Lecture 3.1 - Vectors

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Vector vs. Scalar

Scalar

Full described by a single number

- Mass
- Temperature
- Volume
- Pressure
- Density
- Energy
- Charge
- Voltage

Vector

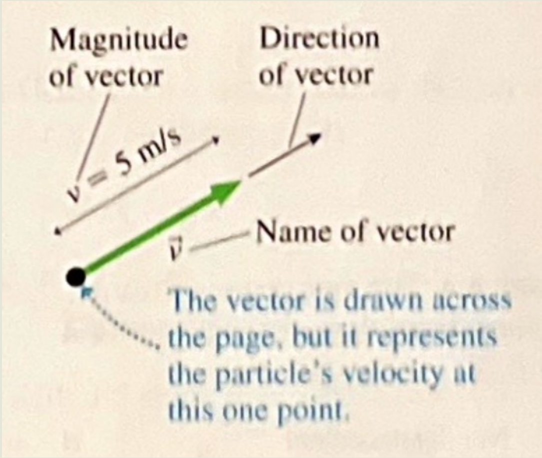
Has both size and direction

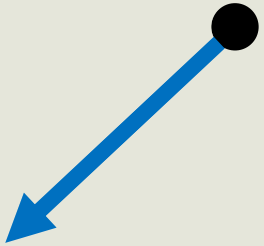
- Position
- Displacement
- Velocity
- Acceleration
- Force
- Momentum
- Electric Field

Geometric Representation of a Vector

- Represented as an arrow
- The tail is placed at the point where the measurement is made.
- The vector "radiates" outward from the point to which it is attached.
- A vector has a magnitude AND a direction
- Describing a vector's magnitude:

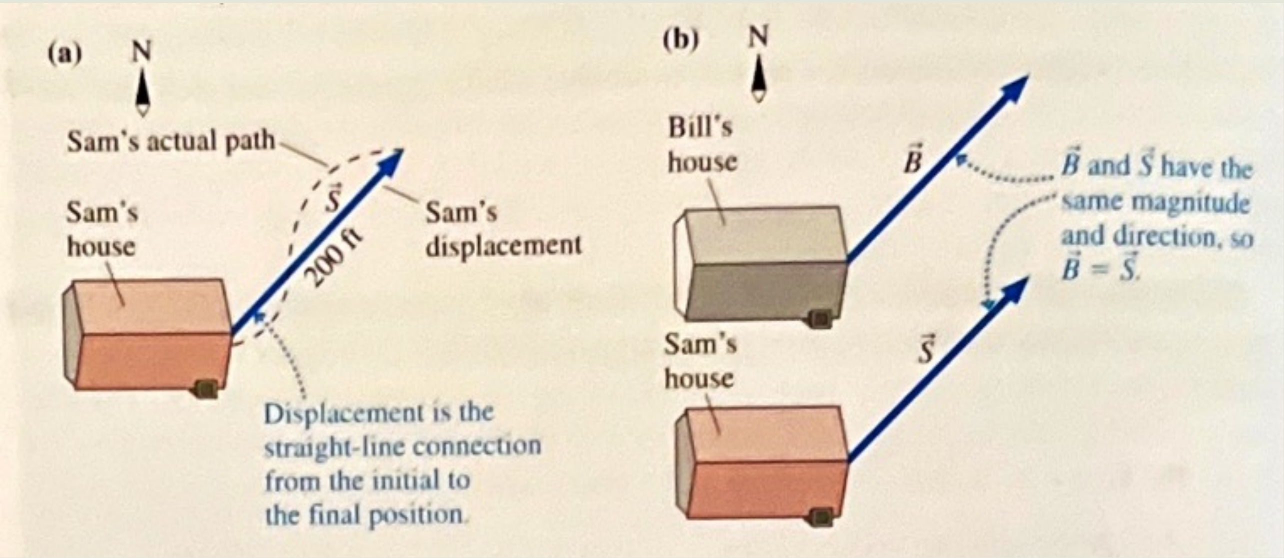
$$v = |\vec{v}|$$





Equal Vectors

- Consider the two vectors representing Bill and Sam's displacements. The path they took does not matter. Their *displacements* are the same.



- two vectors are equal if they have the same magnitude and direction. This is true regardless of the starting points of the vectors.

