The Central Limit Theorem

MAT 152 – STATISTICAL METHODS I
LECTURE 3
INSTRUCTOR: DUSTIN ROTEN
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Using the Central Limit Theorem

Finding the probability of the mean (Use CLT for means)

Finding the probability of a sum or total (Use CLT for sums)

Finding the probability of an individual value (DO NOT use CLT!)

Law of Large Numbers:

As the sample size (*n*) increases, the mean of each sample will get closer to μ.

The larger *n* gets, the smaller the sample standard deviation gets.

Example

A study involving stress is conducted among the students on a college campus. **The stress scores follow a uniform distribution** with the lowest stress score equal to one and the highest equal to five. Using a sample of 75 students, find:

a. The probability that the **mean stress score** for the 75 students is less than two.
b. The 90^{th} percentile for the **mean stress score** for the 75 students.
c. The probability that the **total of the 75 stress scores** is less than 200.
d. The 90^{th} percentile for the **total stress score** for the 75 students.

Let X = one stress score.

Before addressing (a), identify what is known.

Stress scores follow a uniform distribution such that $X \sim U(1,5)$.

The mean and standard deviations of the population can be determined.

$\mu_X = \frac{a+b}{2} = \frac{1+5}{2} = \frac{6}{2} = 3$ and $\sigma_X = \sqrt{\frac{(b-a)^2}{12}} = \sqrt{\frac{(5-1)^2}{12}} = 1.15$

Example (a)

A study involving stress is conducted among the students on a college campus. **The stress scores follow a uniform distribution** with the lowest stress score equal to one and the highest equal to five. Using a sample of 75 students, find:

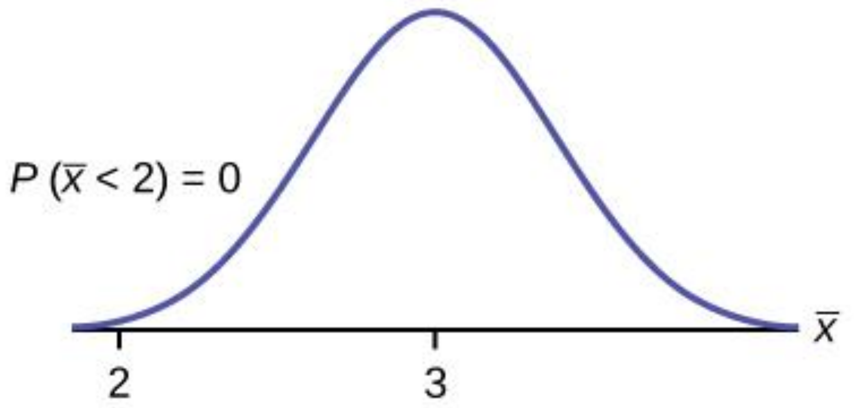
a. The probability that the **mean stress score** for the 75 students is less than two.

If we randomly sample 75 students, what is the probability that the sample mean is less than two? $P(\bar{x} < 2) = ?$

What is the sampling distribution of the means?

$$\bar{X} \sim N\left(\mu_X, \frac{\sigma_X}{\sqrt{n}}\right)$$

$$\bar{X} \sim N\left(3, \frac{1.15}{\sqrt{75}}\right)$$



Example (b)

