



**Politecnico
di Torino**

POLITECNICO DI TORINO
B.Sc. IN ELECTRONIC AND COMMUNICATIONS
ENGINEERING

Final Project

Simulation of a communication system

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Abstract

This project presents a comprehensive study on the simulation of a communication system that transmits and processes two audio signals. Each signal is analyzed in both the time and frequency domains to determine its bandwidth and identify any unwanted disturbance. Two filtering approaches are examined for disturbance removal: a fourth-order Bessel low-pass filter and a narrowband rejection filter with conjugate poles and zeros. Performance is evaluated using the Signal-to-Interference Ratio (SIR), revealing that the choice of filter depends on the relative bandwidth of the signal and the disturbance frequency. Additionally, a multiplexing strategy is implemented to share a channel already occupied by a low-bandwidth baseband signal. By selecting appropriate carrier frequencies and applying amplitude modulation, both audio signals are efficiently transmitted in parallel with minimal interference. The results confirm that careful selection of filter parameters, carrier frequencies, and modulation techniques can greatly improve signal quality, as measured by SIR, while preserving desired audio content.

1 Analysis of Audio Signals

In this section, we present a detailed analysis of two audio tracks to examine their characteristics in both the time and frequency domains. The signals were sampled at a frequency of 44,100 Hz, which is the standard for high-quality audio recordings.

To gain insights into the temporal and spectral properties of the signals, we plotted the waveforms in the time domain, as well as their corresponding frequency spectra. These

visualizations provide a comprehensive view of the amplitude variations over time and the distribution of frequency components present in the audio signals.

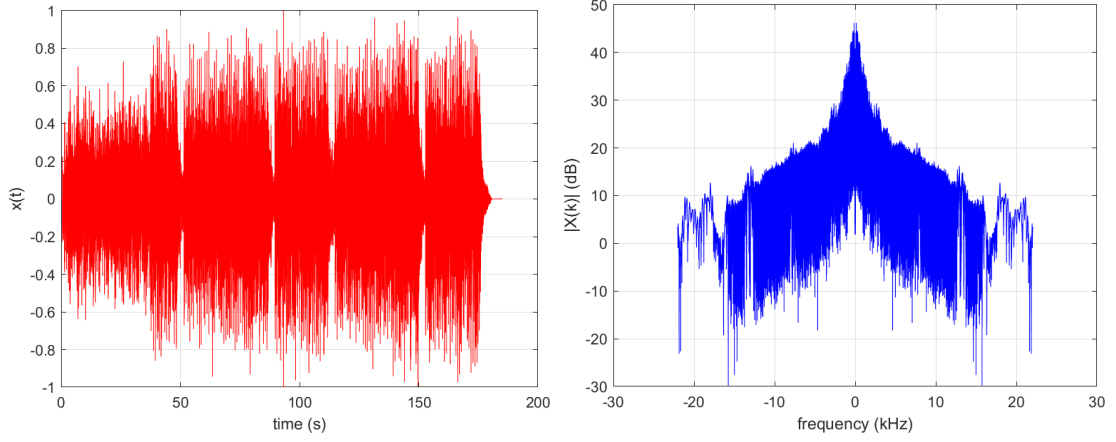


Figure 1: Time and Frequency domain representation of the first song

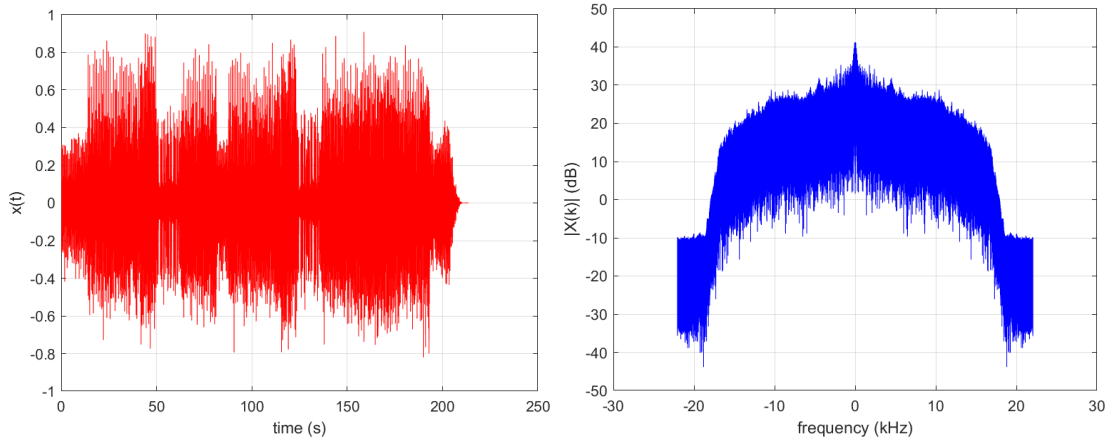


Figure 2: Time and Frequency domain representation of the second song

Bandwidth is defined as the frequency range that contains 95% of the total energy of the signal. This measure provides an effective representation of the spectral content while excluding negligible high-frequency components.

For the first song, the calculated bandwidth is approximately **970 Hz**, indicating a relatively narrow frequency range with concentrated energy. In contrast, the second song exhibits a significantly broader spectral range with a bandwidth of approximately **7270 Hz**. This suggests a more complex harmonic structure and a wider variety of frequency components.

2 Disturbance Analysis and Removal

Both audio signals are affected by a common disturbance. To effectively design a suitable filter for noise removal, a comprehensive analysis of the disturbance is necessary. This involves studying the disturbance in both the time and frequency domains for each signal.

Analysis of disturbance in the first song

The first song is analyzed using both time-domain and spectral analysis techniques to understand the characteristics of the disturbance. This will help us design a filter to attenuate the disturbance.

- **Time-Domain Analysis:** The waveform of the first song is plotted to observe variations in amplitude, which may indicate the presence of the disturbance.
- **Frequency-Domain Analysis:** The Power Spectral Density (PSD) is calculated and visualized to identify the frequency components associated with the disturbance.

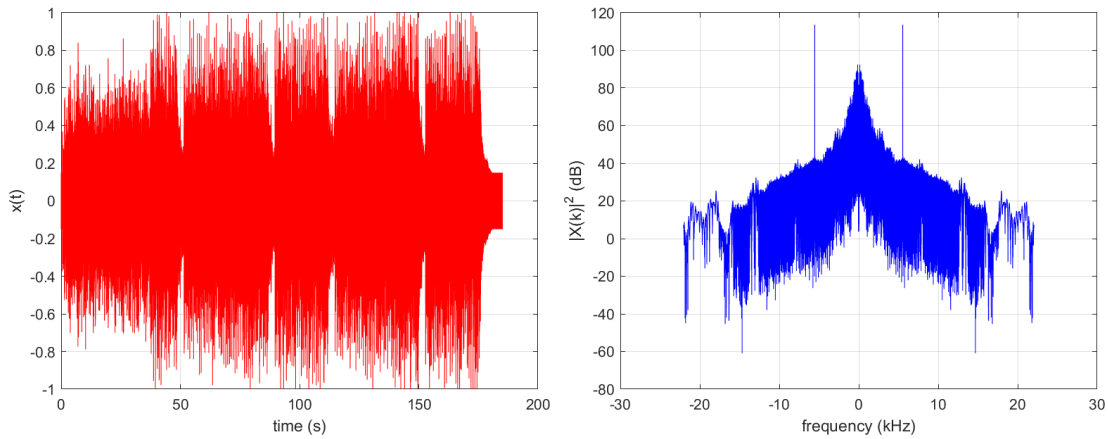


Figure 3: Time and Frequency domain Representation of the first song with disturbance. The time-domain plot (left) shows amplitude increase with respect to the signal without disturbance, while the spectral density plot (right) reveals a noticeable peak around 5 kHz, corresponding to the disturbance.

As illustrated in Figure 3, the time-domain plot of the first disturbance song shows an increased amplitude compared to the original song in the time domain shown in Figure 1. This suggests sharper variations, which are typically indicative of the presence of a high-frequency component (exceeding the bandwidth of the original signal) with significant amplitude superimposed on the original signal.

