Ejercicio 2.

$$\begin{cases} x+3y-2z=3 \\ x+y+2z=0 \\ 3x-y-z=-1 \end{cases}$$

a) $\underline{\mathbb{Z}_5}$ pueden dejarse los coeficientes como están.

[Recordemos que $-2=3$ en $\mathbb{Z}_5$
$-1=4$ en $\mathbb{Z}_5$]

$$\left[\begin{array}{ccc|c} 1 & 3 & -2 & 3 \\ 1 & 1 & 2 & 0 \\ 3 & -1 & -1 & -1 \end{array}\right] \sim \left[\begin{array}{ccc|c} \boxed{1} & 1 & 2 & 0 \\ \cancel{1} & 3 & -2 & 3 \\ \cancel{3} & -1 & -1 & -1 \end{array}\right] \sim \begin{array}{l} E_2-E_1\to E_2 \\ E_3+2E_1\to E_3 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} \boxed{1} & 1 & 2 & 0 \\ 0 & 2 & -4 & 3 \\ 0 & 1 & 3 & -1 \end{array}\right] \xrightarrow{3\cdot E_2\to E_2} \left[\begin{array}{ccc|c} \boxed{1} & 1 & 2 & 0 \\ 0 & \boxed{1} & 3 & 4 \\ 0 & 1 & 3 & -1 \end{array}\right]$$

son iguales
$-1=4$

$$\sim \left[\begin{array}{ccc|c} \boxed{1} & 0 & -1 & -4 \\ 0 & \boxed{1} & 3 & 4 \\ 0 & 0 & 0 & 0 \end{array}\right] = \left[\begin{array}{ccc|c} \boxed{1} & 0 & 4 & 1 \\ 0 & \boxed{1} & 3 & 4 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\underline{\text{C.I.}}$

$$\begin{cases} x=1+\lambda \\ y=4+2\lambda \\ z=\lambda \end{cases} \qquad \lambda\in\mathbb{Z}_5$$

Solución:
$\{ \underset{\lambda=0}{(1,4,0)};\ \underset{\lambda=1}{(2,3,1)};\ \underset{\lambda=2}{(3,0,2)};$
$\underset{\lambda=3}{(4,1,3)};\ \underset{\lambda=4}{(0,2,4)} \}$

5 soluciones.

b) $\underline{\mathbb{Z}_7}$

$$\left[\begin{array}{ccc|c} 1 & 3 & -2 & 3 \\ 1 & 1 & 2 & 0 \\ 3 & -1 & -1 & -1 \end{array}\right] \sim \left[\begin{array}{ccc|c} \boxed{1} & 1 & 2 & 0 \\ \cancel{1} & 3 & -2 & 3 \\ \cancel{3} & -1 & -1 & -1 \end{array}\right] \sim \begin{array}{l} E_2-E_1\to E_2 \\ E_3-3E_1\to E_3 \end{array}$$

$$\left[\begin{array}{ccc|c} \boxed{1} & 1 & 2 & 0 \\ 0 & 1 & -4 & 3 \\ 0 & \underset{= 3}{-4} & 0 & \underset{= 6}{-1} \end{array}\right] \underset{5 \cdot E_3 \to E_3}{\sim} \left[\begin{array}{ccc|c} \boxed{1} & 1 & 2 & 0 \\ 0 & 1 & -4 & 3 \\ 0 & \boxed{1} & 0 & 3 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} \boxed{1} & \cancel{1} & 2 & 0 \\ 0 & \boxed{1} & 0 & 3 \\ 0 & \cancel{1} & -4 & 3 \end{array}\right] \sim \left[\begin{array}{ccc|c} \boxed{1} & 0 & 2 & 4 \\ 0 & \boxed{1} & 0 & 3 \\ 0 & 0 & 3 & 0 \end{array}\right] \underset{5 \cdot E_3 \to E_3}{\sim}$$

$$\sim \left[\begin{array}{ccc|c} \boxed{1} & 0 & \cancel{2} & 4 \\ 0 & \boxed{1} & 0 & 3 \\ 0 & 0 & \boxed{1} & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} \boxed{1} & 0 & 0 & 4 \\ 0 & \boxed{1} & 0 & 3 \\ 0 & 0 & \boxed{1} & 0 \end{array}\right] \quad \text{C. D.}$$

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Solución: $\{(4, 3, 0)\}$

c) usamos algunas transformaciones anteriores

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 2 & 0 \\ 0 & 1 & -4 & 3 \\ 0 & -4 & 0 & \end{array}\right]$$

no sirve

$$\left[\begin{array}{ccc|c} \boxed{1} & 1 & 2 & 0 \\ \cancel{1} & 3 & -2 & 3 \\ \cancel{3} & -1 & -1 & -1 \end{array}\right] \sim \left[\begin{array}{ccc|c} \boxed{1} & 1 & 2 & 0 \\ 0 & 2 & -4 & 3 \\ 0 & -4 & -7 & -1 \end{array}\right] \sim$$

$$\left[\begin{array}{ccc|c} \boxed{1} & \cancel{1} & 2 & 0 \\ 0 & \boxed{1} & -2 & 3/2 \\ 0 & \cancel{-4} & -7 & -1 \end{array}\right] \sim \left[\begin{array}{ccc|c} \boxed{1} & 0 & 4 & -3/2 \\ 0 & \boxed{1} & -2 & 3/2 \\ 0 & 0 & -15 & 5 \end{array}\right] \sim$$

$$\sim \left[\begin{array}{ccc|c} \boxed{1} & 0 & 4 & -3/2 \\ 0 & \boxed{1} & -2 & 3/2 \\ 0 & 0 & \boxed{1} & -1/3 \end{array}\right] \sim \left[\begin{array}{ccc|c} \boxed{1} & 0 & 0 & -1/6 \\ 0 & \boxed{1} & 0 & 5/6 \\ 0 & 0 & \boxed{1} & -1/3 \end{array}\right] \quad \text{C. D.}$$

Solución $\left\{\left(-\frac{1}{6}, \frac{5}{6}, -\frac{1}{3}\right)\right\}$