Lecture 7.1 – Kinetic and Potential Energy

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Energy

- Two fundamental forms of energy are *kinetic* and *potential* energy.
  - Kinetic Energy – An energy of *motion*
  - Potential Energy – An energy of *position*

- One of the greatest scientific discoveries is that of *energy*.
  - It is important to understand how energy can be *stored* and/or *transformed* from one form to another.
  - Energy can be transferred into or out of a system or transformed within a system.

- The fact that nature “conserves” energy is one of the most important concepts in science!

- Ideas about energy extend to heat, chemical energy, and molecular energy.

Motion, Position, and Energy

Consider a ball in free-fall. The only force acting on it is $\vec{F}_G$ and $a = -g$.

Just like in previous lectures, we will consider the "before" and "after" pictures of the situation.

There is one kinematic equation that relates before and after information without regard to time:

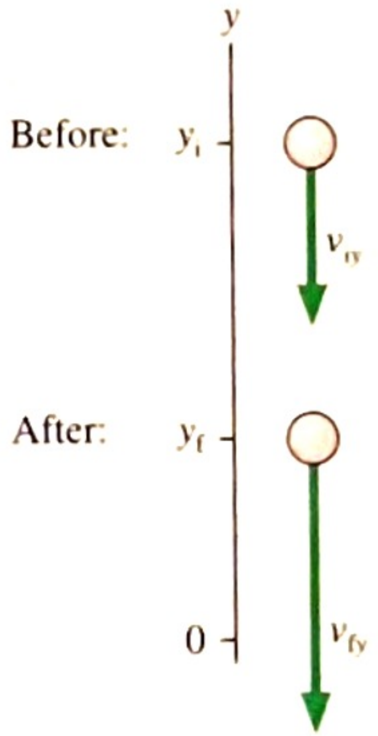
$$v_f^2 = v_i^2 + 2a\Delta y$$

Terms can be rearranged such that all the "final" and "initial" are separated.

$$v_f^2 = v_i^2 + 2a(y_f - y_i) = v_i^2 + 2ay_f - 2ay_i$$
$$v_f^2 + 2gy_f = v_i^2 + 2gy_i$$

$v^2 + 2gy$ is the same before AND after. This is a conservation law!

FIGURE 10.3 The before-and-after representation of an object in free fall.



Rewriting Newton's Second Law

Applying Newton's Second Law to the free-fall problem:

$$F_{\text{net}} = ma = m\frac{dv}{dt} = -mg$$

Using the chain rule:

$$\frac{dv}{dt} = \frac{dv}{dy}\frac{dy}{dt} = v\frac{dv}{dy}$$

NSL becomes:

$$F_{net} = ma = mv\frac{dv}{dy} = -mg$$

We can now look at the differentials in the equation:

$$mvdv = -mgdy$$

$$mvdv = -mgdy$$

We can integrate from "before" to "after":

$$m\int_{v_i}^{v_f} vdv = -mg\int_{y_i}^{y_f} dy$$

$$\frac{1}{2}mv^2\Big|_{v_i}^{v_f} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = -mgy\Big|_{v_i}^{y_f} = -mgy_f + mgy_i$$

We can group the "before" and "after" terms:

$$\frac{1}{2}mv_f^2 + mgy_f = \frac{1}{2}mv_i^2 + mgy_i$$

Compare to: $v_f^2 + 2gy_f = v_i^2 + 2gy_i$

Kinetic and Potential Energy

$$\frac{1}{2}mv_f^2 + mgy_f = \frac{1}{2}mv_i^2 + mgy_i$$

Kinetic Energy (motion): $\mathrm{K} = \frac{1}{2}mv^2$

Potential Energy (position): $U = mgy$

$$K_f + U_f = K_i + U_i$$

Note that energy is a scalar, NOT a vector!

Kinetic energy is never negative!

What are the units of energy?

Kinetic Energy

mass multiplied by velocity squared $[\mathrm{kg}] \cdot \left[\frac{\mathrm{m}^2}{\mathrm{s}^2}\right] = \left[\mathrm{kg}\frac{\mathrm{m}^2}{\mathrm{s}^2}\right]$

Potential Energy

mass multiplied by acceleration multiplied by position $[\mathrm{kg}] \cdot \left[\frac{\mathrm{m}}{\mathrm{s}^2}\right] \cdot [\mathrm{m}] = \left[\mathrm{kg}\frac{\mathrm{m}^2}{\mathrm{s}^2}\right]$

