Ejercicio 1

$\boxed{1}$ $\mathbb{R}$

$$\left[\begin{array}{ccc|c} 0 & 1 & -2 & -4 \\ 1 & 1 & -1 & 0 \\ 2 & -1 & 1 & 3 \end{array}\right] \underset{E_1 \leftrightarrow E_2}{\sim} \left[\begin{array}{ccc|c} \boxed{1} & 1 & -1 & 0 \\ 0 & 1 & -2 & -4 \\ \cancel{2} & -1 & 1 & 3 \end{array}\right] \; E_3-2E_1 \to E_3$$

$$\left[\begin{array}{ccc|c} \boxed{1} & \cancel{1} & -1 & 0 \\ 0 & \boxed{1} & -2 & -4 \\ 0 & \cancel{-3} & 3 & 3 \end{array}\right] \underset{\substack{E_1-E_2 \to E_1 \\ E_3+3E_2 \to E_3}}{\sim} \left[\begin{array}{ccc|c} \boxed{1} & 0 & 1 & 4 \\ 0 & \boxed{1} & -2 & -4 \\ 0 & 0 & -3 & -9 \end{array}\right] \underset{-\frac{1}{3} \cdot E_3 \to E_3}{\sim}$$

$$\left[\begin{array}{ccc|c} \boxed{1} & 0 & \cancel{1} & 4 \\ 0 & \boxed{1} & \cancel{-2} & -4 \\ 0 & 0 & \boxed{1} & 3 \end{array}\right] \underset{\substack{E_1-E_3 \to E_1 \\ E_2+2E_3 \to E_2}}{\sim} \left[\begin{array}{ccc|c} \boxed{1} & 0 & 0 & 1 \\ 0 & \boxed{1} & 0 & 2 \\ 0 & 0 & \boxed{1} & 3 \end{array}\right]$$

Solución Es C.D. $\{(1,2,3)\}$ Solución

ó $\begin{cases} x_1 = 1 \\ x_2 = 2 \\ x_3 = 3 \end{cases}$

$\boxed{1}$ $\mathbb{Z}_3$ En primer lugar los coeficientes se sustituyen por sus representantes más sencillos (0, 1 ó 2)

$$\left[\begin{array}{ccc|c} 0 & 1 & 1 & 2 \\ 1 & 1 & 2 & 0 \\ 2 & 2 & 1 & 0 \end{array}\right] \underset{E_1 \leftrightarrow E_2}{\sim} \left[\begin{array}{ccc|c} \boxed{1} & 1 & 2 & 0 \\ 0 & 1 & 1 & 2 \\ \cancel{2} & 2 & 1 & 0 \end{array}\right] \; E_3-2E_1 \to E_3$$

$$\sim \left[\begin{array}{ccc|c} \boxed{1} & \cancel{1} & 2 & 0 \\ 0 & \boxed{1} & 1 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

[Si la ecuación 1: $1 \quad 1 \quad 2 \mid 0$ se multiplica por 2 (en $\mathbb{Z}_3$) $2 \quad 2 \quad 1 \mid 0$]

$$\underset{E_1-E_2 \to E_1}{\sim} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

C.I. $\begin{cases} x_1 = 1 + 2\lambda \\ x_2 = 2 + 2\lambda \\ x_3 = \lambda \end{cases}$

Como $\lambda \in \mathbb{Z}_3$,

$\boxed{\lambda = 0}$ $x_1 = 1$, $x_2 = 2$, $x_3 = 0$

$\boxed{\lambda = 1}$ $x_1 = 0$, $x_2 = 1$, $x_3 = 1$

$\boxed{\lambda = 2}$ $x_1 = 2$, $x_2 = 0$, $x_3 = 2$

La solución del sistema es

$\{(1,2,0);\ (0,1,1);\ (2,0,2)\}$

$\boxed{1}$ $\mathbb{Z}_5$

$$\left[\begin{array}{ccc|c} 0 & 1 & 3 & 1 \\ 1 & 1 & 4 & 0 \\ 2 & 4 & 1 & 3 \end{array}\right] \underset{E_1 \leftrightarrow E_2}{\sim} \left[\begin{array}{ccc|c} \boxed{1} & 1 & 4 & 0 \\ 0 & 1 & 3 & 1 \\ \cancel{2} & 4 & 1 & 3 \end{array}\right]$$

$$\underset{E_3 - 2E_1 \to E_3}{\sim} \left[\begin{array}{ccc|c} \boxed{1} & \cancel{1} & 4 & 0 \\ 0 & \boxed{1} & 3 & 1 \\ 0 & \cancel{2} & 3 & 3 \end{array}\right] \underset{\substack{E_1 - E_2 \to E_1 \\ E_3 - 2E_2 \to E_3}}{\sim} \left[\begin{array}{ccc|c} \boxed{1} & 0 & 1 & 4 \\ 0 & \boxed{1} & 3 & 1 \\ 0 & 0 & 2 & 1 \end{array}\right]$$

$$\underset{3 \cdot E_3 \to E_3}{\sim} \left[\begin{array}{ccc|c} \boxed{1} & 0 & \cancel{1} & 4 \\ 0 & \boxed{1} & \cancel{3} & 1 \\ 0 & 0 & \boxed{1} & 3 \end{array}\right] \underset{\substack{E_1 - E_3 \to E_1 \\ E_2 - 3E_3 \to E_2}}{\sim} \left[\begin{array}{ccc|c} \boxed{1} & 0 & 0 & 1 \\ 0 & \boxed{1} & 0 & 1 \\ 0 & 0 & \boxed{1} & 3 \end{array}\right]$$

