January through June, total highway miles driven declined every month. The U.S. Department of Transportation estimated a total decline of 20 billion miles traveled during the first half of 2008. At the same time, commuter rail usage increased. Gasoline prices also affected car sales. Purchases of gas-guzzling vehicles such as SUVs (Sport Utility Vehicles) and pickup trucks fell approximately 40 percent in May, and again in June. Relative sales of smaller cars rose. In addition, sales of diesel cars increased (the price of diesel gasoline did not rise as sharply).

In the next section we will combine budget lines with the utility theory from Chapter 3. After studying that section, you will be able to explain why consumers changed their spending habits in response to the rise in gasoline prices as described here.

LEARNING-BY-DOING EXERCISE 4.1

Good News/Bad News and the Budget Line

Suppose that a consumer's income (I) doubles and that the prices (P_x and P_y) of both goods in his basket also double. He views the doubling of income as good news because it increases his purchasing power. However, the doubling of prices is bad news because it decreases his purchasing power.

Problem What is the net effect of the good and bad news?

Solution The location of the budget line is determined by the x and y intercepts. Before the doubling of income and prices, the y intercept was I/P_y; afterward, the y intercept is $2I/2P_y = I/P_y$, so the y intercept is unchanged. Similarly, the x intercept is unchanged. Thus, the location of the budget line is unchanged, as is its slope, since $-(2P_x/2P_y) = -(P_x/P_y)$. The doubling of income and prices has no net effect on the budget line, on the trade-off between the two goods, or on the consumer's purchasing power.

Similar Problems: 4.1, 4.2.

We have learned that the consumer can choose any basket on or inside the budget line. But which basket will he choose? We are now ready to answer this question.

4.2 OPTIMAL CHOICE

optimal choice
Consumer choice of a basket of goods that (1) maximizes satisfaction (utility) while (2) allowing him to live within his budget constraint.

If we assume that a consumer makes purchasing decisions rationally and we know the consumer's preferences and budget constraint, we can determine the consumer's **optimal choice**—that is, the optimal amount of each good to purchase. More precisely, optimal choice means that the consumer chooses a basket of goods that (1) maximizes his satisfaction (utility) and (2) allows him to live within his budget constraint.

Note that an optimal consumption basket must be located on the budget line. To see why, refer back to Figure 4.1. Assuming that Eric likes more of both goods (food and clothing), it's clear that a basket such as F cannot be optimal because basket F doesn't require Eric to spend all his income. The unspent income could be used to increase satisfaction with the purchase of additional food or clothing.[2] For this reason, no point inside the budget line can be optimal.

Of course, consumers do not always spend all of their available income at any given time. They often save part of their income for future consumption. The introduction of time into the analysis of consumer choice really means that the consumer is making choices over more than just two goods, including for instance the consumption of food

[2]This observation can be generalized to the case in which the consumer is considering purchases of more than two goods, say N goods, all of which yield positive marginal utility to the consumer. At an optimal consumption basket, all income must be exhausted.

today, clothing today, food tomorrow, and clothing tomorrow. For now, however, let us keep matters simple and assume that there is no tomorrow. Later, we will introduce time (with the possibility of borrowing and saving) into the discussion.

To state the problem of optimal consumer choice, let $U(x, y)$ represent the consumer's utility from purchasing x units of food and y units of clothing. The consumer chooses x and y, but must do so while satisfying the budget constraint $P_x x + P_y y \leq I$. The optimal choice problem for the consumer is expressed like this:

$$\max_{(x,y)} U(x, y) \qquad (4.2)$$
$$\text{subject to: } P_x x + P_y y \leq I$$

where the notation "$\max_{(x,y)} U(x, y)$" means "choose x and y to maximize utility," and the notation "subject to: $P_x x + P_y y \leq I$" means "the expenditures on x and y must not exceed the consumer's income." If the consumer likes more of both goods, the marginal utilities of food and clothing are both positive. At an optimal basket all income will be spent (i.e., the consumer will choose a basket *on* the budget line $P_x x + P_y y = I$).

Figure 4.4 represents Eric's optimal choice problem graphically. He has an income of $I = \$800$ per month, the price of food is $P_x = \$20$ per unit, and the price of clothing is $P_y = \$40$ per unit. The budget line has a vertical intercept at $y = 20$, indicating that if he were to spend all his income on clothing, he could buy 20 units of clothing each month. Similarly, the horizontal intercept at $x = 40$ shows that Eric could buy 40 units of food each month if he were to spend all his income on food. The slope of the budget line is $-P_x/P_y = -1/2$. Three of Eric's indifference curves are shown as U_1, U_2, and U_3.

