Lecture 7.3 – Elastic Collisions

Dustin Roten, Ph.D.

Wilkes Community College

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Perfectly Elastic Collisions

At a molecular level, the chemical bonds making up objects act like tiny springs.

In terms of energy, the kinetic energy of colliding objects is transformed into elastic potential energy in the molecular bonds of the objects. This elastic potential energy is then transformed back into kinetic energy as the objects “spring” apart.

A collision in which <u>all</u> the kinetic energy is transferred to elastic potential energy in the bonds then back into kinetic energy is called a *perfectly elastic collision*.

Most real-world collisions fall somewhere between perfectly elastic and inelastic; however, these types of “ideal” collisions can be used as approximations.

Perfectly Elastic Collisions

Perfectly elastic collisions must obey two conservation laws.

conservation of momentum (obeyed in any collision)

conservation of energy (obeyed in perfectly elastic collisions)

In this type of collision, the mechanical energy before and after is purely kinetic.

Momentum conservation:

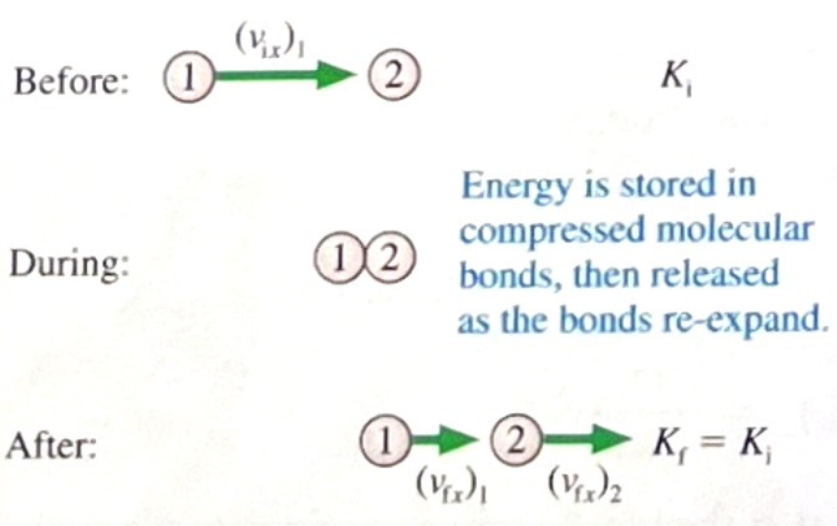
$$m_1 v_{1,i} = m_1 v_{1,f} + m_2 v_{2,f}$$

Energy conservation:

$$\frac{1}{2} m_1 v_{1,i}^2 = \frac{1}{2} m_1 v_{1,f}^2 + \frac{1}{2} m_2 v_{2,f}^2$$

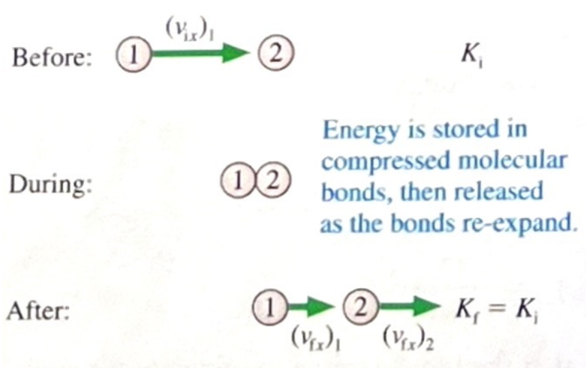
Both conservation laws are needed since there are two unknowns (final velocities) in this type of collision.

FIGURE 10.24 A perfectly elastic collision.



Deriving Equations for Final Velocities (Ball 2 at Rest)

FIGURE 10.24 A perfectly elastic collision.



We need to determine the final velocities of each object in this collision.