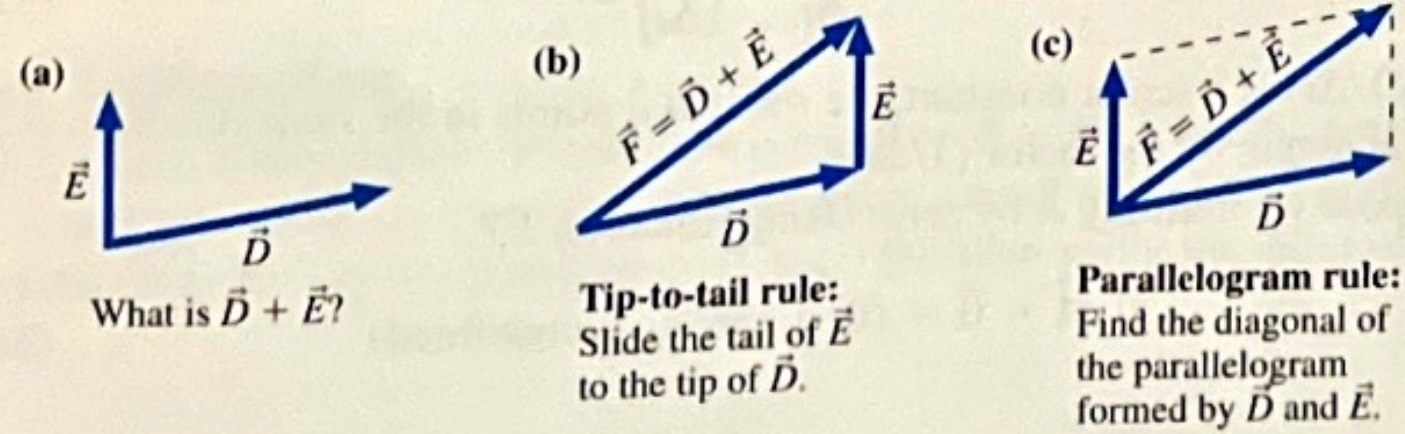
- A vector is unchanged if you move it to a different point on the page (if you don't change its length or the direction it points).

Vector Addition

- The sum of multiple vectors is called the *resultant vector.*
  - Vectors can be added in any order: $\vec{A} + \vec{B} = \vec{B} + \vec{A}$

- Vectors can be added *graphically* using the "tip-to-tail" method or the "parallelogram" method.

FIGURE 3.6 Two vectors can be added using the tip-to-tail rule or the parallelogram rule.

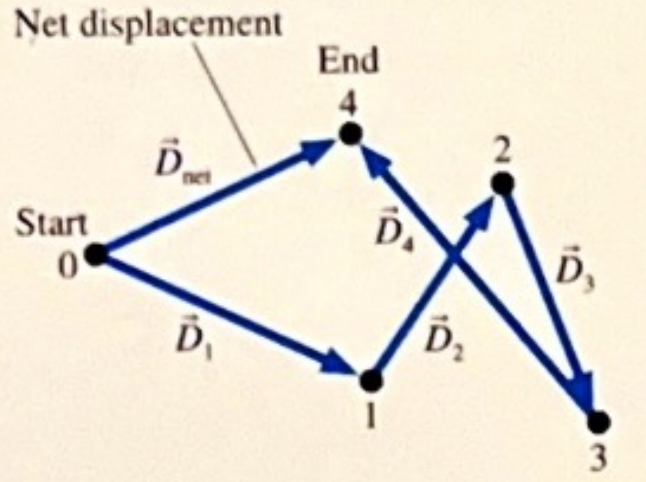


Graphical Addition Example

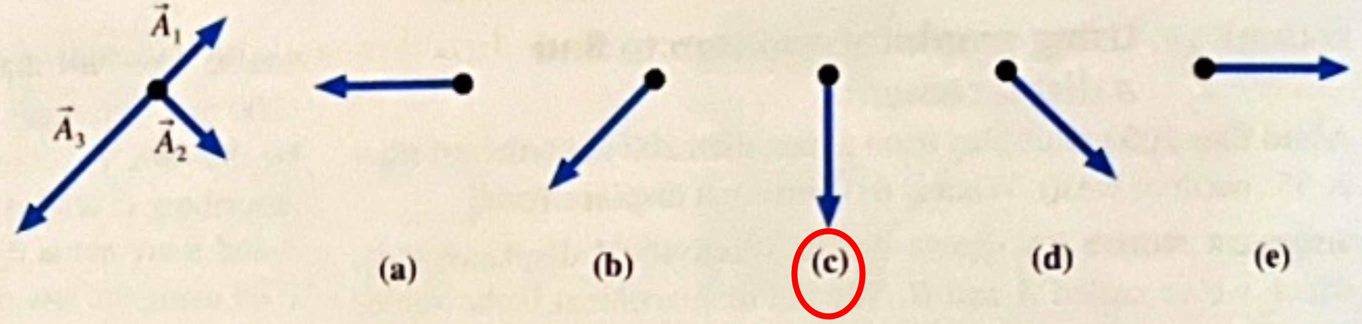
- The vectors $\vec{D}_1$, $\vec{D}_2$, $\vec{D}_3$, and $\vec{D}_4$ can be added graphically.

$$\vec{D}_{\text{net}} = \vec{D}_1 + \vec{D}_2 + \vec{D}_3 + \vec{D}_4$$

FIGURE 3.7 The net displacement after four individual displacements.