In the frequency domain, a narrow and prominent spectral peak appears around **5 kHz**, which is absent in the original signal shown in Figure 1, confirming the presence of the disturbance. The increased spectral density at this frequency indicates that the disturbance introduces a frequency component beyond the original song's bandwidth of 970 Hz.

The above analysis and logical reasoning provide a foundation for designing a filter that effectively attenuates the disturbance.

Analysis of Disturbance in the second song

Similarly, the second song is analyzed using methods in the time and frequency domains to examine the characteristics of the disturbance.

In the frequency domain, a noticeable spectral peak is observed around **5 kHz**, which is absent in the original signal depicted in Figure 2. This confirms the presence of the

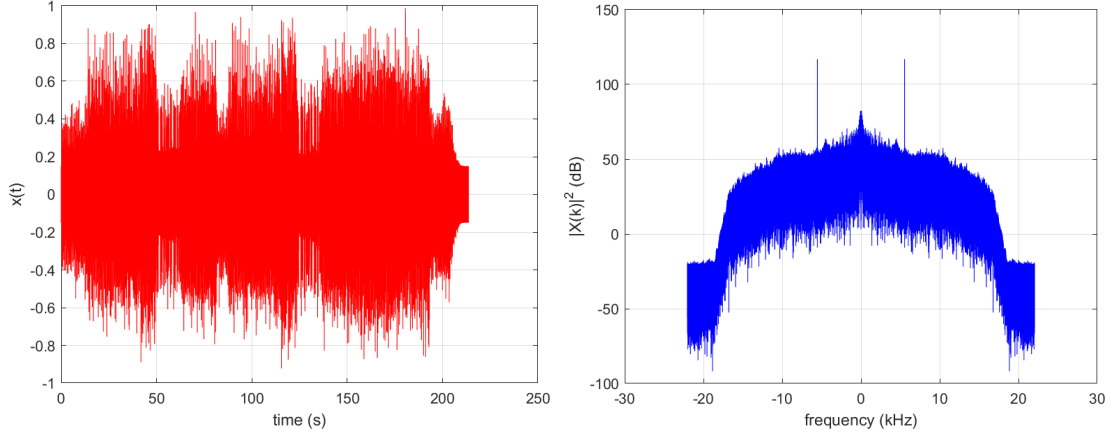


Figure 4: Time and Frequency domain representation of the second song

same disturbance as in the first song. The increased spectral density at this frequency indicates that the disturbance introduces an additional frequency component below the original song's bandwidth of 7270 Hz.

Signal-to-Interference Ratio (SIR)

To optimize filter parameters, the Signal-to-Interference Ratio (SIR) is used as a performance metric. A higher SIR indicates superior signal quality, as it reflects a greater proportion of the desired signal relative to interference or noise.

An equivalent noise is defined as the error signal resulting from the discrepancy between the output signal and the reference signal. This error signal quantifies the unwanted components that deviate from the ideal output.

The SIR is mathematically expressed as:

$$SIR = \frac{P_S}{P_I} = \frac{\mathbb{E} [|x_{out}[n]|^2]}{\mathbb{E} [|e[n]|^2]}$$

where:

- P_S is the power of the output signal.
- P_I is the power of the interference or noise.
- $x_{out}[n]$ is the output signal at discrete time n .
- $e[n]$ is the error signal, defined as:

$$e[n] = x_{out}[n] - x_{ref}[n]$$

Here, $x_{ref}[n]$ represents the reference (original) signal. By minimizing the power of the error signal, the SIR is maximized, leading to an optimized filter design with enhanced signal quality.

2.1 Disturbance Removal

To mitigate the effects of disturbance, we design a filter based on the disturbance analysis. Identifying the logical frequency range of the cut-off frequency, we optimize the filter parameters using the signal-to-interference ratio (SIR).

2.1.1 Disturbance Removal Using a Bessel Low-Pass Filter

A fourth-order Bessel low-pass filter is employed to suppress the disturbance while preserving signal quality.

First song. As previously analyzed, the disturbance frequency lies above the song's bandwidth. Thus, the cutoff frequency of the Bessel filter must be higher than the bandwidth to maintain signal integrity while being low enough to attenuate the disturbance effectively. Based on this consideration, the cutoff frequency is chosen within the range of 1000 to 5000 Hz.

To determine the optimal cut-off frequency, we use SIR to perform parameter optimization.

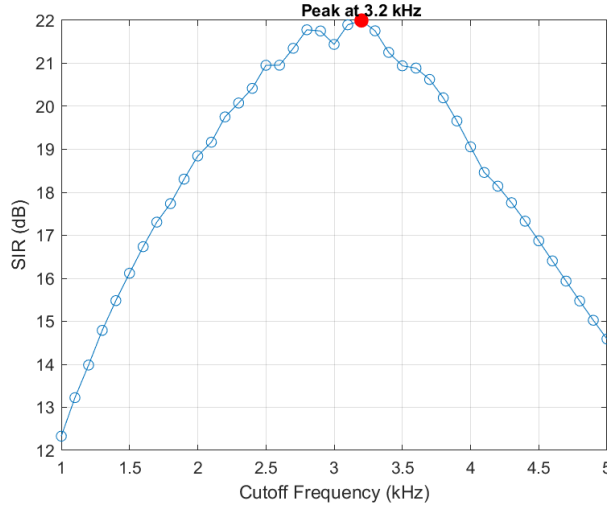


Figure 5: Optimization process using SIR for the first song

After selecting the optimal cut-off frequency, the fourth-order Bessel low-pass filter is applied to the song. In the following, we present the transfer function in the form of a magnitude Bode plot, representing the digital filter that mimics the analog transfer function. In addition, we compare the filtered signal with the original signal.

As observed in Figure 6, the filter exhibits a sharp attenuation beyond the cutoff frequency $f_c = 3.2$ kHz. A small portion of the filtered signal, within the range of $-f_c$ to $+f_c$, closely matches the original signal, as expected. The disturbance peak is significantly attenuated compared to the unfiltered signal; however, its spectral components remain higher than most of the song's spectrum.

Second Song

Similarly, we use SIR optimization to determine the optimal cut-off frequency for the Bessel low-pass filter to attenuate the disturbance in the second song. The key difference is that the second song has a wider bandwidth, which extends beyond the disturbance frequency.

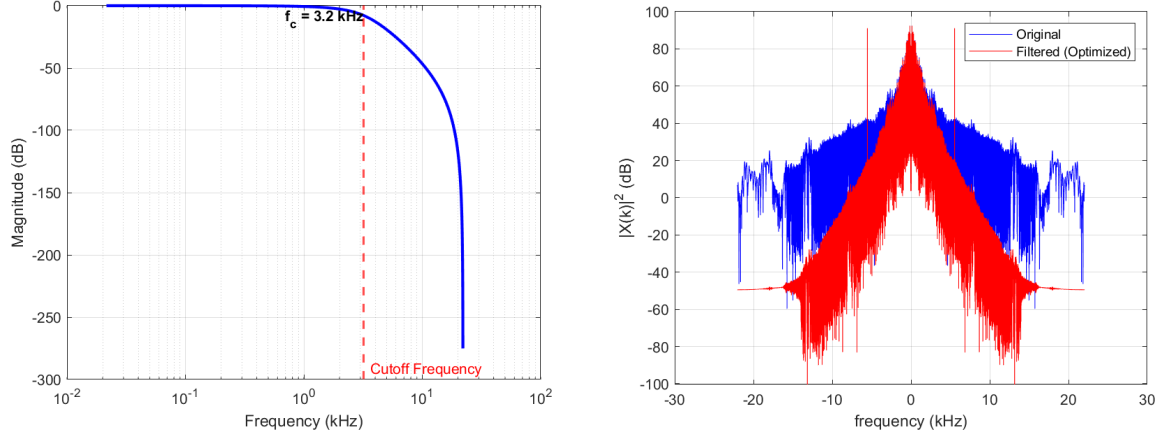


Figure 6: The magnitude Bode plot (left) represents the digital Bessel filter, while the comparison plot (right) shows the filtered song alongside the original song.

