LECTURE 8.3 - ENERGY & POWER

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Fall 2023

CONSERVATION OF ENERGY

Recall that the work-kinetic energy theorem:

$$\Delta K = W_{\text{net}}$$

The net work acting on a system comes from both conservative and nonconservative sources:

$$W_{\text{net}} = W_{\text{c}} + W_{\text{nc}}$$

The work done by conservative forces can be represented by a potential energy:

$$W_{\text{c}} = -\Delta U$$

Work done by nonconservative forces can also be broken down into *dissipative* and *external* forces:

$$W_{\text{nc}} = W_{\text{diss}} + W_{\text{ext}}$$

$$\Delta K = W_{\text{net}}$$

$$\Delta K = W_c + W_{\text{nc}}$$

$$\Delta K = W_c + W_{\text{diss}} + W_{\text{ext}}$$

Examples:

Picking a book up off the floor and placing it on a table adds gravitational potential energy but $\Delta K = 0$.

"Reaching in" to the system and lifting the book is an example of an *external* force doing work. (W_{ext}).

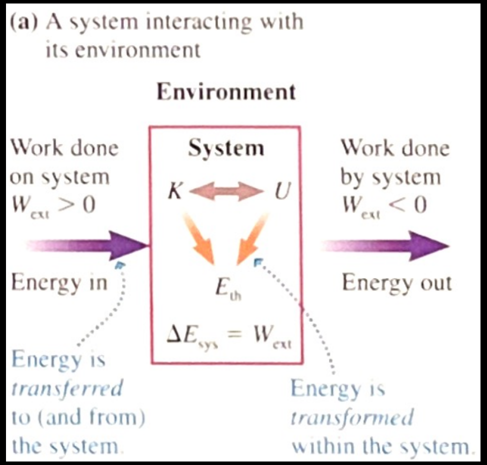
CONSERVATION OF ENERGY

Example (External Force):

- Picking a book up off the floor and placing it on a table adds gravitational potential energy but $\Delta K = 0$.
- "Reaching in" to the system and lifting the book is an example of an *external* force doing work. (W_{ext}).
- Energy is transferred to/from the system.

Examples (Dissipative Force):

- Dragging a block across the floor with a rope changes the kinetic energy of the system but not the potential energy.
- Recall that thermal energy is a part of the system! Energy is transformed within the system.
- Work done by the system can raise the temperature of the system: $\Delta E_{\text{th}} = -W_{\text{diss}}$.



$$\Delta K = W_c + W_{\text{diss}} + W_{\text{ext}}$$

Becomes:

$$\Delta K = -\Delta U - \Delta E_{\text{th}} + W_{\text{ext}}$$

THE ENERGY EQUATION

$$\Delta K = -\Delta U - \Delta E_{\text{th}} + W_{\text{ext}}$$

Can now be written as:

$$\Delta K + \Delta U + \Delta E_{\text{th}} = W_{\text{ext}}$$

$$\Delta E_{\text{mech}} + \Delta E_{\text{th}} = \Delta E_{\text{sys}} = W_{\text{ext}}$$

$$K_i + U_i + W_{\text{ext}} = K_f + U_f + \Delta E_{\text{th}}$$

Work done by external forces changes the energy of the system.

In an isolated system, $W_{ext} = 0J$, so the total energy of the system is conserved.

If there are no dissipative forces, $\Delta E_{th} = 0J$, then the mechanical energy is also conserved!