- Beginning with conservation of momentum:

$$m_1 v_{1,i} = m_1 v_{1,f} + m_2 v_{2,f}$$

$$m_1 v_{1,f} = m_1 v_{1,i} - m_2 v_{2,f}$$

$$v_{1,f} = v_{1,i} - \frac{m_2}{m_1} v_{2,f}$$

Now from conservation of energy:

$$\frac{1}{2} m_1 v_{1,i}^2 = \frac{1}{2} m_1 v_{1,f}^2 + \frac{1}{2} m_2 v_{2,f}^2$$

$$\frac{1}{2} m_1 v_{1,i}^2 = \frac{1}{2} m_1 \left(v_{1,i} - \frac{m_2}{m_1} v_{2,f} \right)^2 + \frac{1}{2} m_2 v_{2,f}^2$$

$$\frac{1}{2} m_1 v_{1,i}^2 = \frac{1}{2} m_1 \left(v_{1,i} - \frac{m_2}{m_1} v_{2,f} \right) \left(v_{1,i} - \frac{m_2}{m_1} v_{2,f} \right) + \frac{1}{2} m_2 v_{2,f}^2$$

$$\frac{1}{2} m_1 v_{1,i}^2 = \frac{1}{2} m_1 v_{1,i}^2 - m_2 v_{1,i} v_{2,f} + \frac{m_2^2}{2m_1} v_{2,f}^2 + \frac{1}{2} m_2 v_{2,f}^2$$

$$\frac{m_2^2}{2m_1} v_{2,f}^2 + \frac{1}{2} m_2 v_{2,f}^2 - m_2 v_{1,i} v_{2,f} = 0$$

$$v_{2,f} \left[\left(1 + \frac{m_2}{m_1} \right) v_{2,f} - 2 v_{1,i} \right] = 0$$

Deriving Equations for Final Velocities

$$v_{2,f}\left[\left(1+\frac{m_2}{m_1}\right)v_{2,f}-2v_{1,i}\right]=0$$

Consider the zero-product property: $ab=0$. This indicates that a or b must be zero. Using this property, $v_{2,f}$ or $\left(1+\frac{m_2}{m_1}\right)v_{2,f}-2v_{1,i}$ equals zero.

$v_{2,f}=0\frac{m}{s}$ is a "trivial" solution. If the "after" velocity of the second object is zero, this implies that the collision never happened.

On the other hand, if $\left(1+\frac{m_2}{m_1}\right)v_{2,f}-2v_{1,i}=0$ then:

$$v_{2,f}=\frac{2v_{1,i}}{\left(1+\frac{m_2}{m_1}\right)}=\frac{2m_1}{m_1+m_2}(v_{1,i})$$

$$v_{2,f}=\frac{2m_1}{m_1+m_2}(v_{1,i})$$

Substituting back into the conservation of momentum equation:

$$v_{1,f}=v_{1,i}-\frac{m_2}{m_1}v_{2,f}$$

$$v_{1,f}=v_{1,i}-\frac{m_2}{m_1}\left[\frac{2m_1}{m_1+m_2}(v_{1,i})\right]=v_{1,i}-\left(\frac{2m_2}{m_1+m_2}(v_{1,i})\right)$$

$$v_{1,f}=\left(1-\frac{2m_2}{m_1+m_2}\right)(v_{1,i})$$

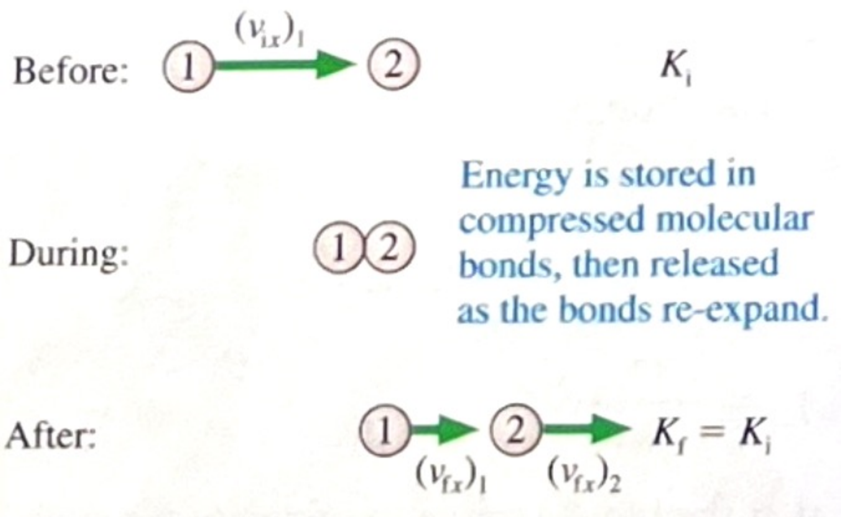
$$v_{1,f}=\left(\frac{m_1+m_2}{m_1+m_2}-\frac{2m_2}{m_1+m_2}\right)(v_{1,i})=\frac{m_1-m_2}{m_1+m_2}(v_{1,i})$$

$$v_{1,f}=\frac{m_1-m_2}{m_1+m_2}(v_{1,i})$$

Example #1
(Ball 2 at Rest)

Ball 1, with a mass of 100g and traveling at $10\,\frac{\mathrm{m}}{\mathrm{s}}$, collides head-on with ball 2, which has a mass of 300g and is initially at rest. What is the final velocity of each ball if the collision is perfectly elastic?

