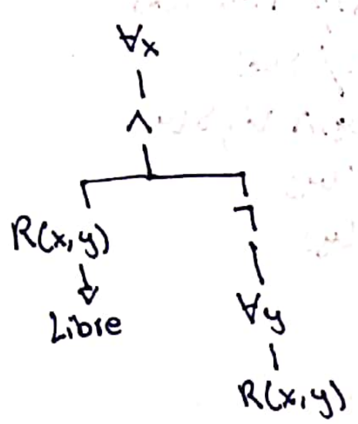
David Martínez Díaz.

- Relacion Tema 5:

5.1.

→ $\forall x\, (R(x,y) \wedge \neg \forall y\, R(x,y))$

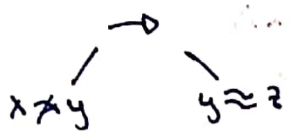


Subfórmulas

- $\forall x\, (R(x,y) \wedge \neg \forall y\, R(x,y))$
- $R(x,y) \wedge \neg \forall y\, R(x,y)$
- $R(x,y)$
- $\neg \forall y\, R(x,y)$
- $\forall y\, R(x,y)$

*Como la y es libre, la fórmula no es sentencia.

→ $x \not\approx y \rightarrow y \approx z$

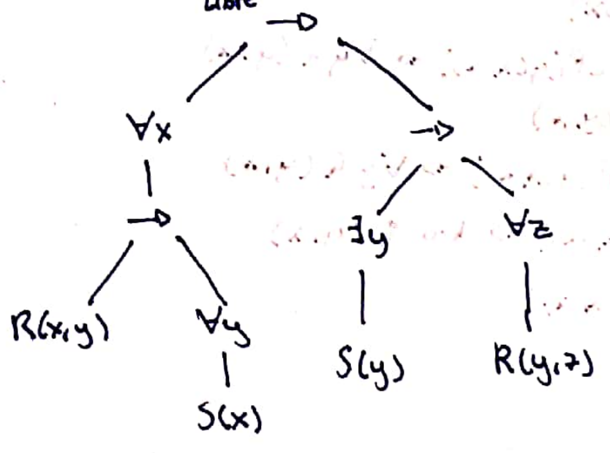


Subfórmulas

- $x \not\approx y \rightarrow y \approx z$
- $x \not\approx y$
- $y \approx z$

*Como no aparecen cuantificadores, todas las ocurrencias son libres y por tanto no es una sentencia.

→ $\forall x\, (\underset{\text{Libre}}{R(x,y)} \rightarrow \forall y\, S(x)) \rightarrow (\exists y\, S(y) \rightarrow \forall z\, \underset{\text{Libre}}{R(y,z)})$