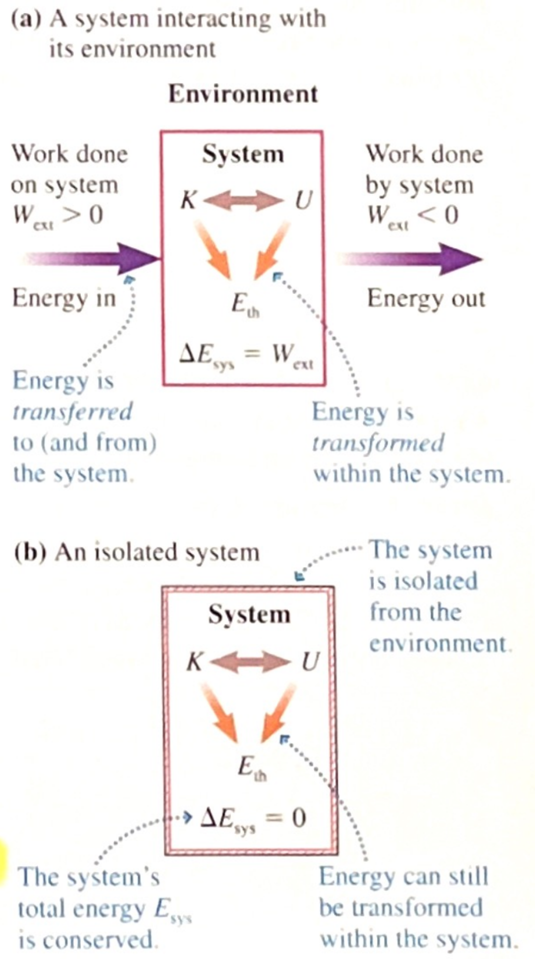
LAW OF CONSERVATION OF ENERGY

The total energy ($E_{\text{sys}} = E_{\text{mech}} + E_{\text{th}}$) of an isolated system is constant.

The kinetic, potential, and thermal energy ($E_{\text{sys}} = \Delta K + \Delta U + E_{\text{th}}$) within an isolated system can be transformed into each other, but their sum cannot change.

The mechanical energy $E_{\text{mech}} = K + U$ is conserved if the system is both isolated and nondissipative.

FIGURE 11.30 The basic energy model is a pictorial representation of the energy equation.



A FEW NOTES ON HEAT

Work is not the only way to transfer energy to a system.

Heat (Q) is the transfer of energy when there is a temperature difference between the system and the environment.

The energy of a system is changed by external (mechanical) forces and/or adding heat:

$$\Delta E_{sys} = W_{ext} + Q$$

This is the First Law of Thermodynamics!

POWER

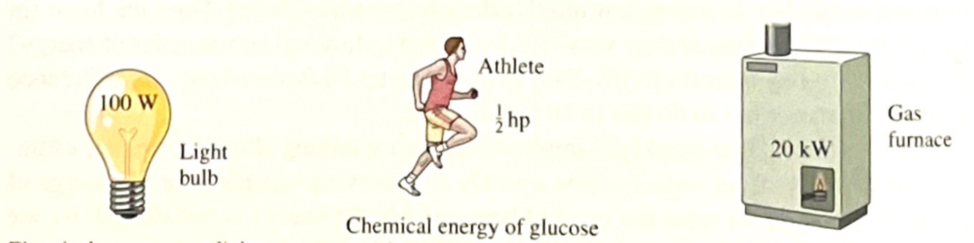
Work is a transfer of energy between the environment and a system. *But how fast/slow is the energy transferred?*

What is the *rate* of energy transfer?

This is the *power*: $P = \frac{d}{dt}E_{sys}$ (Units: *watt*; 1 joule per second)

In terms of mechanical forces, $P = \frac{dW}{dt}$. For a constant force, $P = Fv\cos\theta$

FIGURE 11.36 Examples of power.



Electrical energy ⟶ light and heat at 100 J/s.

Chemical energy of glucose and fat ⟶ mechanical energy at ≈350 J/s ≈ $\frac{1}{2}$ hp.

Chemical energy of gas ⟶ thermal energy at 20,000 J/s.

REVIEW BY EXAMPLES!

