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How to Use This Book

The best way of understanding the laws of physics and learning how to solve physics problems is through practice. This book features almost three hundred problems and solutions worked out in detail. In Part I, *Problems* are arranged thematically, starting in Chapter 1 with problems about **mechanics**, the branch of physics concerned with the behaviour of physical bodies when subjected to forces or displacements, and the subsequent effect of the bodies on their environment. Chapter 2 offers problems in **thermodynamics**, the study of energy conversion between heat and mechanical work, while the **electrodynamics** problems in Chapter 3 deal with the phenomena associated with moving electrical charges and their interaction with electric and magnetic fields. Chapter 4's problems on **magnetism** seek to understand how materials respond on the microscopic level to an applied magnetic field. Lastly, the **optics** problems in Chapter 5 address the branch of physics that studies the behaviour and physical properties of light.

While the problems are arranged by topic, the problems within a single topic are often arranged by increasing level of difficulty. Indeed, many of these physics problems are difficult – yet we encourage students to try and solve the problems on their own, and to only consult the *Solutions* section in order to compare their own attempts with the correct results. We encourage creativity in problem-solving, and these physics problems are intended as a means of developing the student's knowledge of physics by applying them to concrete problems.

Physical Constants and Other Data

Gravitational constant	G	6.673×10^{-11} Nm^{2}kg^{-2}
Speed of light (in vacuum)	c	2.998×10^{8} ms^{-1}
Elementary charge	e	1.602×10^{-19} C
Electron mass	m_e	9.109×10^{-31} kg (511.0 keV)
Proton mass	m_p	1.673×10^{-27} kg (938.3 MeV)
Neutron mass	m_n	1.675×10^{-27} kg (939.6 MeV)
Charge-to-mass ratio of electron	e/m_e	1.759×10^{11} Ckg^{-1}
Unified atomic mass constant	m_u	1.661×10^{-27} kg
Boltzmann constant	k	1.381×10^{-23} JK^{-1}
Plank constant	h	6.626×10^{-34} Js
Avogadro constant	N_A	6.022×10^{23} mol^{-1}
Gas constant	R	8.315 Jmol^{-1} K^{-1}
Permittivity of free space	ε_0	8.854×10^{-12} CV$^{-1}m^{-1}$
Permeability of free space	μ_0	$4\pi \times 10^{-7}$ Vs$^2C^{-1}m^{-1}$
Coulomb constant	$k = 1/4\pi\varepsilon_0$	8.987×10^{9} VmC^{-1}
Compton wavelength of electron	λ_c	2.426×10^{-12} m
Mean radius of the Earth	R	6371 km
Sun-Earth distance (Astronomical Unit, AU)		1.49×10^{8} km
Mean density of the Earth	ρ	5520 kgm^{-3}
Acceleration due to gravity	g	9.807 ms^{-2}
Mass of the Earth		5.978×10^{24} kg
Mass of the Sun		1.989×10^{30} kg
1 light year		9.461×10^{15} m
Surface tension of water	γ	0.073 Nm^{-1}
Heat of vaporisation of water	L	2256 kJkg^{-1} = 40.6 kJmol^{-1}
Tensile strength of steel	σ	500–2000 MPa

Part I

PROBLEMS

Chapter 1

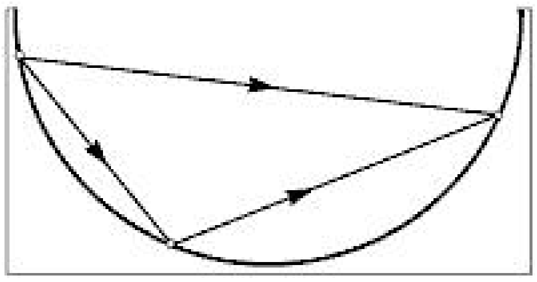
Mechanics Problems

1.1 Kinematics

Problem 1. A train is moving at a speed of v towards the railwayman next to the rails. The train whistles for a time of T. How long does the railwayman hear the whistle? The speed of sound is $c = 330\,\mathrm{m/s}$; $v = 108\ \mathrm{km/hour} = 30\,\mathrm{m/s}$, $T = 3\,\mathrm{s}$; the train does not reach the railwayman until the end of the whistle.

Problem 2. The speed of a motorboat in still water is four times the speed of a river. Normally, the motorboat takes one minute to cross the river to the port straight across on the other bank. One time, due to a motor problem, it was not able to run at full power, and it took four minutes to cross the river along the same path. By what factor was the speed of the boat in still water reduced? (Assume that the speed of the water is uniform throughout the whole width of the river.)

Problem 3. Consider a trough of a semicircular cross section, and an inclined plane in it that leads from a point A to point B lying lower than A. Prove that wherever point C is chosen on the arc AB, an object will always get from A to B faster along the slopes ACB than along the original slope AB. The change of direction at C does not involve a change in speed. The effects of friction are negligible.



Problem 4. The acceleration of an object is uniformly increasing, and it is $a_0 = 2\ \mathrm{m/s^2}$ at $t_0 = 0$ s and $a_1 = 3\ \mathrm{m/s^2}$ at $t_1 = 1$ s. The speed of the object at $t_0 = 0$ s is $v_0 = 1\ \mathrm{m/s}$.

a) Determine the speed of the object at $t_2 = 10$ s.

b) Determine the $v-t$ function of the motion, and then plot it in the $v-t$ coordinate system.

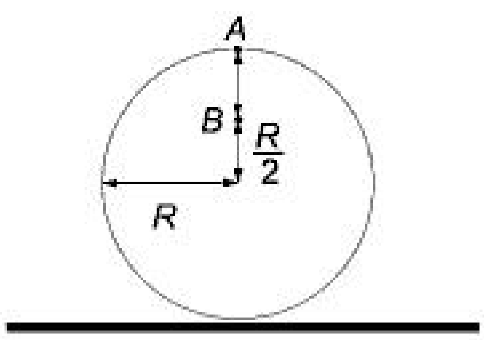
c) Estimate the distance covered by the object in the first and last second of the time interval $0 < t < 10$ s.

Problem 5. An object moves on a circular path such that its distance covered is given by the function: $s = 0.5t^2\,\mathrm{m} + 2t\,\mathrm{m}$. The ratio of the magnitudes of its accelerations at times $t_1 = 2\,\mathrm{s}$ and $t_2 = 5\,\mathrm{s}$ is $1:2$. Find the radius of the circle.

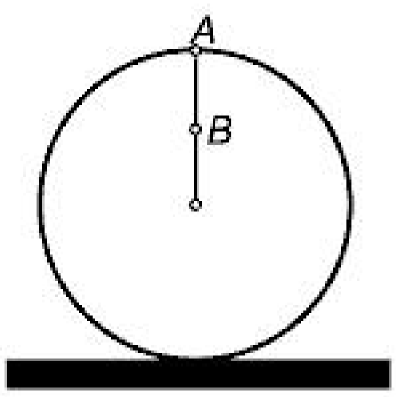
Problem 6. The radius of the tire of a car is R. The valve cap is at distance r from the axis of the wheel. The car starts from rest without skidding, at constant acceleration. Is it possible, in some way, that the valve cap has no acceleration

a) in the $\frac{1}{8}$ turn following the bottom position,

b) in the $\frac{1}{8}$ turn preceding the bottom position?



Problem 7. A disc of diameter $20\,\mathrm{cm}$ is rolling at a speed of $4\ \mathrm{m/s}$ on the ground, without slipping. How long does it take until the speed of point A first becomes equal to the present value of the speed of point B?



Problem 8. A disc of radius $R = 1\ \mathrm{m}$ rolls uniformly, without skidding on horizontal ground. The speed of its centre is $v = 0.5\,\mathrm{m/s}$. Let A stand for the topmost point at $t = 0$ and B for the mid-point of the corresponding radius.

a) At what time will the speed of point A first equal the speed of point B?

b) Following on from part a) above, when the speed of point A first equals the speed of point B, what is this speed?

c) Following on from part a) above, find the distance travelled by the centre of the disk up to the time when the speed of point A first equals the speed of point B.

Problem 9. A cart moves on a muddy road. The radius of its wheels is $R = 0.6\ \mathrm{m}$. A small bit of mud detaches from the rim at a height $h = \frac{3}{2}R$ from the ground.

a) Find the speed of the cart if the bit of mud falls back on the wheel at the same height.

b) Find the length of the arc on the rim that connects the points of detaching and falling back.

c) Find the distance covered by the car in the meantime.

Problem 10. A balloon is rising vertically from the ground in such a way that with high accuracy its acceleration is a linearly decreasing function of its altitude above the ground level. At the moment of release the velocity of the balloon is zero, and its acceleration is a_0.

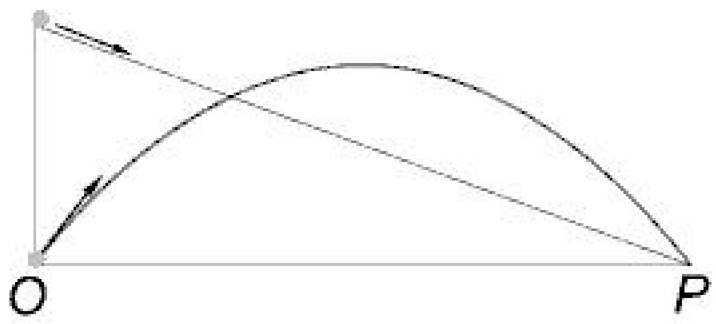
a) Determine the speed of the balloon at the height H, where its acceleration becomes zero.

b) What is the speed of the balloon at half of the altitude H?

c) How long does it take the balloon to reach the altitude H?

Problem 11. A massive ball is falling down from an initial height of $h = 20$ m. With a gun held horizontally, $d = 50$ m far from the trajectory of the falling ball, at the height of $h' = 10$ m, we are going to shoot at the falling ball. The bullet leaves the gun at a speed of $v = 100$ m/s. At what time after the start of the fall should the gun be fired in order to hit the falling ball with the bullet? (The air resistance is negligible.)

Problem 12. Two objects, one sliding down from rest on a smooth (frictionless) slope, the other being thrown from the point O, start their motion at the same instant. Both get to the point P at the same time and at the same speed. Determine the initial angle of the throw.

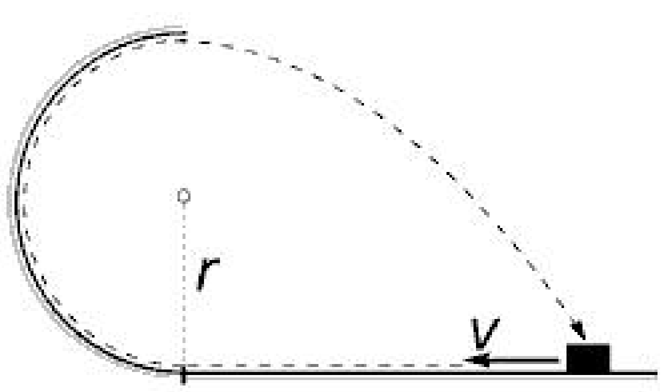


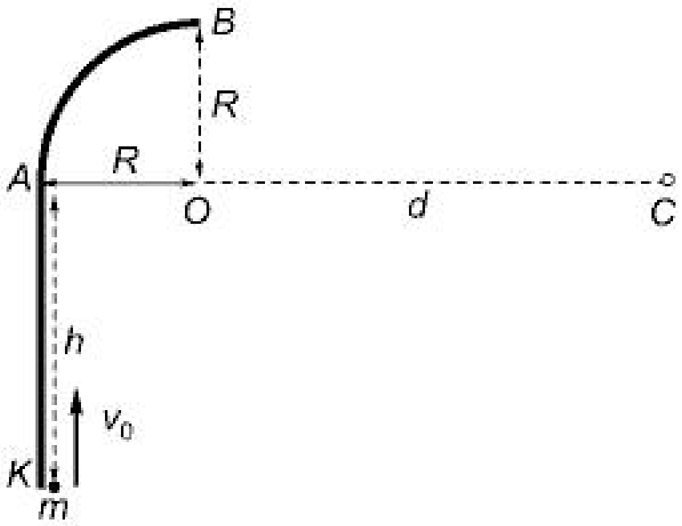
Problem 13. A projectile is projected on the level ground at an angle of 30° with an initial speed of 400 m/s. At one point during its trajectory the projectile explodes into two pieces. The two pieces reach the ground at the same moment; one of them hits the ground at exactly where it was projected with a speed of 250 m/s. At what height did the explosion occur? (Air drag and the mass of the explosive material is negligible, the acceleration due to gravity can be considered as 10 m/s^2.)

Problem 14. The bullet of a poacher flying at a speed of $v = 680$ m/s passes the gamekeeper at a distance $d = 1$ m. What was the distance of the bullet from the gamekeeper when he began to sense its shrieking sound? The speed of propagation of sound is $c = 340$ m/s.

1.2 Dynamics

Problem 15. A frictionless track consists of a horizontal part of unknown length, which connects to a vertical semicircle of radius r as shown. An object, which is given an initial velocity v, is to move along the track in such a way that after leaving the semicircle at the top it is to fall back to its initial position. What should the minimum length of the horizontal part be?





Problem 16. A pointlike object of mass m starts from point K in the figure. It slides along the full length of the smooth track of radius R, and then moves freely and travels to point C.

a) Determine the vertical initial velocity of the pointlike object.

b) What is the minimum possible distance $OC = d$, necessary for the object to slide along the entire length of the track?

c) Find the normal forces exerted by the track at points A and B.

(Let $R = 1\,\mathrm{m}$, $h = 2\,\mathrm{m}$, $d = 3\,\mathrm{m}$, $m = 0.5\,\mathrm{kg}$, use $g = 10\,\mathrm{m/s^2}$)

Problem 17. A small object starts with a speed of $v_0 = 20\,\mathrm{m/s}$ at the lowest point of a circular track of radius $R = 8.16\,\mathrm{m}$. The small object moves along the track. How big a part of the circular track can be removed, if you want to carry out the same trick? (Neglect friction, $g = 9.8\,\mathrm{m/s^2}$.)

Problem 18. A small object of mass $m = 0.5\,\mathrm{kg}$ that hangs on a string of length $L = 5.6\,\mathrm{m}$ is given a horizontal velocity of $v_0 = 14\,\mathrm{m/s}$. The string can withstand a maximum tension of $40\,\mathrm{N}$ without breaking. Where is the stone when the string breaks? Use $g = 10\,\mathrm{m/s^2}$.

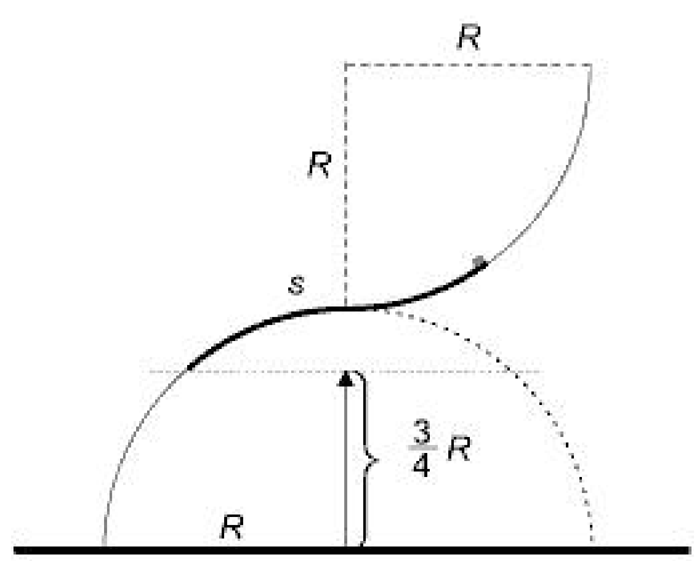
Problem 19. An object slips down the frictionless surface of a cylinder of radius R.

a) Find the position in which the acceleration of the object is two thirds of the gravitational acceleration G.

b) Find the direction of the object's acceleration in that position.

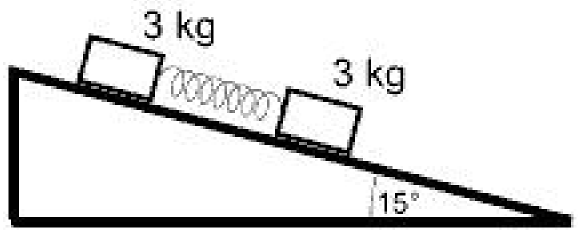
Problem 20. Two horizontal tracks are connected through two circular slopes the radii of which are equal and $R = 5\,\mathrm{m}$. The tracks and the slopes are in a vertical plane and they join without a break or sharp corner. The height difference between the horizontal tracks is $h = 2\,\mathrm{m}$. An object moves from the track at the top onto the bottom one without friction. What is the maximum initial speed of the object when it starts, in order for it to touch the path at all times during its motion?

Problem 21. A small object is moving on a special slope consisting of a concave and a convex circular arc, both of which have a right angle at the centre and radius $R = 0.5\,\mathrm{m}$, and they join smoothly, with horizontal common tangent, as it is shown in the figure. Determine the distance covered by the object on the slope, provided that it started from rest and it detaches from the slope at the altitude $\frac{3}{4}R$. (The friction is negligibly small.)



Problem 22. A pendulum, whose cord makes an angle 45° with the vertical is released. Where will the bob reach its minimum acceleration?

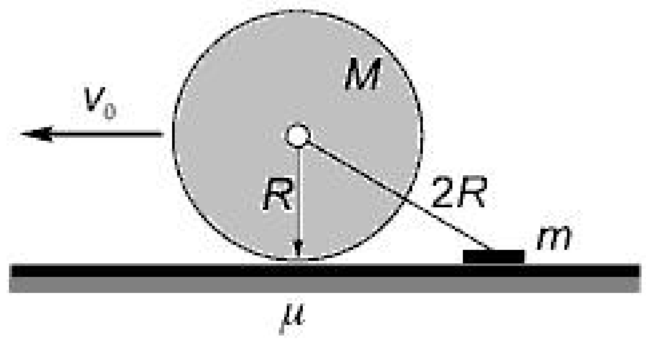
Problem 23. Two blocks, each of mass 3 kg, are connected by a spring, whose spring constant is $200\,\mathrm{N/m}$. They are placed onto an inclined plane of angle 15°. The coefficient of friction between the upper block and the inclined plane is 0.3, while between the lower block and the inclined plane it is 0.1. After a while, the two blocks move together with the same acceleration. Use $g = 10\,\mathrm{m/s^2}$.



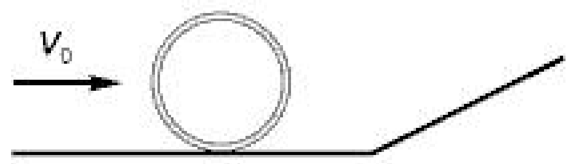
a) Find the value of their acceleration.

b) Find the extension of the spring.

Problem 24. A solid cylinder of mass M and radius R, rolling without sliding on a rough horizontal plane, is pulled at its axis with a horizontal velocity of v_0. By means of a string of length $2R$ attached to its axis, the cylinder is dragging a thin plate of mass $m = 2M$ lying on the plane. If the system is released, how long does it take to stop, and what is the stopping distance? ($\mu = 0.4$; $v_0 = 2$ m/s; $R = 0.5\,\mathrm{m}$, use $g = 10$ m/s^2)



Problem 25. A rigid surface consists of a rough horizontal plane and an inclined plane connecting to it without an edge. A thin hoop of radius $r = 0.1\,\mathrm{m}$ is rolling towards the slope without slipping, at a velocity of $v_0 = 3.5\ \dfrac{\mathrm{m}}{\mathrm{s}}$, perpendicular to the base of the slope.

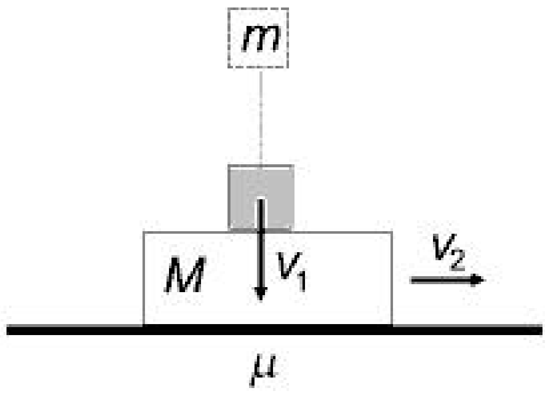


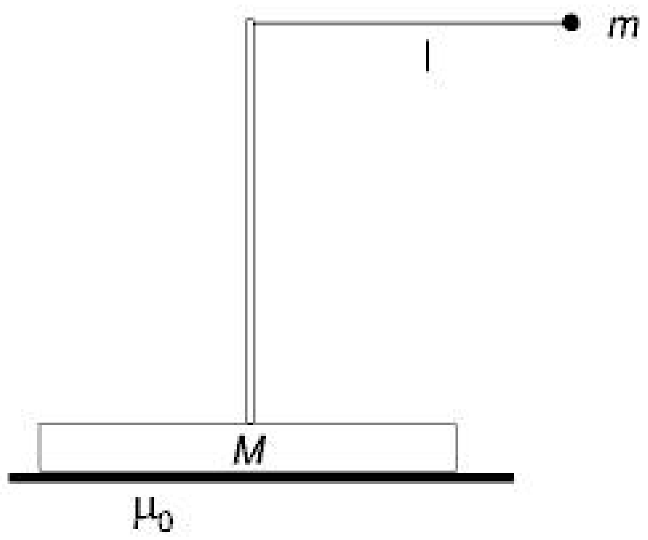
a) In which case will the hoop get higher up the slope: if there is friction on the slope or if there is not?

b) Assume that the slope is ideally smooth. At a time $t = 2.4$ s after arriving at the slope, what will be the speed of the hoop returning from the slope?

(The coefficients of both static and kinetic friction between the horizontal plane and the hoop are $\mu = 0.2$. The slope connects to the horizontal plane with a smooth curve of radius $R > r$, which is considered part of the slope. The hoop does not fall on its side during the motion.)

Problem 26. A block of mass $M = 5$ kg is moving on a horizontal plane. An object of mass $m = 1$ kg is dropped onto the block, hitting it with a vertical velocity of $v_1 = 10$ m/s. The speed of the block at the same time instant is $v_2 = 2$ m/s. The object sticks to the block. The collision is momentary. What will be the speed of the block after the collision if the coefficient of friction between the block and the horizontal plane is $\mu = 0.4$?





Problem 27. A pointlike ball of mass m is tied to the end of a string, which is attached to the top of a thin vertical rod. The rod is fixed to the middle of a block of mass M lying at rest on a horizontal plane. The pendulum is displaced to a horizontal position and released from rest.

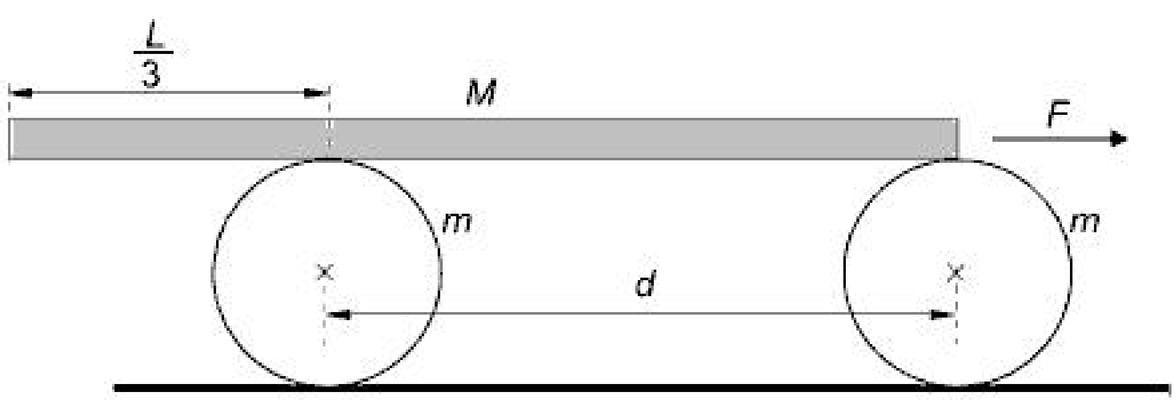
If the coefficient of static friction between the block and the ground is μ_s, what angle will the string create with the vertical rod at the time instant when the block starts to slide? ($M = 2$ kg, $m = 1$ kg, $\mu_s = 0.2$.)

Problem 28. Two small cylinders of equal radius are rotating quickly in opposite directions. Their spindles are parallel and lie on the same horizontal plane. The distance between the spindles is 2L. Place a batten of uniform density onto the top of the two cylinders so that the batten is perpendicular to the spindles, and its centre of mass is at a distance of x from the perpendicular bisector of the segment between the two spindles, which is perpendicular to the spindles. What type of motion does the batten undergo?

Problem 29. An object is pulled up uniformly along an inclined plane which makes an angle of α with the horizontal. The angle between the force with which it is pulled up and the plane of the incline is β. The coefficient of friction between the plane and the object is μ. In what interval can the angle β vary to allow the force to pull up the object?

Problem 30. A coin is placed onto a phonograph turntable at a distance of $r = 10$ cm from the centre. The coefficient of static friction between the coin and the turntable is $\mu = 0.05$. The turntable, which is initially at rest, starts to rotate with a constant angular acceleration of $\beta = 2\ \mathrm{s}^{-2}$. How much time elapses before the coin slips on the turntable?

Problem 31. A rigid rod of length $L = 3$ m and mass $M = 3$ kg, whose mass is distributed uniformly, is placed on two identical thin-walled cylinders resting on a horizontal table. The axes of the two cylinders are $d = 2$ m from each other. As for the rod, one of its endpoints is directly above the axis of one cylinder, while its trisector point (closer to its other end) is directly above the axis of the other cylinder. The mass of the cylinders is $m = 1$ kg each. A constant horizontal pulling force $F = 12$ N acts on the rod. Both cylinders roll without friction.

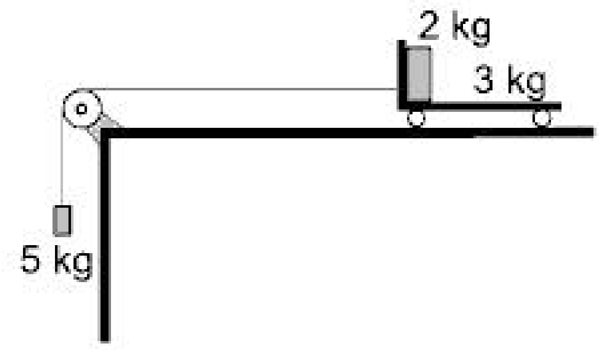


a) Find the final speed of the rod, when its leftmost end is exactly above the axis of the left cylinder.

b) Find the friction force and the minimum coefficient of friction required between the cylinders and the rod for pure rolling.

c) Find the minimum coefficient of friction between the table and the cylinders.

Problem 32. A cart of mass $3\,\mathrm{kg}$ is pulled by a $5\,\mathrm{kg}$ object as shown. The cart, whose length is $40\,\mathrm{cm}$ moves along the table without friction. There is a brick of mass $2\,\mathrm{kg}$ on the cart, which falls from it $0.8\,\mathrm{s}$ after the start of the motion. Find the coefficient of kinetic friction between the cart and the brick. Use $g = 10\,\mathrm{m/s^2}$.



Problem 33. A small solid sphere of mass $m = 8\,\mathrm{kg}$ is placed inside a rigid hollow sphere of mass $M = 8\,\mathrm{kg}$. The hollow sphere is then dropped from a great height. Air drag is in direct proportion to the square velocity: $F = kv^2$. If speed and force are measured in $\mathrm{m/s}$ and Newton respectively, then $k = 0.1$. Draw a graph that represents the force exerted by the small sphere on the hollow sphere in terms of velocity. Use $g = 10\,\mathrm{m/s^2}$.

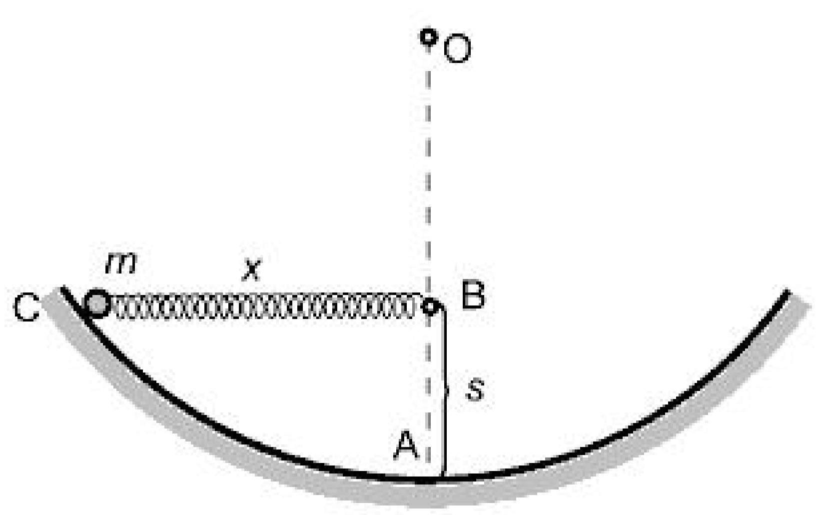
Problem 34. A small body that is fixed to the end of a string of length $l = 20\,\mathrm{cm}$ is forced to move along a circle on a slope whose angle of inclination is $\alpha = 30^\circ$. The body starts from the lowest position in such a way that its speed at the topmost position is $v = 3\,\mathrm{m/s}$.

a) Find the initial velocity, if at the topmost point, the tension in the string is half of what it is at the moment of starting.

b) Find the coefficient of friction.

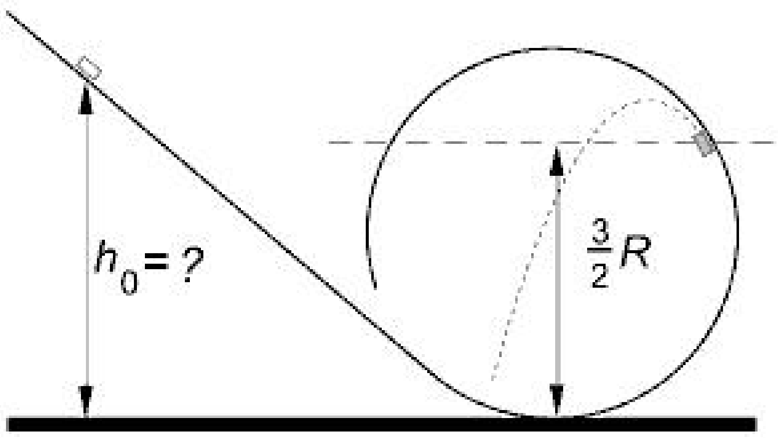
c) Find the distance travelled by the body until stopping, if after $5/4$ turns the string is released and the body remains on the slope throughout its motion.

