LECTURE 4.3 – RELATIVE MOTION

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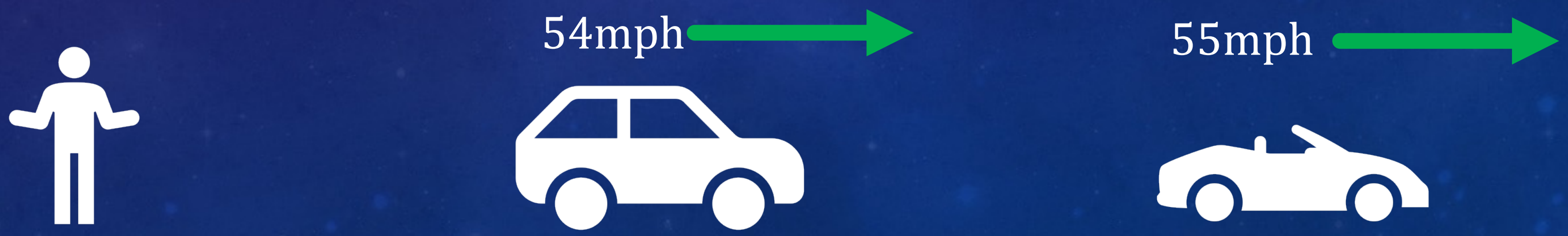
RELATIVE MOTION

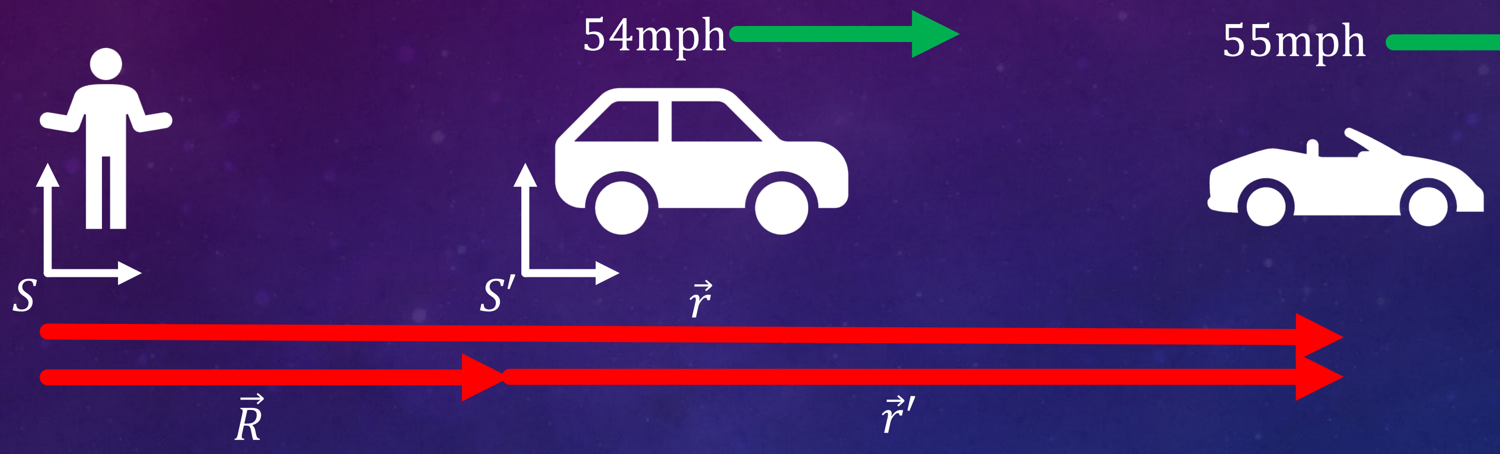
Let’s consider motion in two different reference frames. Suppose a sportscar and minivan pass a pedestrian standing on the sidewalk. The sportscar is maintaining a constant speed of 55mph and the minivan is trying to keep up. However, the minivan isn’t quite fast enough and can only maintain a speed of 54mph.

While the pedestrian sees the cars travel away at 55mph and 54mph respectively, the driver of the minivan sees something much different.

The driver of the minivan sees the distance between him and the sportscar slowly grow. It appears that the sportscar is moving away at 1mph.