STOP TO THINK 3.1 Which figure shows $\vec{A}_1 + \vec{A}_2 + \vec{A}_3$?

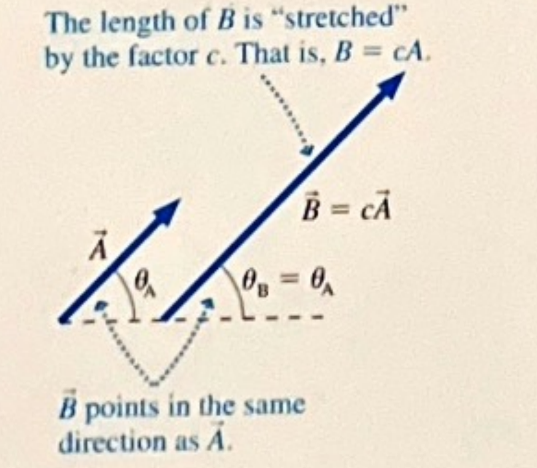


Multiplication by a Scalar

Multiplying a vector by a positive scalar *gives another vector* of different magnitude but pointing in the same direction.

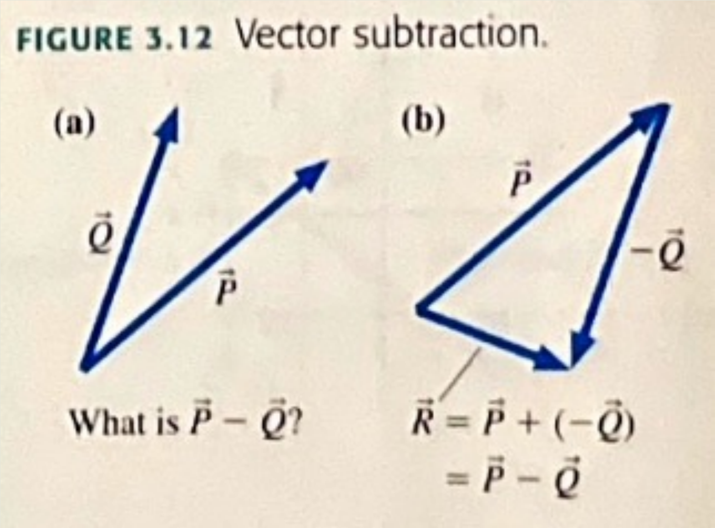
$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t} = \left(\frac{1}{\Delta t}\right) \Delta \vec{v}$$

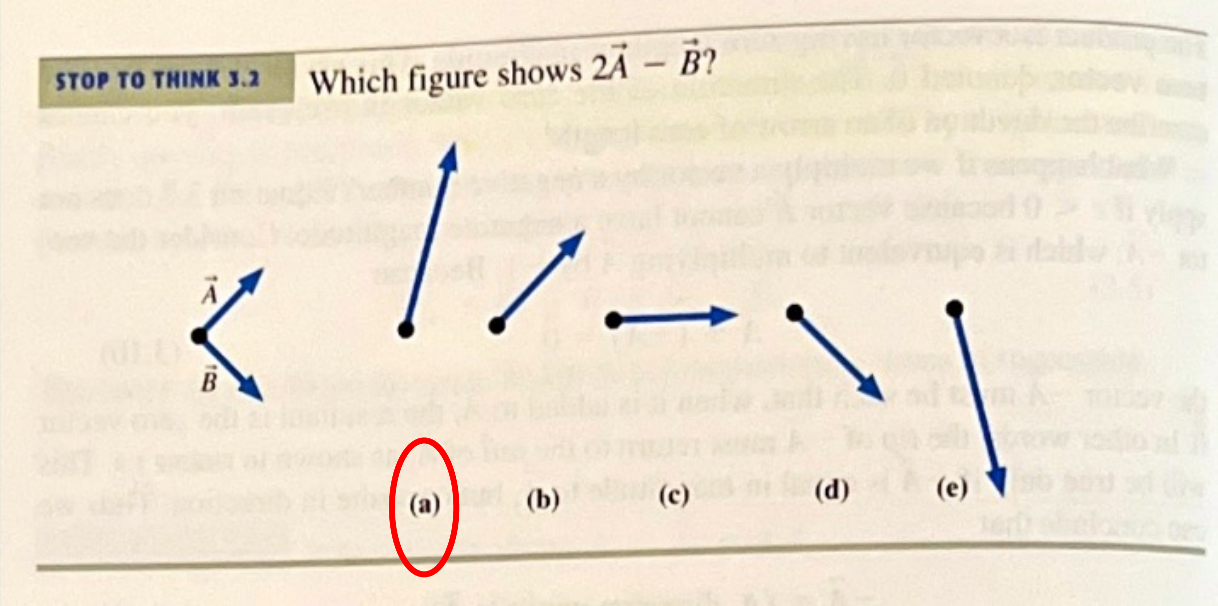
FIGURE 3.8 Multiplication of a vector by a scalar.



