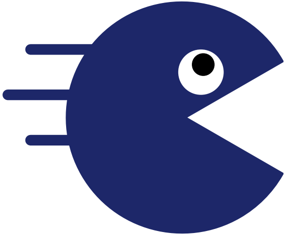

Multi-threading on CPUs with OpenMP and Metrics for Performance Analysis of Applications

Polytech Sorbonne – EI5-SE – Calcul haute performance (EPU-F9-IHP)

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Programming Multi-core CPUs

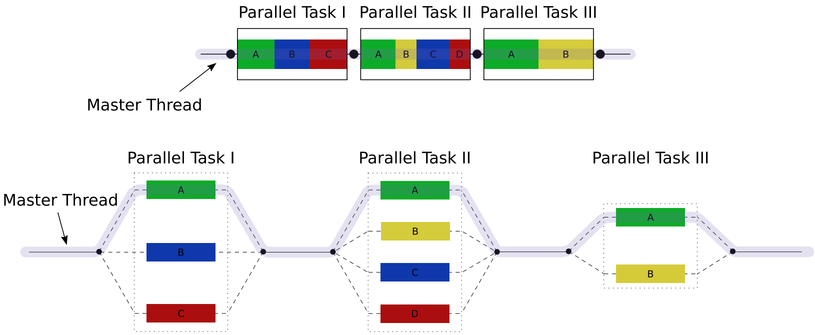
- **Nowadays, multi-core architecture is well spread** in High Performance Computing (HPC) and in embedded targets
- There are two main ways to use multi-core architectures
  1. Create **multiple processes** (= distributed memory model)
     ➔ MPI standard, Unix inter-processes communications, sockets, ...
  2. Create **multiple threads** (= shared memory model)
     ➔ Threads POSIX, OpenMP, ...
- In this session we will not talk about the multiple processes model
- And **we will go deeper into the multi-threaded model**

- OpenMP is a language dedicated to setup multi-threaded codes
- It is based on **compiler directives** (`#pragma`)
  - Those directives describe how to perform the parallelism
  - The main advantage of directives is to **not modify sequential code** (in theory...)

```c
void add_vectors(const float* A, const float* B, float* C, const size_t n)
{
#pragma omp parallel // directive for the creation of a parallel zone (= threads creation)
{ // <- beginning of the parallel zone
#pragma omp for // directive for distribution of for-loop indices among threads
  for (size_t i = 0; i < n; i++)
    C[i] = A[i] + B[i];
} // <- end of the parallel zone
}
```

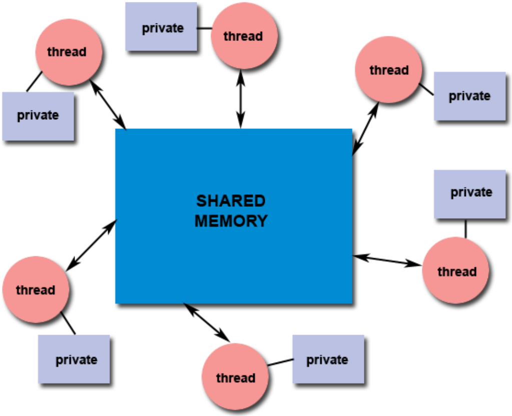
Simple `add_vectors` OpenMP implementation

Fork-join Model



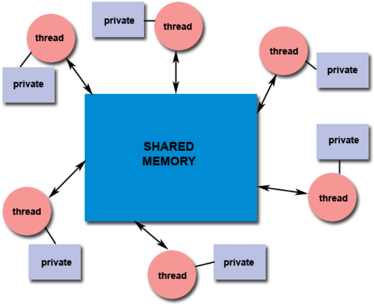
- When using `#pragma omp parallel` directive: threads are **created** (= **fork**)
- At the end of a parallel zone
  - Threads are **destroyed** (= **join**), except for the master thread
  - There is an implicit barrier

Shared Memory Model



- Each thread can access the global memory zone
  - This is called the shared memory (or the RAM of the CPUs)
- But threads also own private data
  - Not completely shared model
  - Very often, the key for achieving performance is to keep the memory private when possible...

