HYPOTHESIS TESTING WITH ONE SAMPLE

MAT 152 – Statistical Methods I
Lecture 3
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FULL HYPOTHESIS TEST EXAMPLES

- Three different distributions for hypothesis testing
  - Normal Distribution (known population standard deviation)
  - T-Distribution (unknown population standard deviation)
  - Binomial Distribution

EXAMPLE (NORMAL DISTRIBUTION)

- A college football coach records the mean weight that his players can bench press as 275 pounds, with a standard deviation of 55 pounds. Three of his players thought that the mean weight was **more than** that amount. They asked 30 of their teammates for their estimated maximum lift on the bench press exercise. $\bar{x} = 286.2$lbs (from the sample)

- Conduct a hypothesis test using a 2.5% level of significance to determine if the bench press mean is **more than 275 pounds**.

$H_0: \mu = 275$

$H_a: \mu > 275$

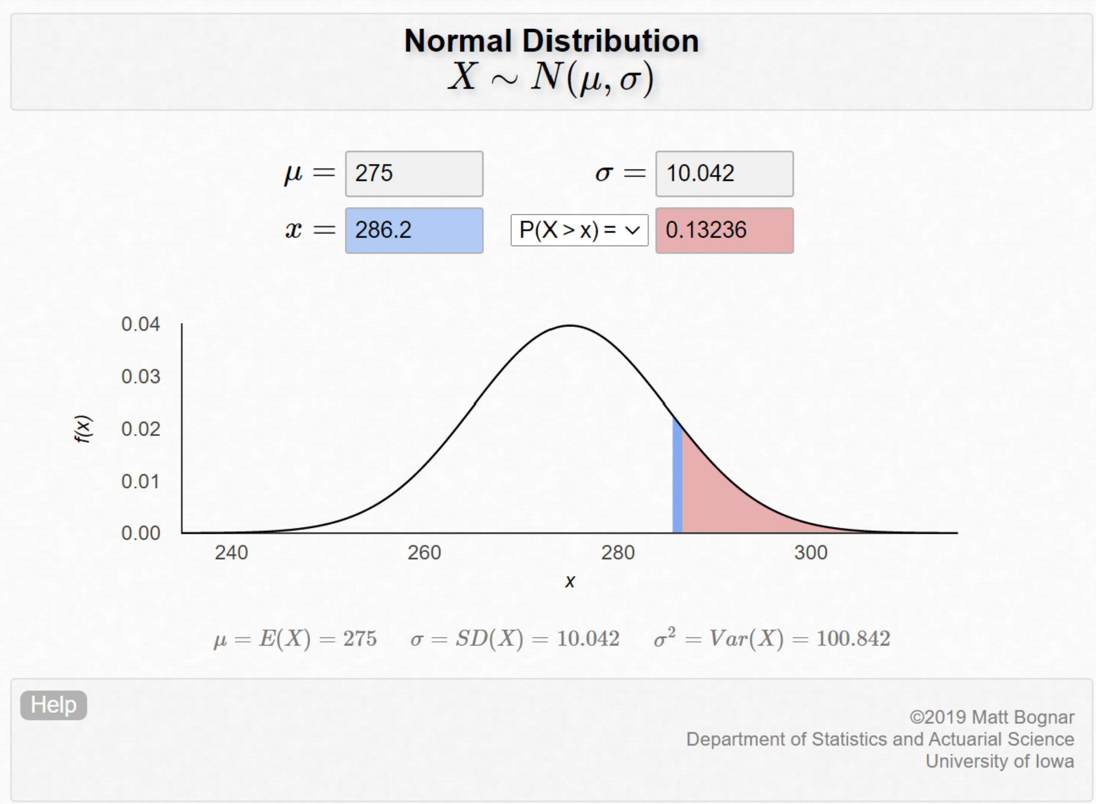
EXAMPLE (NORMAL DISTRIBUTION)

Since we know the standard deviation, a normal distribution will be used. The distribution must be constructed for the SAMPLE MEANS.

$$\bar{X} \sim N\left(275, \frac{55}{\sqrt{30}}\right) = N(275, 10.042)$$

Now, we calculate the probability of our sample mean: $P(\bar{X} > 286.2) = 0.13236$

$(H_a: \mu > 275$ mean right-tailed test)



EXAMPLE (NORMAL DISTRIBUTION)

Interpretation of the p-value:

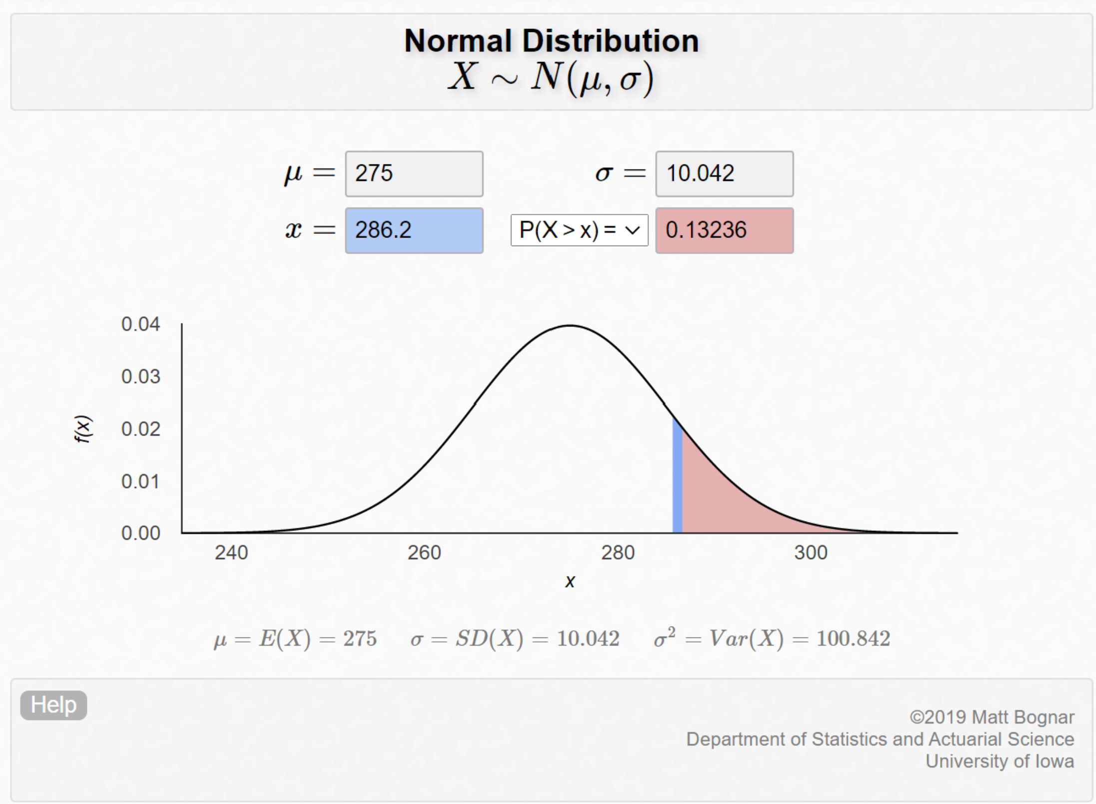
If H_0 is true, then there is a 0.13236 (13.24%) that the football players can lift a mean weight of 286.2 pounds or more. Because a 13.23% chance is large enough, a mean weight lift of 286.2 pounds or more is not an extreme event.

$$0.025 < 0.13236$$

Do not reject the null hypothesis!

Full conclusion:

At the 2.5% level of significance, from the sample data, there is not sufficient evidence to conclude that the true mean weight lifted is more than 275 pounds.



EXAMPLE (T-DISTRIBUTION)

Statistics students believe that the mean score on the first statistics test is 65. A statistics instructor thinks the mean score is higher than 65. He samples ten statistics students and obtains the score. He performs a hypothesis test using a 5% level of significance. (Since the data are grades, they are assumed to be normal.) The statistics from the sample are: $\bar{x} = 67$ and $s \approx 3.2$.

