

## Ejercicios Tema 2:

1.

$$\left. \begin{array}{l} x_1 = N^{\circ} \text{ días mina 1} \\ x_2 = N^{\circ} \text{ días mina 2} \\ x_3 = N^{\circ} \text{ días mina 3} \end{array} \right\}$$

$$\left. \begin{array}{l} \min f = 20x_1 + 22x_2 + 28x_3 \\ \text{s.a. } 4x_1 + 6x_2 + x_3 \geq 84 \\ 4x_1 + 4x_2 + 6x_3 \geq 65 \\ x_1 + x_2 + x_3 \leq 7 \\ x_1, x_2, x_3 \geq 0 \end{array} \right.$$

2.

$$\left. \begin{array}{l} x_1 = N^{\circ} \text{ Unidades Modelo 1} \\ x_2 = N^{\circ} \text{ Unidades Modelo 2} \\ x_3 = N^{\circ} \text{ Unidades Modelo 3} \\ x_4 = N^{\circ} \text{ Unidades Modelo 4} \end{array} \right\}$$

$$\left. \begin{array}{l} \max f = 7x_1 + 7x_2 + 6x_3 + 9x_4 \\ \text{s.a. } 2x_1 + 2.5x_2 + 3x_3 + 3x_4 \leq 30.000 \\ 4x_1 + 5x_2 + 3x_3 + 5x_4 \leq 20.000 \\ x_1, x_2, x_3, x_4 \geq 0 \end{array} \right.$$

3.  $x_1 = \text{Artículo 1}$

$$x_2 = \text{Artículo 2}$$

$$x_3 = \text{Artículo 3}$$

$$x_4 = \text{Artículo 4}$$

$$x_5 = \text{Artículo 5}$$

$$\left. \begin{array}{l} \max f = 100x_1 + 60x_2 + 70x_3 + 15x_4 + 15x_5 \\ \text{s.a. } 52x_1 + 23x_2 + 35x_3 + 15x_4 + 7x_5 \leq 60 \\ x_1, x_2, x_3, x_4, x_5 \geq 0 \\ x_1, x_2, x_3, x_4, x_5 \leq 1 \end{array} \right.$$

4.  $x_1 = N^{\circ} \text{ Juegos Tipo 1}$

$$x_2 = N^{\circ} \text{ Juegos Tipo 2}$$

$$\left. \begin{array}{l} \max f = 29x_1 + 38x_2 \\ \text{s.a. } 3.5x_1 + 4x_2 \leq 50 \\ x_1, x_2 \geq 0 \end{array} \right.$$

5.  $x_1 = N^{\circ}$  Alimentos A

$x_2 = N^{\circ}$  Alimentos B

$x_3 = N^{\circ}$  Alimentos C

$x_4 = N^{\circ}$  Alimentos D

$x_5 = N^{\circ}$  Alimentos E

$x_6 = N^{\circ}$  Alimentos F

} Min  $f = 2x_1 + 3x_2 + 5x_3 + 6x_4 + 8x_5 + 8x_6$

s.a.  $20x_1 + 30x_2 + 40x_3 + 40x_4 + 45x_5 + 30x_6 \geq 70$

$30x_1 + 30x_2 + 20x_3 + 25x_4 + 50x_5 + 20x_6 \geq 100$

$4x_1 + 9x_2 + 11x_3 + 10x_4 + 9x_5 + 10x_6 \geq 20$

$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$

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6.  $x_1 = N^{\circ}$  Unidades Prod 1

$x_2 = N^{\circ}$  Unidades Prod 2

$x_3 = N^{\circ}$  Unidades Prod 3

$x_4 = N^{\circ}$  Unidades Prod 4

} Max  $f = 6x_1 + 4x_2 + 6x_3 + 8x_4$

s.a.  $2x_1 + 3x_2 + 2x_3 + 4x_4 \leq 480$

$x_1 + x_2 + 2x_3 + 3x_4 \leq 400$

$2x_1 + x_2 + 2x_3 + x_4 \leq 400$

$x_1 \geq 50 \quad x_4 \leq 25$

$x_1 + x_2 + x_3 \geq 100$

$x_1, x_2, x_3, x_4 \geq 0$

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7.  $x_1 = N^{\circ}$  Días Planta A

$x_2 = N^{\circ}$  Días Planta B

$x_3 = N^{\circ}$  Días Planta C

}

Min  $f = 210.000x_1 + 190.000x_2 + 182.000x_3$

s.a.  $110x_1 + 65x_2 \geq 1500$

$35x_1 + 53x_3 \geq 1100$

$x_1 + x_2 + x_3 \leq 30$

$x_1, x_2, x_3 \geq 0$

8.a

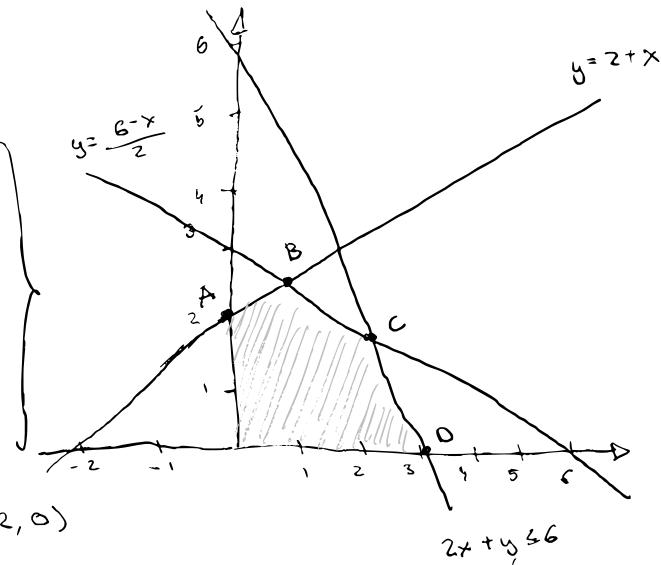
$$\text{Max } x+y$$

$$\text{s.a. } -x+y \leq 2$$

$$x+2y \leq 6$$

$$2x+y \geq 6$$

$$x, y \geq 0$$



$$y = 2+x \Rightarrow (0,2)/(-2,0)$$

$$y = \frac{6-x}{2} \Rightarrow (0,6)/(6,0)$$

$$y = 6-2x \Rightarrow (0,6)/(3,0)$$

$$\text{Punto A: } (0,2) \Rightarrow 2$$

$$\text{Punto B: } \left(\frac{2}{3}, \frac{8}{3}\right) \Rightarrow \frac{10}{3}$$

$$\left. \begin{array}{l} y = \frac{6-x}{2} \\ y = 2+x \end{array} \right\} 4+2x = 6-x \Rightarrow 3x = 2 \Rightarrow x = \frac{2}{3}$$

$$\boxed{\begin{array}{l} x = \frac{2}{3} \\ y = \frac{8}{3} \end{array}}$$

$$\text{Punto C: } (2,2) \Rightarrow 4 \Rightarrow \boxed{\text{Máximo}}$$

$$\left. \begin{array}{l} y = 6-2x \\ y = \frac{6-x}{2} \end{array} \right\} 12-4x = 6-x \Rightarrow 6 = 3x \Rightarrow x = 2$$

$$y = 2$$

$$\text{Punto D: } (3,0) \Rightarrow 3$$

8. b

$$\text{Max } x + 3y$$

$$\text{sa } x + y \leq 6$$

$$-x + 2y \leq 8$$

$$x, y \geq 0$$

$$y = 6 - x \Rightarrow (0, 6) / (6, 0)$$

$$y = \frac{8+x}{2} \Rightarrow (0, 4) / (-8, 0)$$

Punto A:  $(0, 4) \Rightarrow 12$

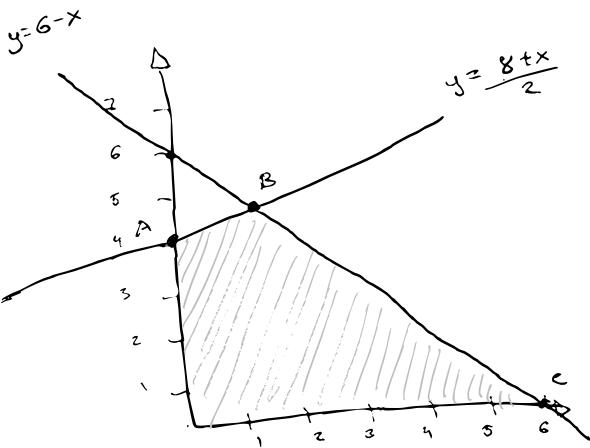
Punto B:  $(\frac{4}{3}, \frac{14}{3}) \Rightarrow 15\frac{1}{3} \Rightarrow \boxed{\text{Máximo}}$

$$\begin{cases} y = \frac{8+x}{2} \\ y = 6 - x \end{cases}$$

$$12 - 2x = 8 + x \Rightarrow 4 = 3x \Rightarrow$$

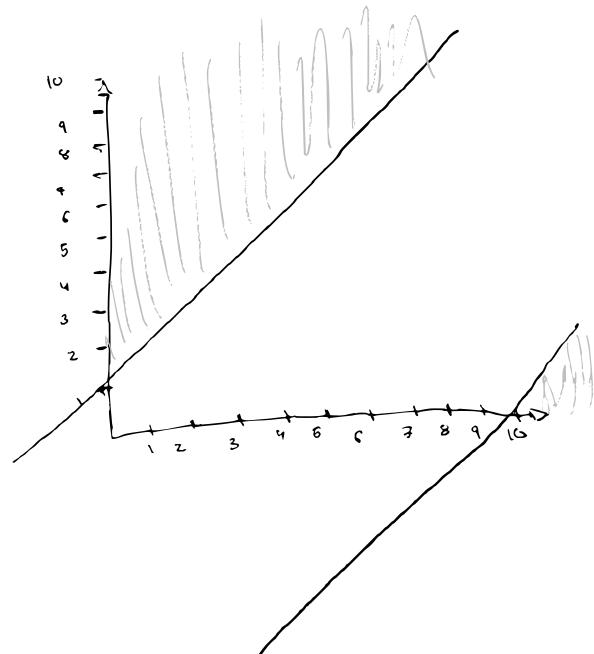
$$\boxed{\begin{array}{l} x = \frac{4}{3} \\ y = \frac{14}{3} \end{array}}$$

Punto C:  $(6, 0) \Rightarrow 6$



8.c

$$\left. \begin{array}{l} \text{Max } -x-y \\ \text{s.a } x-y \geq 10 \\ -x+y \geq 2 \\ x, y \geq 0 \end{array} \right\}$$



$$y = x - 10 \Rightarrow (0, -10) / (10, 0)$$

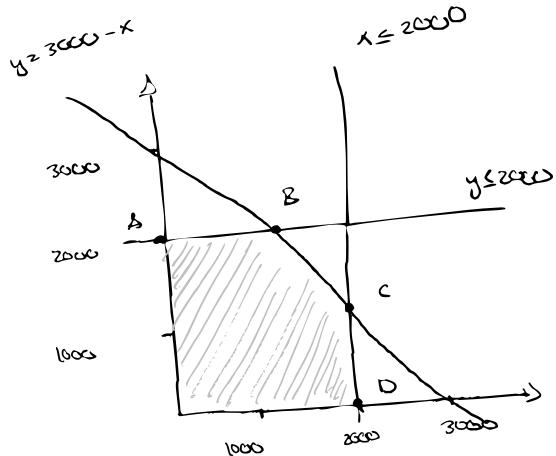
$$y = 2 + x \Rightarrow (0, 1) / (-1, 0)$$

\* Regiones Incompatibles

9. Max  $f = 10x + 15y$

s.a.

$$\begin{aligned} x + y &\leq 3000 \\ x &\leq 2000 \\ y &\leq 2000 \\ x, y &> 0 \end{aligned}$$



$$y = 3000 - x \Rightarrow (0, 3000) / (3000, 0)$$

Punto A:  $(0, 2000) \Rightarrow 30.000$

Punto B:  $(2000, 2000) \Rightarrow 40.000 \Rightarrow \boxed{\text{Máximo}}$

$$\begin{aligned} y &= 2000 \\ y &= 3000 - x \end{aligned} \quad \left\{ \begin{array}{l} 2000 = 3000 - x \Rightarrow \\ \boxed{x = 2000} \\ \boxed{y = 2000} \end{array} \right.$$