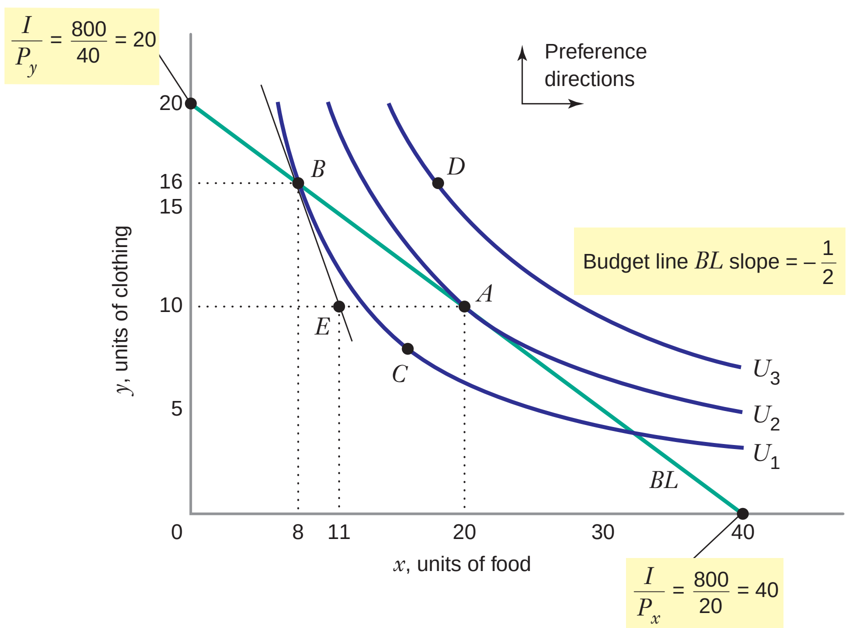

FIGURE 4.4 Optimal Choice: Maximizing Utility with a Given Budget
Which basket should the consumer choose if he wants to maximize utility while living within a budget constraint limiting his expenditures to \$800 per month? He should select basket *A*, achieving a level of utility U_2. Any other basket on or inside the budget line *BL* (such as *B*, *E*, or *C*) is affordable, but leads to less satisfaction. A basket outside the budget line (such as *D*) is not affordable.

At the optimal basket *A* the budget line is tangent to an indifference curve. The slope of the indifference curve U_2 at point *A* and the slope of the budget line are both $-1/2$.

To maximize utility while satisfying the budget constraint, Eric will choose the basket that allows him to reach the highest indifference curve while being on or inside the budget line. In Figure 4.4 that optimal basket is A, where Eric achieves a level of utility U_2. Any other point on or inside the budget line will leave him with a lower level of utility.

To further understand why basket A is the optimal choice, let's explore why other baskets are *not* optimal. First, baskets outside the budget line, such as D, cannot be optimal because Eric cannot afford them. We can therefore restrict our attention to baskets on or inside the budget line. Any basket inside the budget line, such as E or C, is also not optimal, since, as we have shown, an optimal basket must lie on the budget line.

If Eric were to move along the budget line away from A, even by a small amount, his utility would fall because the indifference curves are bowed in toward the origin (in economic terms, because there is diminishing marginal rate of substitution of x for y). At the optimal basket A, the budget line is just tangent to the indifference curve U_2. This means that the slope of the budget line $(-P_x/P_y)$ and the slope of the indifference curve are equal. Recall from equation (3.5) that the slope of the indifference curve is $-MU_x/MU_y = -MRS_{x,y}$. Thus, at the optimal basket A, this tangency condition requires that

$$\frac{MU_x}{MU_y} = \frac{P_x}{P_y} \tag{4.3}$$

or $MRS_{x,y} = P_x/P_y$. In Appendix 1, we show how this condition can be derived using formal mathematical tools.

interior optimum An optimal basket at which a consumer will be purchasing positive amounts of all commodities.