Set up the hypothesis test:

$H_0: \mu = 65$

$H_a: \mu > 65$ (A right-tailed test is required)

Random variable: $\bar{X}$ = average score on the first statistics test

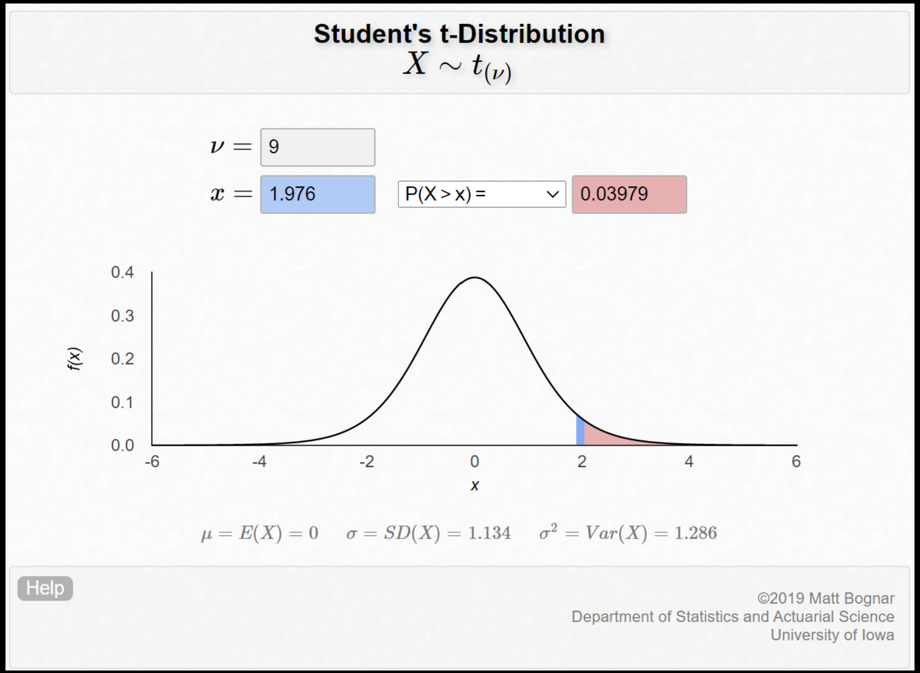
EXAMPLE (T-DISTRIBUTION)

Since no population standard deviation is given and the data are assumed to be normal, a t-test is needed. We will use $T \sim t_{df}$.

We must determine $P(\bar{x} > 67)$ using the t-Distribution.

It is important to note that our t-Distribution calculator only accepts t-scores, not the actual values.

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{67 - 65}{\frac{3.2}{\sqrt{10}}} \approx 1.976$$



$$\text{p} - \text{value} = P(\bar{x} > 67) = 0.03979$$

EXAMPLE (T-DISTRIBUTION)

Decision:

Since $p < \alpha$ where $0.0397 < 0.05$ we reject H_0.

Thus, we believe that the mean is greater than 65.

Conclusion:

At a 5% level of significance, the sample data show sufficient evidence that the mean (average) test score is more than 65, just as the math instructor thinks.

EXAMPLE (BINOMIAL DISTRIBUTION)

June believes that 50% of first-time brides in the United States are younger than their grooms. She performs a hypothesis test to determine if the percentage is the same or different from 50%. June samples 100 first-time brides and 53 reply that they are younger than their grooms. For the hypothesis test, she uses a 1% level of significance.

$H_0: p = 0.50$

$H_a: p \neq 0.50$ (Can be above or below, so two-tailed test)

$\alpha = 0.01$

Random variable: P' = percent of first-time brides who are younger than their grooms.

EXAMPLE (BINOMIAL DISTRIBUTION)

This is a proportion problem so a binomial distribution must be used.

From June's assumptions, we can build an approximate normal distribution:

$$P' \sim N\left(p, \sqrt{\frac{p \cdot q}{n}}\right) = N\left(0.5, \sqrt{\frac{0.5 \cdot 0.5}{100}}\right) = N(0.5, 0.05)$$

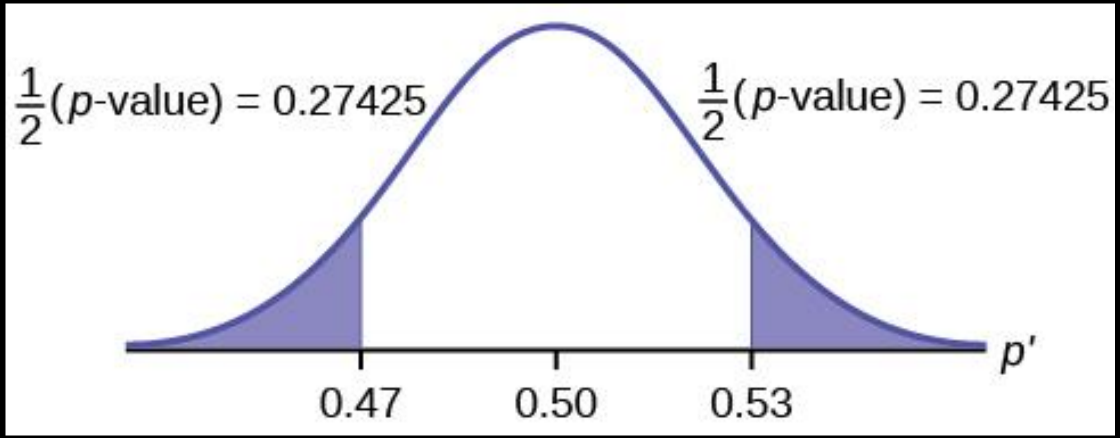
We need to examine the likelihood of obtaining her sample proportion