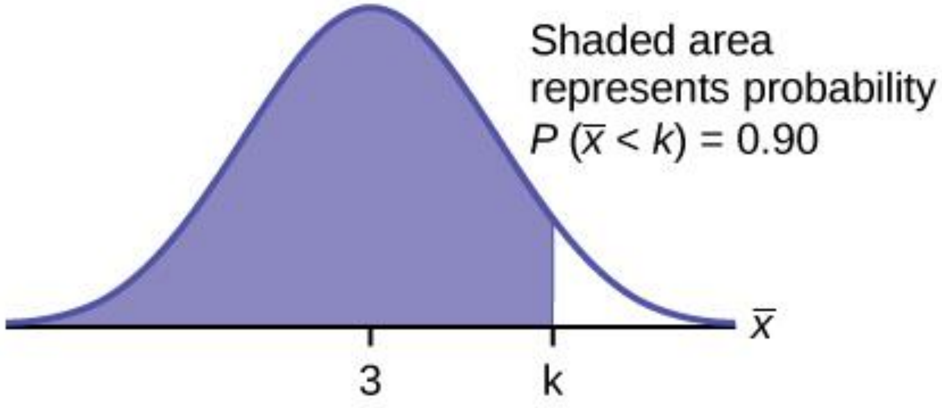
A study involving stress is conducted among the students on a college campus. **The stress scores follow a uniform distribution** with the lowest stress score equal to one and the highest equal to five. Using a sample of 75 students, find:

b. The 90th percentile for the **mean stress score** for the 75 students.

Using the same distribution as before, the value *k* must be determined such that the area to the left (blue) is equal to 0.90. $P(\bar{x} < k) = 0.90$.

$$\bar{X} \sim N\left(3, \frac{1.15}{\sqrt{75}}\right)$$

Using technology, k=3.2



Example (c)

A study involving stress is conducted among the students on a college campus. **The stress scores follow a uniform distribution** with the lowest stress score equal to one and the highest equal to five. Using a sample of 75 students, find:

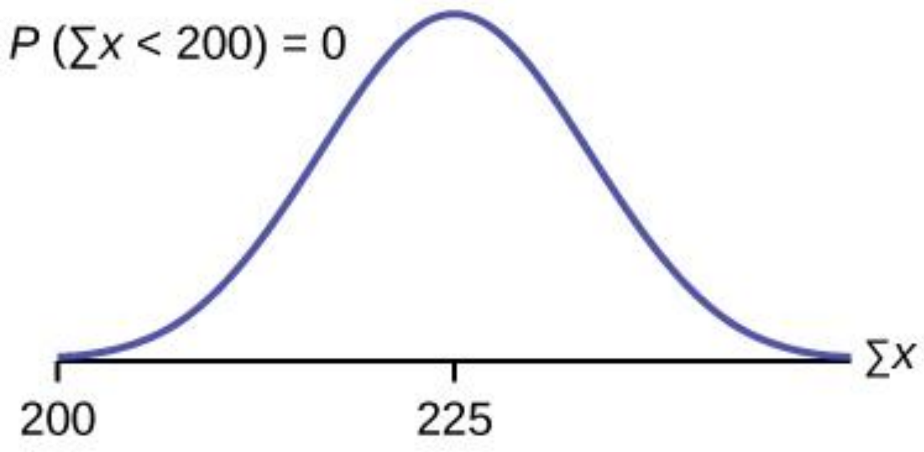
c. The probability that the **total of the 75 stress scores** is less than 200.

Let $\sum X =$ the sum of 75 stress scores.

If we randomly sample 75 students, what is the probability that the total of the stress scores is less than 200? $P(\sum x < 200) = ?$

What is the sampling distribution of the means?

$$\sum X \sim N\left((n)(\mu_X), (\sqrt{n})(\sigma_X)\right)$$



The mean of the sum of 75 test scores is $(n)(\mu_X) = (75)(3) = 225$

The s.d. of the sum of 75 test scores is $(\sqrt{n})(\sigma_X) = (\sqrt{75})(1.15) = 9.96$

Example (d)

A study involving stress is conducted among the students on a college campus. **The stress scores follow a uniform distribution** with the lowest stress score equal to one and the highest equal to five. Using a sample of 75 students, find:

d. The 90th percentile for the **total stress score** for the 75 students.

Using the same distribution as before, the value *k* must be determined such that the area to the left (blue) is equal to 0.90. $P(\bar{x} < k) = 0.90$.

$$\sum X \sim N(225, 9.96)$$

Using technology, k=237.8