Lecture 1.1
Units, Conversions, Significant Figures, Scientific Notation

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Units

- Traditionally, most students beginning a physics course find the topic of units and conversions “tedious” and “boring”; however, good knowledge of this topic is essential for understanding (and communicating about) the world around us.

- Understanding the natural world and built environment relies on experimental measurements. These measurements may be obtained in different ways or different systems. Therefore, it is crucial to understand these systems and connect between them.

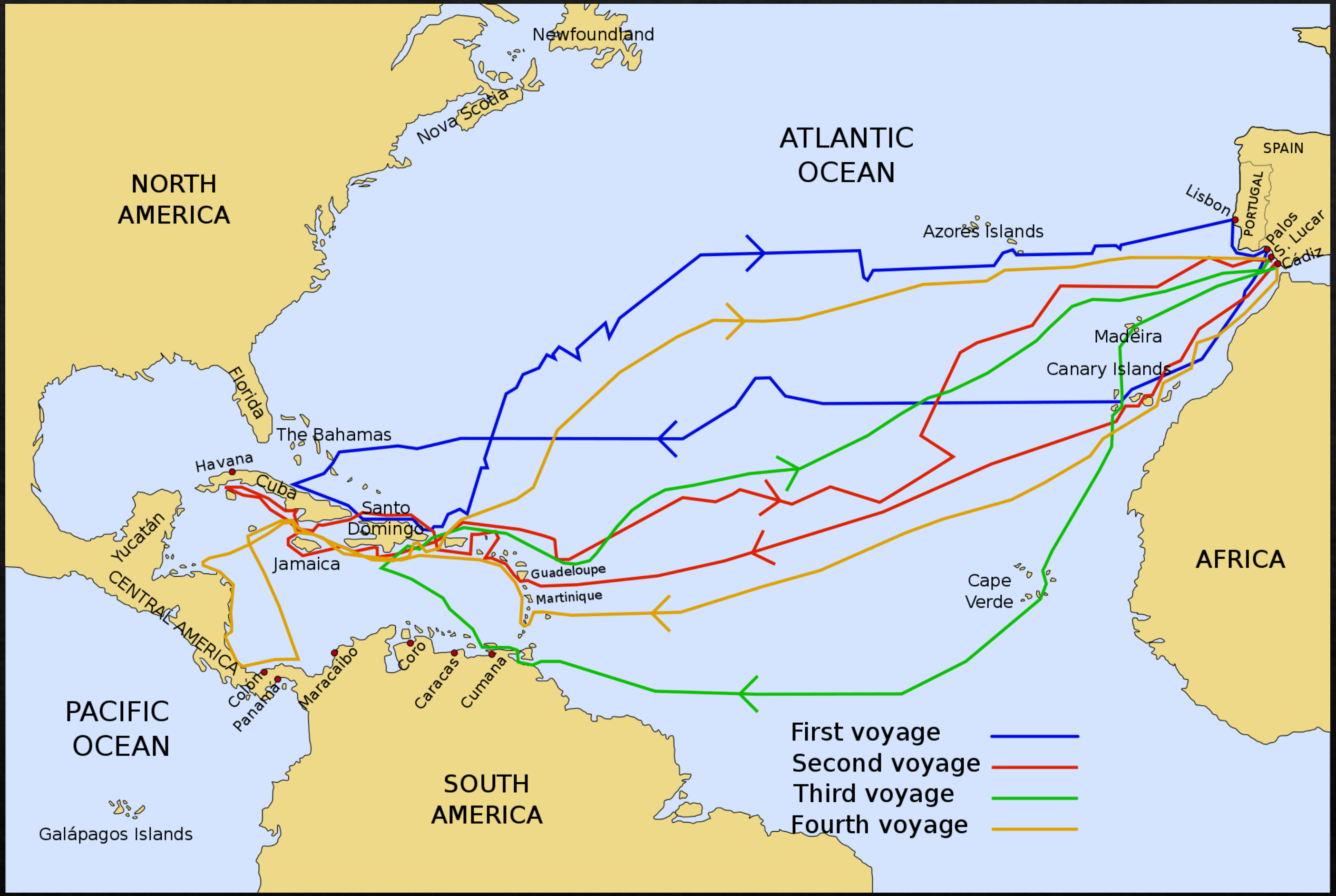
Examples of Measurements Gone Wrong

- Christopher Columbus’s voyage to “India” (1490’s)
- The Swedish Warship “Vasa” (1628)
- Air Canada emergency landing (1983)
- The Mars Climate Orbiter crash (1998)

Humble Pi: When Math goes Wrong in the Real World
M. Parker, 2019

Christopher Columbus (1490's)

- Columbus calculated the distance from Spain to Asia using Italian miles instead of Arab miles, making his calculated distance much shorter than the actual distance. Luckily, he encountered the Americas on his voyage.