FIGURE 10.24 A perfectly elastic collision.



$$v_{1,f} = \frac{m_1 - m_2}{m_1 + m_2}\left(v_{1,i}\right) = \frac{0.1\mathrm{kg} - 0.3\mathrm{kg}}{0.1\mathrm{kg} + 0.3\mathrm{kg}}\left(10\,\frac{\mathrm{m}}{\mathrm{s}}\right) = -5.0\,\frac{\mathrm{m}}{\mathrm{s}}$$

$$v_{2,f} = \frac{2m_1}{m_1 + m_2}\left(v_{1,i}\right) = \frac{2 \cdot 0.1\mathrm{kg}}{0.1\mathrm{kg} + 0.3\mathrm{kg}}\left(10\,\frac{\mathrm{m}}{\mathrm{s}}\right) = 5.0\,\frac{\mathrm{m}}{\mathrm{s}}$$

Using Reference Frames

When both objects have initial velocities, solving for the final velocities of the objects can be *messy*.

Using the previous "trick" of a Galilean transformation, the mathematics can be simplified to a problem like the last example!

The transformation to the "prime" reference frame:

$$v' = v - V$$

Allow the traveling reference frame to "ride along" with the second object.

FIGURE 10.27 A perfectly elastic collision in which both balls have an initial velocity.

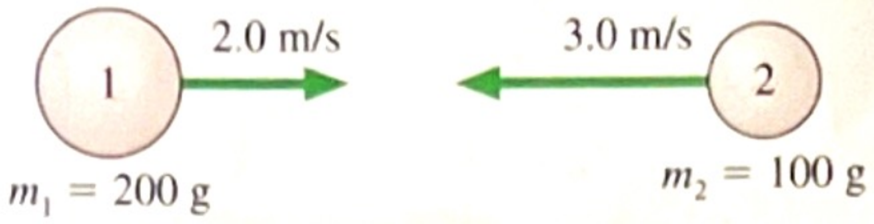
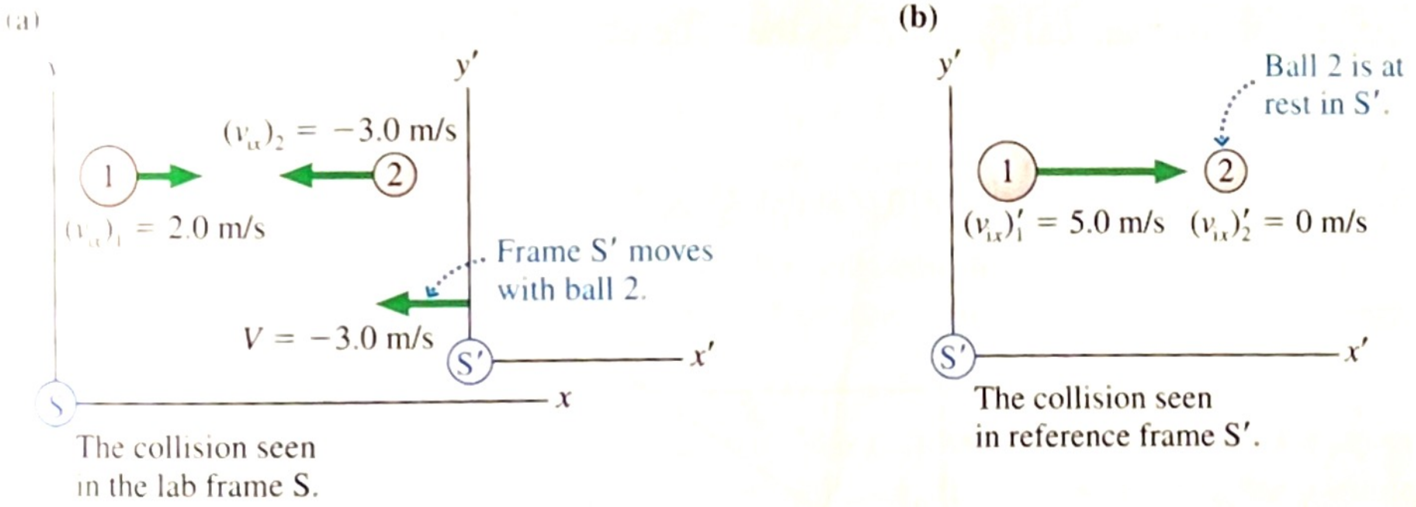


FIGURE 10.28 The collision seen in two reference frames, S and S′.



