Education

Basic Options Terminology

An **option** is the right, but not the obligation, to buy or sell an underlying asset (in this case, the Phoenix Index) at a specified price on or before a specified date. There are two types of options – the **call**, which gives the right to buy the underlying, and the **put**, which gives the right to sell the underlying. The price of an option is often referred to as the **option premium** to reflect the price paid for the right to transact.

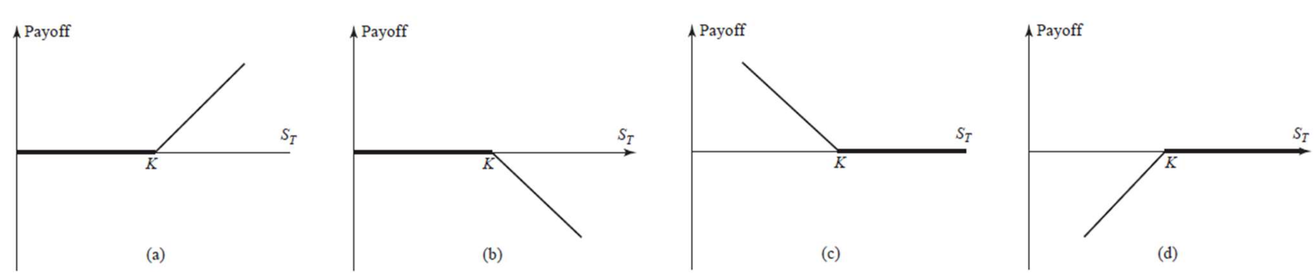
Options have several key parameters apart from their type that define their contract specification:

- **Strike price:** the price at which you are allowed to buy or sell
- **Expiration date:** last date the option exists on; cannot exercise or trade beyond this date
- **Exercise style: American** options allow exercise on or before the expiration date, while **European** options can only be exercised on the expiration date. (Other exercise styles exist, but we will not mention them here.) This case will deal with only European options.
- **Contract unit:** amount of the underlying asset that the options holder receives or has to deliver on exercise. In this case, the contract unit is simply the cash value of the index at expiry.

Exercising an option means choosing to buy the stock at the strike price for a call or choosing to sell the stock at the strike price for a put. An option is referred to as **in-the-money** if exercising it would be profitable (underlying price > strike price for a long call and underlying price < strike price for a long put), **at-the-money** if strike price = underlying price, and **out-of-the-money** otherwise.

The payoff of a long European call at expiry with strike price K and underlying price S_T is $\max\{S_T - K, 0\}$. The payoff of a long European put at expiry with the same parameters is $\max\{K - S_T, 0\}$. The **intrinsic value** of an option is the option's payoff if it expired at the current underlying price S – for a long European call with strike K, $\max\{S - K, 0\}$, and for a long European put with the same strike, $\max\{K - S, 0\}$. Options also can have a **time value** that reflects potential future benefits of holding the option, and the price of the option should reflect the sum of intrinsic value and time value.

Figure 9.5 Payoffs from positions in European options: (a) long call; (b) short call; (c) long put; (d) short put. Strike price = K; price of asset at maturity = S_T.



Source: Hull (2010)

Options Arbitrage Relationships

We can provide theoretical bounds on the prices of European calls and puts using **arbitrage arguments**. To show these relationships hold, we assume that the current price violates the relationship and construct a portfolio that results in a risk-free profit. The **principle of no arbitrage** states that opportunities for a risk-free profit are rarely (if ever) present in the market, so we arrive at a contradiction and so the original price must follow the specified relationship.

Note if these opportunities do exist, they are referred to as **arbitrages**. **Arbitrageurs** hypothetically will take advantage of these opportunities to profit until the mispricing is corrected. In real markets, there may appear to be arbitrage opportunities in the market but trading them is actually unprofitable due to transactions costs. Although you will likely not make money by trying to find these arbitrage opportunities during the case, it is worth thinking through these derivations to gain an understanding of how calls and puts behave. A more extensive discussion with examples can be found in the Appendix. In the Appendix, we use an arbitrage argument to derive the famous **put-call parity** result, which states the following relationship for the prices of a European call c and put p with the same strike K and time to expiry:

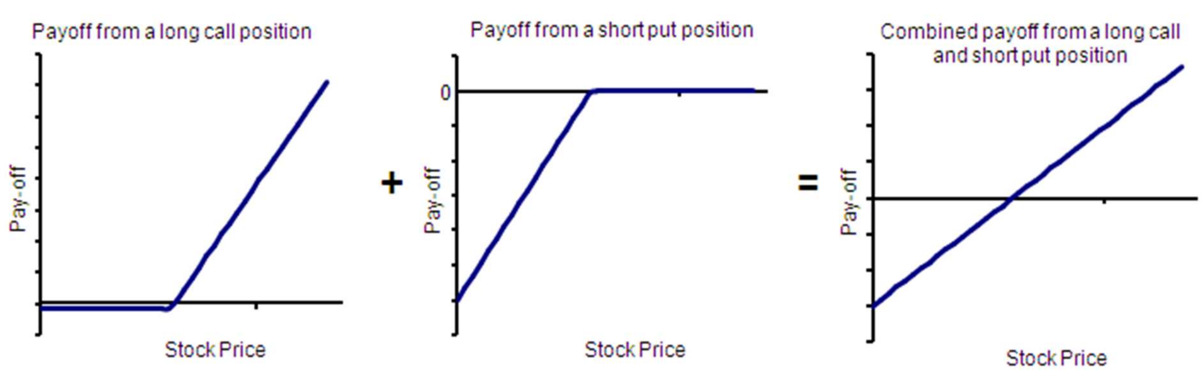
$$c + K = p + S_0$$

or

$$c - p = S_0 - K$$

where S_0 denotes the current underlying price. This shows a no-arbitrage relationship between calls and puts of this nature and also shows how to construct **synthetic calls and puts** from the other three instruments – a portfolio that has the same payoff as a vanilla call or put.

The arbitrages we have used to prove these relationships are examples of **static, model-independent** arbitrages. A **static** arbitrage is one in which positions do not need to be rebalanced, and these arbitrages are **model-independent** since we have not used any mathematical pricing model to determine the price of the instrument. However, as noted above, these types of opportunities are rarely available and are even less commonly profitable. We will need to consider strategies that are **dynamic** and **model-dependent** to take advantage of mispricings, and to do so, we need to build an understanding of options pricing models.



Source: www.riskprep.com

The Black-Scholes-Merton (BSM) Option Pricing Model

The derivation of the BSM model requires relatively advanced mathematics (stochastic calculus and stochastic differential equations) which we do not assume competency in for the purposes of this competition – we outline a more intuitive approach to the derivation in the Appendix and present the results of the derivation here. The fundamental idea of the derivation is creating a portfolio that consists of an option with some value V and some number of shares Δ of the underlying (Δ could be positive or negative) such that no matter how the underlying moves the portfolio maintains the same value. (Such a portfolio is referred to as **delta-hedged**.) This riskless property is only preserved for a very short period of time, and **dynamic hedging** (rebalancing of the underlying) is required to maintain a **delta-neutral** position. By looking at how such a portfolio changes over time we get the **Black-Scholes-Merton differential equation**:

$$\frac{1}{2}\sigma^2 S_t^2 \Gamma + \theta - r(V - \Delta \cdot S_t) = 0$$

Considering the intrinsic value boundary conditions of $c_T = \max\{S_T - K, 0\}$ and $p_T = \max\{K - S_T, 0\}$ at expiry gives the **Black-Scholes-Merton pricing formulae** for European calls and puts:

$$c = S_0 N(d_1) - Ke^{-rT} N(d_2)$$
$$p = Ke^{-rT} N(-d_2) - S_0 N(-d_1)$$
$$d_1 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r + \frac{\sigma^2}{2}\right) T}{\sigma\sqrt{T}}$$
$$d_2 = d_1 - \sigma\sqrt{T}$$

where $N(\cdot)$ denotes the normal(0,1) cumulative density function, S_0 is the current underlying price, K is the strike price, r is the risk-free rate, T is the time to expiry (in years), and σ is the annualized volatility of the underlying.

The Black-Scholes model relies on a wide range of assumptions, some more realistic than others:

- The underlying price is lognormally distributed (continuously compounded returns of the stock are normally distributed) with constant drift and volatility.
- Short selling of securities with full use of proceeds is permitted.
- No transactions costs or taxes; all securities perfectly divisible.
- No dividends during the life of the derivative.
- No riskless arbitrage opportunities.
- Continuous/smooth pricing and trading.
- Risk-free rate of interest is constant and same across maturities.

Two notable assumptions that are commonly violated in the real world are constant volatility and continuous pricing. Be aware of how these assumptions may not be accurate and how that may affect your model of prices.