Punto C:  $(2000, 1000) \Rightarrow 35.000$

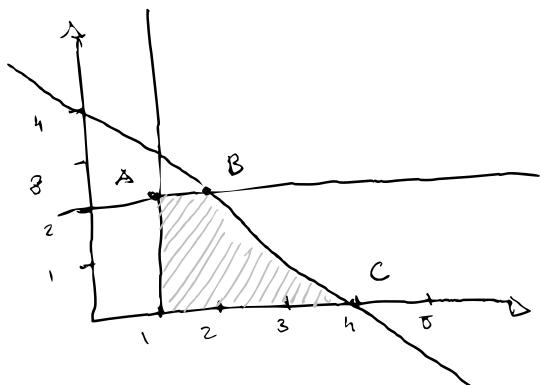
$$\begin{aligned} x &= 2000 \\ x &= 3000 - y \end{aligned} \quad \left\{ \begin{array}{l} 2000 = 3000 - y \Rightarrow \\ y = 1000 \end{array} \right.$$

$$\boxed{\begin{array}{l} y = 1000 \\ x = 2000 \end{array}}$$

Punto D:  $(2000, 0) \Rightarrow 20.000$

20. a

$$\left. \begin{array}{l} \text{Max } x+y \\ \text{s.a. } x+y \leq 4 \\ x \geq 2 \\ y \leq 2 \\ x, y \geq 0 \end{array} \right\}$$



$$y = 4 - x \Rightarrow (0,4) / (4,0)$$

Punto A:  $(2,2) \Rightarrow 3$

Punto B:  $(2,0) \Rightarrow 4$

Punto C:  $(0,4) \Rightarrow 4$

Solución segmentos B-C

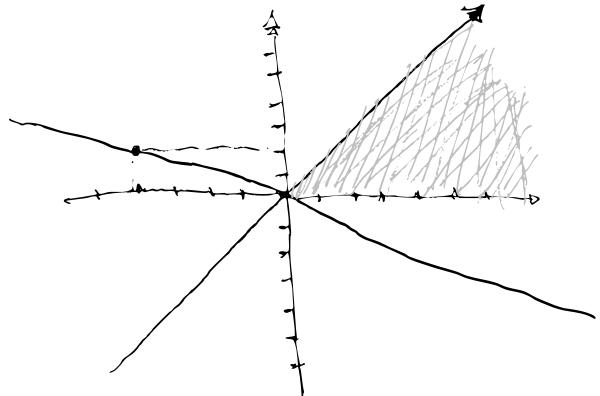
20. b.

$$\left. \begin{array}{l} \text{Max } x+y \\ \text{s.a. } x+2y \geq 0 \\ x-y \geq 0 \\ x, y \geq 0 \end{array} \right\}$$

$$y = \frac{-x}{2} \Rightarrow (0,0) / (-4,2)$$

$$y = x$$

\*No óptima

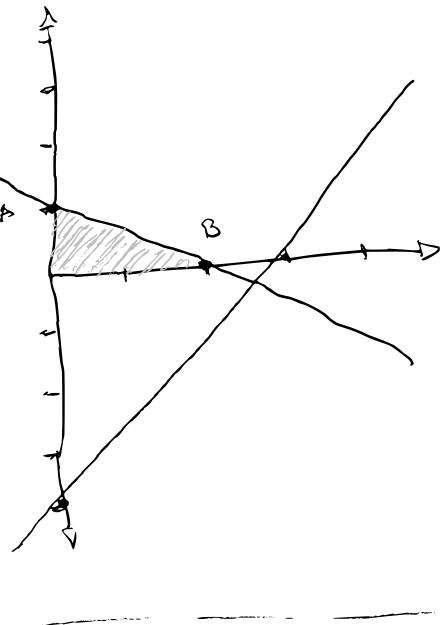


10. c.

$$\begin{array}{l} \text{Max } 3x + 4y \\ \text{s.a. } x + 2y \leq 2 \\ \quad 4x - 3y \leq 12 \\ \quad x, y \geq 0 \end{array}$$

$$y = \frac{2-x}{2} \Rightarrow (0, 1) / (2, 0)$$

$$y = \frac{12-4x}{-3} \Rightarrow (0, -4) / (3, 0)$$



Punto A:  $(0, 1) \Rightarrow 4$

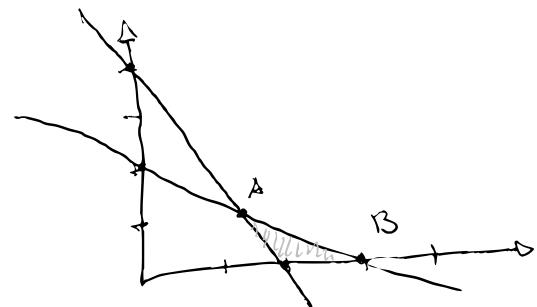
Punto B:  $(2, 0) \Rightarrow 6 \Rightarrow \boxed{\text{Máximo}}$

10. d.

$$\begin{array}{l} \text{Max } x+y \\ \text{s.a. } 3x + 2y \geq 6 \\ \quad 2x + 4y \leq 8 \\ \quad x, y \geq 0 \end{array}$$

$$y = \frac{6-3x}{2} \Rightarrow (0, 3) / (2, 0)$$

$$y = \frac{8-2x}{4} \Rightarrow (0, 2) / (4, 0)$$



Punto A:  $(1, 3/2) \Rightarrow 5/2$

$$y = \frac{6-3x}{2} \quad 24 - 12x = 16 - 4x \Rightarrow 8 = 8x \Rightarrow \boxed{\begin{array}{l} x = 1 \\ y = 3/2 \end{array}}$$

Punto B:  $(0, 3) \Rightarrow 3 \Rightarrow \boxed{\text{Máximo}}$

- Exercise 2:

$$\left\{ \begin{array}{l} x \Rightarrow \text{late trips A} \\ y \Rightarrow \text{late trips B} \end{array} \right\}$$

$$\text{Max } 32.25x + 12.5y$$

$$\text{s.a. } x + 2y \leq 400$$

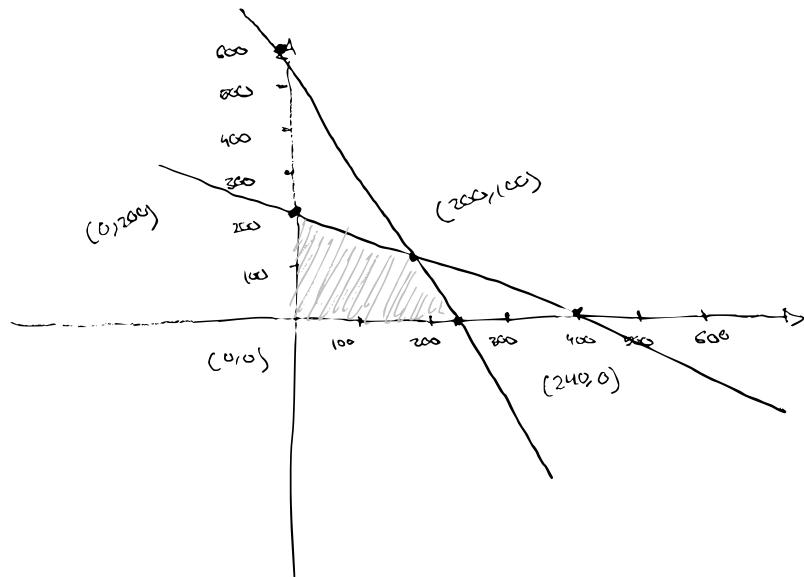
$$5x + 2y \leq 2200$$

$$x, y \geq 0$$


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$$y = \frac{400 - x}{2} \Rightarrow (0, 200) / (400, 0)$$

$$y = \frac{1200 - 5x}{2} \Rightarrow (0, 600) / (240, 0)$$



$$\left. \begin{array}{l} y = \frac{400 - x}{2} \\ y = \frac{1200 - 5x}{2} \end{array} \right\} 800 - 2x = 2400 - 10x \Rightarrow 8x = 1600$$

$$\boxed{\begin{array}{l} x = 200 \\ y = 100 \end{array}}$$

- Ejercicio 2:

$x$  = Avión Tipo A

$y$  = Avión Tipo B

$$\text{Min } 80.000x + 20.000y$$

$$\text{s.a. } 600x + 300y \leq 4800$$

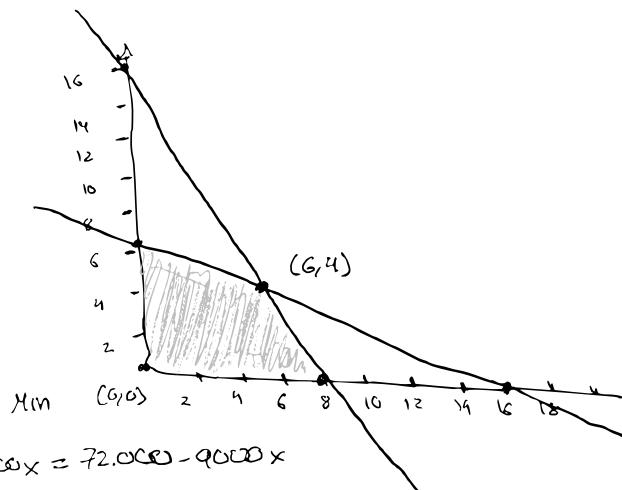
$$6x + 15y \leq 96$$

$$x \leq 22$$

$$y \leq 32$$

$$y = \frac{4800 - 600x}{300} \Rightarrow (0, 16) / (8, 0)$$

$$y = \frac{96 - 6x}{15} \Rightarrow (0, 6.4) / (16, 0)$$



$$28.800 - 1800x = 72.000 - 9000x$$

$$7200x = 43.200$$

$$\boxed{\begin{array}{l} x=6 \\ y=4 \end{array}} \quad \Rightarrow 560.000$$

$$(6, 4) \Rightarrow 128.000$$

640.000

Ejercicio 3:

$$\begin{cases} x = \text{Albaricoque} \\ y = \text{Cítrica} \end{cases}$$

$$\text{Max } 60x + 80y$$

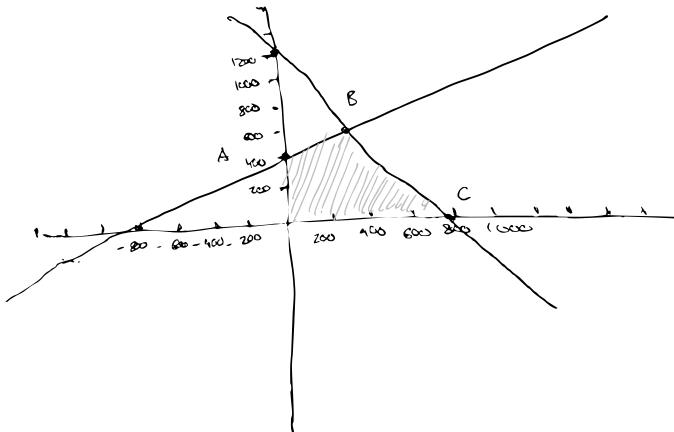
$$\text{s.a. } 2y \leq x + 800$$

$$3x + 2y \leq 2400$$

$$x, y \geq 0$$

$$y = \frac{x + 800}{2} \Rightarrow (0, 400) / (-800, 0)$$

$$y = \frac{-2400 - 3x}{2} \Rightarrow (0, -1200) / (800, 0)$$



Punto A (0, 400)  $\Rightarrow 32.000 \text{ €}$

Punto C (800, 0)  $\Rightarrow 48.000 \text{ €}$

Punto B: (400, 600)  $\Rightarrow 72.000 \text{ €}$   $\Rightarrow$  Máximo

$$2x + 1600 = 4800 - 6x \Rightarrow 8x = 3200 \Rightarrow$$

$x = 400$
$y = 600$

## Métodos Cuantitativos Tema 3:

Ejercicio 3: Expresa en forma canónica y extender

$$\text{Max } z = x + y$$

$$\text{s.a. } -x + y = 2$$

$$x + 2y \leq 6$$

$$2x + y \geq 6$$

$$x \geq 0 \quad y \leq 0$$

- Forma extender:

$$\text{Max } f = x - y_1$$

$$\text{s.a. } -x - y_1 = 2$$

$$x - 2y_1 + a = 6$$

$$2x - y_1 - b = 6$$

$$x \geq 0 \quad y_1 \geq 0$$

$$a, b \geq 0$$

$$\boxed{y = -y_1}$$

- Forma canónica:

$$\text{Max } f = x - y_1$$

$$\boxed{y = -y_1}$$

$$\text{s.a. } -x - y_1 \leq 2$$

$$x + y_1 \geq -2$$

$$x - 2y_1 \leq 6$$

$$-2x + y_1 \leq 6$$

$$x, y_1 \geq 0$$

## Relación ejercicios tema 3:

2. b) Max  $2x + 3y + z$   
s.a.  $4x + 3y + z \leq 20$   
 $x + y \leq 20$   
 $x, y \geq 0$