REFERENCE FRAMES



- The pedestrian's reference frame, S, is stationary.
- The minivan's reference frame, S', is moving along with the driver.
- In the pedestrian's reference frame, the location of the sportscar is identified by $\vec{r}$.
- We can also write $\vec{r}$ as a combination of $\vec{R}$ and $\vec{r}'$. $\vec{r} = \vec{R} + \vec{r}'$
- $\vec{R}$ represents the position of the traveling reference frame, S', with respect to the pedestrian.
- $\vec{r}'$ represents the position of the sportscar in the *traveling* reference frame.

REFERENCE FRAMES

- A *reference frame* is a coordinate system where an experimenter makes *position* and *time* measurements of physical events.

- Two reference frames can be moving relative to each other with velocity $\vec{V} = \frac{d\vec{R}}{dt}$.

- For our work:
  - The frames are oriented the same way
  - $t = 0s$ is the time origin for both frames
  - $\vec{V} = \frac{d\vec{R}}{dt} = \text{constant}$ (constant velocity)

- Reference frames that move with constant speed in a straight line are called *inertial reference frames*.

A SIMPLE EXAMPLE

Two different groups observe a flash of light and measure its position in their respective reference frames.

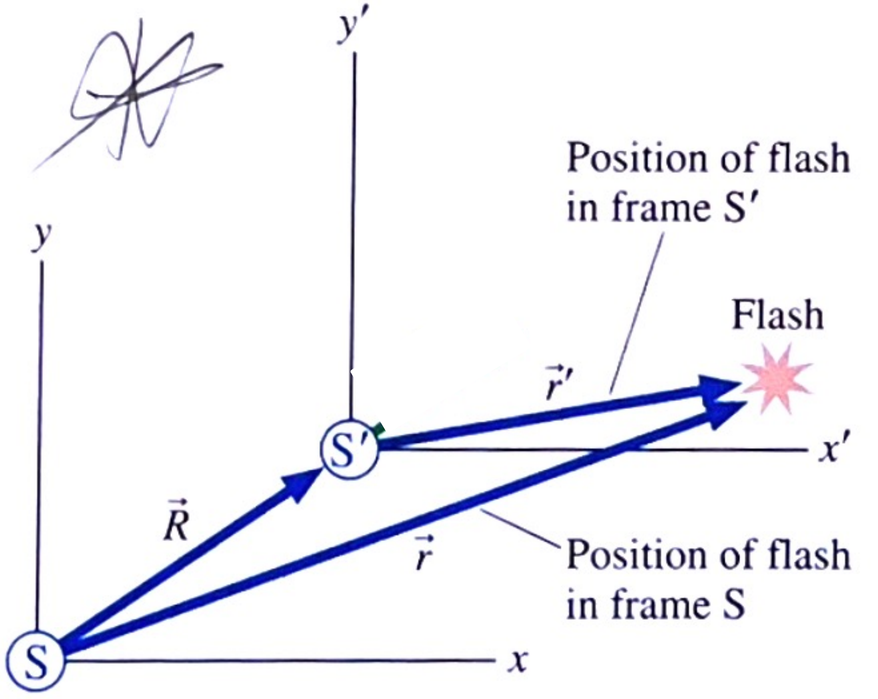
Group S measures the position of the flash at $\vec{r} = x\hat{\imath} + y\hat{\jmath}$

Group S', which is standing much closer to the flash, measures it at $\vec{r}' = x'\hat{\imath} + y'\hat{\jmath}$

The relationship between the positions of S and S' is related by $\vec{R}$.

Thus, $\vec{r} = \vec{R} + \vec{r}'$

FIGURE 4.26 Measurements made in frames S and S′.



