CONTINUOUS RANDOM VARIABLES

MAT 152 – Statistical Methods I

Lecture 2

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Fall 2020

The Uniform Distribution

The uniform distribution is a continuous probability distribution

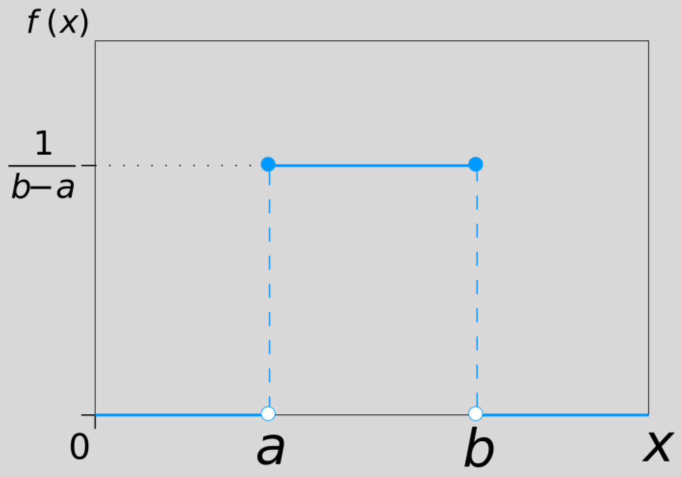
It is used to model events that are equally likely.

The PDF is given by:

$$f(x) = \begin{cases} \dfrac{1}{b-a} & a \leq x \leq b \\ 0 & \text{else} \end{cases}$$

a = lowest value of x
b = highest value of x

If a continuous random variable follows a uniform distribution, it can be written as $X \sim U(a, b)$.



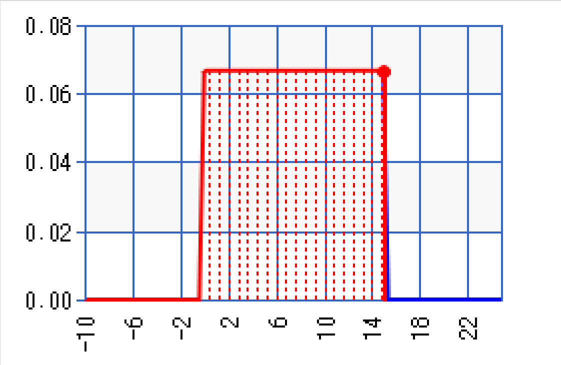
$$\mu = \frac{a+b}{2}$$

$$\sigma = \sqrt{\frac{(b-a)^2}{12}}$$

Example

The amount of time (in minutes) that a person must wait for a bus follows the uniform distribution: $X{\sim}U(0{,}15)$.

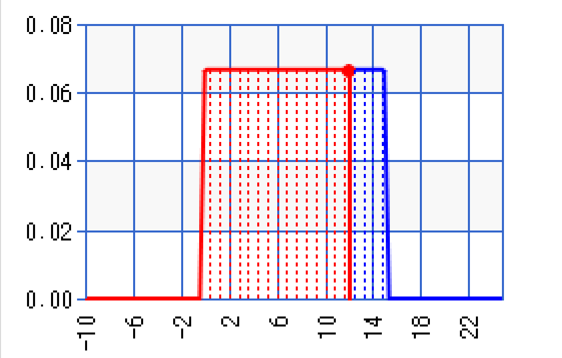
Plot the probability distribution



What is the probability of waiting less than 1 minute for a bus? $P(x < 1)$.

$$P(x < 1) = (1 - 0)\left(\frac{1}{15}\right) = \frac{1}{15}$$

What is the probability of waiting more than 12 minutes on a bus?