Shared Memory Model – Code Example




```c
void add_vectors(const float* A, const float* B, float* C,
                 const size_t n)
{
#pragma omp parallel
{
#pragma omp for
  // 'i' is private because it is declared
  // after the omp parallel directive
  for (size_t i = 0; i < n; i++) // <- 'n' is shared
    // 'A', 'B' and 'C' are shared!
    C[i] = A[i] + B[i];
}
}
```

- By default, variables that are **declared before a parallel zone** are **shared** (here `A`, `B`, `C` and `n`)
- And variables **declared inside a parallel zone** are **private** (here `i`)

Control Data Range

- OpenMP provides data range control
  - **`private`**: local to the thread,
  - **`firstprivate`**: local to the thread and initialized
  - **`shared`**: shared by all the threads, in C/C++ this is the default behavior

- Here `alpha` is a constant, we can put it in the private memory of each thread

- Efficient parallelism comes with minimal synchronizations
  - Shared data can generate a lot of synchronizations
  - Privacy increases thread independence

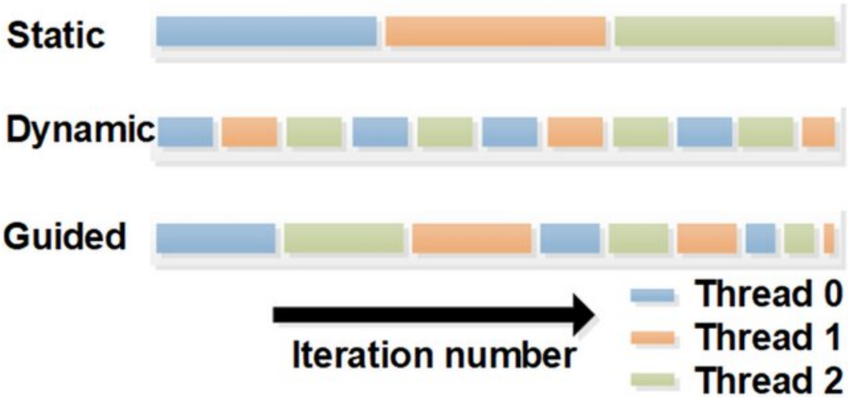
```c
void dot(const float* A,
                  float* B,
            const float  alpha,
            const size_t n)
{
#pragma omp parallel      \
            shared(A, B) \
            firstprivate(alpha, n)
{
#pragma omp for
  // 'i' is still private because
  // it is declared after the
  // parallel zone
  for (size_t i = 0; i < n; i++) {
    B[i] = alpha * A[i];
  }
}
}
```

`for-loop` Indices Distribution (1)

- `for-loop` indices distribution can be controlled by the **`schedule`** clause
  - **`static`**: indices distribution is precomputed (at compilation time), and the amount of indices is the same for each thread
  - **`dynamic`**: indices distribution is done in real time along the loop execution, work load balancing can be better than with the `static` scheduling but `dynamic` scheduling costs some additional resources in order to attribute indices at real time. This is a **greedy algorithm**.
- There are other types of scheduling and it is also possible to choose the scheduler at runtime with `OMP_SCHEDULE` environement variable and the `schedule(runtime)` primitive

```c
// ...
#pragma omp for schedule(static, 128) // we statistically attribute 128 per 128 indices to each threads
  for(int i = 0; i < n; i++)
    B[i] = alpha * A[i];
// ...
```

for-loop Indices Distribution (2)



From DOI: 10.1007/s10291-022-01266-8.

https://doi.org/10.1007/s10291-022-01266-8

Sections

- OpenMP proposes **sections** to enable threads to execute different "parts" (= **section**) of the code

```c
#pragma omp parallel
{ // <- multiple threads have been created
#pragma omp sections
{ // <- entering in a 'sections' zone
  #pragma omp section
  { // this code is executed by a single thread
    printf("id = %d,", omp_get_thread_num());
  }
  #pragma omp section
  { // this code is executed by a single thread too
    printf("id = %d,", omp_get_thread_num());
  }
} // <- end of the 'sections' zone
} // <- threads are destroyed
```

- The code will print: `id = 0, id = 1,`
- But not necessarily in this order

Single and master directives


```c
void dot(const float* A, float* B, const float  alpha,
         const size_t n) {
#pragma omp parallel shared(A, B) \
                     firstprivate(alpha, n)
{
#pragma omp for
  for (size_t i = 0; i < n; i++) {
    B[i] = alpha * A[i];
#pragma omp single
{ // executed by only one thread
    printf("B[i] = %f," B[i]);
} // <- there is an implicit barrier here, all the
  // threads are waiting
  }
}
}
```

```c
void dot(const float* A, float* B, const float  alpha,
         const size_t n) {
#pragma omp parallel shared(A, B) \
                     firstprivate(alpha, n)
{
#pragma omp for
  for (size_t i = 0; i < n; i++) {
    B[i] = alpha * A[i];
#pragma omp master
{ // executed by only the master thread
    printf("B[i] = %f," B[i]);
} // <- there is NO implicit barrier here, other
  // threads will not wait for the master thread
  }
}
}
```