Problem 35. The inner radius of a frictionless spherical shell is $OA = 0.8\,\mathrm{m}$. One end of a spring of relaxed length $L = 0.32\,\mathrm{m}$ and spring constant $D = 75\,\mathrm{N/m}$ is fixed to point B, which is $0.48\,\mathrm{m}$ below the centre of the sphere. A ball of mass $m = 3.2$ kg is attached to the other end of the spring, while the spring is extended in a horizontal position to reach point C. Then the ball is released. ($g = 10\,\mathrm{m/s^2}$)



a) Find the speed of the ball when it has traveled furthest down the cylinder.

b) Find the force exerted by the ball on the spherical shell at that point.



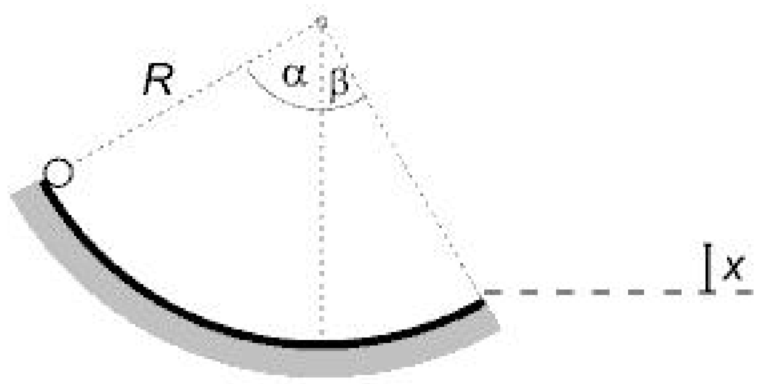
Problem 36. A tangentially attached slope leads to a circular match-box track with radius $R = 32$ cm set in a vertical plane. The toy car starts from rest at the top of the slope, runs down the slope and detaches from the track at height $h = \dfrac{3}{2}R$ measured from the bottom.

a) Find the height the car starts from.

b) Find the maximum height reached by it after it reaches the bottom of the track.

(Assume that the toy car is point-like, neglect drag and friction.)

Problem 37. A small wheel, initially at rest, rolls down a ramp in the shape of a quarter circle without slipping. The radius of the circle is $R = 1$ m and $\alpha = 60°$, $\beta = 30°$. Find the height x reached by the wheel after leaving the track.

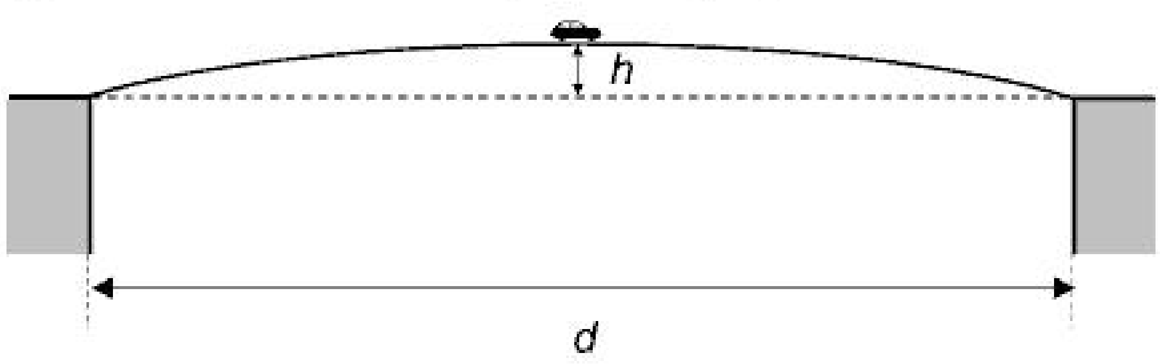


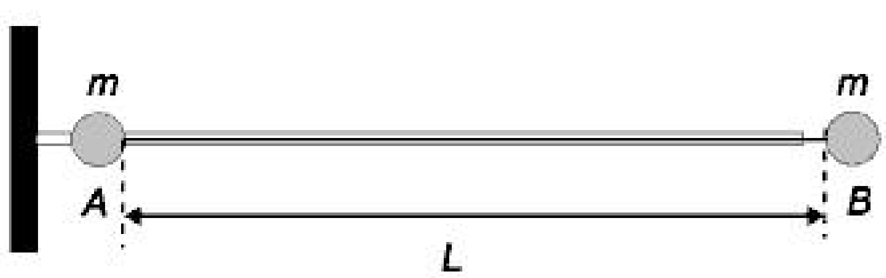
Problem 38. Two banks of a river whose width is $d = 100$ m are connected by a bridge whose longitudinal section is a parabola arc. The highest point of the path is $h = 5$ m above the level of the banks (see the figure). A car with mass $m = 1000$ kg traverses the bridge at a constant speed of $v = 20$ m/s. Find the magnitude of the force that the car exerts on the bridge

a) at the highest point of the bridge,

b) at 3/4 of the distance between the two banks.

(Drag can be neglected. Calculate with $g = 10$ m/s^2.)



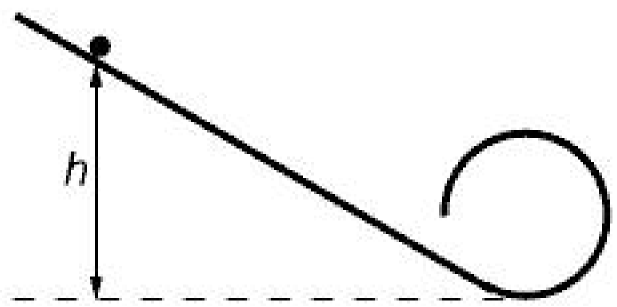


Problem 39. An iron ball (A) of mass $m = 2$ kg can slide without friction on a fixed horizontal rod, which is led through a diametric hole across the ball. There is another ball (B) of the same mass m attached to the first ball by a thin thread of length $L = 1.6$ m. Initially the balls are at rest, the thread is horizontally stretched to its total length and coincides with the rod, as is shown in the figure. Then the ball B is released with zero initial velocity.

a) Determine the velocity and acceleration of the balls (A) and (B) at the time when the thread is vertical.

b) Determine the force exerted by the rod on the ball (A) and the tension in the thread at this instant. (In the calculations take the gravitational acceleration to be $g = 10\ \mathrm{m/s^2}$.)

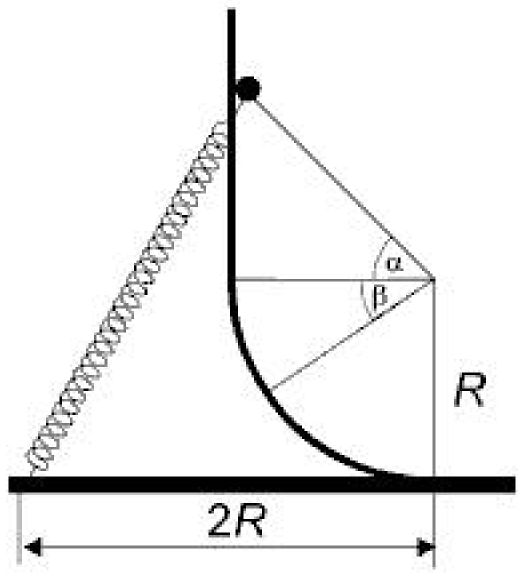
Problem 40. A plane inclined at an angle of $30°$ ends in a circular loop of radius $R = 2\ \mathrm{m}$. The plane and the loop join smoothly. A marble of radius $r = 1\ \mathrm{cm}$ and of mass $m = 20\ \mathrm{g}$ is released from the slope at a height of $h = 3R$. What is the lowest value of the coefficient of friction if the marble rolls along the path without sliding?



Problem 41. The vertical and horizontal parts of a track are connected by a quarter of a circular arc whose radius is $R = 0.2\ \mathrm{m}$. A ball slides on the track with negligible friction; it is pulled through a slit along the track by a stretched spring as is shown in the figure. The length of the unstretched spring is $0.2\ \mathrm{m}$, the spring constant is $100\ \mathrm{N/m}$. The sliding ball starts from a point that is higher than $\alpha = 45°$ above the horizontal part of the track when viewed from the centre of the arc and reaches the maximum velocity at angle $\beta = 34°$ below the horizontal part of the track.

a) Find the mass of the ball.

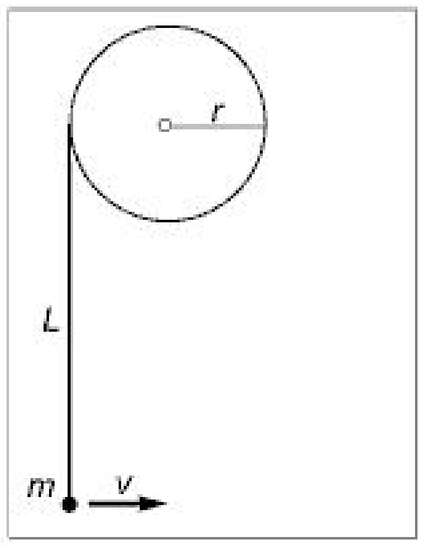
b) Find the maximum speed of the ball.



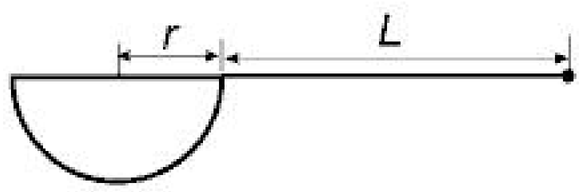
Problem 42. A horizontal disk of radius $r = 0.2\ \mathrm{m}$ is fixed onto a horizontal frictionless table. One end of a massless string of length $L = 0.8\ \mathrm{m}$ is fixed to the perimeter of the disk, while the other end is attached to an object of mass $m = 0.6\ \mathrm{kg}$, which stands on the table as shown. The object is then given a velocity of magnitude $v = 0.4\ \mathrm{m/s}$ in a direction perpendicular to the string.

a) At what time will the object hit the disk?

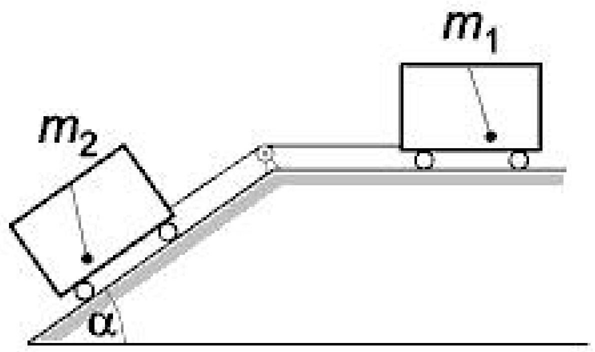
b) Find the tension in the string as a function of time.



Problem 43. A semi-cylinder of radius $r = 0.5$ metres is fixed in horizontal position. A string of length L is attached to its edge. The object tied to the end of the string is released from a horizontal position. When the object at the end of the string is rising, at a certain point the string becomes slack. When the string becomes slack, the length of the free part of the string is $s = 0.96r = 0.48$ metres. What is the total length of the string?



Problem 44. On a horizontal table with the height $h = 1\,\mathrm{m}$ there is a block of mass $m_1 = 4\,\mathrm{kg}$ at rest. The block is connected by a long massless string to a second block of mass $m_2 = 1\,\mathrm{kg}$ which hangs from the edge of the table. The blocks are then released. Find the distance between the points where the two blocks hit the ground. Neglect friction.

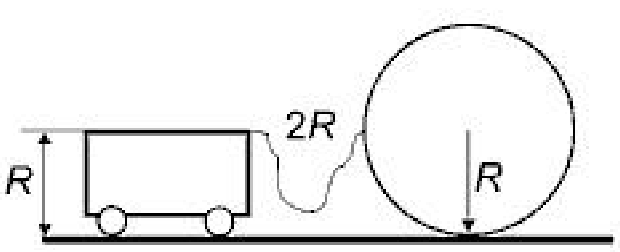


Problem 45. Two carts of masses $m_1 = 8$ kg and $m_2 = 17$ kg are connected by a cord that passes over a pulley as shown. Cart m_2 stands on an incline with an angle $\alpha = 36°52'$. If the system is released, what would be the positions of the pendulums inside the two carts? Neglect friction.

Problem 46. A solid cube of mass $m = 8\,\mathrm{kg}$ and edge $l = 20\,\mathrm{cm}$ is lying at rest on a smooth horizontal plane. A string of length l is attached to the midpoint of one of its base edges.

With the other end of the string kept on the plane, the cube is pulled with the string at an acceleration of $a = 3g$. The string stays perpendicular to the edge of the cube that it pulls on. Find the constant force exerted by the cube on the ground and the force exerted by the string on the cube.

Problem 47. A uniform solid disc of radius R and mass m is pulled by a cart on a horizontal plane with a string of length $2R$ attached to its perimeter. The other end of the string is attached to the cart at a height R above the ground. In the case of equilibrium, what angle does the string create with the horizontal plane if

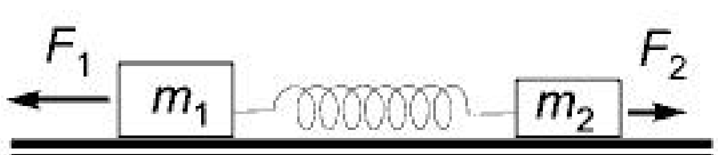


a) there is no friction,

b) there is friction?

The axis of the disc is perpendicular to both the string and the velocity.

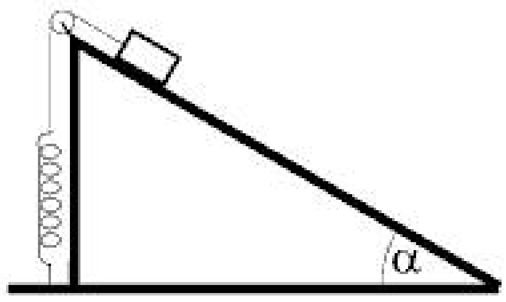
Problem 48. The system shown in the figure undergoes uniformly accelerated motion. Data: $m_1 = 10$ kg, $F_1 = 20$ N, $m_2 = 2$ kg, $F_2 = 10$ N. Find the reading on the spring scale:



a) in this arrangement,

b) if the forces F_1 and F_2 are swapped,

c) if $m_1 = m_2 = 6$ kg. How does the result change in cases a) and b) if m_2 is negligibly small in comparison to m_1, for example $m_2 = 10$ g? Friction is negligible and the mass of the spring is negligible as well.

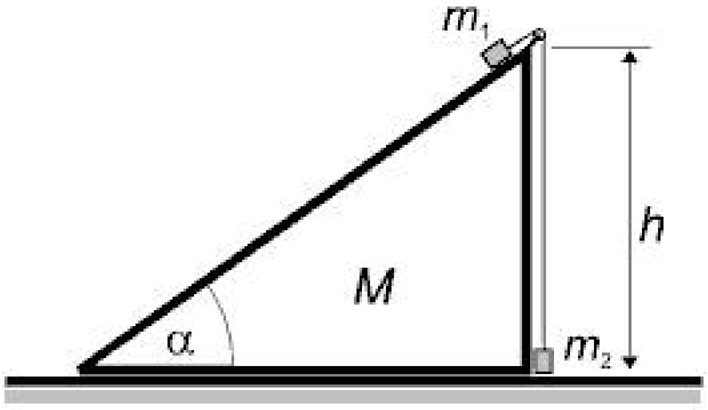


Problem 49. A block of mass $m=3\,\mathrm{kg}$ is connected to a spring and held on top of an inclined plane of angle $\alpha=$ $=30^\circ$ as shown. The spring, whose spring constant is $D=$ $=80\,\mathrm{N/m}$ is in its relaxed state when the block is released. The coefficient of friction is very small. Use $g=10\,\mathrm{m/s^2}$.

a) What is the greatest depth reached by the block?

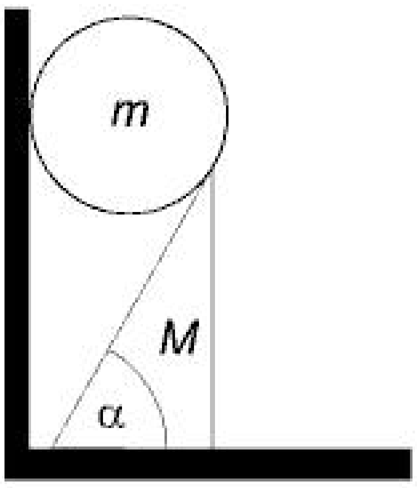
b) Where will the very small friction make the block stop?

Problem 50. A body of mass m is placed on a wedge whose angle of inclination is α and whose mass is M. Find the horizontal force F that should be applied on the wedge in order for the body of mass to slide from the top to the bottom of the wedge in twice as much time as it would if the wedge were stationary. The friction between the wedge and the horizontal ground can be neglected, the coefficient of friction between the wedge and the body is μ. Initially both bodies are at rest. ($M=1\,\mathrm{kg}$, $m=1\,\mathrm{kg}$, $\alpha=30^\circ$, $\mu=0.2$, $g=9.81\,\mathrm{m/s^2}$)



Problem 51. A block of mass $m_1=7$ kg is placed on top of a $h=1$ m high inclined plane with an angle $\alpha=36.87^\circ$ and mass $M=2\,\mathrm{kg}$ which is connected by a cord of length h over a massless, frictionless pulley to a second block of mass $m_2=1$ kg hanging vertically as shown. The inclined plane can move without friction in the horizontal direction. The blocks are then released. After how long will the two blocks be nearest to each other? Neglect friction and use $g=10\,\mathrm{m/s^2}$.

Problem 52. A sphere of mass m is placed between a vertical wall and a wedge of mass M and angle α, in such a way that the sphere touches the wedge tangentially at the topmost point of the wedge, as is shown in the figure. The wedge is standing on a horizontal plane, and both the sphere and the wedge move without friction.

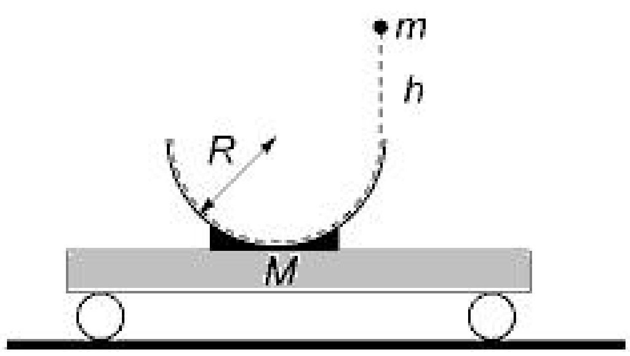


a) How should the mass ratio M/m and the angle α be chosen so that the wedge does not tilt after releasing the sphere?

b) Determine the speed reached by the sphere by the time it slides along a segment of length $l=20$ cm of the wedge, provided that $\alpha=60^\circ$ and $M/m=12$.

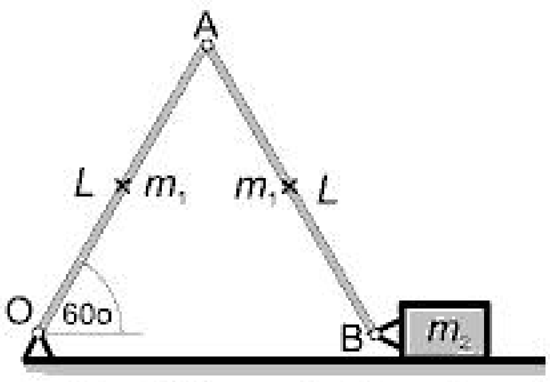
Problem 53. A large, closed box slides down on a very long, inclined plane. An observer inside the box wants to determine the angle of the inclined plane (α) and the coefficient of kinetic friction. What experiments should he do and what should he measure in order to be able to calculate the above quantities?

Problem 54. A thin, rigid wooden rod of height h is fixed to the ground and is standing vertically. A simple pendulum of length $l < h$ and mass m is attached to its upper end. The pendulum is moved to a horizontal position and released. Determine the torque that the fixed lower end must bear to keep the rod in position. (Let $h = 1.2\,\mathrm{m}$, $m = 0.5\,\mathrm{kg}$, use $g = 10\,\mathrm{m/s^2}$)



Problem 55. As shown in the figure, a smooth hemisphere of radius R is fixed to the top of a cart that can roll smoothly on a horizontal ground. The total mass of the cart is M, and it is initially at rest. A pointlike ball of mass m is dropped into the hemisphere tangentially, from a point $h = R$ above its edge. The ball slides all the way along the hemisphere with negligible friction.

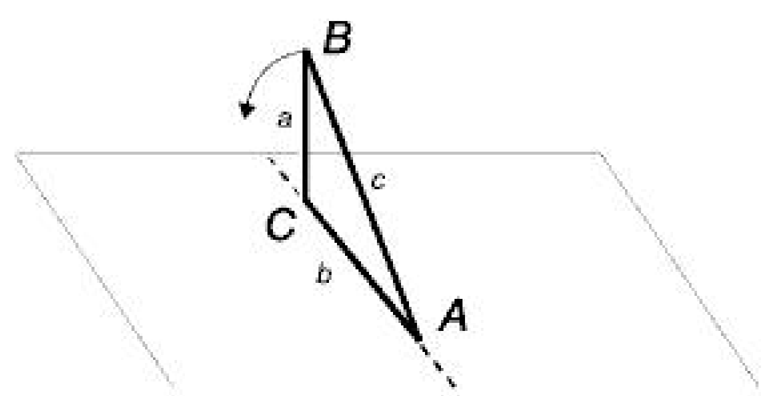
a) Where will the ball be when it reaches the maximum height during its motion?
b) With what force will the ball press on the hemisphere at its lowermost point?
(Let $R = 0.5\,\mathrm{m}$, $M = 2\,\mathrm{kg}$, $m = 0.5\,\mathrm{kg}$, use $g = 10\,\mathrm{m/s^2}$)



Problem 56. Two rods, each of length $L = 0.5\,\mathrm{m}$ and mass $m_1 = 1\,\mathrm{kg}$, are joined together by hinges as shown. The bottom end of the left rod is connected to the ground, while the bottom end of the right one is connected to a block of mass $m_2 = 2\,\mathrm{kg}$. The block is then released to a position where the rods form a 60° angle with the horizontal plane. Friction is negligible.

a) Find the velocity of point A as it hits the ground.
b) Find the acceleration of mass m_2 at that moment.

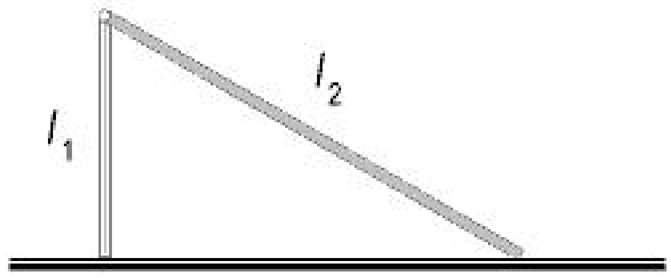
Problem 57. A right triangle of side lengths a, b and c is formed using three thin rods of the same material, which are firmly fixed to each other. The triangle, which is initially placed vertically onto a horizontal plane on its edge b, tumbles down from this unstable position. $a = 30$ cm, $c = 50$ cm.



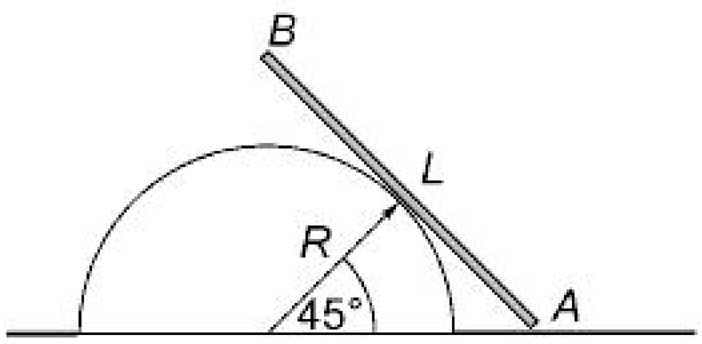
a) Determine the velocity of the vertex B when it hits the horizontal plane, provided that the triangle does not slide along ground.

b) Determine the position and velocity of the vertex B when it hits the horizontal plane, provided that the friction is negligible!

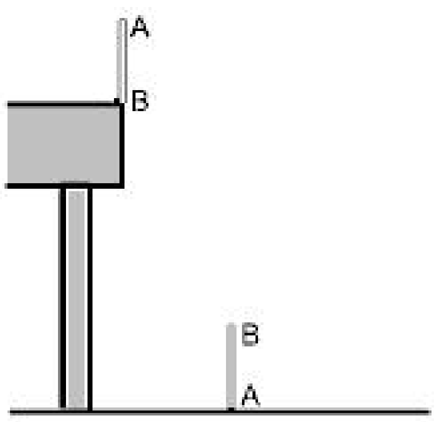
Problem 58. Two thin rods of identical material and cross-section with lengths $l_1 = 0.6$ m and $l_2 = 1$ m are connected by a frictionless joint. The structure slides from its unstable equilibrium position in such a way that the rods remain on a vertical plane and the angle enclosed by them decreases. Find the place where the joint reaches the ground and find its speed upon impact.



Problem 59. As shown in the figure, a thin and solid rod of length $L = 2R$ is leaning against a smooth semi-cylinder of radius $R = 1$ m that is fixed to a horizontal plane. The lower end of the rod A is held on the ground and then released from rest. The rod falls, sliding along the side of the cylinder. What will be the speed of its upper end B at the time instant when it reaches the surface of the cylinder? (Neglect all friction.)



Problem 60. A rod with length L stands on the edge of a table in such a way that its bottom is propped against a smooth (frictionless) peg. Then the rod tilts and falls. Find the height of the table if the rod reaches the floor in a vertical position, with its top end hitting the floor first.



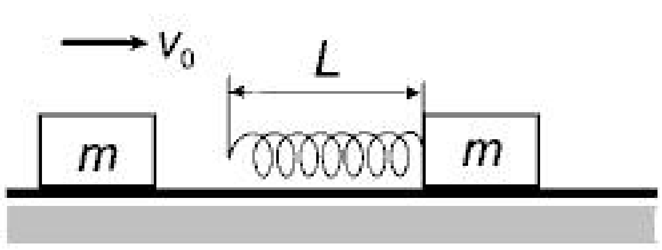
Problem 61. The following forces act on a body, which is initially at rest: $F_1 = 10$ N for $t_1 = 4$ s, then $F_2 = 4$ N acting in the same direction for $t_2 = 14$ s, then $F_3 = 15$ N acting in the opposite direction for $t_3 = 2$ s.

Find the magnitude of the constant force that causes the same final velocity of the body:

a) at the same time,

b) at the same distance.

Problem 62. A block of mass m with a spring fastened to it rests on a horizontal frictionless surface. The spring constant is D_0, the relaxed length of the spring is L and the spring's mass is negligible. A second block of mass m moves along the line of axis of the spring with constant velocity v_0 and collides with the spring as shown.



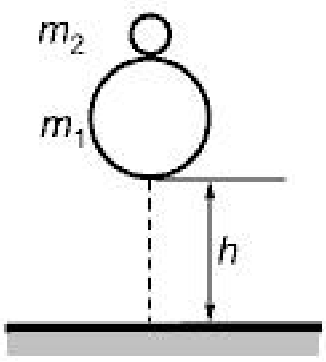
a) What is the shortest length of the spring during the collision?

b) The second block then sticks to the left end of the spring. What is the frequency of oscillation of the system?

Values: $m = 1\,\text{kg}$, $L = 0.2\,\text{m}$, $D_0 = 250\,\text{N/m}$, $v_0 = 0.8\,\text{m/s}$.

Problem 63. Our model rocket is a trolley on which several spring launchers are installed. Each spring is compressed and therefore stores $E = 100$ J of elastic energy. The system, whose total mass is $M = 100$ kg is initially at rest. Find the velocity of the trolley if the structure shoots out three balls with mass $m = 5$ kg in succession and in the same direction along the longitudinal axis.

Problem 64. A ball of mass m and of speed v collides with a stationary ball of mass M. The collision was head-on but not totally elastic. Determine the kinetic energy which is lost during the collision as a function of the speeds as well as the given masses before and after the collision. Based on the result, define a quantity which characterizes the elasticity of the collision.

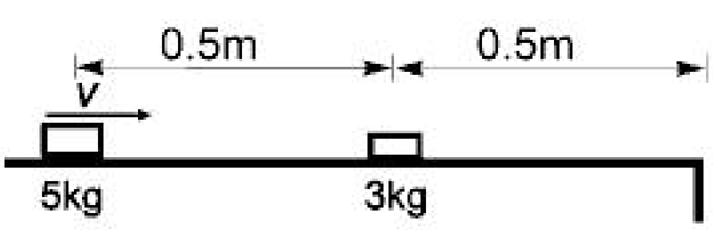


Problem 65. An object of mass m_1 and another of mass m_2 are dropped from a height h, the second one immediately following the first one. All collisions are perfectly elastic and occur along a vertical line.

a) For what ratio of the masses will the object of mass m_1 remain at rest after the collisions?

b) According to a), how high will the object of mass m_2 rise?

Problem 66. Two blocks of masses $m_1 = 5\,\text{kg}$ and $m_2 = 3\,\text{kg}$ are at rest on a table at a distance of $s_1 = 0.5\,\text{m}$ from each other. Block m_2 is at a distance of $s_2 = 0.5\,\text{m}$ from the edge of the table as shown. The coefficient of friction is $\mu = 0.102 = 1/9.8$.

