LECTURE 8.2 – FORCE, WORK, AND POTENTIAL ENERGY

Dustin Roten, Ph.D.
Wilkes Community College
Fall 2023

FORCE AND WORK

What is the work done by gravity on an object sliding down a frictionless path?

Using the definition of work:

$$W_G = \vec{F}_G \cdot \Delta\vec{r} = F(\Delta r)\cos\theta$$

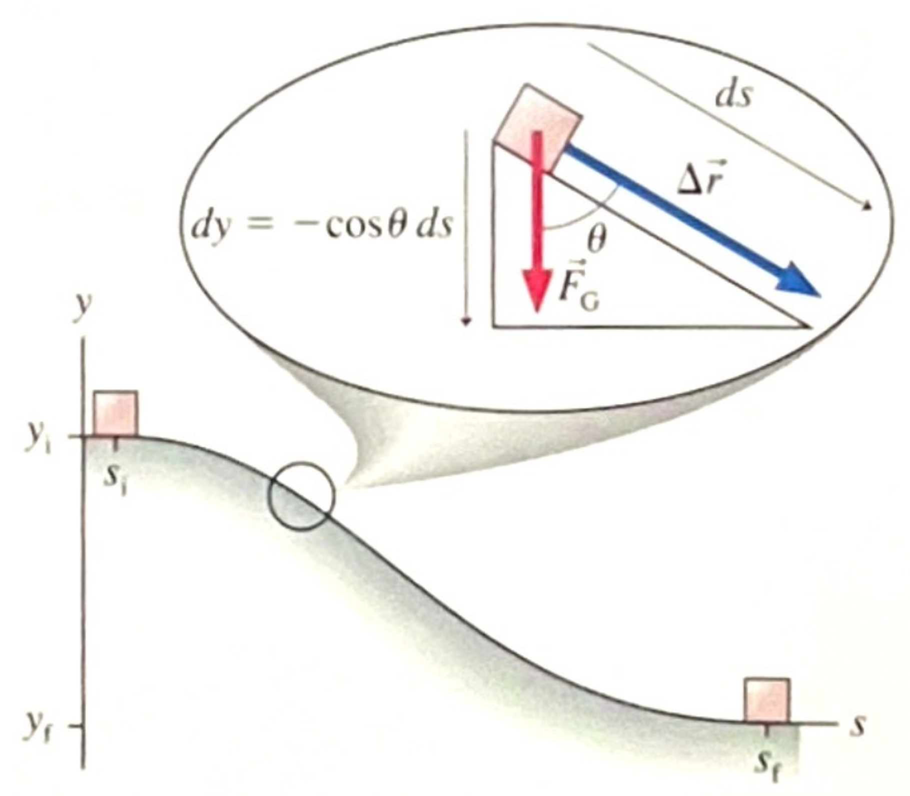
Consider the work done over a small distance $\Delta r = ds$:

$$dW_G = \vec{F}_G \cdot \Delta\vec{r} = F_G \cos\theta \, ds$$

We see from the diagram that the small change in height can be written as $dy = -\cos\theta \, ds$. Thus:

$$dW_G = -mgdy$$

FIGURE 11.19 An object moves along an arbitrarily shaped path.



FORCE AND WORK

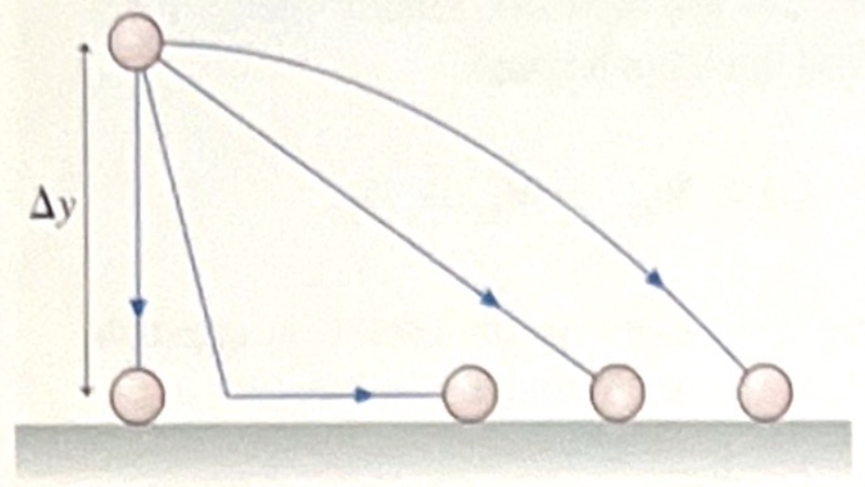
$$dW_G = -mgdy$$

This equation can now be integrated:

$$W_G = -mg\int_{y_i}^{y_f} dy = -mg(y_f - y_i) = -mg\Delta y$$

The work done by gravity is *independent* of θ.
The shape of the path taken doesn't matter!

