Hypothesis Testing with Two Samples

MAT 152 – STATISTICAL METHODS I

LECTURE 1

INSTRUCTOR: DUSTIN ROTEN

FALL 2020

Revisiting a Previous Example

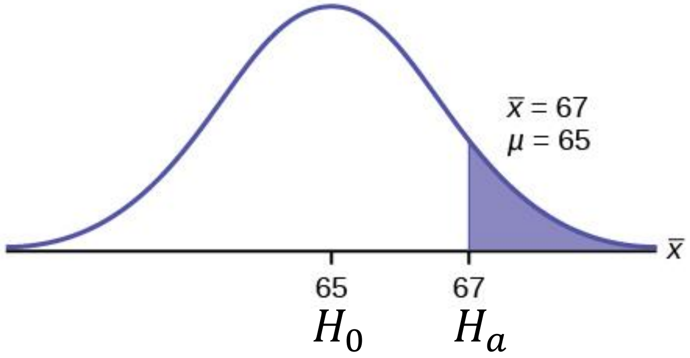
Statistics students believe that the mean score on the first statistics test is 65. A statistics instructor thinks the mean score is higher than 65. He samples 10 statistics students and obtains the score. He performs a hypothesis test using a 5% significance level. (Since the data are grades, they are assumed to be normally distributed.) The statistics from the sample are: $\bar{x} = 67$ and $s \approx 3.2$.

$H_0: \mu = 65$

$H_a: \mu > 65$

$\bar{X}$ = mean score of the first statistics test

$\alpha = 0.05$



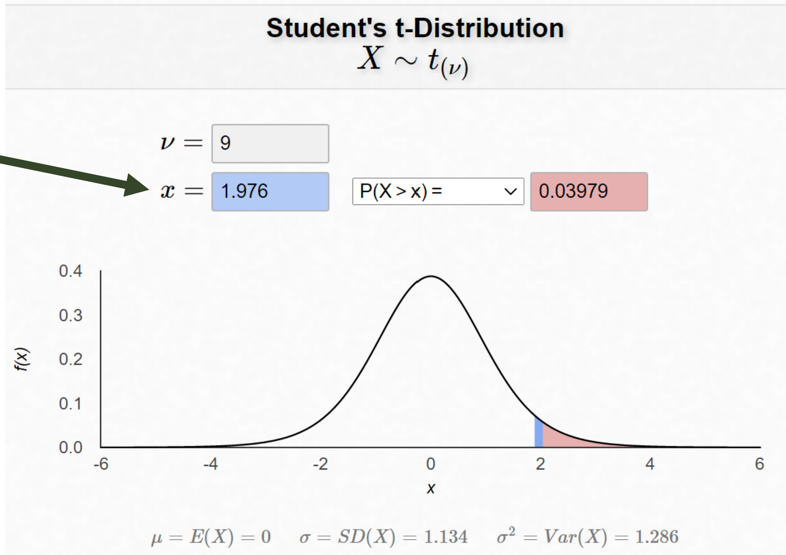
No population standard deviation is given, so a t-Distribution is required.

Revisiting a Previous Example (Cont.)

Remember, we first want to assume that $H_0: \mu = 65$ is true. How likely is the occurrence of our sample (and other samples like it)? $P(\bar{x} > 67)$?

We need the t-score of our value under the assumption that H_0 is true.

$$t = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}} = \frac{67-65}{\frac{3.2}{\sqrt{10}}} \approx 1.976$$



$P(\bar{x} > 67) = 0.03979$ so if the mean really is 65, then there is only a 3.979% chance of our sample occurring.

Since $p < \alpha$ $(0.03979 < 0.05)$ we REJECT H_0.

Conclusion:

At a 5% significance level, the sample data show sufficient evidence that the mean test score is more than 65.

Hypothesis Testing with Two Samples

One-sample hypothesis testing is great for testing claims about a population mean or proportion; however, **studies often compare two groups**.

The process for two-sample hypothesis testing of two groups follows similar logic as the one-sample method.

The two groups can be:

Independent - Samples selected from one population are not related in any way to sample values selected from the other population.

Matched pair - Two samples that are dependent. (Before and after comparison)

Paired Samples

In a hypothesis test for matched or paired samples, subjects are matched in pairs and **differences** are calculated. These differences are used to conduct a t-test.

The new t-score:

$$t = \frac{\Delta\bar{x} - \Delta\mu}{\frac{\Delta s}{\sqrt{n}}}$$

$$\Delta\bar{x} = \frac{1}{n}\sum_{i=1}^{n}\left(x_{i,1} - x_{i,2}\right)$$

$$\Delta\mu = \mu_1 - \mu_2$$

$$\Delta s = s_1 - s_2$$

Example

A study was conducted to investigate the effectiveness of hypnotism in reducing pain. 8 subjects were randomly selected for the study. Each participant was asked to rate their pain before and after a session. The data are presented below.

Subject:	A	B	C	D	E	F	G	H
Before	6.6	6.5	9.0	10.3	11.3	8.1	6.3	11.6
After	6.8	2.4	7.4	8.5	8.1	6.1	3.4	2.0

Here, the subjects A – H have paired values. Each value in the “Before” group is paired with a value in the “After” group. Now, the differences between the groups must be calculated.

Here, the differences are calculated as After – Before.

Subject:	A	B	C	D	E	F	G	H
Difference	0.2	-4.1	-1.6	-1.8	-3.2	-2	-2.9	-9.6

$\Delta s = 2.91$

$\Delta \bar{x} = -3.13$

Example (Cont.)

A study was conducted to investigate the effectiveness of hypnotism in reducing pain. 8 subjects were randomly selected for the study. Each participant was asked to rate their pain before and after a session.