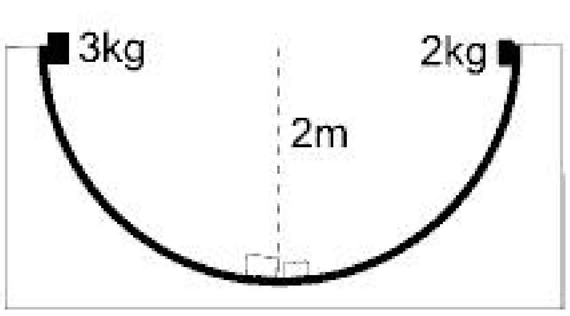


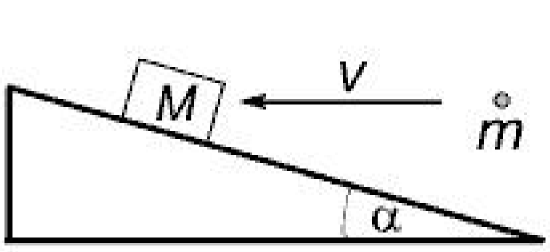
Find the velocity that should be given to block m_1 if after the elastic collision of the two blocks

a) block m_1,

b) block m_2 is to reach the edge of the table and stop there.

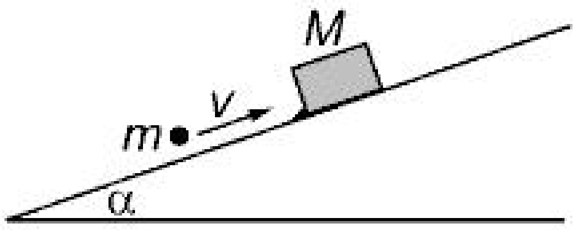
Problem 67. At the rim of a hollow hemisphere of diameter 4 metres two objects of masses $m_1 = 3\,\text{kg}$ and $m_2 = 2\,\text{kg}$ are released at the same moment. Initially the two objects are at the two endpoints of a diameter of the hemisphere. They collide totally elastically. After the first collision what are the greatest heights the blocks can reach? The friction is negligible.





Problem 68. A block of mass $M = 1.6\,\text{kg}$ is lying on a plane inclined at an angle of $\alpha = 16.25^\circ$ to the horizontal. The coefficient of friction is $\mu = 0.2$. At the same time that the block on the inclined plane is released, a shell of mass $m = 0.4\,\text{kg}$ is fired into it horizontally with a speed of $v = 12\,\text{m/s}$. How much will the block of mass and the shell of mass slide up the incline? ($g = 10\,\text{m/s}^2$)

Problem 69. A block of mass M, supported by a buffer, stays at rest on a plane inclined at an angle α to the horizontal. From below, parallel to the inclined plane, a bullet of mass m is shot into the block at a speed of v. How long does it take for the block to reach the buffer again? The coefficient of friction between the block and the plane is μ. The bullet penetrates into the block. During the penetration the displacement of the block is negligible. The coefficients of static and kinetic friction can be considered equal.



Problem 70. A ball made of a totally inelastic material is hung between two heavy iron rods, which are also hung as pendulums. The mass of the ball is negligible with respect to that of the rods. The masses of the rods are: m_1 and m_2, $(m_1 > m_2)$. One of the rods is pulled out, so that its centre of mass rises to a height of h, and then it is released. The plastically deformable ball becomes flat due to the collision. Which rod should be raised in order to cause the greater compression of the ball if h is the same in both cases. Based on the result, draw a conclusion about the efficiency of deforming an object by hammering it.

Problem 71. A projectile thrown upwards explodes at the top of its path into two parts of masses $m_1 = 3$ kg and $m_2 = 6$ kg. The two parts reached the ground at equal distances from the position of the projection, and with a time difference of $T = 4$ seconds. At what height did the projectile explode? (Neglect air resistance.)

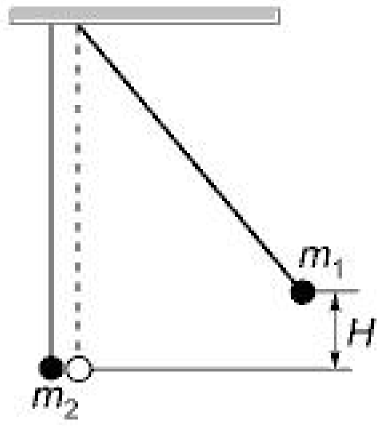
Problem 72. From a horizontal ground a projectile is shot at an initial speed of $v_0 = 150$ m/s and at an angle of $\alpha = 60°$ from the horizontal ground. After a time of $t_1 = 10$ s the projectile explodes and breaks up into two pieces of masses m and $2m$. At the moment $\Delta t = 10$ s after the explosion the piece of mass m hits the ground at a distance of $d = 500$ m behind the place of shooting, in the plane of the trajectory of the unexploded projectile. At this instant how far is the other piece of mass $2m$ from the cannon?

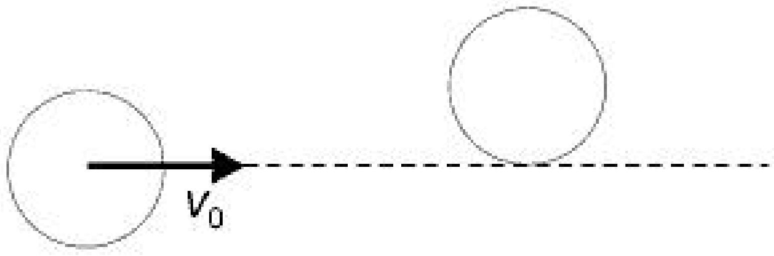
Problem 73. A trolley of mass $M = 20$ kg is travelling at a speed of $V = 10$ m/s. A spring, initially compressed, launches an object of mass $m = 2$ kg off the trolley in a forward direction in such a way that after the launch the speed of the object is $v = 2$ m/s relative to the trolley. Determine the kinetic energy of the object relative to the ground.

Problem 74. Two elastic balls are suspended at the same height; one has mass $m_1 = 0.2$ kg, the other has mass m_2. If the system is left alone in the position shown in the figure, we find that – after an elastic and central collision – both balls rise to the same height.

a) Find the mass of the other ball.

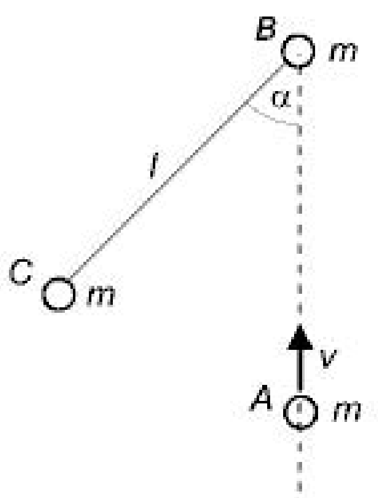
b) At what fraction is height h reached by the balls after the collision of H?

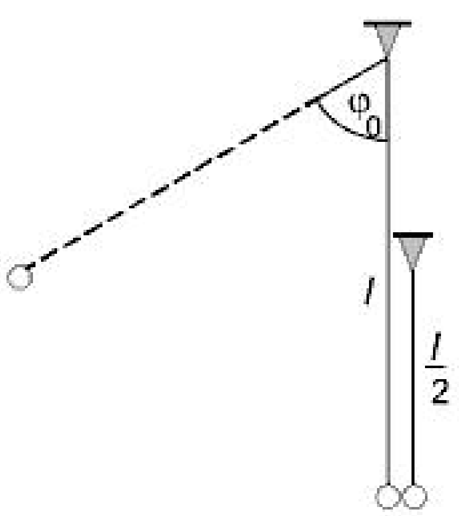




Problem 75. There are two thin, homogeneous disks of the same radius and mass lying on a horizontal air cushion table. One of the disks is at rest, while the other is moving at a speed of $v_0 = 1$ m/s. The line going through the centre of the moving disk, which follows the direction of its velocity, touches the other disk tangentially. The two disks collide elastically. Determine the velocities of the disks after the collision. The directions of the velocities can be described by angles relative to the initial velocity $\vec{v}_0$. In the process investigated friction is negligible everwhere.

Problem 76. There are three thin disks of identical mass ($m_A = m_B = m_C = m$) and radius lying at rest on a smooth horizontal plane. The disks B and C are connected by a thin thread of length $l = 1$ m. Initially the thread is straight, but not stretched, and it makes an angle of $45°$ with the line going through the midpoints of the disks A and B. Now we push the disk A at a speed $v = 2$ m/s in such a way that it centrally collides with the disk B. The collisions are elastic and instantaneous. At what time after the collision of the disks A and B will the line connecting the centres of the disks B and C be parallel to the trajectory of the disk A? At this instance, determine the distance of the disk A from B and C. (The disks can be considered pointlike.)

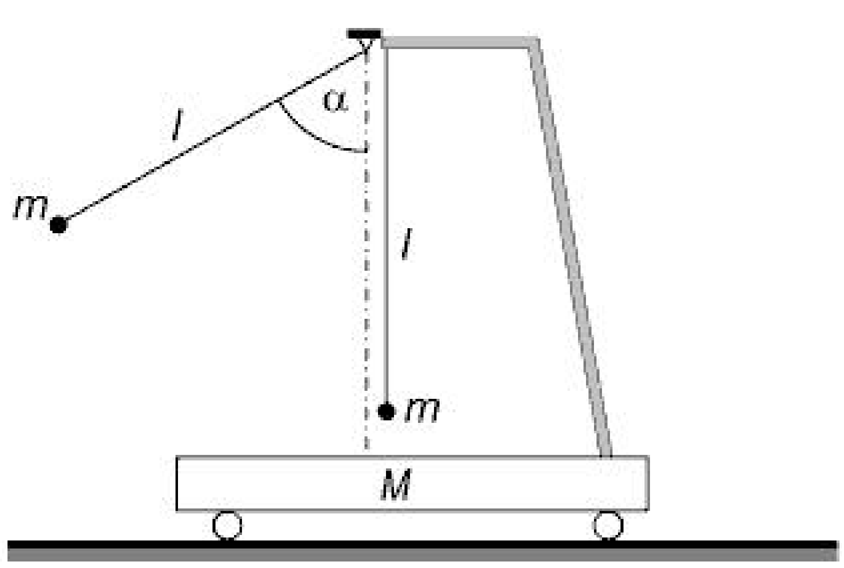




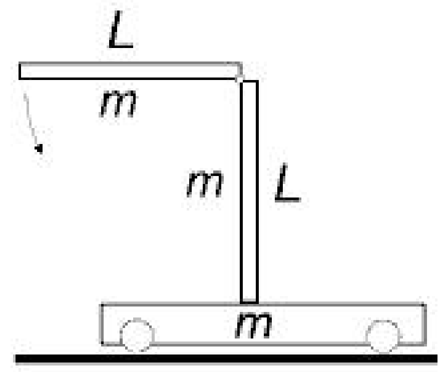
Problem 77. There are two identical balls of mass $m = 0.2$ kg suspended on two threads of lengths $l = 1$ m and $l/2$. The threads are made of the same material, and in their vertical position the two balls touch each other. If the ball hanging on the longer thread is released from an initial angle of $\varphi_0 = 60°$ with respect to the vertical, then the thread breaks just before the collision.

What is the maximum initial angle from which this ball can be released, so that none of the threads break after the totally elastic collision?

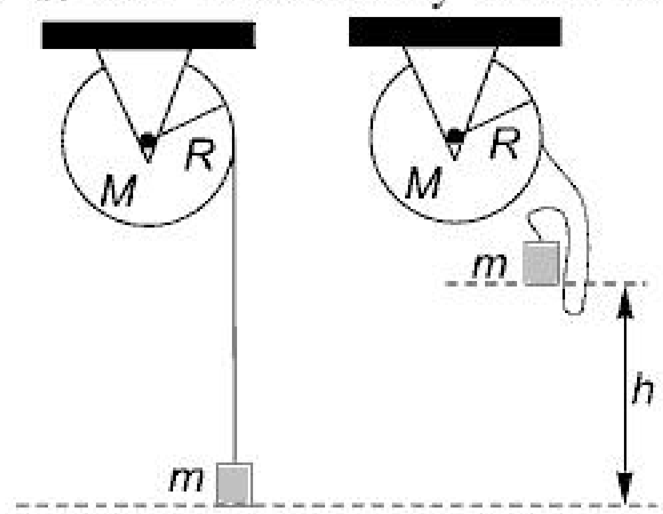
Problem 78. A mathematical pendulum of length l and mass m is suspended on a smoothly running trolley of mass M. Another pendulum, also of length l and mass m is suspended from the ceiling, displaced through angle α and then released without initial velocity. The two pendulums collide centrally and perfectly elastically. Find the angle φ through which the pendulum suspended from the trolley swings out.
($M = 3$ kg, $m = 2$ kg, $\alpha = 60°$.)



Problem 79. A small cart of mass m is at rest on a horizontal track. A vertical column of length $L = 2\,\mathrm{m}$ and the same mass m is fixed to the cart. A rod of the same mass m and length $L = 2\,\mathrm{m}$ is attached to its upper end with a hinge, and released from a horizontal position. At what speed will the end of the rod hit the base of the column? ($g = 10\,\mathrm{m/s^2}$.)



Problem 80. A cylinder of mass M and radius R can rotate freely about its horizontal axis. A thread is wound around its lateral surface, and a weight of mass m is attached to the free end of the thread. Initially the thread below the cylinder is vertical, and unstrained. Then the weight is lifted to a height h, and released from that position at zero initial speed. At what time after its release does the weight cover the distance $2h$?

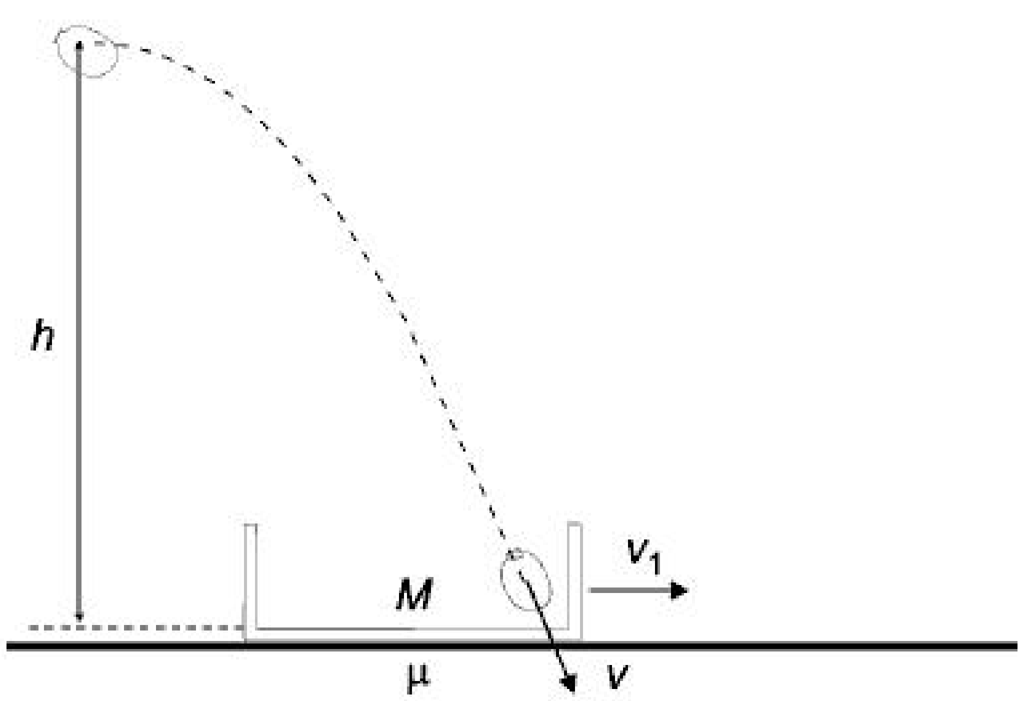


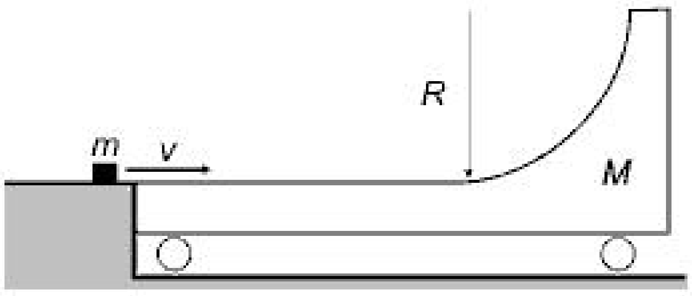
(The thread is unstretchable, and the interaction is instantaneous and totally inelastic.

Data: $M = 2$ kg, $R = 0.2$ m, $m = 3$ kg, $h = 1.2$ m.)

Problem 81. An inclined plane of angle α and mass M can move on the ground without friction. A small object of mass m and vertical velocity v collides with the stationary inclined plane. Assuming that the collision is elastic, find the velocity of the object (u) after the collision, the angle (φ) formed by this velocity and the horizontal ground. Find the speed (c) of the inclined plane after the collision. Data: $\alpha = 36.87°$, $m = 6\,\mathrm{kg}$, $M = 18\,\mathrm{kg}$, $v = 14\,\mathrm{m/s}$.

Problem 82. A big chest of mass $M = 50$ kg is sliding on the horizontal ground, and a sand bag of the same mass is falling into it. The bag was projected from the initial height of $h = 3$ m at certain a horizontal speed, and when it hits the chest, the speed of the chest is $v_1 = 5$ m/s. The velocity of the chest is in the plane of the trajectory of the bag, and the bag hits the chest in such a way, that its velocity makes an angle of $60°$ with the velocity of the chest. The coefficient of kinetic friction between the chest and the ground is $\mu = 0.4$. The collision of the bag is instantaneous. Determine the distance covered by the chest from the collision until it stops. What would the distance be if the bag was not thrown into the chest?





Problem 83. A quarter-round slope of radius $R = 0.5$ m is attached tangentially to a freely rolling trolley of mass $M = 3$ kg, originally at rest. A small-sized body of mass $m = 2$ kg slides onto the trolley at velocity $v = 15$ m/s.

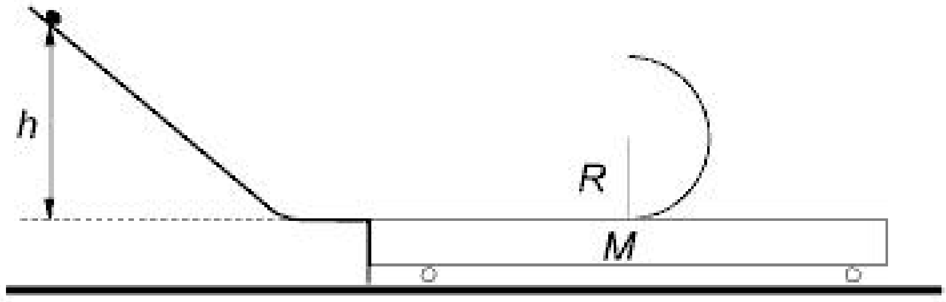
a) Find the velocity of the trolley when the small body leaves it.

b) Find the distance travelled by the trolley from parting to reunion with the body.

c) Find the velocities of the trolley and the body when they part from each other again. (Friction and drag can be neglected. Calculate with $g = 10\,\mathrm{m/s^2}$.)

Problem 84. A small object of mass $m = 1$ kg is released from rest at the top of an inclined plane that connects to a horizontal plane without an edge. It slides onto a cart of mass M that has a semi-cylindrical surface of radius $R = 0.36$ m fixed to the middle of it, as shown in the figure.

The small object reaches the topmost point of the semi-cylinder and stops there. In continues to move vertically with free fall and hits the cart exactly at the edge. All friction can be neglected.

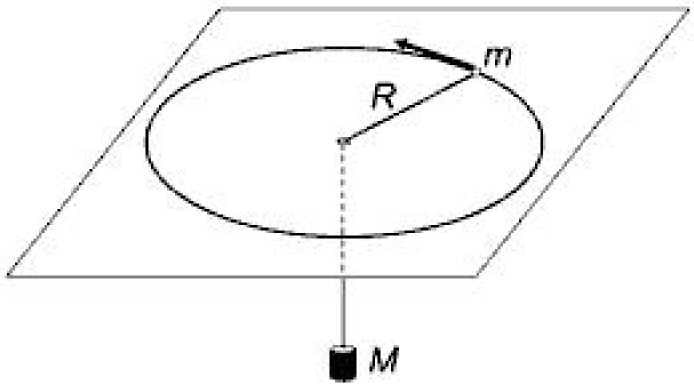


a) What is the minimum possible length of the cart?

b) What is the mass of the cart?

c) At what height h was the small object released?

Problem 85. An 80 kg man stands on the rim of a 300 kg rotating disk with radius 5 m. The disk initially rotates at 0.1 s^{-1} around a vertical axis. Then the man walks from the rim to the centre of the disk. Find the change in the energy of the system.



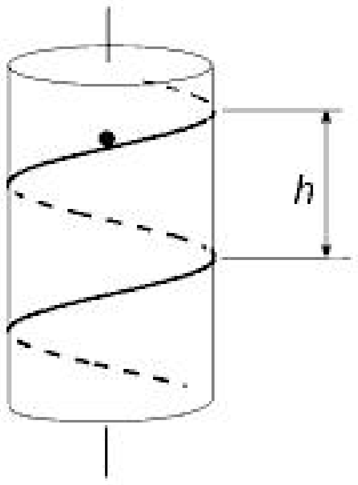
Problem 86. An object of mass $m = 1$ kg attached to a string is moving in a circle of radius $R = 40$ cm on a horizontal surface. The other end of the string is threaded through a hole at the centre of the circle and a mass of $M = 2$ kg is hung from it.

If the mass M is released, the closest approach of the mass m to the centre will be $r = 10$ cm.

a) Find the smallest and largest speeds of the mass m.

b) What is the speed of each object when the mass m is at a distance of $R/2$ from the centre?

c) Find the accelerations of the mass M at the highest and lowest points. (Neglect all friction, use $g = 10\,\mathrm{m/s^2}$.)

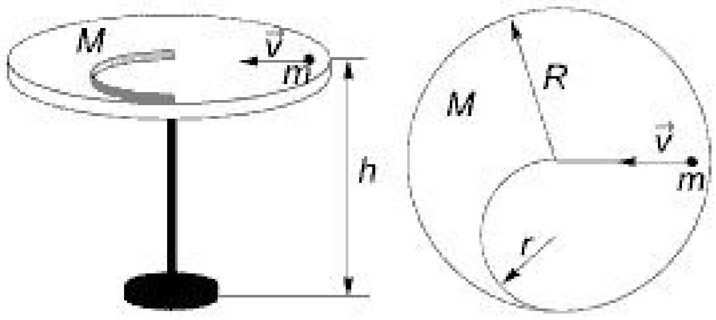


Problem 87. A solid cylinder of radius $R = 0.2$ metres is supported at the endpoints of its axis by frictionless pin bearings. An object slides down a frictionless helical track threaded around the cylindrical surface. The mass of the object is one fifth of the mass of the cylinder. The pitch of the track is $h = 0.2$ metres. $g = 10\,\mathrm{m/s^2}$.

a) What will be the speed of the object when it has descended through a height of $h = 0.2$ metres below the starting point?

b) How long will it take to attain that speed?

Problem 88. A disc of mass $2\,\mathrm{kg}$ and radius $R = 0.5\,\mathrm{m}$ can rotate freely around a vertical axis supported by bearings at a height $h = 1\,\mathrm{m}$ from the ground. A constraining vertical surface of negligible mass, whose shape is a semicircular arc of radius $r = R/2$, is fixed on the disc as shown in the figure. A small ball of mass $m = 1\,\mathrm{kg}$ is placed on the stationary disc and is bowled at a speed $v = 3\,\mathrm{m/s}$ in such a way that it reaches the internal side of the constraining surface tangentially.

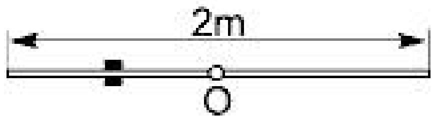


a) Find the distance from the rim of the disc where the ball reaches the ground.

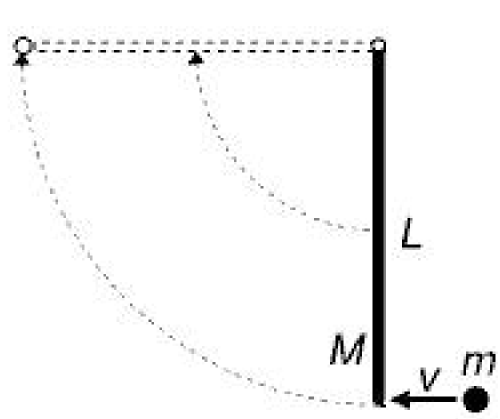
b) How far is the ball at the moment of reaching the ground from the point of leaving on the disc? (Every type of friction can be neglected.)

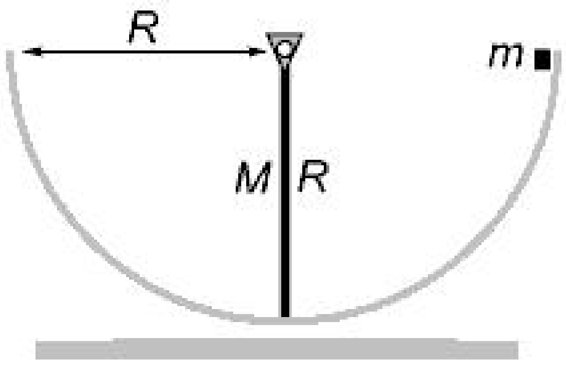
Problem 89. A pointmass moves on the frictionless inner surface of a spherical shell, whose inner radius is $R = 1.4\,\mathrm{m}$. Its velocity reaches its maximum and minimum at heights $h_1 = 0.1\,\mathrm{m}$ and $h_2 = 0.3\,\mathrm{m}$ respectively. Find the maximum and minimum values of the velocity.

Problem 90. A $2\,\mathrm{m}$ long rod of negligible mass is free to rotate about its centre. An object of mass $3\,\mathrm{kg}$ is threaded into the rod at a distance of $0.5\,\mathrm{m}$ from its end in such a way that the object can move on the rod without friction. The rod is then released from its horizontal position. Find the speed of the rod's end in the rod's vertical position. Use $g = 10\,\mathrm{m/s^2}$.



Problem 91. A board of length $L = 3.06\,\mathrm{m}$ and mass $M = 12\,\mathrm{kg}$ hangs vertically on a hinge that is connected to one of its ends. A bullet of mass $m = 0.25\,\mathrm{kg}$ is fired into the bottom end of the board, making the board swing up. What should the velocity of the bullet be if the board is to swing up to the horizontal position?

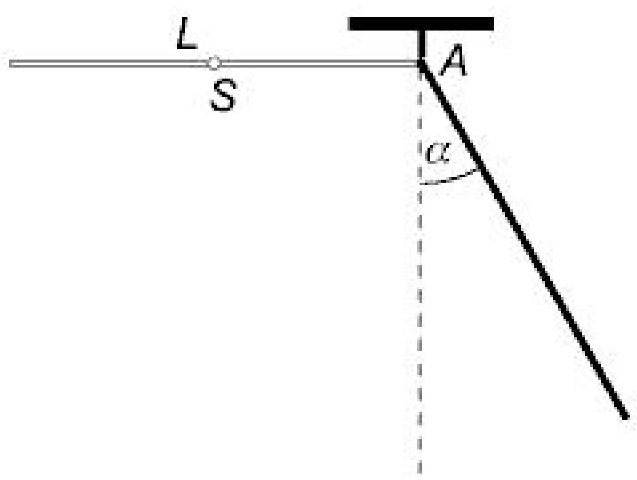




Problem 92. A rod of mass M and length R is fixed to a horizontal axis of rotation above a track with a semicircular cross section and a radius R, as is shown in the figure.

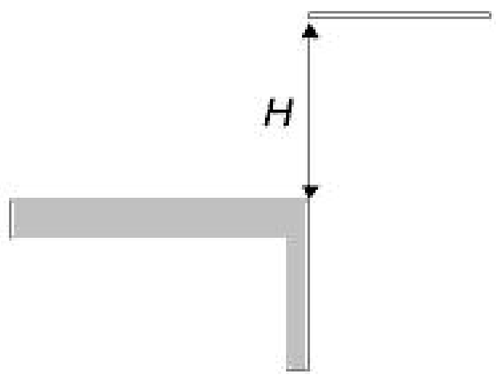
a) Find the mass of the rod relative to the mass of a point-like body that starts on the track at a height of R if it stops after an elastic collision with the rod.

b) Find the angular displacement of the rod after the collision. (Friction is negligible everywhere.)



Problem 93. A thin, homogeneous stick has a length $L = 1$ m. An axis perpendicular to this stick is fixed at one end (A) of the stick; the stick is hung on the axis by a hook. The stick is sent into a horizontal position and then released without an initial velocity. The hook forms a (small) arc which allows the stick to leave it when it encloses an angle of $\alpha = 30°$ with the vertical plane.

Find the angle enclosed by the stick and the horizontal at the moment when its centre of mass (S) is at the highest point after detachment from the hook.

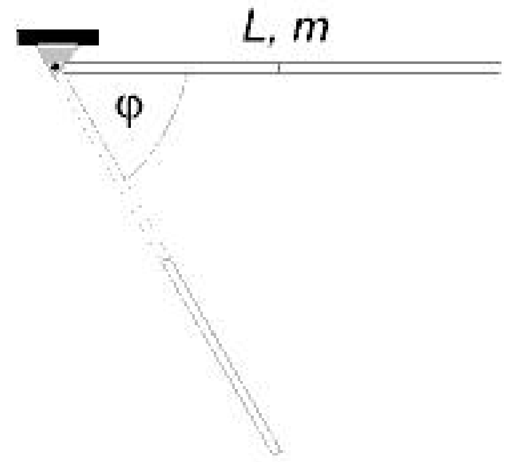


Problem 94. A thin rod of length L is falling freely in horizontal position from a height H above the surface of the table, in such a way that the end of the rod just hits the edge of the table. This collision is instantaneous and totally elastic. At what time after the collision does the rod perform a whole revolution? Where is its centre at that moment?

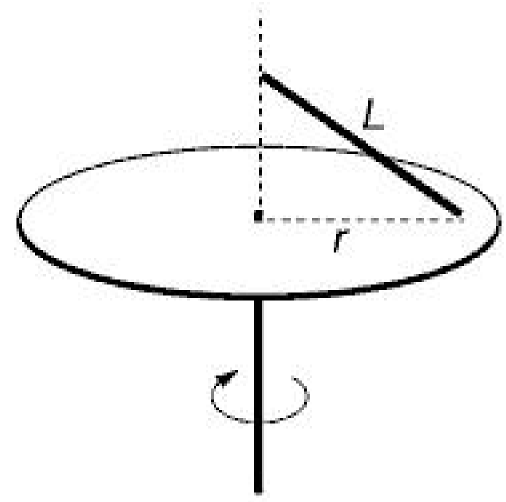
($H = 80$ cm, $L = 40$ cm, calculate with free fall acceleration $g = 10$ m/s.)