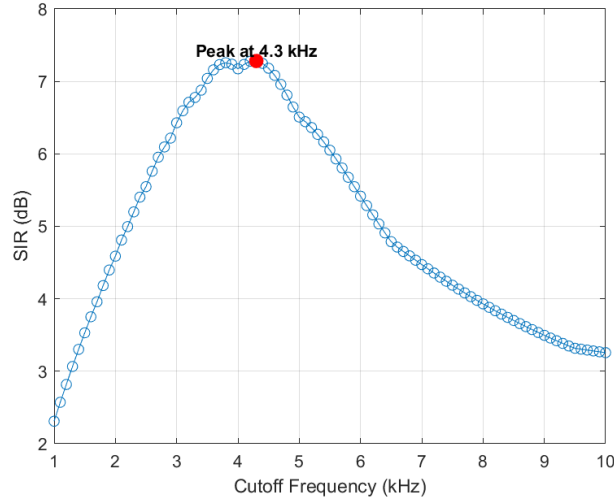


Figure 7: Optimization process using SIR for the second song.

Following the optimization process, we apply the Bessel low-pass filter to the second song. As observed in Figure 8, the Bessel low-pass filter attenuated the disturbance and most of the bandwidth part of the second song.

2.2 Disturbance Removal Using a Narrow Frequency Range Rejection Filter with Conjugate Poles and Zeros

From the disturbance analysis, we observe that the disturbance consists of very narrow frequency components. When applying a fourth-order Bessel low-pass filter, the disturbance is significantly attenuated; however, this also results in the attenuation of a considerable portion of the signal's bandwidth, leading to a loss of useful information.

To overcome this issue, we design a filter that strongly attenuates a narrow frequency range where the disturbance is present while preserving all other frequency components. The frequency-domain analysis indicates that the disturbance is symmetrical, meaning it appears at both $\pm f_d$. Consequently, the filter should have conjugate zeros around

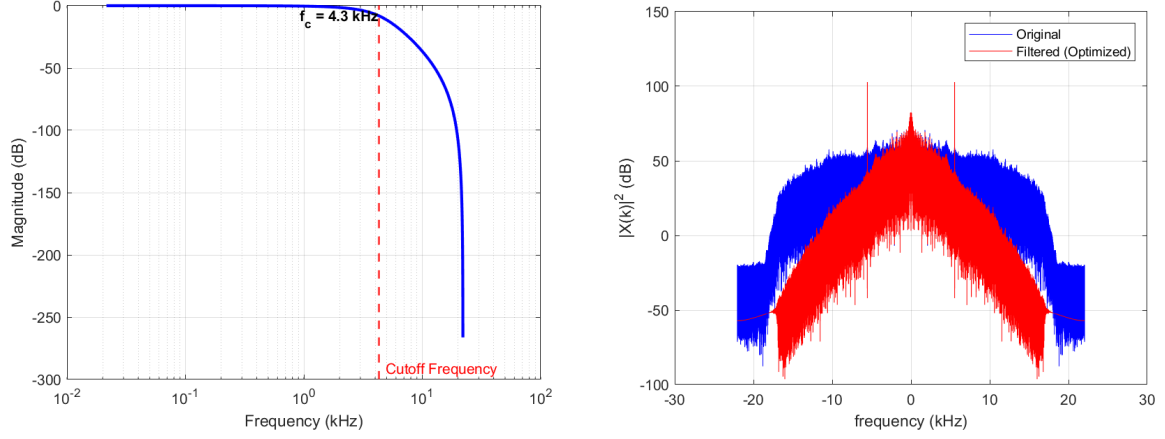


Figure 8: The magnitude Bode plot (left) represents the digital filter response, while the comparison plot (right) shows the filtered song and the original song.

the disturbance frequency to effectively suppress it while maintaining conjugate poles near the zeros. This ensures that other frequency components remain unaffected by the attenuation caused by the zeros. For stability, the poles must be located inside the unit circle in the complex plane.

The transfer function of the filter is given by:

$$H(z) = \frac{(z - z_1)(z - z_1^*)}{(z - p_1)(z - p_1^*)} \quad (1)$$

where:

$$z_1 = e^{j2\pi f_d/F_s}, \quad p_1 = 0.99e^{j2\pi f_d/F_s}$$

Here, f_d represents the frequency to be attenuated, and F_s is the sampling frequency. To obtain the precise value of f_d , we perform an optimization process using the signal-to-interference ratio (SIR).

2.2.1 First Song

The optimization process is illustrated in Figure 12. To improve accuracy and speed of the optimization, we use a two-stage optimization approach: 1. A larger frequency range with a big step size is used to quickly approximate the disturbance frequency. 2. A smaller range with a small step size refines the estimation of the exact disturbance frequency. After optimization, we apply the designed filter. Below, Figure 10 presents the filter's transfer function in the Z-plane and its frequency response.

As seen in the frequency response, the filter acts as a specific frequency rejection filter, effectively attenuating components around **5537.8 Hz** while keeping other frequency components intact.

To evaluate the effect of filtering, we compare the filtered song with the original song. From Figure 11, it is evident that the filtering results are remarkable. The original and filtered songs are almost identical, confirming that the disturbance has been removed without significantly affecting the desired signal.

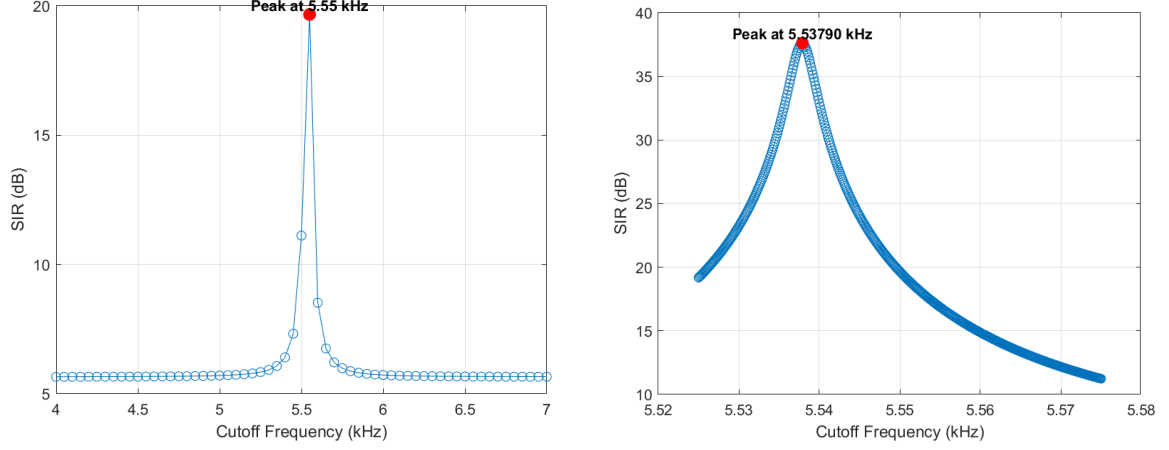


Figure 9: Optimization process to determine the best disturbance frequency f_d . (Left) optimization over a larger range. (Right) optimization for precise zero placement.

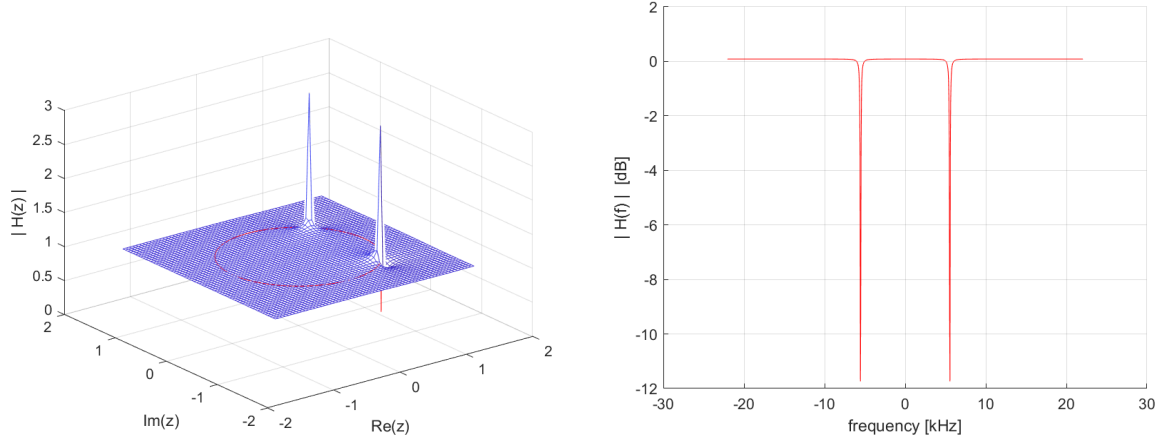


Figure 10: Designed narrow frequency rejection filter. (Left) Transfer function of filter in the Z-plane. (Right) Frequency response of the filter showing strong attenuation around 5537.8 Hz.

