# Learning to Rank Patients Severity of Illness with Partially Observable Data

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#### **Overview**

- Ranking the severity of illness with only partially observable data.
- ► Submodular optimization *Maximum Coverage* with cardinality constraint.
- ► Learning from data *Distributional Optimization from Samples* (DOPS) [2].

# **Backgrounds**

- ► *ICU is extremely expensive*. In 2005, cost of ICU was roughly \$82 billion (0.66% of the United States GDP). To reduce cost, ICU resources will become limited. Hence, most effective allocation of resources is imperative.
- Learning to Rank Allowing general structured output labels, Structured SVM [3] becomes a handy ranking model for data mining and information retrieval.
- Submodularity & Optimization from Samples
  Submodular function is not always accessible but
  learnable. DOPS provides reliable, efficient and scalable
  submodular optimization by learning from data within
  same distribution.

# Maximum Coverage Problem

- A classical NP-hard problem with widespread application.
- The corresponding coverage function is a simple, yet important and widely used class of submodular functions.
- Extension: Weighted Maximum Coverage. Let  $\mathcal{U}$  be a ground truth set containing d items with corresponding non-negative weights  $\Theta$  and C be a collection of n subsets of  $\mathcal{U}$ , i.e.,  $\mathcal{U} = \{u_i\}_{i=1}^d, \Theta = \{\theta_i\}_{i=1}^d, C = \{C_j\}_{j=1}^n$ , where  $\forall i \in [d], \theta_i \geq 0$  and  $\forall j \in [n], C_j \subseteq \mathcal{U}$ . Given S which is a subset of C, a function  $f_{\theta}: 2^{[n]} \to \mathbb{R}$  is a coverage function if:

$$f_{\Theta}(S) = \sum_{u_i \in C(S)} \theta_i, \quad C(S) = \bigcup_{C_j \in S} C_j, \quad |S| \le k$$
 (1)

The objective is to find S with cardinality constraint such that  $f_{\theta}(S)$  is maximized.

## **Methods**

## **Optimize Patients Severity of Illness**

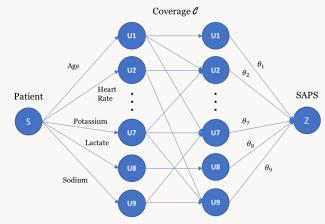
#### Motivation

 Demographic and physiological indices are innately connected.

#### Measurement Transformation

- Two-level transformation continuous to binary.
- Multi-level transformation continuous to one-hot encodings of multiple levels.

#### Formulation: State-cover-State



# Learnable Optimization Problem

- Given the training set  $S = \{S_i\}_{i=1}^n$  and test set  $T = \{T_i\}_{i=1}^m$ , where  $S_i$  and  $T_i$  contain k observed measurements, we use DOPS to learn f from S, and find  $T = \operatorname{argmax}_{T_i \in T} f_{\Theta}(T_i)$ .
- With the spirit of DOPS and Structured SVM, we first shuffle data and divide into  $N = \lfloor \frac{n}{m} \rfloor$  *m*-tuple sample sets  $S = \{(S^i, \mathbf{z}^i)\}_{i=1}^N$ , then solve the following program:

$$\min_{\Theta \in \mathbb{R}^{d}, \xi_{i} \in \mathbb{R}} \quad \frac{1}{2} ||\Theta||^{2} + \frac{\lambda}{N} \sum_{i=1}^{N} \xi_{i}$$
s.t.  $\phi_{\Theta}(\mathbf{S}^{i}, \mathbf{z}^{i}) - f_{\Theta}(S_{y}^{i}) + \xi_{i} \geq \Delta_{\alpha}(\mathbf{z}^{i}, y), \ \forall i \in [N], \forall y \in [m]$ 

$$\xi_{i} \geq 0, \ \forall i \in [N]$$

$$\theta_{i} \geq 0, \ \forall i \in [d]$$

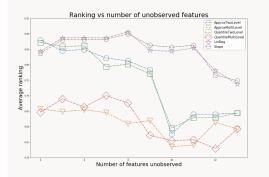
where  $\Delta_{\alpha}(\mathbf{z}^{i}, y) = \mathbb{1}_{\{y \notin \alpha(\mathbf{z}^{i})\}}, \alpha(\mathbf{z}) = \{y \in [m] : z_{y} \geq \alpha \max \mathbf{z}\}$  and  $\phi_{\Theta}(\mathbf{S}^{i}, \mathbf{z}^{i}) = \frac{1}{|\alpha(\mathbf{z}^{i})|} \sum_{y \in \alpha(\mathbf{z}^{i})} f_{\Theta}(S_{y}^{i})$ .

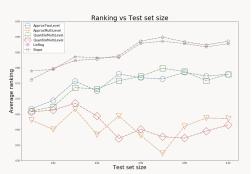
 By converting (2) to convex and unconstrained program with average hinge loss, it is easy to be solved using subgradient descent.

# **Experiments**

- ► **MIMIC-III Dataset** 13,500 ICU patients records [1].
- Measurement Masking set values of "unobservable" measurements to 0.
- Baseline Linear Regression

## **Results and Discussion**





#### Results

- Linear regression outperforms both DOPS models with different number of unobservable measurements and test size.
- Two-level coverage model is better than multi-level one.
- As the number of unobservable measurements increases, the ranking performance deteriorate

#### Future Works

- Use Neural Network to learn the binary representation of current continuous input vectors.
- Establish coverage C rigorously by consulting medical experts.
- Incorporate treatment effect into our current maximum coverage framework to predict patient's severity score.

# Reference

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- [3] I. Tsochantaridis, T. Hofmann, T. Joachims, and Y. Altun. Support vector machine learning for interdependent and structured output spaces. In *Proceedings of the twenty-first international conference on Machine learning*, page 104. ACM, 2004.