(*) $\boxed{[2]_5^{-1} = 3}$

C. D. $\{(1,1,3)\}$ Solución.

Ejercicio 1

$\boxed{2}$ $\mathbb{C}$

Es un sistema HOMOGÉNEO (todos los t.i. son 0) y por tanto tiene la solución trivial: (0,0,0,0,0) y como hay 3 ecuaciones y 5 incógnitas, al menos 2 serán parámetros

C. I.

$$\left[\begin{array}{ccccc|c} \boxed{1} & -1 & 1 & 1 & 1 & 0 \\ \cancel{1} & 1 & 1 & 1 & -1 & 0 \\ \cancel{-1} & -1 & 1 & 1 & -1 & 0 \end{array}\right] \sim \left[\begin{array}{ccccc|c} \boxed{1} & -1 & 1 & 1 & 1 & 0 \\ 0 & 2 & 0 & 0 & -2 & 0 \\ 0 & -2 & 2 & 2 & 0 & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{ccccc|c} \boxed{1} & \cancel{-1} & 1 & 1 & 1 & 0 \\ 0 & \boxed{1} & 0 & 0 & -1 & 0 \\ 0 & \cancel{-2} & 2 & 2 & 0 & 0 \end{array}\right] \sim \left[\begin{array}{ccccc|c} \boxed{1} & 0 & 1 & 1 & 0 & 0 \\ 0 & \boxed{1} & 0 & 0 & -1 & 0 \\ 0 & 0 & 2 & 2 & -2 & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{ccccc|c} \boxed{1} & 0 & \cancel{1} & 1 & 0 & 0 \\ 0 & \boxed{1} & 0 & 0 & -1 & 0 \\ 0 & 0 & \boxed{1} & 1 & -1 & 0 \end{array}\right] \sim \left[\begin{array}{ccccc|c} \boxed{1} & 0 & 0 & 0 & 1 & 0 \\ 0 & \boxed{1} & 0 & 0 & -1 & 0 \\ 0 & 0 & \boxed{1} & 1 & -1 & 0 \end{array}\right]$$

$$\begin{cases} x = -\lambda_2 \\ y = \lambda_2 \\ z = -\lambda_1 + \lambda_2 \\ t = \lambda_1 \\ v = \lambda_2 \end{cases} \qquad \lambda_1, \lambda_2 \in \mathbb{C}$$

C. I. (el mismo razonamiento)

$\boxed{2}$ $\mathbb{Z}_3$

$$\left[\begin{array}{ccccc|c} \boxed{1} & 2 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 2 & 0 \\ 2 & 2 & 1 & 1 & 2 & 0 \end{array}\right] \sim \begin{array}{l} E_2 - E_1 \to E_2 \quad (\text{ó } E_2 + 2E_1 \to E_2) \\ E_3 + E_1 \to E_3 \end{array}$$

→ porque $2+1=0$

$$\left[\begin{array}{ccccc|c} \boxed{1} & 2 & 1 & 1 & 1 & 0 \\ 0 & 2 & 0 & 0 & 1 & 0 \\ 0 & 1 & 2 & 2 & 0 & 0 \end{array}\right] \underset{E_2 \leftrightarrow E_3}{\sim} \left[\begin{array}{ccccc|c} \boxed{1} & \cancel{2} & 1 & 1 & 1 & 0 \\ 0 & \boxed{1} & 2 & 2 & 0 & 0 \\ 0 & \cancel{2} & 0 & 0 & 1 & 0 \end{array}\right]$$

$$\underset{\substack{E_1 + E_2 \to E_1 \\ E_3 + E_2 \to E_1}}{\sim} \left[\begin{array}{ccccc|c} \boxed{1} & 0 & 0 & 0 & 1 & 0 \\ 0 & \boxed{1} & 2 & 2 & 0 & 0 \\ 0 & 0 & 2 & 2 & 1 & 0 \end{array}\right] \sim 2 \cdot E_3 \to E_3$$

$$\left[\begin{array}{ccccc|c} \boxed{1} & 0 & 0 & 0 & 1 & 0 \\ 0 & \boxed{1} & 2 & 2 & 0 & 0 \\ 0 & 0 & \boxed{1} & 1 & 2 & 0 \end{array}\right] \underset{E_2+E_3 \to E_2}{\sim} \left[\begin{array}{ccccc|c} \boxed{1} & 0 & 0 & 0 & 1 & 0 \\ 0 & \boxed{1} & 0 & 0 & 2 & 0 \\ 0 & 0 & \boxed{1} & 1 & 2 & 0 \end{array}\right]$$

$$\begin{cases} x = 2\lambda_2 \\ y = \lambda_2 \\ z = 2\lambda_1 + \lambda_2 \\ t = \lambda_1 \\ v = \lambda_2 \end{cases}$$