Vector Subtraction

- *Vector subtraction* works the same way as *vector addition;* however, the vector being subtracted must be turned 180° THEN added using the tip-to-tail method.





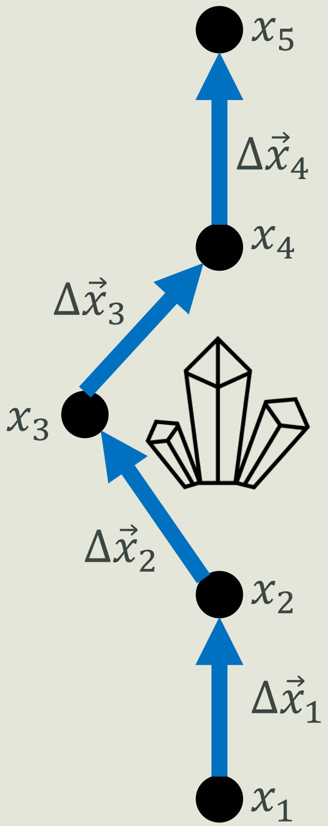
Reviewing Position, Velocity, and Acceleration

So far, we have discussed position, velocity, and acceleration in one dimension.

- Displacement (Change in position vectors)
$$\Delta\vec{x} = \vec{x}_f - \vec{x}_i$$
- Velocity (Rate of change of position)
$$\vec{v} = \frac{\Delta\vec{x}}{\Delta t} = \frac{\vec{x}_f - \vec{x}_i}{\Delta t} = \left(\frac{1}{\Delta t}\right)\Delta\vec{x}$$
- Acceleration (Rate of change of rate of change)
$$\vec{a} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

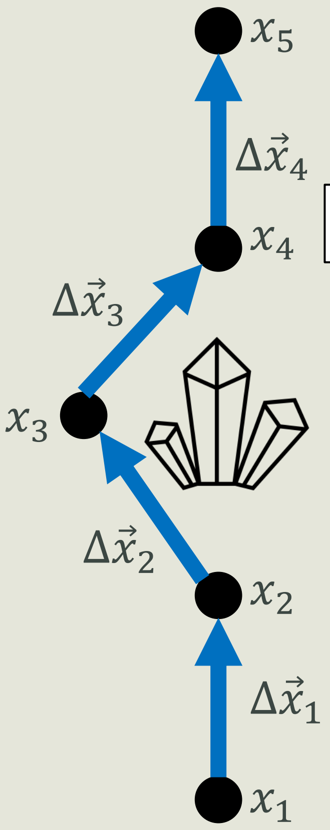
What about position, velocity, and acceleration in two dimensions?

Consider the displacement vectors of a bicyclist dodging a large boulder…



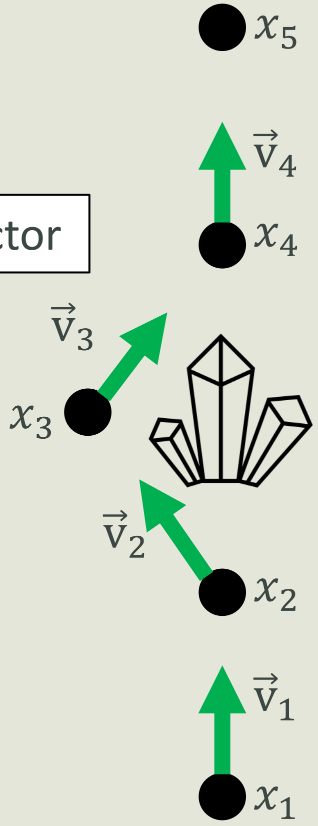
An Accelerating Biker

Consider the displacement vectors of a bicyclist dodging a large boulder…



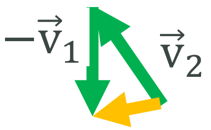
Scaled Displacement Vector

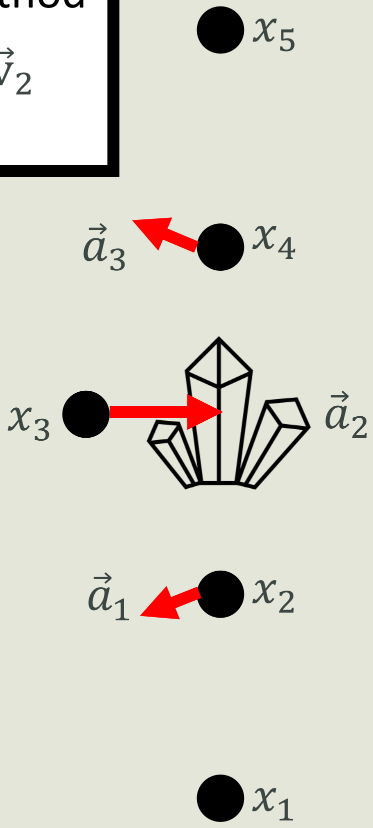
$$\vec{v} = \left(\frac{1}{\Delta t}\right)\Delta\vec{x}$$