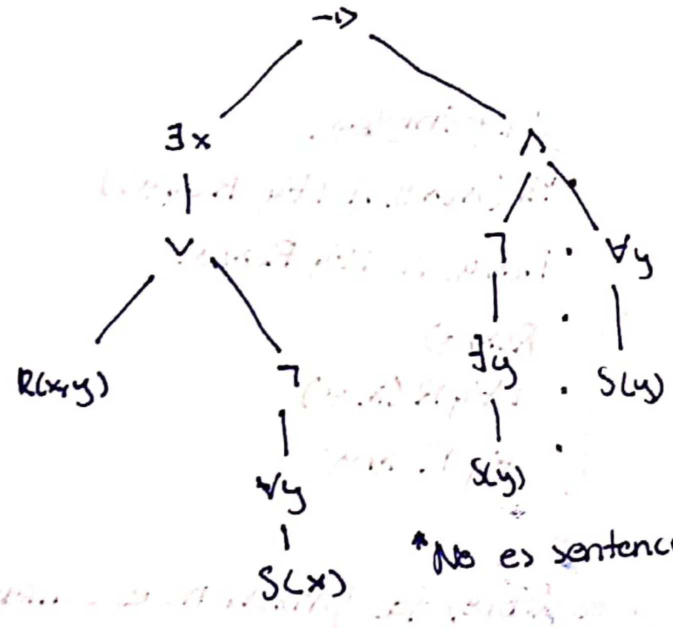
Subfórmulas

- $\forall x\, (R(x,y) \rightarrow \forall y\, S(x)) \rightarrow (\exists y\, S(y) \rightarrow \forall z\, R(y,z))$
- $\forall x\, (R(x,y) \rightarrow \forall y\, S(x))$ / $\exists y\, S(y)$
- $\forall x\, R(x,y)$ / $S(y)$
- $\forall y\, S(x)$ / $\forall z\, R(y,z)$
- $R(x,y)$ / $R(y,z)$
- $S(x)$

*No es una sentencia.

4.→ $\exists x\,(R(x,y) \lor \neg \forall y\, S(x)) \rightarrow (\neg \exists y\, S(y) \land \forall y\, S(y))$

↳ Libre



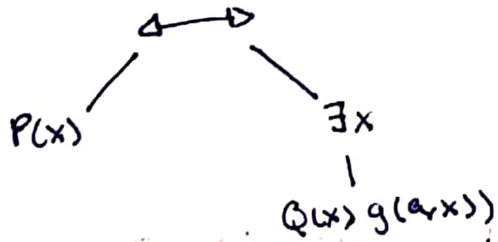
*No es sentencia

Subformulas

- $\exists x\,(R(x,y) \lor \neg \forall y\, S(x))$
- $R(x,y) \lor \neg \forall y\, S(x)$
- $R(x,y)$ / $\neg \forall y\, S(x)$
- $\forall y\, S(x)$ / $S(x)$
- $\neg \exists y\, S(y) \land \forall y\, S(y)$
- $\neg \exists y\, S(y)$
- $\forall y\, S(y)$
- Sy

→ $P(x) \leftrightarrow \exists x\, Q(x, g(a,x))$

Libre



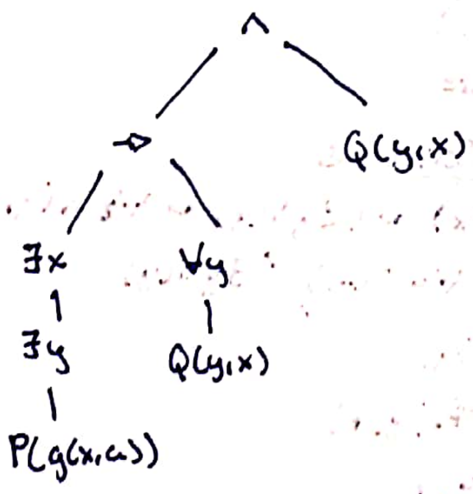
Subformulas

- $P(x) \leftrightarrow \exists x\, Q(x, g(a,x))$
- $P(x)$
- $\exists x\, Q(x, g(a,x))$
- $Q(x, g(a,x))$

*No es una sentencia

→ $\exists x \exists y\, (P(g(x,a)) \rightarrow \forall y\, Q(y,x)) \land Q(y,x)$

↳ Libre



Subformulas

- $Q(y,x)$
- $\exists x \exists y\, (P(g(x,a)) \rightarrow \forall y\, Q(y,x)$
- $\forall y\, Q(y,x)$
- $\exists y\, (P(g(x,a)) \rightarrow \forall y\, Q(y,x)$
- $P(g(x,a)) \rightarrow \forall y\, Q(y,x)$
- $P(g(x,a))$

5.2. a = Antonio b = Begoña c = Carmen

$$\left\{\begin{array}{l} H(x) = x \text{ es hombre.} \\ M(x) = x \text{ es mujer} \\ P(x,y) = x \text{ es progenitor de } y. \\ A(x,y) = x \text{ es antepasado de } y. \\ Hr(x,y) = x \text{ es hermano de } y. \end{array}\right\}$$