• Forma estándar:

$$\begin{aligned} \text{Max } & 2x - 3y_1 + (z_1 - z_2) + 0 \cdot a + 0 \cdot b \\ \text{s.a. } & 4x - 3y_1 + (z_1 - z_2) + a = 20 \\ & x - y_1 + b = 20 \\ & x, y_1, z_1, z_2, a, b \geq 0 \end{aligned}$$

$$z = z_1 - z_2$$

$$y = -y_1$$

• Forma canónica:

$$\begin{aligned} \text{Max } & 2x - 3y_1 + (z_1 - z_2) \\ \text{s.a. } & 4x - 3y_1 + (z_1 - z_2) \leq 20 \\ & x - y_1 \leq 20 \\ & x, y_1, z_1, z_2 \geq 0 \end{aligned}$$

$$z = z_1 - z_2$$

$$y = -y_1$$

## Ejercicio 2 c)

$$\text{Min } x + cy$$

$$\text{s.a. } -x + y \leq 2$$

• Forma estandar:

$$x = x_1 - x_2 \quad y = y_1 - y_2$$

$$\text{Max } f = -x_1 + x_2 - y_1 + y_2$$

$$\text{s.a. } -x_1 + x_2 + y_1 - y_2 + s_1 = 2$$

$$x_1, x_2, y_1, y_2, s_1 \geq 0$$

• Forma canónica:

$$x = x_1 - x_2 \quad y = y_1 - y_2$$

$$\text{Max } f = -x_1 + x_2 - y_1 + y_2$$

$$\text{s.a. } -x_1 + x_2 + y_1 - y_2 \leq 2$$

$$x_1, x_2, y_1, y_2 \geq 0$$

## Ejercicio 2:

$$x = x_1 - x_2$$

$$\left. \begin{array}{l} \text{Max } -x + y \\ \text{s.a } -2x + y \leq 4 \\ x + y \leq 2 \\ y \geq 0 \end{array} \right\}$$

$$\left. \begin{array}{l} \text{Max } f = -x_1 + x_2 + 0s_1 + 0s_2 \\ \text{s.a } -2x_1 + 2x_2 + y + s_1 = 4 \\ x_1 - x_2 + y + s_2 = 2 \\ x_1, x_2, y, s_2, s_1 \geq 0 \end{array} \right.$$

### Método Simplex:

	-2	1	1	0	0	
V.B	$x_1$	$x_2$	$y$	$s_1$	$s_2$	$x_B$
0	$s_1$	-2	2	1	1	0
0	$s_2$	1	-1	1	0	1
	$z_j - c_j$	2	-1	-2	0	0

### Pivote 1:

	-2	1	1	0	0	
V.B	$x_1$	$x_2$	$y$	$s_1$	$s_2$	$x_B$
0	$s_1$	-3	3	0	1	-1
2	$y$	1	-1	1	0	1
	$z_j - c_j$	2	-2	0	0	2

$-F_p$   
 $F_p$

• Pivote 3:

	-2	1	2	0	0		
	V.B	$x_1$	$x_2$	$y$	$s_1$	$s_2$	$x_B$
2	$x_2$	-1	1	0	$\frac{1}{3}$	$-\frac{1}{3}$	1
2	$y$	0	0	1	$\frac{1}{3}$	$\frac{2}{3}$	2
	$z_j - c_j$	0	0	0	$\frac{2}{3}$	$\frac{1}{3}$	3

$$\left. \begin{array}{l} \text{Solución} \Rightarrow x_2 = 2, y = 2, x_1 = 0, s_1 = 0, s_2 = 0 \\ x_B = 2 \cdot 2 + 2 \cdot 1 = 3 \quad \text{Multiple} \end{array} \right\}$$

### Ejercicios 3 del Tema 3:

$$\left. \begin{array}{l}
 \text{Opt} \quad x - 2y + 3z \\
 \text{s.a.} \quad x + 2y + z \leq 4 \\
 \quad \quad \quad 2x + y - z \leq 2 \\
 \quad \quad \quad x, y, z \geq 0
 \end{array} \right\} \quad \begin{array}{l}
 \text{Max} \quad f = x - 2y + 3z \\
 \text{Min} \quad f = \text{Max} - g = -x + 2y - 3z \\
 \text{s.a.} \quad x + 2y + z + s_1 = 4 \\
 \quad \quad \quad 2x + y - z + s_2 = 2 \\
 \quad \quad \quad x, y, z, s_1, s_2 \geq 0
 \end{array}$$

#### Método Simplex:

	1	-2	3	0	0		
V.B.	x	y	z	s <sub>1</sub>	s <sub>2</sub>	x <sub>B</sub>	
0	s <sub>1</sub>	2	2	1	2	0	4
0	s <sub>2</sub>	2	2	-2	0	2	2
	$\bar{z}_j - c_j$	-2	2	-3	0	0	0

#### Pivote (-3):

	1	-2	3	0	0		
V.B.	x	y	z	s <sub>1</sub>	s <sub>2</sub>	x <sub>B</sub>	
3	z	1	2	1	1	0	4
0	s <sub>2</sub>	3	3	0	1	2	6
	$\bar{z}_j - c_j$	2	8	0	3	9	12

$$\left. \begin{array}{l}
 \text{Solución} \Rightarrow z = 4, s_2 = 6, x = 0, y = 0, z = 0 \\
 x_B = 3 \cdot 4 + 6 \cdot 0 \Rightarrow 12
 \end{array} \right\}$$

$$\begin{aligned}
 F_1 &= \frac{F_1}{P} \\
 F_2 &= +F_1
 \end{aligned}$$

### Ejercicio 3 $\Rightarrow$ Modelo Min

$$\left. \begin{array}{l} \text{Opt} \quad x - 2y + 3z \\ \text{s.a.} \quad x + 2y + z \leq 4 \\ \quad \quad \quad 2x + y - z \leq 2 \\ \quad \quad \quad x, y, z \geq 0 \end{array} \right\} \quad \begin{array}{l} \text{Min} \quad g = \text{Max} - g = -x + 2y - 3z \\ \text{s.a.} \quad x + 2y + z + s_1 = 4 \\ \quad \quad \quad 2x + y - z + s_2 = 2 \\ \quad \quad \quad x, y, z, s_1, s_2 \geq 0 \end{array}$$

### Método Simplex:

	-2	2	-3	0	0	
V.B.	x	y	z	$s_1$	$s_2$	$x_B$
0	$s_1$	2	2	2	2	0
0	$s_2$	2	1	0	2	2
$z - c_j$	3	-2↑	3	0	0	0

### Pivote (1):

	-2	2	-3	0	0	
V.B.	x	y	z	$s_1$	$s_2$	$x_B$
0	$s_1$	-3	0	3	1	-2
2	y	2	1	-2	0	2
$z - c_j$	5	0	1	0	2	-4

$$\left. \begin{array}{l} \Rightarrow \text{Solución} = [s_1 = 0, y = 2, s_2 = 0, x = 0, z = 0] \\ x_{B_{\min}} = -4 \quad \text{Óptima Única} \Rightarrow (\text{Solo hay } 2 \text{ ceros}) \\ \text{Mas ceros} \Rightarrow \text{Solución Múltiple} \end{array} \right\}$$

$$F_1 = -2F_2$$

$$F_2 = \frac{F_1}{-2}$$

## Ejercicio 4 del tema 3:

$$\left. \begin{array}{l}
 \text{a) Max } f = 3x + 2y + z \\
 \text{s.a. } 2x - 3y + 2z \leq 3 \\
 \quad \quad -x + y + z \leq 5 \\
 \quad \quad x, y, z \geq 0
 \end{array} \right\} \quad \left. \begin{array}{l}
 \text{Max } f = 3x + 2y + z \\
 \text{s.a. } 2x - 3y + 2z + s_1 = 3 \\
 \quad \quad -x + y + z + s_2 = 5 \\
 \quad \quad x, y, z \geq 0
 \end{array} \right.$$

- Método Simplex:

	3	2	2	0	0	
V.B.	x	y	z	s <sub>1</sub>	s <sub>2</sub>	x <sub>B</sub>
0	s <sub>1</sub>	2	-3	2	2	3
0	s <sub>2</sub>	-1	1	1	0	5
$z_j - c_j$	-3↑	-2	-2	0	0	0

- Pivote 2:

	3	2	2	0	0	
V.B.	x	y	z	s <sub>1</sub>	s <sub>2</sub>	x <sub>B</sub>
3	x	1	-3/2	1/2	1/2	3/2
0	s <sub>2</sub>	0	-1/2	2	1/2	13/2
$z_j - c_j$	0	-13/2	2	3/2	0	9/2

$$F_P = \frac{F_1}{2}$$

$$F_2 = F_2 + F_P$$

\* Problema No Acotado

Puede entrar y pero no puede salir ninguna variable

### Ejercicio 8c:

$$\text{Min } x_1 + x_2$$

$$\text{s.a. } x_1 + x_2 \leq 1$$

$$4x_1 + 2x_2 \geq 6$$

$$x_1, x_2 \geq 0$$

Max

$$-x_1 - x_2 - 0 \cdot s_1 - 0 \cdot s_2 - t_1$$

$$\text{s.a. } x_1 + x_2 + s_1 = 1$$

$$4x_1 + 2x_2 - s_2 + t_1 = 6$$

$$x_1, x_2, s_1, s_2, t_1 \geq 0$$

	-2	-2	0	0	-M	
	$x_1$	$x_2$	$s_1$	$s_2$	$t_1$	
0	$s_1$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	1
-M	$t_1$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	6
		$-4x_1 + 1$	$-2x_2 + 2$	0	M	0

• Private  $\frac{1}{2}$ :

	-2	-2	0	0	-M	
	$x_1$	$x_2$	$s_1$	$s_2$	$t_1$	
-2	$x_1$	$\frac{1}{2}$	$\frac{1}{2}$	0	0	1
-M	$t_1$	0	-2	-4	-1	2
		$2x_1$	$4x_2 - 4$	M	0	$-1 - 2M$
						$\frac{1}{2} + 2M$

• Como  $t_1 > 0$  es in factible

- Ejercicio 9. a.

$$\begin{aligned}
 \text{Min} \quad & 20x_1 + 25x_2 \\
 \text{s.a} \quad & 2x_1 + 3x_2 \geq 28 \\
 & x_1 + 3x_2 \geq 22 \\
 & 4x_1 + 3x_2 \geq 24 \\
 & x_1, x_2 \geq 0
 \end{aligned}$$

$$\left. \begin{aligned}
 \text{Max} \quad & 0x_1 + 0x_2 + t_1 + t_2 + t_3 \\
 & 2x_1 + 3x_2 - s_1 + t_1 = 28 \\
 & x_1 + 3x_2 - s_2 + t_2 = 22 \\
 & 4x_1 + 3x_2 - s_3 + t_3 = 24 \\
 & x_1, x_2, s_1, s_2, s_3, t_1, t_2, t_3 \geq 0
 \end{aligned} \right\}$$

	0	0	0	0	0	1	2	3	
$v_B$	$x_1$	$x_2$	$s_1$	$s_2$	$s_3$	$t_1$	$t_2$	$t_3$	
$t_1$	2	3		1	0	0	1	0	18
$t_2$	1	3	0	-1	0	0	1	0	12
$t_3$	4	3	0	0	-1	0	0	1	24
$\bar{z}_0$	7	9	-11	-1	-1	0	0	0	54



Ejercicio 9. b:

$$\left. \begin{array}{l} \text{Max } f = 4x_1 + 3x_2 \\ \text{s.a. } \begin{aligned} 3x_1 + 4x_2 &\leq 12 \\ x_1 + x_2 &\geq 4 \\ 4x_1 + 2x_2 &\leq 8 \\ x_1, x_2 &\geq 0 \end{aligned} \end{array} \right\} \quad \begin{array}{l} \text{Max } f = 4x_1 + 3x_2 \\ \text{s.a. } \begin{aligned} 3x_1 + 4x_2 + s_1 &= 32 \\ x_1 + x_2 - s_2 + t_1 &= 4 \\ 4x_1 + 2x_2 + s_3 &= 8 \\ x_1, x_2, s_1, s_2, s_3, t_1 &\geq 0 \end{aligned} \end{array}$$

	0	0	0	0	0	-1	
	$x_1$	$x_2$	$s_1$	$s_2$	$s_3$	$t_1$	$x_B$
0	$s_1$	3	4	2	0	0	0
-1	$t_1$	1	1	0	-1	0	1
0	$s_3$	1	2	0	0	1	0
$c_j - z_j$		-1	-1	0	1	0	-4

Pivote el 4:

	0	0	0	0	0	-1	
	$x_1$	$x_2$	$s_1$	$s_2$	$s_3$	$t_1$	$x_B$
0	$s_1$	0	2.5	1	0	0.25	0
-1	$t_1$	0	0.5	0	-1	-0.25	1
0	$x_1$	1	0.5	0	0	0.25	0
$c_j - z_j$	0	-0.5	0	1	0.25	0	-2

Pivote 2<sup>5</sup>:

	0	0	0	0	0	-1	
	$x_1$	$x_2$	$s_1$	$s_2$	$s_3$	$t_1$	$x_B$
0	$x_2$	0	1	0.4	0	-0.3	0
-1	$t_1$	0	0	-0.2	-1	-0.1	1
0	$s_3$	1	0	-0.2	0	0.4	0
$c_j - z_j$	0	0	0.2	1	0.1	0	-0.8

- Primero comprobamos si hay alguna variable artificial entre las básicas:

{  $\Rightarrow$  Si es 0 o positiva, pasemos a la fase 2.  
 $\Rightarrow$  Si es negativa, problema infelible.