A SIMPLE EXAMPLE

But what if S' is moving!?

$$\vec{r} = \vec{r}' + \vec{R}$$

$\vec{r}$ will NOT change over time

$\vec{R}$ will change over time

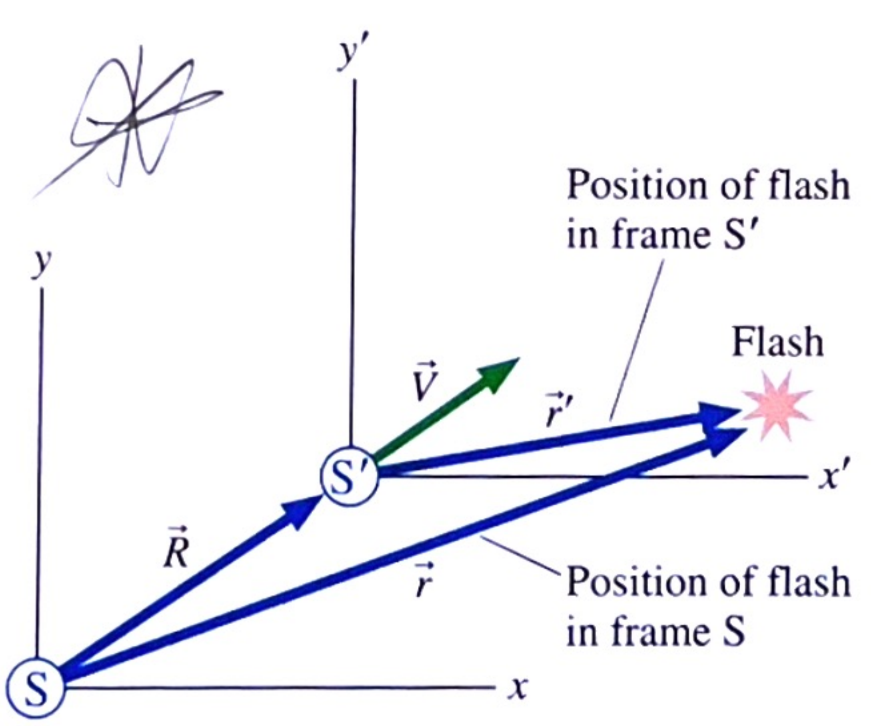
$\vec{r}'$ will NOT change over time

So,

$$\vec{r} = \vec{r}' + \vec{V}t$$

This is called the *Galilean transformation of position*.

FIGURE 4.26 Measurements made in frames S and S′.



If we know *where* and *when* an event occurred in one reference frame, we can *transform* that position into any other reference frame that moves relative to the first with constant $\vec{V}$.

ANOTHER TRANSFORMATION

The rate of change of an object's position is, of course, given by $\frac{d\vec{r}}{dt}$.

Starting with $\vec{r} = \vec{r}' + \vec{R}$, we see that:

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d\vec{r}'}{dt} + \frac{d\vec{R}}{dt} = \frac{d\vec{r}'}{dt} + \frac{d}{dt}\left(\vec{V}t\right) = \vec{v}' + \vec{V}$$

This is called a *Galilean transformation of velocity*.

$\vec{v}$ and $\vec{v}'$ are the velocities of objects, $\vec{V}$ is the velocity of the reference frame.

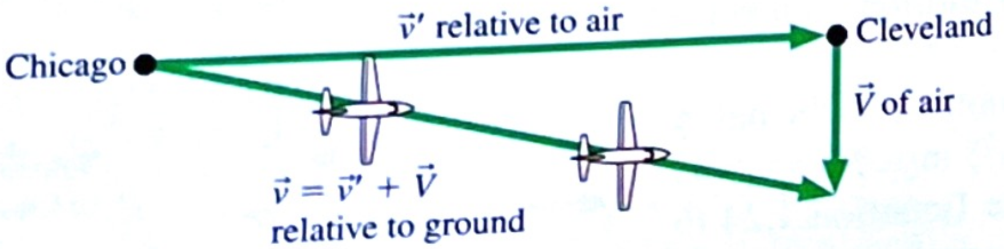
EXAMPLE #1

EXAMPLE 4.9 Flying to Cleveland I

Cleveland is 300 miles east of Chicago. A plane leaves Chicago flying due east at 500 mph. The pilot forgot to check the weather and doesn't know that the wind is blowing to the south at 50 mph. What is the plane's ground speed? Where is the plane 0.60 hour later, when the pilot expects to land in Cleveland?

MODEL Let the earth be reference frame S. Chicago and Cleveland are at rest in the earth's frame. Let the air be frame S′. If the x-axis points east and the y-axis north, then the air is moving with respect to the earth at $\vec{V} = -50\hat{j}$ mph. The plane flies in the air, so its velocity in frame S′ is $\vec{v}' = 500\hat{\imath}$ mph.