The Greeks and Volatility

The Greeks describe different risk exposures of options positions and are calculated by taking derivatives of the price given by the BSM model with respect to different parameters. Portfolio Greeks can generally be calculated by summing the Greeks for each position in the portfolio. Some of the Greeks are described below:

- **Delta (Δ)** measures the rate of change of the theoretical option value with respect to changes in the underlying asset's price. This can be mathematically expressed as $\frac{\partial V}{\partial S}$. We note the delta of owning the long underlying is 1, the delta of calls is strictly positive, and the delta of puts is strictly negative. We have seen the delta of an option before as the **hedge ratio** to create a riskless portfolio. Calculating the delta of your portfolio and choosing when to hedge and rebalance with the underlying is essential to success in this case.

- **Gamma (Γ)** measures the rate of change of delta with respect to changes in the underlying asset's price. This can be mathematically expressed as $\frac{\partial \Delta}{\partial S} = \frac{\partial^2 \mathrm{V}}{\partial S^2}$. Gamma is positive for all options, highest for at-the-money options, and is roughly normally distributed with respect to strike. Gamma can be thought of as a measure of the nonlinearity of an option or as how quickly a delta hedge must be adjusted. Note that the underlying does not have any gamma, so a **gamma hedge** requires taking positions in other options contracts. An example of a strategy with high absolute gamma is the **straddle**.

- **Theta (θ)** measures the sensitivity of the value of the option to the change in time (in years). Theta is mathematically expressed as $\frac{\partial V}{\partial t}$. Theta is often known as "time decay" because it reflects the fact that the time value of the option decreases as the option gets closer to expiration. Theta is negative for both puts and calls and is commonly normalized to reflect per day option value decay (divide raw theta by number of days per year). Theta is most negative for at-the-money options, and in a delta-neutral portfolio, theta is a proxy for gamma.

- **Vega (ν)** measures the option's sensitivity to implied volatility (see below). Mathematically, this is $\frac{\partial V}{\partial \sigma}$. Vega is notable because it involves taking a derivative with respect to a parameter of the model as opposed to a variable – we assumed volatility was constant when deriving the BSM model. This turns out to be a decent estimator of vega calculated from a more theoretically sound model, such as a stochastic volatility model. Vega is positive for all options, highest for at-the-money options, and decreases as the option approaches expiry. Note that the underlying does not have any vega, so a **vega hedge** requires taking positions in different options contracts. Portfolios that are gamma neutral are not generally vega neutral, and vice versa, but can be made so by trading two different options contracts on the same underlying.

Volatility is a measure of uncertainty about the returns provided by a stock, usually mathematically defined as the standard deviation of the return provided by the stock in one year when the return is expressed using continuous compounding. **Realized/historical volatility** can be measured by observing the stock price S_i, $i = 0,1,\ldots,n$ over some period T (time between measurements, measured in years), calculating the log return $u_i = \ln\left(\frac{S_i}{S_{i-1}}\right)$ in each period, and taking the standard deviation of the u_i:

$$s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(u_i - \bar{u})^2}$$

Then s is an estimator for the standard deviation of the daily returns $\sigma\sqrt{T}$, so $\hat{\sigma} = s/\sqrt{T}$. (Commonly with daily measurements for a one-year period the number of trading days per year, 252, is used instead of the number of calendar days per year to convert from days to years.)

Implied volatility (IV) is the value of σ that, when fed into the BSM model with the other observable parameters, will output the current observed option premium in the market. Since all other parameters (strike, interest rate, underlying price, time to expiry) are observable and relatively deterministic, estimates of volatility are the main driver of options pricing disagreements for a given instrument, and options prices are often quoted in terms of implied volatilities for this reason. Implied volatility estimates affect only the time value of the option, not the intrinsic value. IV can be numerically found by inverting the Black-Scholes model with the current market price and iteratively searching for the correct σ (for example, with Newton's method) – no closed-form/analytic solution exists to determine σ. IV reflects current market sentiment with regards to future volatility.

A plot of the implied volatility of an option against its strike price (holding time to expiry constant) is referred to as a **volatility smile**. The "smile" name comes from the fact that far out-of-the-money strikes tend to have higher implied volatilities than at-the-money strikes. Using the volatility smile to model options prices assigns a higher probability of extreme moves than the BSM model predicts and thus values options struck at extreme values more than they would be worth using the BSM model. The volatility smile effect arises from real-world violations of the BSM assumptions, namely constant volatility and smooth price changes, which both increase the likelihood of extreme outcomes. Equity options often display a **volatility skew**, where implied volatility decreases as strike increases – the implied distribution has a heavier left tail and a less heavy right tail, implying the probability of extreme downward moves is much higher than predicted by BSM, while extreme upward moves are less likely. Potential explanations for this effect include leverage or fear of stock market crashes.

