MAT 2240 - Linear Algebra
Project #1

Directions: Complete the following exercises *after* the accompanying lecture has been given by instructor. A combination of computer work and hand-written work may be required. All solutions should be printed and/or written neatly and submitted.

1. Construct the following augmented matrix in Octave. Use the random integer function, **randi(...)**, to assign values to a, b, c, d, e, and f. All values generated should be > -10 and < 10. (*Note: Be sure you understand all of the possible arguments for the* ***randi()*** *function.*)

$$A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$$

2. Using your generated matrix from above, complete the following:

   (a) Write down the linear system of equations expressed by the matrix.

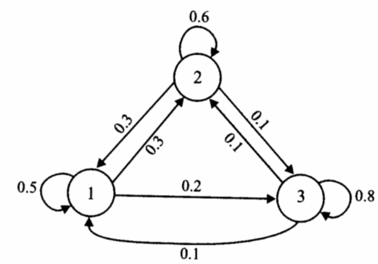
   (b) Does this linear system have one solution, no solution, or an infinite number of solutions? Use **ezplot(...)** to graphically determine the number of solutions by plotting **both** equations in Octave.

   (c) Add appropriate labels to your plot. Include a plot title, x-axis label, y-axis label, and legend.

3. Consider the following two vectors $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\mathbf{v}_2 = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$. Develop a strategy to determine a linear combination of $\mathbf{v}_1$ and $\mathbf{v}_2$ that produces vector $\mathbf{v}_3 = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$. Confirm your solution using Octave.

4. Neutrinos are elementary particles that are abundantly present throughout the universe. Three different types (or "flavors") of neutrinos exist: the muon neutrino, the tau neutrino, and the electron neutrino. Among other places, they are produced in fusion reactions that take place inside stars and the earth's core. A *neutrino oscillation* is a phenomenon where a neutrino with an initial flavor can be measured at a later time to have a different flavor. Spend some time reviewing *Markov Chains*, *state diagrams*, and *transition matrices* and apply your knowledge to the following exercises.

   (a) Suppose the accompanying state diagram represents the probability of a certain neutrino being of the 1) muon, 2) tau, or 3) electron flavor. Construct the corresponding transition matrix.



   (b) Suppose the transition matrix generated above represents repeated, equally-timed measurements. If initially there was a 50% chance of measuring a muon neutrino, a 30% chance of measuring a tau neutrino, and a 20% chance of measuring an electron neutrino ($\mathbf{X}_0 = \begin{bmatrix} 0.5 \\ 0.3 \\ 0.2 \end{bmatrix}$) what would be the most likely state of the neutrino after the first measurement, X_1? Use Octave to determine the most likely state measured during the 10th measurement, X_{10}.