- Ejercicio 9.c:

$$\begin{aligned} \text{Max } & x_1 - 2x_2 + 3x_3 \\ \text{s.a. } & x_1 + x_2 + x_3 = 6 \\ & x_3 \leq 2 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

$$\left. \begin{aligned} \text{Max } & 0x_1 - 0x_2 + 0x_3 + 0s_1 - 2t_1 \\ & x_1 + x_2 + x_3 + t_1 = 6 \\ & x_3 + s_1 = 2 \\ & x_1, x_2, x_3, t_1, s_1 \geq 0 \end{aligned} \right\}$$

		0	0	0	0	-2	
	$v_B$	$x_1$	$x_2$	$x_3$	$s_1$	$t_1$	$x_B$
-1	$t_1$	1	1	1	0	1	6
0	$s_1$	0	0	1	1	0	2
		-1	-1	-1	0	0	-6

Pivote 2:

		0	0	0	0	-1	
	$v_B$	$x_1$	$x_2$	$x_3$	$s_1$	$t_1$	
0	$x_1$	1	1	1	0	1	6
0	$s_1$	0	0	1	1	0	2
		0	0	0	0	1	0

$\Rightarrow$  Vemos que por ahora es factible pasamos  
a la fase 2.

• Fase 2: (Desaparece la t)

		1	-2	3	0	
	$v_B$	$x_1$	$x_2$	$x_3$	$s_1$	
2	$x_1$	1	1	1	0	6
0	$s_1$	0	0	1	1	2
		0	3	-2	0	6

• Pivote 2:

		1	-2	3	0	
	$v_B$	$x_1$	$x_2$	$x_3$	$s_1$	
1	$x_1$	1	1	0	-1	4
3	$x_3$	0	0	1	1	2
		0	3	0	2	10

Solución óptima única:

$$\left. \begin{array}{l} x_1 = 4 \\ x_3 = 2 \\ x_2 = 0 \end{array} \right\} x_B = 20$$

- Ejercicios q.d:

$$\left. \begin{array}{l}
 \text{Max } x_1 + x_2 + 10x_3 \\
 \text{s.a } x_2 + 4x_3 = 2 \\
 -2x_1 + x_2 - 6x_3 = 2 \\
 x_1, x_2, x_3 \geq 0
 \end{array} \right\} \rightarrow \begin{array}{l}
 x_2 + 4x_3 + f_1 = 2 \\
 -2x_1 + x_2 - 6x_3 + f_2 = 2 \\
 x_1, x_2, x_3, f_1, f_2 \geq 0
 \end{array}$$

Fase 2:  $\hat{g}^z = 0x_1 + 0x_2 + 0x_3 - f_1 - f_2$

	1	0	0	0	-1	-1
1						
-1	$f_1$	0	1	4	1	0
-1	$f_2$	-2	1	-6	0	1
	2	-2	2	0	0	4

Pivote 1:

	1	0	0	0	-1	-1
1						
0	$x_2$	0	1	4	1	0
-1	$f_2$	-2	0	-10	-1	1
	2	0	10	2	0	0

• Fase 2:  $\mathcal{F} = x_1 + x_2 + 10x_3$

	1	1	10	
	$x_1$	$x_2$	$x_3$	
2	$x_2$	0	1	4
0	$t_2$	-2	0	$-10$
		-1	0	-6

$$\left\{ \begin{array}{l} F_P = F_2 / -10 \\ F_{IN} = F_1 - F_P \end{array} \right.$$

• Pivote  $-10$ :

	1	1	10	
	$x_1$	$x_2$	$x_3$	
2	$x_2$	-0.8	1	0
10	$x_3$	0.2	0	1
		0.2	0	0

Sol. Óptima Única

$$x_1 = 0$$

$$x_2 = 2 \quad f = 2$$

$$x_3 = 0$$

Ejercicio q.e.:

$$\text{Max } x_1 + 2x_2$$

$$\text{s.a. } x_1 + x_2 = 4$$

$$2x_1 - 3x_2 = 3$$

$$3x_1 - x_2 = 8$$

$$x_1, x_2 \geq 0$$

$$\text{Max } 0x_1 + 0x_2 - t_1 - t_2 - t_3$$

$$x_1 + x_2 + t_1 = 4$$

$$2x_1 - 3x_2 + t_2 = 3$$

$$3x_1 - x_2 + t_3 = 8$$

$$x_1, x_2, t_1, t_2, t_3 \geq 0$$

Fase 1:

	$x_1$	$x_2$	$t_1$	$t_2$	$t_3$	
$t_1$	1	1	1	0	0	4
$t_2$	2	-3	0	1	0	3
$t_3$	3	-1	0	0	1	8
	-6	3	0	0	0	

Pivote 2:

	$x_1$	$x_2$	$t_1$	$t_2$	$t_3$	
$t_1$	0	$\frac{5}{2}$	1	$-\frac{1}{2}$	0	$\frac{5}{2}$
$x_1$	2	$-\frac{3}{2}$	0	$\frac{1}{2}$	0	$\frac{3}{2}$
$t_3$	0	$\frac{7}{2}$	0	$-\frac{3}{2}$	1	$\frac{7}{2}$
	0	-6	0	1	0	

Pivot 2:

	$x_1$	$x_2$	$t_1$	$t_2$	$t_3$	
$v_B$						
0 $x_2$	0	1	$\frac{3}{5}$	$-\frac{1}{5}$	0	1
0 $x_1$	1	0	$\frac{3}{5}$	-1	0	3
-1 $t_3$	0	0	$\frac{3}{5}$	$-\frac{9}{5}$	1	0
	0	0	1	$\frac{14}{5}$	0	

Fase 2:

	$x_1$	$x_2$	
$v_B$			
2 $x_2$	0	1	1
1 $x_1$	1	0	3
0 $t_3$	0	0	0
	0	0	

Solución óptima única:

$$\begin{aligned} x_2 &= 1 \\ x_1 &= 3 \end{aligned} \quad \left\{ \quad x_B = 5 \right.$$

	$x_1$	$x_2$	$t_1$	$t_2$	$t_3$	
$v_B$						
-1 $t_1$	0	$\frac{5}{2}$	1	$-\frac{1}{2}$	0	$\frac{5}{2}$
0 $x_1$	2	$-\frac{3}{2}$	0	$\frac{1}{2}$	0	$\frac{3}{2}$
-1 $t_3$	0	$\frac{9}{2}$	0	$-\frac{3}{2}$	1	$\frac{7}{2}$
	0	-6	0	1	0	

### Ejercicio 30. a:

$$\text{Max } 6x_1 + 4x_2$$

$$\text{s.a. } x_1 \leq 700$$

$$3x_1 + x_2 \leq 2400$$

$$x_1 + 2x_2 \leq 1600$$

$$x_1, x_2 \geq 0$$

— — — — —

$$c = \begin{pmatrix} 6 \\ 4 \end{pmatrix}$$

$$A = \begin{pmatrix} x_1 & x_2 \\ 1 & 0 \\ 3 & 1 \\ 1 & 2 \end{pmatrix} \Rightarrow A' = \begin{pmatrix} y_1 & y_2 & y_3 \\ 1 & 3 & 1 \\ 0 & 1 & 2 \end{pmatrix}$$

$$b = \begin{pmatrix} 700 \\ 2400 \\ 1600 \end{pmatrix}$$

$\Rightarrow$  Dual:

$$\text{Min } w = 700y_1 + 2400y_2 + 1600y_3$$

$$\text{s.a. } y_1 + 3y_2 + y_3 \geq 6$$

$$y_2 + 2y_3 \geq 4$$

$$y_1, y_2, y_3 \geq 0$$

Ejercicio 30. b:

$$\begin{aligned} \text{Max } & 4'5x_1 + 3x_2 + 1'5x_3 \\ \text{s.a. } & x_1 + 2x_2 - x_3 \leq 4 \\ & 2x_1 - x_2 + x_3 = 8 \\ & x_1 - x_2 \leq 6 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

$$\left. \begin{aligned} \text{Max } & 4'5x_1 + 3x_2 + 1'5x_3 \\ \text{s.a. } & x_1 + 2x_2 - x_3 \leq 4 \\ & 2x_1 - x_2 + x_3 \leq 8 \\ & -2x_1 + x_2 - x_3 \leq -8 \\ & x_1 - x_2 \leq 6 \\ & x_1, x_2, x_3 \geq 0 \end{aligned} \right\}$$

Matrices:

$$C = \begin{pmatrix} 4'5 \\ 3 \\ 1'5 \end{pmatrix} \quad b = \begin{pmatrix} 4 \\ 8 \\ -8 \\ 6 \end{pmatrix}$$

$$\left. \begin{aligned} \text{Max } & z = c^T x \\ \text{s.a. } & Ax \leq b \\ & x \geq 0 \end{aligned} \right\}$$

$$A = \begin{pmatrix} x_1 & x_2 & x_3 \\ 1 & 2 & -1 \\ 2 & -1 & 1 \\ -2 & 1 & -1 \\ 1 & -1 & 0 \end{pmatrix} \Rightarrow A' = \begin{pmatrix} y_1 & y_2 & y_3 & y_4 \\ 1 & 2 & -2 & 1 \\ 2 & -1 & 1 & -1 \\ -1 & 1 & -1 & 0 \end{pmatrix}$$

Forma dual:

$$\text{Min } w = 4y_1 + 8y_2 - 8y_3 + 6y_4$$

$$\text{s.a. } y_1 + 2y_2 - 2y_3 + y_4 \geq 4'5$$

$$2y_1 - y_2 + y_3 - y_4 \geq 3$$

$$-y_1 + y_2 - y_3 \geq 1'5$$

$$y_1, y_2, y_3, y_4 \geq 0$$

$$\left. \begin{aligned} \text{Min } & w = b^T y \\ \text{s.a. } & A^T y \geq c \\ & y \geq 0 \end{aligned} \right\}$$

Ejercicio 20.c:

$$\begin{array}{ll}
 \text{Min} & 6x_1 + 4x_2 \\
 \text{s.a} & x_1 \leq 700 \\
 & 3x_1 + x_2 \geq 2400 \\
 & x_1 + 2x_2 \leq 1600 \\
 & x_1, x_2 \geq 0
 \end{array}
 \rightleftharpoons
 \begin{array}{ll}
 \text{Max} & -6x_1 - 4x_2 \\
 \text{s.a} & -x_1 \geq -700 \\
 & 3x_2 + x_1 \geq 2400 \\
 & -x_1 - 2x_2 \geq -3600 \\
 & x_1, x_2 \geq 0
 \end{array}$$

$$C = \begin{pmatrix} -6 \\ -4 \end{pmatrix} \quad A = \begin{pmatrix} -1 & 0 \\ 3 & 1 \\ -1 & 2 \end{pmatrix} \Rightarrow A' = \begin{pmatrix} -1 & 3 & -1 \\ 0 & 1 & 2 \end{pmatrix}$$

$\Rightarrow$  Dual:

$$\text{Max } w = -700y_1 + 2400y_2 - 1600y_3$$

$$\text{s.a } -y_1 + 3y_2 - 4y_3 \leq 6$$

$$y_2 - 2y_3 \leq 4$$

$$y_1, y_2, y_3 \geq 0$$

Ejercicio 20. d):

$$\text{Max } 4'5x_1 + 3x_2 + 1'5x_3$$

$$\text{s.a. } x_1 + 2x_2 - x_3 \leq 4$$

$$2x_1 - x_2 + x_3 \leq 8$$

$$x_1 - x_2 \leq 6$$

$$x_1, x_3 \geq 0 \quad x_2 \text{ libre}$$

$$c = \begin{pmatrix} 4'5 \\ 3 \\ 1'5 \end{pmatrix} \quad b = \begin{pmatrix} 4 \\ 8 \\ 6 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 2 & -1 \\ 2 & -1 & 1 \\ 1 & -1 & 0 \end{pmatrix} \Rightarrow A^T = \begin{pmatrix} 1 & 2 & 1 \\ 2 & -1 & -1 \\ -1 & 1 & 0 \end{pmatrix}$$

Forma dual:

$$\text{Min } w = 4y_1 + 8y_2 + 6y_3$$

$$\text{s.a. } y_1 + 2y_2 + y_3 \geq 4'5$$

$$2y_1 - y_2 - y_3 = 3$$

$$-y_1 + y_2 \geq 1'5$$

$$y_1, y_2, y_3 \geq 0$$

Ejercicio 30. e:

$$\text{Min } 6x_1 + 4x_2$$

$$\text{s.a } x_1 \leq 700 \Rightarrow -x_1 \geq -700$$

$$3x_1 + x_2 \geq 2400$$

$$x_1 + 2x_2 = 1600$$

$$x_1 \geq 0 \quad x_2 \text{ sin restricciones}$$

$$c = \begin{pmatrix} 6 \\ 4 \end{pmatrix} \quad b = \begin{pmatrix} -700 \\ 2400 \\ 1600 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 0 \\ 3 & 1 \\ 1 & 2 \end{pmatrix} \Rightarrow A' = \begin{pmatrix} 1 & 3 & 2 & 1 \\ 0 & 1 & 2 & 0 \end{pmatrix}$$