Random variable: $\Delta\bar{X}$ = the mean difference of the sensory measurements.

Null hypothesis (worse or no difference):

$H_0: \Delta\mu \geq 0$

Alternative hypothesis (hypnotism reduces pain):

$H_a: \Delta\mu < 0$ (left-tailed test)

Example (Cont.)

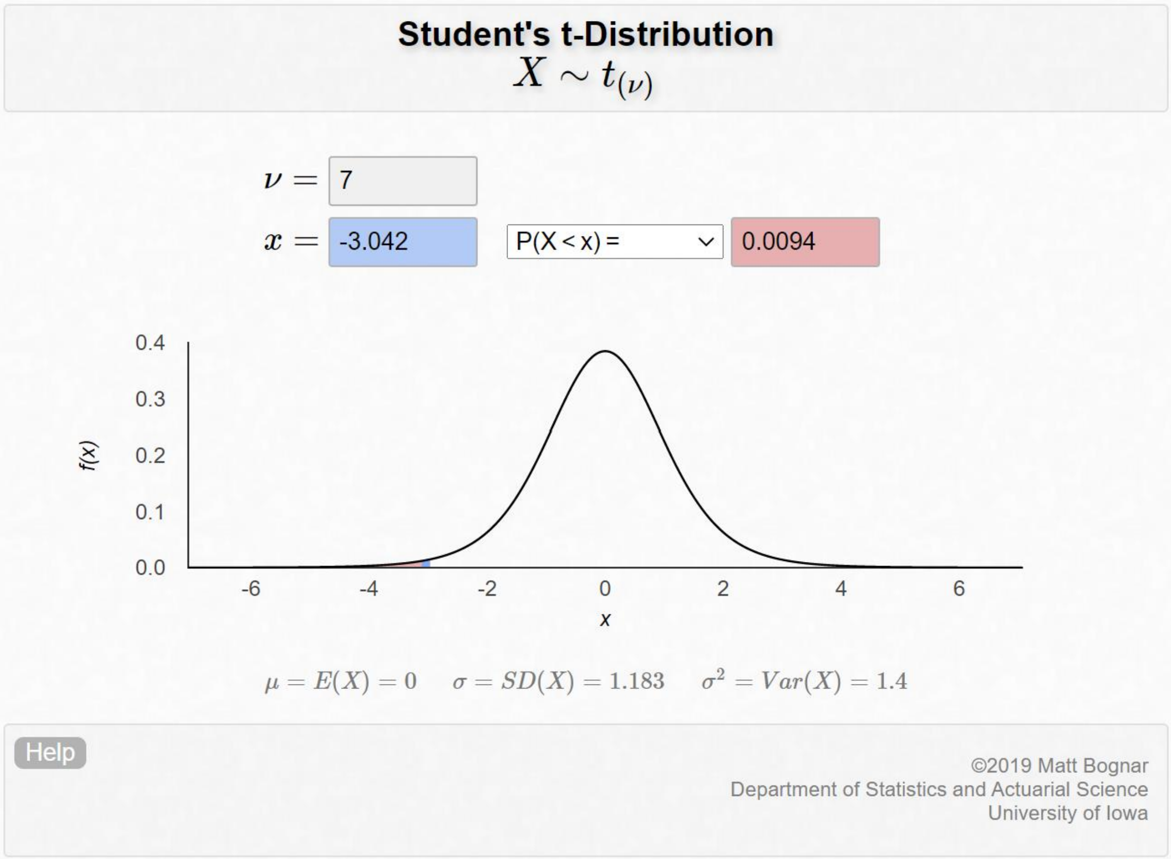
Since the population standard deviation is unknown, a t-distribution must be used.

The degrees of freedom (df or ν) must be determined. $t_\nu = n - 1 = 8 - 1 = 7$.

So, the t-distribution t_7 must be used.

What is the t-score of our value? (We assume the null hypothesis is true so there is no change in the means: $\Delta\mu = 0$.)

$$t = \frac{\Delta\bar{x} - \Delta\mu}{\frac{\Delta s}{\sqrt{n}}} = \frac{-3.13 - 0}{\frac{2.91}{\sqrt{8}}} \approx -3.042$$



Example (Cont.)

A study was conducted to investigate the effectiveness of hypnotism in reducing pain. 8 subjects were randomly selected for the study. Each participant was asked to rate their pain before and after a session.

Using $\Delta\bar{x} = -3.13$ and $\Delta s = 2.91$ from the paired sample, the t-score of $\Delta\bar{x}$ is calculated **under the assumption that the null hypothesis is true**.

The t-score is used to determine the probability of obtaining the differences found in our sample (The p-value is 0.0094)

Assuming $\alpha = 0.05$, we see that the p-value is less than our threshold, so the differences obtained in the study are an extreme event. ($p < \alpha$)

Reject the null hypothesis.

Example (Cont.)

A study was conducted to investigate the effectiveness of hypnotism in reducing pain. 8 subjects were randomly selected for the study. Each participant was asked to rate their pain before and after a session.

In conclusion:

At the 5% level of significance, from the sample data, there is sufficient evidence to conclude that the sensory measurements, on average, are lower after hypnotism. Hypnotism appears to be effective in reducing pain.

Summary of Matched/Paired Samples

When using a hypothesis test for matched/paired samples, the following characteristics should be present:

- Simple random sample is used.
- Sample sizes are often small.
- Two measurements are drawn from the same pair of objects.
- Differences are calculated.
- The differences from the sample that is used for the hypothesis test.
- Either the matched pairs have differences that come from a population that is normal OR the number of differences is sufficiently large.