FIGURE 11.20 The work done by gravity is the same in all four cases.



CONSERVATIVE FORCES

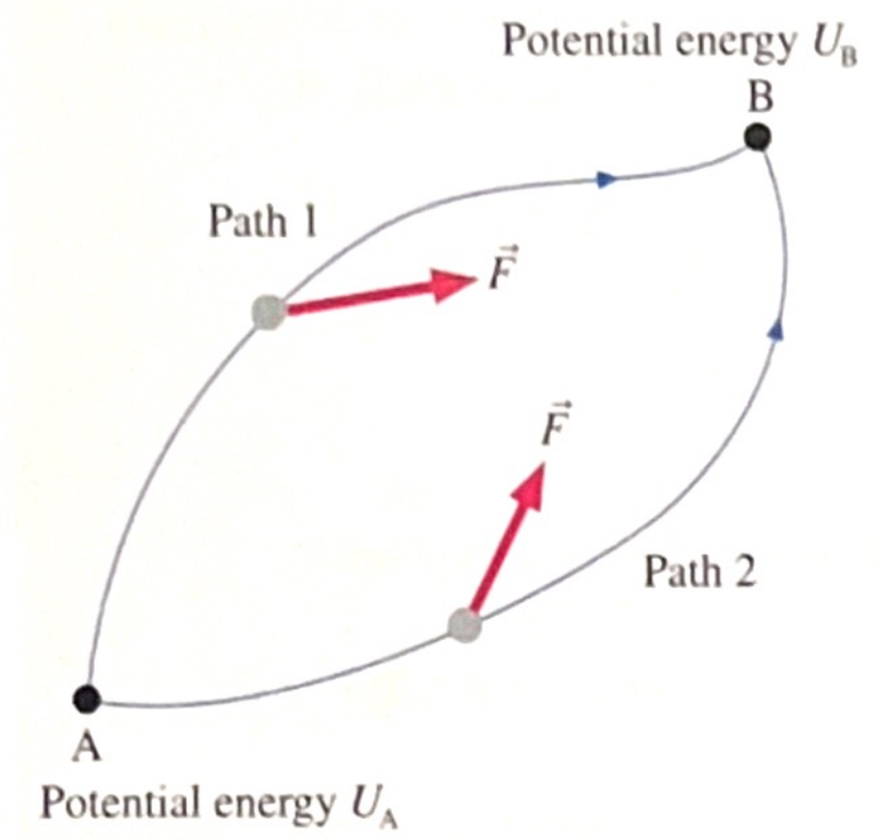


FIGURE 11.21 An object can move from A to B along either path 1 or path 2.

- Potential energy is an energy of position. The system has one value of potential energy at position A and another at position B. $\Delta U = U_B - U_A$ regardless of the path taken.
- Potential energy is transformed into kinetic energy since $\Delta K = -\Delta U$. If ΔU is independent of the path, then ΔK is also independent of the path. No matter the path, the object will have the same kinetic energy at position B.
- Since work and kinetic energy are related, $W = \Delta K$, the work done by a force $\vec{F}$ as an object moves from A to B is also independent of the path.
- A force moving an object from an initial to a final position independently of the path is a *conservative force*.
- A potential energy can be associated with any conservative force.

CONSERVATIVE FORCES (SPRING POTENTIAL ENERGY)

Hooke's Law is also a conservative force.

Given that $F_{\text{sp}} = -k\Delta x$ and $W_{\text{sp},(i\to f)} = \int_{x_i}^{x_f} F_{\text{sp}}\, dx$,

If we assume that the spring starts from equilibrium ($x_e = 0\text{m}$), then:

$$F_{\text{sp}} = -k\Delta x = -k(x - x_e) = -kx$$

$$W_{\text{sp},(i\to f)} = \int_{x_i}^{x_f} F_{\text{sp}}\, dx = \int_{x_i}^{x_f} -kx\, dx = -\left(\frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2\right) = -\frac{1}{2}k(\Delta x)^2 = -\Delta U_{\text{sp}}$$

FORCE FROM POTENTIAL ENERGY?

We can now find the potential energy from a conservative force. How can we go "backwards"?

