Lecture 5.2 – Mass, Weight, and Gravity

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Equilibrium

- An object on which $\vec{F}_{\mathrm{net}} = 0\mathrm{N}$ is in <u>equilibrium</u>.
- This could be an object at rest (*static equilibrium*, engineering statics).
- Or, this could be straight-line motion with constant velocity (*kinematic equilibrium, dynamics*).
- Both situations are the same since $\vec{F}_{\mathrm{net}} = 0\mathrm{N}$ and $\vec{a} = 0\,\frac{\mathrm{m}}{\mathrm{s}^2}$.
  - $(F_{net})_x = \sum_i (F_i)_x = 0\mathrm{N}$
  - $(F_{net})_y = \sum_i (F_i)_y = 0\mathrm{N}$

Equilibrium Problems

- MODEL
  - Make simplifying assumptions, use the particle model when appropriate
- VISUALIZE
  - Set up a coordinate system
  - Define symbols
  - Identify all of the forces using a free-body diagram
  - Identify the unknown
- SOLVE
  - $\vec{F}_{\mathrm{net}} = \sum_i \vec{F}_i = 0\mathrm{N}$
- ASSESS
  - Check that the result has the correct units and is reasonable

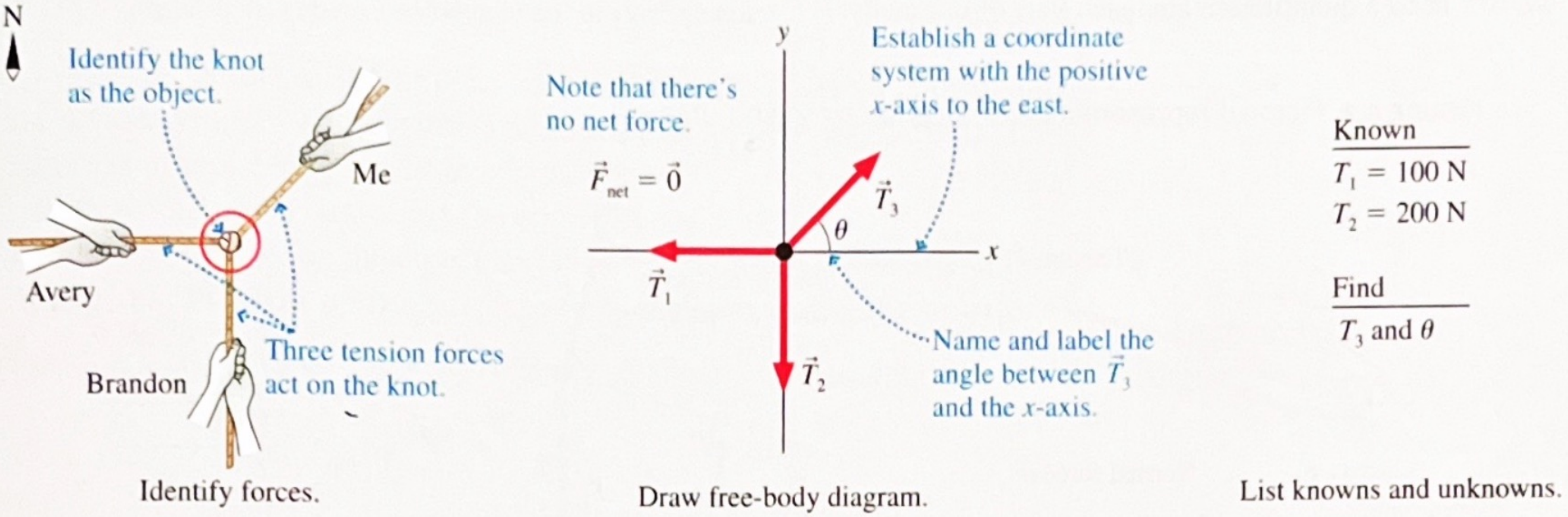
Example #1 (Static Equilibrium)