2.2.2 Second Song

As mentioned earlier, both songs are affected by the same disturbance. Thus, we apply the same filtering approach to the second song.

The optimization results indicate that the best zero placement is the same as for the first song. So designed filter is the same as the filter for first song. The comparison plot in Figure 13 demonstrates that the filtering procedure is highly effective, with the filtered and original songs exhibiting minimal differences.

2.2.3 Discussion

The narrow frequency rejection filter with two conjugate poles and zeros proves to be highly effective in removing disturbances that occupy a narrow frequency band. Unlike the Bessel low-pass filter, which attenuates all frequency components beyond the cutoff frequency, this approach selectively removes the unwanted disturbance while preserving

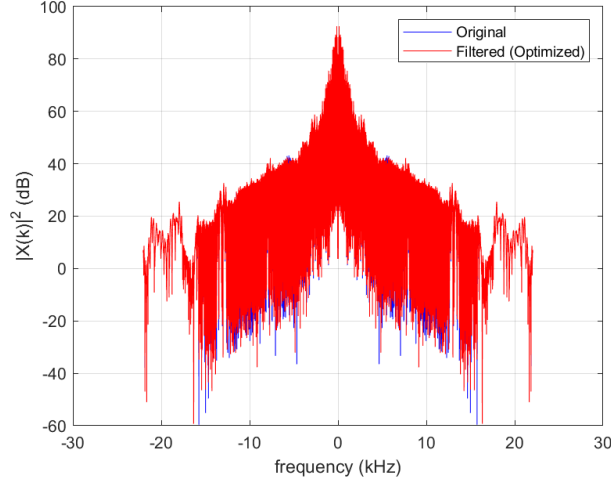


Figure 11: Comparison of the original and filtered first song. The filtering process successfully removes the disturbance while maintaining signal integrity.

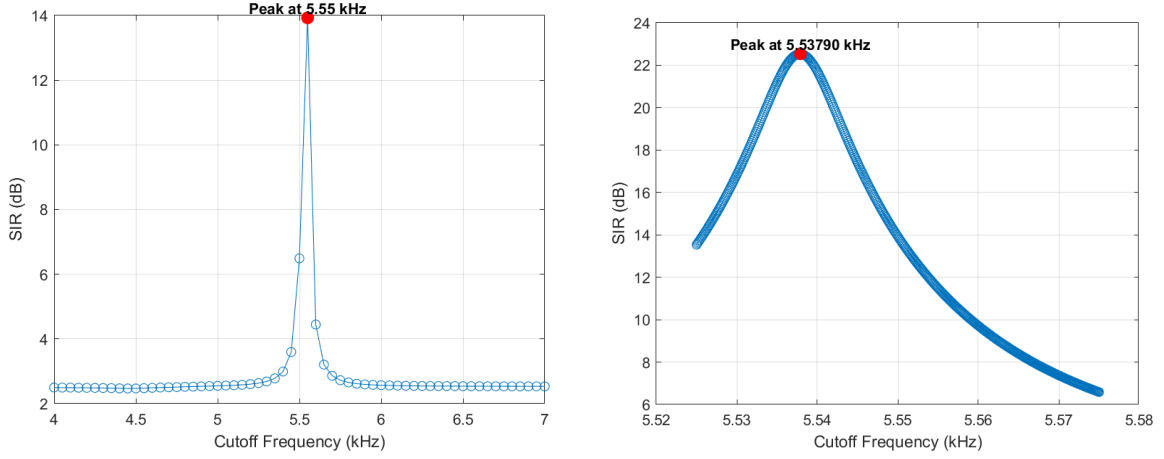


Figure 12: Optimization process to determine the best disturbance frequency f_d . (Left) optimization over a larger range. (Right) optimization for precise zero placement.

the rest of the signal. The results confirm that this method achieves superior performance in maintaining the original signal quality. These results can be easily verified by examining the table below.

The Bessel low-pass filter exhibits better performance for the first song because the disturbance frequency is significantly higher than the song's bandwidth. The optimized cutoff frequency is chosen to balance signal preservation while effectively attenuating the disturbance. However, for the second song, the Bessel low-pass filter performs poorly because the disturbance frequency lies within the song's bandwidth. If the cutoff frequency is set below f_d , it attenuates not only the disturbance but also a portion of the second song's bandwidth, leading to a decrease in signal quality.

The narrowband rejection filter demonstrates better performance for the first song than second song. This is because the disturbance frequency (f_d) is much higher than the bandwidth of the first song, allowing the filter to remove the disturbance with minimal impact on the signal. In contrast, for the second song, the disturbance frequency falls

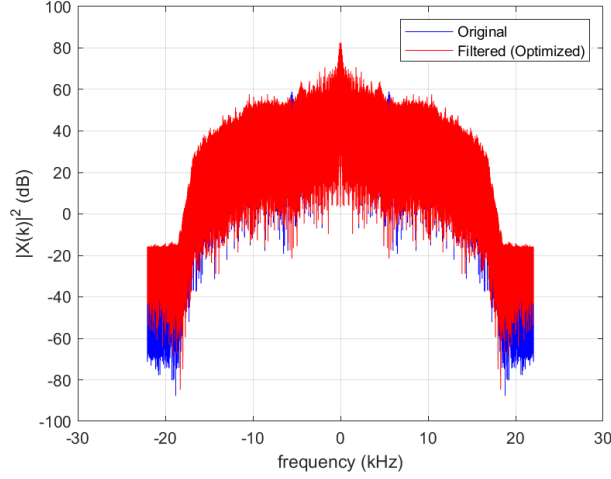


Figure 13: Comparison of the original and filtered second song. The filtering process successfully removes the disturbance while maintaining signal integrity.

Filters	SIR (dB)	Filter Parameters	Bandwidth of song (Hz)
Bessel LPF for 1-song	22	$f_c = 3200$	970
Bessel LPF for 2-song	7.3	$f_c = 4300$	7270
Narrowband rejection filter for 1-song	38	$f_d = 5537.9$	970
Narrowband rejection filter for 2-song	23	$f_d = 5537.9$	7270

Table 1: Filter characteristics

within the song's bandwidth, making the filter attenuate not only the disturbance but also some useful part of the second song. Consequently, the SIR of the narrowband rejection filter is higher for the first song than for the second song.

3 Sharing the Channel

We aim to transmit two audio signals (songs) over a communication channel that is already occupied by a baseband signal modeled as a stochastic process. Since both songs need to be transmitted in parallel, we must efficiently allocate the available bandwidth while ensuring the highest possible quality for at least one received song, minimizing distortion and interference.

3.1 Analysis of the Channel

To efficiently share the channel, we must first analyze its characteristics. In Figure 14, the spectral density of the baseband signal is depicted.

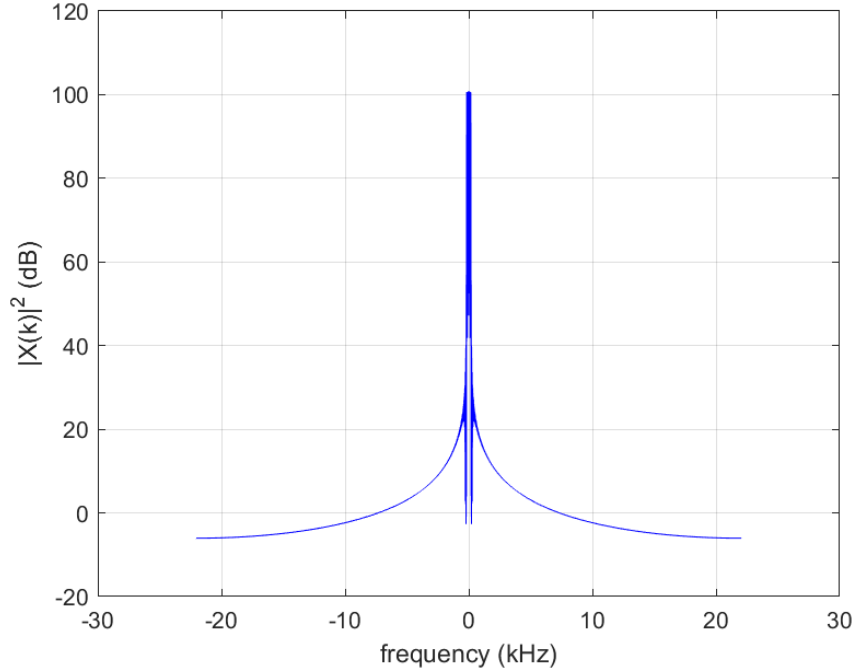


Figure 14: Spectral density of the baseband signal.