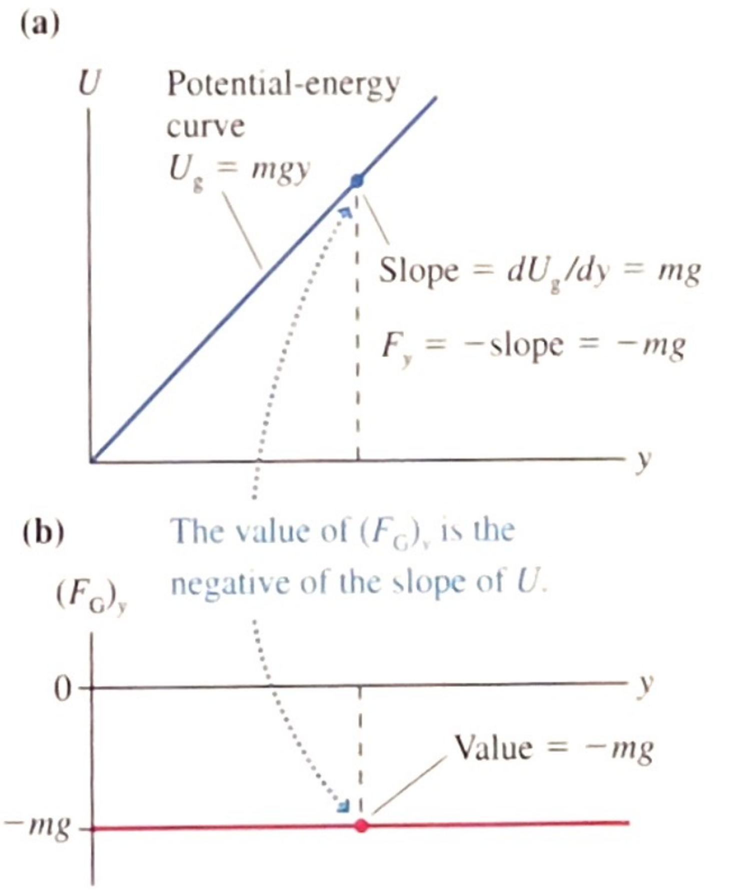
How can we find the force provided by a potential energy?

$$W = F_s \Delta s \text{ and } W = -\Delta U \text{ so } F_s \Delta s = -\Delta U$$

$$F_s = \lim_{\Delta s \to 0} \left(-\frac{\Delta U}{\Delta s} \right) = -\frac{dU}{ds}$$

F_s is the negative of the slope of the U vs. s graph.

FIGURE 11.24 Gravitational potential energy and force diagrams.



NONCONSERVATIVE FORCES (FRICTION)

Not all forces are conservative!

If we consider the work done by friction on a particle across a small distance (ds):

$$dW_f = \vec{f}_k \cdot d\vec{s} = (\mu_k mg)(ds)\cos\alpha$$

Remember: the direction of the friction vector is *always* opposite the direction of motion, so $\cos(180°) = -1$. Here, friction depends on the path taken!

Thus,

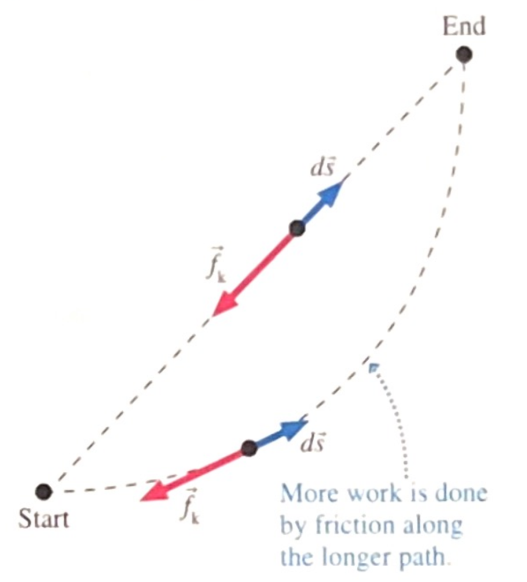
$$dW_f = -\mu_k mg ds$$

If we sum (integrate) across the entire path:

$$W_{f,(i\to f)} = \int_{s_i}^{s_f} -\mu_k mg\, ds = -\mu_k mg \int_{s_i}^{s_f} ds = -\mu_k mg\Delta s$$

A force that *depends* on the path (Δs) taken is called a *nonconservative* force. A potential energy cannot be defined for a nonconservative force. (Energy can't be recovered!)

FIGURE 11.22 Top view of two particles sliding across a surface.



MECHANICAL ENERGY

Both conservative and nonconservative forces do work: W_c and W_{nc}.

