Lecture 5.6 - Gravitation

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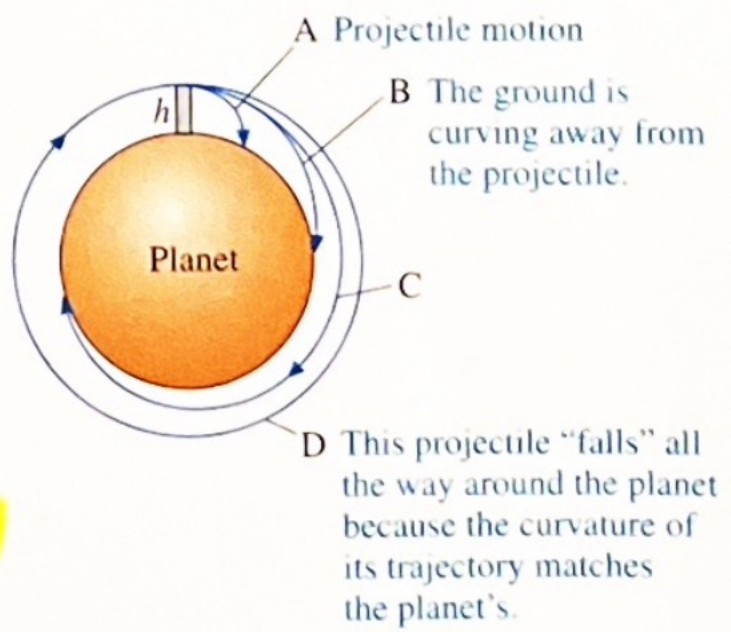
Circular Orbits

- How does a satellite orbit the earth?
- What forces act on it?
- Why does it move in a circle?

Defining an Orbit

- Consider a perfectly smooth, spherical, airless planet with one tower of height *h*.
- A projectile is launched horizontally from the tower. If the launch velocity is very small, the "flat-earth approximation" is valid and our classical 2D kinematic equations can be used.
- As the initial speed is increased, the distance that the projectile travels before finally reaching the ground increases because the projectile must "catch up" with the ground that is curving away from it.
- If the launch speed is sufficiently large, then the curved trajectory of the ball is parallel to the curve of the Earth. In this case, the object continuously falls but never hits the ground. This is called an orbit.

FIGURE 8.12 Projectiles being launched at increasing speeds from height *h* on a smooth, airless planet.



More on Orbits

The most important point of this qualitative analysis is that an orbiting projectile is in free fall.

The only force acting here is gravity.

Its tangential velocity is so large that the curvature of its trajectory matches the curvature of the Earth.

When this happens, the object falls under the influence of gravity but never gets any closer to the surface.

Calculating Orbits

- An object in orbit around the Earth must have a centripetal acceleration that matches the acceleration due to gravity.

$$a_r = \frac{(v_{\text{orbit}})^2}{r} = g$$

- Therefore, if an object moves with speed $v_{\text{orbit}} = \sqrt{rg}$ then the free-fall acceleration provides exactly the centripetal acceleration needed for a circular orbit.

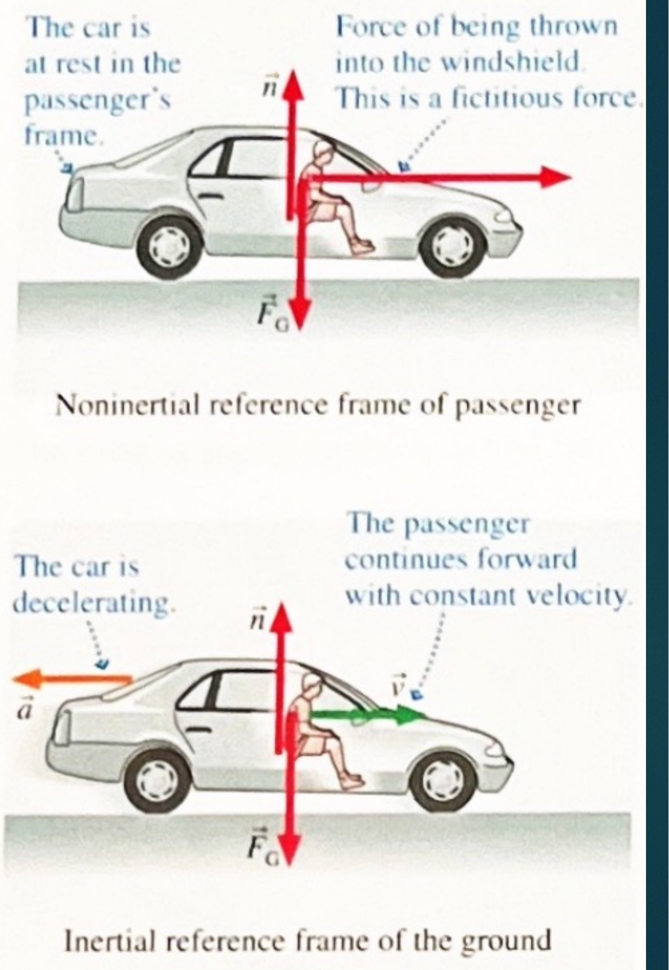
- Additionally, the period of a satellite can be calculated from

