HYPOTHESIS TESTING WITH ONE SAMPLE

MAT 152 – Statistical Methods I

Lecture 2

Instructor: Dustin Roten

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RARE EVENTS

Suppose your cranky grandmother makes the statement "only 10% of kids these days know how to cook!" Your grandfather agrees by adding "$H_0 : p = 0.1$". You argue saying that "I think many of my friends know how to cook", emphasizing that $H_a : p > 0.1$.

Your grandparents' baseless claim annoys you, so you decide to investigate. You survey your classmates in your statistics class and find that 14 of the 20 students (including you) regularly prepare meals for themselves and their family. $p' = \frac{14}{20} = 0.7$

Your result is so different from your grandparents claim ($H_0 : p = 0.1$) that you must reject the null hypothesis.

P-VALUES

How different must a result be for the null hypothesis to be rejected?

What if your survey revealed that only 3 out of 20 classmates cooked ($p' = 0.15$)? Would this have been different enough from your grandparents' statement to prove them wrong?

Sample data is used to calculate the actual probability of getting the test result, called the **p-value**. It states the likelihood of other samples being as extreme (or more extreme) as our current sample.

P-VALUES

- Assuming $H_0: p = 0.1$ is true, our sample of $p' = 0.7$ would have a very small p-value (very extreme).
- Assuming $H_0: p = 0.1$ is true, our fictitious sample of $p' = 0.15$ would have a very large p-value (not extreme).
- The more extreme a sample is, the smaller the p-value.

P-VALUES

A systematic way to decide whether to accept or reject H_0 is to compare the p-value to α (significance level).

- $p < \alpha$ indicates an extreme event. Reject H_0. (Reject initial assumption)
  - The results are significant

- $p > \alpha$ indicate a less extreme event. Do not reject H_0.
  - Results are not significant

When you fail to reject the null hypothesis (H_0), it does not mean that it is true. It means the sample didn't sufficiently prove it false.

HYPOTHESIS TESTING

- The experimenter selects the significance level (α) BEFORE collecting the sample data. (A common standard is to use $\alpha = 0.05$)

- The p-value represents the area in the left, right, or both tails. H_a indicates which tail to use.

- H_a never has a symbol that contains an equal sign.

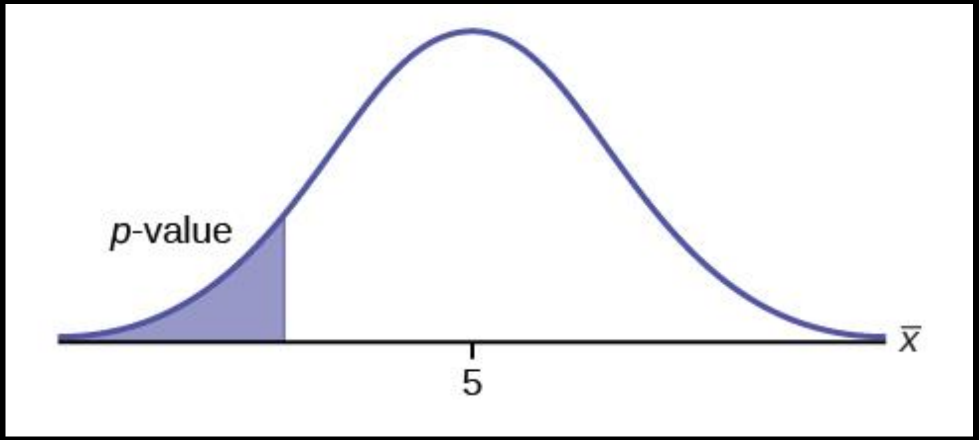
HYPOTHESIS TESTING

Suppose we wish to test the following hypotheses.

$H_0 : \mu = 5$

$H_a : \mu < 5$

H_a tells us that we are interested in the probability of the population mean being lower than the claim of $\mu = 5$. So, this is a left-tailed test.