The baseband signal has a bandwidth of 160 Hz, meaning that 95% of its energy is contained within this frequency range. Given that the channel can allocate a maximum frequency of $F_s/2 = 22050$ Hz, we are left with:

$$22050 - 160 = 21890 \text{ Hz} \quad (2)$$

of available bandwidth for transmitting the two songs.

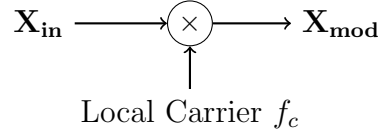
To evaluate the impact of transmitting one song directly over the channel, we attempted transmission without additional signal processing. The resulting signal-to-interference ratio (SIR) is approximately 1 dB, indicating severe degradation of song quality due to interference from the baseband signal.

3.2 Multiplexing Two Songs

Transmission systems commonly use multiplexing to share channels by transmitting multiple signals simultaneously. To successfully transmit both songs over a channel that is already occupied by a baseband signal, we must employ multiplexing techniques. This allows for parallel transmission of both songs while mitigating interference. A fundamental approach is to analyze the channel's spectrum and design an appropriate amplitude modulation (AM) strategy to shift our signals into an unoccupied spectral region.

Modulation for Efficient Transmission To enable simultaneous transmission of both songs, modulation is necessary. This technique shifts each signal into a different frequency band, preventing spectral overlap and interference.

Let $x_1(t)$ and $x_2(t)$ represent the two songs. Using double sideband amplitude modulation (DSB-AM) with carrier frequencies f_{c1} and f_{c2} , the modulated signals are:

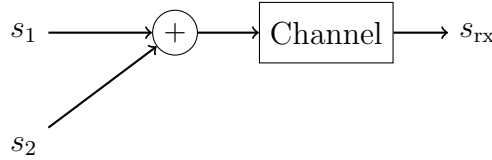


$$s_1(t) = x_1(t) \cos(2\pi f_{c1}t) \quad (3)$$

$$s_2(t) = x_2(t) \cos(2\pi f_{c2}t) \quad (4)$$

The overall transmitted signal is:

$$s(t) = s_1(t) + s_2(t) \quad (5)$$



The modulation process shifts the spectrum of each song to its respective carrier frequency. To ensure efficient utilization of the available channel bandwidth, the carrier frequencies f_{c1} and f_{c2} must be carefully selected to:

- Position the modulated signals within the available channel bandwidth of 21,890 Hz (from Equation (2)).
- Avoid spectral overlap between the two transmitted signals.
- Maximize frequency separation to reduce mutual interference while staying within the channel's maximum bandwidth.

Given that the first song has a bandwidth of 970 Hz and the second song has a bandwidth of 7,270 Hz, suitable carrier frequency choices are:

$$f_{c1} = 2000 \text{ Hz}, \quad f_{c2} = 12000 \text{ Hz}.$$

These values ensure that both signals are placed efficiently within the channel without interference while maintaining sufficient spectral separation.

Figure 15 illustrates the transmission of both songs over the channel. On the positive frequency axis, three distinct peaks represents three signals and their bandwidths don't overlap. This confirms that both songs are successfully modulated and transmitted over the channel, while adhering to the available bandwidth constraints.

This technique ensures that each song occupies a different spectral region, preventing interference with the baseband signal while maintaining signal integrity.

Demodulation. After transmission, the received signal must be properly recovered. This process is known as demodulation, which extracts the original signal from the modulated

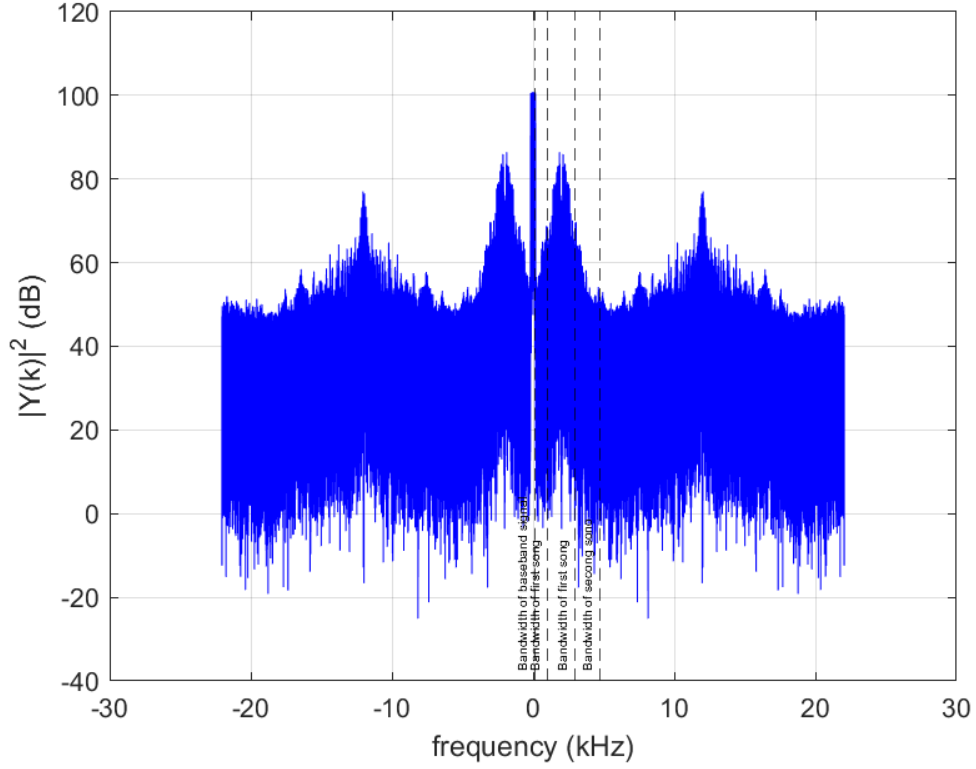
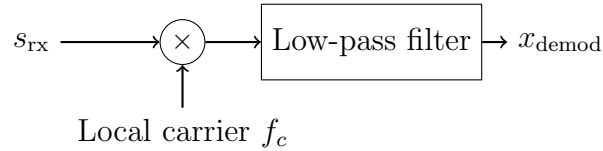


Figure 15: Spectrum of the transmitted songs after modulation. On the positive frequency axis there are three peaks representing three signals

carrier. Demodulation is crucial for ensuring signal integrity and minimizing distortion. A common method for demodulating an amplitude-modulated (AM) signal is to multiply the received signal by a local carrier of the same frequency as the transmitter. The block diagram below illustrates the demodulation process.



Given a transmitted signal in the form of a double-sideband (DSB) modulated signal,

$$s_{rx}(t) = x(t) \cos(2\pi f_c t) \quad (6)$$

where - $x(t)$ is the original signal, - f_c is the carrier frequency.

To recover $x(t)$, we multiply the received signal $s_{rx}(t)$ by a locally generated carrier $\cos(2\pi f_c t)$,

$$s_{mix}(t) = s_{rx}(t) \cos(2\pi f_c t) \quad (7)$$

we obtain

$$s_{mix}(t) = \frac{x(t)}{2} + \frac{x(t) \cos(4\pi f_c t)}{2} \quad (8)$$

Low-pass filtering. The resulting signal consists of two components: 1- term $\frac{x(t)}{2}$, which is the desired signal. 2-term $\frac{x(t)\cos(4\pi f_c t)}{2}$, which is a high-frequency component at $2f_c$. To recover $x(t)$, we apply a low-pass filter, which removes the high-frequency term while preserving the baseband component,

$$x_{\text{demod}}(t) = \frac{x(t)}{2} \quad (9)$$

The demodulation process involves: 1 - Mixing the received signal with a locally generated carrier. 2 - Applying a low-pass filter to extract the original baseband signal. This method ensures accurate signal recovery with minimal distortion, making it a fundamental technique in communication systems.