Example of "Tip-to-Tail" Method

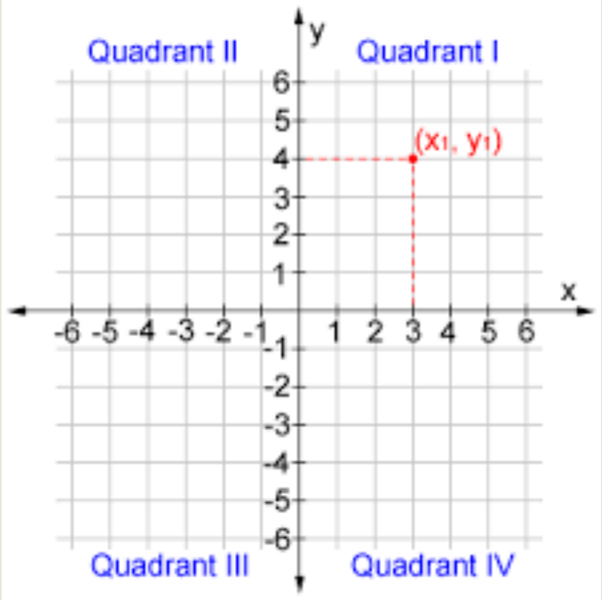
$$\frac{\vec{v}_2 - \vec{v}_1}{\Delta t} = \vec{a}$$





Coordinate Systems

- A coordinate system is artificially imposed.
- Different coordinate systems can be chosen but some coordinate systems may make a problem easier to solve.
- A typical choice is *Cartesian Coordinates* where the axes are perpendicular to each other and form a rectangular grid.
- There is no requirement that the x-axis be horizontal. Tilted axes may be helpful for some problems!



Component Vectors

- Once the directions of the axes are known, we can define the *components* of a vector

- Consider the vector $\vec{A}$

  - We can write this vector in terms of its *x-component* and *y-component.*

  - $\vec{A} = \vec{A}_x + \vec{A}_y$
  - $\vec{A}_x$ determines the *amount of* $\vec{A}$ in the x-direction.
  - $\vec{A}_y$ determines the *amount* of $\vec{A}$ in the y-direction
  - These component vectors also show if these values are (+) or (-).

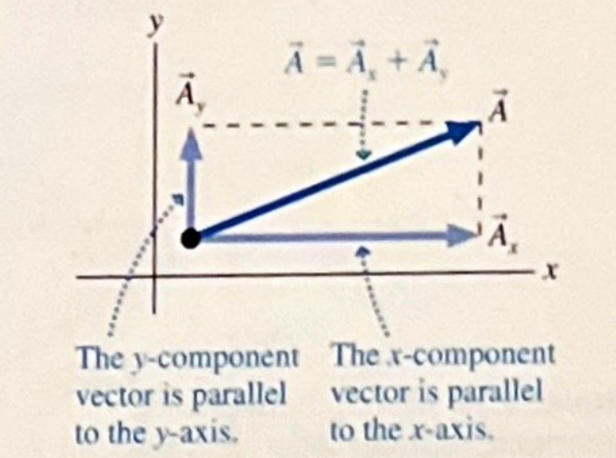


FIGURE 3.14 Component vectors $\vec{A}_x$ and $\vec{A}_y$ are drawn parallel to the coordinate axes such that $\vec{A} = \vec{A}_x + \vec{A}_y$.