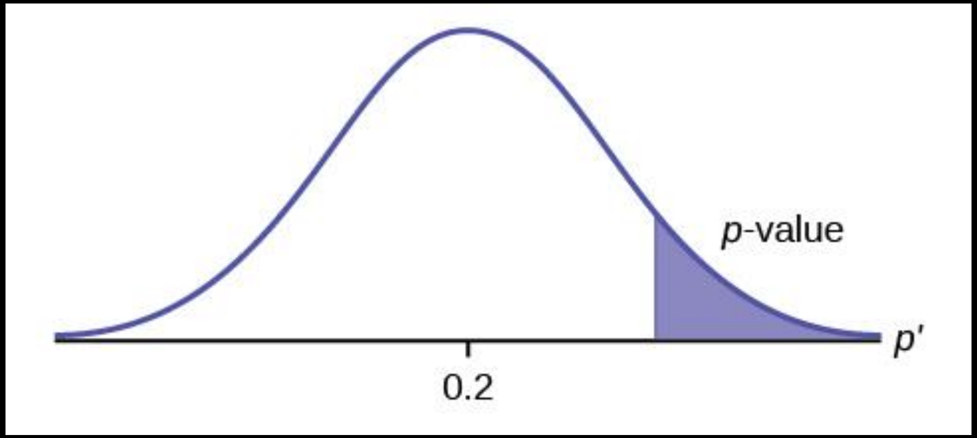
HYPOTHESIS TESTING

Suppose we wish to test the following hypotheses.

$H_0: p \leq 0.2$

$H_a: p > 0.2$

H_a tells us that we are interested in the probability of the population proportion being higher than the claim of $\mathrm{p} \leq 0.2$. So, this is a right-tailed test.



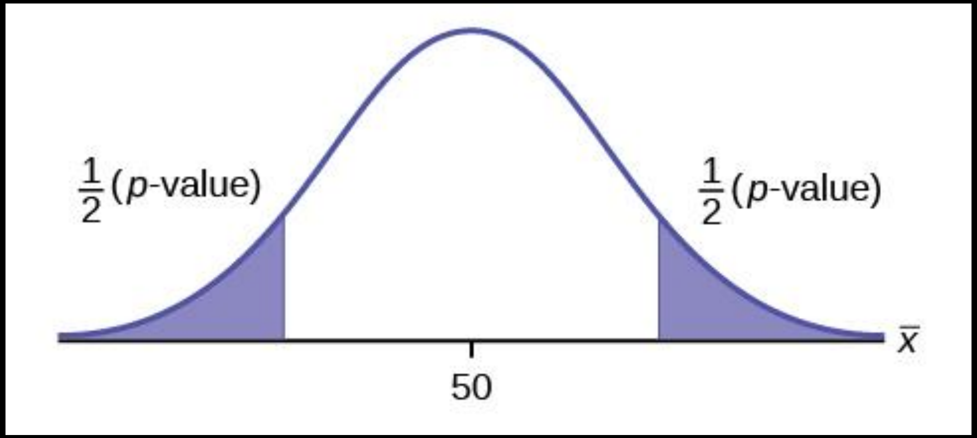
HYPOTHESIS TESTING

Suppose we wish to test the following hypotheses.

$H_0: \mu = 50$

$H_a: \mu \neq 50$

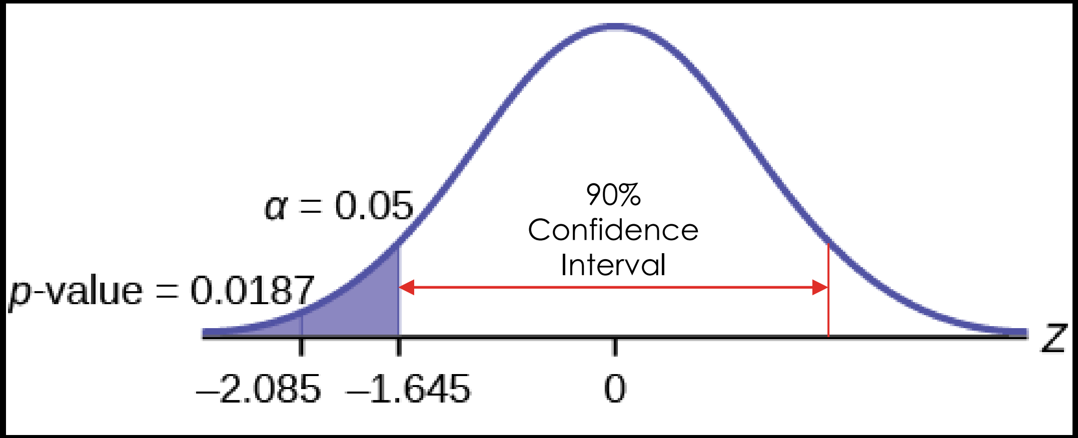
H_a tells us that we are interested in the probability of the population proportion being different than the claim of $\mu = 50$. So, this is a two-tailed test. (The value can be higher OR lower.)