- Directives to be executed by a single thread
- Even if they look similar, there is an implicit barrier after the `single` directive

Environment Variables

- OpenMP take advantage of environment variable to customize the multi-threaded execution
- The most famous one is `OMP_NUM_THREADS`, it enables control the number of threads in the OpenMP parallel zones

```
export OMP_NUM_THREADS=4
./my_omp_program # the code will run on 4 threads

OMP_NUM_THREADS=2 ./my_omp_program # the code will run on 2 threads
```

- It is possible to select which scheduler will be used with the `OMP_SCHEDULE` environment variable

```
export OMP_NUM_THREADS=4
OMP_SCHEDULE="static,3" ./my_omp_program # the code will run on 4 threads and
                                         # with a static scheduling of size 3
```

And Many more Features

- Tasks
- Atomic, critical section
- SIMD
- Accelerators (GPUs)
- ...

Go Further

- Previous slides were a brief overview of the main OpenMP principles
- To have more precise informations you can take a look at the very good OpenMP reference card[1]
  - It could be a very good idea to print it and keep it ;-)
- In the next slides we will pay attention to some **OpenMP use cases**

[1] https://www.openmp.org/resources/refguides/

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Avoid False Sharing

- **False sharing** is a phenomena that occurs when threads write simultaneously data in a same cache line
  - Remember, the cache system works with lines of words: a line is the smallest element in caches coherence mechanism
  - If two or more threads are working on the same line they cannot write data simultaneously!
    - ➔ Stores are serialized and we talk about false sharing

- To avoid false sharing, threads have to work on a bigger amount of data than the cache line size
  - Concretely we have to avoid `(static,1)` or `(dynamic,1)` scheduling
  - Cache lines are not very big ($\approx$ 64 Bytes)
  - Just putting a `(static,16)` or `(dynamic,16)` often resolves the problem
    - Be aware that in some OpenMP implementations, the default scheduling pattern is `(static,1)`!

Threads Synchronizations – Barriers

- In OpenMP there are a lot of **implicit barriers**, after each
  - `#pragma omp parallel` directive
  - `#pragma omp for` directive
  - `#pragma omp single` directive
- But not after `#pragma omp master` directive!
- If we are sure that there is no need to synchronise threads after the `#pragma omp for` directive, we can use the **`nowait`** clause
- Optimally we need only one `#pragma omp parallel` directive in a fully parallel code
  - OpenMP manages a pool of threads in order to reduce the cost of the `#pragma omp parallel` directive but this is not free, each time OpenMP has to reorganize the pool and wakes up the required threads

Threads Synchronizations – Barriers – Example


```c
// A, B & C <- size = n, D <- size = 2n
void kernel_v1(const float *A, const float *B, const float *C,
               float *D, const float alpha, const size_t n) {
// overhead: threads creation and private variables creation
#pragma omp parallel shared(A, B, D) \
                     firstprivate(alpha, n)
{
#pragma omp for schedule(static,16)
{
  for (size_t i = 0; i < n; i++)
    D[i] = alpha * A[i] + B[i];
} // implicit barrier
} // implicit barrier

// overhead: threads attribution and private variables creation
#pragma omp parallel shared(A, C, D) firstprivate(n)
{
#pragma omp for schedule(static,16)
{
  for (size_t i = 0; i < n; i++)
    D[n + i] = A[i] + C[i];
} // implicit barrier
} // implicit barrier
}
```

```c
// A, B & C <- size = n, D <- size = 2n
void kernel_v2(const float *A, const float *B, const float *C,
               float *D, const float alpha, const size_t n) {
// overhead: threads creation and private variables creation
#pragma omp parallel shared(A, B, C, D) \
                     firstprivate(alpha, n)
{
#pragma omp for schedule(static,16) nowait
{
  for (size_t i = 0; i < n; i++)
    D[i] = alpha * A[i] + B[i];
} // no implicit barrier (nowait clause)

#pragma omp for schedule(static,16)
{
  for (size_t i = 0; i < n; i++)
    D[n + i] = A[i] + C[i];
} // implicit barrier
} // implicit barrier
}

/* 'kernel_v2' is a better version than 'kernel_v1':
 *    - only one parallel zone
 *    - no barrier after the first loop (nowait clause) */
```

Threads Synchronizations – Critical Sections

- Sometimes it is not possible to have a fully parallel code and some regions of the code remain intrinsically sequential
- In OpenMP we can specify this kind of region with the `#pragma omp critical` directive
  - In a critical section, all the threads will execute the code but the execution is made one by one
  - Same as mutual exclusion (mutex) zones in POSIX threads
- But we have to use this directive carefully
  - `#pragma omp critical` can be the main cause of slow down in OpenMP codes!