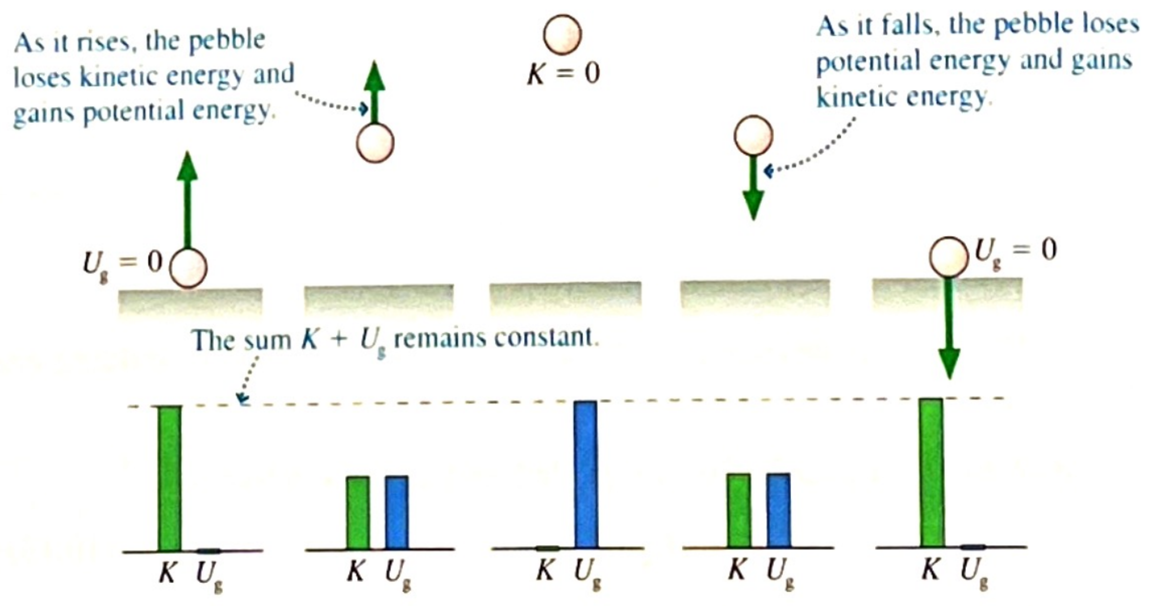
This unit is a Joule.

$$1\mathrm{J} = \left[\mathrm{kg}\frac{\mathrm{m}^2}{\mathrm{s}^2}\right]$$

Mass MUST BE in kilograms and velocities MUST BE in m/s before doing calculations!

Energy Bar Charts

FIGURE 10.6 Simple energy bar chart for a pebble tossed into the air.



- When the pebble is initially tossed upward, it has no gravitational potential energy. The initial energy is all kinetic.
- When the ball reaches its maximum height, its speed is briefly zero. All the pebble's energy is potential energy.
- Finally, the potential energy is transformed into kinetic energy as the pebble returns to the ground.
- The total energy in the system remains the same!

The Zero of Potential Energy

FIGURE 10.8 Amber and Bill use coordinate systems with different origins to determine the potential energy of a rock.