1. $\Rightarrow M(b) \land P(b,c)$;
2. $\Rightarrow H(b) \land \exists x\,(P(x,a) \land Hr(x,b))$;
3. $\Rightarrow \exists x\,(P(a,x) \land P(x,b) \land H(a))$;
4. $\Rightarrow \exists x\,(P(a,x) \land P(x,b) \land M(b))$;
5. $\Rightarrow \forall x \exists y\,(P(y,x) \land H(y))$;
6. $\Rightarrow \forall x\,(\exists y \exists z\,(y \neq z \land P(y,x) \land P(z,x)))$;
7. $\Rightarrow \neg \exists x\, P(x,x)$;
8. $\exists x\,(\neg \exists y\, Hr(y,x))$;
9. $\forall x\,(A(x,b) \rightarrow A(x,c))$;
10. $\exists x\,(\neg \exists y\, P(x,y)) \land \exists x\,(\exists y\, P(x,y))$
11. $\forall x \forall y\,(Hr(x,y) \leftrightarrow [\forall z\, P(z,x) \rightarrow P(z,y)] \land [\forall z\, P(z,y) \rightarrow P(z,x)]$;
12. $\exists x\,(H(x) \land Hr(b,x) \land P(a,x))$;

*Añadimos al lenguaje los elementos:

$$\left\{\begin{array}{l} p(x) = \text{El padre de } x. \\ m(x) = \text{La madre de } x. \end{array}\right\}$$

1.→ $b \approx m(c)$

5.→ $\forall x \exists y (y \approx p(x))$

14. $\forall x \forall y (y \approx p(x) \to A(y, x))$

16. $\forall (\exists y (y \not\approx m(x) \land \neg \exists z (z \approx m(x) \land z \not\approx y))))$

$\ldots \wedge \exists r(b,x) \wedge P(a,x))$;

5.3. Universo $\mathbb{Z}_4$ / $P(x) = \begin{cases} 1 & \text{si } x^2 = 0 \\ 0 & \text{si } x^2 \neq 0 \end{cases}$ $Q(x) = \begin{cases} 1 & \text{si } x^2 = 2 \\ \emptyset & \text{sino} \end{cases}$

$R = \{(0,1), (0,2), (2,3), (2,2), (1,2), (3,0)\}$ / $c = 0$

$S = \{(0,1), (0,2), (0,3), (2,3), (0,0)\}$ / $d = 0$

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1. $P(c) = P(0) = 1 \Rightarrow$ [Verdadera];
2. $\neg P(d) = 1 + P(1) = 1 + 0 = 1 \Rightarrow$ [Verdadera];
3. $P(c) \wedge P(d) = P(0) \cdot P(1) = 1 \cdot 0 = 0 \Rightarrow$ [Falsa]
4. $P(c) \Rightarrow \neg Q(d) = 1 + P(0) + P(0) \cdot (1 + Q(1)) = 1 + Q(1) = 1$ [Verdadera]
5. $\exists x\, Q(x) = 0$

x	$Q(x)$
0	0
1	0
2	0
3	0

Falsa

6. $\neg(\exists x\, Q(x)) = 1 + \exists x Q(x) = 1 + 0 = 1$ [Verdadera]

7. $\exists x \neg Q(x) \Rightarrow I^{U \times 10}_{(\neg Q(x))} = \neg Q(0) = 1 + Q(0) = 1$ [Verdadera]

$\exists x \neg Q(x) = 1$, es verdadera.