Calculating work:

$$W = \int F ds \text{ or } W = \vec{F} \cdot \Delta\vec{r}$$

Energy Equation:

$$K_i + U_i + W_{\text{ext}} = K_f + U_f + \Delta E_{\text{th}}$$

Power:

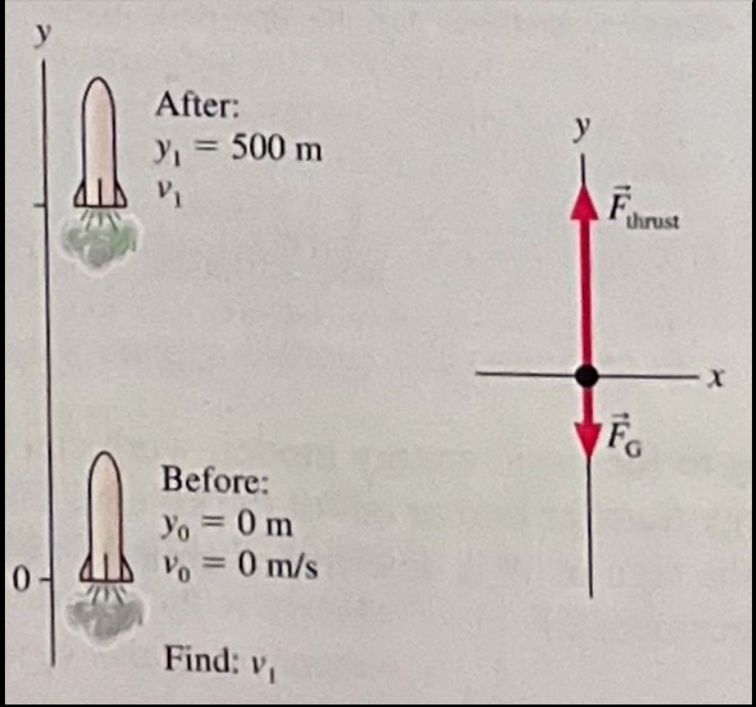
$$P = \frac{d}{dt}(W)$$

CALCULATING WORK

EXAMPLE 11.2 Work during a rocket launch

A 150,000 kg rocket is launched straight up. The rocket motor generates a thrust of 4.0×10^6 N. What is the rocket's speed at a height of 500 m? Ignore air resistance and any slight mass loss.

MODEL Model the rocket as a particle. Thrust and gravity are constant forces that do work on the rocket.



We can use the Work-Kinetic Energy Theorem:

$$\Delta K = W_{\text{net}}$$

What is the work done by the thrust?

$$W_{\text{thrust}} = \int_{y_i}^{y_f} F_{\text{thrust}}\,dy = \int_{y_i}^{y_f} F_{\text{thrust}} \cos\theta\,dy = F_{\text{thrust}} \cos\theta\,\Delta y$$

$$(4.0{\times}10^6\text{N})\cos(0^\circ)\,(500\text{m}) = 2.00{\times}10^9\text{J}$$

What is the work done by the gravity?

$$W_{\text{grav}} = \int_{y_i}^{y_f} F_{\text{grav}}\,dy = \int_{y_i}^{y_f} mg\cos\theta\,dy = mg\cos\theta\,\Delta y$$

$$(1.5{\times}10^5\text{kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)\cos(180^\circ)\,(500\text{m}) = -0.74{\times}10^9\text{J}$$

$$\Delta K = K_f - K_i = W_{net} = W_{thrust} + W_{grav}$$

$$\frac{1}{2}mv_f^2 = (2.00{\times}10^9\text{J} - 0.74{\times}10^9\text{J}) = 1.26{\times}10^9\text{J}$$

$$v_f = \sqrt{\frac{2W_{\text{net}}}{m}} = 130\,\frac{\text{m}}{\text{s}}$$

DOT PRODUCT

$$\vec{A} \cdot \vec{B} = \left(A_x\,\hat{\imath} + A_y\,\hat{\jmath}\right) \cdot \left(B_x\,\hat{\imath} + B_y\,\hat{\jmath}\right)$$

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y$$

The dot product is the sum of the product of the components.

The result is a scalar.

Work can be calculated with a dot product:

$$W = \vec{F} \cdot \Delta\vec{r}$$

$\vec{A} = 3\hat{\imath} + 3\hat{\jmath} = \langle 3,3 \rangle$
$\vec{B} = 4\hat{\imath} - \hat{\jmath} = \langle 4,-1 \rangle$

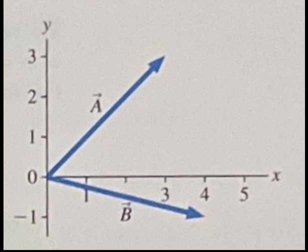


FIGURE 11.15 Vectors $\vec{A}$ and $\vec{B}$.