Threads Synchros – Critical Sections – Example

Scale `A` in `B` and find the minimum value of `B` in `min_val`

```c
float kernel_v1(const float *A, float *B, const size_t n) {
  float min_val = INF;
#pragma omp parallel shared(A, B, min_val) firstprivate(n)
{
#pragma omp for schedule(static,16)
{
  for (size_t i = 0; i < n; i++) {
    B[i] = 0.5f * A[i];
#pragma omp critical  // we want to be sure that only one
{                     // thread can modify min_val
    if (B[i] < min_val)
      min_val = B[i];
}
  }
}
}
  return min_val;
}

/* This code is slow because each loop step contains a
 * sequential part */
```

```c
float kernel_v2(const float *A, float *B, const size_t n) {
  float min_val = INF;
#pragma omp parallel shared(A, B, min_val) firstprivate(n)
{
#pragma omp for schedule(static,16)
{
  for (size_t i = 0; i < n; i++) {
    B[i] = 0.5f * A[i];
    // no more threads synchro to perform the test
    if (B[i] < min_val)
#pragma omp critical
{      // this is very important to re-do the test because
       // an other thread may have modify the min_val value
      if (B[i] < min_val)
        min_val = B[i];
}
  }
}
}
  return min_val;
}
```

Search Algorithms

- In OpenMP 3 there is no optimal solution for search algorithms
- This kind of algorithm typically requires while-loops or do-while-loops
- However there is a tip to fix this lack in OpenMP 3 (see next slide)
- Latest versions of OpenMP (v4 to v6) provides better control of threads
  - We can terminate threads...
  - But this lead to more complex solutions, we will not see them today

Search Algorithms – OpenMP 3 Tip

Search if `val` element is in the `A` array of size `n`

```c
bool search_val_v1(const float *A, const size_t n, float val)
{
  bool found = false;
#pragma omp parallel shared(A, found) firstprivate(val)
{
#pragma omp for schedule(static,16)
{
  for (size_t i = 0; i < n; i++) {
    if (A[i] == val)
      found = true; // no more break, this is valid but
                    // this is also slow, no more early
                    // exit :-(
  }
}
}
  return found;
}
```

```c
bool search_val_v2(const float *A, const size_t n, float val)
{
  bool found = false;
#pragma omp parallel shared(A, found) firstprivate(val)
{
#pragma omp for schedule(static,16)
{
  for (size_t i = 0; i < n; i++) {
    if (!found) // we are doing nothing if we have found
                // the value in the array
      if (A[i] == val)
        found = true;
  }
}
}
  return found;
}
```

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Execution Time

How do you compare two versions of a code that does the same thing (from the functional point of view)?

- Compare the execution time of the two versions
  - The fastest program is the most efficient one
  - Intuitive and worth keeping in mind

- Be careful to compare the same times
  - Classic error: comparing the total execution time of one program with just a sub-part of another program's execution time
    ➔ In this case, the two measured times are not comparable

Execution Time of a Parallel Code

- Let's consider $\mathcal{D}_1$ (or $\mathcal{D}_s$) the sequential time (time on 1 core) of a code
  - With 2 cores, we can hope to divide the time by 2 at best ($\mathcal{D}_2^m \geq \mathcal{D}_s/2$)
  - With 3 cores, we can hope to divide the time by 3 at best ($\mathcal{D}_3^m \geq \mathcal{D}_s/3$)

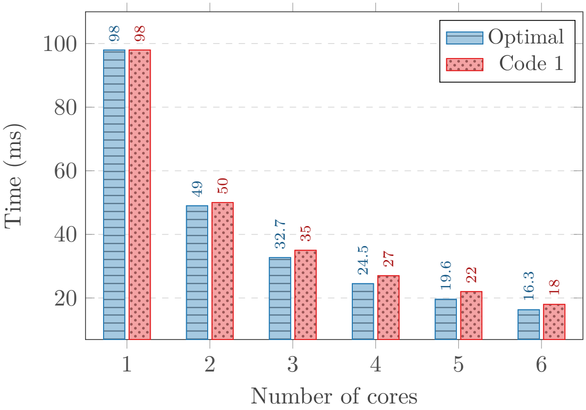
- The following table shows the execution times measured for a `Code 1`:

# core ($\mathcal{C}$)	Measured time ($\mathcal{D}^m$)	Optimal time ($\mathcal{D}^o$)
1	98 ms	98.0 ms
2	50 ms	49.0 ms
3	35 ms	32.7 ms
4	27 ms	24.5 ms
5	22 ms	19.6 ms
6	18 ms	16.3 ms

  - Optimal time $= \mathcal{D}^o = \mathcal{D}_s/\mathcal{C}$

Visualization of the Execution Time

- The previous table is not easy to read
- Let's look at the results on a graph:



Speedup – Definition

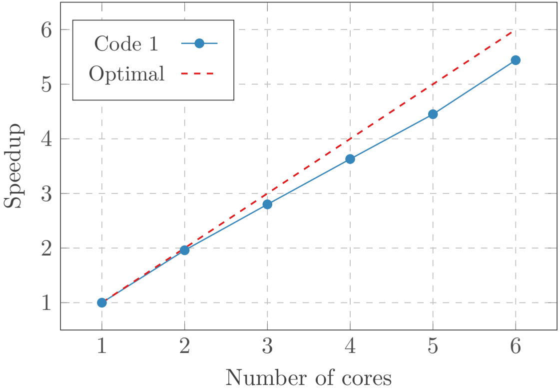
$$\mathcal{S} = \mathcal{D}_s / \mathcal{D}_\mathcal{C},$$

with $\mathcal{D}_s$ the time measured for the 1-core version (= sequential version) of the code and $\mathcal{D}_\mathcal{C}$ the time measured for the parallel version with $\mathcal{C}$ cores.

# cores ($\mathcal{C}$)	Time ($\mathcal{D}^m$)	Speedup ($\mathcal{S}$)
1	98 ms	1.00
2	50 ms	1.96
3	35 ms	2.80
4	27 ms	3.63
5	22 ms	4.45
6	18 ms	5.44

- Sequential time is used as reference time

Speedup – Visualization



- Visually very simple to see if `Code 1` is close to the the optimal
- Optimal speedup is equal to the number of cores used (no more!)

Amdahl's Law

- Can we increase parallelism indefinitely to speedup our codes?
  - Amdahl said no!
  - To be more precise, it depends on the characteristics of the code...
  - If the code is fully parallelizable: speedup is infinite
  - If the code is NOT fully parallelizable: there's a limit

$$\mathcal{S}_{\text{max}} = \frac{1}{1 - f\mathcal{D}_p},$$

with $\mathcal{S}_{\text{max}}$ the maximum achievable speedup and $f\mathcal{D}_p$ the fraction of parallel time in the code ($0 \leq f\mathcal{D}_p \leq 1$).

Amdahl's Law – Example

- Let's take a code composed of two parts:
  - 20 % is intrinsically sequential
  - 80 % can be parallelized

- What is the maximum achievable speedup?

$$\mathcal{S}_{\max} = \frac{1}{1 - f\mathcal{D}_p} = \frac{1}{1 - 0.8} = \frac{1}{0.2} = 5.$$

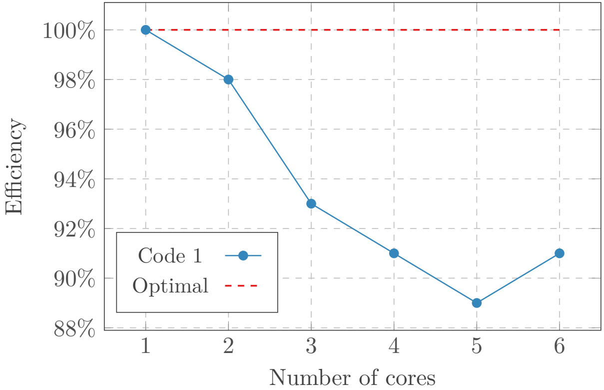
That's not much when you consider architectures with dozens of CPU cores!

Efficiency – Definition

- Several ways to define the efficiency ($\mathcal{E}$) of a code
  — From the speedup: $\mathcal{E} = \mathcal{S}^m / \mathcal{S}^o$
  — From the execution time: $\mathcal{E} = \mathcal{D}^m / \mathcal{D}^o$
- Efficiency is a ratio: $0\% < \mathcal{E} \leq 100\%$
- For `Code 1`, the efficiency as a function of the number of cores:

# cores ($\mathcal{C}$)	Time ($\mathcal{D}^m$)	Speedup ($\mathcal{S}$)	Efficiency ($\mathcal{E}$)
1	98 ms	1.00	100%
2	50 ms	1.96	98%
3	35 ms	2.80	93%
4	27 ms	3.63	91%
5	22 ms	4.45	89%
6	18 ms	5.44	91%

Efficiency – Visualization



- Equivalent to the speedup, at least for now...