8. $\exists x (P(x) \wedge Q(x)) = 0 \Rightarrow$ [Falso]

9. $\forall x\, Q(x) \Rightarrow I^{U \times 10}_{(Q(x))} = Q(0) = 0$, luego $\forall x Q(x) = 0$;
$\Rightarrow$ [Es falsa]

10. $\forall x (P(x) \to Q(x)) = 0$

x	$P(x)$	$Q(x)$	$P(x) \to Q(x)$
$\emptyset$	1	0	0
1	0	0	1
2	1	0	0
3	0	0	1

Falsa

11. $\forall x (Q(x) \to \neg P(x)) = 1$

x	$Q(x)$	$\neg P(x)$	$Q(x) \to \neg P(x)$
0	0	0	1
1	0	1	1
2	0	0	1
3	0	1	1

Verdadera

12. $\forall x (Q(x) \to \exists y (P(x) \vee Q(y)) = 1$

$\Downarrow$

$\neg \exists x Q(x) \Rightarrow$ [Verdadera, $Q(x)$ nunca se da]

13. $\forall x R(c, x) \Rightarrow I^{V \times 13} (R(c,x)) = R(0,3) = 0$

$\hookrightarrow$ luego $\forall x R(c,x) = 0$ la sentencia es falsa.

14. $\forall x\, S(c,x)$

x	$S(0,x)$
0	1
1	1
2	1
3	1

Verdadera.

15. $\forall x\,(R(c,x) \to S(c,x)) = 1$

x	$R(c,x)$	$S(c,x)$	$R(c,x) \to S(c,x)$
0	0	1	1
1	1	1	1
2	1	1	1
3	0	1	1

$\Rightarrow$ [Verdadera]

16. $\exists y\, \forall x\, (R(c,x) \to S(c,x)) = 1$

$I^{v_{y|0}}(\forall x(R(c,x) \to S(c,x)) = \forall x\,(R(c,x) \to S(c,x)) = 1$

$\Rightarrow$ [Verdadera]

17. $\forall x\, \forall y\, (R(x,y) \to S(x,y)) = 0$

x	y	$R(x,y)$	$R(x,y) \to S(x,y)$
1	2	1	0

$\Rightarrow$ [Falsa]

18. $\forall x\, \forall y\, (R(x,y) \to \exists z\,(S(x,z))$

x	y	$R(x,y)$	$\exists z\,(S(x,z))$	$R(x,y) \to \exists z\,(S(x,z))$
1	2	1	0	0

$\Rightarrow$ [Falsa]

19. $\forall x\,(P(x) \rightarrow \exists y\,(R(x,y)) = 1$

x	$P(x)$	$\exists y\,(R(x,y))$	$P(x) \rightarrow \exists y\,(R(x,y))$
0	1	1	1
1	0	1	1
2	1	1	1
3	0	1	1

[Verdadera

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20. $\forall x\,(R(x) \rightarrow \exists y\,(S(x,y) \wedge R(y,x))) = \alpha = 0$

x	$P(x)$	$\exists y\,(S(x,y) \wedge R(y,x))$	$R(x) \rightarrow \alpha$
2	1	0	0

$\Rightarrow$ [Falsa]

5.4. 1. $\{\forall x P(x)\} \models P(a)$

$$\left\{ \begin{array}{l} I = (\mathcal{E}, v) \\ I^v = (\forall x P(x)) = 1 \Rightarrow I^v(P(x)) = 1; \end{array} \right\}$$

[* Para cualquier valor de x luego en particular $I^v(P(a)) = 1$; Verdad]

$\Rightarrow$ La consecuencia lógica es cierta.

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2. $\exists x P(x) \models P(a)$ $\Rightarrow$ [Falso]

$D = \{3, 4\}$; $P(x) =$ "x es par"; $a = 3$;

x	P(a)	P(x)	$\exists x P(x)$
3	0	0	1
4		1	

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4. $\models \forall x P(x) \rightarrow P(a) \overset{TD}{\Leftrightarrow} \forall x P(x) \models P(a)$

[* y esto es verdad por 5.4.1]

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5. $\emptyset \models \forall x (P(x) \rightarrow P(a)) = 0$; $\Rightarrow$ La consecuencia lógica es [Falsa]

$D = \{0, 1\}$ $P(x) =$ "$x = 1$"; $a = 0$; $P = \{1\}$

x	P(x)	P(a)	$P(x) \rightarrow P(a)$	$\forall x (P(x) \rightarrow P(a))$
0	0	0	1	0
1	1	0	0	

6. $\models P(a) \to \exists x P(x) \Leftrightarrow P(a) \models \exists x P(x)$

* Supongamos $P(a)$ verdad, es decir, a cumple P
por tanto tomando $x=a$; $\left.\begin{array}{l} x \text{ tiene la propiedad } P \\ x \text{ cumple } P \\ P(x) \text{ es verdad para } x=a \end{array}\right\}$

$\exists x P(x)$ es verdad.