Successful competitors will be able to create their own models based off of BSM for how volatility should evolve over time and will compare their predictions to current implied volatilities to see if there are any profitable trading opportunities in the market.

Figure 19.1 Volatility smile for foreign currency options.

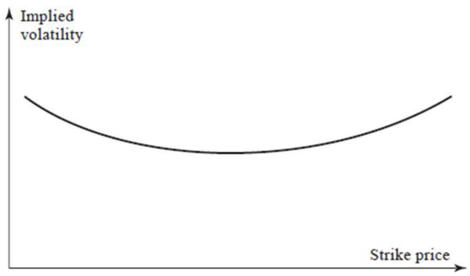


Figure 19.2 Implied and lognormal distribution for foreign currency options.

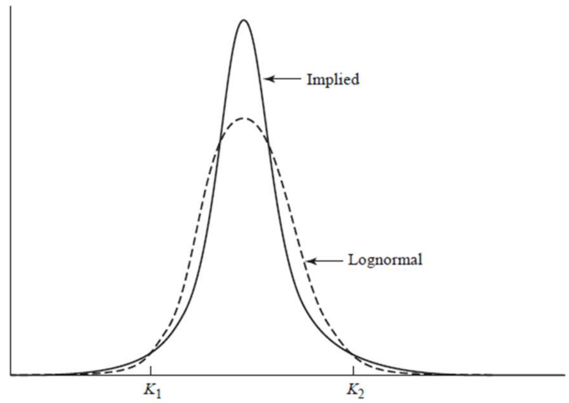


Figure 19.3 Volatility smile for equities.

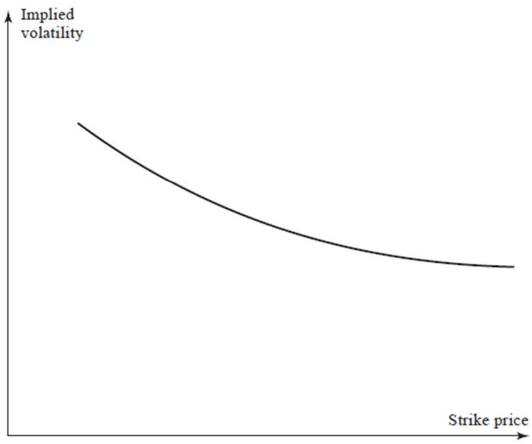
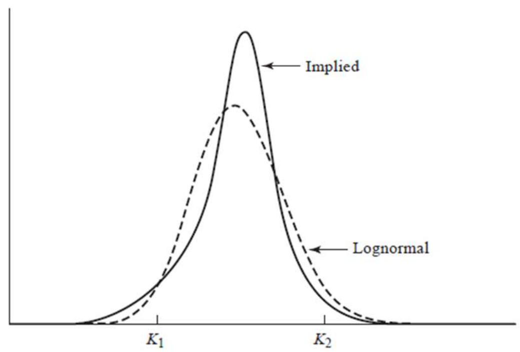


Figure 19.4 Implied distribution and lognormal distribution for equity options.



Source: Hull (2010)

Market-Making Strategy

In our markets for options, there are two types of orders – **market orders** and **limit orders**. **Market orders** request an immediate fill at whatever the prevailing price is (best bid for sales, and best offer for purchases) – these orders are said to **take liquidity** from the market, and these orders pay an implicit fee ("crossing the spread") as a result. **Market makers provide liquidity** in options markets by quoting a **bid** and an **ask** as **limit orders**, a contract that specifies the quantity, price, and direction to be traded. A market maker profits when both the bid and ask are traded against in the same size – in this case, the market maker keeps the spread between the bid and the ask as the profit, with little to no risk. However, in the real world, this does not always happen, and so market makers accumulate **inventory**. Managing this inventory, hedging risk, and adjusting quoted prices to reduce risk are crucial to successful market-making (i.e. making sure the captured spread is not whittled away by market moves). Furthermore, market makers must maintain an idea of where the theoretical price of the asset is and should be ready to adjust that price based on the trades they take part in – if someone is willing to buy 100 calls off a market maker at their listed price, they may need to adjust their theoretical value of those calls up (what does the person trading against them know that they don't?). This is known as **adverse selection** and should be considered when planning a market-making strategy.