EXAMPLE 6.1 Three-way tug-of-war

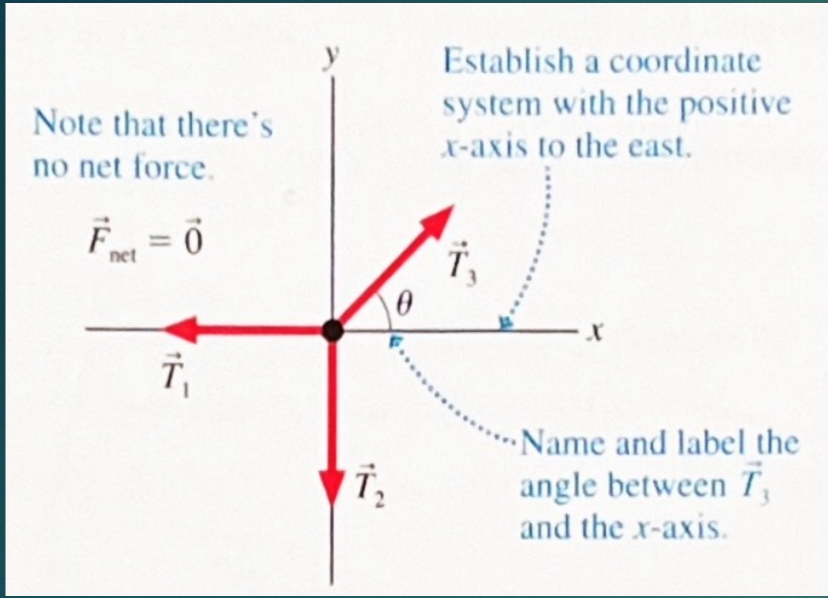
You and two friends find three ropes tied together with a single knot and decide to have a three-way tug-of-war. Avery pulls to the west with 100 N of force, while Brandon pulls to the south with 200 N. How hard, and in which direction, should you pull to keep the knot from moving?

MODEL We'll treat the *knot* in the rope as a particle in static equilibrium.

FIGURE 6.1 Pictorial representation for a knot in static equilibrium.



Example #1 (Static Equilibrium)



$$\vec{F}_{net} = \sum_i \vec{F}_i = \vec{T}_1 + \vec{T}_2 + \vec{T}_3$$

$$(F_{\text{net}})_x = \sum_i \left(\vec{F}_i\right)_x = \left(\vec{T}_1\right)_x + \left(\vec{T}_2\right)_x + \left(\vec{T}_3\right)_x = 0\text{N}$$

$$(F_{\text{net}})_y = \sum_i \left(\vec{F}_i\right)_y = \left(\vec{T}_1\right)_y + \left(\vec{T}_2\right)_y + \left(\vec{T}_3\right)_y = 0\text{N}$$

<u>x-direction</u>

$$\left(\vec{T}_1\right)_x = -T_1$$

$$\left(\vec{T}_2\right)_x = 0\text{N}$$

$$\left(\vec{T}_3\right)_x = T_3 \cos\theta$$

$$\boxed{\begin{gathered}\left(\vec{T}_1\right)_x + \left(\vec{T}_2\right)_x + \left(\vec{T}_3\right)_x = 0\text{N} \\ -T_1 + T_3 \cos\theta = 0\text{N} \\ T_1 = T_3 \cos\theta\end{gathered}}$$