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10. $\forall x P(x) \to Q(a) \models \forall x (P(x) \to Q(a)) \Rightarrow$ [Falso]

$D = \{3, 4\}$ $\left.\begin{array}{l} P(x) = \text{"x es par"}; \\ Q(x) = \text{"x es impar"}; \\ a = 4 \end{array}\right\}$

x	$P(x)$	$\forall x P(x)$	$Q(a)$	$\forall x P(x) \to Q(a)$	$P(x) \to Q(a)$	$\forall x (P(x) \to Q(a))$
3	0	0	0	1	1	0
4	1				0	

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11. $\forall x P(x) \to Q(a) \models \exists x (P(x) \to Q(a)) \Rightarrow$ [Verdadero]

Porque $\forall x P(x) \to Q(a) \equiv \exists x (P(x) \to Q(a))$

[* Segun la chuleta oficial];

Escaneado con CamScanner

5.5. 1. $\forall x\,(R(x,y) \wedge \neg\forall y\, R(x,y)) \Rightarrow \forall x\,(R(x,y) \wedge \exists y\, \neg R(x,y))$

$\Rightarrow \forall x\,(R(x,y) \wedge \exists z\, \neg R(x,z)) \Rightarrow$

$$\left\{\begin{array}{l} \cdot\ \forall x\, \exists z\, (R(x,y) \wedge \neg R(x,z)) \Rightarrow \text{Prenexa} \\ \cdot\ \forall x\, (R(x,y) \wedge \neg R(x, f(x)) \Rightarrow \text{Skolem} \end{array}\right\}$$

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2. $\forall x\,(R(x,y) \rightarrow S(x)) \rightarrow (\exists y\, S(y) \rightarrow \forall z\, R(y,z))$

$\Rightarrow \forall x\,(R(x,y) \rightarrow S(x)) \rightarrow \exists w\,(S(w \rightarrow \forall z\, R(y,z))$

$\Rightarrow \forall x\,(R(x,y) \rightarrow S(x)) \rightarrow \exists w\, \forall z\,(S(w) \rightarrow R(y,z))$

$\Rightarrow \exists w\,(\forall x\,(R(x,y) \rightarrow S(x) \rightarrow \forall z\,(S(w) \rightarrow R(y,z));$

$$\left\{\begin{array}{l} \cdot\ \exists w\, \forall x\, \forall z\, ((R(x,y) \rightarrow S(x)) \rightarrow (S(w) \rightarrow R(y,z)) \Rightarrow \text{Prenexa} \\ \cdot\ \forall x\, \forall z\, ((R(x,y) \rightarrow S(x)) \rightarrow (S(a) \rightarrow R(y,z) \Rightarrow \text{Skolem} \end{array}\right\}$$

3. $\exists x\,(R(x,y) \vee \neg S(x)) \rightarrow (\neg\exists y\, S(y) \wedge \forall y\, S(y));$

$\Rightarrow \exists x\,(R(x,y) \vee \neg S(x)) \rightarrow (\forall y \neg S(y) \wedge \forall y\, S(y));$

$\Rightarrow \exists x\,(R(x,y) \vee \neg S(x)) \rightarrow \forall y\,(S(y) \wedge \neg S(y));$

$\Rightarrow \exists x\,(R(x,y) \vee \neg S(x)) \rightarrow \forall z\,(S(z) \wedge \neg S(z));$