HYPOTHESIS TESTING

Consider the arbitrary p-value from a hypothesis test at the 0.05 significance level.

Here, we see that the p-value is smaller than α, so the results are significant.



EXAMPLE

Jeffrey, as an eight-year old, established a mean time of 16.43 seconds for swimming the 25-yard freestyle, with a standard deviation of 0.8 seconds. His dad, Frank thought that Jeffrey could swim the 25-yard freestyle faster using goggles. Frank bought Jeffrey a new pair of expensive goggles and timed Jeffrey for 15 25-yard freestyle swims. For the 15 swims, Jeffrey's mean time was 16 seconds. Frank thought that the goggles helped Jeffrey to swim faster than the 16.43 seconds. Conduct a hypothesis test using a preset $\alpha = 0.05$. Assume that the swim times for the 25-yard freestyle are normally distributed.

1. Set up the hypothesis test
   - $H_0: \mu = 16.43$
   - $H_a: \mu < 16.43$

Random variable: $\bar{X}$ = the mean time to swim the 25-yard freestyle.

Population mean and standard deviation are known: 16.43s and 0.8s

EXAMPLE (CONT.)

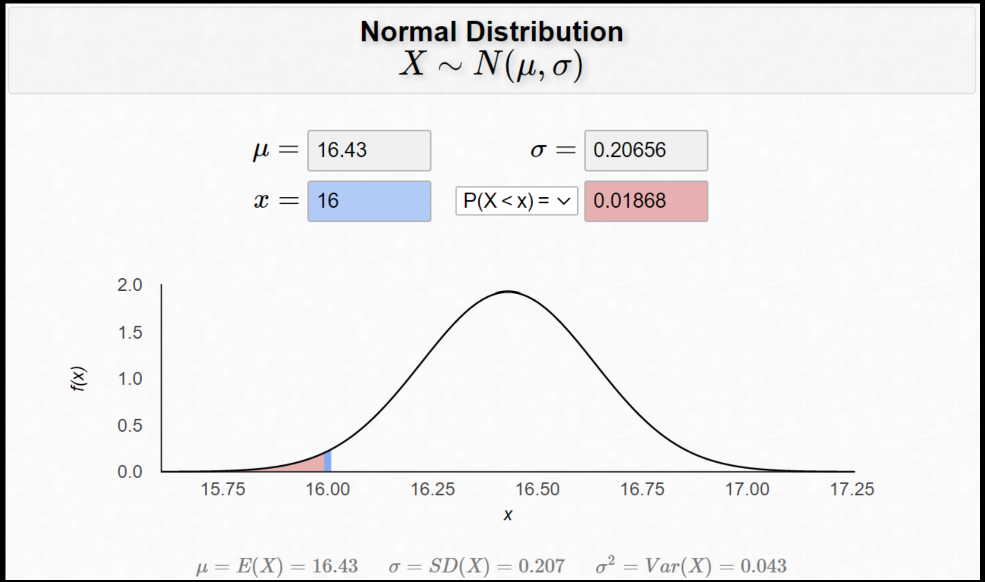
2. Since we know the population standard deviation, we can apply a normal distribution.

$$\bar{X} \sim N\left(\mu, \frac{\sigma_X}{\sqrt{n}}\right) = N\left(16.43, \frac{0.8}{\sqrt{15}}\right) = N(16.43, 0.20656)$$

16.43s comes from H_0, not the data.

Now, we have all the information to perform the test. $P(\bar{x} < 16) = ?$

3. $P(\bar{x} < 16) \approx 0.0187$ (actual significance)



EXAMPLE (CONT.)

$P(\bar{x} < 16) \approx 0.0187$ (actual significance)

$p < \alpha$

Since $p = 0.0187$, there is a 0.0187 (1.87%) chance that Jeffrey's mean time to swim the 25-yard freestyle is 16s or less. Thus, this faster time is an extreme event.

Decision: Since $p < \alpha$, reject H_0. We think Jeffrey swims the 25-yard freestyle faster with goggles.

Conclusion:

At a 5% significance level, we conclude that Jeffrey swims faster using the new goggles. The sample data show there is sufficient evidence that Jeffrey's mean time to swim the 25-yard freestyle is less than 16.43s.

A QUICK REVIEW

- Assume H_0 is true.
- Construct a sample.
- Under this assumption, what is the probability of obtaining another sample like the current sample?
- Compare this probability to a significance level (α).
- Use this comparison to decide: Reject or fail to reject the null hypothesis.