$\lambda_1, \lambda_2 \in \mathbb{Z}_3$

Hay $\underset{3^2}{\overset{9}{\shortparallel}}$ soluciones.

$\boxed{2}$ $\mathbb{Z}_5$

$$\begin{cases} x = 4\lambda_2 \\ y = \lambda_2 \\ z = 4\lambda_1 + \lambda_2 \\ t = \lambda_1 \\ v = \lambda_2 \end{cases}$$

$\lambda_1, \lambda_2 \in \mathbb{Z}_5$

Hay 25 soluciones

Ejercicio 1

$\boxed{3}$ $\mathbb{Z}_3$

$$\left[\begin{array}{ccc|c} 1 & \cancel{1} & 1 & 2 \\ 1 & \cancel{2} & 1 & 1 \\ 0 & \boxed{1} & 0 & 0 \end{array}\right]$$

→ mi ecuación favorita borra todo lo de la columna del pivote sin afectar al resto

$$\sim \left[\begin{array}{ccc|c} \boxed{1} & 0 & 1 & 2 \\ 0 & \boxed{1} & 0 & 0 \\ \cancel{1} & 0 & 1 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 2 \end{array}\right] \sim$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 1 & \cancel{2} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \boxed{1} \end{array}\right]$$

→ mi ec. – favorita también.

$$\sim \left[\begin{array}{ccc|c} \boxed{1} & 0 & 1 & 0 \\ 0 & \boxed{1} & 0 & 0 \\ 0 & 0 & 0 & \boxed{1} \end{array}\right]$$

I.

$\boxed{3}$ $\mathbb{Z}_5$

$$\left[\begin{array}{ccc|c} \boxed{1} & 1 & 1 & 2 \\ \cancel{1} & 2 & 1 & 1 \\ 0 & 1 & 0 & 3 \end{array}\right] \sim \left[\begin{array}{ccc|c} \boxed{1} & 1 & 1 & 2 \\ 0 & 1 & 0 & 4 \\ 0 & 1 & 0 & 3 \end{array}\right]$$

— Incompatible

$$\sim \left[\begin{array}{ccc|c} \boxed{1} & 0 & 1 & 0 \\ 0 & \boxed{1} & 0 & 0 \\ 0 & 0 & 0 & \boxed{1} \end{array}\right]$$

s.e.r.

$\boxed{3}$ $\mathbb{Q}$ $\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$

I.

Ejercicio 1

$\boxed{4}$ $\mathbb{Z}_3$

$$\left[\begin{array}{ccccc|c} \cancel{1} & 1 & 2 & 1 & 2 & 0 \\ \cancel{1} & 2 & 1 & 1 & 1 & 1 \\ \boxed{1} & 0 & 0 & 1 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccccc|c} \boxed{1} & 0 & 0 & 1 & 0 & 1 \\ 0 & \boxed{1} & 2 & 0 & 2 & 2 \\ 0 & \cancel{2} & 1 & 0 & 1 & 0 \end{array}\right] \sim \quad E_3+E_2 \to E_3$$

$$\sim \left[\begin{array}{ccccc|c} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 2 & 0 & 2 & 2 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{array}\right]$$ Incompatible

$$\sim \left[\begin{array}{ccccc|c} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right]$$ s.e.r.

$\boxed{4}$ $\mathbb{Z}_5$

$$\left[\begin{array}{ccccc|c} \cancel{1} & 1 & -1 & 1 & -1 & 0 \\ \cancel{1} & -1 & 1 & 1 & 1 & 1 \\ \boxed{1} & 0 & 0 & 1 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccccc|c} \boxed{1} & 0 & 0 & 1 & 0 & -1 \\ 0 & \boxed{1} & -1 & 0 & -1 & -1 \\ 0 & \cancel{-1} & 1 & 0 & 1 & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{ccccc|c} \boxed{1} & 0 & 0 & 1 & 0 & 1 \\ 0 & \boxed{1} & -1 & 0 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{array}\right]$$ Incompatible

$$\sim \left[\begin{array}{ccccc|c} \boxed{1} & 0 & 0 & 1 & 0 & 0 \\ 0 & \boxed{1} & 4 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & \boxed{1} \end{array}\right]$$

$\boxed{4}$ $\mathbb{Q}$

$$\left[\begin{array}{ccccc|c} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right]$$ I.