<u>y-direction</u>

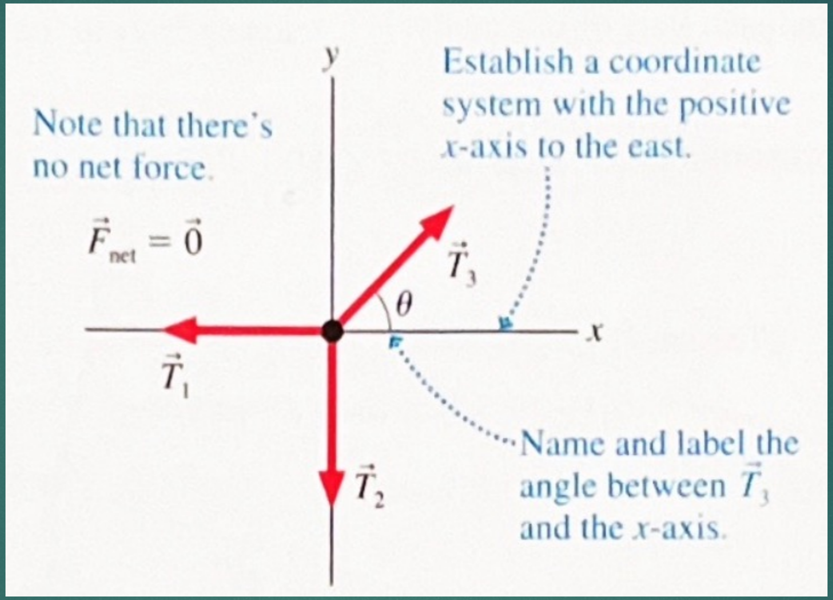
$$\left(\vec{T}_1\right)_y = 0\text{N}$$

$$\left(\vec{T}_2\right)_y = -T_2$$

$$\left(\vec{T}_3\right)_y = T_3 \sin\theta$$

$$\boxed{\begin{gathered}\left(\vec{T}_1\right)_y + \left(\vec{T}_2\right)_y + \left(\vec{T}_3\right)_y = 0\text{N} \\ -T_2 + T_3 \sin\theta = 0\text{N} \\ T_2 = T_3 \sin\theta\end{gathered}}$$

Example #1 (Static Equilibrium)



Angle of θ

$$\theta = \tan^{-1}\left(\frac{\text{opp}}{\text{adj}}\right) = \tan^{-1}\left(\frac{\mathrm{T}_3 \sin\theta}{T_3 \cos\theta}\right) = \tan^{-1}\left(\frac{\mathrm{T}_2}{\mathrm{T}_1}\right) = \tan^{-1}\left(\frac{200\mathrm{N}}{100\mathrm{N}}\right) = 63.4°$$

Magnitude of $\vec{T}_3$

$$T_3 = \frac{T_1}{\cos\theta} = \frac{100\mathrm{N}}{\cos 63.4°} \approx 224\mathrm{N}$$

Example #2 (Dynamic Equilibrium)

EXAMPLE 6.2 Towing a car up a hill

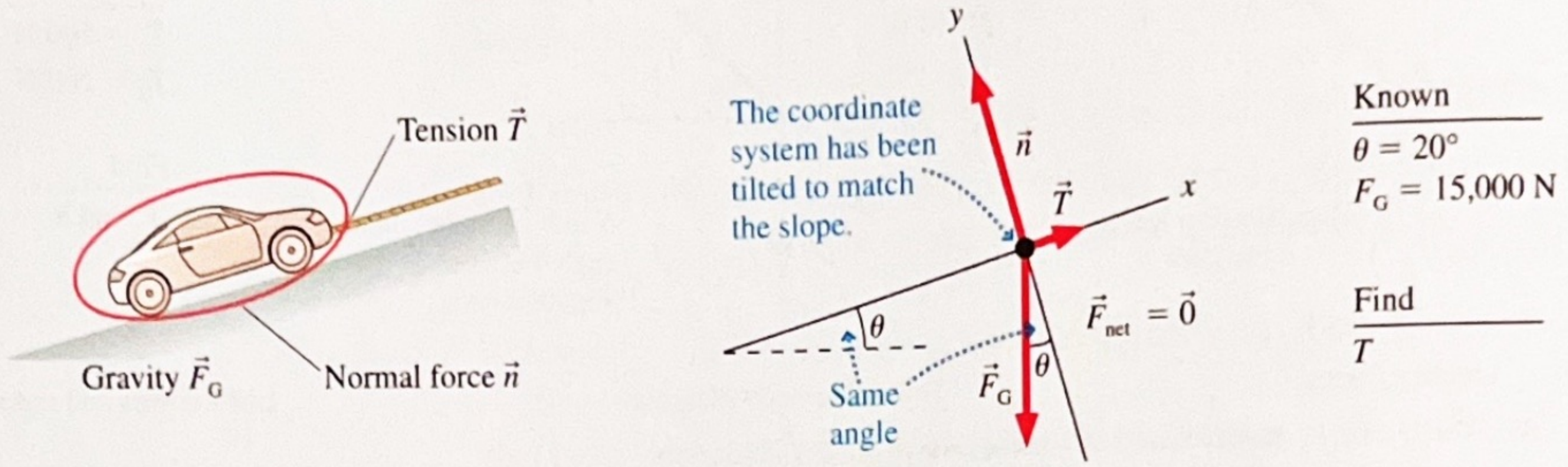
A car with a weight of 15,000 N is being towed up a 20° slope at constant velocity. Friction is negligible. The tow rope is rated at 6000 N maximum tension. Will it break?