$P(x > 12)$ is in blue

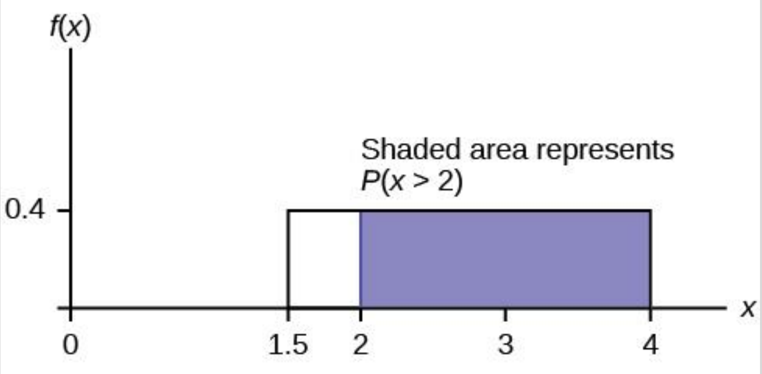
$$P(x > 12) = (15 - 12)\left(\frac{1}{15}\right) = \frac{3}{15} = \frac{1}{5}$$

Example

Ace Heating and Air Conditioning Service finds that the amount of time a repairman needs to fix a furnace is uniformly distributed between 1.5 and four hours. Let $X =$ the time needed to fix a furnace. Then $X \sim U(1.5, 4)$.

1) Find the probability that a randomly selected furnace repair requires more than two hours.

$$f(x) = \begin{cases} \frac{1}{b-a} & a \le x \le b \\ 0 & \text{else} \end{cases}$$



$$f(x) = \begin{cases} \frac{1}{4-1.5} & 1.5 \le x \le 4 \\ 0 & \text{else} \end{cases} = \begin{cases} \frac{1}{2.5} & 1.5 \le x \le 4 \\ 0 & \text{else} \end{cases} = \begin{cases} 0.4 & 1.5 \le x \le 4 \\ 0 & \text{else} \end{cases}$$

$$P(x > 2) = (4-2)(0.4) = 0.8$$

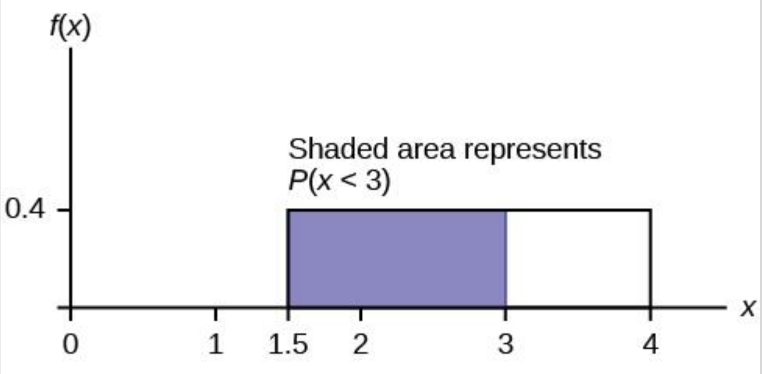
Example (cont.)

Ace Heating and Air Conditioning Service finds that the amount of time a repairman needs to fix a furnace is uniformly distributed between 1.5 and four hours. Let $X =$ the time needed to fix a furnace. Then $X \sim U(1.5, 4)$.

2) Find the probability that a randomly selected furnace repair requires less than three hours.

$$f(x) = \begin{cases} \frac{1}{b-a} & a \leq x \leq b \\ 0 & \text{else} \end{cases}$$

$$f(x) = \begin{cases} \frac{1}{4-1.5} & 1.5 \leq x \leq 4 \\ 0 & \text{else} \end{cases} = \begin{cases} \frac{1}{2.5} & 1.5 \leq x \leq 4 \\ 0 & \text{else} \end{cases} = \begin{cases} 0.4 & 1.5 \leq x \leq 4 \\ 0 & \text{else} \end{cases}$$



$$P(x < 3) = (3 - 1.5)(0.4) = 0.6$$

Example (cont.)

Ace Heating and Air Conditioning Service finds that the amount of time a repairman needs to fix a furnace is uniformly distributed between 1.5 and four hours. Let $X = \text{the time needed to fix a furnace}$. Then $X \sim U(1.5, 4)$.

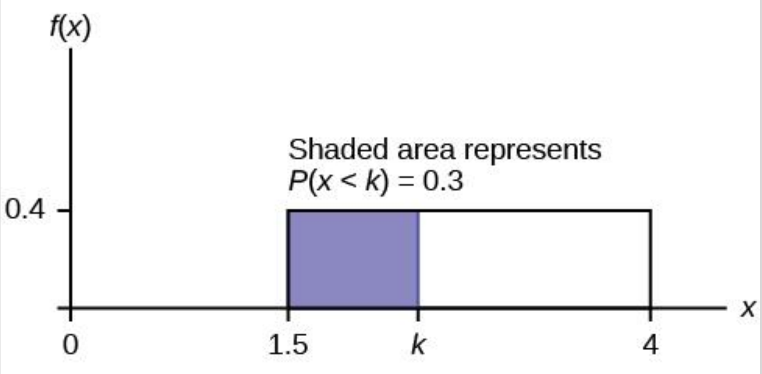
3) Find the 30th percentile of furnace repair times.