$$v_{orbit} = \frac{2\pi r}{T}$$

Fictitious Forces

- A force throwing a crash test dummy into the windshield of a decelerating car cannot be identified yet clearly the dummy moves from "rest" when the car rapidly decelerates.
- Newton's laws are NOT valid!
- Given that the car is decelerating, a coordinate system "riding along" with the car is NOT an inertial reference frame. However, an observer outside the car IS in an inertial reference frame.
- This motion is described by <u>fictitious forces</u>. These are "made up" forces that describe motion in <u>non-inertial reference frames</u>.

FIGURE 8.14 The forces are properly identified only in an inertial reference frame.



Turning a corner: an example

- When you turn a corner in a moving car, your body tries to continue moving in a straight line, obeying Newton's First Law.
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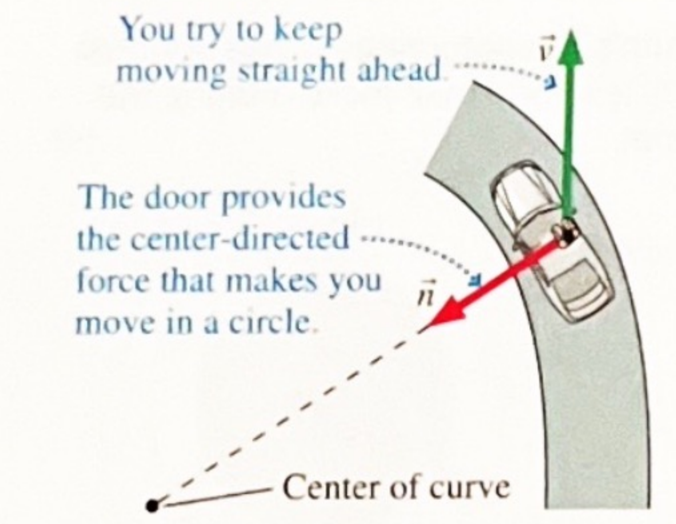
- The door suddenly turns in front of you and runs into you! Only then do you feel the normal force of the door pointing inward toward the center of the curve.
- This normal force causes you to turn the corner.
- You were not "thrown" into the door; the door ran into you.
- This <u>fictitious force</u> is called the <u>centrifugal force</u> and occurs in a <u>non-inertial reference frame</u>.

FIGURE 8.15 Bird's-eye view of a passenger as a car turns a corner.



Gravity on a Rotating Earth

- We often say that the "weight" of a person/object is simply $|\vec{n}|$. However, this assumes a flat, stationary Earth.

- Using this assumption, we can calculate $g = 9.83\,\frac{m}{s^2}$. However, we know that g is closer to $9.80\,\frac{m}{s^2}$. Why is the actual value of gravity slightly smaller than the calculated gravity?

- The key lies in the assumption of a flat, stationary Earth!

- Let's take a look at what is going on when we consider the Earth's rotation.

$$\sum F_r = ma_c = \frac{mv^2}{r} = F_g - n$$

$$n = F_g - \frac{mv^2}{r} = mg - m\omega^2 R$$

- In our reference frame, $m\omega^2 R$ is a fictitious centrifugal force trying to throw us off the Earth. Thus, measured gravity appears to be a bit weaker.