FIGURE 4.30 The wind causes a plane flying due east in the air to move to the southeast relative to the earth.



Velocity relative to ground:

$$\vec{v} = \vec{v}' + \vec{V} = [(500\text{mph})\hat{\imath} + (0\text{mph})\hat{\jmath}] + [(0\text{mph})\hat{\imath} + (-50\text{mph})\hat{\jmath}]$$

$$\vec{v} = (500\text{mph})\hat{\imath} - (50\text{mph})\hat{\jmath}$$

Ground speed:

$$v = \sqrt{v_x^2 + v_y^2} \approx 502\text{mph}$$

Faster than the reported speed of the aircraft!

Location after 0.6hr?

$$x = v_x t = (500\text{mph})(0.60\text{hr}) = 300\text{mi}$$

$$y = v_y t = (-50\text{mph})(0.60\text{hr}) = -30\text{mi}$$

30mi South of Cleveland!

Actual heading of the plane was $\theta = \tan^{-1}\left(\frac{V}{v}\right) = 5.72°$ South of East.

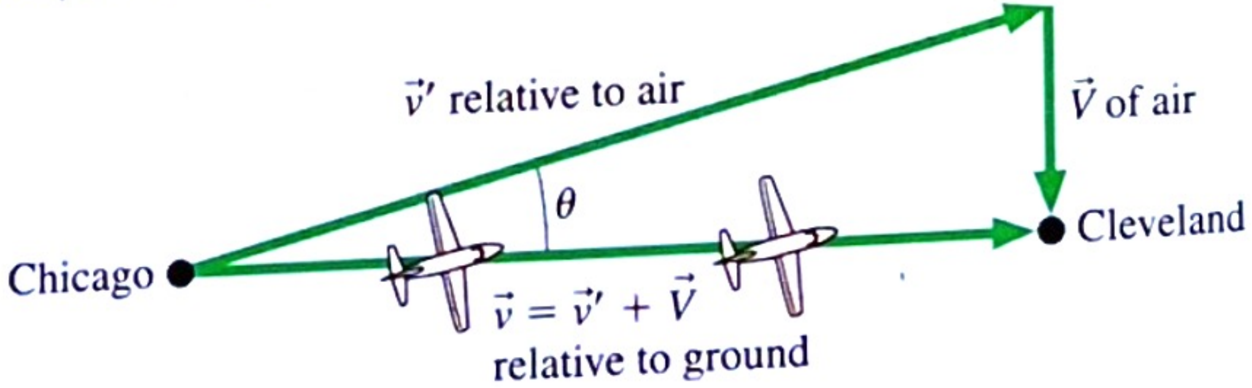
EXAMPLE #1 (CORRECTED FLIGHT PATH)

EXAMPLE 4.10 Flying to Cleveland II

A wiser pilot flying from Chicago to Cleveland on the same day plots a course that will take her directly to Cleveland. In which direction does she fly the plane? How long does it take to reach Cleveland?

MODEL Let the earth be reference frame S. Let the air be frame S′. If the x-axis points east and the y-axis north, then the air is moving with respect to the earth at $\vec{V} = -50\hat{j}$ mph.

FIGURE 4.31 To travel due east in a south wind, a pilot has to point the plane somewhat to the northeast.



What is required?

$$\vec{v} = v_x\hat{\imath} + v_y\hat{\jmath} = (500\text{mph} \cdot \cos\theta)\hat{\imath} + (0\text{mph})\hat{\jmath}$$

Direct path to Cleveland as seen by Chicago/Cleveland

$$v_x = v'_x + V_x = 500\text{mph} \cdot \cos\theta$$

$$v_y = v'_y + V_y = 500\text{mph} \cdot \sin\theta - 50\text{mph} = 0\text{mph}$$

$$\theta = \sin^{-1}\left(\frac{50\text{mph}}{500\text{mph}}\right) = 5.74°$$

$$t = \frac{d}{v_x} = \frac{300\text{mi}}{500\text{mph} \cdot \cos 5.74°} = 0.604\text{hr}$$