- $\exists x \forall z\,(R(x,y) \vee \neg S(x) \rightarrow S(z) \wedge \neg S(z)),$ $\Rightarrow$ Prenexa
- $\forall z\,((R(a,y) \vee \neg S(a)) \rightarrow (S(z) \wedge \neg S(z))$ $\Rightarrow$ Skolem

4. $\exists x\, R(x,y) \vee [S(x) \wedge \forall z \neg R(a,z)]$

$\Rightarrow \exists u\,[R(u,y) \vee (S(x) \wedge \forall z \neg R(a,z))]$

$\Rightarrow \exists u\,[R(u,y) \vee \forall z\,(Sx \wedge \neg R(a,z)]$

- $\exists u \forall z\,[R(u,y) \vee (S(x) \wedge \neg R(a,z))]$ $\Rightarrow$ Prenexa
- $\forall z\,[R(b,y) \vee (S(x) \wedge \neg R(a,z))]$ $\Rightarrow$ Skolem

5. $\exists x\,(S(x) \rightarrow R(x,y)) \rightarrow (\exists y\, A(y) \rightarrow \forall z\, B(y,z))$

$\Rightarrow \exists x[(S(x) \rightarrow R(x,y)) \rightarrow \exists u\,(A(u) \rightarrow \forall z\, B(y,z)]$

$\Rightarrow \exists x \exists u\,[(S(x) \rightarrow R(x,y)) \rightarrow \forall z\,(A(u) \rightarrow B(y,z))]$

- $\Rightarrow \exists x \exists u \forall z\,[(S(x) \rightarrow R(x,y)) \rightarrow (A(u) \rightarrow B(y,z))]$ $\Rightarrow$ Prenexa
- $\Rightarrow \forall z\,[(S(a) \rightarrow R(a,y)) \rightarrow (A(b) \rightarrow B(y,z))]$ $\Rightarrow$ Skolem

6. $\forall x\, R(x,y) \wedge (\neg S(z) \vee \exists z \neg R(x,z));$

$\Rightarrow \forall u\, R(u,y) \wedge (\neg S(z) \vee \exists w \neg R(x,w));$

$\Rightarrow \forall u\, R(u,y) \wedge \exists w\,(\neg S(z) \vee \neg R(x,w));$

$\Rightarrow \exists w\,[\forall u\, R(u,y) \wedge (\neg S(z) \vee \neg R(x,w))];$

- $\exists w \forall u\,[R(u,y) \wedge (\neg S(z) \vee \neg R(x,w))]$ $\Rightarrow$ Prenexa
- $\forall u\,[R(u,y) \wedge (\neg S(z) \vee \neg R(x,a))]$ $\Rightarrow$ Skolem

Escaneado con CamScanner

7. $\forall x P(x) \rightarrow Q(x,b) \vee \exists y\, Q(y,y)$

$\Rightarrow \forall z P(z) \rightarrow \exists y\, (Q(x,b) \vee Q(y,y))$

$$\left\{ \begin{array}{l} \cdot\ \exists y \forall z\, [P(z) \rightarrow Q(x,b) \vee Q(y,y)] \Rightarrow \text{Prenexa} \\ \cdot\ \forall z\, [P(z) \rightarrow Q(x,b) \vee Q(a,a)] \Rightarrow \text{Skolem} \end{array} \right\}$$

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8. $P(x) \leftrightarrow \exists x\, Q(x, g(a,x))$

$(P(x) \rightarrow \exists x\, Q(x, g(a,x))) \wedge (\exists x\, Q(x, g(a,x)) \rightarrow P(x))$

$[P(x) \rightarrow \exists y Q(y, g(a,y))] \wedge [\exists z\, Q(z, g(a,z)) \rightarrow P(x)]$