MODEL We'll treat the car as a particle in dynamic equilibrium. We'll ignore friction.

VISUALIZE This problem asks for a yes or no answer, not a number, but we still need a quantitative analysis. Part of our analysis of the problem statement is to determine which quantity or quantities allow us to answer the question. In this case the answer is clear: We need to calculate the tension in the rope. FIGURE 6.2 shows the pictorial representation. Note the similarities to Examples 5.2 and 5.6 in Chapter 5, which you may want to review.

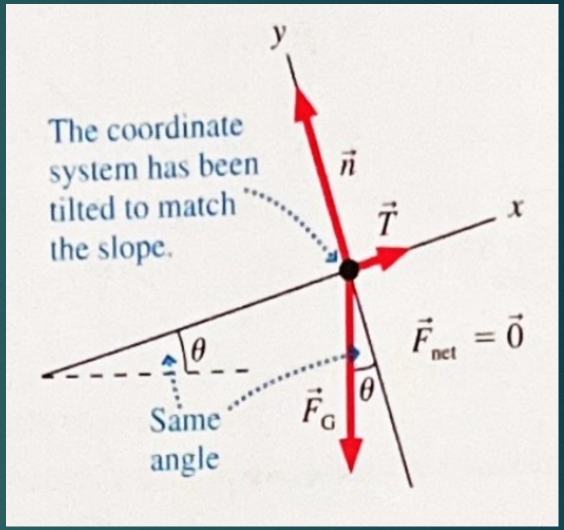
We noted in Chapter 5 that the weight of an object at rest is the magnitude F_G of the gravitational force acting on it, and that information has been listed as known. We'll examine weight more closely later in the chapter.

FIGURE 6.2 Pictorial representation of a car being towed up a hill.



Example #2 (Dynamic Equilibrium)

$$\vec{F}_{\text{net}} = \sum_i \vec{F}_i = \vec{T} + \vec{n} + \vec{F}_G$$



$$\left(\vec{F}_{\text{net}}\right)_x = \textstyle\sum_i F_x = T_x + n_x + \left(\vec{F}_G\right)_x = 0\text{N}$$

$$\left(\vec{F}_{\text{net}}\right)_y = \textstyle\sum_i F_y = T_y + n_y + \left(\vec{F}_G\right)_y = 0\text{N}$$