Scalability of a Code – Definition

- The scalability of a code is its capacity to be efficient when the number of cores increases
  - A code scales if it is able to benefit from the power of several cores
- How do we measure a code's scalability? How do we know if a code doesn't scale?
  - No simple answer
- 2 widely used models for characterizing the scalability of parallel code:
  - The "strong" scalability
  - The "weak" scalability

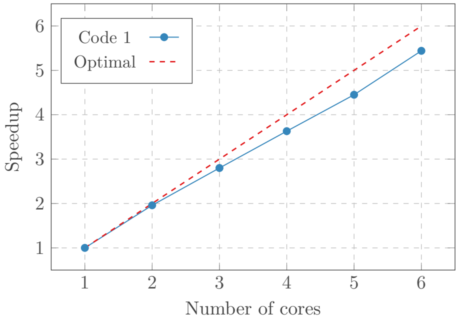
Strong Scalability – `Code 1` Example

- Measures execution time as a function of the number of cores
- With **a constant problem size**
- For example, for `Code 1`, with a problem of size 100:

# cores	Size	Time	Speedup
1	100	98 ms	1.00
2	100	50 ms	1.96
3	100	35 ms	2.80
4	100	27 ms	3.63
5	100	22 ms	4.45
6	100	18 ms	5.44

Strong Scalability – `Code 1` Visualization

Strong scalability is generally observed on an speedup graph:



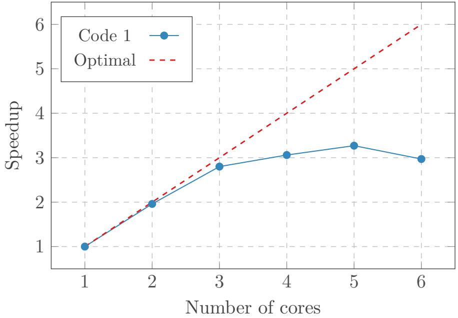
Here, for 6 cores, `Code 1` achieves a speedup of 5.4, so we can conclude that this code scales well up to 6 cores.

Strong Scalability – `Code 2` Example

- Let's consider a new `Code 2`
- Here are the measurements for this code:

# cores	Size	Time	Speedup
1	100	98 ms	1.00
2	100	50 ms	1.96
3	100	35 ms	2.80
4	100	32 ms	3.06
5	100	30 ms	3.27
6	100	33 ms	2.97

Strong Scalability – `Code 2` Visualization



- We can see that the strong scalability of `Code 2` is poor
- Above a certain number of cores, parallelism can no longer speedup code :-(

Weak Scalability

- This model considers the execution time as a function of the number of cores
- **And the problem size increases in proportion to the number of cores!**
- Compute the *speedup* makes no sense if the problem size is not constant
- BUT it is possible to compute the efficiency: $\mathcal{E} = \mathcal{D}^s/\mathcal{D}^m$

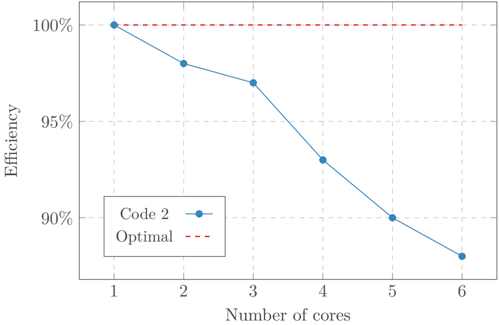
Intuition: if we can't compute a problem of a given size any faster, can we compute a bigger problem in the same time?

Most of the time, yes, and it's easier! This is what happens most of the time in high performance computing: scientific models become more and more refined = the size of the problem increases.

Weak Scalability – `Code 2` Example

Measures for `Code 2`:

# cores	Size	Time	Efficiency
1	100	098 ms	100%
2	200	100 ms	98%
3	300	101 ms	97%
4	400	105 ms	93%
5	500	109 ms	90%
6	600	111 ms	88%

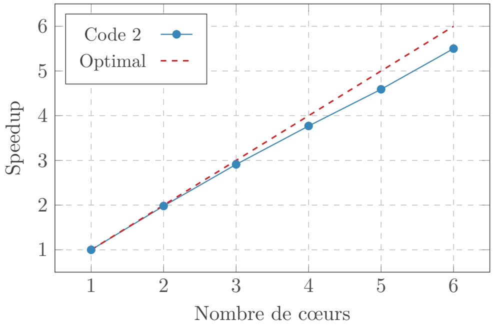


- The weak scalability of `Code 2` is good ($\approx$ 90% for 6 cores)
- Why is strong scalability bad?
  - Amdahl's law: not enough parallelism for a small problem size

Strong Scalability – `Code 2` AGAIN!