Forma dual:

$$\text{Max } w = -700y_1 + 2400y_2 + 1600y_3$$

$$\text{s.a } y_1 + 3y_2 + y_3 \geq 6$$

$$y_2 + 2y_3 = 4$$

$$y_1, y_2 \geq 0 \quad y_3 \text{ sin restricciones}$$

• Ejercicio 22:

$$\left. \begin{array}{l}
 \text{c) Max } 2x_1 + x_2 + 3x_3 \\
 \text{s.a. } \begin{aligned}
 2x_1 - x_2 + 3x_3 &\leq 6 \\
 x_1 + 3x_2 + 5x_3 &\leq 10 \\
 2x_1 + x_3 &\leq 7 \\
 x_1, x_2, x_3 &\geq 0
 \end{aligned}
 \end{array} \right\} \begin{array}{l}
 2x_1 - x_2 + 3x_3 + s_1 = 6 \\
 x_1 + 3x_2 + 5x_3 + s_2 = 10 \\
 2x_1 + x_3 + s_3 = 7 \\
 x_1, x_2, x_3 \geq 0
 \end{array}$$

$V_B$	$x_1$	$x_2$	$x_3$	$s_1$	$s_2$	$s_3$	
0	$s_1$	2	-1	3	1	0	0
0	$s_2$	1	3	5	0	1	0
0	$s_3$	2	0	3	0	0	1
		-2	-1	-3	0	0	0

• Pivote

Ejercicio 82.a:

$$\begin{array}{l}
 \text{Men} \quad z = x_1 + 2x_2 \\
 \text{s.a} \quad x_1 + x_2 \geq 4 \\
 \quad \quad \quad 2x_1 + x_2 \leq 6 \\
 \quad \quad \quad x_1, x_2 \geq 0
 \end{array}
 \quad \left. \begin{array}{l}
 \text{Max} \\
 \text{s.a}
 \end{array} \right\} \quad \begin{array}{l}
 -x_1 - 2x_2 \\
 -x_1 - x_2 + s_1 = 4 \\
 -2x_1 - x_2 + s_2 = 6 \\
 x_1, x_2, s_1, s_2 \geq 0
 \end{array}$$

	$x_1$	$-x_2$	$0$	$0$	
VB	$x_1$	$x_2$	$s_1$	$s_2$	
$0$	$x_1$	$-1$	$-1$	$0$	$-4$
$0$	$s_1$	$-1$	$1$	$0$	$-6$
$0$	$s_2$	$-2$	$0$	$4$	$-6$
	$1$	$2$	$0$	$0$	

Pivote -2:

	$x_1$	$-x_2$	$0$	$0$	
VB	$x_1$	$x_2$	$s_1$	$s_2$	
$0$	$s_1$	$0$	$-\frac{1}{2}$	$1$	$-\frac{1}{2}$
$-1$	$x_1$	$1$	$\frac{1}{2}$	$0$	$-\frac{1}{2}$
	$0$	$\frac{3}{2}$	$0$	$\frac{1}{2}$	$-3$

Pivote  $-\frac{1}{2}$ :

	$-1$	$-2$	$0$	$0$	
VB	$x_1$	$x_2$	$s_1$	$s_2$	
$0$	$s_2$	$0$	$1$	$-2$	$2$
$-1$	$x_1$	$2$	$2$	$-1$	$0$
	$0$	$1$	$2$	$0$	$-4$

Solución óptima única  $\begin{cases} x_1 = 4 \\ x_2 = 2 \end{cases}$   $x_{1B} = 4$   
 $x_2 = s_1 = 0$

• Ejercicio 22.b:

$$\text{Min } x_1 + x_2 + x_3 + x_4$$

$$\text{s.a. } 2x_1 + x_4 \geq 250$$

$$3x_2 \geq 1000$$

$$3x_2 + 10x_3 + 6x_4 \geq 750$$

$$x_1, x_2, x_3, x_4 \geq 0$$

$$\left. \begin{array}{l} \text{Max } -x_1 - x_2 - x_3 - x_4 + s_1 + s_2 + s_3 \\ -2x_1 - x_4 + s_1 = -250 \\ -3x_2 + s_2 = -1000 \\ -3x_2 - 10x_3 - 6x_4 + s_3 = -750 \\ x_1, x_2, x_3, x_4 \geq 0 \end{array} \right\}$$

$$-2x_1 - x_4 + s_1 = -250$$

$$-3x_2 + s_2 = -1000$$

$$-3x_2 - 10x_3 - 6x_4 + s_3 = -750$$

$$x_1, x_2, x_3, x_4 \geq 0$$

$v_B$	$x_1$	$x_2$	$x_3$	$x_4$	$s_1$	$s_2$	$s_3$	
0	$s_1$	-2	0	0	-1	1	0	-250
0	$s_2$	0	-3	0	0	0	1	-1000
0	$s_3$	0	-3	-10	-6	0	0	-750
	1	1	1	1	0	0	0	

• Pivote -3:

$v_B$	$x_1$	$x_2$	$x_3$	$x_4$	$s_1$	$s_2$	$s_3$	
0	$s_1$	-2	0	0	-1	1	0	-250
-1	$x_2$	0	1	0	0	0	$-\frac{1}{3}$	$\frac{1000}{3}$
0	$s_3$	0	0	-10	-6	0	-1	250
	1	0	1	1	0	$\frac{1}{3}$	0	

Pivote -2-

$v_B$	-1	-1	-1	-1	0	0	0	
$x_1$	1	0	0	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	125
$x_2$	0	1	0	0	0	$-\frac{1}{3}$	0	$\frac{1000}{3}$
$s_3$	0	0	-10	-6	0	-1	1	250
	0	0	1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{3}$	0	$-458\frac{1}{3}$

Solución óptima única:

$$\left. \begin{array}{l} x_1 = 125 \quad x_2 = \frac{1000}{3} \quad s_3 = 250 \\ x_3 = x_4 = s_1 = s_2 = 0 \end{array} \right\} x_B = 458\frac{1}{3}$$

Ejercicio 32.c.

$$\begin{array}{ll}
 \text{Max} & -4x_1 - 6x_2 \\
 \text{s.a.} & \left. \begin{array}{l} 3x_1 + x_2 \geq 2400 \\ x_1 + 2x_2 \geq 1600 \\ x_1, x_2 \geq 0 \end{array} \right\} \rightarrow \begin{array}{l} -3x_1 - x_2 + s_1 = -2400 \\ -x_1 - 2x_2 + s_2 = -1600 \\ x_1, x_2, s_1, s_2 \geq 0 \end{array}
 \end{array}$$

		-4	-6	0	0	
	V_B	x <sub>1</sub>	x <sub>2</sub>	s <sub>1</sub>	s <sub>2</sub>	x <sub>B</sub>
0	s <sub>1</sub>	<span style="border: 1px solid black; padding: 2px;">-3</span>	-2	2	0	<span style="border: 1px solid black; border-radius: 50%; padding: 2px;">-2400</span>
0	s <sub>2</sub>	-1	-2	0	2	<span style="border: 1px solid black; border-radius: 50%; padding: 2px;">-1600</span>
		4	6	0	0	

Pivot en -3:

		-4	-6	0	0	
	V_B	x <sub>1</sub>	x <sub>2</sub>	s <sub>1</sub>	s <sub>2</sub>	x <sub>B</sub>
-4	x <sub>1</sub>	2	$\frac{1}{3}$	$-\frac{1}{3}$	0	800
0	s <sub>2</sub>	0	<span style="border: 1px solid black; padding: 2px;"><math>-\frac{5}{3}</math></span>	$-\frac{1}{3}$	1	<span style="border: 1px solid black; border-radius: 50%; padding: 2px;">-800</span>
		0	$\frac{14}{3}$	$\frac{4}{3}$	0	

•  $\frac{4/3}{-1/3} \Rightarrow -4$

•  $\frac{\frac{14}{3}}{-\frac{5}{3}} \Rightarrow [-2^8]$

• Pivote  $-1/2$ :

VB	$x_1$	$x_2$	$s_1$	$s_2$	
$s_2$	0	1	-2	1	2
$x_1$	1	1	-1	0	4
	0	1	1	0	-4

Solución óptima única:

$$\left. \begin{array}{l} x_1 = 4 \\ x_2 = 0 \\ s_1 = 0 \\ s_2 = 2 \end{array} \right\}$$

Con  $-z^* = -4 \Rightarrow z^* = 4$

a) Var. no. básica  $\Rightarrow x_1 = \hat{c}_1$

$$z_1 - \hat{c}_1 \geq 0 \Rightarrow (2 \ 0) \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix} - c_1 \Rightarrow 2 - c_1 \geq 0$$

$$x_2 = \hat{c}_2$$

$$z_2 - \hat{c}_2 \geq 0 \Rightarrow (2 \ 0) \begin{pmatrix} 1 \\ 0 \end{pmatrix} - c_2 \Rightarrow 2 - c_2 \geq 0$$

$$2 \geq c_2$$

### Ejercicio 24.a:

$$\begin{array}{ll}
 \text{Max} & x_1 + 2x_2 \\
 \text{s.a} & x_1 + x_2 \leq 4 \\
 & 2x_1 + x_2 \leq 6 \\
 & x_1, x_2 \geq 0
 \end{array}$$

	1	2	0	0	
v_B	x <sub>1</sub>	x <sub>2</sub>	s <sub>1</sub>	s <sub>2</sub>	
x <sub>2</sub>	1	1	1	0	4
s <sub>2</sub>	1	0	-1	1	2
	1	0	2	0	8

a) Var. Nu. básica:  $x_1 \rightarrow \boxed{\hat{C}_1}$

$$z_1 - \hat{C}_1 \geq 0 \Rightarrow (2 \ 0) \begin{pmatrix} 1 \\ 1 \end{pmatrix} - \hat{C}_1 \geq 0$$

$$\Rightarrow 2 - \hat{C}_1 \geq 0 \Rightarrow \boxed{\hat{C}_1 \leq 2}$$

↳ Tabla óptima

b) Var. básica:  $x_2 \rightarrow \boxed{\hat{C}_2}$

$$z_1 - \hat{C}_1 \geq 0 \Rightarrow (\hat{C}_2 \ 0) \begin{pmatrix} 1 \\ 2 \end{pmatrix} - 1 \geq 0 \rightarrow \boxed{\hat{C}_2 \geq 1}$$

$$z_3 - C_3 \geq 0 \Rightarrow (\hat{C}_2 \ 0) \begin{pmatrix} 1 \\ -2 \end{pmatrix} - 0 \geq 0 \rightarrow \hat{C}_2 \geq 0$$

$$z = (\hat{C}_2 \ 0) \begin{pmatrix} 4 \\ 2 \end{pmatrix} = 4\hat{C}_2$$

↳ Tabla opt

Ejercicio 24.c

$$\begin{array}{ll} \text{Max} & x_1 + 2x_2 \\ \text{s.a} & x_1 + x_2 \leq 4 \\ & 2x_1 + x_2 \leq 6 \\ & x_1, x_2 \geq 0 \end{array}$$

	1	2	0	0	
$v_B$	$x_1$	$x_2$	$s_1$	$s_2$	
2	$x_2$	1	1	1	0
0	$s_2$	1	0	-1	1
		1	0	2	0
		1	0	2	8

$$b = \begin{pmatrix} 4 \\ 6 \end{pmatrix} \rightarrow b_1$$

$$b = \begin{pmatrix} 4 \\ 6 \end{pmatrix} \rightarrow b_2$$

$$\boxed{\hat{b}_1} \quad x_B = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix} \cdot \begin{pmatrix} \hat{b}_1 \\ 6 \end{pmatrix} = \begin{pmatrix} \hat{b}_1 \\ -\hat{b}_1 + 6 \end{pmatrix} \geq 0$$

$$0 \leq \hat{b}_1 \leq 6 \rightarrow \text{Tabla opt}$$

$$\boxed{\hat{b}_2} \quad x_B = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ \hat{b}_2 \end{pmatrix} = \begin{pmatrix} 4 \\ -4 + \hat{b}_2 \end{pmatrix} \geq 0$$

$$\boxed{\hat{b}_2 \geq 4} \rightarrow \text{Tabla opt}$$

$$z = (2 \ 0) \begin{pmatrix} 4 \\ -4 + \hat{b}_2 \end{pmatrix} = 8$$

$$\boxed{\hat{b}_1 \text{ y } \hat{b}_2}$$

$$x_B = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} \hat{b}_1 \\ \hat{b}_2 \end{pmatrix} = \begin{pmatrix} \hat{b}_1 \\ -\hat{b}_1 + \hat{b}_2 \end{pmatrix} \geq 0 \quad \left. \begin{array}{l} 0 \leq \hat{b}_1 \leq \hat{b}_2 \\ 0 \leq \hat{b}_2 \end{array} \right\}$$

$$z = (2 \ 0) \begin{pmatrix} \hat{b}_1 \\ -\hat{b}_1 + \hat{b}_2 \end{pmatrix} = 2\hat{b}_2$$

## Ejercicio 24. d:

$$\begin{array}{ll} \text{Max} & x_1 + 2x_2 \\ \text{s.a} & x_1 + x_2 \leq 4 \\ & 2x_1 + x_2 \leq 6 \\ & x_1, x_2 \geq 0 \end{array}$$

	1	2	0	0	
v_B	x_1	x_2	s_1	s_2	
1	1	1	1	0	4
2	x_2	1	1	1	2
0	s_2	1	0	-1	1
	1	0	2	0	8