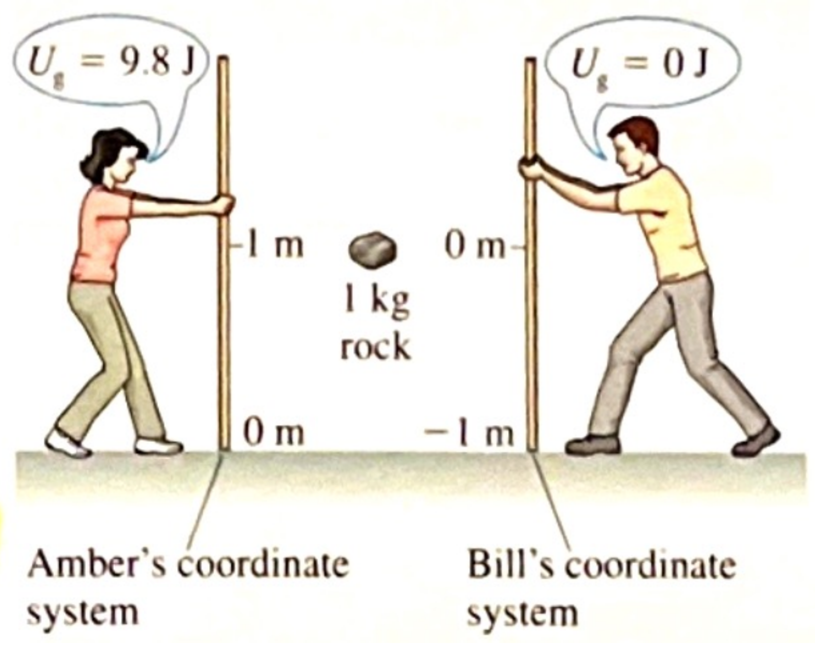
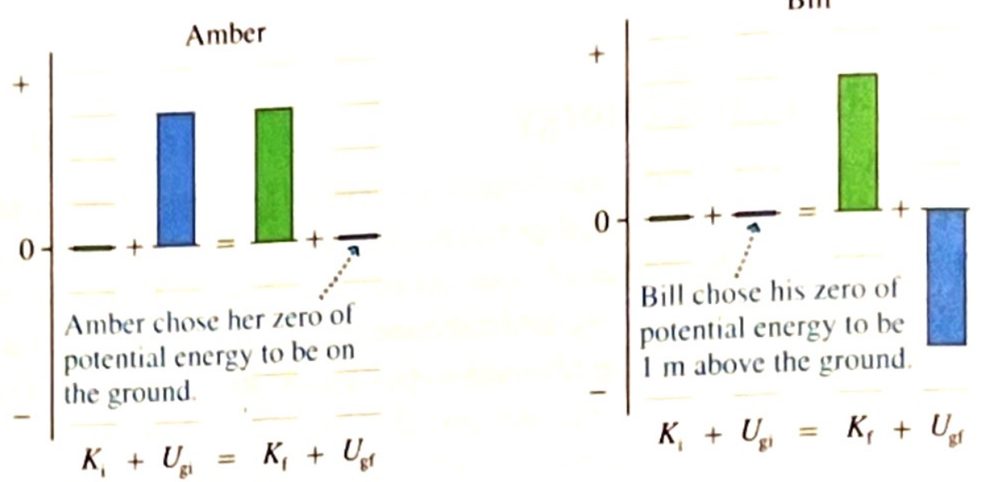


FIGURE 10.10 Amber's and Bill's energy bar charts for the falling rock.



- Placing the origin of the coordinate system allows us to pick the "zero of potential energy".
- Although Amber and Bill may have different values of U_f and U_i, their values of ΔU will be the same.

Gravitational Potential Energy along a Path

Consider NSL for a particle being pushed up a ramp. The "s-axis" follows the incline.

Once again, apply the chain rule.

$$F_{\text{net}} = ma_s = m\frac{dv_s}{dt} = mv_s\frac{dv_s}{ds} = -mg\sin\theta$$

Rearranging:

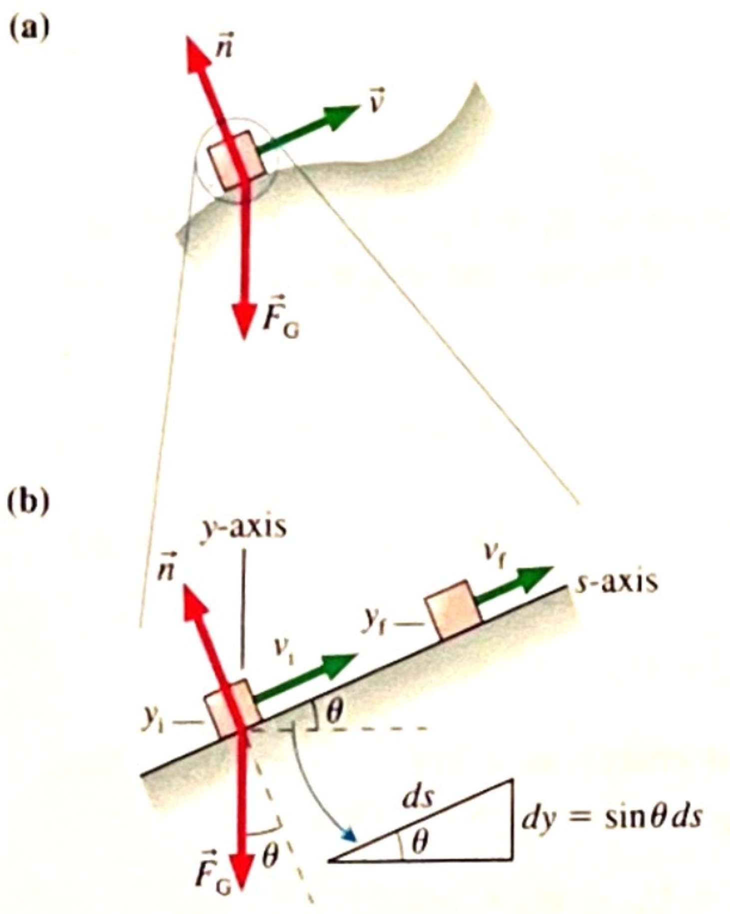
$$mv_s dv_s = -mg\sin\theta\, ds$$

If we consider the geometry of the situation (bottom right of the figure), its clear that a change in position up the ramp (ds) is accompanied by an increase in elevation $dy = \sin\theta\, ds$

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{dy}{ds}$$

$$mv_s dv_s = -mg\sin\theta\, ds = -mgdy$$

FIGURE 10.11 A particle moving along a frictionless surface of arbitrary shape.