- New problem size: 600

# cores	Size	Time	Speedup
1	600	611 ms	1.00
2	600	308 ms	1.98
3	600	210 ms	2.91
4	600	162 ms	3.77
5	600	133 ms	4.59
6	600	111 ms	5.50



- For larger problem sizes, strong scalability is good
- Not always possible to test for strong scalability: lack of time
- Not always possible to test for weak scalability: impossible to have ever larger problem sizes

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4 Kernel Performance Analysis

Number of Arithmetic Operations

- The number of arithmetic operations in a code is an **important characteristic**
- Example of the number of float operations (*flops*) in a kernel that performs a sum:

```c
float sum(float *values, size_t n) {
  float sum = 0.f;

  // total flops = n * 1
  for (size_t i = 0; i < n; i++)
    sum = sum + values[i]; // 1 flop because of the addition

  return sum;
}
```

Number of Operations per Second

- Metric widely used in high performance computing, particularly the number of **floating-point** operation per second (flop/s)
  - The same is defined for integer operation (iop/s)
  - Or simply the number of operation per second (op/s), or *Million Instructions Per Second* (MIPS)
    - ➔ This metrics is used in the kernel Linux (`cat /proc/cpuinfo`)

- The flop/s ratio can be directly compared with the peak performance of a computing architecture

- A metric to estimate the good use (or not) of the hardware architecture

Processor Peak Performance

- **The processor's maximum computational capacity**
- It can be deduced from our knowledge of the hardware architecture:

$$peakPerf = nOps \times freq \times nCores,$$

with $nOps$ the number of operations the architecture can achieve in one cycle (ILP), $freq$ the processor frequency and $nCores$ the number of processor cores.

Processor Peak Performance – Example

CPU name	Core i7-2630QM
Architecture	Sandy Bridge
Vect. inst.	AVX-256 bit (4 double, 8 single)
Frequency	2 GHz
Nb. cores	4

Peak performance in floating-point single precision:

$$peakPerf_{sp} = nOps \times freq \times nCores = (2 \times 8) \times 2 \times 4 = 128\ Gflop/s$$

Peak performance in floating-point double precision:

$$peakPerf_{dp} = nOps \times freq \times nCores = (2 \times 4) \times 2 \times 4 = 64\ Gflop/s$$

- $nOps = 2 \times vectorSize$ because the architecture back-end allows to issue 2 instructions par cycle (`vadd` et `vmul`)

Arithmetic Intensity

- Sometimes (even often) measured op/s are far from peak performance
  - Code is poorly optimized
  - Peak performance cannot be achieved
  - In most cases, both are true...
- With **arithmetic intensity** we take into account **memory accesses**:

$$AI = \frac{ops}{memops}.$$

Arithmetic Intensity – Example


```c
float sum(float *values, size_t n) {
  float sum = 0.f; // do not count the memory acces to 'sum' because it will be optimized in register
  // total flops = n * 1  ||  total memops = n * 1
  for (size_t i = 0; i < n; i++)
    sum = sum + values[i]; // 1 flop because of the addition, 1 memop because of one acces in
                           // the 'values' array
  return sum;
}
```

- Arithmetic intensity of `sum` is: $AI_{\texttt{sum}} = \frac{n \times 1}{n \times 1} = 1$
- The higher the arithmetic intensity, the more the code is limited by computational units
- The lower the arithmetic intensity, the more the code is limited by memory accesses

Operational Intensity

- Compared to arithmetic intensity, operational intensity takes into account **the size of the data in memory**:

$$OI = \frac{flops}{memops \times sizeOfData} = \frac{AI}{sizeOfData},$$

$sizeOfData$ depends on the datatype, `int` and `float` use 4 bytes, `long long int` and `double` use 8 bytes.