3.3 Recovering Songs Using a Bessel Low-Pass Filter

To recover the transmitted signal, we apply a Bessel low-pass filter. The first step in designing the filter is selecting an optimal cutoff frequency to balance signal preservation while effectively filtering out unwanted components.

3.3.1 First Song

To transmit the first song, a carrier frequency of $f_c = 10$ kHz is chosen. When the signal is multiplied by $\cos(2\pi f_c t)$, the baseband signal shifts to f_c . Therefore, to recover the original song, a filter must be designed to attenuate components at f_c . The optimization process for determining the best cutoff frequency for the first song is illustrated in Figure 12.

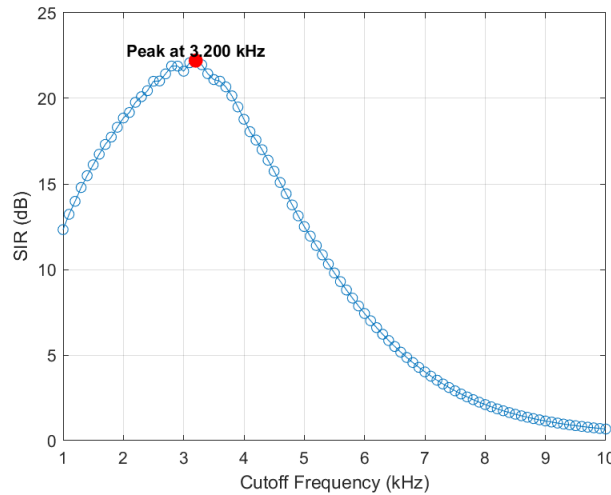


Figure 16: Optimization process for selecting the cutoff frequency of the Bessel low-pass filter for the first song.

Once the optimal cutoff frequency is determined, we implement the Bessel low-pass filter and analyze its performance. Figure 17 presents the transfer function of the designed filter and comparison of the original and recovered song.

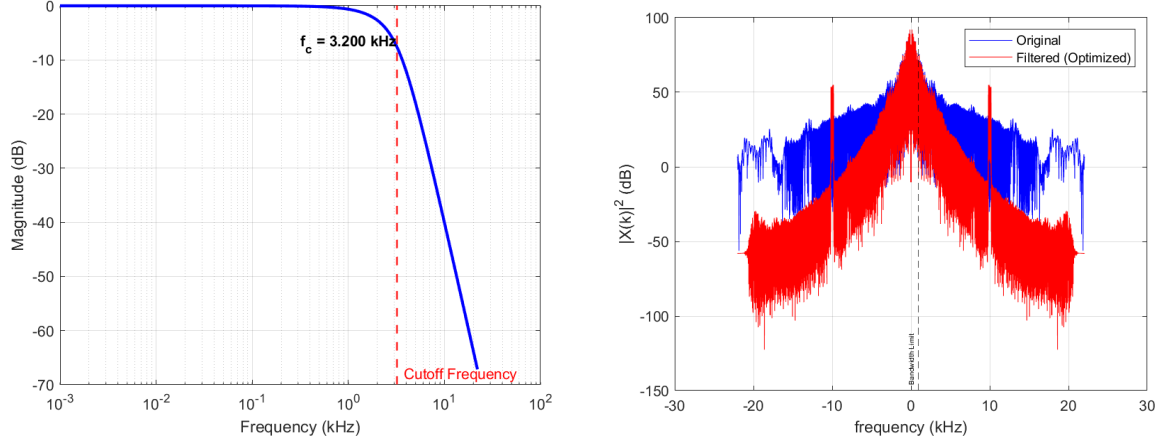


Figure 17: (Left) Transfer function of the Bessel low-pass filter. (Right) Comparison between the original and recovered first song.

As observed in Figure 17, within the song's bandwidth, the original and recovered signals closely match, demonstrating the effectiveness of the Bessel low-pass filter in preserving signal integrity while eliminating undesired components.

Second Song

A similar procedure is applied to the second song to determine the optimal cutoff frequency for the Bessel low-pass filter. The optimization process for selecting the best cutoff frequency is illustrated in Figure 18.

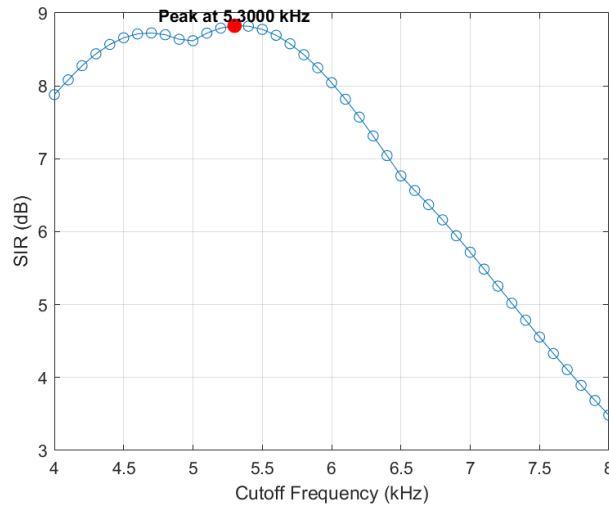


Figure 18: Optimization process for selecting the cutoff frequency of the Bessel low-pass filter for the second song.

After optimization, the Bessel low-pass filter is applied to recover the transmitted signal. As observed in Figure 19, the Bessel low-pass filter effectively recovers the second song while attenuating some portions of the bandwidth. It preserves all frequency components below f_c while strongly attenuating higher frequencies.

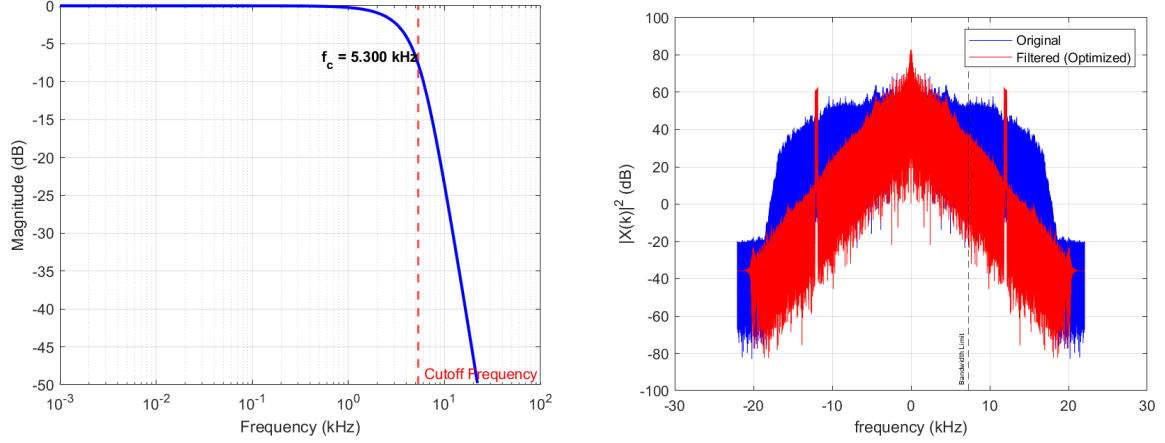


Figure 19: (Left) Transfer function of the Bessel low-pass filter. (Right) Comparison between the original and recovered second song.

3.4 Recovering song using low-pass filter with conjugate poles and zeros

To demodulate the modulated signal, it is multiplied by a cosine wave with the local carrier frequency. After this multiplication, a low-pass filter is required to extract the desired baseband signal while attenuating unwanted high-frequency components.

First song

To transmit the first song, a carrier frequency of $f_c = 10$ kHz is chosen. When the signal is multiplied by $\cos(2\pi f_c t)$, the baseband signal shifts to f_c . Therefore, to recover the original song, a filter must be designed to attenuate components at f_c .

The design of a low-pass filter with two conjugate poles and zeros follows this principle:

- Poles are placed near frequencies that should be amplified.
- A zero is placed at f_c to attenuate the undesired frequency component.