Problem 95. One end of a thin and heavy rod of length $L = 1$ m is attached to a horizontal axis at a height of $2L$ above the ground, and the rod is held in a horizontal position. One of two pointlike objects of negligible mass is placed on the free end of the rod and another is held against it from below, as shown in the figure. The coefficient of friction between the small objects and the rod is $\mu = 0.841$. The system is released from rest. At what distance from each other will the small objects hit the ground?

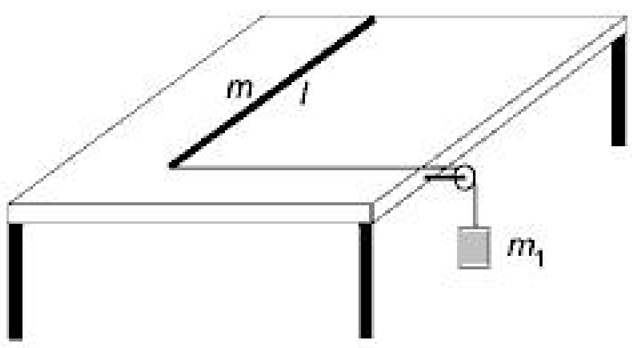
Problem 96. A thin, homogeneous rod of length L and mass m is suspended on a hinge at one end and then displaced into a horizontal position as is shown in the figure. The rod is released without an initial velocity. Find the magnitude and the direction of the force exerted by one half of length $L/2$ of the rod on the other half of $L/2$ when the angular displacement is $\varphi = 60°$.



Problem 97. A disk rotates at constant angular velocity around its vertical axis of symmetry. A rod of length $L = 1$ m is placed onto the disk in a way that its one end touches the disk at a distance of $r = 0.8$ m from the centre, while its other end is above the centre as shown. The rod is then released and rotates together with the disk in this position. Find the angular velocity of the disk. Use $g = 10\,\mathrm{m/s^2}$.



Problem 98. There is a rod of length l, mass m lying on a horizontal table. A cord is led through a pulley, and its horizontal part is attached perpendicularly to one end of the rod, while its vertical part is attached to a weight of mass m_1. The mass of the pulley and the friction are negligible.



a) Which point of the rod has zero acceleration at the moment of releasing the weight?

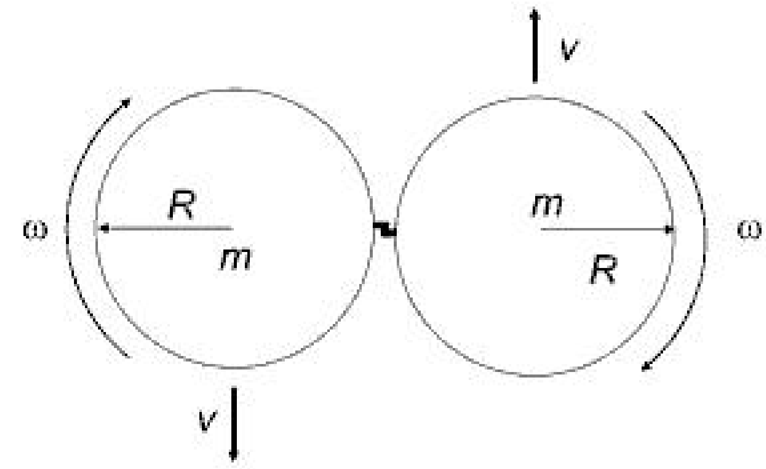
b) At what mass ratio is the acceleration of the centre of the rod maximal at the moment of releasing the weight? Determine this acceleration.

Problem 99. A thin rod of length l, mass m and uniform mass distribution is lying on a smooth tabletop. One end is given a sudden horizontal impulse in a direction perpendicular to the length of the rod. How long will the rod slide along the table as it makes two complete revolutions?

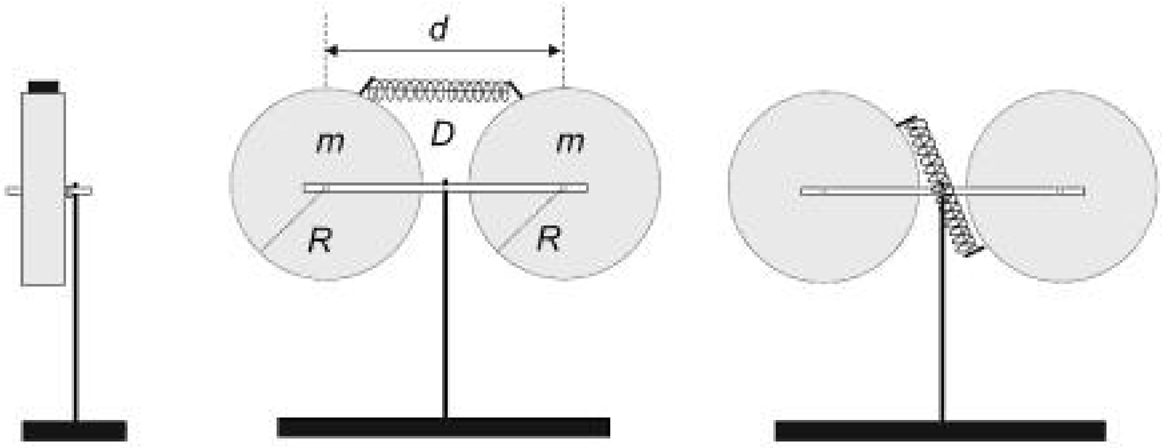
Problem 100. Two discs of radius $R = 4$ cm rotating in the same direction at angular velocity $\omega = 2\ \mathrm{s^{-1}}$ move in opposite directions at velocity $v = 10$ cm/s on an air-cushioned table as shown in the figure. The discs collide along the spikes that are located on their circumferences and whose dimensions are negligible. Determine the velocities after the collision if the discs

a) stick together firmly after a perfectly inelastic collision,

b) part after a perfectly elastic, instantaneous collision.

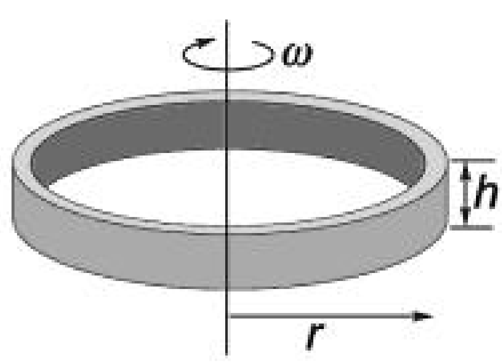


Problem 101. There are two homogeneous, solid disks of radius $R=10$ cm and mass $m=4$ kg mounted by two parallel, horizontal axes at the ends of a horizontal rod of negligible mass. The distance between these axes is $d=25$ cm and the disks can freely rotate around them. The rod itself, with the disks mounted on it, can also freely rotate around a horizontal axis in its midpoint. (See the figures. All the three axes are perpendicular to the rod.) On the rim of each disk there is a small pin, and between them there is a spring of spring constant $D=1800\ \mathrm{N/m}$, which is initially compressed by $\Delta l=5$ cm. Determine the angular velocity of the disks after we burn the thread that holds the spring in its compressed position, provided that its initial position corresponds to figure a) or figure b). (The spring is in contact with the pins until it extends to its unstretched position, and then falls down.)

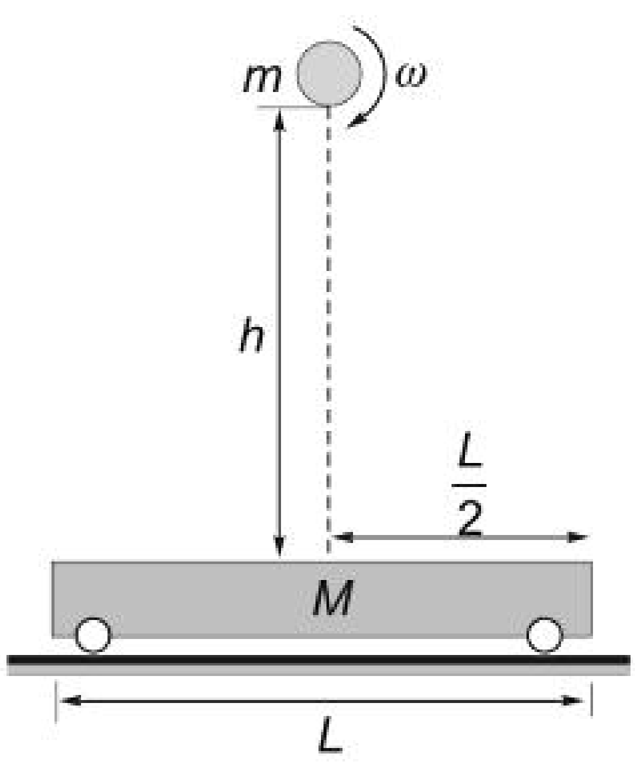


a) b)

Problem 102. A thin ring of radius $r=10$ cm, rotating in a horizontal plane, is dropped onto a tabletop from a height of $h=20$ cm. At the instant when it starts to fall, the angular speed of the ring is $\omega_0=2\ \mathrm{s}^{-1}$ around its vertical axis. The collision is inelastic and takes a very short time. The coefficient of friction between the ring and the tabletop is $\mu=0.3$. $g=10\ \mathrm{m\,s^{-2}}$. How many revolutions will the ring make from the start of its fall until it finally stops?



Problem 103. A solid and rigid sphere of mass $m=80$ kg and radius $R=0.2$ m is spun about a horizontal axis at an angular speed of ω, and then dropped without an initial speed onto a stationary cart of mass $M=200$ kg from a height of $h=$ $=1.25$ m. It hits the cart exactly at the centre. (The longitudinal axis of the cart lies in the plane of the rotation.) The cart can roll smoothly, its deformation in the collision is perfectly elastic, and the collision is momentary. The sphere keeps sliding throughout the entire duration of the collision. The coefficient of kinetic friction between the sphere and the cart is $\mu=0.1$. The sphere rebounds from the cart and falls back onto it again.



a) What is the minimum possible length of the cart?

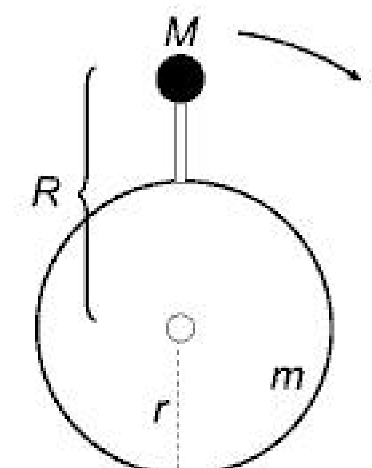
b) What is the minimum possible initial angular speed of the sphere?

c) Provided that the sphere is started at the minimum angular speed as in question b), how much mechanical energy is dissipated in each of the first and second collisions?

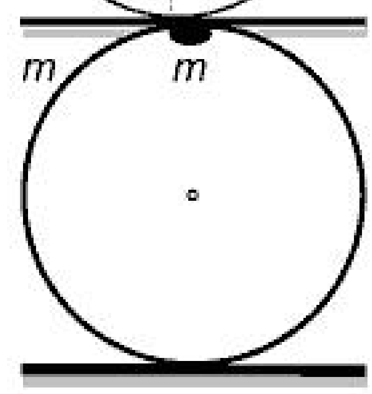
d) Find the total work done by the friction force and the works done by the sphere on the cart and by the cart on the sphere.

e) How much translational kinetic energy does each object gain? What is the change of the rotational kinetic energy?

Problem 104. A small ball of mass $M = 4$ kg is attached to a solid cylinder of radius $r = 3$ dm and mass $m = 40$ kg by a massless rod as shown. The ball is at a distance of $R = 5$ dm above the centre of the cylinder. The system is then tipped from its unstable equilibrium position. Find the speed of the ball when it hits the ground. The cylinder rolls without slipping. Use $g = 10\ \mathrm{m/s^2}$.



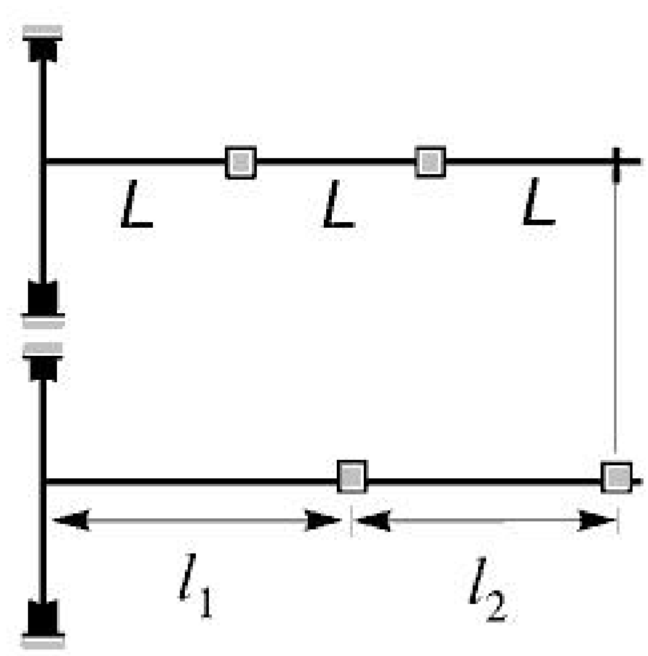
Problem 105. A weight of mass $m = 5$ kg is fixed to the perimeter of a hoop of the same mass $m = 5$ kg and radius $r = 1$ m. The hoop is placed on a horizontal plane. Friction is negligible. $g = 10\ \mathrm{m/s^2}$. Initially, the weight is at the top. Then the hoop is released.

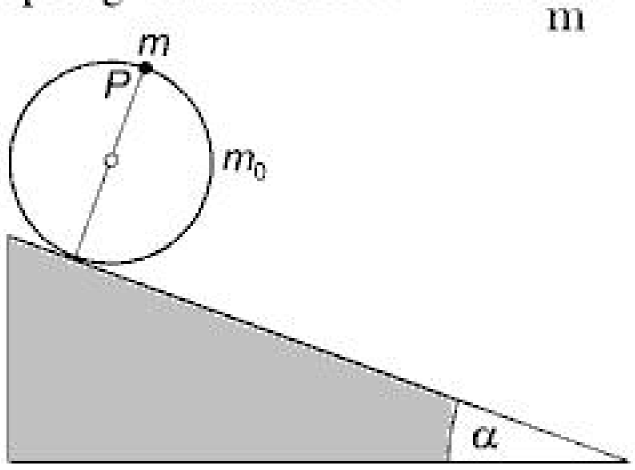


a) Find the acceleration of the centre of the hoop when the weight is level with the centre.

b) With what force does the hoop press on the ground at that time instant?

Problem 106. A horizontal rod is fastened to a vertical axis as shown. There are two identical particles beaded onto the rod, each of mass 1 kg. The particles are connected to each other and to the axis by two springs, each of which have a length of $L = 0.1$ m in their relaxed states. The particles can move on the rod without friction. What should the angular velocity of the system be if the distance of the outer particle from the axis is to be $3L$? The spring constant is $D = 10\ \frac{\mathrm{M}}{\mathrm{m}}$.

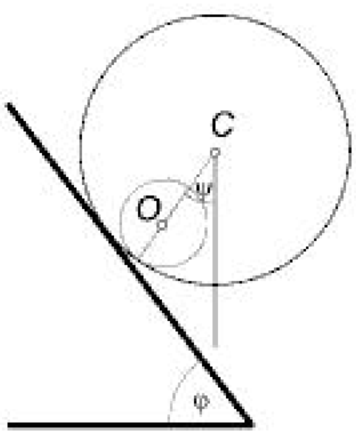




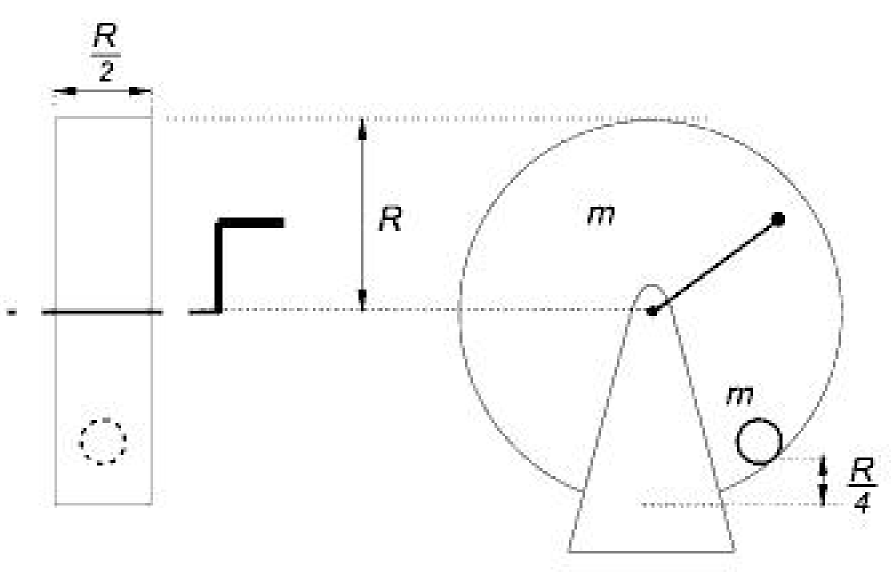
Problem 107. A ring of mass m_0 rolls along a slope with angle of inclination α without sliding. When it begins, a beetle of mass m lands at point P. Find the force with which the beetle should hang on to the ring after $5/4$ turns in order to remain on the ring. ($\alpha = 20^\circ$, $m = 1$ g, $m_0 \gg m$.)

Problem 108. A hoop of radius r and of mass m is thrown above the ground in such a way that the plane of the hoop is vertical. The hoop is rotating backwards, about its centre with an angular speed of ω_0 and the velocity of its centre is v_0 in the forward direction. What must the angular speed of the hoop be if after reaching the ground during the course of its motion the hoop turns back (moves backward)? At what angular speed of ω_0 will the speed of the hoop moving backward be v_0?

Problem 109. A ping-pong ball of mass $m = 3$ g is hit back in such a way that it gains a horizontal velocity at a height of $h = 20$ cm above the table. There is a spin put on the ball causing it to rotate about a horizontal axis that is perpendicular to its velocity. After hitting the table the ball bounces back in the vertical direction without rotation. The collision is elastic, and due to the unevenness of the surfaces the coefficient of kinetic friction between the ball and the table is not zero, but $\mu = 0.25$. Therefore, what is the maximum heat produced during the collision of the ball with the table? (Use $g = 10\ \mathrm{m/s^2}$.)



Problem 110. A hollow rim of radius $r_1 = 1$ m and mass $m_1 = 670$ g rolls down an inclined plane of angle $\varphi = 53^0 08'$. Inside the rim there is a solid cylinder of radius $r_2 = 0.3$ m and mass m_2. The centre of mass of the cylinder remains at rest relative to the centre of mass of the rim so that the line connecting the two centres (points O and C) forms an angle $\psi = 36°52'$ with the vertical throughout the motion. Find the mass of the cylinder if both objects roll without slipping.



Problem 111. For a freely rotating wheel of fortune of mass m, the base and the nappe of the cylinder, whose radius is R and height is $R/2$, are made of a plate of uniform width and material. Within the originally stationary wheel there is a solid ball of radius $r = R/6$ and the same mass m, which is in touch with the surface of the cylinder at a height $R/4$ and is initially at rest.

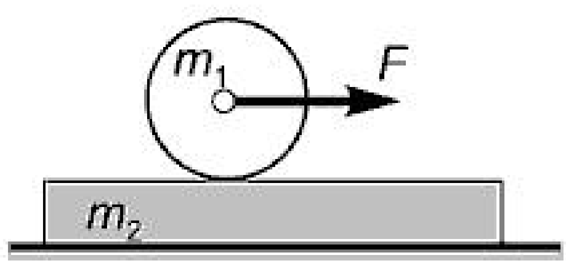
a) Find the torque that should be applied on the wheel of fortune in order to have the centre of mass of the ball in it stay at rest.

b) Find the work done this way in 2 s.

c) Find the angular acceleration of the ball and the wheel of fortune.

(The ball rolls without skidding. Let $R = 0.54$ m and $m = 2$ kg. The mass of the driving rod is negligible.)

Problem 112. A cylinder of mass $m_1 = 30$ kg and radius $r = 8$ cm lies on a board of mass $m_2 = 60$ kg. The ground is frictionless and the coefficient of friction (both static and kinetic) between the board and the cylinder is $\mu = 0.1$. The centre of mass of the cylinder is pulled with a force of $F = 44.15$ N for two seconds. Find the work done by force F.

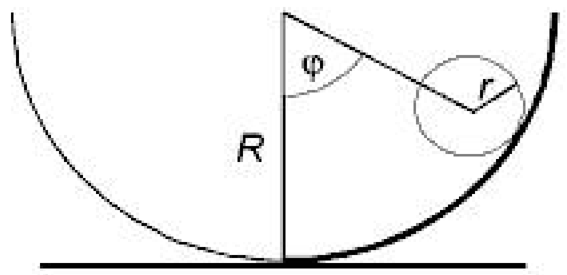


Problem 113. A solid sphere is rolling down, without sliding, on an incline of angle $30°$. The angle of the incline is variable.

a) The experiment is repeated with a hollow sphere, containing a concentric, spherical hole of half radius inside. Determine the slope of the incline so that the time of the motion is the same as in the previous experiment, provided that the two spheres are started from the same point on the incline.

b) In which case is larger the minimal static friction coefficient necessary for the slide free rolling?

Problem 114. One half of a semi-cylinder of radius $R = 1$ m has a rough inner surface, while the other half of the surface is frictionless. A solid sphere of radius $r = 0.2$ m is released from the position described by the initial angle $\varphi = 60°$ on the rough part of the semi-cylinder. Determine how high the centre of the sphere gets on the other, frictionless part of the semi-cylinder, in respect to the lowest point of the circular ramp. (On the rough part of the surface the static friction is strong enough for rolling without slipping, and the rolling friction is negligible.)

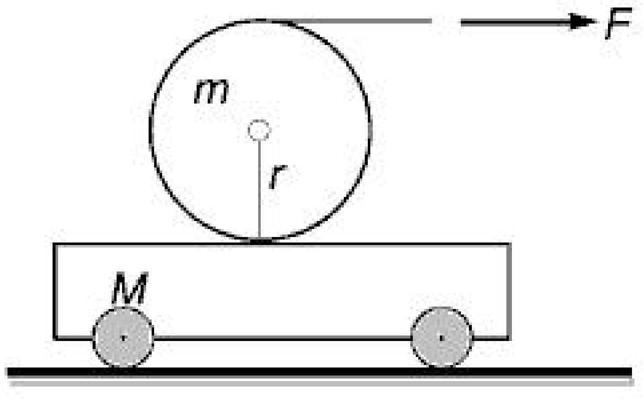


Problem 115. A disk of mass $m = 10$ kg and radius $r = 0.2$ m is placed on top of a cart of mass $M = 5$ kg that stands on a frictionless surface. A massless string is wrapped around the disk.

a) Find the accelerations of the disk and the cart, if the free end of the string is pulled with a constant horizontal force of magnitude $F = 100$ N. The coefficient of friction between the cart and the disk is $\mu = 0.1$.

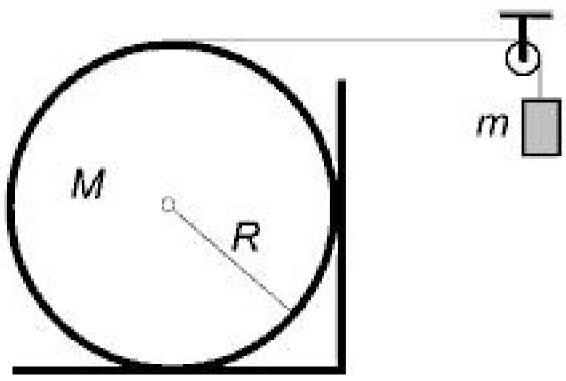
b) Find the kinetic energies of the two objects at the instant when the length of the unwound string is $L = 2$ m.

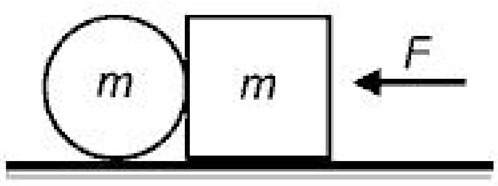
c) Find the work done by force F until that moment.



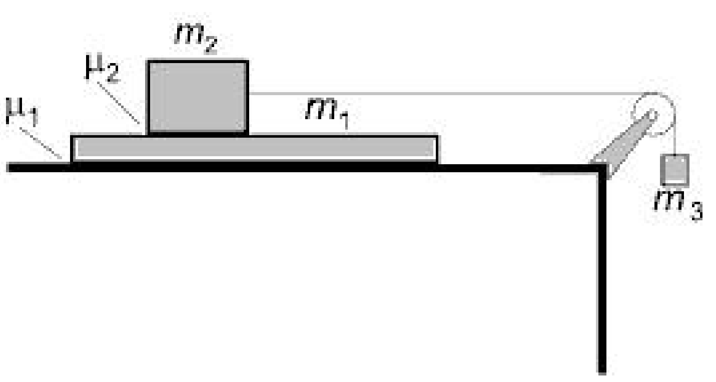
Problem 116. There is a ball of mass m at the middle of the top of a block of mass M and of length $2l$. A constant force of F is exerted on the block from the initial time 0 till time t. Then the exerted force is ceased. Friction between the horizontal surface and the block is negligible. The static friction between the ball and the block ensures that the ball rolls without sliding. Find the time T which elapses until the ball falls off the block. (When will the ball reach the end of the block?) The rolling resistance exerted on the ball is negligible.

Problem 117. A cylinder of mass M and radius R lies in a corner so that it touches both the wall and the ground as shown. A massless chord passes around the cylinder, over a pulley, and is attached to a small object of mass m. The coefficient of kinetic friction is μ for all surfaces. Find the acceleration of the object attached to the string. Data: $\mu = 0.5$, $m = 11\,\mathrm{kg}$, $M = 8\,\mathrm{kg}$, $R = 0.4\,\mathrm{m}$, $g = 10\,\mathrm{m/s^2}$.

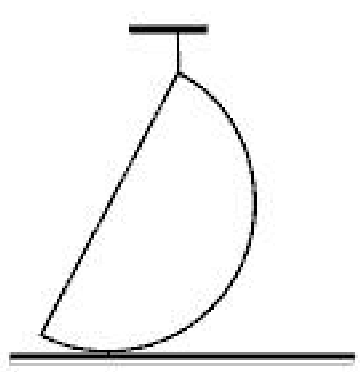




Problem 118. A cube and a cylinder are placed on a horizontal surface such that a generator of the cylinder touches the side of the cube as shown. The radius of the cylinder is equal to the side length of the cube and the masses of the two objects are also equal. For all surfaces the coefficients of static and kinetic friction are μ_0 and μ respectively ($\mu_0 > \mu$). With what force should the cube be pushed if the two objects are to move together in such a way that the cylinder's motion remains purely translational? Data: $m = 12\,\mathrm{kg}$, $\mu = 0.2$, $\mu_0 = 0.6$, $g = 10\,\mathrm{m/s^2}$.

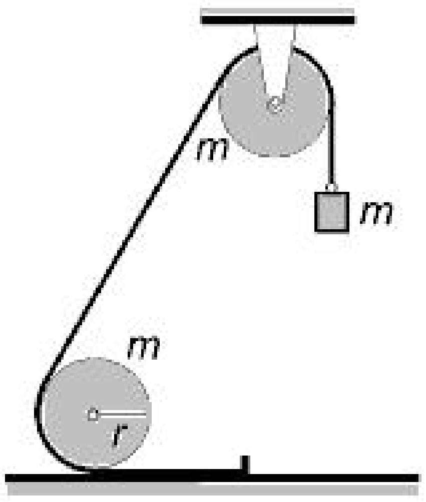


Problem 119. Describe the motion of the system shown in the figure. The coefficient of friction between the board of mass m_1 and the table is μ_1, while the coefficient of friction between the board and the brick of mass m_2 is μ_2. (The coefficients of static and kinetic friction are the same.) Data: $m_1 = 2\,\mathrm{kg}$, $m_2 = 2\,\mathrm{kg}$, $m_3 = 1\,\mathrm{kg}$, $\mu_1 = 0.1$, $\mu_2 = 0.35$.

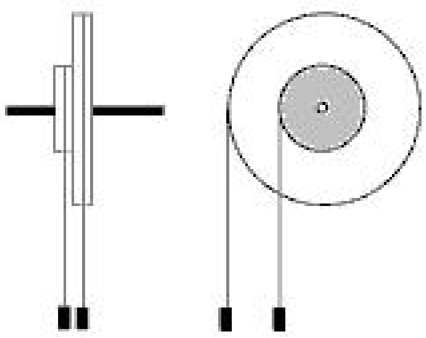


Problem 120. A homogeneous full hemisphere is suspended by a string at a point on its edge is such a way that it touches but does not push the ragged surface beneath it. Find the minimum value of the coefficient of friction at which the hemisphere will not slip after burning the string. The centre of mass of a hemisphere is at $3/8$th of its radius.

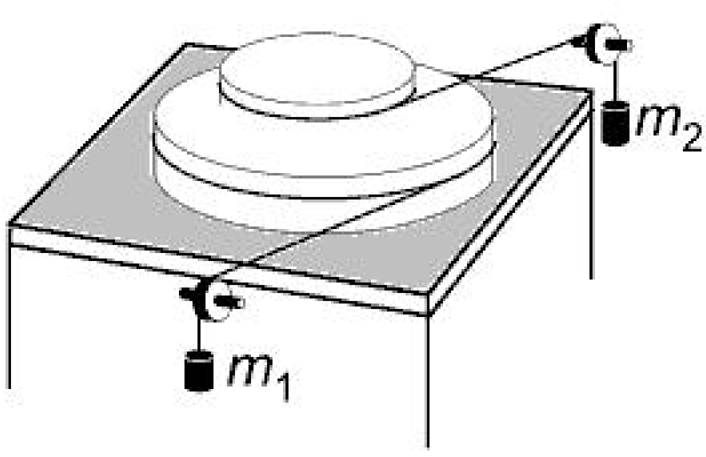
Problem 121. A massless cord that has one of its ends attached to a peg on the ground passes below a cylinder, over a pulley, and is attached to a small object of mass $m = 8\,\mathrm{kg}$ as shown. The rotating part of the pulley is identical to the cylinder on the ground, both having a radius of $r = 25\,\mathrm{cm}$ and a mass of $m = 8$ kg. The cord between the pulley and the cylinder (that are at a great distance from each other) forms $60°$ with the horizontal. Find the acceleration of the hanging object at the moment when it is released. (The cord does not slip on the pulley.)