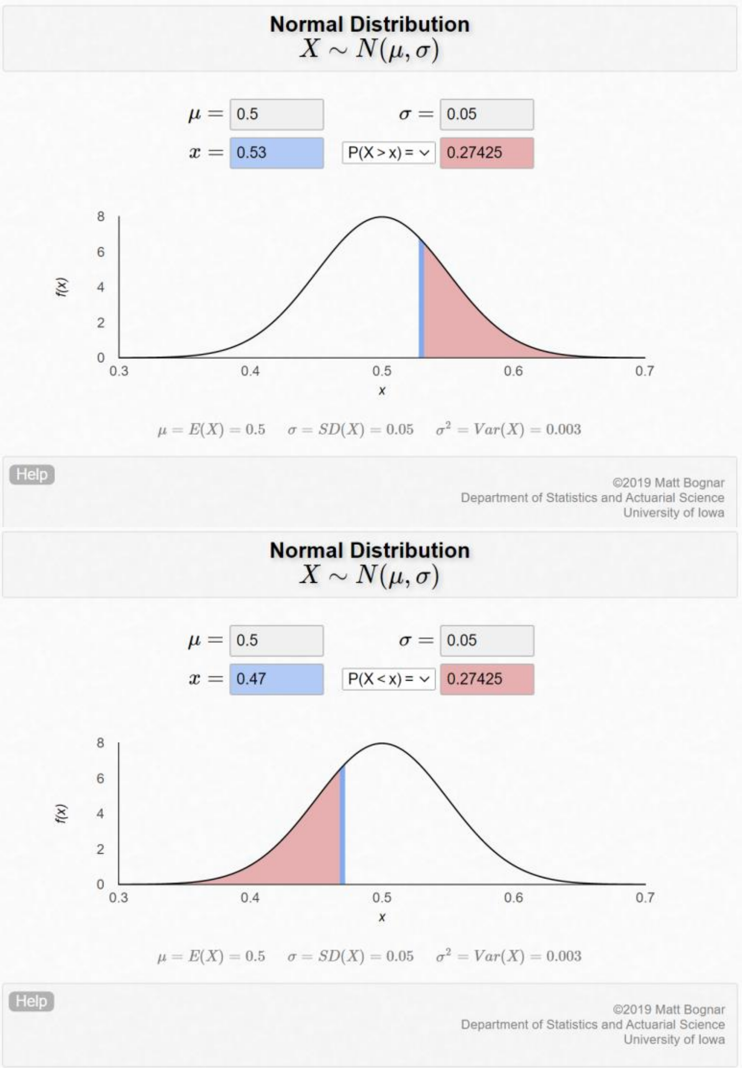
$$p' = \frac{53}{100} = 0.53$$

EXAMPLE (BINOMIAL DISTRIBUTION)

This is a two-tailed test so the probability must be investigated on BOTH sides of the distribution.

$$P(p' < 0.47 \mid p' > 0.53) = P(p' < 0.47) + P(p' > 0.53) = \ 0.5485$$





EXAMPLE (BINOMIAL DISTRIBUTION)

Interpretation:

If the null hypothesis is true, then there is 0.5485 (54.85%) probability that the sample proportion p' is 0.53 or more OR 0.47 or less.

Decision:

Since $\alpha < p$ (0.01 < 0.5485), we cannot reject the null hypothesis.

Conclusion:

At the 1% level of significance, the sample data do not show sufficient evidence that the percentage of first-time brides who are younger than their grooms is different from 50%.

So, June may be wrong be we don't have sufficient evidence to prove it.