle θ.

Using the dot product (with a constant force):

$$W = \vec{F}\cdot\Delta\vec{r} = F(\Delta r)\cos\theta$$

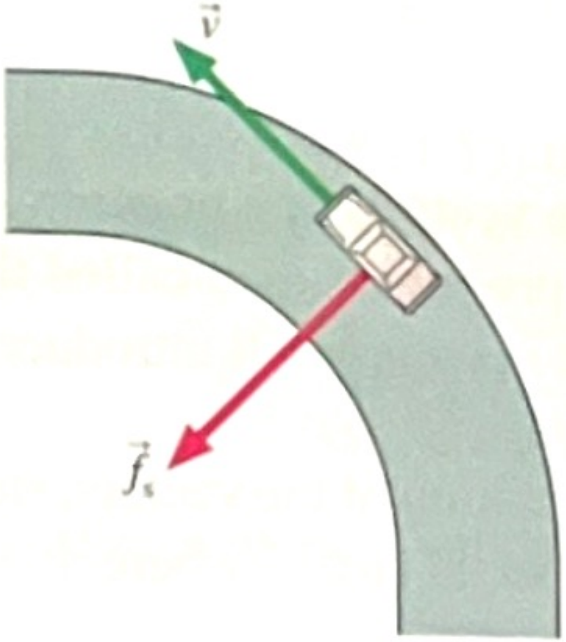
TACTICS BOX 11.1 Calculating the work done by a constant force

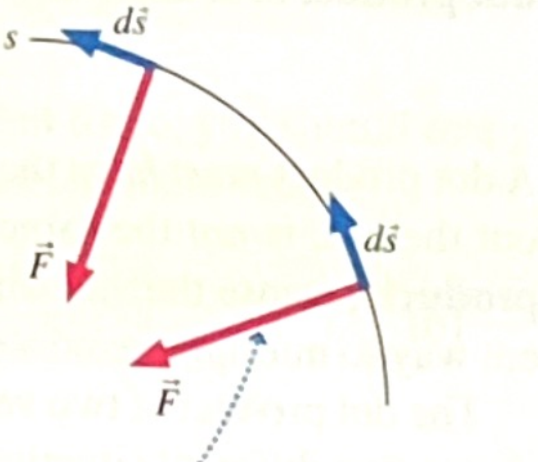
MP

Force and displacement	θ	Work W	Sign	Energy transfer
	0°	$F(\Delta r)$	+	Energy is transferred into the system. The particle speeds up. K increases.
	<90°	$F(\Delta r)\cos\theta$	+	
	90°	0	0	No energy is transferred. Speed and K are constant.
	>90°	$F(\Delta r)\cos\theta$	−	Energy is transferred out of the system. The particle slows down. K decreases.
	180°	$-F(\Delta r)$	−	

Exercises 3–10

Consider a car being held in a curve by friction.

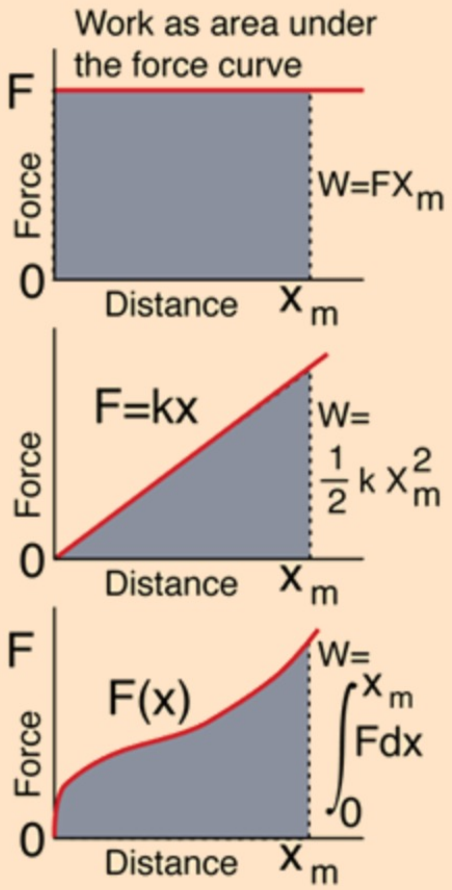




The force is everywhere perpendicular to the car's displacement, so it does no work.

WORK DONE BY VARIABLE FORCE

Work done by a variable force



The basic work relationship W=Fx is a special case which applies only to constant force along a straight line. That relationship gives the area of the rectangle shown, where the force F is plotted as a function of distance. In the more general case of a force which changes with distance, the work may still be calculated as the area under the curve. For example, for the work done to stretch a spring, the area under the curve can be readily determined as the area of the triangle. The power of calculus can also be applied since the integral of the force over the distance range is equal to the area under the force curve:

$$\text{Work} = \int_0^{x_m} F(x)\,dx = \int_0^{x_m} kx\,dx = \tfrac{1}{2}k x_m^2$$

For any function of x, the work may be calculated as the area under the curve by performing the integral

$$\text{Work} = \int_{x_1}^{x_2} F(x)\,dx$$

More general path