Problem 122. Two coaxial pulleys of the same thickness and of the same material have radii $10\,\mathrm{cm}$ and $20\,\mathrm{cm}$ respectively. The total mass of the pulley-system is $5\,\mathrm{kg}$. The blocks hanging from the pulleys have a mass of $9\,\mathrm{kg}$ each. Find the times the hanging blocks need to travel down to a depth of $4.9\,\mathrm{m}$ from their original positions.

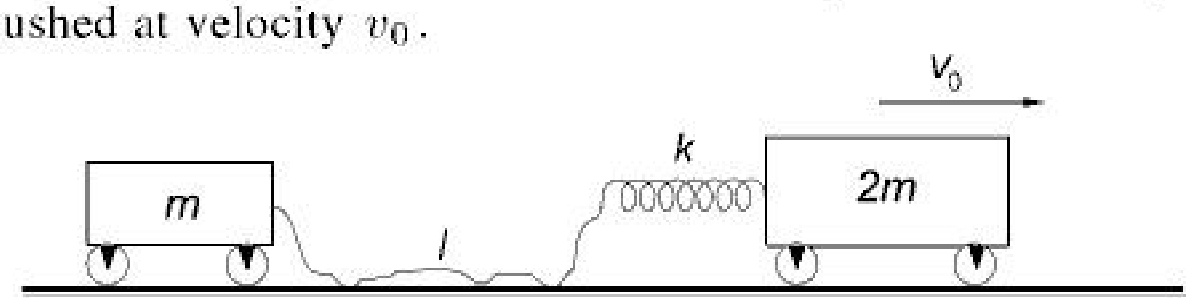


Problem 123. Two disks with radii $r_1 = 0.3\,\mathrm{m}$ and $r_2 = 0.2\,\mathrm{m}$ are fixed together so that their centres are above each other. The rotational inertia of the disk-system is $\Theta = 0.25\ \mathrm{kgm}^2$. The greater disk stands on a frictionless table. Massless chords that are wrapped around the greater and the smaller disks pass over pulleys and are attached to small objects of masses $m_1 = 5$ kg and m_2 respectively as shown. Find the value of m_2 at which the axis of symmetry of the disk-system remains stationary. Use $g = 10\,\mathrm{m/s}^2$.



Problem 124. A cylinder of radius R has two disks, both of radius $r = R/3$ fixed onto its two base surfaces. The system is suspended on two massless chords that are wrapped around the disks. There are inked letters placed all around the cylindrical surface. With what acceleration should the end of the cords be moved if our task is to print the letters clearly onto a vertical wall? Neglect the mass of the disks.

Problem 125. A loosely hanging thread of length l is attached to a freely rolling trolley of mass m; the other end of it is attached to a cylindrical spring with spring constant k whose other end is attached to a trolley of mass $2m$ as shown in the figure. The spring can also be compressed and its axis always remains straight. The cart of mass $2m$ is pushed at velocity v_0.

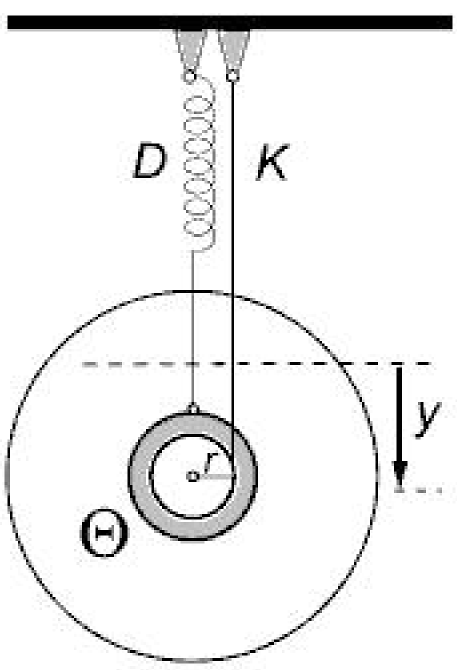


a) Find the time elapsed from the stretching of the thread to the rear trolley reaching the spring.

b) Find the time after which the thread stretches again.
($m = 8\,\mathrm{kg}$, $k = 23.3\,\mathrm{N/m}$, $l = 1\,\mathrm{m}$, $v_0 = 2\,\mathrm{m/s}$.)

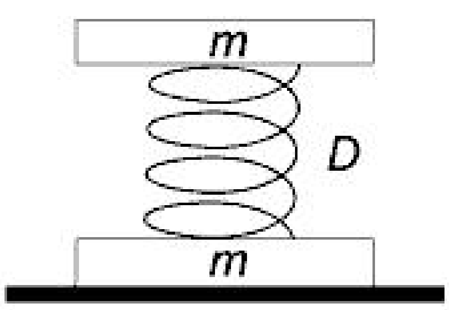
Problem 126. A spring balances a disc of radius R and of moment of inertia Q, which is able to rotate about a horizontal axle, so that the torque exerted by the spring is proportional to the angle turned. A thread which is attached to the spring is wound around the disc and a small body of mass m is hung from its other end. What type of motion will this system undergo if it is moved a little bit out of its equilibrium position? Neglect friction and air resistance.

Problem 127. An axle is attached to a disk at its centre perpendicularly to the plane of the disc. Then two pieces of thread are wound round the two ends of the axle. The ends of the threads are kept vertically and attached to the ceiling, while the disc is held at rest. Symmetrically to the disc two frictionless rings are placed on the axle, and two springs are attached to the rings. The other ends of the springs are fixed to the ceiling so that they hang vertically. The springs are not extended at this position.



Then the system is released. How much time elapses until the disc reaches its lowest position?

(Numerical data: the mass of the disc is $m = 2$ kg, its moment of inertia $\Theta = 0.01\ \mathrm{kgm^2}$, radius of the axle $r = 2\,\mathrm{cm}$, spring constant of one spring (the springs are alike) $D = 1.5\ \mathrm{N/m}$, $g = 10\,\mathrm{m/s^2}$.)



Problem 128. Two slabs of mass $m = 0.1$ kg are connected by a spring of spring constant $k = 20\ \mathrm{N/m}$, whose unstretched length is $l_0 = 0.3$ m as shown in the figure. The upper slab is pushed down by 0.15 m and then released. Find the maximum distance between the two slabs.

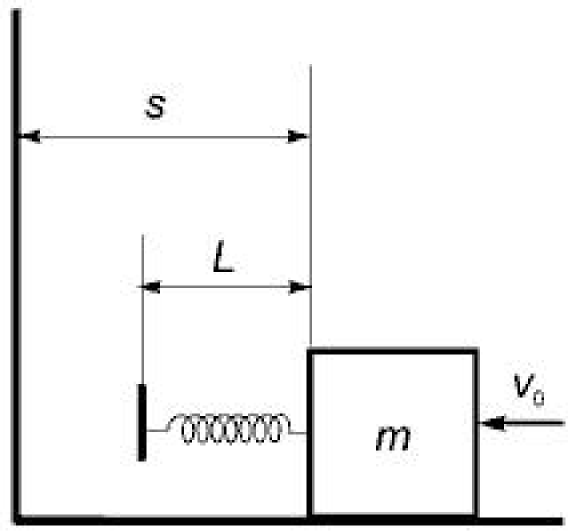
(The mass of the spring is negligible. Calculate with $g = 10\ \mathrm{m/s^2}$.)

Problem 129. An object hangs from a spring in the cockpit of a truck and causes an elongation of $\Delta l = 0.1$ m of the spring. The truck arrives at a highway that was built from concrete plates of length $x = 20$ m fitted next to each other, but the fittings are not perfect. When the truck runs at speed v, the hanging body oscillates with very high amplitude. What is the speed of the truck?

Problem 130. An object of mass $m = 1\,\mathrm{kg}$ moving on a horizontal ground is given an initial speed of $v_0 = 2\,\mathrm{m/s}$. Initially, the distance of the object from the wall is $s = 1$ metre. A spring of length $L = 8\,\mathrm{cm}$ and spring constant $D = 100\,\mathrm{N/m}$ is attached to the object. The coefficient of friction is $\mu = 0.2$. $g = 10\,\mathrm{m/s^2}$.

a) Where will the block stop?

b) When will the block stop?

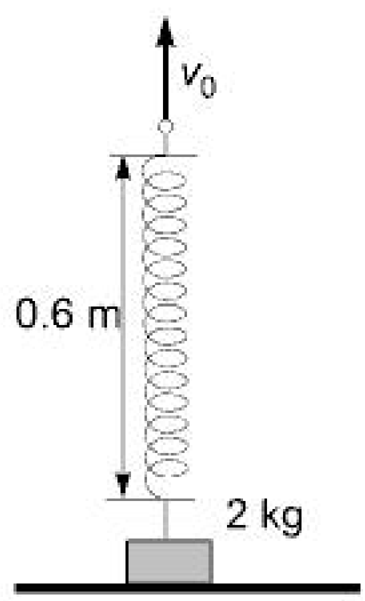


Problem 131. The unstretched length of a spring is $L_0 = 0.6$ metres and the spring constant is $D = 80\,\mathrm{N/m}$. The lower end of the spring is attached to an object of mass $m = 2\,\mathrm{kg}$ lying on the ground, and the upper end is held at a height of 0.6 metres vertically above the object. Initially, the spring is unstretched. Then the upper end is lifted at a uniform speed of $v_0 = 0.5\,\mathrm{m/s}$. $g = 10\,\mathrm{m/s^2}$.

a) How high will the object be lifted in 1.75 seconds?

b) What is the work done by the lifting force?

c) Describe the variation of power as a function of time.



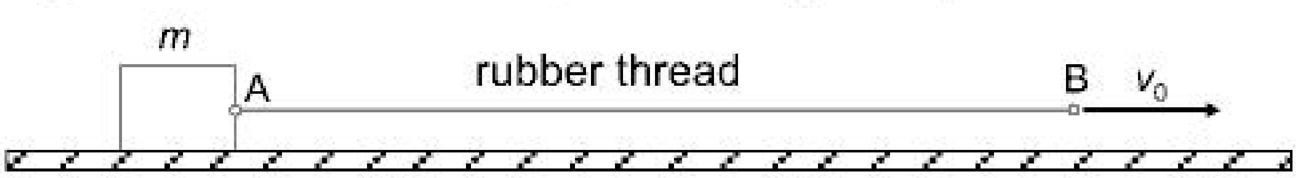
Problem 132. A body of mass $m = 1.25$ kg is suspended vertically by a spring of spring constant $D = 250$ N/m and unstretched length $l = 1$ m. It is released at zero initial speed from the unstretched position of the spring. Determine the time when the speed of the body reaches the value $v = 0.5$ m/s first.

Problem 133. A body of mass $m = 1$ kg is at rest on a horizontal, frictionless ground. A thin rubber thread is attached to the side of the body at the point A. The unstretched length of the thread is $L_0 = 50$ cm. Initially the other end of the thread (point B) is at a distance L_0 from A in horizontal direction. When the rubber thread is stretched, it behaves as if it had a spring constant $D = 100$ N/m, but it is impossible to "compress" the thread, since then it loosens and exerts no force.

At a given moment we start to pull the end B of the rubber thread horizontally, at a constant speed $v_0 = 1$ m/s to the right (see the figure), and continuously maintain this uniform pull.

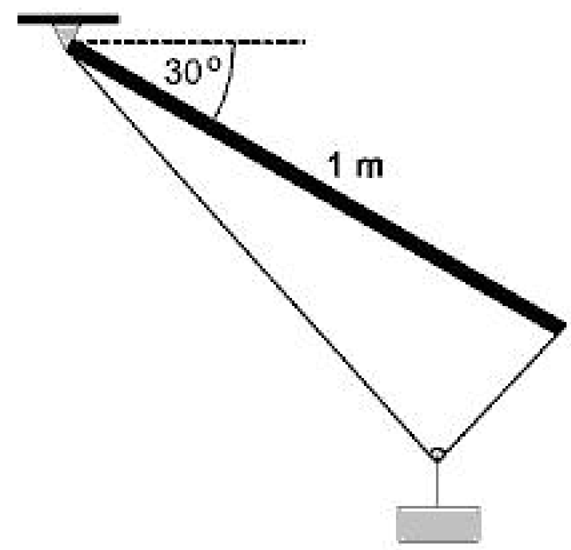
a) Determine the longest distance between the points A and B.

b) How long does it take for the body to catch up with point B?



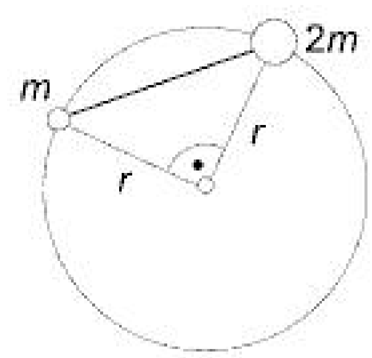
1.3 Statics

Problem 151. One end of a rod of mass 1 kg and of length 1 m can freely rotate about a fixed horizontal axis. Initially the rod makes an angle of $30°$ to the horizontal. A thread of length 1.3 m is attached to the two ends of the rod. A small pulley can run without friction along the thread, and a weight of mass 0.2 kg is suspended on the axis of the pulley.



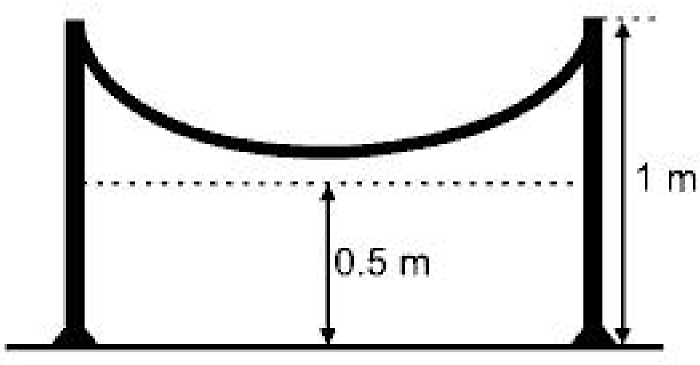
Determine the work needed to lift the rod to a horizontal position. (Use the value $g = 10\ \mathrm{m/s}^2$ for the acceleration due to gravity.)

Problem 152. Two beads of masses m and $2m$ can move on a circular vertical loop of radius $r = 0.5$ m. The beads are connected by a massless string, and if the string is taut, it keeps the beads on the ends of a quarter-circle as shown. The coefficient of friction is 0.15. Find the positions in which the beads are in equilibrium with the string being taut.



Problem 153. An analytical balance is used with brass weights. Find the mass of a body made of Plexiglas, whose two measurements result in a difference of at least one mark if one measurement is performed in dry weather and the other in wet weather? In both cases the room temperature is 23 °C and the atmospheric pressure is 10^5 Pa. In wet weather the pressure of the water vapour in the air is $2\cdot 10^3$ Pa. The sensitivity of the balance is 0.1 mg/scalemark. ($\varrho_{\mathrm{Cu}} = 8.5\cdot 10^3\ \mathrm{kg/m^3}$; $\varrho_{\mathrm{Plexiglas}} = 1.18\cdot 10^3\ \mathrm{kg/m^3}$.)

Problem 154. The two ends of a homogeneous chain of mass 2 kg are fixed to columns of height 1 m as shown in the figure. The chain is clutched in the middle and is pulled down until it becomes tight. In the meantime 0.5 J of work is done. The lowest point of the chain is then 0.5 m from the ground then. Where was the centre of mass of the chain initially?



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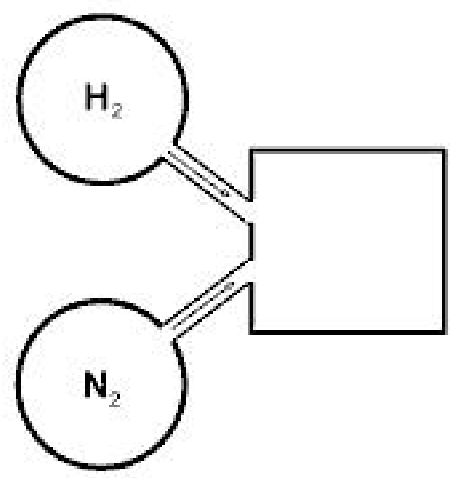
2.2 Ideal gas processes

Problem 168. For an experiment a mixture of gases containing 50 volume percents hydrogen and 50 volume percents nitrogen should be continuously provided at a speed of $0.5\ \mathrm{kg/min}$. The cross section of the gas tubes is $10\ \mathrm{cm}^2$. Determine the speed of gas flow in the tubes, provided that the pressure is 10^5 Pa and the temperature is $27\ ^\circ\mathrm{C}$ in the tubes.



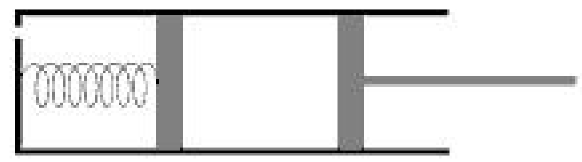
Problem 169. A cylinder with base area $1\ \mathrm{dm}^2$ lies on its side on a horizontal surface and is divided into two parts of volumes 0.8 litre and 4.2 litre by a frictionless vertical piston as shown. The pressure in each part is $0.02\,\mathrm{N/cm}^2$. The masses of the cylinder and piston are $0.8\,\mathrm{kg}$ and $0.2\,\mathrm{kg}$ respectively. The cylinder is then pushed by a constant horizontal force of magnitude $2.5\,\mathrm{N}$ to the left. What will the new position of the piston be? (Assume constant temperature.)

Problem 170. A cylinder of base area $10\,\mathrm{cm}^2$ in which a $47\,\mathrm{cm}$ high air column is enclosed by a piston is floating upside down in a container. The piston is connected by a cord to the bottom of the container, which is filled with mercury and has a base area of $20\,\mathrm{cm}^2$. The closed end of the cylinder is $10\,\mathrm{cm}$ below mercury level.

a) Find the new position of the cylinder if the cord is shortened by $6\,\mathrm{cm}$.

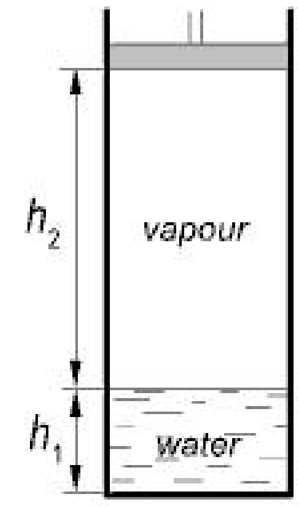
b) Find the volume of mercury that should be poured into the container to set the mercury level back into its original height.

Problem 171. The cylindrical vessel shown in the figure has two pistons in it. The piston on the left touches a spring attached to the wall of the vessel. The wall has a hole in it. The volume of the air between the pistons is $2000\,\mathrm{cm}^3$ and its pressure is initially equal to the external atmospheric pressure of $10^5\,\mathrm{N/m}^2$. The piston on the right is slowly pressed inwards, maintaining constant temperature, until its inner surface is at the position where the inner surface of the piston on the left was initially. What will be the final volume of the air between the pistons?



The cross-sectional area of the cylinder is $100\,\mathrm{cm}^2$, and a force of $10\,\mathrm{N}$ compresses the spring by $1\,\mathrm{cm}$.

Problem 185. A vertical cylinder with cross-sectional area $A = 1\ \mathrm{dm}^2$ contains $h_1 = 25$ cm of water at the bottom. The space above it is filled with the saturated vapour of the water, which is separated from the external space by a piston. The bottom of the piston is $h_2 = 75$ cm above the water level. The density of water at this temperature is $n = 2$ times the density of saturated vapour.

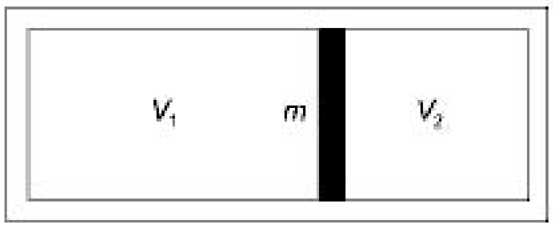


a) If temperature is held constant, by how much should the piston be pushed down in order to decrease the volume of vapour to $V = 4.5\ \mathrm{dm}^3$?

b) If temperature is held constant, by how much should the piston be pushed down in order to have the vapour condense completely?

(The sum of the masses of water and vapour is constant throughout the process.)

Problem 186. Helium gas whose volume is $V_1 = 3$ litres, pressure is $p_1 = 4 \cdot 10^5$ Pa and temperature is $T_1 = 1092$ K is separated from helium gas whose volume is $V_2 = 2$ litres, pressure is $p_2 = 2.5 \cdot 10^5$ Pa and temperature is $T_2 = 1365$ K by a highly insulated wall of mass $m = 2$ kg in an insulated cylinder. The partition wall is released, it can move without friction. Find the maximum speed acquired by the partition wall.



Problem 187. 4 grams of helium and 16 grams of oxygen are enclosed by a piston in a cylinder. The a temperature of the gas is $0\ ^\circ\mathrm{C}$ and its pressure is 10^5 Pa. The cylinder walls and the piston are good thermal insulators. The pressure is increased to $2 \cdot 10^5$ Pa. What will be the final temperature and volume of the gas? The molar specific heats of helium are $C_{vh} = 12.3\ \mathrm{J/(mod \cdot K)}$, $C_{ph} = 20.5\ \mathrm{J/(mol \cdot K)}$; and those of oxygen are $C_{vo} = 20.5\ \mathrm{J/(mol \cdot K)}$, $C_{po} = 28.7\ \mathrm{J/(mod \cdot K)}$.

Problem 188. A smoothly moving, fixed piston made of good insulating material separates two gases in an insulated cylinder whose cross-sectional area is $A = 1\ \mathrm{dm}^2$. One part contains helium, the other part contains hydrogen. The initial data of helium is: its pressure is $p_1 = 2 \cdot 10^5$ Pa, its volume is $V_1 = 4\ \mathrm{dm}^3$, its temperature is $T_1 = 350$ K, the corresponding data of hydrogen are: $p_2 = 3 \cdot 10^5$ Pa, $V_2 = 5\ \mathrm{dm}^3$ and $T_2 = 280$ K. Find the displacement of the piston when it is released and it reaches an equilibrium

a) if the piston does not allow the gases to mix,

b) if the piston is permeable and particles can diffuse through it slowly,

c) if the piston allows only the helium to diffuse through it.

Problem 189. Three identical containers, each containing 32 g of oxygen gas at a temperature of $200\ ^\circ\mathrm{C}$ and pressure of $10\ \mathrm{N/cm}^2$ are connected by thin tubes. The container on the left is then cooled down to $100\ ^\circ\mathrm{C}$, the one on the right is heated to $300\ ^\circ\mathrm{C}$, while the temperature of the middle one remains $200\ ^\circ\mathrm{C}$.

a) Find the new pressure of the system.

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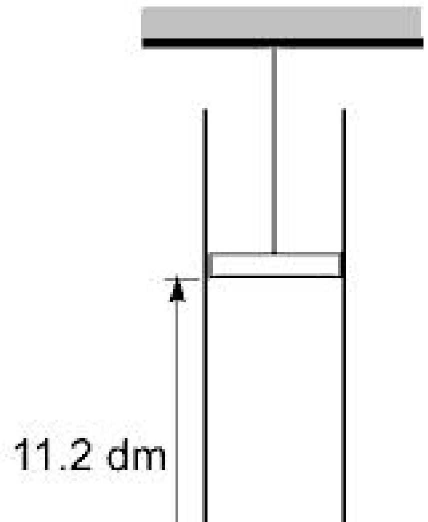
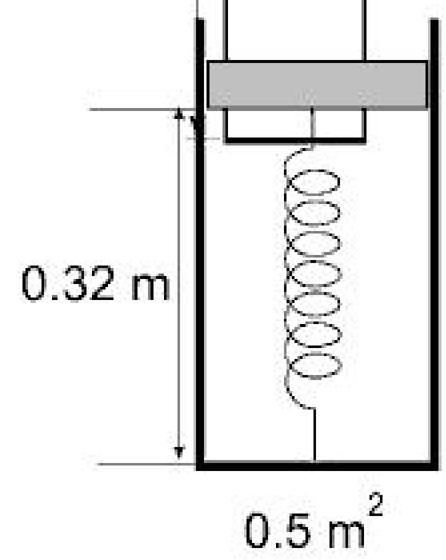
Problem 202. A cylinder of mass $8\,\mathrm{kg}$ and cross-sectional area $20\,\mathrm{cm}^2$ is hanging, suspended on its piston. The cylinder contains helium of temperature $27\ ^\circ\mathrm{C}$. The temperature is slowly decreasing. How much heat is necessary to extract from the helium so that the initial length $11.2\,\mathrm{dm}$ of the gas column decreases to $8.96\,\mathrm{dm}$? The external air pressure is $10^5\ \mathrm{Pa}$, $g = 10\ \mathrm{m/s}^2$. The molar specific heat of helium at constant volume is $C_v = 12300\,\mathrm{J/(kmol\cdot K)}$.



Problem 203. A cylindrical container of base area $0.5\,\mathrm{m}^2$ contains helium gas at $218.4\ \mathrm{K}$. The gas is enclosed by a frictionless piston of mass $600\ \mathrm{kg}$ that is connected to the base of the container by a spring, whose spring constant is $2.67\cdot 10^5\ \mathrm{N/m}$. Initially the piston is at a height of $0.32\,\mathrm{m}$, which is the relaxed length of the spring. Atmospheric pressure is $10^5\,\mathrm{Pa}$, the molar specific heat of helium is $C_v = 12.3\,\mathrm{joule/(mol\,K)}$, $g = 10\,\mathrm{m/s}^2$. The wall of the container is a good thermal conductor causing the temperature of the gas to change until it reaches the external temperature. The work done by the gas is found to be 1800 joules.

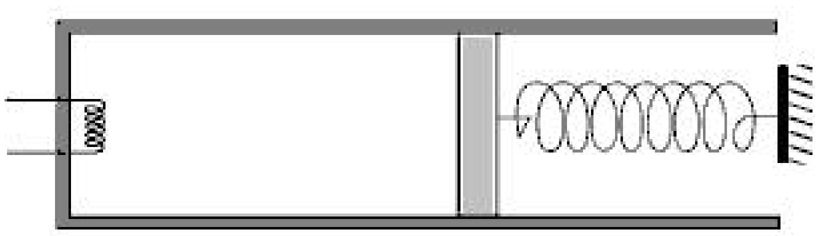
a) Find the external temperature.

b) Find the heat given to the helium gas.



Problem 204. In an $11.2\,\mathrm{dm}$ high cylindrical container, whose base area is $1\ \mathrm{dm}^2$, a frictionless piston of mass $8\,\mathrm{kg}$ is held at a height of $5.6\,\mathrm{dm}$. The piston encloses $1\ \mathrm{mol}$ of helium at $273\ ^\circ\mathrm{C}$. The wall of the container is insulated. Find the maximum height reached by the piston after being released. The molar specific heat of helium at constant volume is $C_v = 12.6\ \mathrm{J/(molK)}$, while at constant pressure it is $C_p = 21\ \mathrm{J/(molK)}$. The atmospheric pressure is $10.12\ \mathrm{N/cm}^2$.

Problem 205. A piston encloses some air in the cylindrical vessel with horizontal longitudinal axis as shown in the drawing. The initial pressure of the air is equal to the external atmospheric pressure of $10^5\ \mathrm{Pa}$. The cross-sectional area of the piston is $0.03\ \mathrm{m}^2$. An originally unstretched spring with spring constant $2000\ \mathrm{N/m}$ is attached to the piston. The walls of the vessel and the piston are perfectly insulated. The initial volume of the enclosed air is $0.024\ \mathrm{m}^3$, its initial temperature is $300\ \mathrm{K}$. The air is heated to $360\ \mathrm{K}$ with a heating filament built into the vessel.



a) Find the displacement of the piston caused by the heating.

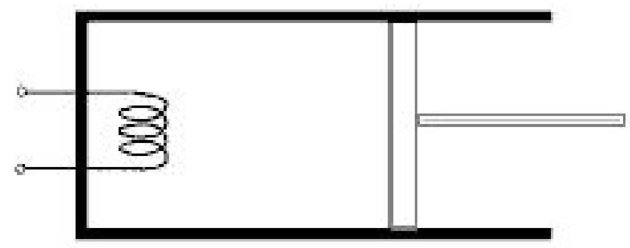
b) Find the energy delivered by the heating filament.

Problem 218. Initially $n = 10$ mol of an ideal gas has the pressure $p_1 = 10^5$ Pa, volume $V_1 = 249.42\ \mathrm{dm}^3$ and temperature $T_1 = 300$ K. Then the gas is heated, and in an isobaric process it reaches the temperature T_2. During this process the work done by the gas is 68% of the increase of its internal energy. If, however, from the same initial state an adiabatic compression is used to increase the temperature of the gas to T_2, then $W = 36.85$ kJ has to be done on the gas.

a) What type of gas is the experiment performed with?

b) Determine the final temperature T_2.

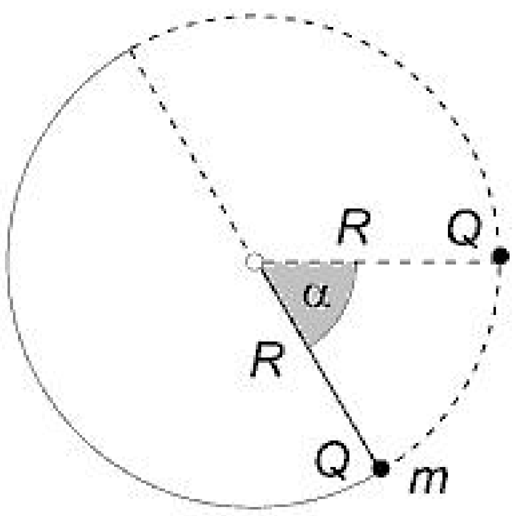
Problem 219. There is 5 g of a certain diatomic gas in a container closed with a frictionless piston. The gas is heated for 25 seconds by an electric resistor of 50 Ω built in the container, applying a voltage of 220 V. While the gas expands at constant pressure, its temperature increases by 250 °C. The efficiency of the electric heater is 75%. What kind of gas can be found in the container?