What value of *k* will have 30% of the distribution to the left and 70% of the distribution on the right?

$P(x < k) = 0.3$

$P(x < k) = (k - 1.5)(0.4) = 0.3$

k = 2.25 is the 30th percentile



Example (cont.)

Ace Heating and Air Conditioning Service finds that the amount of time a repairman needs to fix a furnace is uniformly distributed between 1.5 and four hours. Let $X =$ the time needed to fix a furnace. Then $X \sim U(1.5, 4)$.

4) Find the mean and standard deviation.

$$\mu = \frac{a+b}{2} \text{ and } \sigma = \sqrt{\frac{(b-a)^2}{12}}$$

$$\mu = \frac{1.5+4}{2} = 2.75$$

$$\sigma = \sqrt{\frac{(4-1.5)^2}{12}} = 0.7217$$

Example (cont.)

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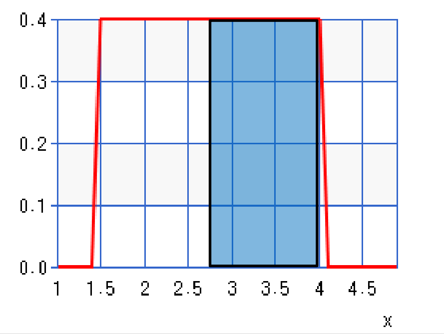
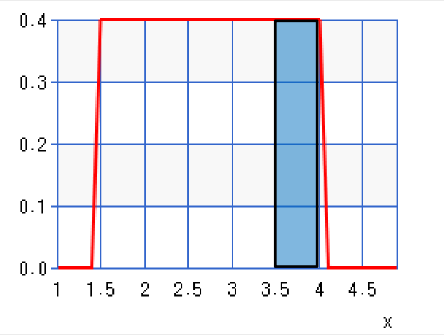
5) If a repairman has already been working for 2.75 hours, what is the probability that it will last more than 3.5 hours?

Determine $P(x > 3.5 | x > 2.75)$



$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(x > 3.5 \cap x > 2.75)}{P(x > 2.75)}$$

$P(x > 3.5 \cap x > 2.75)$ is the shared area between the two probabilities.
$P(x > 3.5 \cap x > 2.75) = P(x > 3.5)$



$$P(B|A) = \frac{P(A \cap B)}{P(B)} = \frac{P(x > 3.5 \cap x > 2.75)}{P(x > 2.75)} = \frac{P(x > 3.5)}{P(x > 2.75)} = 0.4$$

Review

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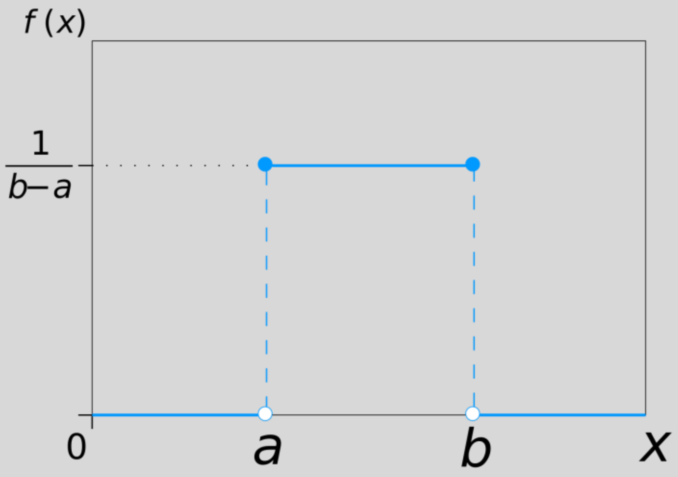

$$\mu = \frac{a+b}{2}$$

$$\sigma = \sqrt{\frac{(b-a)^2}{12}}$$