In Figure 4.4 the optimal basket A is said to be an **interior optimum**, that is, an optimum at which the consumer will be purchasing both commodities ($x > 0$ and $y > 0$). The optimum occurs at a point where the budget line is tangent to the indifference curve. In other words, at an interior optimal basket, the consumer chooses commodities so that the ratio of the marginal utilities (i.e., the marginal rate of substitution) equals the ratio of the prices of the goods.

We can also express the tangency condition by rewriting equation (4.3) as follows:

$$\frac{MU_x}{P_x} = \frac{MU_y}{P_y} \tag{4.4}$$

This form of the tangency condition states that, at an interior optimal basket, the consumer chooses commodities so that the marginal utility per dollar spent on each commodity is the same. Put another way, at an interior optimum, the extra utility per dollar spent on good x is equal to the extra utility per dollar spent on good y. Thus, at the optimal basket, each good gives the consumer equal "bang for the buck."

Although we have focused on the case in which the consumer purchases only two goods, such as food and clothing, the consumer's optimal choice problem can also be analyzed when the consumer buys more than two goods. For example, suppose the consumer chooses among baskets of three commodities. If all of the goods have positive marginal utilities, then at the optimal basket the consumer will spend all of his income. If the optimal basket is an interior optimum, the consumer will choose the goods so that the marginal utility per dollar spent on all three goods will be the same. The same principles apply to the case in which the consumer buys any given number of goods.

TWO WAYS OF THINKING ABOUT OPTIMALITY

We have shown that basket A in Figure 4.4 is optimal for the consumer because it answers this question: *What basket should the consumer choose to maximize utility, given a budget constraint limiting expenditures to \$800 per month?* In this case, since the consumer chooses the basket of x and y to maximize utility while spending no more than \$800 on the two goods, optimality can be described as follows:

$$\max_{(x,y)} \text{Utility} = U(x, y) \qquad (4.5)$$
$$\text{subject to: } P_x x + P_y y \leq I = 800$$

In this example, the endogenous variables are x and y (the consumer chooses the basket). The level of utility is also endogenous. The exogenous variables are the prices P_x and P_y and income I (i.e., the level of expenditures). The graphical approach solves the consumer choice problem by locating the basket on the budget line that allows the consumer to reach the highest indifference curve. That indifference curve is U_2 in Figure 4.4.

There is another way to look at optimality, by asking a different question: *What basket should the consumer choose to minimize his expenditure* ($P_x x + P_y y$) *and also achieve a given level of utility* U_2? Equation (4.6) expresses this algebraically:

$$\min_{(x,y)} \text{expenditure} = P_x x + P_y y \qquad (4.6)$$
$$\text{subject to: } U(x, y) = U_2$$

expenditure minimization problem Consumer choice between goods that will minimize total spending while achieving a given level of utility.

This is called the **expenditure minimization problem**. In this problem the endogenous variables are still x and y, but the exogenous variables are the prices P_x, P_y, and the required level of utility U_2. The level of expenditure is also endogenous. Basket A in Figure 4.5 is optimal because it solves the expenditure minimization problem. Let's see why.

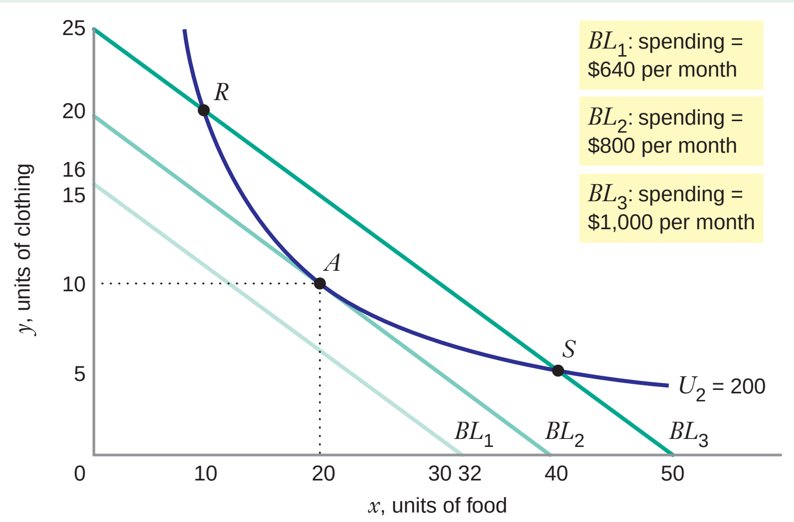

FIGURE 4.5 Optimal Choice: Minimizing Expenditure to Achieve a Given Utility
Which basket should the consumer choose if he wants to minimize the expenditure necessary to achieve a level of utility U_2? He should select basket A, which can be purchased at a monthly expenditure of \$800. Other baskets on U_2 will cost the consumer more than \$800. For example, to purchase R or S (also on U_2), the consumer would need to spend \$1,000 per month (since R and S are on BL_3). Any total expenditure less than \$800 (e.g., \$640, represented by BL_1) will not enable the consumer to reach the indifference curve U_2.