The net work is responsible for changing the system's kinetic energy (W-K Energy Theorem):

$$\Delta K = W_{net} = W_c + W_{nc}$$

Work done by conservative forces can now be associated with a potential energy, $W_c = -\Delta U$.

The work kinetic energy theorem can now be restated as:

$$\Delta K + \Delta U = \Delta E_{mech} = W_{nc}$$

Mechanical energy is conserved if there are no nonconservative forces.

THERMAL ENERGY

Mechanical energy deals with the *macrophysics* of a system (the large objects)

Thermal energy deals with the *microphysics* of a system (the heat-retaining atomic bonds within the objects).

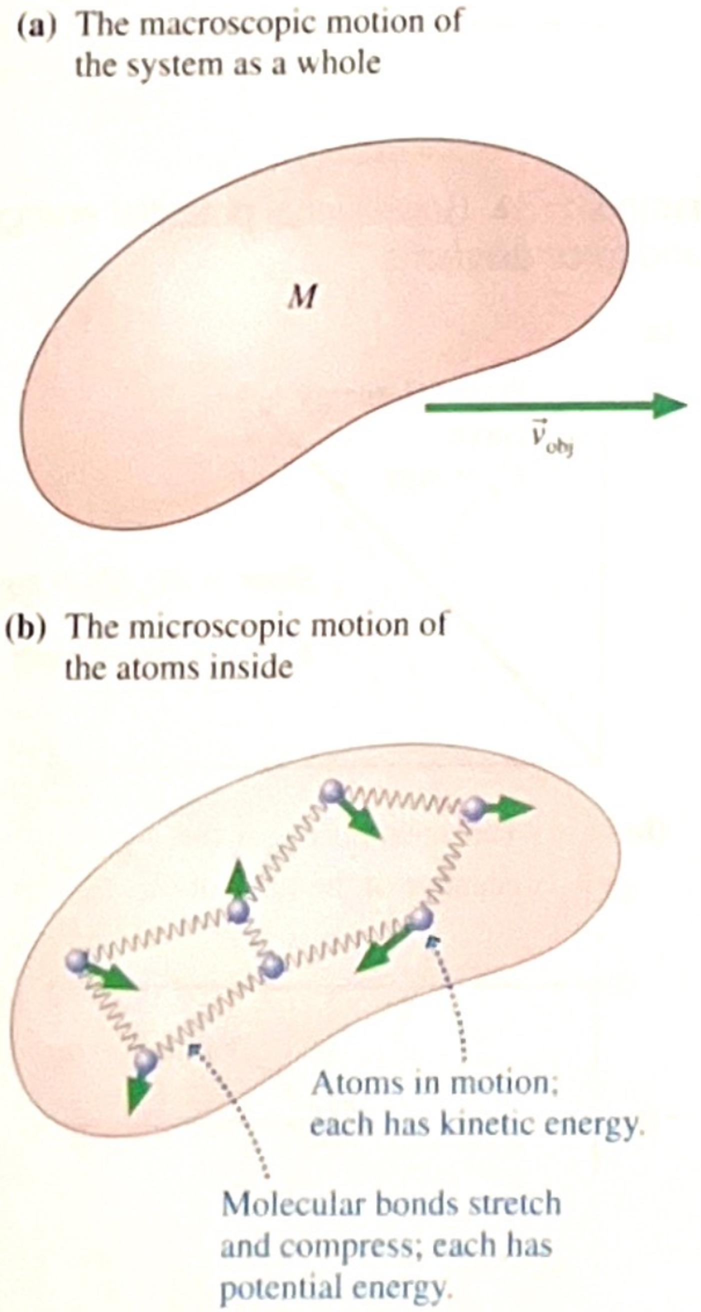
The total kinetic energy of all the atoms at the microscopic level is K_{micro}. Similarly, the stored potential energy in these bonds is U_{micro}.

The total thermal energy can be defined as:

$$\mathrm{E_{th}} = K_{\text{micro}} + U_{\text{micro}}$$

Summed over all atoms making up an object, E_{th} can be large! ($\sim 0.12\text{MJ}$)

FIGURE 11.26 Two perspectives of motion and energy.