Var. no básica  $\rightarrow x_1$

$$y_{11} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \xrightarrow{\rightarrow} \begin{pmatrix} a_{11} \\ a_{21} \end{pmatrix}$$

$$\boxed{\hat{a}_{11}} \quad \hat{y}_{11} = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} \hat{a}_{11} \\ 2 \end{pmatrix} = \begin{pmatrix} \hat{a}_{11} \\ -\hat{a}_{11} + 2 \end{pmatrix}$$

$$z_1 - c_1 = (2 \ 0) \begin{pmatrix} \hat{a}_{11} \\ -\hat{a}_{11} + 2 \end{pmatrix} - 2 =$$

$$2\hat{a}_{11} - 2 \geq 0 \Rightarrow \boxed{\hat{a}_{11} \geq \frac{1}{2}}$$

$$\boxed{\hat{a}_{21}} \quad \hat{y}_{12} = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} \hat{a}_{21} \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 + \hat{a}_{21} \end{pmatrix}$$

$$z_1 - c_1 = (2 \ 0) \begin{pmatrix} 1 \\ -1 + \hat{a}_{21} \end{pmatrix} - 2 \geq 0$$

$$z_1 - c_1 \Rightarrow \boxed{12 \geq 0}$$

$$\boxed{\hat{a}_{11} \ y \ \hat{a}_{21}} \quad \hat{y}_{11} = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} \hat{a}_{11} \\ \hat{a}_{21} \end{pmatrix} = \begin{pmatrix} \hat{a}_{11} \\ -\hat{a}_{11} + \hat{a}_{21} \end{pmatrix}$$

$$z_1 - c_1 = (2 \ 0) \begin{pmatrix} \hat{a}_{11} \\ -\hat{a}_{11} + \hat{a}_{21} \end{pmatrix} - 2$$

$$\Rightarrow 2\hat{a}_{11} - 2 \geq 0 \Rightarrow \boxed{\hat{a}_{11} \geq \frac{1}{2}}$$

Ejercicio 24.c:

Solución óptima  $\Rightarrow x_1 = 0; x_2 = 4$

$$4x_1 + x_2 \leq 5$$

$$4 \cdot 0 + 4 = 4 \leq 5$$

↳ Verifica la restricción

Table óptima

Ejercicio 24.g:

$$x_1 + 4x_2 \leq 3$$

$$0 + 4 \cdot 4 = 16 \neq 3 \rightarrow \text{Tabla no óptima}$$

$$x_1 + 4x_2 + s_3 = 3$$

	$x_1$	$x_2$	$s_1$	$s_2$	$s_3$	
$x_2$	1	1	1	0	0	$k$
$s_2$	1	0	-1	1	0	2
$s_3$	1	4	0	0	1	3
$x_2$	1	1	1	0	0	4
$s_2$	1	0	-1	1	0	2
$s_3$	(-3)	0	-4	0	1	-13
$z_j - c_j$	1	0	2	0	0	

$f_3 - 4f_1$

Simplex dual

→ Solución óptima única:

$$\left. \begin{array}{l} x_1 = 3 \\ x_2 = 0 \\ s_1 = 2 \\ s_2 = s_3 = 0 \end{array} \right\} \boxed{z = 3}$$

## Examen 3 (2 fases y dual):

### Ejercicio 2:

$$\text{Max } z = 7x_1 + 6x_2 + 3x_3$$

$$\text{s.a. } 4x_1 + 4x_2 - 8x_3 \geq 7 \Rightarrow -4x_1 - 4x_2 + 8x_3 \leq -7$$

$$6x_1 - 4x_2 + 6x_3 \leq 7$$

$$9x_1 - 3x_2 + 7x_3 = 7$$

$$x_1, x_2 \geq 0 \quad x_3 \text{ libres}$$

$$C = \begin{pmatrix} 7 \\ 6 \\ 3 \end{pmatrix} \quad b = \begin{pmatrix} -7 \\ 7 \\ 7 \end{pmatrix}$$

$$A = \begin{pmatrix} x_1 & x_2 & x_3 \\ -4 & -4 & +8 \\ 6 & -4 & 6 \\ 9 & -3 & 7 \end{pmatrix} \Rightarrow A' = \begin{pmatrix} y_1 & y_2 & y_3 \\ -4 & 6 & 9 \\ -4 & -4 & -3 \\ 8 & 6 & 7 \end{pmatrix}$$

$$\text{Min } w = -7y_1 + 7y_2 + 7y_3$$

$$\text{s.a. } -4y_1 + 6y_2 + 9y_3 \geq 7$$

$$-4y_1 - 4y_2 - 3y_3 \geq 6$$

$$8y_1 + 6y_2 + 7y_3 = 3$$

$$y_1, y_2 \geq 0 \quad y_3 \text{ libre}$$

## Ejercicio 2:

$$\text{Max } z = 4x_1 + 5x_2 + 7x_3$$

$$\text{s.a. } 6x_1 + 3x_2 - 8x_3 = 9$$

$$7x_1 - 6x_2 + 6x_3 \leq 6 \rightarrow -7x_1 + 6x_2 - 6x_3 \geq -6$$

$$8x_1 - 2x_2 + 8x_3 \geq 6$$

$x_1, x_3 \geq 0, x_2$  libre

$$c = \begin{pmatrix} 4 \\ 5 \\ 7 \end{pmatrix} \quad b = \begin{pmatrix} 9 \\ -6 \\ 6 \end{pmatrix}$$

$$A = \begin{pmatrix} 6 & 3 & -8 \\ -7 & 6 & -6 \\ 8 & -2 & 8 \end{pmatrix} \Rightarrow \begin{pmatrix} 6 & -7 & 8 \\ 3 & 6 & -2 \\ -8 & -6 & 8 \end{pmatrix}$$

$$\text{Max } w = 9x_1 - 6x_2 + 6x_3$$

$$\text{s.a. } 6x_1 + 3x_2 - 8x_3 = 9$$

$$7x_1 - 6x_2 + 6x_3 \leq 6$$

$$-8x_1 + 6x_2 + 8x_3 \leq 7$$

$x_1$  libre

## Ejercicio 3:

$$\text{Max } x_1 - 2x_2 + x_3$$

$$\text{s.a. } x_1 + x_2 + x_3 \leq 2$$

$$2x_1 + x_2 - x_3 \geq 4$$

$$x_1 + x_2 + x_3 + s_1$$

$$2x_1 + x_2 - x_3 - s_2 + e_1$$

$$z = 0x_1 + 0x_2 + 0x_3 + 0s_1 + 0s_2 - e_1$$

Ejercicio 25:

$$\text{Max } 20x_1 + 30x_2 + 16x_3$$

$$\text{s.a. } 5x_1 + 3x_2 + x_3 \leq 3$$

$$x_1 + 3x_2 + 2x_3 \leq 4$$

$$x_1, x_2, x_3 \geq 0$$

VB	$x_1$	$x_2$	$x_3$	$s_1$	$s_2$	$\theta$
30		3	1	0	$\frac{2}{3}$	$\frac{1}{3}$
16		-4	0	3	-2	1
	6	0	0	4	6	36

a) Var. no. básicas:  $x_1 = \hat{c}_1$

$$z_1 - \hat{c}_1 \geq 0 \Rightarrow (30 \ 16) \begin{pmatrix} 3 \\ -4 \end{pmatrix} - \hat{c}_1 \geq 0$$

$$90 - 64 = 26 - c_1 \geq 0 \Rightarrow 26 \geq c_1$$

b) Var. básicas =

$$x_2 = \hat{c}_2$$

$$x_3 = \hat{c}_3$$

$$x_2: z_1 - c_1 \geq 0 \Rightarrow (c_2 \ 16) \begin{pmatrix} 3 \\ -4 \end{pmatrix} - 20 \geq 0$$

$$3c_2 - 64 - 20 \geq 0 \Rightarrow c_2 \geq \frac{80}{3}$$

$$z_4 - c_4 \geq 0 \Rightarrow (c_2 \ 16) \begin{pmatrix} 2/3 \\ -1 \end{pmatrix} - 0 \geq 0$$

$$\frac{2c_2}{3} - 16 \geq 0 \Rightarrow c_2 \geq 24$$

$$z_5 - c_5 \geq 0 \Rightarrow (c_2 \ 16) \begin{pmatrix} -1/3 \\ 1 \end{pmatrix} - 0 \geq 0$$

$$-\frac{c_2}{3} + 16 \geq 0 \Rightarrow c_2 \leq 48$$

$$24 \leq c_2 \leq 48$$

$$\Rightarrow x_3 = \hat{c}_3 :$$

$$\bullet z_1 - c_1 \geq 0 \Rightarrow (30 \hat{c}_3) \begin{pmatrix} 3 \\ -4 \end{pmatrix} - 20 \geq 0$$

$$90 - 4c_3 - 20 \geq 0 \Rightarrow \boxed{\frac{70}{4} \geq c_3}$$

$$\bullet z_4 - c_4 \geq 0 \Rightarrow (30 \hat{c}_3) \begin{pmatrix} 2/3 \\ -1 \end{pmatrix} - 0 \geq 0$$

$$20 - \hat{c}_3 \geq 0 \Rightarrow \boxed{20 \geq c_3}$$

$$\bullet z_5 - c_5 \geq 0 \Rightarrow (30 \hat{c}_3) \begin{pmatrix} -1/3 \\ 1 \end{pmatrix} - 0 \geq 0$$

$$-10 + c_3 \geq 0 \Rightarrow \boxed{c_3 \geq 10}$$

$$\boxed{10 \leq c_3 \leq \frac{70}{4}}$$

c) Respeto a los recursos:

$$b = \begin{pmatrix} 3 \\ 4 \end{pmatrix} \xrightarrow{b_1} \quad B^{-1} = \begin{pmatrix} 2/3 & -1/3 \\ -1 & 1 \end{pmatrix}$$

$$\bullet \hat{b}_1 \Rightarrow \begin{pmatrix} 2/3 & -1/3 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} b_1 \\ 4 \end{pmatrix} \Rightarrow \left( \frac{2b_1}{3} - \frac{4}{3} \right) \geq 0 \quad \left( -b_1 + 4 \right) \geq 0$$

$$\left. \begin{array}{l} 2b_1 \geq 4 \Rightarrow b_1 \geq 2 \\ -b_1 + 4 \geq 0 \Rightarrow 4 \geq b_1 \end{array} \right\} \quad \boxed{2 \leq b_1 \leq 4}$$

$$b_2: \begin{pmatrix} 2/3 & 1/3 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} 3 \\ b_2 \end{pmatrix} \Rightarrow \begin{pmatrix} 2 + \frac{b_2}{3} \\ -3 + b_2 \end{pmatrix} \geq 0$$

$$\left. \begin{array}{l} 2 + \frac{b_2}{3} \geq 0 \Rightarrow b_2 \geq -6 \\ -3 + b_2 \geq 0 \Rightarrow b_2 \geq 3 \end{array} \right\} \boxed{-6 \leq b_2 \leq 3}$$

d) Respecto al recurso tecnológico  $a_{11}$ :

$$\begin{pmatrix} 2/3 & 1/3 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} a_{11} \\ 1 \end{pmatrix} \xrightarrow{\text{Tabla mixta}} \begin{pmatrix} \frac{2a_{11}}{3} - \frac{4}{3} \\ -a_{11} - 4 \end{pmatrix}$$

↳ Para que la tabla sea siendo óptima:

$$(30 \ 26) \begin{pmatrix} \frac{2a_{11}-4}{3} \\ -a_{11}-4 \end{pmatrix} \Rightarrow 20a_{11} - 40 - 16a_{11} - 64 \\ 4a_{11} - 104 - 20 \geq 0$$

$$4a_{11} - 124 \geq 0 \Rightarrow a_{11} \geq 31$$

## Ejercicio 27:

$$\text{Max } 600x_1 + 300x_2$$

$$\text{s.a. } 2x_1 + 1.5x_2 \leq 1200$$

$$x_1 + 0.25x_2 \leq 375$$

$$x_1, x_2 \geq 0$$

				600	300	0	0
VB	x <sub>1</sub>	x <sub>2</sub>	x <sub>3</sub>	S <sub>1</sub>	S <sub>2</sub>		
300		1		1	-2	450	
600	2	0	-1/4	3/2	225		
	0	0	-5/4	150	300	292500	
			3 - C <sub>j</sub>	0	0		

Sol. óptima  $\rightarrow x_1 = 225, x_2 = 450$

Sol. dual  $\rightarrow y_1 = 250, y_2 = 300$

a) Restricciones dual excedida

$$2y_1 + 0.25y_2 > 200$$

$$2 \cdot 150 + 0.25 \cdot 300 = 375 > 200$$

$\Rightarrow$  La solución se mantiene opt

Verifica la restricción  $\Rightarrow$  La solución se mantiene opt

b)  $y_1 + 0.15y_2 \geq 200$

$$\Rightarrow 150 + 0.15 \cdot 300 = 395 \neq 200 \Rightarrow$$
 Tabla no opt

$$\hat{y}_3 = \begin{pmatrix} 1 & -2 \\ -1/4 & 3/2 \end{pmatrix} \begin{pmatrix} 2 \\ 0.15 \end{pmatrix} \Rightarrow \begin{pmatrix} 7/10 \\ -1/40 \end{pmatrix}$$

$$z_3 - C_3 \Rightarrow (300 \text{ ecc}) \begin{pmatrix} 7/10 \\ -1/40 \end{pmatrix} - 200 = \boxed{-5}$$

