Appendix Viscosity

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A Parameter tracking for the viscosity νnu

Throughout *all* proofs we write Navier–Stokes in the dimensional form

$$\partial_t u + (u\cdot\nabla)u + \nabla p = \nu\,\Delta u, \qquad \nabla\cdot u = 0, \tag{1}$$

with a fixed positive viscosity $\nu > 0$.

A. Conventions

[label=**A.0**, leftmargin=2em]**Scaling macro.** All files load `\newcommand{\nucl}{\nu}`, so replacing "`\nucl`" in any line shows exactly where ν enters. **Parabolic rescaling.** To pass from the physical system eq:NS-nu to the normalised $\nu = 1$ version used inside many lemmas, apply

$$x' = x, \quad t' = \nu t, \quad u'(x',t') = u(x,t), \quad p'(x',t') = \nu^{-1}p(x,t).$$

Under this map Δ picks up the factor ν^{-1}, cancelling the right-hand side of eq:NS-nu. The Sobolev norms transform as $\|u\|_{H^s} = \nu^0\|u'\|_{H^s}$. **Constant tracking rule.** Unless stated otherwise, every constant $C(\cdots)$ in an estimate satisfies

$$C(\nu, data) = \nu^{-k}\,C_0(data),$$

with an integer exponent $k \geq 0$ indicated in the surrounding text.

B. Global contraction of the heat semigroup

[Strict contraction] For every $s \geq 0$ and $\alpha > 0$,

$$\|e^{\alpha\Delta}f\|_{H^s} \;\leq\; e^{-\alpha\,\lambda_s}\,\|f\|_{H^s}, \qquad \lambda_s = 14(2\pi)^{2-2s}.$$

Consequently the suppression operator $L_\alpha = e^{\alpha\Delta}$ is a strict contraction on each Sobolev space H^s, independently of ν.

Diagonalise in Fourier; $e^{\alpha\Delta}\hat{f}(k) = e^{-\alpha|k|^2}\hat{f}(k)$ and $|k|^2 \geq (2\pi)^2$ for $k \neq 0$ on [3]. Substitute into $\|\cdot\|_{H^s}^2 = \sum_k k^{2s}|\hat{f}(k)|^2$.

C. Quick reference table

Symbol	Meaning / location
ν	Physical viscosity; macro defined in the preamble.
$\int_0^T \|\omega\|_{L^\infty}$	BKM integral; final estimates depend on ν^{-1} linearly.
$C_m(\nu)$	Constants in Lemma **??**; scale like ν^{-m}.
$e^{\alpha\Delta}$	Contraction factor $e^{-\alpha\lambda_s}$ independent of ν.

With these unified rules every proof can safely set $\nu = 1$ inside technical lemmas and restore ν at the statement level via the parabolic rescaling map in **A.2**. A search for "`\nucl`" lists the exact lines where ν appears.