The Vasa (1628)

- The Vasa was built using two different rulers: the Swedish-feet ruler and the Amsterdam-feet ruler. The difference between these systems are slightly different. The accidental use of two different systems caused the ship to be asymmetric. Thus, it sank almost immediately.

https://www.nytimes.com/2023/04/08/world/europe/sweden-vasa-warship-woman.html

Air Canada Emergency Landing (1983)

- An Air Canada flight was scheduled to fly from Montreal to Edmonton with a stop in Ottawa. Ground maintenance in Montreal performed fuel calculations in pounds instead of Kilograms which left the plane drastically under-fueled. Luckily, the flight made it to Ottawa where the same error calculation happened again! This time, the aircraft ran out of fuel before making it to Edmonton. It landed at an abandoned military base in the town of Gimli.

https://admiralcloudberg.medium.com/a-mathematical-miracle-the-story-of-air-canada-flight-143-or-the-gimli-glider-9e99545d9b3d

The Mars Climate Orbiter (1998)

- The Mars Climate Orbiter, launched by NASA, was supposed to fly through the upper atmosphere of Mars. However, a unit conversion error recorded thrust data in pounds instead of Newtons (Imperial vs. metric). This caused the spacecraft to approach too close to Mars and burn up in the atmosphere.

https://news.cornell.edu/stories/1998/12/two-nasa-spacecraft-launches-one-mars-and-other-research-birth-stars-involve

The Metric System

- Much of science and engineering uses *Le Systéme Internationale d'Unités* (SI units).

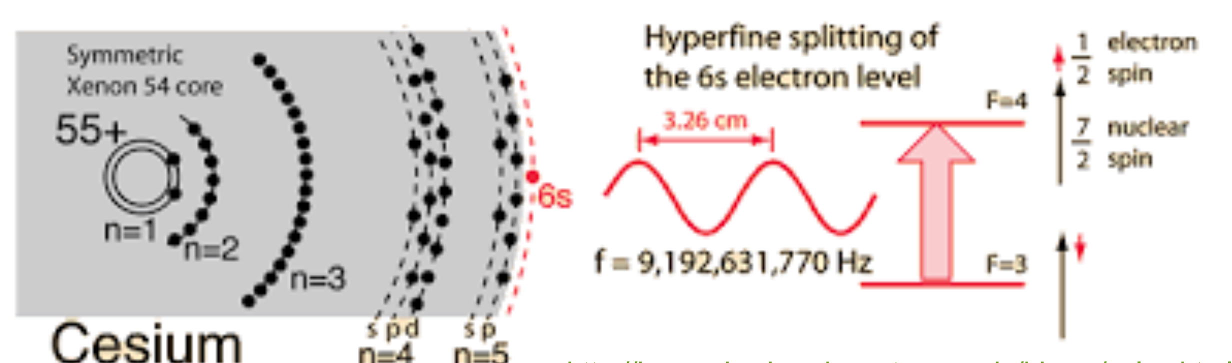
- There are seven base units in the SI system. The three that we will deal with first are time, length, and mass.

Base Unit	*Symbol*	Description
Second	*s*	Time
Meter	*m*	Length
Kilogram	*kg*	Mass
Ampere	*A*	Electric current
Kelvin	*K*	Temperature
Mole	*mol*	Amount of a substance
Candela	*cd*	Luminous intensity

Time [second (s)]

- Last definition in 1967

- The time required for 9,192,631,770 oscillations of the radiowave absorbed by the cesium-133 atom.



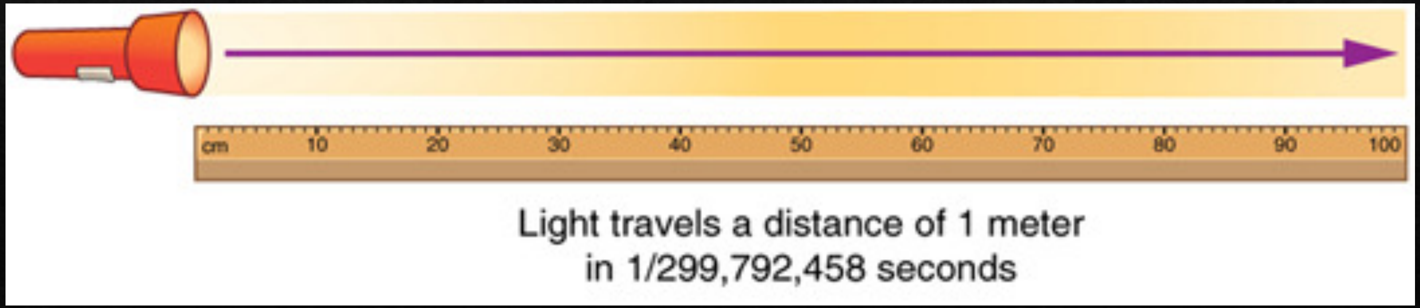
http://hyperphysics.phy-astr.gsu.edu/hbase/acloc.html