Problem 220. A container, closed by a freely moving piston, contains a mixture of hydrogene and helium gas of total mass $m = 180$ g. A heat of $Q = 156$ kJ is transferred to the gas at constant pressure. Due to this the gas performs 56 kJ work. Determine the mass of hydrogene in the mixture. Determine the temperature change of the system.

Problem 221. The state of helium gas is changed in such a way that its graph is a straight line segment on the pressure–volume plane. During this process the total heat transferred to the gas is equal to the heat necessary to double the absolute temperature of the gas at constant volume. By what ratio may the volume of the gas most increase? (The expression "total heat" refers to the signed sum of heat absorbed and heat released during the process.)

Problem 222. A small ball of mass $m = 1$ g and charge Q is attached to the end of a string of length $R = 10$ cm. Level with the suspension point of the pendulum, at a distance of $R = 10$ cm there is a small fixed object of the same charge Q. If the pendulum is released from a position $\alpha = 60°$ below the horizontal, the string will become slack when the pendulum bob has covered a semicircle exactly. Find the magnitude of the charge Q.



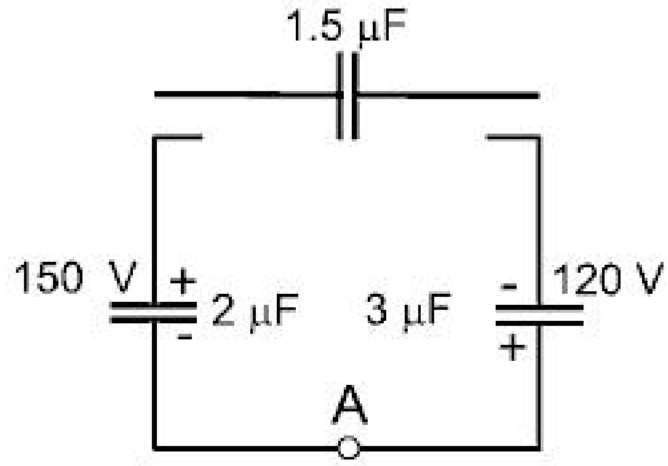
a) In the first part, two plates are placed into the capacitor as shown. The plates, that are initially neutral, are connected by a wire and are positioned such that the four plates are at equal distance from each other. Find the electric field strengths between the plates.

b) In the second part, the two initially neutral plates that are connected are positioned as shown in figure b). The plates are at equal distances from each other in this case as well. Find the electric field strengths between the plates.

Problem 236. One plate of a $2\text{-}\mu\mathrm{F}$ capacitor charged to 150 volts is connected to the oppositely charged plate of a $3\text{-}\mu\mathrm{F}$ capacitor charged to 120 volts. The other plate of each capacitor ends in a free wire. An uncharged capacitor of $1.5\text{-}\mu\mathrm{F}$ is dropped onto the free ends.

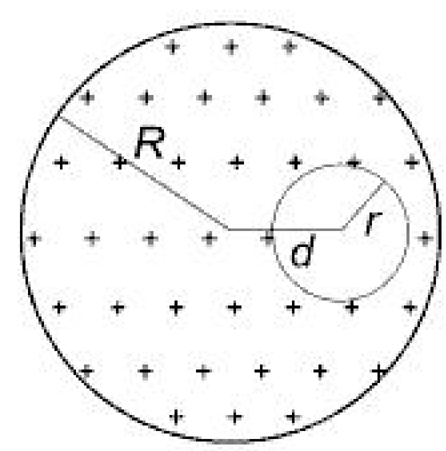
a) What will be the potential difference across each capacitor?

b) How much charge will pass through point A and in what direction?

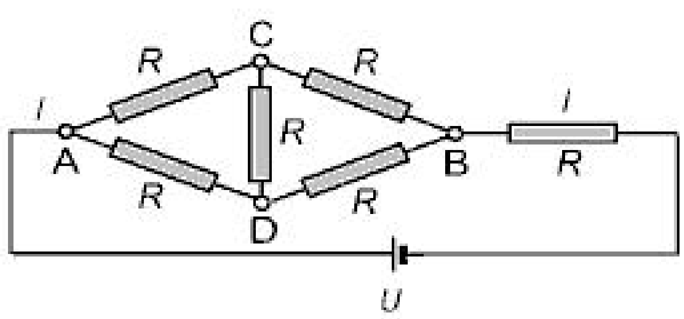


Problem 237. In ancient times, people believed that the Earth was a big, flat disc. Let us imagine that the Earth is not actually sphere with radius R but a flat disc with a very large radius and a thickness of H. What thickness H is needed to experience the same gravitational acceleration on the surface of the disc (far from its rim) as on the surface of the spherical Earth? ($R = 6370\,\mathrm{km}$. Let us consider the densities in the two 'Earth' models to be constant and equal to each other.)

Problem 238. A long insulating cylinder of radius R has a cylindrical bore of radius r in it. The axes of the cylinder and the bore are parallel, separated by a distance d. The insulator carries a positive charge of uniform distribution with a charge density of ϱ. The relative dielectric constant of the material is 1. Find the electric field inside the bore.



Problem 239. If the values of the resistance of the resistors shown in the figure are equal then the current in the main branch is I. By what factor does this current change if the resistance of the two resistors, which are diagonally opposite each other, is doubled?

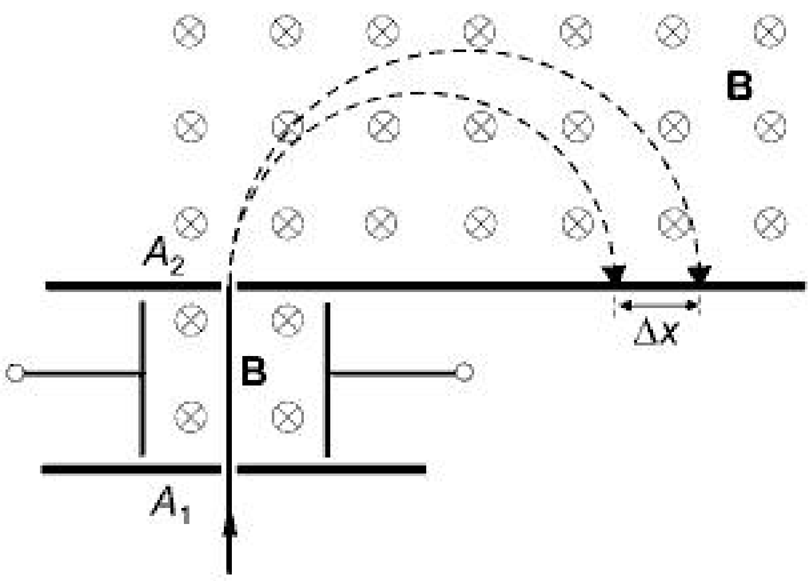


Chapter 4

Magnetism Problems

4.1 Magnetic field

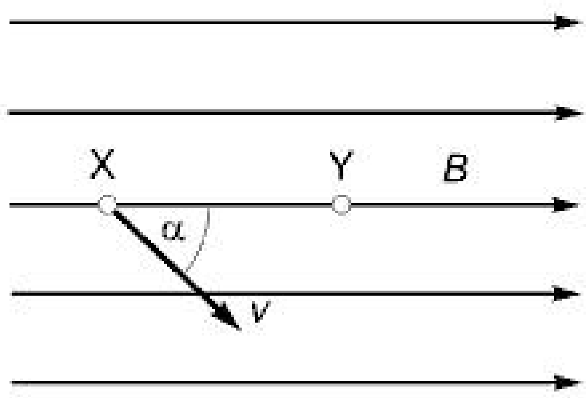
Problem 254. In a mass spectrometer the Cl^- ions, after passing through the diaphragm A_1 at different speeds, first travel in perpendicular (homogeneous) electric and magnetic fields. Then, after passing diaphragm A_2, they move further in a magnetic field only. The ^{35}Cl and ^{37}Cl isotopes hit the photo plate at points $\Delta x = 4$ cm apart from each other. The magnetic induction is $B = 0.02$ T (in both regions).



a) Determine the speed of the Cl isotopes when they pass through the diaphragm A_2.

b) Determine the magnitude and the direction of the electric field between the two diaphragms A_1 and A_2.

Problem 255. Two points lying on the same field line are separated by a distance $\overline{XY} = L = 10$ cm in vacuum, in a uniform magnetic field where the magnitude of the magnetic induction vector is $B =$ $= 0.02$ tesla. An electron accelerated by a potential difference of 800 volts passes through the point X. Its velocity encloses an angle α with the field lines. What should be the measure of the angle α so that the electron also passes through the point Y? The charge of an electron is $1.6 \cdot 10^{-19}$ coulombs, and its mass is $9 \cdot 10^{-31}$ kg.



Problem 256. A sphere of radius 1 cm is charged to a voltage of 900 V. The sphere is mounted to a 30 cm long insulating handle and is rotated, the number of revolutions is 18000/minute. Determine the magnetic induction which can be observed at the position of the axis of the rotation. (Consider the rotating small sphere as a pointlike charge.) The magnetic field at the centre of a current carrying single loop is: $H = \dfrac{I}{2r} \left(\dfrac{\mathrm{A}}{\mathrm{m}}\right)$.

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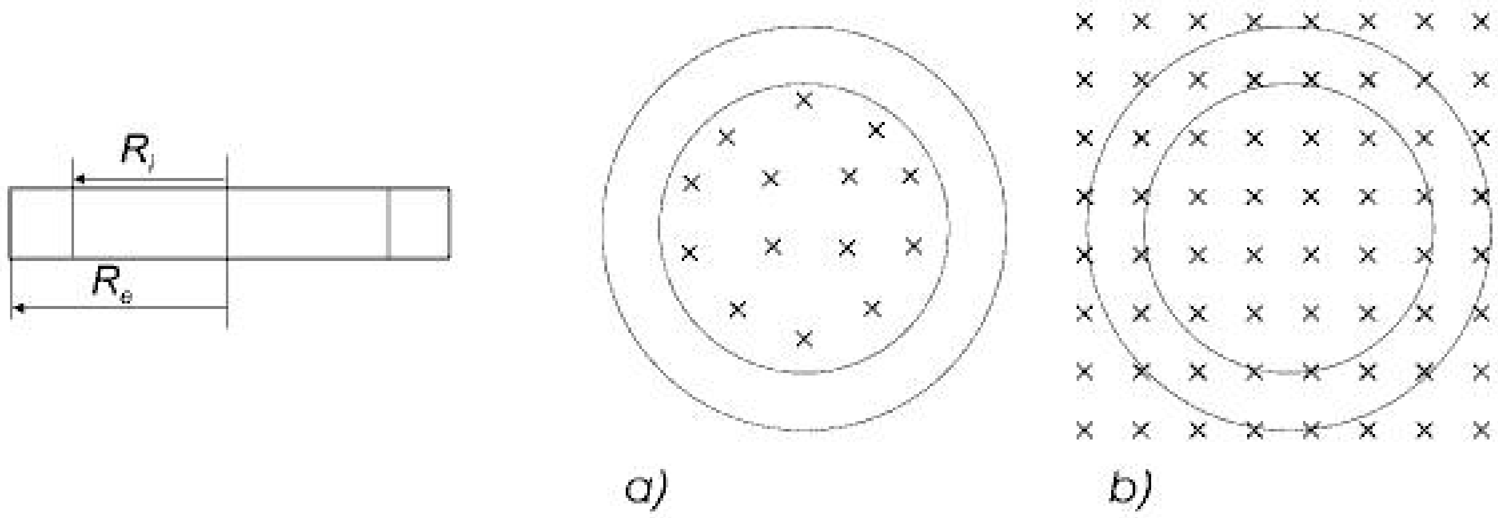
4.3 Induction (transformer emf)

Problem 268. A solid copper ring of square cross section has an internal radius $R_i = 5\,\text{cm}$ and an external radius $R_e = 7\,\text{cm}$. The ring is in a uniform magnetic field parallel to its axis. The magnetic induction $B = 0.2$ T of the field changes uniformly to its reverse in a time interval $\Delta t = 2\,\text{s}$. Express the drift speed v and angular speed ω of the conduction electrons in the ring in terms of the distance r from the axis if

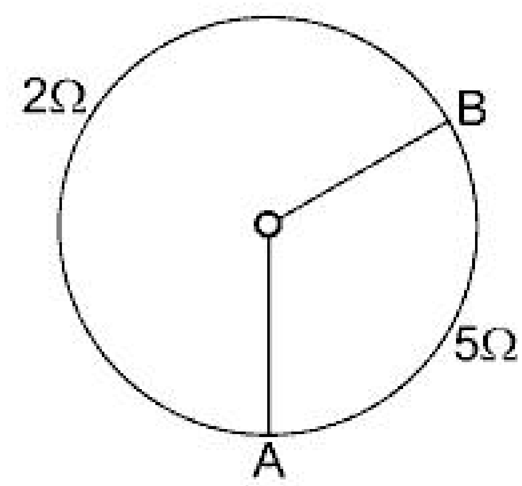
a) the uniform field only fills the interior of radius R_i of the ring,

b) the entire ring is in the magnetic field.



Problem 269. A copper ring of radius $R = 8$ cm and circular cross-sectional area $A = 2\ \text{mm}^2$ is in a homogeneous magnetic field whose induction is perpendicular to its plane and changes uniformly. At $t = 0$ the induction is $B_0 = 0$ and in $t = 0.2$ s it increases to $B = 2$ T. Find the angular velocity ω at which the ring should be rotated uniformly in order not to have tensile stress in it at time instant $t_1 = 0.1$ s. Can this problem be solved if magnetic induction changes from 2 T to 0? (Self induction can be neglected.)

Problem 270. The resistance of one third of a circular conducting loop is 5 ohms, and the resistance of the remaining two thirds is 2 ohms. The area of the circle is $0.3\,\text{m}^2$. The points where the two parts join are connected with radial wires to an ammeter of small size placed at the centre of the circle. The resistance of the ammeter is 0.5 ohms. The loop is in a uniform magnetic field perpendicular to its plane. The magnitude of the magnetic induction vector changes uniformly with time:



$$\frac{B}{t} = 0.4\ \frac{\text{T}}{\text{s}}.$$

a) What current does the ammeter read?

b) The ammeter is replaced by an ideal voltmeter. What voltage does it read?

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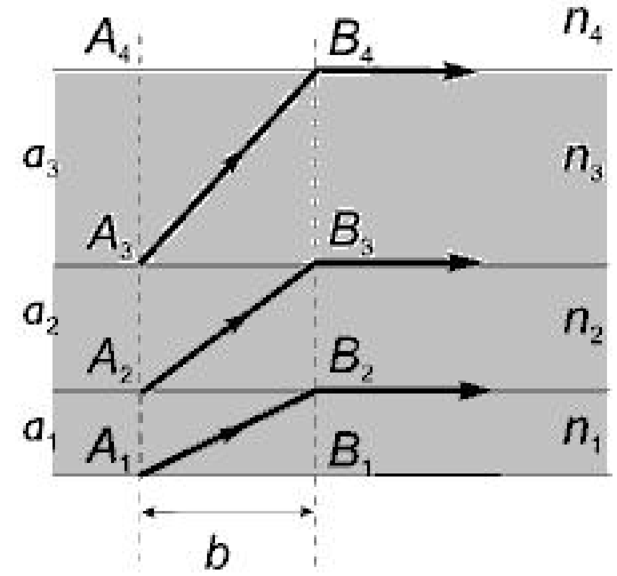
Chapter 5

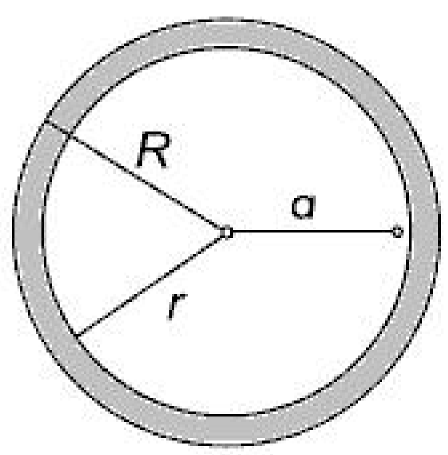
Optics Problems

Problem 284. Two power supplies with the same output voltage U are connected in a series. We then gain a power supply whose output voltage is also U. Can it happen?

Problem 285. Four layers of glass plates are placed on top of each other in such a way that the bottom one has thickness a_1 and refractive index $n_1 = 2.7$, the next one has thickness a_2 and refractive index $n_2 = 2.43$, and the third one and the top one have thicknesses a_3 and a_4 and refractive indices n_3 and n_4 respectively. Three rays of light starting simultaneously from points A_1, A_2, A_3 reach points B_2, B_3, B_4 at the same time, with their angles of incidence being the critical angles as shown. $A_2B_2 = A_3B_3 = A_4B_4 = b = 10$ mm. Find thicknesses a_1, a_2, a_3 and refractive indices n_3, n_4.



Problem 286. A glass spherical shell with an outer radius of $R = 7.5$ cm and an inner radius of $r = 6.5$ cm has a refractive index $n_2 = 1.5$. The inside of the shell is filled with carbon disulphide, whose refractive index is $n_1 = 1.6$. A source of light is placed at a distance of $a = 6$ cm from the centre. What percent of the energy of the light source leaves the system?



Problem 287. The optical model of an endoscope is an optical fibre of refractive index n_1, which is covered by a cladding of refractive index n_2. The end of the fibre is flat and it is in contact with the surrounding material of refractive index n_3. (The refractive indices are with respect to air.) How should the value of n_1 be chosen if through the fibre the whole half-space below the end of the fibre is to be visible

a) $n_2 = n_3 = 1$,

b) $n_2 = 1$ and $n_3 = 4/3$?

Problem 288. We have three equal lenses of focal length f. By placing these lenses at distances d_a and d_b from each other we build an optical system. With this optical system the image of an object is detected on a screen, which is at distance A from the object. We observe that when moving the optical system along the optical axis back and forth the image on the screen remains sharp. By what values of the geometric data is this possible?

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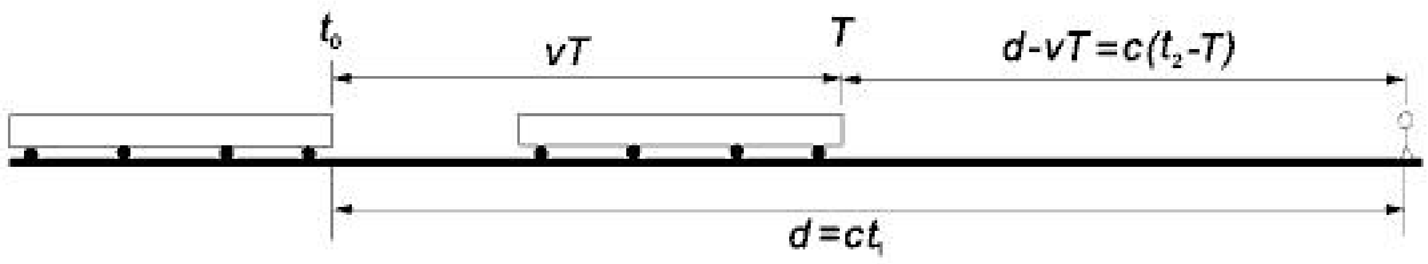
Chapter 6

Mechanics Solutions

6.1 Kinematics

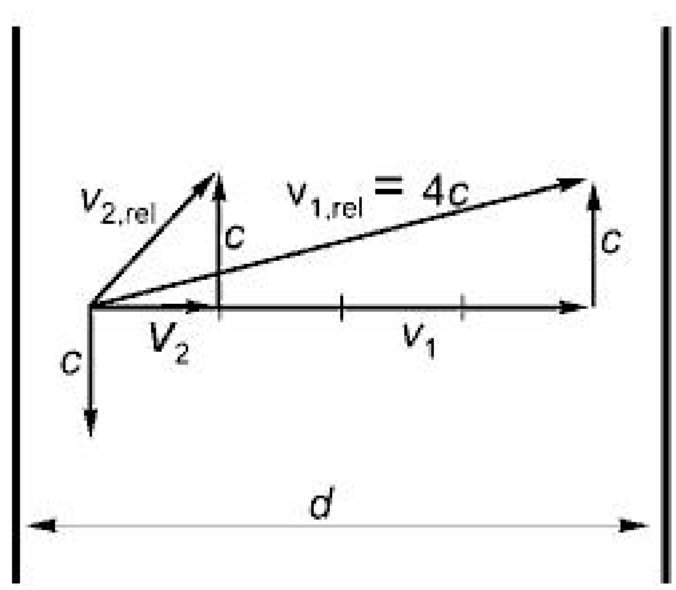
Solution of Problem 1. The railwayman hears the signal during the time which elapses between the moments when the beginning and the end of the signal reach him. The time is measured from the initial moment when the sound is emitted. The beginning of the whistle reaches the railwayman after a time of $t_1 = d/c$. Time T elapses between the moments when the beginning and the end of the whistle are emitted, and the end of the whistle covers a distance of $d - vT$, thus $(d - vT)/c$ time elapses. So, the total time which elapses until the railway man hears the end of the whistle is $t_2 = T + (d - vT)/c$. The railwayman hears the signal for a time of $\Delta t = t_2 - t_1$, which is

$$\Delta t = t_2 - t_1 = T + \frac{d - vT}{c} - \frac{d}{c} = \frac{c - v}{c} T = \frac{330 - 30}{330} \cdot 3\,\mathrm{s} = 2.727\,\mathrm{s}.$$



Solution of Problem 2. Let d and c denote the width and speed of the river. Let v_1 and v_2 be the speeds of the boat relative to the ground in the two cases, and let $4c = v_{1_{\mathrm{rel}}}$ and $v_{2_{\mathrm{rel}}}$ denote its speeds relative to the water. The task is to find the ratio $v_{2_{\mathrm{rel}}}/v_{1_{\mathrm{rel}}}$.

In both cases, the component parallel to the riverbanks is equal to c. The speeds of the boat relative to the ground in the two cases are



$$v_1 = \frac{d}{t_1}, \qquad \text{and} \qquad v_2 = \frac{d}{t_2} = \frac{d}{4t_1} = \frac{v_1}{4}.$$

As shown in the figure, the speeds are related as follows:

$$v_{2_{\mathrm{rel}}}^2 = c^2 + v_2^2 = c^2 + \frac{v_1^2}{16}. \qquad (1)$$

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Remark: The legitimacy of the approximations can be checked by comparing the previous results with the exact ones obtained by integration. The exact values of the two distances are, by integration:

$$s_1 = \int_0^1 \left(\frac{1}{2}t^2 + 2t + 1\right) dt = \left[\frac{1}{2}\cdot\frac{t^3}{3} + 2\cdot\frac{t^2}{2} + t\right]_0^1 = 2.17 \text{ m},$$

and

$$s_2 = \int_9^{10} \left(\frac{1}{2}t^2 + 2t + 1\right) dt = \left[\frac{1}{2}\cdot\frac{t^3}{3} + 2\cdot\frac{t^2}{2} + t\right]_9^{10} = 65.17 \text{ m}.$$

The relative errors in the two cases are:

$$\frac{\Delta s_1}{s_1} = \frac{2.25 - 2.17}{2.17} = 0.0369 = 3.69\%,$$

and

$$\frac{\Delta s_2}{s_2} = \frac{65.25 - 65.17}{65.17} = 0.00123 = 0.123\%.$$

(The second error is smaller, since the curvature of the parabola is less.)

Solution of Problem 5. Since the distance covered is a quadratic function of time, velocity must be a linear function of time. In general, if the distance covered can be written in the form of

$$s = \frac{1}{2}a_t t^2 + v_0 t,$$

then

$$v = v_0 + a_t t,$$

where v_0 is the initial velocity, a_t is the tangential acceleration and t is the time elapsed. Comparing the parametric equation with the one given in this case:

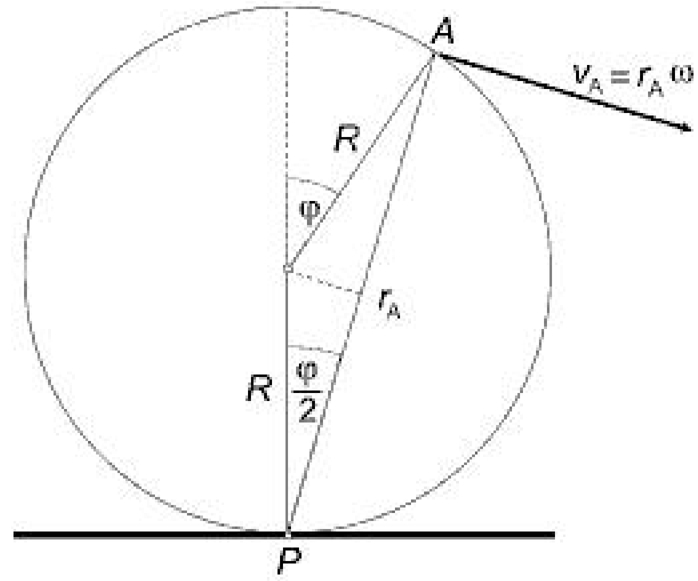
$$s = 0.5t^2 + 2t$$

it follows that $a_t/2 = 0.5\,\text{m/s}^2$ so $a_t = 1\,\text{m/s}^2$ and $v_0 = 2\,\text{m/s}$.

The accelerations at times $t_1 = 2$ s and $t_2 = 5$ s can be calculated as the resultant vector of the tangential and normal (centripetal) accelerations, whose magnitude is:

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{a_t^2 + \left(\frac{v^2}{R}\right)^2},$$

where $v = v_0 + a_t t$.



Second solution of Problem 7. *With an instantaneous axis of rotation.*

The figure shows that the instantaneous axis of rotation is the point P where the disc touches the ground. The speed of point A is thus

$$v_A = r_A \omega,$$

where

$$r_A = 2R\cos\left(\frac{\varphi}{2}\right) \qquad \text{and} \qquad \omega = \frac{v_0}{R}.$$

The expression (2) is obtained again by substituting $\varphi = \omega t = \frac{v_0}{R} t$:

$$v_A = 2R\cos\left(\frac{v_0}{2R}\cdot t\right)\cdot\frac{v_0}{R} = 2v_0\cos\left(\frac{v_0}{2R}\cdot t\right),$$

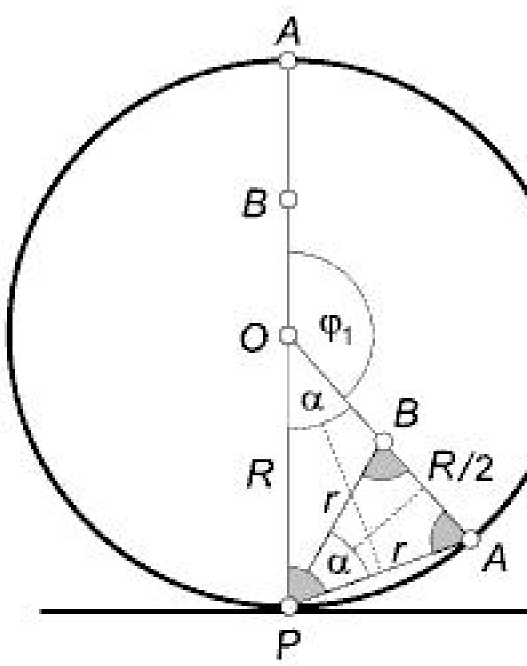
and hence the requirement of the problem leads to the same result as above.

Solution of Problem 8. a) According to the condition $v_A = v_B$. Since the magnitude of the velocity for a point on the circumference $0 \le v_A \le 2v$, there will be momentary instants when it is equal to v_B, which is always greater than zero and always less than $2v$ (its direction will obviously be different). This requirement is fulfilled when $r_A\omega = r_B\omega$, where r_A and r_B are the momentary radii of rotation drawn to A and B respectively, which are the distances of A resp. B measured from the point that is stationary at the moment (the point that is exactly touching the ground). Since the angular speed is the same for each point of a rigid disc, in order to fulfil the condition

$$r_A = r_B = r$$

should be true.

From the figure it can be seen where the points should be to fulfil this condition. (From the figure it is clear that this state is reached in the second quarter of the angular displacement first.) Triangles OAP and PAB are similar because they are isosceles and their angles on the base are common.