Using Figure 4.5, let's look for a basket that would require the lowest expenditure to reach indifference curve U_2. (In this figure, U_2 corresponds to a utility level of 200.)

In the figure, we have drawn three different budget lines. All baskets on the budget line BL_1 can be purchased if the consumer spends \$640 per month. Unfortunately, none of the baskets on BL_1 allows him to reach the indifference curve U_2, so he will need to spend more than \$640 to achieve the required utility. Could he reach the indifference curve U_2 with a monthly expenditure of \$1,000? All baskets on budget line BL_3, such as baskets R and S, can be purchased by spending \$1,000 a month. But there are other baskets on U_2 that would cost the consumer less than \$1,000. To find the basket that minimizes expenditure, we have to find the budget line that is tangent to the indifference curve U_2. That budget line is BL_2, which is tangent to BL_2 at point A. Thus, the consumer can reach U_2 by purchasing basket A, which costs only \$800. Any expenditure less than \$800 will not be enough to purchase a basket on indifference curve U_2.

The utility maximization problem of equation (4.5) and the expenditure minimizing problem of equation (4.6) are said to be *dual* to one another. The basket that maximizes utility with a given level of income leads the consumer to a level of utility U_2. That *same* basket minimizes the level of expenditure necessary for the consumer to achieve a level of utility U_2.

We have already seen that a basket such as B in Figure 4.6 is not optimal because the budget line is not tangent to the indifference curve at that basket. How might the consumer improve his choice if he is at basket B, where he is spending \$800 per month and realizing a level of utility $U_1 = 128$? We can answer this question from either of our dual perspectives: utility maximization or expenditure minimization. Thus, the consumer could ask, "If I spend \$800 per month, what basket will maximize my satisfaction?" He will choose basket A and realize a higher level of utility U_2. Alternatively, the consumer might say, "If I am content with a level of utility U_1, what is the least amount of money I will need to spend?" As the graph shows, the answer to this question is basket C, where he needs to spend only \$640 per month.

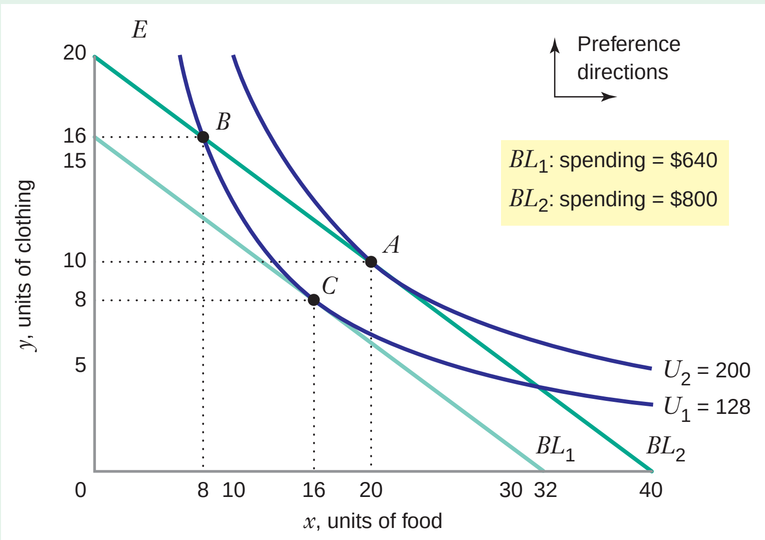

FIGURE 4.6 Nonoptimal Choice At basket B the consumer spends \$800 monthly and realizes a level of utility U_1. There are two ways to see that basket B is not an optimal choice. The consumer could continue to spend \$800 per month but realize greater utility by choosing basket A, reaching indifference curve U_2. Or the consumer could continue to achieve U_1 but spend less than \$800 per month by choosing basket C.