History: https://www.bbc.com/news/magazine-29476893

Length [meter (*m*)]

- Last definition in 1983

- The distance traveled by light *in a vacuum* during 1/299,792,458 of a second.



https://pressbooks.bccampus.ca/humanbiomechanics/chapter/1-2-physical-quantities-and-units-2/

Extra: https://spark.iop.org/speed-light-connects-time-and-distance#:~:text=The%20speed%20of%20light%20is,travels%20in%201299%2C792%2C458%20second

Mass [kilogram (*kg*)]

- The mass of the international standard kilogram, a polished platinum-iridium cylinder stored in Paris. *The kilogram is the only SI unit still defined by a manufactured object.*

- The standard kilogram appears to be losing mass!

Mass loss: https://www.nbcnews.com/id/wbna20744160

Unit Prefixes

- It is often the case that measurements of length, time, or mass are much smaller/larger than the standard unit. In this case, prefixes are used to denote powers of 10.

- For example, if we wanted to talk about the circumference of the Earth (~40 million meters), we may use a prefix designated for large values:
  - $1\ kilo\mathrm{meter} = 1{,}000\mathrm{m} = 1\times10^{3}\mathrm{m}$
  - $1\ Mega\mathrm{meter} = 1{,}000{,}000\mathrm{m} = 1\times10^{6}\mathrm{m}$

$$C_{earth} = \frac{40{,}000{,}000\mathrm{m}}{1{,}000} = \frac{40{,}000{,}000\mathrm{m}}{1\times10^{3}} = 40{,}000\mathrm{km}$$

$$C_{earth} = \frac{40{,}000{,}000\mathrm{m}}{1{,}000{,}000} = \frac{40{,}000{,}000\mathrm{m}}{1\times10^{6}} = 40\mathrm{Mm}$$

- Note that the exponent indicates the number of decimal places “absorbed” by the prefix.

Unit Prefixes

- A whole number without a prefix may become a decimal when a prefix is added.
  - For example, consider 1,247m.

$$1{,}247\text{m} = \frac{1{,}247\text{m}}{1000} = \frac{1{,}247\text{m}}{1\times 10^3} = 1.247\text{km}$$

$$1{,}247\text{m} = \frac{1{,}247\text{m}}{1{,}000{,}000} = \frac{1{,}247\text{m}}{1\times 10^6} = 0.001247\text{Mm}$$

Prefix	Meaning	Abbreviation
femto-	10^{-15}	f
pico-	10^{-12}	p
nano-	10^{-9}	n
micro-	10^{-6}	u
milli-	10^{-3}	m
centi-	10^{-2}	c
kilo-	10^{3}	k
Mega-	10^{6}	M
Giga-	10^{9}	G
Terra-	10^{12}	T

Unit Prefixes

- Quantities given with prefixed units must be converted to base SI units! Otherwise, calculations may be incorrect.
- Other systems of measurement must be converted to SI units before calculations can take place.

Unit Conversions

- Example: Given that $1 \text{ mile} = 1.609\text{km}$, convert 1,247m to miles
  - Notice that there are two relationships that must be dealt with: meters/kilometers and kilometers/miles
    - Relationship #1: $1\text{km} = 1{,}000\text{m}$
      - Divide both sides of the relationship: $\frac{1\text{km}}{1{,}000\text{m}} = \frac{1{,}000\text{m}}{1{,}000\text{m}} = 1.$
    - Relationship #2: $1 \text{ mile} = 1.609\text{km}$
      - $\frac{1\ mile}{1.609km} = 1$
  - Since both relationships are equal to 1, we can multiply the foreign quantity by them without changing its value.

$$1{,}247\text{m}\times\left(\frac{1\text{km}}{1{,}000\text{m}}\right)\times\left(\frac{1 \text{ mile}}{1.609\text{km}}\right) = 1{,}247\cancel{\text{m}}\times\left(\frac{1\cancel{\text{km}}}{1{,}000\cancel{\text{m}}}\right)\times\left(\frac{1 \text{ mile}}{1.609\cancel{\text{km}}}\right) = \left[1{,}247\times\left(\frac{1}{1{,}000}\right)\times\left(\frac{1}{1.609}\right)\right] \text{ miles}$$