- In the previous code (`sum`), the memory accesses are made on `float` and the operational intensity is: $OI_{\texttt{sum}} = \frac{n \times 1}{(n \times 1) \times 4} = \frac{1}{4}$

Operational Intensity – Example

Sum in single precision:

```c
// AI = 1  ||  OI = 1/4
float sum1(float *values, size_t n) {
  float sum = 0.f;
  for (size_t i = 0; i < n; i++)
    sum = sum + values[i];
  return sum;
}
```

Sum in double precision:

```c
// AI = 1  ||  OI = 1/8
double sum2(double *values, size_t n) {
  double sum = 0.0;
  for (size_t i = 0; i < n; i++)
    sum = sum + values[i];
  return sum;
}
```

- `sum1` and `sum2` kernels have the same arithmetic intensity
- The operational intensity of `sum1` is higher than that of `sum2`
  - `sum2` kernel is more limited by memory access than `sum1` kernel

Roofline Model

- Roofline[1] is model that limits **the maximum achievable performance**
- Takes into account
  - Memory bandwidth (RAM)
  - Processor peak performance
- Depending on operational intensity, code is limited either by memory bandwidth or by processor peak performance

$$\text{Attainable op/s} = \min \begin{cases} \text{Peak CPU op/s performance,} \\ \text{Peak memory bandwidth} \times OI. \end{cases}$$

[1] S. Williams, A. Waterman, and D. Patterson. "Roofline: An Insightful Visual Performance Model for Multicore Architectures". In: *ACM Communications* 52.4 (Apr. 2009), pp. 65–76. DOI: 10.1145/1498765.1498785.

Memory Bandwidth Measurement

- Memory bandwidth or memory throughput represents the number of bytes that can be read/written from RAM in one second (o/s or GB/s)
- How to know the memory bandwidth?
  - It is possible to compute its theoretical value
  - But we often prefer to use a micro-benchmark program: "STREAM"[1] or "bandwidth"[2]
- STREAM (University of Virginia) is a small, relatively simple code for measuring memory bandwidth → C code
- bandwidth (Sorbonne University) targets the same features as STREAM but in a more friendly-user and accurate way → C++ code

[1] STREAM: https://www.cs.virginia.edu/stream/

[2] bandwidth: https://github.com/alsoc/bandwidth

Roofline Model – Example – Part 1

Let's take the same processor as before:

CPU name	Core i7-2630QM
Peak perf sp	128 GFlop/s
Peak perf dp	64 GFlop/s
Mem. bandwidth	17.6 GB/s

Single precision sum:

```c
// AI = 1  ||  OI = 1/4
float sum1(float *values, size_t n) {
  float sum = 0.f;
  for (size_t i = 0; i < n; i++)
    sum = sum + values[i];
  return sum;
}
```

Double precision sum:

```c
// AI = 1  ||  OI = 1/8
double sum2(double *values, size_t n) {
  double sum = 0.0;
  for (size_t i = 0; i < n; i++)
    sum = sum + values[i];
  return sum;
}
```

Roofline Model – Example – Part 2

Peak perf sp	128 GFlop/s
Peak perf dp	64 GFlop/s
Mem. bandwidth	17.6 GB/s

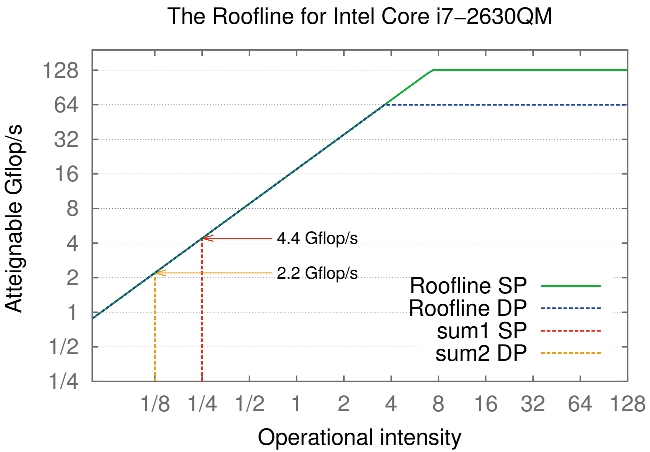
For `sum2`, operational intensity is: $OI_{sum2} = 1/8$.
Let's apply the Roofline model:

$$\text{Attainable Gflop/s} = \min \begin{cases} \text{Peak floating point performance,} \\ \text{Peak memory bandwidth} \times OI. \end{cases}$$

$$\Rightarrow$$

$$\text{Attainable Gflop/s}_{\texttt{sum2}} = \min \begin{cases} 64 \text{ Gflop/s,} \\ 17.6 \times \frac{1}{8} \text{ Gflop/s.} \end{cases} = 2.2 \text{ Gflop/s}$$

Roofline Model – Example – Visualization



The Roofline for Intel Core i7–2630QM

Q&A

Thank you for listening!
Do you have any questions?

Bibliography
5 References

[1] S. Williams, A. Waterman, and D. Patterson. "Roofline: An Insightful Visual Performance Model for Multicore Architectures". In: *ACM Communications* 52.4 (Apr. 2009), pp. 65–76. DOI: 10.1145/1498765.1498785.