DISSIPATIVE FORCES

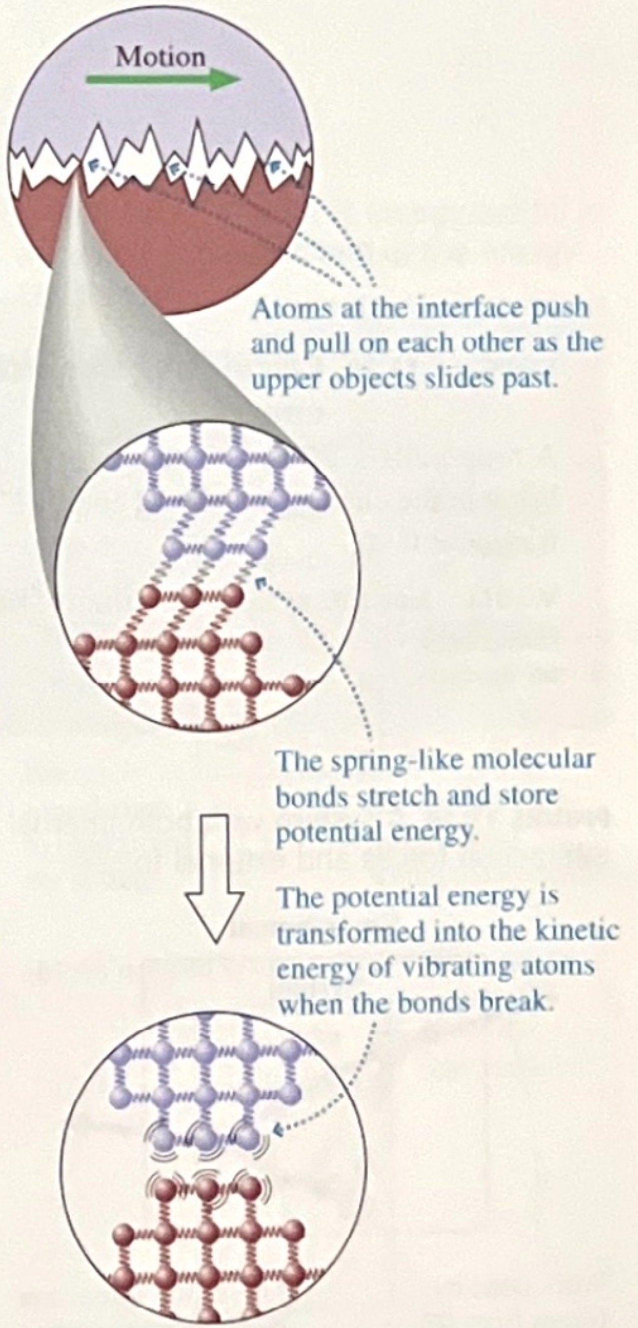
If a book is shoved across a table, it gradually slows down and eventually stops. Where did the kinetic energy go?

Molecular bonds get stretched on *both* sides of the boundary as two objects slide against each other. When these temporary bonds break, the atoms snap back into position and start vibrating with kinetic energy.

Atomic interactions at the boundary transform the kinetic energy of the moving object into atomic/molecular energy $[K \rightarrow (E_{th} = K_{micro} + U_{micro})]$

This is perceived as temperature increases for both objects.

FIGURE 11.27 The atomic-level view of friction.



DISSIPATIVE FORCES

Forces such as friction and drag cause the macroscopic kinetic energy of a system to be “dissipated” as thermal energy.

Dissipative forces are always nonconservative forces.

Dissipative forces always increase the thermal energy; they never decrease it.

A CONCEPTUAL EXAMPLE

Consider a box being pulled at a constant speed across the floor.

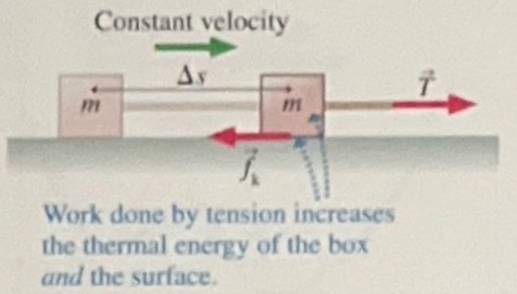
Both the surface and the box get warmer due to friction, but the kinetic and potential energy remains the same. *Where is the thermal energy coming from?*

Remember that work is energy transferred to a system by the environment. So, if the force from the rope is part of the environment (external) it is the source of the increase in energy of the system (box + floor).

$E_{th} = W_{tension} = T\Delta s$. Since $\vec{F}_{net} = 0N$, $E_{th} = W_{tension} = T\Delta s = f_k \Delta s$

Note: There is no energy transfer *between* the box and the floor. Energy is transferred to *both* from outside the system!

FIGURE 11.28 Work done by tension is dissipated as increased thermal energy.



SUMMARY

- Potential energy is the energy of position. Kinetic energy is the energy of motion.
- Conservative forces can be related to a potential. (Gravity)
- Work done by conservative forces is path independent.
- Nonconservative forces have no related potential (Friction)
- Work done by external forces can raise the temperature of the system.