1. a) Demostrar que $\Theta_\epsilon + \Theta_{-\epsilon} = 1$
1. b) Demostrar que $\Theta_\epsilon - \Theta_{-\epsilon} = \frac{1}{2}\epsilon\{\epsilon(V) - \epsilon(U)\}$
2. a) Demostrar que $t_\epsilon = \left(t + \frac{1}{2}\frac{i\pi}{k_0}\right)\Theta_\epsilon + \left(t - \frac{1}{2}\frac{i\pi}{k_0}\right)\Theta_{-\epsilon}$
2. b) Demostrar que $t_{\epsilon'\epsilon} = \left(t + \frac{1}{2}\frac{i\pi}{k_0}\epsilon'\right)\Theta_\epsilon + \left(t - \frac{1}{2}\frac{i\pi}{k_0}\epsilon'\right)\Theta_{-\epsilon}$
3. Demostrar que $a_\Omega^{(\epsilon)} = e^{-iG} b_\Omega^{(\epsilon)} e^{iG}$
4. a) Demostrar que $e^{-i\omega t_\epsilon} = e^{-i\omega t}\left(e^{\frac{1}{2}\frac{\pi\omega}{k_0}}\Theta_\epsilon + e^{-\frac{1}{2}\frac{\pi\omega}{k_0}}\Theta_{-\epsilon}\right)$
4. b) Demostrar que $e^{-i\omega t_{\epsilon\epsilon'}} = e^{-i\omega t}\left(e^{\frac{1}{2}\frac{\pi\omega}{k_0}\epsilon'}\Theta_\epsilon + e^{-\frac{1}{2}\frac{\pi\omega}{k_0}\epsilon'}\Theta_{-\epsilon}\right)$
5. Si $\epsilon' = \epsilon(\omega)$, demostrar que: $e^{-i\omega t_{\epsilon(\omega)\epsilon}} = e^{-i\omega t}\left(e^{\frac{1}{2}\frac{\pi|\omega|}{k_0}}\Theta_\epsilon + e^{-\frac{1}{2}\frac{\pi|\omega|}{k_0}}\Theta_{-\epsilon}\right)$
6. Definiendo $\chi = \chi(\omega)$ por $\tanh\chi = e^{-\frac{\pi|\omega|}{k_0}}$, demostrar que: $e^{-i\omega t_{\epsilon(\omega)\epsilon}} = e^{-i\omega t}\left[\left(\frac{\cosh\chi}{\operatorname{senh}\chi}\right)^{1/2}\Theta_\epsilon + \left(\frac{\operatorname{senh}\chi}{\cosh\chi}\right)^{1/2}\Theta_{-\epsilon}\right]$
7. Demostrar que: $[(\operatorname{senh}\chi)(\cosh\chi)]^{1/2} e^{-i\omega t_{\epsilon(\omega)\epsilon}} = e^{-i\omega t}\left[\cosh\chi\Theta_\epsilon + \operatorname{senh}\chi\Theta_{-\epsilon}\right]$
8. Deducir la Transformación de Bogoliubov: $\chi_\Omega^{(\epsilon)}(x) = \varphi_\Omega^{(\epsilon)}(x)\cosh\chi + \varphi_\Omega^{(-\epsilon)}(x)\sinh\chi$ a partir de:
$\varphi_\Omega(\underline{x}, t) = \varphi_\Omega(\underline{x})\frac{1}{\sqrt{2|\omega|}}e^{-i\omega t}$ y $\varphi_\Omega^{(\epsilon)}(x) = \varphi_\Omega(\underline{x}, t)\Theta_\epsilon(x)$
9. Deducir: $\chi_\Omega^{(\epsilon)}(x) = \sqrt{\frac{\sinh\chi\cosh\chi}{2|\omega|}}\varphi_\Omega(\underline{x})e^{-i\omega t_{\epsilon(\omega)\epsilon}}$
10. Con la identidad BCH demostrar que: $e^{-iG}|0\rangle \;\; = \frac{1}{\cosh\chi}e^{(\tanh\chi)b_\Omega^{(+)\dagger}b_\Omega^{(-)}}|0\rangle_{\mathrm{B}}$