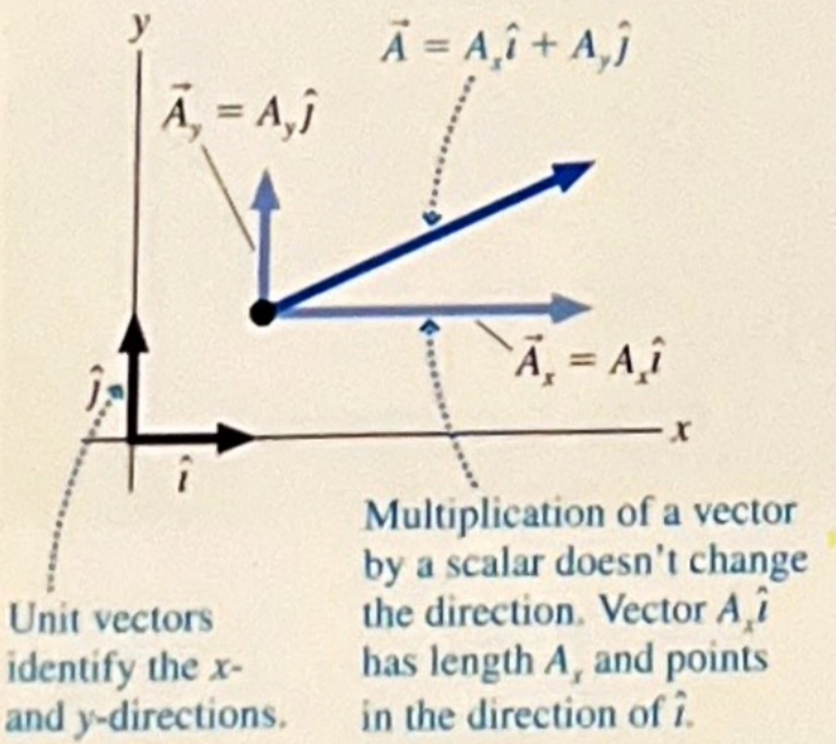
Unit Vectors

- *Unit vectors* are parallel to the coordinate axes and have a magnitude of 1. They have special symbols: $\hat{\imath}$ and $\hat{\jmath}$.

- So, we can write $\vec{A}$ in terms of its components:

$$\vec{A} = \vec{A}_x + \vec{A}_y = |\vec{A}_x|\hat{\imath} + |\vec{A}_y|\hat{\jmath} = A_x\hat{\imath} + A_y\hat{\jmath}$$

FIGURE 3.22 The decomposition of vector $\vec{A}$ is $A_x\hat{\imath} + A_y\hat{\jmath}$.



Algebraic Addition

Vectors must be added component-wise

$$\vec{D} = \vec{A} + \vec{B} + \vec{C}$$

$$\vec{D} = \left(A_x\hat{\imath} + A_y\hat{\jmath}\right) + \left(B_x\hat{\imath} + B_y\hat{\jmath}\right) + \left(C_x\hat{\imath} + C_y\hat{\jmath}\right)$$

$$\vec{D} = D_x\hat{\imath} + D_y\hat{\jmath} = (A_x + B_x + C_x)\hat{\imath} + \left(A_y + B_y + C_y\right)\hat{\jmath}$$

We can perform vector addition by adding the x-components of the individual vectors to give the x-component of the *resultant vector*. We add the y-components of the individual vectors to get the y-component of the *resultant* vector.

Vector Components

We can use trigonometry to get the components of a vector.

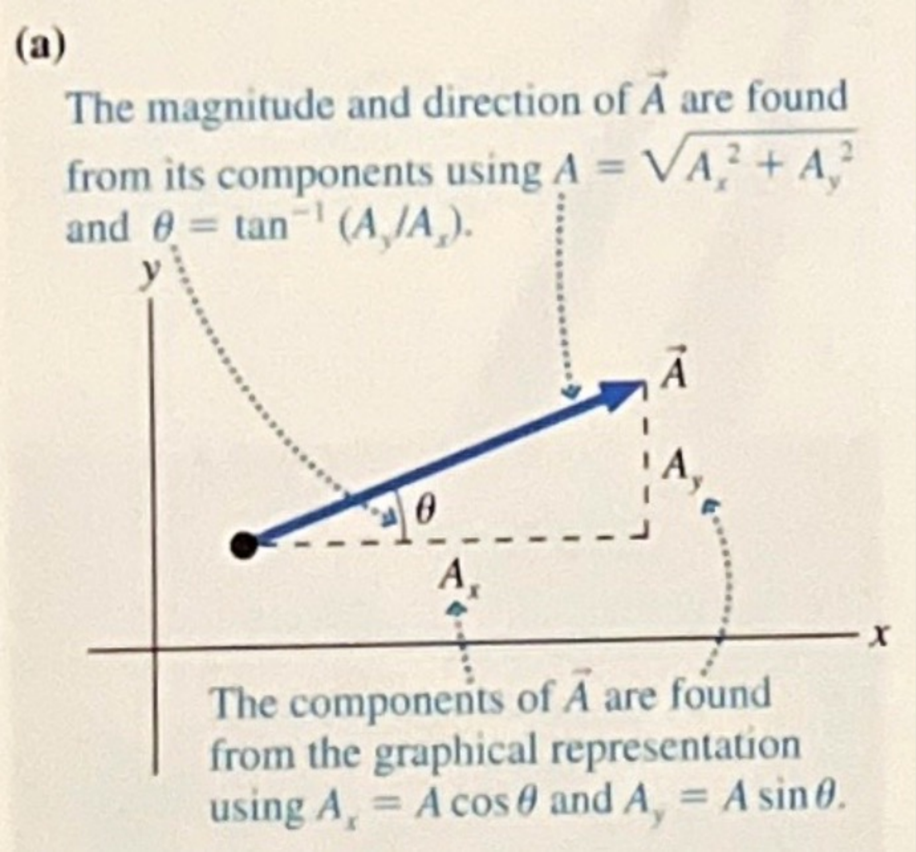
Trig identities:

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2}$$



Putting it All Together

$\vec{A} + \vec{B} = ?$

Write the resultant vector in terms of the magnitude and θ

$\vec{A} = 5\hat{\imath} + 5\hat{\jmath}$

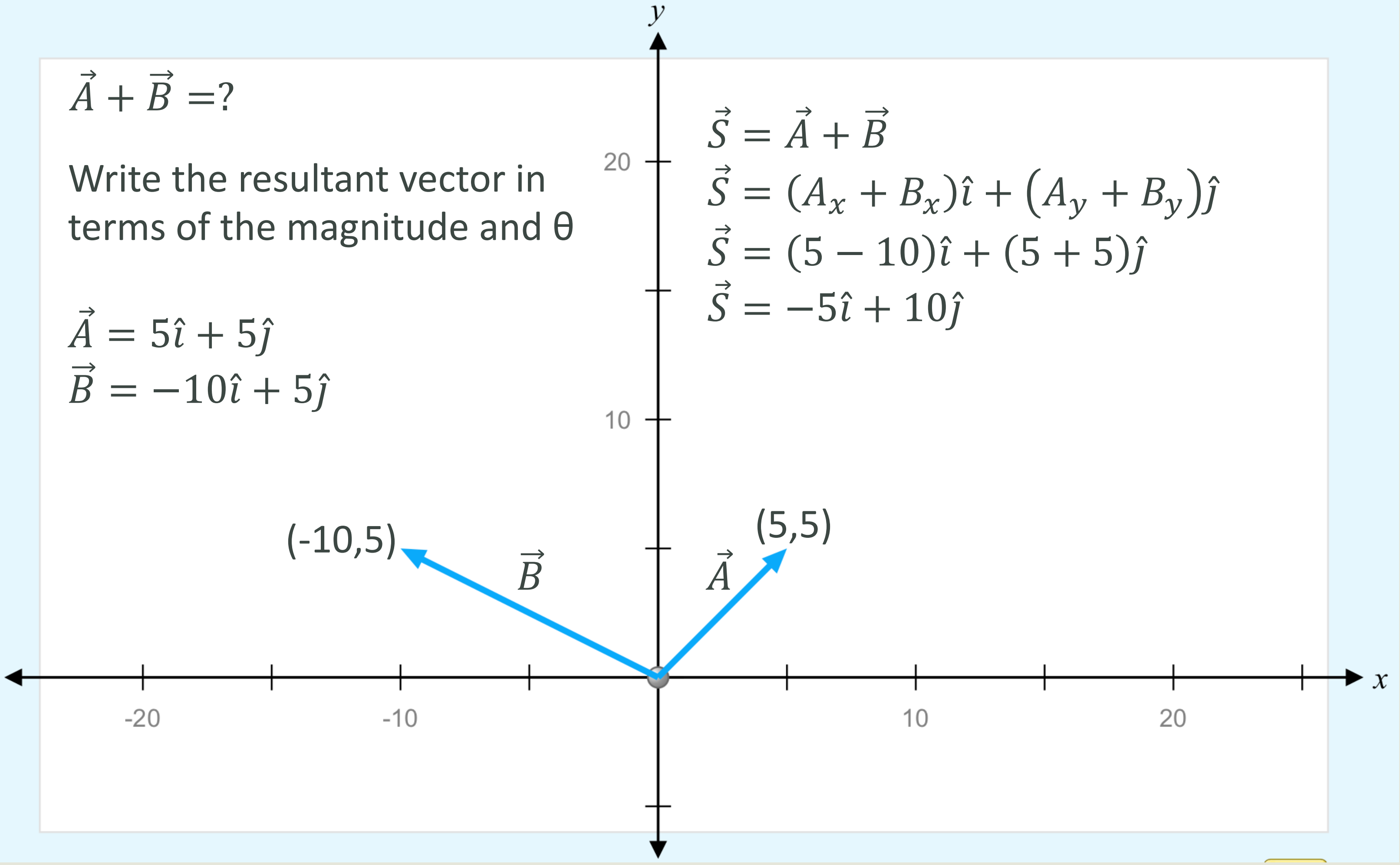
$\vec{B} = -10\hat{\imath} + 5\hat{\jmath}$