To ensure uniform amplification of all useful frequency components and minimize distortion, the poles should not be placed too close to the unit circle in the z-plane. The pole location is given by:

$$p_1 = 0.8 \cdot e^{j2\pi f/F_s}$$

where f is optimized using the signal-to-interference ratio (SIR) to determine the best placement.

The zero is placed at f_c to effectively attenuate the shifted baseband signal since it has high spectrum within small range of frequency:

$$z_1 = e^{j2\pi f_c/F_s}$$

The complete transfer function of the low-pass filter is:

$$H(z) = \frac{(z - z_1)(z - z_1^*)}{(z - p_1)(z - p_1^*)}$$

Since poles amplify certain frequency components, the filtered signal must be renormalized after processing.

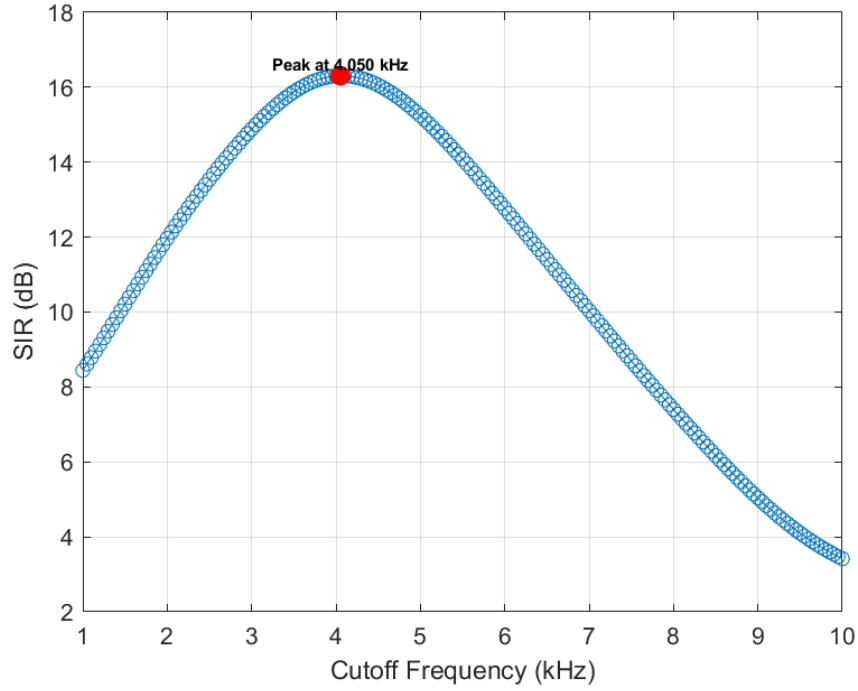


Figure 20: Optimization process for selecting the best pole frequency for the first song.

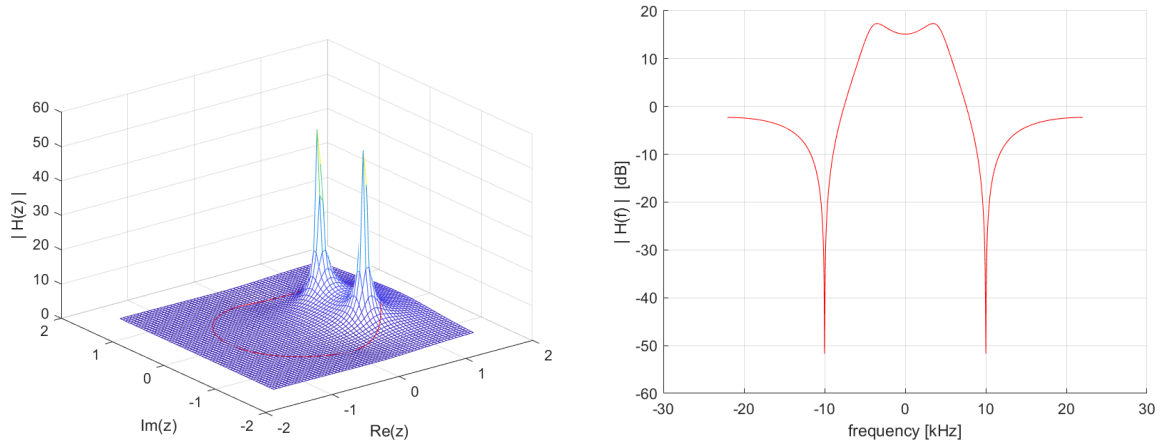


Figure 21: (Left) Transfer function $H(z)$ of the low-pass filter. (Right) Frequency response $H(f)$ of the filter for the first song.

Second song

A similar approach is used for designing the low-pass filter for the second song. The pole is positioned to amplify the useful components:

$$p_1 = 0.6 \cdot e^{j2\pi f/F_s}$$

where f is optimized using the SIR method.

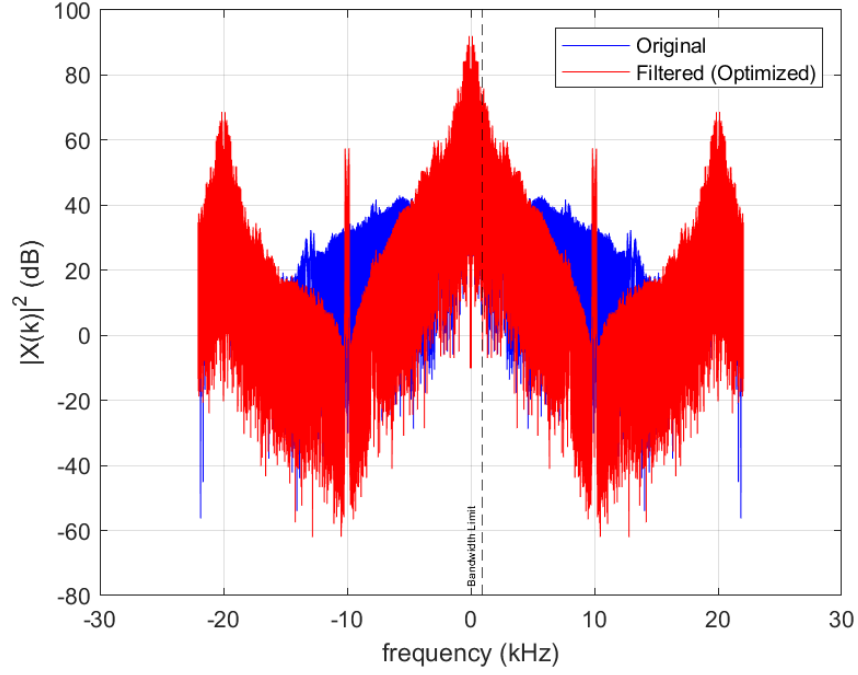


Figure 22: Comparison between the original and recovered first song.

The zero is placed at the carrier frequency f_{c1} to attenuate the undesired baseband signal component:

$$z_1 = e^{j2\pi f_{c1}/F_s}$$

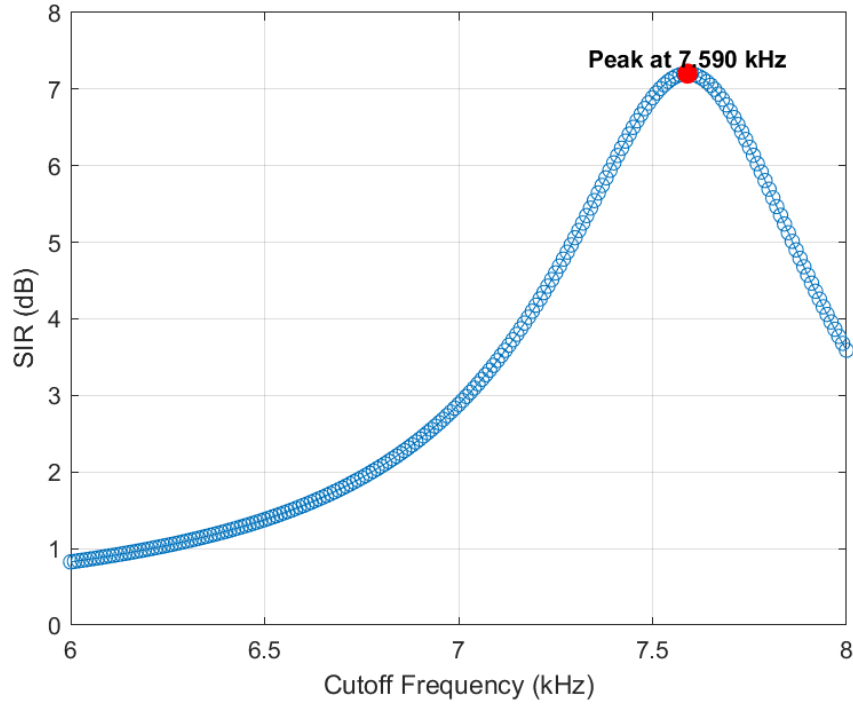


Figure 23: Optimization process for selecting the best pole frequency for the second song.