$\exists y\, [P(x) \rightarrow Q(y, g(a,y))] \wedge \exists z\, [Q(z, g(a,z)) \rightarrow P(x)]$

$$\left\{ \begin{array}{l} \cdot\ \exists y \exists z\, [(P(x) \rightarrow Q(y, g(a,y)) \wedge (Q(z, g(a,z)) \rightarrow P(x)] \Rightarrow \text{Prenexa} \\ \cdot\ P(x) \rightarrow Q(b, g(a,b))] \wedge [Q(c, g(a,c)) \rightarrow P(x)] \Rightarrow \text{Skolem} \end{array} \right\}$$

5.6. 1. $\forall x\, S(x) \to \exists z \forall y\, R(z,y) \Rightarrow \exists z\, [\forall x\, S(x) \to \forall y\, R(z,y)]$

- $\exists z \forall x \forall y\, [S(x) \to R(z,y)] \Rightarrow$ Prenexa
- $\forall x \forall y\, [S(x) \to R(a,y)] \Rightarrow$ Skolem
- $\forall x \forall y\, (\neg S(x) \vee R(a,y)) \Rightarrow$ Clausular

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2. $\exists x\, [R(x) \to \forall y\, \neg T(x,y)] \wedge \forall z\, \neg [\forall u\, P(u,z) \to \forall u\, Q(u,z)]$

$\Rightarrow \exists x \forall y\, [R(x) \to \neg T(x,y)] \wedge \forall z \exists u \forall u\, [P(u,z) \wedge \neg Q(u,z)]$

$\Rightarrow \exists x \forall y\, ([R(x) \to \neg T(x,y)] \wedge \exists u \forall u\, [P(u,y) \wedge \neg Q(u,y)]$

- $\exists x \forall y \exists u \forall u\, ([R(x) \to \neg T(x,y)] \wedge [P(u,y) \wedge \neg Q(u,y)]] \Rightarrow$ Prenexa
- $\forall y \forall u\, ([R(a) \to \neg T(a,y)] \wedge [P(u,y] \wedge \neg Q\, (f(y), y)]] \Rightarrow$ Skolem

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$\alpha = (R(a) \to \neg T(a,y)) \wedge P(u,y) \wedge \neg Q\, (f(y), y)$

$\Rightarrow [\neg R(a) \vee \neg T(a,y)] \wedge P\, (u,y) \wedge \neg Q(f(y), y)$

- $\forall y\, (\neg R(a) \vee \neg T(a,y)) \wedge \forall y \forall u\, P(u,y) \wedge \forall y\, \neg Q(f(y), y)$

  $\hookrightarrow$ Clausular

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3. $\forall x\, [P(x) \to (Q(x) \vee \neg R(x))] \wedge \exists y\, Q(y)$

- $\exists y \forall x\, [P(x) \to Q(x) \vee \neg R(x)) \wedge Q(y)] \Rightarrow$ Prenexa
- $\forall x\, [(P(x) \to Q(x) \vee \neg R(x)) \wedge Q(a)] \Rightarrow$ Skolem

$\alpha = [\neg P(x) \vee Q(x) \vee \neg R(x)] \wedge \boxed{Q(a)}$

$\forall x\, (\neg P(x) \vee Q(x) \vee \neg R(x)) \wedge Q(a) \Rightarrow$ Clausular

Entonces $\forall x \alpha \equiv$

4. $\forall x\, (P(x) \to Q(x)) \to (\forall y\, P(y) \to \forall z\, P(z))$

$\Rightarrow \forall x\, [(P(x) \to Q(x)) \to \forall y \forall z\, (P(y) \to P(z))]$

$$\left\{\begin{array}{l} \cdot\ \forall x \forall y \forall z\, [(P(x) \to Q(x)) \to (P(y) \to P(z))] \Rightarrow \text{Prenexa} \\ \cdot\ \forall x \forall y \forall z\, [(P(x) \to Q(x)) \to (P(y) \to P(z))] \Rightarrow \text{Skolem} \end{array}\right\}$$