$\vec{S} = \vec{A} + \vec{B}$

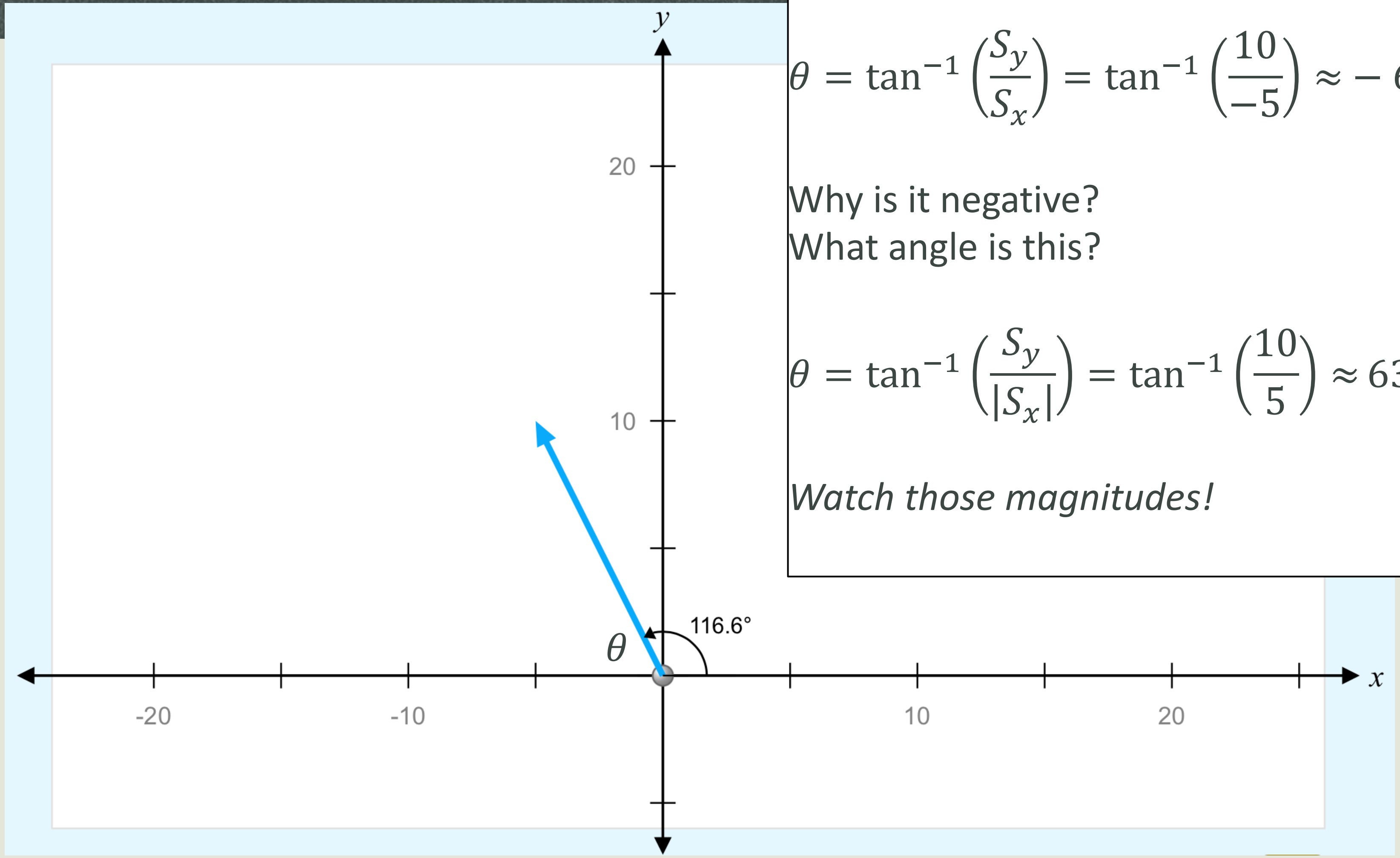
$\vec{S} = (A_x + B_x)\hat{\imath} + (A_y + B_y)\hat{\jmath}$

$\vec{S} = (5 - 10)\hat{\imath} + (5 + 5)\hat{\jmath}$

$\vec{S} = -5\hat{\imath} + 10\hat{\jmath}$



Putting it All Together



$$|\vec{S}| = \sqrt{S_x^2 + S_y^2} = \sqrt{(-5)^2 + (10)^2} \approx 11.18$$

$$\theta = \tan^{-1}\left(\frac{S_y}{S_x}\right) = \tan^{-1}\left(\frac{10}{-5}\right) \approx -63.43°$$

Why is it negative?
What angle is this?

$$\theta = \tan^{-1}\left(\frac{S_y}{|S_x|}\right) = \tan^{-1}\left(\frac{10}{5}\right) \approx 63.43°$$

Watch those magnitudes!