		600	300	200	0	0	
	x <sub>1</sub>	x <sub>2</sub>	x <sub>3</sub>	x <sub>4</sub>	S <sub>1</sub>	S <sub>2</sub>	
300		1	<u>7/10</u>	1	-2	450	$\Rightarrow$
600	2	0	-1/40	-1/4	3/2	225	
	0	0	-5/4	150	300	292500	
			3 - C <sub>j</sub>	1/7	-20/7	450/7	
				10/7	10/7	10/7	
	x <sub>3</sub>	0	10/7	1	10/7	-20/7	
	x <sub>1</sub>	1	1/28	0	-3/14	10/7	
		0	50/7	0	1100/7	2070000/7	

Ejercicios Tipo Examen:

Ejercicio 2:

• Vemos la variable básica:  $x_2 = c_2$

$$\cdot z_3 - c_3 \Rightarrow (c_2 \geq) \begin{pmatrix} 1 \\ -1 \end{pmatrix} - 2 \geq 0$$

$$c_2 - 2 - 2 \geq 0 \Rightarrow c_2 \geq 3$$

$$\cdot z_4 - c_4 \Rightarrow (c_2 \geq) \begin{pmatrix} 1 \\ -1 \end{pmatrix} - 0 \Rightarrow c_2 - 2 \geq 0 \Rightarrow c_2 \geq 2$$

$$\cdot z_5 - c_5 \Rightarrow (c_2 \geq) \begin{pmatrix} -1 \\ 2 \end{pmatrix} - 0 \Rightarrow -c_2 + 4 \geq 0 \Rightarrow c_2 \leq 4$$

⇒ Solución:

$$3 \leq c_2 \leq 4$$

Ejercicio 2:

• Var. no básica:  $x_3 = c_3$

$$z_3 - c_3 = (3 \ 4) \begin{pmatrix} -1 \\ 2 \end{pmatrix} - c_3 \Rightarrow -3 + 8 - c_3 \geq 0$$

$$5 - c_3 \geq 0 \Rightarrow c_3 \leq 5$$

• Ejercicio 3:

• Recursos:

$$b = \begin{pmatrix} 10 \\ 20 \end{pmatrix} \rightarrow b_1 \\ \rightarrow b_2 \quad B^{-1} = \begin{pmatrix} 1 & -y_2 \\ 0 & y_2 \end{pmatrix}$$

$$B^{-1} \cdot b \Rightarrow \begin{pmatrix} 1 & -y_2 \\ 0 & y_2 \end{pmatrix} \cdot \begin{pmatrix} 10 \\ b_2 \end{pmatrix} \Rightarrow \begin{pmatrix} 10 - 5 \\ \frac{b_2}{2} \end{pmatrix} \geq 0$$

$$\frac{b_2}{2} \geq 0 \Rightarrow \boxed{b_2 \geq 0}$$


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• Ejercicio 4:

$$\boxed{x_1 = 2, \quad x_2 = 3, \quad x_3 = 0}$$

$$2x_1 + 4x_2 + x_3 \leq 10 \Rightarrow 2 \cdot 2 + 4 \cdot 3 + 0 \leq 10$$

↳ No se cumple

$v_B$	4	3	3	0	0	0	
$x_2$	0	1	-1	1	-1	0	3
$x_1$	1	0	2	0	1	0	2
$s_3$	2	4	1	0	0	1	10

• Hago cero en  $c_1$ :

$v_B$	$x_1$	$x_2$	$x_3$	$s_1$	$s_2$	$s_3$	
$x_2$	0	1	-1	1	-1	0	3
$x_1$	1	0	2	0	1	0	2
$s_3$	0	4	-3	0	-2	1	6

• Hago ceros en  $c_2$ :

$v_B$	$x_1$	$x_2$	$x_3$	$s_1$	$s_2$	$s_3$	
$x_2$	0	1	-1	1	-1	0	3
$x_1$	1	0	2	0	1	0	2
$s_3$	0	0	2	-4	12	1	-6

•  $\frac{3}{4} =$

• Pivote -6:

$v_B$	$x_1$	$x_2$	$x_3$	$s_1$	$s_2$	$s_3$	
$x_2$	0	1	$-\frac{3}{4}$	0	$-\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{2}$
$x_1$	1	0	2	0	1	0	2
$s_1$	0	0	$-\frac{1}{4}$	1	$-\frac{1}{2}$	$-\frac{1}{4}$	$\frac{6}{4}$

$$\left[ x_1 = 2, \quad x_2 = \frac{3}{2}, \quad x_3 = 0, \quad z = \frac{25}{2} \right]$$

Ejercicio 5:

→ Coeficiente factores evi:

$$A = \begin{pmatrix} 4 \\ 5 \end{pmatrix} \Rightarrow a_{12} \quad B^{-1} = \begin{pmatrix} 1 & -1/2 \\ 0 & 1/2 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 1 & -1/2 \\ 0 & 1/2 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ a_{22} \end{pmatrix} \Rightarrow \begin{pmatrix} 4 - \frac{a_{22}}{2} \\ \frac{a_{22}}{2} \end{pmatrix}$$

$$\Rightarrow (0 \ 5) \begin{pmatrix} 4 - \frac{a_{22}}{2} \\ \frac{a_{22}}{2} \end{pmatrix} - 2 \Rightarrow \frac{5}{2} a_{22} - 2 \geq 0$$
$$\Rightarrow \frac{5}{2} a_{22} \geq 2$$
$$\Rightarrow a_{22} \geq \frac{4}{5}$$

Ejercicio 6:

$$g^* = (0, 1)$$

$$2y_1 + 10y_2 \geq 15$$

↳

$$2 \cdot 0 + 10 \cdot 1 \leq 15 \Rightarrow \text{No se cumple}$$

$$B^{-1} = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 10 \end{pmatrix} \Rightarrow \begin{pmatrix} 2 - 10 \\ 10 \end{pmatrix} \Rightarrow \begin{pmatrix} -8 \\ 10 \end{pmatrix}$$

$$z = (0, 1) \begin{pmatrix} -8 \\ 10 \end{pmatrix} \Rightarrow 20 - 15 = 5$$

	$x_1$	$x_2$	$x_3$	$s_1$	$s_2$	
$v_B$	1	0	0	0	0	
$s_1$	-3	0	-8	2	-1	3
$x_2$	2	2	10	0	1	1
	2	0	-5	0	1	1

Pivot row:

	$x_1$	$x_2$	$x_3$	$s_1$	$s_2$	
$v_B$	1	0	0	0	0	
$s_1$	$-\frac{11}{5}$	$\frac{8}{10}$	0	1	$-\frac{1}{10}$	$\frac{19}{10}$
$x_3$	$\frac{1}{10}$	$\frac{1}{10}$	1	0	$\frac{1}{10}$	$\frac{1}{10}$
	$\frac{1}{2}$	$\frac{1}{2}$	0	0	$\frac{15}{10}$	$\frac{15}{10}$

Ejercicio 7:

$$g^* = (0, 2^5)$$

$$5y_1 + 3y_2 \geq 6 \Rightarrow \text{No se cumple}$$

$$\begin{pmatrix} 1 & -1/2 \\ 0 & 1/2 \end{pmatrix} \begin{pmatrix} 5 \\ 3 \end{pmatrix} - 6 \Rightarrow \begin{pmatrix} 5 - \frac{3}{2} \\ \frac{3}{2} \end{pmatrix} \begin{pmatrix} 7/2 \\ 3/2 \end{pmatrix}$$

$$(0 \ 5) \begin{pmatrix} 5 - \frac{3}{2} \\ \frac{3}{2} \end{pmatrix} \Rightarrow \frac{15}{2} - 6 \Rightarrow \frac{3}{2}$$

Creamos la Tabla:

	5	2	6	0	0
5	$x_1$	$x_2$	$x_3$	$s_1$	$s_2$
0	$s_1$	6	$\frac{3}{2}$	$\frac{7}{2}$	1
5	$x_1$	1	$\frac{5}{2}$	$\frac{3}{2}$	0
	0	$\frac{1}{2}$	$\frac{3}{2}$	0	$\frac{5}{2}$

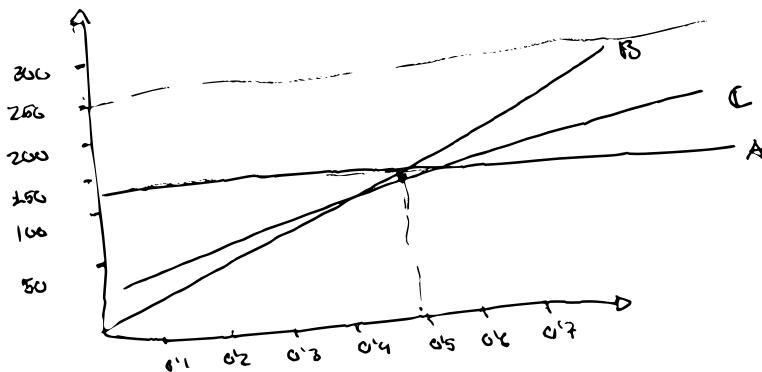
$$\left[ \begin{array}{cccc} x_1 = 10 & x_2 = 0 & x_3 = 0 & z = 50 \end{array} \right]$$

## Métodos Cuantitativos : Parte 2.

### Ejercicio 2.2:

→ Nuestro agente es el observador.

	Ganar	Perder	La pieza	Wald	óptimo	Hornik
Opción A	125	125	125	125	125	$125\alpha + 125(1-\alpha)$
Opción B	260	0	125	0	250	$250\alpha$
Opción C	200	50	225	50	200	$200\alpha + 50(1-\alpha)$



{ Para  $\alpha < 0.5 \Rightarrow$  Opción A }  
 { Para  $\alpha > 0.5 \Rightarrow$  Opción B }

Sacado:

	B	P	Sacado
A	125	0	$A = 125$
B	0	125	$B = 125$
C	50	75	$C = 75 \rightarrow$ Éxito

• Ejercicio 8.2:

$$\text{Max } f = 10x_1 + 8x_2 + 2x_3$$

$$\text{s.a. } 3x_1 + 2x_2 + 4x_3 \leq 2000000$$

$$x_1, x_2, x_3 \geq 0$$

$x_1, x_2, x_3$  enteros

---

Ejercicios 2.3:

	↑ 30%	↑ 20%	=	↓ 5%
Brasil	1000	900	800	400
China	1300	1100	700	300
España	1200	9400	600	-1000
Alemania	1000	900	700	700

a)

Indicadores de incertidumbre:

Laplace	Wald	Optimo	Hauschek
725	900	1000	$1000\alpha + 400(1-\alpha)$
850	300	1300	$1300\alpha + 300(1-\alpha)$
775	100	1400	$1400\alpha - 300(1-\alpha)$
820	825	1000	$1000\alpha + 700(1-\alpha)$

Savage:

↑ 30%	↑ 20%	=	↓ 5%	PUE
900	300	200	300	240
1000	200	0	400	120
0	0	100	800	120
400	300	0	0	370

b)

VME:

$$\text{Brasil} = 1000 \cdot 0.2 + 900 \cdot 0.3 + 800 \cdot 0.4 + 400 \cdot 0.1 = 750$$

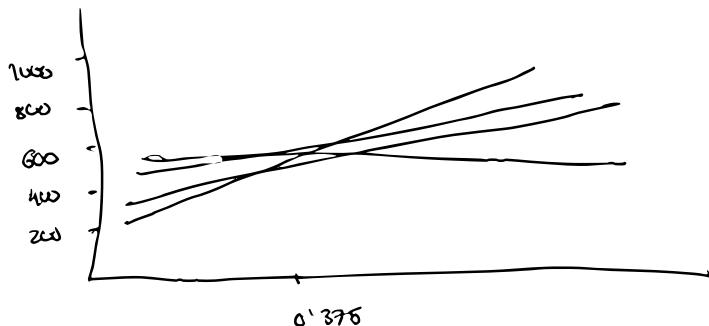
$$\text{China} = 1300 \cdot 0.2 + 1100 \cdot 0.3 + 700 \cdot 0.4 + 300 \cdot 0.1 = 900$$

$$\text{España} = 1200 \cdot 0.2 + 9400 \cdot 0.3 + 600 \cdot 0.4 - 100 \cdot 0.1 = 870$$

$$\text{Alemania} = 1000 \cdot 0.2 + 900 \cdot 0.3 + 700 \cdot 0.4 + 700 \cdot 0.1 = 820$$

Ejercicio 1.4:

	V. crecientes	V. decrecientes	
Horas extras	500	100	$500p + 200(1-p)$
Contratar mano	700	0	$700p + 0(1-p)$
Alquiler	900	-100	$900p - 100(1-p)$
Nada	400	200	$400p + 200(1-p)$