x-direction

$T_x = T$

$n_x = 0\text{N}$

$(F_G)_x = -F_G \sin\theta$

y-direction

$T_y = 0\text{N}$

$n_y = n$

$(F_G)_y = -F_G \cos\theta$

$T - F_G \sin\theta = 0\text{N}$

$n - F_G \cos\theta = 0\text{N}$

$T = F_G \sin\theta = (15{,}000N)\sin 20^\circ = 5100\text{N}$

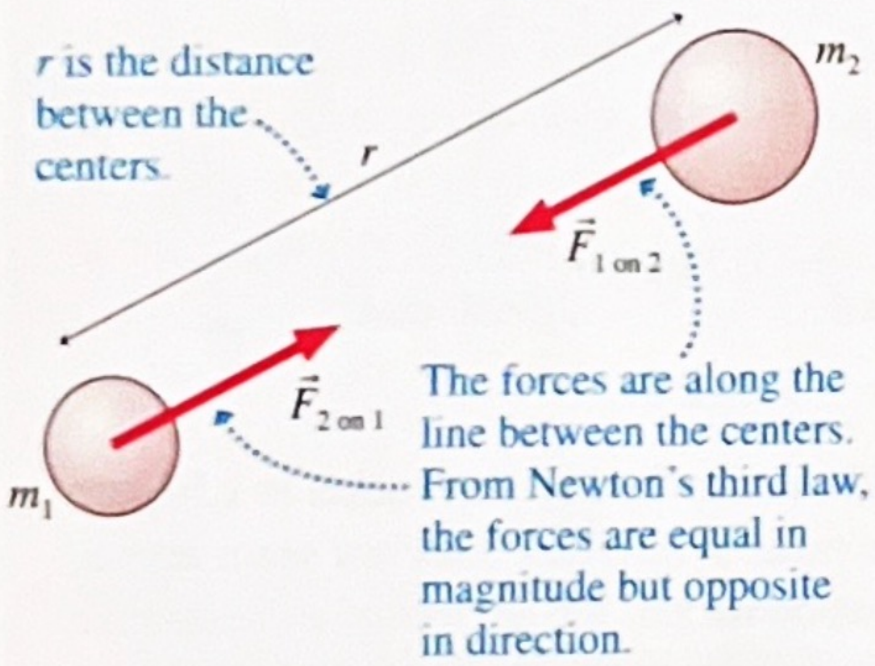
The rope will not break.

Mass, Weight, and Gravity

- Mass is an intrinsic property of an object
- Gravity is an attractive, long-range force between any two objects.
- Gravity is a force that acts on mass.
- Each object pulls on the other with a force given by Newton's law of gravity:

$$F_{1\text{ on }2} = F_{2\text{ on }1} = \frac{Gm_1m_2}{r^2}$$

FIGURE 6.6 Newton's law of gravity.



More on Gravity (Newton's Law of Gravity)

Consider an object on the surface of the Earth.

$M \equiv$ Mass of the Earth

$m \equiv$ Mass of the object

$R \equiv$ Radius of the Earth

$G \equiv$ Gravitational Constant $\left(6.67\times10^{-11}\ \mathrm{N}\frac{\mathrm{m}^2}{\mathrm{kg}^2}\right)$

$$\vec{F}_G = \vec{F}_{\text{planet on } m} = \frac{Gm_1m_2}{r^2} = \frac{GMm}{R^2}$$

(assuming the height of the object is $h \ll R$ and can be ignored)

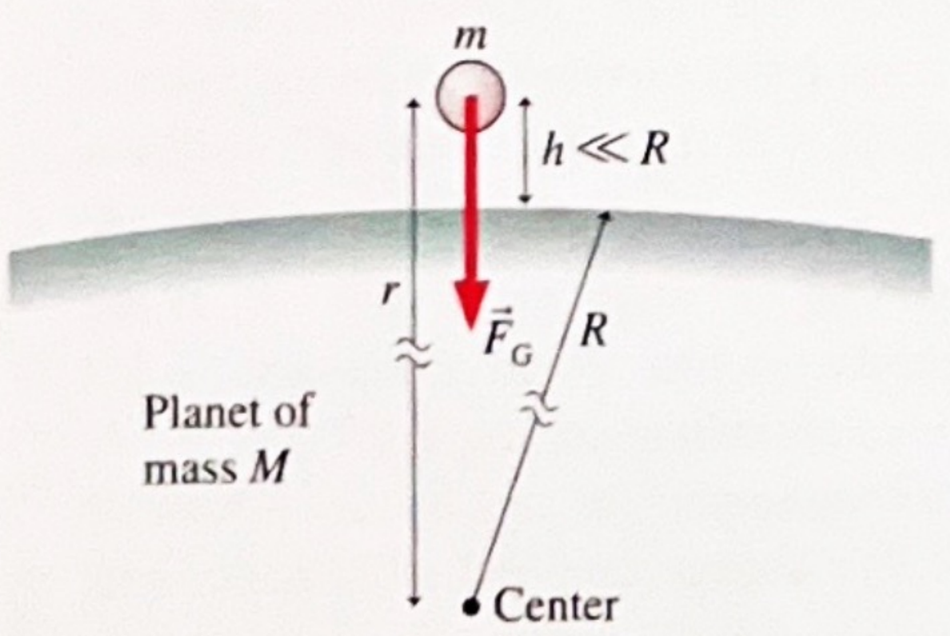
Using Newton's Second Law and free fall ($\vec{a} = -g$)

$$\vec{F}_{net} = \sum_i \vec{F}_i = m\vec{a} = -mg = -\left(\frac{GM}{R^2}\right)m$$