Units work like algebraic quantities: $\frac{a}{ab} = \frac{1}{b}$

"Fence Post" Notation

- One method of unit conversion is "fence post" notation. Here, conversion factors are "blocked off" for easy reading.

$$1{,}247\text{m}\times\left(\frac{1\text{km}}{1{,}000\text{m}}\right)\times\left(\frac{1\text{ mile}}{1.609\text{km}}\right)$$

$$\begin{array}{c|c|c} 1{,}247\cancel{\text{m}} & 1\cancel{\text{km}} & 1\ \underline{\text{mile}} \\ \hline 1 & 1{,}000\cancel{\text{m}} & 1.609\cancel{\text{km}} \end{array} = \frac{\text{<Multiply Across>}}{\text{<Multiply Across>}} = \frac{1{,}247 * 1 * 1\text{ miles}}{1*1{,}000*1.609} = \frac{1{,}247\text{ miles}}{1.609} = 0.77501554\text{ miles} \approx 0.78\text{ miles}$$

- Important Question: How many decimal places must we include?

Significant Figures

- It is important in science and engineering to not inadvertently include more accuracy in calculations than what is available.

- Significant figures must be considered (digits that are reliably known).

- The appropriate number of significant figures for a calculation is determined by the data provided.

Significant Figures

Scientific measurements are reported so that every digit is certain except the last, which is estimated. All digits of a measured quantity, including the certain one, are called ***significant figures***.

Counting Significant Figures	Examples
1. All non-zero digits are **always significant**.	1.54 (3 sig. figs.) 45 (2 sig. figs.)
2. Interior zeros (zeros between nonzero numbers) are **significant**.	0.025**0**3 (4 sig. figs.) 4**0**2 (3 sig. figs.) 3.**00**674 (6 sig. figs.)
3. Leading zeros (zeros at the beginning of a number) are **NOT significant.**	0.103 (3 sig. figs.) 0.000002 (1 sig. fig.)
4. Trailing zeros (zeros at the end of the number): ✓ are **significant** *if and only if* there is a decimal point present in the number OR they carry overbars. ✓are **NOT** significant otherwise.	1.**050** (4 sig. figs.) 1.**00** x 10^3 (3 sig. figs.) 1**0** (2 sig. figs.) 1***000*** (1 sig. fig.) 19***0*** (2 sig. figs.)
5. Exact numbers have an unlimited number of significant figures.	10 dm = 1m (unlimited sig. figs.)

https://www.montgomerycollege.edu/_documents/academics/support/learning-centers/ackerman-learning-center-rockville/significant-figures.pdf

Rules for Significant Figures

- Multiplication and Division
  - The number of *significant figures* in the answer should match the number of *significant figures* of the least precisely known number used in the calculation.

- Addition and Subtraction
  - The number of *decimal places* in the answer should match the smallest number of *decimal places* of any number used in the calculation.

- It is acceptable to keep extra digits during calculations as long as the answer is reported with the appropriate number of significant figures.

Significant Figures

How many significant figures are in each term?

a) 34.620 (5)

b) 5010.0 (5)

c) 0.05407 (4)

d) 2.047×10^5 (4)

Read the number from right to left. The first non-zero number is the first significant figure.

Simplify each expression with the correct number of significant figures.

a) $5032 - 45.6 + 96.77$ 5083

b) 57×7.368 4.2×10^2

c) $8.578 / 4.33821$ 1.977

Putting it all together…

Alice and Bob recently purchased a small rectangular piece of property and would like to know its area and perimeter. Alice quickly measures one side to be 32ft. Bob measures the longer side with more care, determining it to be 18.1m. (Be mindful of significant figures!)

- Determine the area of the property
  - First, convert Alice's measurement to SI units

$$32\text{ft} \cdot \left(\frac{12\text{in}}{1\text{ft}}\right) \cdot \left(\frac{2.54\text{cm}}{1\text{in}}\right) \cdot \left(\frac{1\text{m}}{100\text{cm}}\right) = 9.7536\text{m}$$

  - Next, multiply: $\text{Area} = \text{A} \times \text{B} = 9.7536\text{m} \times 18.1\text{m} = 176.54016 \approx 180\text{m}^2$

- Determine the perimeter of the property
  - Add up the sides: $2\text{A} + 2\text{B} = 2(9.7536\text{m}) + 2(18.1\text{m}) = 55.7072\text{m} \approx 56\text{m}$

Summary

- Units
  - The importance of units
  - Prefixes
  - Unit Conversions
- The SI system
  - Base units
- Significant Figures