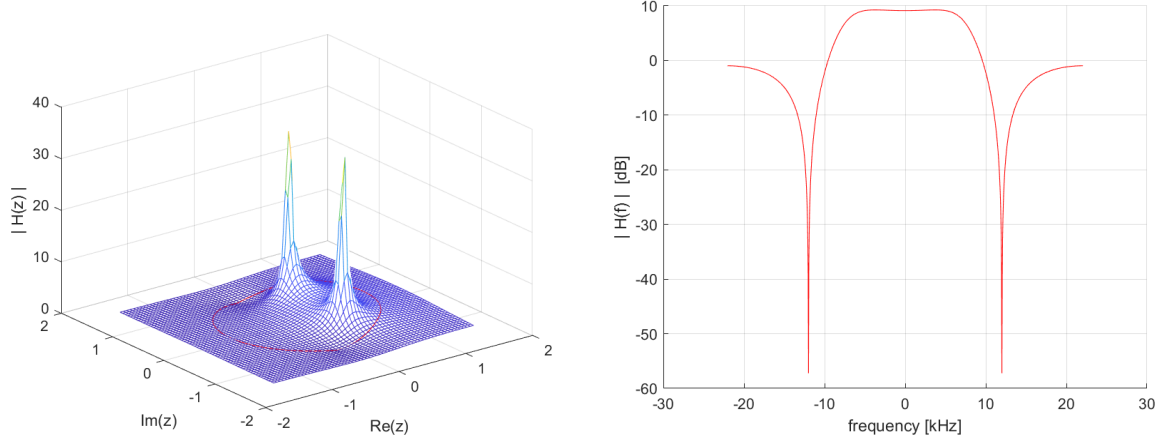


Figure 24: (Left) Transfer function $H(z)$ of the low-pass filter. (Right) Frequency response $H(f)$ of the filter for the second song.

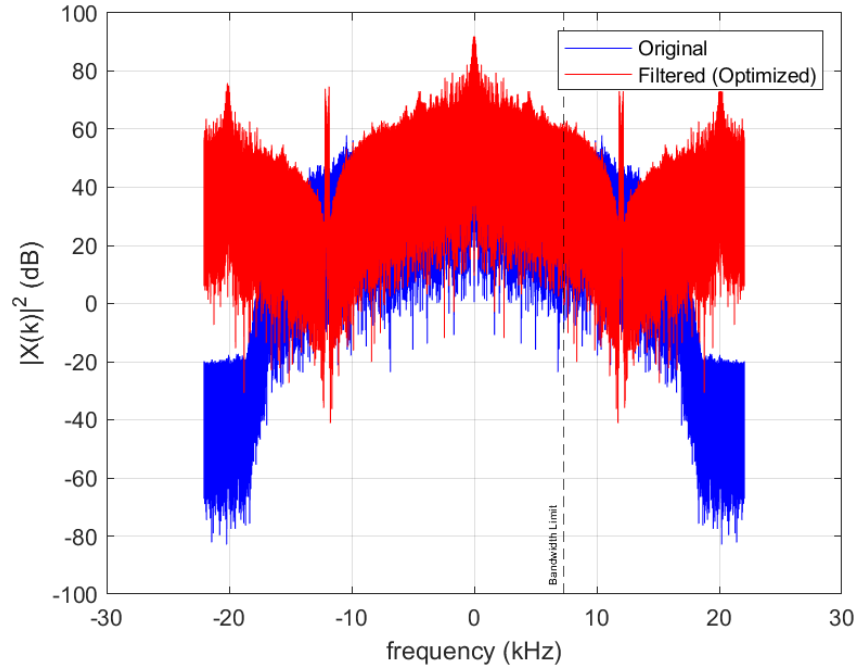


Figure 25: Comparison between the original and recovered second song.

3.5 Discussion

A comparison of different low-pass filters is presented in Table 2. Bessel low pass filter shows almost the same performance for two songs.

From this table, we can conclude that the Bessel low-pass filter exhibits better performance in demodulation.

One of the reasons for the lower performance of the low-pass filter with two conjugate poles and zeros is that it can only attenuate a narrow range of frequencies. In the comparison plots of this filter, it is evident that the signal component at $2f_c$ is not sufficiently attenuated, while only the component at f_c is effectively suppressed.

Additionally, the choice of f_c significantly affects the SIR. If f_c is set too low, the two

Filter	SIR (dB)	Cutoff frequency (Hz)	Bandwidth of Song (Hz)
Bessel LPF for first song	22	760	970
Bessel LPF for second song	9	5300	7270
LPF with conjugate poles and zeros for first song	16	4050	970
LPF with conjugate poles and zeros for second song	7	7590	7270

Table 2: Comparison of filter characteristics and performance.

signals within the channel might overlap, making it difficult to extract the desired signal, as attenuation of one signal could interfere with the other. On the other hand, if f_c is too high, the SIR may suffer due to aliasing effects. Therefore, to determine the optimal f_c , it is recommended to conduct an optimization study.

For the first song, $f_{c1} = 10$ kHz, resulting in a wider separation between the first song and the baseband signal. In contrast, for the second song, $f_c = 12$ kHz, leading to a shorter gap between the second song and the baseband signal, as the second song has a bandwidth of 7,270 Hz. When comparing the SIR values after filtering, the first song achieves a significantly higher SIR than the second song. This indicates that a wider gap between signals within the channel facilitates better recovery of the desired signal.

4 Conclusions

This work demonstrates a complete workflow for analyzing, transmitting, and recovering two audio signals in the presence of both a low-bandwidth baseband occupant and a narrowband disturbance at approximately 5 kHz. The key findings are summarized as follows:

1. Disturbance Analysis and Removal:

- A fourth-order Bessel low-pass filter is effective when the disturbance lies above the signal bandwidth, as it preserves the desired audio components while attenuating unwanted frequencies.
- A narrow frequency rejection filter with conjugate poles and zeros selectively suppresses the disturbance at around 5 kHz without significantly affecting other frequency components. This is particularly useful when the disturbance is within the signal bandwidth and a broader filter would cause excessive attenuation of the desired signal.

2. Channel Sharing:

- The channel already hosts a 160 Hz baseband signal, leaving 21,890 Hz of usable bandwidth.

- By choosing suitable carrier frequencies (for instance, 2,000 Hz and 12,000 Hz) and employing double sideband amplitude modulation, both audio signals are transmitted in parallel without overlap or interference.

3. Demodulation and Recovery:

- Standard AM demodulation is performed by multiplying the received signal by a local carrier and applying a low-pass filter.
- Bessel low-pass filters and low-pass filters with conjugate poles and zeros are both tested for recovering the baseband signals; their performance depends on the position of the disturbing frequencies relative to the desired bandwidth.
- The SIR indicates that wider separation between signals in the channel and careful filter design yield better recovery of the audio content.

4. Performance Comparison:

- The Bessel low-pass filter tends to work best when the disturbance is outside the main bandwidth of the audio signal.
- The narrowband rejection filter provides more targeted attenuation when the disturbance lies inside the signal's band.
- Proper choice of carrier frequency is crucial to prevent excessive overlap or aliasing within the channel, especially for signals with larger bandwidth.

In summary, the results underscore the importance of selecting the correct filter type, carrier frequency, and modulation method when designing a communication system with parallel audio transmissions and an existing baseband occupant. Optimizing these parameters can significantly improve the final signal-to-interference ratio and the overall quality of recovered audio signals.