Therefore,

$$|g| = \frac{GM}{R^2} \text{ and } \vec{F}_G = m\vec{g}$$

FIGURE 6.7 Gravity near the surface of a planet.



More on Gravity

When an object is in free-fall, Newton's Second Law predicts that the acceleration of the object is:

$$\vec{a} = \frac{\vec{F}_{\text{net}}}{m} = \frac{\vec{F}_G}{m} = -\frac{mg}{m} = -g$$

g is the *gravitational field* of a planet and its value is influenced by its mass and size.

All objects on the same planet will have the same free-fall acceleration regardless of mass!

Weight

Mass and weight are not the same thing! Mass is an *intrinsic* property of an object. Weight depends on the strength of gravity (and the situation in which it is measured).

Spring scales measure this value by sensing the compression of an internal spring. The amount of compression determines the weight of the object. In this case, the force due to gravity is acting downward and the spring force is acting upward. The measured spring force is equal to the weight of the object.

For a stationary object,

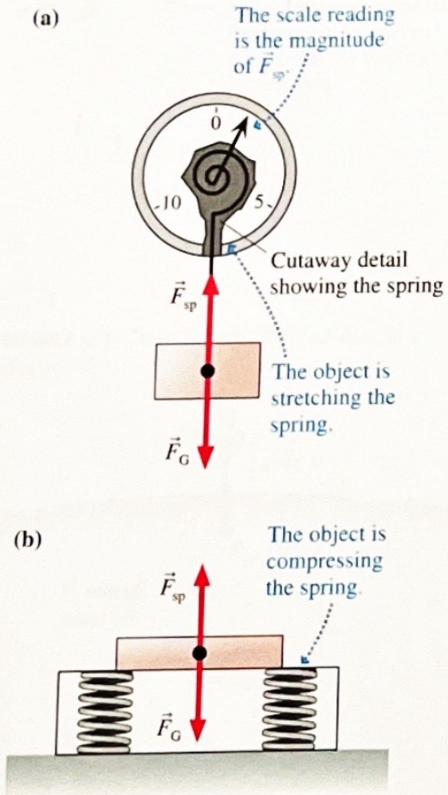
$$\vec{F}_{\text{net}} = \sum_i \vec{F}_i = \vec{F}_{\text{sp}} - \vec{F}_G = \vec{F}_{\text{sp}} - mg = 0\text{N}$$

Therefore,

$$\text{weight} = \vec{F}_{sp} = mg$$

A spring scale, such as the familiar bathroom scale, measures weight, not mass.

FIGURE 6.9 A spring scale measures weight.



Example #3 (Weight)

An astronaut's weight while standing on Earth is 800N. What is his weight on Mars ($g = 3.76\,\frac{\mathrm{m}}{\mathrm{s}^2}$)?

First, what is his mass? This will be the same on ANY planet.

$$m = \frac{\vec{F}_{\mathrm{G,Earth}}}{g} = \frac{800\mathrm{N}}{9.8\,\frac{\mathrm{m}}{\mathrm{s}^2}} = 81.63\mathrm{kg}$$

Now, calculate his weight on Mars.

$$\vec{F}_{\mathrm{G,Mars}} = (81.63\mathrm{kg}) \cdot 3.76\,\frac{\mathrm{m}}{\mathrm{s}^2} \approx 307\mathrm{N}$$

Summary

- Static Equilibrium
- Dynamic Equilibrium
- Weight and mass are not the same thing!
- Newton's Law of Gravity $\vec{F}_G = \frac{Gm_1m_2}{r^2}$