Integrating the Equation

$$mv_s dv_s = -mg\sin\theta\, ds = -mgdy$$

Integrating from “before” to “after”:

$$\int_{v_i}^{v_f} mv_s\, dv_s = -mg\sin\theta \int_{s_i}^{s_f} ds = -mg \int_{y_i}^{y_f} dy$$

$$\frac{1}{2}mv^2\Big|_{v_i}^{v_f} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = -mgy\Big|_{v_i}^{y_f} = -mgy_f + mgy_i$$

$$\frac{1}{2}mv_f^2 + mgy_f = \frac{1}{2}mv_i^2 + mgy_i$$

This is the SAME equation as in the free-fall derivation!

What does this mean!?

$$K_f + U_f = K_i + U_i$$

This is the case for a particle moving along *any* frictionless surface, regardless of the shape.

<u>Provided the surfaces are frictionless</u>, the equations are the same no matter what path the particle takes!

Conservation of Mechanical Energy

K is the total kinetic energy of all the particles in a system.

U is the total potential energy of all the particles in a system.

Kinetic and potential energy can change but there sum remains constant.

Thus,

$$\Delta \mathrm{E}_{\mathrm{mech}} = \Delta K + \Delta U = 0$$

This is the *law of conservation of mechanical energy*!

This also shows us that, if $\Delta E_{mech} = 0$, then $\Delta K = -\Delta U$. This is true on *frictionless* surfaces. If kinetic energy is gained, potential energy is lost. If potential energy is gained, kinetic energy is lost.

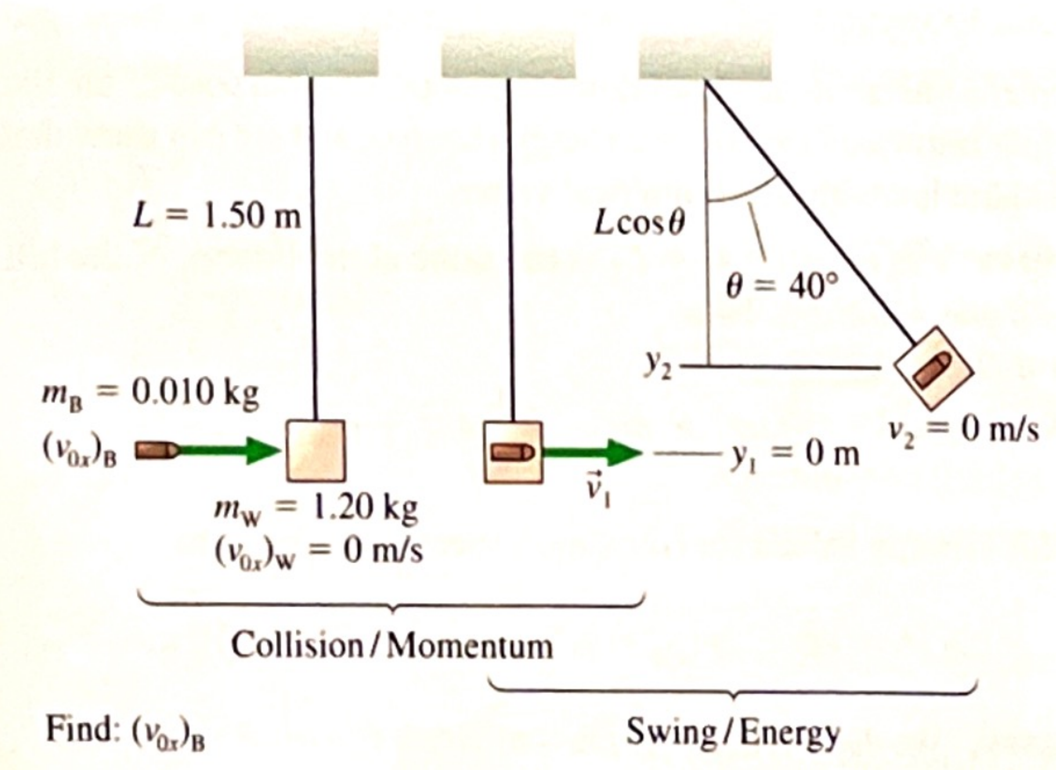
The Basic Energy Model

- Kinetic energy is associated with the motion of a particle
- Potential energy is associated with its position.
- Kinetic energy can be transformed into potential energy, and potential energy can be transformed into kinetic energy.
- Under some circumstances the mechanical energy $E_{\text{mech}} = K + U$ is conserved.