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y = (3)(4) + (3)(-1) = 9$$

Or, $\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}| \cos\theta$

The angle between the vectors can be found:

$|\vec{A}| = \sqrt{3^2 + 3^2} \approx 4.2426$
$|\vec{B}| = \sqrt{4^2 + (-1)^2} \approx 3.873$

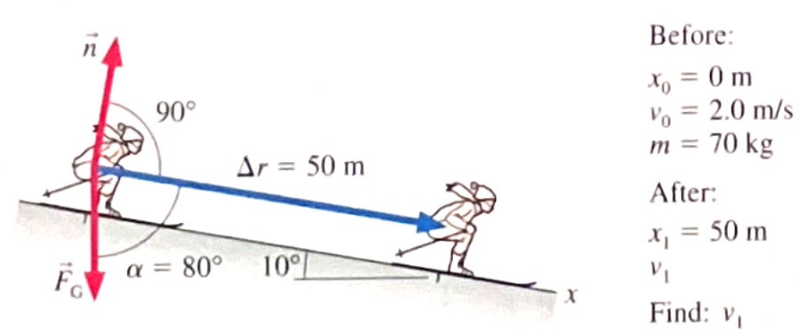
$$\theta = \cos^{-1}\left(\frac{\vec{A} \cdot \vec{B}}{|\vec{A}||\vec{B}|}\right) = \cos^{-1}\left(\frac{9}{4.2426 \cdot 3.873}\right) = 56.87^\circ$$

ENERGY EQUATION (SIMPLIFIED)

A 70kg skier is gliding at $2.0\frac{m}{s}$ when he starts down a very slippery 50-m-long, 10° slope. What is is speed at the bottom if the wind exerts a steady 50N force opposite his motion?

Interpret "very slippery" as frictionless

FIGURE 11.16 Pictorial representation of the skier.



Note: The initial height is $y_i = 50m \sin 10°$

$$W_{\text{net}} = W_{\text{c}} + W_{\text{nc}} = W_{\text{c}} + W_{\text{diss}} + W_{\text{ext}}$$

$$W_{\text{net}} = -\Delta U + 0J + W_{\text{wind}}$$

$$W_{\text{net}} = -\Delta U + W_{\text{wind}} = -\Delta U + \vec{F} \cdot \Delta\vec{r}$$

$$\Delta K = W_{\text{net}} = -\Delta U + \vec{F} \cdot \Delta\vec{r}$$

$$\Delta K + \Delta U = \vec{F} \cdot \Delta\vec{r} = F\Delta r$$

$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 + mgy_f - mgy_i = F\Delta r$$

Solve for v_f

$$v_f = \sqrt{v_i^2 + 2gy_i - \frac{2F\Delta r}{m}} = 10.13\frac{m}{s}$$

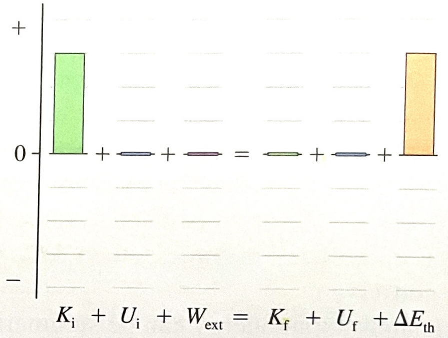
DIFFERENT TYPES OF ENERGY

EXAMPLE 11.11 Energy bar chart I

A speeding car skids to a halt. Show the energy transfers and transformations on an energy bar chart.

SOLVE The car has an initial kinetic energy K_i. That energy is transformed into the thermal energy of the car and the road. The potential energy doesn't change and no work is done by external forces, so the process is an energy transformation $K_i \rightarrow E_{th}$. This is shown in FIGURE 11.32. E_{sys} is conserved but E_{mech} is not.

FIGURE 11.32 Energy bar chart for Example 11.11.