Using Reference Frames

FIGURE 10.27 A perfectly elastic collision in which both balls have an initial velocity.

Ball #1

$$v_1' = v_1 - V$$

V is the speed of the reference frame.

$V = -3.0\,\dfrac{\text{m}}{\text{s}}$

$$v_1' = 2.0\,\frac{\text{m}}{\text{s}} - \left(-3.0\,\frac{\text{m}}{\text{s}}\right) = 5.0\,\frac{\text{m}}{\text{s}}$$

$$v_{1,f} = \frac{m_1 - m_2}{m_1 + m_2}\left(v_{1,i}'\right) = \frac{0.2\text{kg} - 0.1\text{kg}}{0.2\text{kg} + 0.1\text{kg}}\left(5.0\,\frac{\text{m}}{\text{s}}\right) = 1.7\,\frac{\text{m}}{\text{s}}$$

Ball #2

$$v_2' = v_2 - V$$

V is the speed of the reference frame.

$V = -3.0\,\dfrac{\text{m}}{\text{s}}$

$$v_2' = -3.0\,\frac{\text{m}}{\text{s}} - \left(-3.0\,\frac{\text{m}}{\text{s}}\right) = 0\,\frac{\text{m}}{\text{s}}$$

$$v_{2,f} = \frac{2m_1}{m_1 + m_2}\left(v_{1,i}'\right) = \frac{2 \cdot 0.2\text{kg}}{0.2\text{kg} + 0.1\text{kg}}\left(5.0\,\frac{\text{m}}{\text{s}}\right) = 6.7\,\frac{\text{m}}{\text{s}}$$

REMEMBER: These velocities are in a RELATIVE FRAME!

Ball #1

$$v_1' = v_1 - V$$

V is the speed of the reference frame.

$V = -3.0\,\frac{\text{m}}{\text{s}}$

$$v_1' = 2.0\,\frac{\text{m}}{\text{s}} - \left(-3.0\,\frac{\text{m}}{\text{s}}\right) = 5.0\,\frac{\text{m}}{\text{s}}$$

$$v_{1,f} = \frac{m_1 - m_2}{m_1 + m_2}\left(v_{1,i}'\right) = \frac{0.2\text{kg} - 0.1\text{kg}}{0.2\text{kg} + 0.1\text{kg}}\left(5.0\,\frac{\text{m}}{\text{s}}\right) = 1.7\,\frac{\text{m}}{\text{s}}$$

Transform the velocities back to the standard frame!

$$v_1 = v_1' + V$$

$$v_1 = 1.7\,\frac{\text{m}}{\text{s}} + \left(-3.0\,\frac{\text{m}}{\text{s}}\right) = -1.3\,\frac{\text{m}}{\text{s}}$$

Ball #2

$$v_2' = v_2 - V$$

V is the speed of the reference frame.

$V = -3.0\,\frac{\text{m}}{\text{s}}$

$$v_2' = -3.0\,\frac{\text{m}}{\text{s}} - \left(-3.0\,\frac{\text{m}}{\text{s}}\right) = 0\,\frac{\text{m}}{\text{s}}$$

$$v_{2,f} = \frac{2m_1}{m_1 + m_2}\left(v_{1,i}'\right) = \frac{2 \cdot 0.2\text{kg}}{0.2\text{kg} + 0.1\text{kg}}\left(5.0\,\frac{\text{m}}{\text{s}}\right) = 6.7\,\frac{\text{m}}{\text{s}}$$

Transform the velocities back to the standard frame!

$$v_2 = v_2' + V$$

$$v_1 = 6.7\,\frac{\text{m}}{\text{s}} + \left(-3.0\,\frac{\text{m}}{\text{s}}\right) = 3.7\,\frac{\text{m}}{\text{s}}$$

FIGURE 10.29 The post-collision velocities in the lab frame.