Example #1 (Ballistic Pendulum)

A 10g bullet is fired into a 1200g wood block hanging from a 150cm-long string. The bullet embeds itself into the block, and the block then swings out to an angle of $40°$. What was the speed of the bullet?

FIGURE 10.13 A ballistic pendulum is used to measure the speed of a bullet.



Some things to notice:

First, the bullet and the wood block undergo an inelastic collision. They “stick together”. This is a *conservation of momentum* problem.

Once the bullet is embedded in the block, they both act like a single object.

Next the combined object “goes against” gravity, indicating that its kinetic energy is being converted to gravitational potential energy. This is a *conservation of energy* problem.

Both concepts must be used here!

Example #1 (Ballistic Pendulum)

Part 1: Momentum

Consider the "before" and "after" of the collision. Remember, the initial velocity of the bullet ($v_{i,B}$) is needed.

The objects are separate "before" but combined "after". Thus, they will have a common velocity, v_f.

$$m_B v_{i,B} + m_W v_{i,W} = (m_B + m_W) v_f$$

Since $v_{i,W}$ is zero,

$$v_{i,B} = \left(\frac{m_B + m_W}{m_B}\right) v_f$$

We don't know v_f!

Part 2: Energy

The initial speed of the energy problem is the final speed of the momentum problem!

Assuming the system is isolated, we know that energy will be conserved. Thus, the "before" and "after" energies of the combined object are:

$$K_i + U_i = K_f + U_f$$

The block begins at the vertical origin ($U_i = 0\mathrm{J}$) and "ends" (at rest) higher than where it started ($K_f = 0\mathrm{J}$).

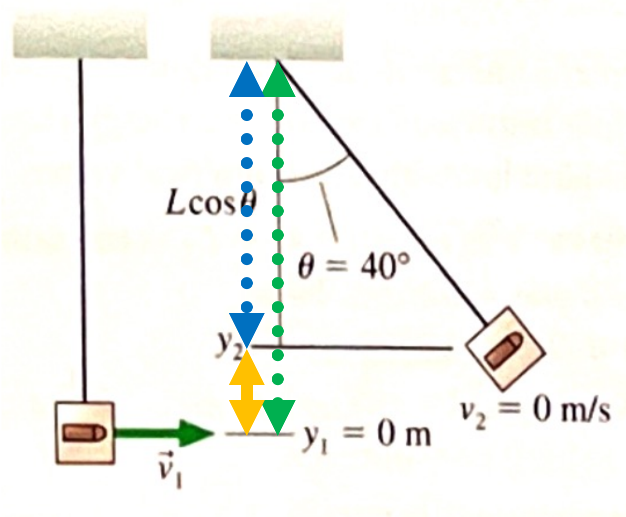
$$\frac{1}{2}(m_B + m_W) v_i^2 = (m_B + m_W) g y_f$$

$$v_i = \sqrt{2 g y_f}$$

Example #1 (Ballistic Pendulum)

We now have an equation to determine the initial speed of the bullet:

$$v_{i,B} = \left(\frac{m_B + m_W}{m_B}\right)\sqrt{2gy_f}$$



The last thing: How do we determine the gained height (y_f)? (See diagram)

$$\text{String Length} - \text{Final Height} = \text{Height Gained}$$

$$y_f = L - L\cos\theta = L(1-\cos\theta)$$

$$v_{i,B} = \left(\frac{m_B + m_W}{m_B}\right)\sqrt{2gL(1-\cos\theta)}$$

$$v_{i,B} = \left(\frac{0.01\text{kg} + 1.2\text{kg}}{0.01\text{kg}}\right)\sqrt{2\left(9.8\frac{\text{m}}{\text{s}^2}\right)(1.5\text{m})(1-\cos 40°)} = 320\,\frac{\text{m}}{\text{s}}$$

Summary

- Kinetic energy is associated with the motion of a particle
  - $K = \frac{1}{2}mv^2$
- Potential energy is associated with its position.
  - $U = mgy$
- Kinetic energy can be transformed into potential energy, and potential energy can be transformed into kinetic energy.
- Under some circumstances the mechanical energy $E_{\mathrm{mech}} = K + U$ is conserved.