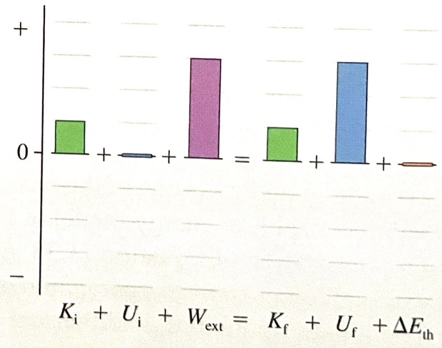


EXAMPLE 11.12 Energy bar chart II

A rope lifts a box at constant speed. Show the energy transfers and transformations on an energy bar chart.

SOLVE The tension in the rope is an external force that does work on the box, increasing the potential energy of the box. The kinetic energy is unchanged because the speed is constant. The process is an energy transfer $W_{ext} \rightarrow U_f$, as FIGURE 11.33 shows. This is not an isolated system, so E_{sys} is not conserved.

FIGURE 11.33 Energy bar chart for Example 11.12.



POWER

Simplified calculations:

(approximate $g \approx 10\,\frac{\mathrm{m}}{\mathrm{s}^2}$)

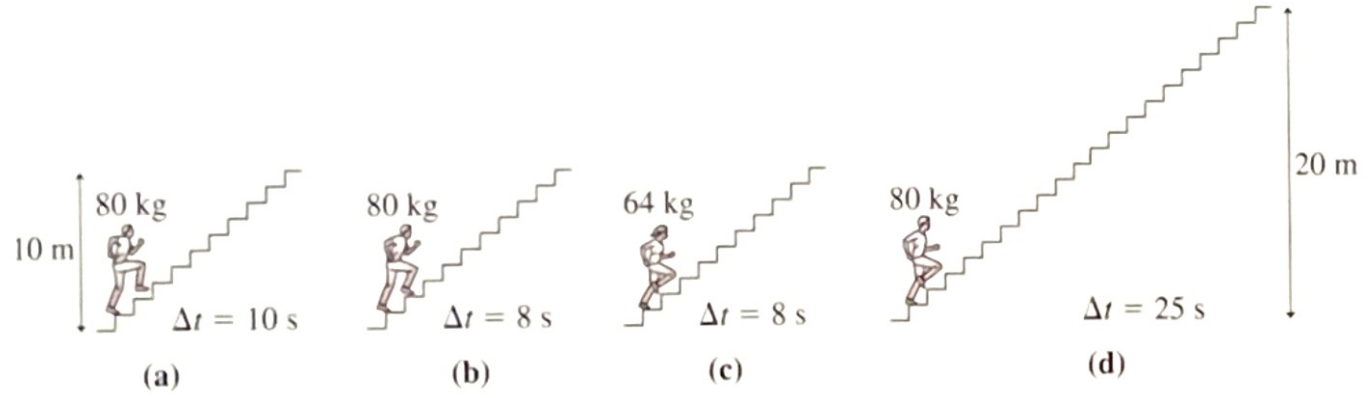
a) $P_a = \frac{\Delta W}{\Delta t} = \frac{mg\Delta y}{\Delta t} = \frac{800\mathrm{N}\cdot 10\mathrm{m}}{10\mathrm{s}} = 800\mathrm{W}$

b) $P_b = \frac{\Delta W}{\Delta t} = \frac{mg\Delta y}{\Delta t} = \frac{800\mathrm{N}\cdot 10\mathrm{m}}{8\mathrm{s}} = 1\mathrm{kW}$

c) $P_c = \frac{\Delta W}{\Delta t} = \frac{mg\Delta y}{\Delta t} = \frac{640\mathrm{N}\cdot 10\mathrm{m}}{8\mathrm{s}} = 800\mathrm{W}$

d) $P_d = \frac{\Delta W}{\Delta t} = \frac{mg\Delta y}{\Delta t} = \frac{800\mathrm{N}\cdot 20\mathrm{m}}{25\mathrm{s}} = 640\mathrm{W}$

STOP TO THINK 11.7 Four students run up the stairs in the time shown. Rank in order, from largest to smallest, their power outputs P_a to P_d.



$$P_b > P_a = P_c > P_d$$