{
   
 Si  $p < 0'375 \Rightarrow$  Nada
   
 Si  $p > 0'375 \Rightarrow$  Alquiler mecanizado

Ejercicios Prueba Continua

Ejercicio 2:

	Ganador	Finalista	Normal	$E[x_{ci}]$
Modelo 1	94.000	86.000	32.000	55780
Modelo 2	91.000	65.000	51.000	58840
Modelo 3	98.000	52.000	34.000	44966
Modelo 4	78.000	76.000	59.000	66456
Probabil.	70%	36%	57%	
	98.000	86.000	59.000	

⇒ a) 66.456 → Valor Esperado → Beneficio

b) Servicio

	Ganador	Finalista	Normal	POE
Modelo 1	4000	0	27.000	15418
Modelo 2	7000	21.000	8.000	12616
Modelo 3	0	34.000	25.000	26490
Modelo 4	20.000	10.000	0	5000
Probabil.	70%	36%	57%	

⇒ b) ⇒ 5000 ← → P.G.E Óptima

$$c) V.M.E = 66.450 \text{ €}$$

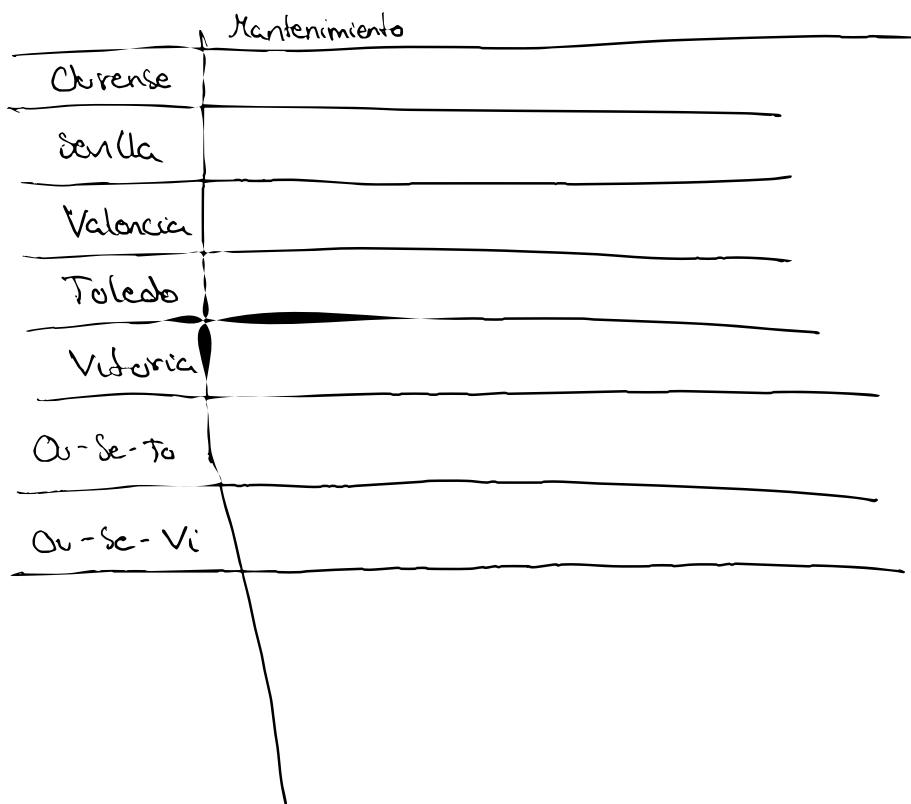
$$V.M.E.I.P = 98.000.027 + 86.000.036 + 59.000 \\ , 057$$

$$V.M.E.I.P = 71450 \text{ €}$$

La empresa está dispuesta a pagar:

$$V.M.G - V.M.E.I.P = 66.450 - 71450 = \boxed{500 \text{ €}}$$

Ejercicio 2:



Ejercicios del examen

1. 22. Ejercicio

Cada CD a 2\$ y cuesta 0'50\$ producirlo.  
 Si no se vende en la quincena a 0'50\$.

La mitad de los sobrantes no se venden.

Demanda o's	0'4	0'4	0'4	0'2	VME	Yes. Prod.
oferta	3	4	5	6		
3	38	38	18	18	28	18
4	25	24	24	24	28'2	24
5	12	21	30	30	24'6	30
6	9	18	27	36	22'5	27
POE	(18)	(24)	(30)	(36)		
	0	6	12	18	9	
	3	0	6	12	3'9	
	6	3	0	6	(2'4)	→ V
	9	6	3	0	4'5	

→ Cuando se demanda y gasta 3 docenas:

$$3 \text{ docenas} \Rightarrow 3 \cdot 12 \Rightarrow 36 \text{ CDs}$$

$$\text{Beneficio Total} \Rightarrow 36 \cdot 2\$ - 36 \cdot 0'5\$ \Rightarrow 18\$$$

→ Cuando demanda 4 docenas y oferta 3:

$$48 \text{ CDs}$$

$$\text{Beneficio} \Rightarrow 36 \cdot 2\$$$

• Oferta 4K y Demanda 3K:

$$4 \cdot 32 \Rightarrow 48 \text{ artes}$$

$$\text{Beneficio} = 36 \cdot 2\$ - 48 \cdot 0.5\$ + 6 \cdot 0.5\$ - 6 \cdot 0.2\$ = 25\$$$

— — — — y 5 y 6K

• Oferta 4K y Demanda 4K:

$$\text{Beneficio} = 48 \cdot 2\$ - 48 \cdot 0.5\$ = 24 \$$$

— — — — y 5 y 6K

• Oferta 5K y Demanda 3K:

$$\text{Beneficio} = 36 \cdot 2\$ - 60 \cdot 0.5\$ + 12 \cdot 0.5\$ = 22 \$$$

Ejercicio 2.4:

Primeras Tablas:

	Precio	distancia	m <sup>2</sup>
GV - P.N	40 <sup>+</sup>	50 <sup>-</sup>	0
GV - L.R	140 <sup>+</sup>	50 <sup>+</sup>	10 <sup>-</sup>
P.N - L.R	100 <sup>+</sup>	100 <sup>-</sup>	10 <sup>+</sup>
	II $q = 200$ 50%	III $P = 200$ 26%	II $g = 75$ 25%
Table 2:			

	+	0'5 <sup>-</sup>	0
GV - PN	0 <sup>+</sup>	0'5 <sup>-</sup>	0
GV - LR	1 <sup>+</sup>	0'5 <sup>+</sup>	0
PN - LR	0 <sup>+</sup>	1 <sup>-</sup>	0 <sup>+</sup>

Perdedores ↓

Ganadores →

	GV	P.N	LR	Φ <sup>+</sup>
GV	-	0	0'625	0'3125
PN	0'125	-	0	0'0625
LR	0	0'25	-	0'125
Φ <sup>-</sup>	0'625	0'725	0'3125	
	Φ <sup>+</sup>	Φ <sup>-</sup>	Φ	

PN - LR - GV

GV	0'3125	0'625	(-0'25)
PN	0'625	0'125	(-0'0625)
LR	0'125	0'53125	(-0'1875)

Examen Adelantado 20-29:

3.

$$\begin{pmatrix} 12 & -\frac{d}{2} & 25 \\ 15 & 20 & 5d \\ 9 & 25 & -10 \end{pmatrix}$$

a)  $5d < 25 \Rightarrow d < 3$

- - -

b)

$$\begin{pmatrix} 12 & -7 & 25 \\ 16 & 20 & 70 \\ 9 & 25 & -10 \end{pmatrix}$$

Ej. Nesh = 25

Ejercicio 4:

$$\left\{ \begin{array}{l} \phi = \phi^+ - \phi^- \\ \phi^- = \phi^+ - \phi \end{array} \right\}$$

	$\phi^+$	$\phi^-$	$\phi$	
A	0.2699	0.2816	-0.0117	PROMETHEE II $C \rightarrow B \rightarrow A \rightarrow D \rightarrow E$
B	0.3539	0.3647	-0.0108	
C	0.4662	0.2425	0.2237	
D	0.2379	0.3262	-0.0883	
E	0.2105	0.3235	-0.1130	

$$A = B$$

$$A < C$$

$$A > D$$

$$A > E$$

$$B < C$$

$$B = D$$

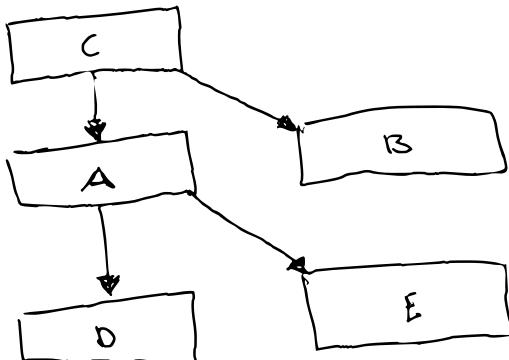
$$B = E$$

$$C > D$$

$$C > E$$

$$D = E$$

PROMETHEE I



	$E_{21}$	$E_{22}$	$E_{23}$
$E_{11}$	0	$\alpha \rightarrow N_o$	$\alpha > 3 \rightarrow$ Pase por Max Col
$E_{12}$	$\alpha$	0	$3 \rightarrow N_o$
$E_{13}$	1	3	0

$\alpha > 3 \rightarrow N_o$

- Para que haya Ig. Puro  $\Rightarrow$  Max Col y Min Fil.
  - No existe valor de  $\alpha$  que produzcan Ig. Puro de Nash
- 

Prc  $\alpha = 2$

$$\begin{array}{c|ccc}
 & E_{21} & E_{22} & E_{23} \\
 \hline
 E_{11} & 0 & 1 & 2 \\
 E_{12} & -2 & 0 & 3 \\
 E_{13} & -1 & -3 & 0
 \end{array} \Rightarrow \begin{pmatrix} -1 & 3 \\ 1 & 0 \end{pmatrix}$$

$J_2 \Rightarrow \text{Max } v$

$$v \leq -2E_{12} + E_{13} \Rightarrow v \leq -E_{12} + (2 - E_{22})$$

$$v \leq 3E_{12}$$

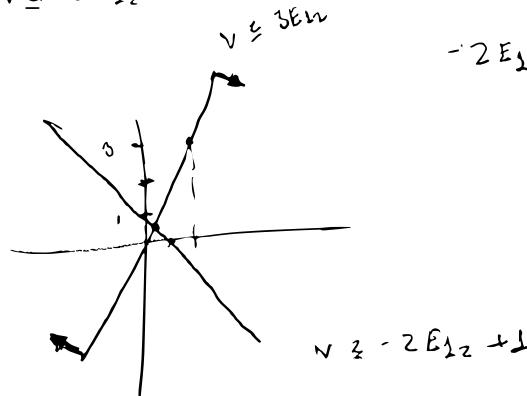
$$\begin{aligned}
 v &\leq -2E_{12} + 2 & \begin{cases} (v=0, E_{12}=\frac{1}{2}) / (v=1, E_{12}=0) \\ (v=0, E_{12}=0) / (v=3, E_{12}=1) \end{cases} \\
 v &\leq 3E_{12}
 \end{aligned}$$

$$v \leq 3E_{12} \quad -2E_{12} + 2 = 3E_{12}$$

$$5E_{12} = 2$$

$$E_{12} = \frac{2}{5}$$

$$N = 3 \cdot \frac{1}{5} = \boxed{\frac{3}{5}}$$



2                    2                    3  
 familia          amigos          vecinos

S	J	$P_3(S)$	$J_1$	$J_2$	$J_3$
$\emptyset$	0	$1/3$	0	0	0
1	0	$1/6$	-	750	900
2	0	$1/6$	750	-	0
3	0	$1/6$	900	0	-
2,2	750	$1/3$	-	-	450
2,3	900	$1/3$	-	300	-
2,3	0	$1/3$	1200	-	-
2,2,3	1200	-	675	225	300

$$P(0) = \frac{0! (3-0-1)!}{n!} \Rightarrow \frac{1}{3}$$

$$P(1) = \frac{1! (3-1-1)!}{n!} \Rightarrow \frac{1}{6}$$

$$P(2) = \frac{2! (3-2-1)!}{n!} \Rightarrow \frac{1}{3}$$

1  
Proprietario2  
Empleados3  
Constructores

S	J	$P_3(S)$	J <sub>1</sub>	J <sub>2</sub>	J <sub>3</sub>
Ø	0	1/3	350.000	0	0
1	350.000	1/6	-	350.000	425.000
2	0	1/6	700.000	-	0
3	0	1/6	775.000	0	-
1,2	700.000	1/3	-	-	75.000
1,3	775.000	1/3	-	0	-
2,3	0	1/3	775.000	-	-
1,2,3	?	-			

$$P_{(0)} = \frac{0! (3-0-1)!}{n!} \Rightarrow \frac{1}{3}$$

$$P_{(1)} = \frac{1! (3-1-1)!}{3!} \Rightarrow \frac{1}{6}$$

$$P_{(2)} = \frac{2! (3-2-1)!}{3!} \Rightarrow \frac{1}{3}$$