Let φ stand for the angular displacement of the radius belonging to point A and α for its supplementary angle. The momentary radius of rotation from the big triangle is

$$r = 2R\sin\frac{\alpha}{2}.$$

and from the small triangle

$$r = \frac{R}{4\sin\frac{\alpha}{2}}.$$

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The change of the kinetic energy is equal to the work done by this force:

$$\frac{ma_0}{2}H = \frac{1}{2}mv^2,$$

thus, the speed at height H is:

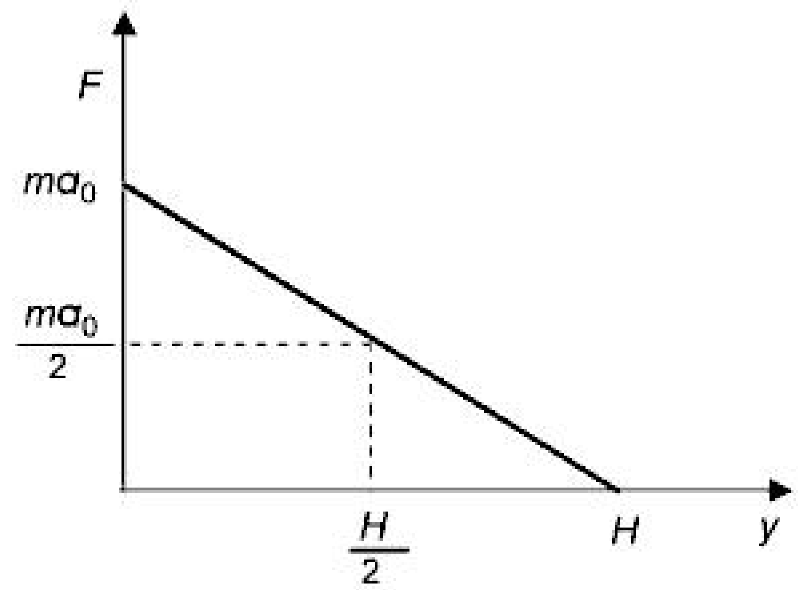
$$v(H) = \sqrt{a_0 H}.$$

From ground level to $H/2$ the average force is:

$$\frac{ma_0 + \frac{ma_0}{2}}{2} = 0.75 ma_0,$$

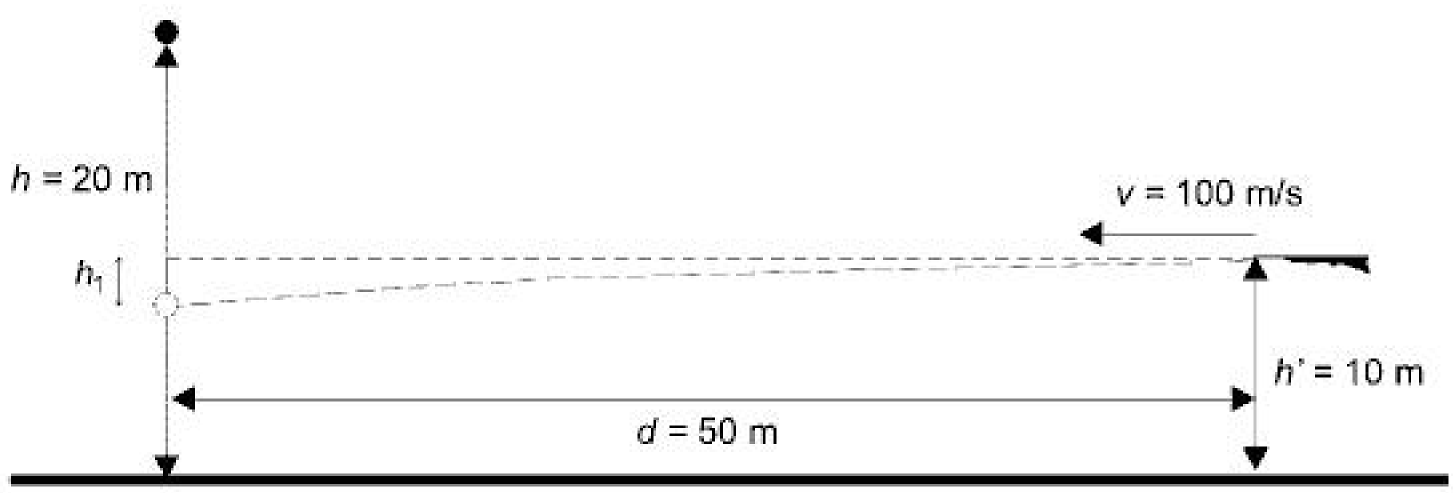
so the speed at height $H/2$ is

$$v = \sqrt{0.75 a_0 H}.$$



The simplest, most elementary way to determine the time needed for the balloon to rise is to apply the analogy to the harmonic oscillation.

Solution of Problem 11. First we have to determine the time it takes the bullet, moving at a constant horizontal velocity component, to arrive at the trajectory of the falling ball. During this time, in a vertical direction, the bullet performs a free fall and loses its height of h_1. However, because the bullet hits the ball, the ball covers the distance $(h-h')+h_1$. The time in question is therefore simply the difference between the time of the free fall of the ball and the time of the flight of the bullet.



It takes the bullet

$$t_1 = \frac{d}{v} = \frac{50 \text{ m}}{100 \ \frac{\text{m}}{\text{s}}} = 0.5 \text{ s}$$

to reach the trajectory of the falling ball. During this time it loses a height of

$$h_1 = \frac{1}{2} g \cdot t_1^2 = \frac{1}{2} \cdot 9.81 \ \frac{\text{m}}{\text{s}^2} \cdot 0.25 \text{ s}^2 \approx 1.23 \text{ m}.$$

The time of the free fall of the ball, until being hit by the bullet, is:

$$t = \sqrt{\frac{2(h-h'+h_1)}{g}} = \sqrt{\frac{2 \cdot 11.23 \text{ m}}{9.81 \ \frac{\text{m}}{\text{s}^2}}} = 1.51 \text{ s}.$$

The solution of the equation gives the time between the projection and the explosion: $t_1 = 12.087\,\mathrm{s}$.

Thus the height of the explosion is:

$$h_1 = v_{0y}t_1 - \frac{1}{2}gt_1^2 = 200\,\frac{\mathrm{m}}{\mathrm{s}} \cdot 12.087\,\mathrm{s} - \frac{1}{2} \cdot 10\,\frac{\mathrm{m}}{\mathrm{s}^2} \cdot 12.087^2\ \mathrm{s}^2 = 1686.9\,\mathrm{m}.$$

Second solution of Problem 13. The problem can be easily solved if we use the fact that phenomena in mechanics are reversible. If the velocity of the impact was reflected, that is, if the splinter was projected with the velocity opposite to the the velocity at which the splinter hits the ground then the splinter, which falls back to the initial position, would move from the place of the projection to the place of the expolosion along the same trajectory upon which it moves when it falls back to the place of the projection from the position of the explosion. Thus the paths of the projectile, which does not explode, and the splinter which falls back, intersect each other at the place where the projectile explodes. Therefore, we only have to write the equations of both paths and to solve the equation system to y.

The equation of the path (of the centre of mass) of the original projectile is

$$y = \tan\alpha_1 \cdot x - \frac{g}{2v_0^2 \cos\alpha_1^2} \cdot x^2,$$

The equation of the path of the splinter which falls back to the place of the projection, and which is "projected back", is

$$y = \tan\alpha_2 \cdot x - \frac{g}{2v_{imp}^2 \cos\alpha_2^2} \cdot x^2,$$

where $\alpha_1 = 30°$, $\tan\alpha_2 = 200/150$, $\cos\alpha_2 = 150/250$.

The trivial solution of the equation system is $x = 0$, which is the initial point of both paths, and the solution which we would like to find when $x \neq 0$ can be calculated from the linear equation, which we gain if x is cancelled.

$$\tan\alpha_1 - \frac{g}{2v_0^2 \cos^2\alpha_1}x = \tan\alpha_2 - \frac{g}{2v_{\mathrm{imp}}^2 \cos^2\alpha_2}x.$$

Substituting the numerical data:

$$\tan 30° - \frac{1}{2 \cdot 400^2 \cos^2 30°}x = \frac{200}{150} - \frac{10}{2 \cdot 250^2 \left(\frac{150}{250}\right)^2}x.$$

The solution for x:

$$x = 4187\,\mathrm{m},$$

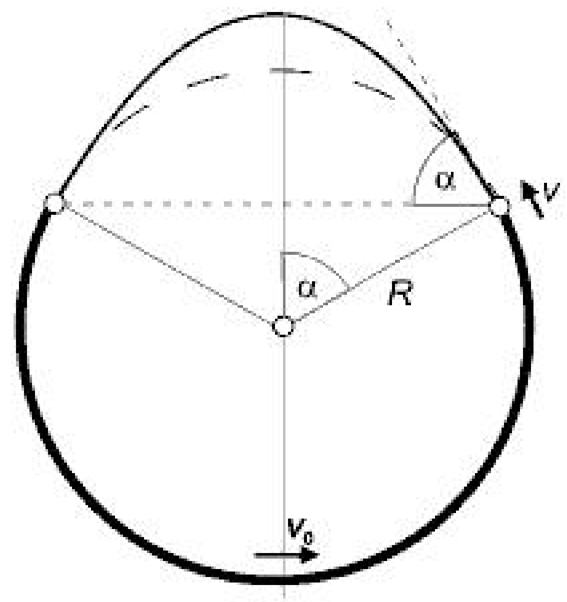
and substituting this result into the first equation the height at which the explosion occurred is:

$$h_1 = y_1 = \tan 30° \cdot 4187\,\mathrm{m} - \frac{10}{2 \cdot 400^2 \cos^2 30°} \cdot 4187^2\,\mathrm{m} = 1686.9\,\mathrm{m}.$$

(The velocity of the splinter which moves forward and the ratio of the masses of the splinters cannot be calculated from the given data.)

Solution of Problem 17. Let us describe the instantaneous position of the object with the angle α between the vertical and the radius drawn to the small object. Where the wall ends, the object undergoes projectile motion with an initial angle of α as well. The omission of some part of the wall does not lead to the failure of the trick if the downward part of the parabola smoothly fits to the circle again.

From this, it is derived that the missing part of the circular track must be symmetrical along the vertical diameter of the circle. This condition can be considered as the horizontal component of the displacement of the object during the time of the ascent of the object (half of the range of the projectile motion) and is the same as the half of the chord which belongs to the arc cut off the circle. Let us determine the central angle subtended by the arc cut off. Let the magnitude of this angle be 2α.



The time of the ascent of the projectile motion is:

$$t_a = \frac{v \sin\alpha}{g}.$$

The distance covered during this time is:

$$v\cos\alpha \cdot t_a = \frac{v\cos\alpha \cdot v\sin\alpha}{g} = R\sin\alpha$$

From this $\cos\alpha = \dfrac{Rg}{v^2}$.

The initial speed of the projectile motion can be calculated using the work-energy theorem:

$$-mgR(1+\cos\alpha) = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2,$$

from which: $v^2 = v_0^2 - 2gR(1+\cos\alpha)$, and $\cos\alpha = \dfrac{Rg}{v_0^2 - 2Rg(1+\cos\alpha)}$.

This is a quadratic equation for the cosine of half of the asked angle:

$$\cos^2\alpha + \left(1 - \frac{v_0^2}{2Rg}\right)\cos\alpha + \frac{1}{2} = 0.$$

In our case $\dfrac{v_0^2}{2Rg} = \dfrac{400}{2\cdot 8.16\cdot 9.8} = 2.5$, which can be substituted into our equation, such that:

$$\cos^2\alpha - 1.5\cos\alpha + \frac{1}{2} = 0,$$

from which 2 solutions are gained for $\cos\alpha$:

$$\cos\alpha_1 = 1, \qquad \text{and} \qquad \cos\alpha_2 = 0.5.$$

which yields

$$v^2 = 2gR(1-\cos\alpha).$$

Substituting this into the formula for acceleration and assuming that the acceleration should be two thirds of g, we get:

$$a = g\sqrt{\sin^2\alpha + 4(1-\cos\alpha)^2} = \frac{2}{3}\cdot g.$$

after some algebra, we get a quadratic equation for $\cos\alpha$:

$$27\cos^2\alpha - 72\cos\alpha + 41 = 0,$$

the only solution to the equation that has a physical meaning is:

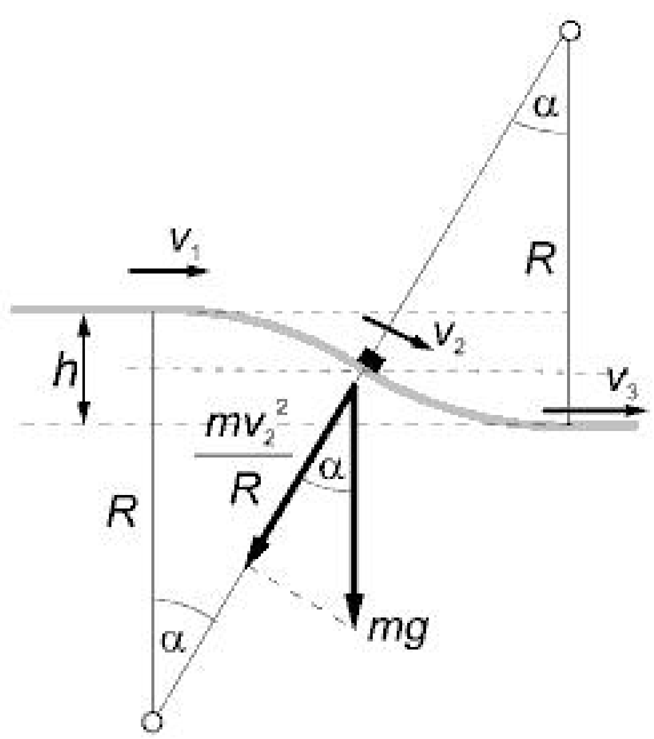
$$\cos\alpha = \frac{12-\sqrt{21}}{9} = 0.8242, \qquad \rightarrow \qquad \alpha = 34.5°.$$

The net acceleration and the tangent line form an angle φ, for which we get:

$$\cos\varphi = \frac{g\sin\alpha}{2g/3} = 1.5\sin\alpha = 0.8496, \quad \rightarrow \quad \varphi = 31.8°.$$

The angle enclosed by the acceleration and the horizontal is $\varepsilon = \varphi + \alpha = 66.3°$.

Solution of Problem 20. The object leaves the slope when the normal reaction becomes zero. In our case this may happen along the part of the path which is concave downward, with the chance of it happening being greatest at the lowest points. Let us assume that the object is a point-like one. The critical position is the inflexion point of the path, so if the normal reaction becomes zero there for a moment, then it will increase abruptly to a high value because of the opposite curvature. Let us consider this case as the critical case, because if the object reaches this point at a greater speed than the speed which belongs to the critical case then it will surely leave a finite segment of the path.



The question can be stated in the following way as well: What should the initial speed of an object which slides down on a smooth circular path of radius R be, so that it it leaves the path just as it descends from a height of $h/2$?

Let the speed of the object at the top of the path be v_1, and v_2 after descending a height of $h/2$. During the motion of the object the resultant force is the vector sum of the normal reaction and the gravitational force. At the moment when the normal reaction ceases the resultant force is equal too the gravitational force. At this moment

The bob will reach its minimum acceleration when the expression under the square root takes its minimum value. This expression is the second degree function of $\cos\alpha$:

$$3\cos^2\alpha - 8\cos\alpha_0\cos\alpha + (1+4\cos^2\alpha_0). \tag{3}$$

The general form of a quadratic function of x is:

$$ax^2+bx+c.$$

By calculating the zeros of this function, you will find the x-intercepts of a parabola, with an axis parallel to the y-axis. These intercepts are symmetrical to the axis (and to the vertex) of the parabola, therefore the x-coordinate of the minimum point of the parabola is the arithmetic mean of the two zeros:

$$x_{\min} = (x_1+x_2)/2,$$

where

$$x_1 = \frac{-b+\sqrt{b^2-4ac}}{2a}, \qquad \text{and} \qquad x_2 = \frac{-b-\sqrt{b^2-4ac}}{2a}.$$

Calculating their arithmetic mean gives:

$$x_{\min} = -\frac{b}{2a}.$$

Let us apply this to the second degree expression of $\cos\alpha$. The angle at which the bob reaches its minimum acceleration is given by:

$$\cos\alpha_{\min} = \frac{8\cos\alpha_0}{6} = \frac{4}{3}\cos\alpha_0. \tag{5}$$

Using this calculation in our case the initial angle is $\alpha_0 = 45°$ and the minimum acceleration is reached when:

$$\cos\alpha_{\min} = \frac{4}{3}\cdot\frac{\sqrt{2}}{2} = \frac{2\sqrt{2}}{3} = 0.9428$$

from which the angle formed by the cord and the vertical is:

$$\alpha_{\min} = 19.47°.$$

The question set in the problem is therefore answered.

Let us calculate some additional data regarding the minimum acceleration. Substituting equation (5) for equation (2), we arrive at a minimum acceleration of:

$$a_{\min} = g\cdot\sqrt{1+3\cdot\frac{16\cos^2\alpha_0}{9} - 8\cdot\frac{4\cos^2\alpha_0}{3} + 4\cos^2\alpha_0} =$$

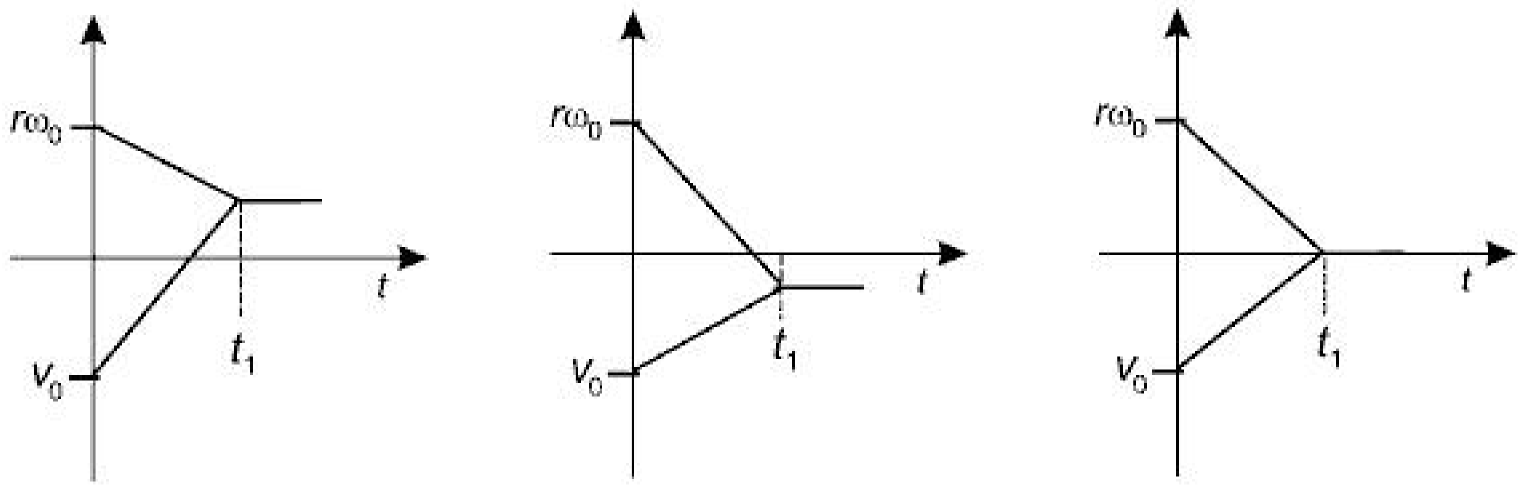
$$= \frac{g}{9}\cdot\sqrt{(48-96+36)\cos_0^2+9} = \frac{g}{9}\cdot\sqrt{9-12\cos^2\alpha}. \tag{6}$$

In the absence of friction (since there is no external torque acting on the hoop), the angular momentum of the hoop, and thus the angular speed too, will remain constant all the way. The hoop will slide up to a height of

$$h = v_0^2/2g,$$

where its speed is reduced to zero, and where it will then slide back to the base of the slope. It arrives at the base of the slope at a velocity of v_0 again, but in the opposite direction. Its sense of rotation does not change. Therefore, the hoop arriving back to the horizontal plane keeps sliding, and will continue to slide until the torque of the friction forces adjust the angular velocity to a value of $\omega = v/r$ 'in tune' with the instantaneous velocity and in the appropriate direction. In principle, this may occur in three different ways. Friction may reverse the sense of rotation, it may reverse the direction of velocity, or, in a special case it may reduce both to zero at the same time. The result will depend on the magnitude of rotational inertia. To solve the problem, this question needs to be investigated.

The diagrams below represent the three cases graphically. ($t = 0$ at the instant when the hoop arrives back down to the base of the slope.)



Calculations:
The acceleration of the centre of mass of the hoop after it has slid back down on the horizontal plane:

$$a = \mu g = \text{const.}$$

The angular acceleration of the hoop:

$$\beta = -\frac{\mu m g r}{m r^2} = -\frac{\mu g}{r} = \text{const.}$$

The speed and angular speed of the centre of the hoop expressed in terms of time:

$$v = -v_0 + at = -v_0 + \mu g t, \qquad \text{and} \qquad \omega = \omega_0 + \beta t = \omega_0 - \frac{\mu g}{r} t.$$

Sliding stops in a time t_1, such that

$$-v_0 + \mu g t_1 = \omega_0 r - \mu g t_1,$$

hence,

$$2\mu g t_1 = 2v_0, \qquad \text{and} \qquad t_1 = \frac{v_0}{\mu g} = \frac{3.5\,\mathrm{m/s}}{0.2 \cdot 10\,\mathrm{m/s}^2} = 1.75\,\mathrm{s}.$$

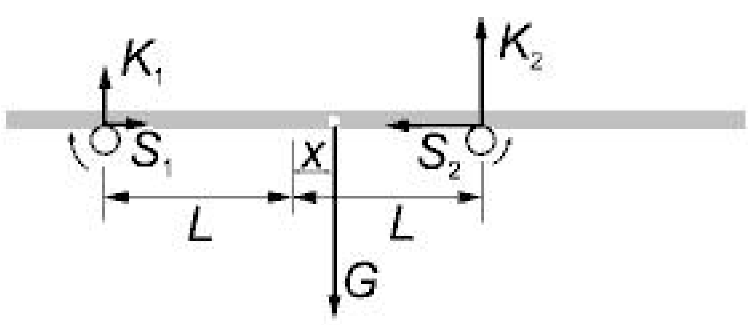
(Any angle α in the interval

$$\alpha_2 = \arcsin 0.33004 = 19.27^\circ < \alpha < 82.038^\circ = \alpha_1,$$

would satisfy the condition of sliding, provided that the block is held fixed until the string reaches that angle. However, under the present conditions the block will start to slide at 82.038°, so any other angle is irrelevant for answering the question.)

Remark. The line of action of the tension force in the string does not pass through the centre of mass of the block, thus it also represents a torque about the centre of mass. As a result, the 'front' of the block is pressed harder against the ground than its rear end, and the value of the static friction force may become uncertain. However, since the solution is independent of the length of the pendulum and the only variable that the torque depends on in addition to the angle and the magnitude of the force is the height h of the rod, it can be made negligibly small. If the block is long enough relative to the rod, the effect of the torque becomes very small. (Note that the size of the ball imposes a limitation on shortening the string: the diameter of the ball must remain negligible relative to the length of the string, otherwise rotational kinetic energy should be considered in the work-energy theorem, too.)

Solution of Problem 28. The batten does not accelerate in the vertical direction, so Newton's second law for this direction is:



$$G - K_1 - K_2 = 0,$$

where G is the gravitational force exerted on the batten, K_1 and K_2 are the absolute values of the normal reactions exerted by the cylinders.

The batten does not rotate, thus the equation for the torques calculated for an axis through the centre of mass is:

$$K_1(L + x) - K_2(L - x) = 0.$$

Based on the above equations, the magnitudes of the normal reactions are:

$$K_1 = \frac{G(L - x)}{2L}, \qquad \text{and} \qquad K_2 = \frac{G(L + x)}{2L}.$$

Because, according to the problem, the cylinders rotate quickly, the relative speed of the batten and the surface of the cylinders is not zero, causing dynamic friction to be exerted between them and giving them each magnitudes of:

$$S_1 = \mu K_1, \qquad \text{and} \qquad S_2 = \mu K_2,$$

and although they are opposite, both are exerted towards the centre of the batten. Newton's second law for the horizontal forces is:

$$\mu K_2 - \mu K_1 = ma.$$

Let us make use of the fact that the rotational inertia of a thin-walled cylinder for its axis is $\Theta_{\mathrm{cm}} = mr^2$:

$$F_{f_2} = \frac{mr^2 - mr^2}{mr^2 + mr^2} F_{f_1} = 0,$$

meaning there is no friction force between the table and the cylinders, and the minimum coefficient of friction required is $\mu = 0$. So in our case the cylinders roll without skidding even on a completely smooth surface.

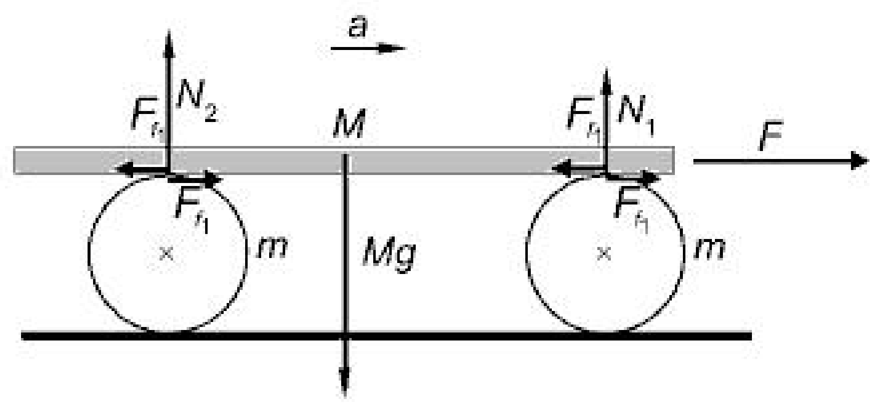
Let us turn to the relationship between the cylinders and the rod now. The rod also moves on the cylinders without skidding, meaning the velocity and the acceleration of the topmost points of the cylinders are the same as those of the rod. But if there is no friction on the ground, the friction forces acting on the points that are touching the rod should be the same for both cylinders, considering this is the only way their accelerations can be the same. This is true despite the fact that as a result of the motion of the rod the normal forces N_1 and N_2 acting between the rod and the cylinders change continuously. The magnitude of N_1 starts from $Mg/4$ and its maximum value is $3Mg/4$ at the end of the process while for N_2 the opposite holds: it decreases from $3Mg/4$ to $Mg/4$, while $N_1 + N_2 = Mg$ is always true. Obviously, this can only be true if the coefficient of static friction between the cylinders and the rod is big enough to provide the relevant friction force F_{f_1} even in the case of the minimum normal force.

The equation of the motion of the rod is

$$F - 2F_{f_1} = Ma. \tag{7}$$

The law of the motion of the centre of mass for either cylinder is

$$a_{\mathrm{cm}} = \frac{F_{f_1}}{m}.$$



The rolling on both surfaces without skidding requires that $a_{\mathrm{cm}} = a/2$ holds. So

$$\frac{a}{2} = \frac{F_{f_1}}{m},$$

that is, $2F_{f_1} = ma$. Substituting this into (7) gives

$$F - ma = Ma,$$

that is,

$$a = \frac{F}{m+M} \quad \left(= 3\,\frac{\mathrm{m}}{\mathrm{s}^2}\right), \qquad \text{and} \qquad S = \frac{m}{2(m+M)} F.$$

With the given numerical values

$$F_{f_1} = \frac{12\,\mathrm{N} \cdot 1\,\mathrm{kg}}{2(1\,\mathrm{kg} + 3\,\mathrm{kg})} = 1.5\,\mathrm{N}.$$

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From this

$$x=\frac{v_0^2-(5\pi\mu\cos\alpha+2\sin\alpha)Rg}{2(\sin\alpha+\mu\cos\alpha)g}=$$

$$=\frac{3.88^2-(5\pi\cdot 0.2\cdot\cos 30^\circ+2\cdot\sin 30^\circ)\cdot 0.2\cdot 9.81}{2\ (\sin 30^\circ+0.2\cos 30^\circ)\cdot 9.81}\,\mathrm{m}\approx 0.59\,\mathrm{m}.$$

(Those who calculated with $g=10\,\mathrm{m/s}^2$, acquired $v_0=3.87\,\mathrm{m/s}$ for the initial velocity, $\mu=0.18$ for the coefficient of friction and $x=0.62$ metres for the distance covered.)