$\alpha = (P(x) \to Q(x)) \to (P(y) \to P(z))$

$(\neg P(y) \vee P(z) \vee P(x)) \wedge (\neg P(y) \vee P(z) \vee \neg Q(x))$;

$$\left\{\begin{array}{l} \forall x \forall y \forall z\, (P(x) \vee \neg P(y) \vee P(z)) \wedge \forall x \forall y \forall z\, (\neg Q(x) \vee \neg P(y) \vee P(z)) \\ \hookrightarrow \text{Forma Clausular} \end{array}\right\}$$

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5. $\forall x\, P(x) \to \exists x\, Qx \Rightarrow \neg \forall x\, P(x) \vee \exists x\, Q(x)$ [* Propiedad]

$\Rightarrow \exists x\, \neg P(x) \vee \exists x\, Q(x)$:

$$\left\{\begin{array}{l} \cdot\ \exists x\, (\neg P(x) \vee Q(x)) \Rightarrow \text{Prenexa} \\ \cdot\ \neg P(a) \vee Q(a) \Rightarrow \text{Skolem / Clausular} \end{array}\right\}$$

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6. $\forall x \forall y\, [\exists z\, (p(x), z) \wedge P(y, z)) \to \exists u\, Q(x, y, u)]$

$\forall x \forall y \exists z\, [(p(x, z) \wedge P(y, z)) \to \exists u\, Q(x, y, u)]$

$$\left\{\begin{array}{l} \cdot\ \forall x \forall y \exists z \exists u\, [(p(x, z) \wedge P(y, z)) \to Q(x, y, u)] \Rightarrow \text{Prenexa} \\ \cdot\ \forall x \forall y\, \cancel{\exists u}\, [P(x, f(x, y)) \wedge P(y, f(x, y)) \to Q(x, y, h(x, y))] \Rightarrow \text{Skolem} \end{array}\right\}$$

$\alpha = \neg (P(x, f(x, y)) \wedge P(y, f(x, y)) \vee Q(x, y, h(x, y))$

$\neg P(x, f(x, y)) \vee \neg P(y, f(x, y)) \vee Q(x, y, h(x, y))$

$$\left\{\cdot\ \forall x \forall y\, [\neg P(x) f(x, y)) \vee \neg P(y, f(x, y)) \vee Q(x, y, h(x, y))] \Rightarrow \text{Clausular}\right\}$$

7. $\forall x\,[P(x) \wedge \forall y\,(\neg Q(x,y) \to \forall z R(a,x,y)]$

$\Rightarrow \forall x\,[P(x) \wedge \forall y\,(\neg Q(x,y) \to R(a,x,y)]$

- $\forall x \forall y\,[P(x) \wedge (Q(x,y) \vee R(a,x,y)] \Rightarrow$ Prenexo/ Skolem
  $\alpha = P(x) \wedge Q(x,y) \vee R(a,x,y);$
- $\forall x P(x) \wedge \forall x \forall y\,(Q(x,y) \vee R(a,x,y);$ $\Rightarrow$ Clausular

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8. $\forall x \forall y\,[\exists z P(z) \wedge \exists u\,(Q(x,u) \to \exists u \cdot Q(y,u)]$

$\Rightarrow [\exists z P(z) \wedge \forall x \forall y \exists u\,(Q(x,u) \to \exists v Q(y,u))]$

$\Rightarrow \exists z [P(z) \wedge \forall x \forall y \exists u \exists v\,[Q(x,u) \to Q(y,v)]]$

- $\exists z \forall x \forall y \exists u \exists v\,[P(z) \wedge (Q(x,u) \to Q(y,u))] \Rightarrow$ Prenexa
- $\forall x \forall y\,[P(a) \wedge (Q(x,f(x,y)) \to Q(y,h(x,y))] \Rightarrow$ Skolem
- $P(a) \wedge \forall x \forall y\,(\neg Q(x,f(x,y)) \vee Q(y,h(x,y))$ $\Rightarrow$ Clausular