Solution of Problem 35. Assume the ball is a pointmass as suggested by the figure. According to the work-energy theorem, the kinetic energy of the ball in point A equals the sum of the work done by the gravitational force and the spring:

$$mgs+\frac{1}{2}D(\Delta l)^2=\frac{1}{2}mv^2,$$

where $\Delta l=x-L$, x and L are the extended and relaxed lengths of the spring. We used the notion that in its vertical position the spring will reach its relaxed length. Length x is provided by the Pythagorean theorem:

$$x=\sqrt{\overline{OC}^2-\overline{OB}^2}=\sqrt{0.8\,\mathrm{m}^2-0.48\,\mathrm{m}^2}=\sqrt{0.4096\,\mathrm{m}^2}=0.64\,\mathrm{m}.$$

The vertical displacement of the ball is $s=OA-OB=0.8\,\mathrm{m}-0.48\,\mathrm{m}=0.32\,\mathrm{m}$. Using this, the speed turns out to be:

$$v=\sqrt{2gs+\frac{D}{m}(x-L)^2},$$

substituting known values, we find:

$$v=\sqrt{2\cdot 10\,\frac{\mathrm{m}}{\mathrm{s}^2}\cdot 0.32\,\mathrm{m}+\frac{75\,\mathrm{N/m}}{3.2\,\mathrm{kg}}(0.64\,\mathrm{m}-0.32\,\mathrm{m})^2}=2.97\,\frac{\mathrm{m}}{\mathrm{s}}.$$

Let us now determine the force exerted on the shell by the ball. At point A the only forces acting on the ball are the gravitational and normal forces, since the spring is in its relaxed state at that moment. Applying Newton's second law to the ball at that point, we obtain:

$$\vec{K}+\vec{G}=m\vec{a},$$

With the positive direction pointing upwards, we have:

$$K-mg=m\frac{v^2}{r},$$

from which the normal force (K) is:

$$K=mg+m\frac{v^2}{r}=3.2\,\mathrm{kg}\cdot 10\,\frac{\mathrm{m}}{\mathrm{s}^2}+3.2\,\mathrm{kg}\cdot\frac{2.97^2\,\mathrm{m}^2/\,\mathrm{s}^2}{0.8\,\mathrm{m}}=67.3\,\mathrm{N}.$$

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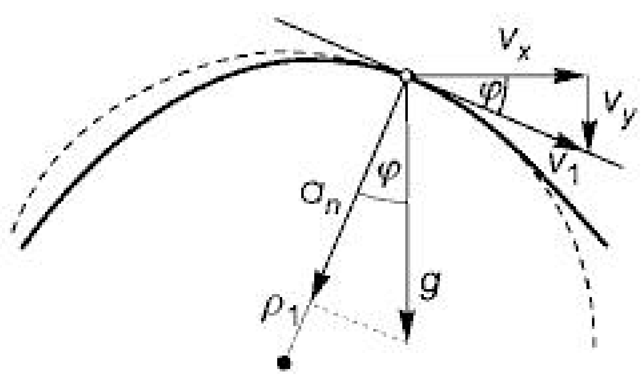
Because at the highest point of the path the velocity of the stone is horizontal, its normal acceleration is the same as acceleration itself (which is g throughout the motion), therefore

$$a_n = g = \frac{v_x^2}{\varrho} \quad \rightarrow \quad \varrho = \frac{v_x^2}{g} = 250\,\text{m}.$$

With this the unknown normal force is

$$N = m\left(g - \frac{v_{\text{car}}^2}{\varrho}\right) = 1000\ \text{kg}\left(10 - \frac{400}{250}\right)\frac{\text{m}}{\text{s}^2} = 8400\,\text{N}.$$

b) The radius of curvature at the parabola point that belongs to 3/4 of the horizontal distance can be determined from the fact that the horizontal velocity component of the thrown stone is constant, therefore 3/4 of the time of the projectile flight belongs to 3/4 of the horizontal distance:



$$t_1 = \frac{3}{4}t_{\text{proj}} = \frac{3}{4}\cdot\frac{2v_0\sin\alpha}{g} = \frac{3}{2}\frac{v_0\sin\alpha}{g}.$$

The normal acceleration of the stone at this point of the path is

$$a_n = g\cos\varphi = \frac{v_1^2}{\varrho_1},$$

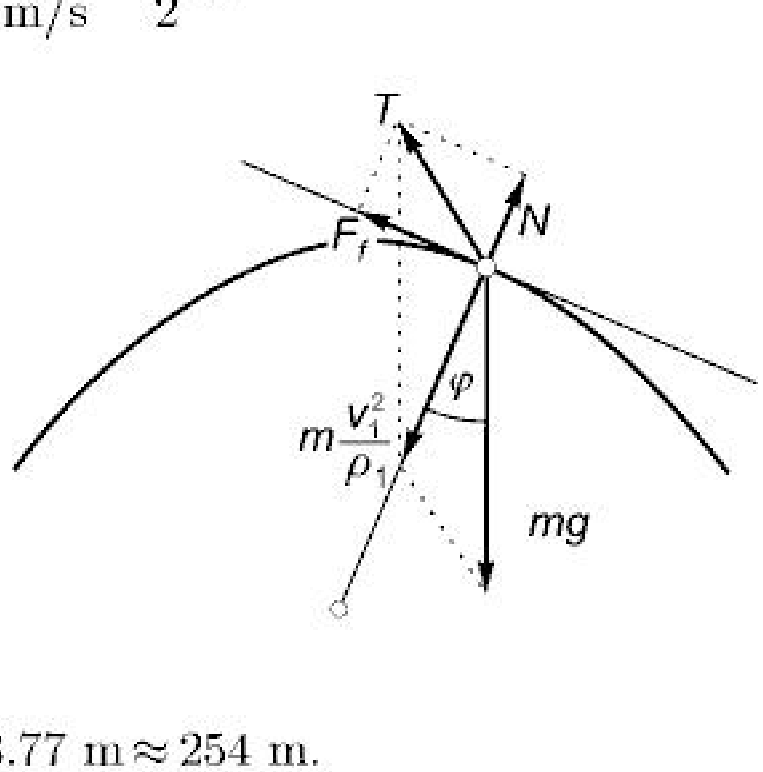
and according to the figure, the radius of curvature belonging to this point is

$$\varrho_1 = \frac{v_1^2}{g\cos\varphi} = \frac{v_1^2}{g\cdot\frac{v_x}{v_1}} = \frac{v_1^3}{gv_x},$$

where v_1 is the instantaneous velocity of the stone at the point in concern at time instant t_1. The time in concern is

$$t_1 = \frac{x_1}{v_x} = \frac{3}{4}\frac{d}{v_x} = \frac{300\ \text{m}}{4\cdot 50\ \text{m/s}} = \frac{3}{2}\ \text{s}.$$

With this, the square of the velocity of the stone at the point in concern is



$$v_1^2 = v_x^2 + v_y^2 = v_0^2\cos^2\alpha + (v_0\sin\alpha - gt_1)^2 =$$
$$= [50^2 + (10-15)^2]\frac{\text{m}^2}{\text{s}^2} = 2525\ \frac{\text{m}^2}{\text{s}^2},$$

and its instantaneous velocity is

$$v_1 = \sqrt{2525}\ \text{m/s} \approx 50.25\ \text{m/s}.$$

Therefore, the unknown radius of curvature is

$$\varrho_1 = \frac{v_1^3}{gv_x} = \frac{50.25^3}{10\cdot 50}\ \text{m} = 253.77\ \text{m} \approx 254\ \text{m}.$$

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Although it was not required in the problem, we deduced the equation of the orbit of the ball B. Because of the similarity of the two right triangles in the figure:

$$\frac{x}{\frac{L}{2}} = \frac{\sqrt{L^2 - y^2}}{L}$$

from which:

$$4x^2 + y^2 = L^2,$$

follows, which can also be written in the form

$$\frac{4x^2}{L^2} + \frac{y^2}{L^2} = 1, \text{ or } \frac{x^2}{(L/2)^2} + \frac{y^2}{L^2} = 1.$$

This means that the orbit is an ellipse with semimajor axis $a = L/2$ and semiminor axis $b = L$:

$$\frac{4x^2}{L^2} + \frac{y^2}{L^2} = 1, \text{ or } \frac{x^2}{(0.8\text{ m})^2} + \frac{y^2}{(1.6\text{ m})^2} = 1.$$

Solution of Problem 40. First, let us examine whether or not the marble can roll along the path at all (assuming that it rolls without sliding). In order for it to do this, it is necessary that the speed of its centre of mass at the top of the circular path exceeds the critical speed v_{crit} for which:

$$mg + K = ma_n = m\frac{v^2}{R-r},$$

where the normal reaction K becomes zero, making the value of the critical speed:

$$v_{\text{crit}} = \sqrt{(R-r)g}$$

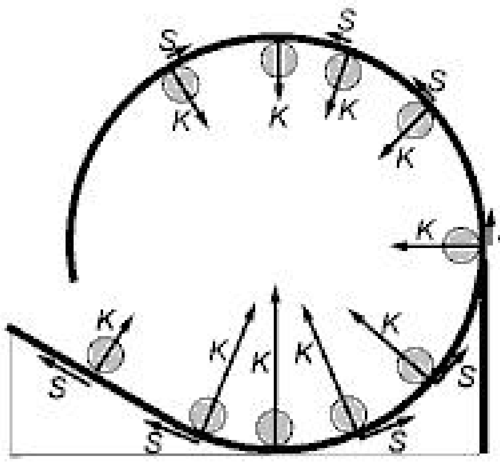
In order to answer this, we have to apply the work-energy theorem:

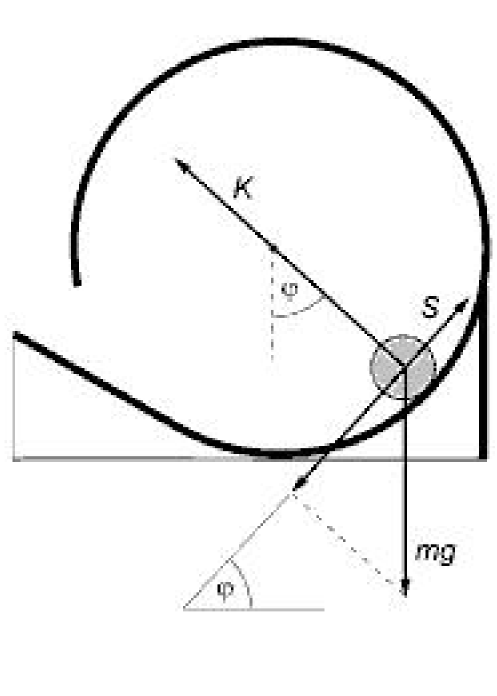
$$mg[3R - (2R - r)] = \frac{1}{2}mv^2 + \frac{1}{2}\cdot\frac{2}{5}mr^2\omega^2,$$

from this we can express the speed, using the work-energy theorem, because the marble rolls without sliding $r\omega = v$, thus:

$$v^2 = \frac{10}{7}g(R+r) > (R-r)g = v_{\text{crit}}^2,$$

which is definitely greater than the critical speed, meaning that even at the topmost point the normal reaction is not zero.





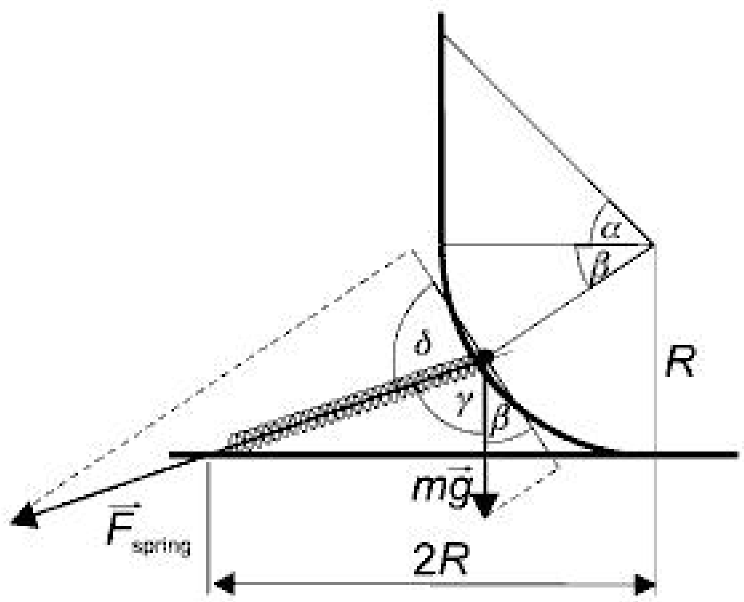
only where their sum is zero. Only gravitational force and spring force may have components parallel with the track (as the ball slides without friction, the force exerted by the track is perpendicular to the track), so at the place where speed has a maximum, the absolute values of the parallel components of gravitational force and spring force are equal:

$$F_{\text{spring}}\cos\delta = mg\cos\beta.$$

From this equation the mass of the ball can be determined if spring force and angle δ are known. The former can be determined from the elongation of the spring, the latter from the geometrical data of the arrangement. At the place where velocity is maximum, the length of the spring can be determined using the Pythagorean theorem:

$$l_2 = \sqrt{(R-R\sin\beta)^2+(2R-R\cos\beta)^2} = R\sqrt{(1-\sin\beta)^2+(2-\cos\beta)^2} =$$

$$= R\sqrt{6-2\sin\beta-4\cos\beta} = 0.2\,\text{m}\sqrt{6-2\sin 34^\circ-4\cos 34^\circ} = 25.02\,\text{cm}.$$



The elongation of the spring is

$$\Delta l_2 = l_2 - l_0 = 25.02\,\text{cm} - 20\,\text{cm} = 5.02\,\text{cm},$$

with it, the magnitude of spring force is

$$F_{\text{spring}} = R\Delta l = 100\frac{\text{N}}{\text{m}}\cdot 5.02\,\text{cm} = 5.02\,\text{N}.$$

Angle δ can be determined from angle γ shown in the figure:

$$\cos\gamma = \frac{R(1-\sin\beta)}{l_2} = \frac{0.2\,\text{m}\cdot(1-\sin 34^\circ)}{0.25\,\text{m}} =$$
$$= 0.3523.$$

From this, $\gamma = 69.4^\circ$, so

$$\delta = 180^\circ - (\beta+\gamma) = 180^\circ - (34^\circ + 69.4^\circ) = 76.6^\circ.$$

With these the mass of the ball is

$$m = \frac{F_{\text{spring}}\cos\delta}{g\cos\beta} = \frac{5\,\text{N}\cdot\cos 76.6^\circ}{9.81\frac{\text{N}}{\text{kg}}\cdot\cos 34^\circ} = 143\,\text{g}.$$

b) The maximum speed of the ball can be determined from the work-kinetic energy theorem. The work done by the force exerted by the track is zero (because it is perpendicular to the track and therefore to velocity at any time), the kinetic energy of the ball is given by the work of the gravitational force and of the spring force:

$$mg(h_1-h_2) + \frac{1}{2}k\cdot\left[(\Delta l_2)^2 - (\Delta l_1)^2\right] = \frac{1}{2}mv^2.$$

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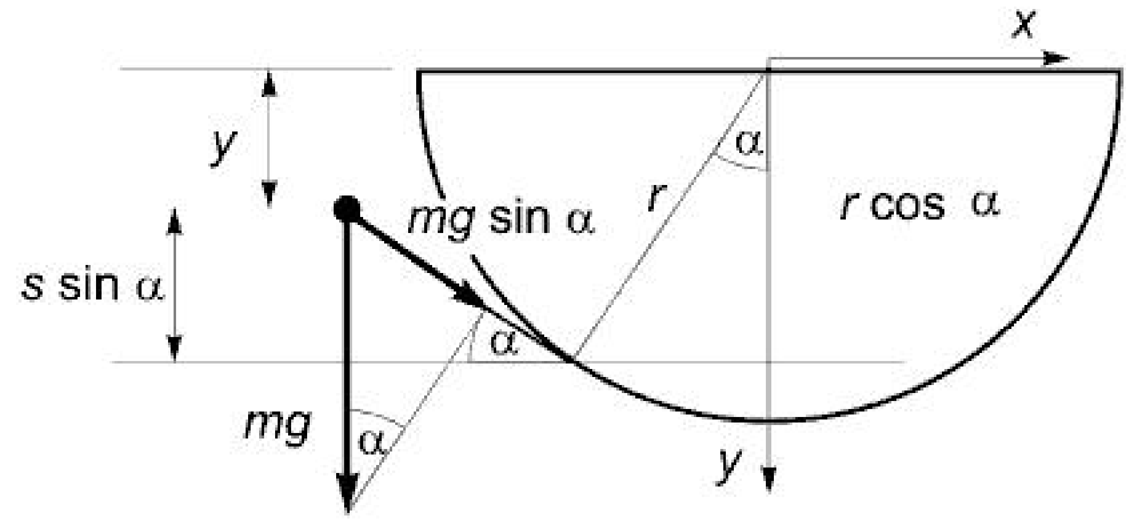
always in the line of the normal to the trajectory. The normal force is provided by the combined action of the string and the force of gravity. At the position of the object where the normal component of the gravitational force alone is able to supply the centripetal force required, the string will become slack. While the free part of the string winding on the semi-cylinder is taut, it remains tangential to the cylindrical surface. Notice that when the string becomes slack, the part that has already wound up on the cylinder is longer than a quarter circle, that is, the object attached to the end has risen higher than the lowermost point of the semi-cylinder. It is only during the rising part of the motion that the force of gravity has a component that can pull the object along the direction of the string (that is, towards the point of suspension).



The position of the small object at the end of the string can be described in terms of the angle α enclosed by the free (straight) part of the string and the horizontal. The same angle is enclosed between the vertical and the radius drawn to the point where the line of the string touches the cylindrical surface. If the x axis of the coordinate frame is attached to the flat horizontal face of the semi-cylinder (in a direction perpendicular to the axis of the cylinder) and the y axis is set vertical, then the y coordinate of the point at the end of the string is

$$y = r \cdot \cos\alpha - s \cdot \sin\alpha. \tag{1}$$

The work-energy theorem can be used to express the instantaneous speed of the object in terms of y:

$$mgy = \frac{1}{2}mv^2.$$

Division by m and the substitution of y from (1) gives the following expression for the square of the speed:

$$v^2 = 2g(r \cdot \cos\alpha - s \cdot \sin\alpha). \tag{2}$$

At the time instant when the string becomes slack, the tension force in the string is zero, and the centripetal force required for moving in an orbit of instantaneous radius of curvature s is provided by the component of the gravitational force in the direction of the string:

$$mg \cdot \sin\alpha = m\frac{v^2}{s}.$$

Hence

$$s \cdot \sin\alpha = v^2/g,$$

and with the use of (2):

$$s \cdot \sin\alpha = 2(r\cos\alpha - s\sin\alpha).$$

Let F denote the force of the string, its vertical component being F_y, and let K be the normal force of the table. Based on the figure, Newton's second law can be used to set up equations for forces in the horizontal and vertical directions and for torques about the centre of the cube. With the notations of the figure,

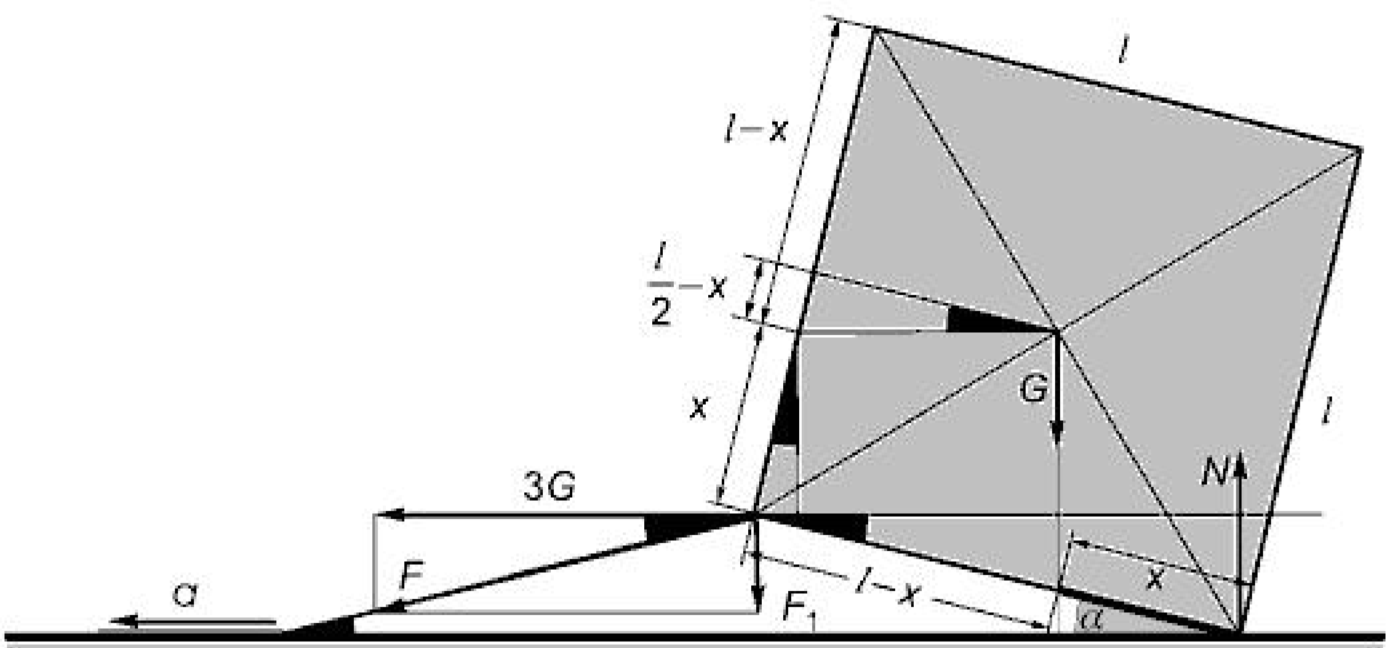
$$K - F_y - G = 0, \tag{1}$$

$$F_y \cot\alpha = 3G, \tag{2}$$

$$Kx\cos\alpha + F_y(l-x)\cos\alpha = 3Gx\cos\alpha. \tag{3}$$

Furthermore,

$$\frac{l}{2} - x = \frac{l}{2}\tan\alpha. \tag{4}$$



From (2) and (4),

$$F_y = 3G\left(l - \frac{2x}{l}\right). \tag{5}$$

From (1) and (3),

$$(F_y + G)x + f_y(l-x) - 3Gx = 0. \tag{6}$$

Then the magnitude of the vertical component of the string force can be obtained from (5) and (6):

$$F_y = \frac{3}{4}G.$$

The force pressing on the ground is equal in magnitude to K:

$$K = F_y + G = \frac{7}{4}mg = 140\,\mathrm{N}.$$

Thus the force exerted by the string is

$$F = \sqrt{F_y^2 + (3G)^2} = \sqrt{\frac{9}{16}G^2 + 9G^2} = \frac{3}{4}\sqrt{17}G = \frac{3}{4}\sqrt{17}\cdot 80\,\mathrm{N} = 247.4\,\mathrm{N}.$$

The base of the cube is lifted through an angle of

$$\alpha = \arctan\frac{F_y}{3G} = \arctan\frac{1}{4} = 14^\circ$$

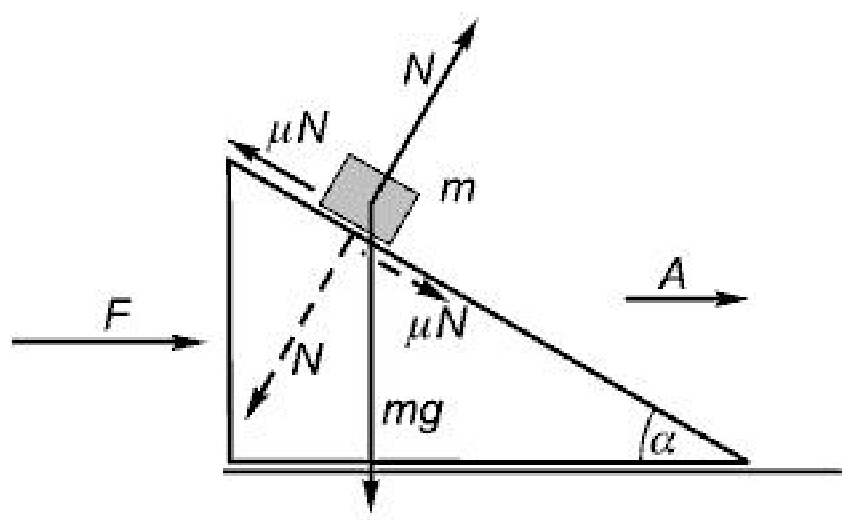
substituting known values, we find:

$$0.19-0.323\mu \leq x \leq 0.19+0.323\mu.$$

If the block is not simply placed onto the inclined plane in an equilibrium position, but reaches its final position from a state of motion as it does in our case, the exact place where it stops inside the above interval can also be determined, but that is a more difficult task to do.

Solution of Problem 50. The vertical displacement of the body that slides down the slope is the same in both cases, but in the case of a moving slope it requires twice as much time.

According to the quadratic distance relationship, the vertical acceleration component of the body is four times as much in the case of the stationary slope as in the case of the moving slope because for the distances travelled vertically in the two cases:



$$s_y=\frac{1}{2}a_{1y}t_1^2=\frac{1}{2}a_{2y}(2t_1)^2,$$

from which

$$a_{1y}=4a_{2y}.$$

Let us state the equation of the motion of the bodies, the constraining condition and the equation that expresses the special condition of the problem. Let us handle the horizontal and vertical components of the motion of the sliding body separately. As from here on the acceleration of the small body is used in the case of the moving slope, we will omit the indices 2. Using the notations of the figure, for the x component of the motion of the small body

$$N\sin\alpha-\mu N\cos\alpha=ma_x \qquad (1),$$

for the y component of the motion

$$mg-N\cos\alpha-\mu N\sin\alpha=ma_y \qquad (2),$$

for the wedge:

$$F-N\sin\alpha+\mu N\cos\alpha=MA \qquad (3)$$

the constraint condition:

$$a_y=(a_x-A)\tan\alpha \qquad (4)$$

the condition of the problem:

$$4a_y=g(\sin\alpha-\mu\cos\alpha)\sin\alpha \qquad (5).$$

Constraint condition (4) and condition (5) can be understood from the following figures:

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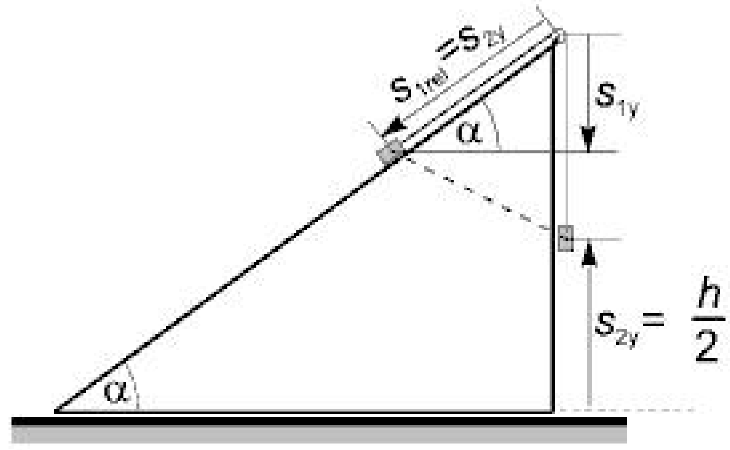
hence

$$a_{1y} = (a_{1x} + A)\tan\alpha,$$

as was stated in equation (6).

As the length of the cord remains constant, the displacements of the two blocks relative to the inclined plane must be equal: $s_{1\mathrm{rel}} = s_{2\mathrm{rel}}$. Using that $s_{1\mathrm{rel}} = s_{1y}/\sin\alpha$, we have

$$s_{2y} = \frac{s_{1y}}{\sin\alpha}.$$



Assuming again that the initial velocities are zero and the accelerations are constant, we find that:

$$a_{2y} = \frac{a_{1y}}{\sin\alpha},$$

as was stated in equation (7).

Let us now solve the system of equations. Multiplying equation (3) by $\cos\alpha$ and equation (4) by $\sin\alpha$, we obtain:

$$N\sin\alpha\cos\alpha - K\cos^2\alpha = m_1 a_{1x}\cos\alpha,$$

$$m_1 g\sin\alpha - N\cos\alpha\sin\alpha - K\sin^2\alpha = m_1 a_{1y}\sin\alpha.$$

After adding these two equations, we find that $N\sin\alpha\cos\alpha$ cancels out, and by factoring out K, its coefficient will be $\sin^2\alpha + \cos^2\alpha = 1$:

$$m_1 g\sin\alpha - K = m_1 a_{1x}\cos\alpha + m_1 a_{1y}\sin\alpha.$$

Let us solve equation (5) for K and substitute it into the above equation:

$$m_1 g\sin\alpha - m_2 a_{2y} - m_2 g = m_1 a_{1x}\cos\alpha + m_1 a_{1y}\sin\alpha. \tag{8}$$

The right hand sides of equations (2) and (3) are equal:

$$(M+m)A = m_1 a_{1x},$$

thus

$$A = \frac{m_1}{M+m_2} a_{1x},$$

inserting this into equation (6), we have:

$$a_{1y} = \left(a_{1x} + \frac{m_1}{M+m_2} a_{1x}\right)\tan\alpha = \frac{M+m_1+m_2}{M+m_2}\tan\alpha\cdot a_{1x},$$

which yields

$$a_{1x} = \frac{M+m_2}{M+m_1+m_2}\cdot\frac{a_{1y}}{\tan\alpha} = \frac{M+m_2}{M+m_1+m_2}\,\frac{\cos\alpha}{\sin\alpha}\cdot a_{1y}.$$

Let us now insert a_{1x} as expressed above and $a_{1y} = a_{2y}\sin\alpha$ from equation (7) into equation (8). The only unknown in this equation will be a_{2y}:

$$m_1 g\sin\alpha - m_2 a_{2y} - m_2 g = m_1 a_{2y}\sin\alpha\cdot\frac{M+m_2}{M+m_1+m_2}\cdot\frac{\cos^2\alpha}{\sin\alpha} + m_1 a_{2y}\sin^2\alpha.$$

Plugging this into equation (2), K can be expressed:

$$K = M\frac{a}{\tan\alpha\cdot\sin\alpha}. \tag{4}$$

Inserting this into the equation (1), we get:

$$mg - M\frac{a}{\tan\alpha\cdot\sin\alpha}\cdot\cos\alpha = ma.$$

Rearranging this, and using the identity $\cos\alpha/\sin\alpha = 1/\tan\alpha$, we obtain:

$$mg = \left(\frac{M}{\tan^2\alpha} + m\right)\cdot a,$$

from which the acceleration of the sphere is:

$$a = \frac{mg}{\frac{M}{\tan^2\alpha} + m} = \frac{g\tan^2\alpha}{\frac{M}{m} + \tan^2\alpha}. \tag{5}$$

According to the figure a), the condition assuring that the wedge does not tilt, expressed in terms of torques relative to the centre of mass, reads as follows:

$$\frac{2}{3}Kh\sin\alpha + \frac{1}{3}K\cos\alpha\frac{h}{\tan\alpha} \le (Mg + K\cos\alpha)\frac{h}{3\tan\alpha}.$$

Here, we have used the fact that in the marginal case the line of action of the force exerted by the ground on the wedge is shifted to the right edge of the wedge.

After simplifying this equation by $h/3$ and multiplying by $\tan\alpha$, we get:

$$2K\sin\alpha\cdot\tan\alpha + K\cos\alpha \le Mg + K\cos\alpha.$$

Omitting the same terms on both sides, and inserting the expression of K from equation (4), we get:

$$2M\frac{a}{\tan\alpha\cdot\sin\alpha}\sin\alpha\cdot\tan\alpha \le Mg,$$

so the maximal acceleration of the sphere is:

$$a \le \frac{g}{2}.$$

Putting this into (5) we obtain the desired mass ratio M/m:

$$a = \frac{g\tan^2\alpha}{\frac{M}{m} + \tan^2\alpha} \le \frac{g}{2}, \qquad \text{from which} \qquad 2\tan^2\alpha \le \frac{M}{m} + \tan^2\alpha,$$

and finally the relation

$$\frac{M}{m} \ge \tan^2\alpha$$

is obtained. Thus the ratio M/m and the angle α should satisfy the inequality above.

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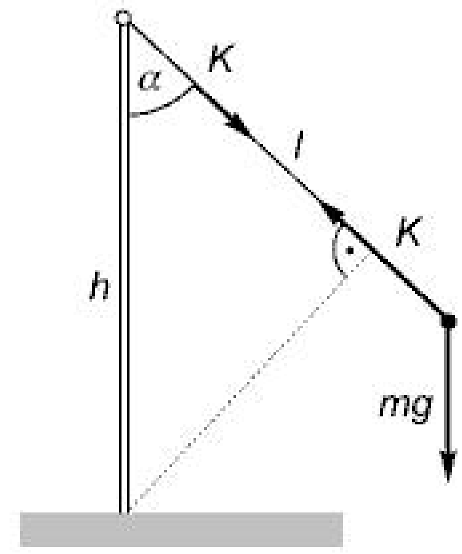
From (1) and (2):

$$K - mg\cos\alpha = 2mg(\cos\alpha - \cos\alpha_0),$$

and hence the tension K is

$$K = mg(3\cos\alpha - 2\cos\alpha_0).$$



The torque of the string is the only torque that the grip at the lower end needs to counteract: it must be able to exert an equal torque in the opposite direction. The figure shows that the torque of the tension force with respect to the lower end is

$$\tau = Kh\sin\alpha = mg((3\cos\alpha - 2\cos\alpha_0)\cdot h\cdot\sin\alpha.$$

Since the pendulum started from an initial position of $\alpha_0 = 90^\circ$, $\cos\alpha_0 = 0$ and the torque in question is

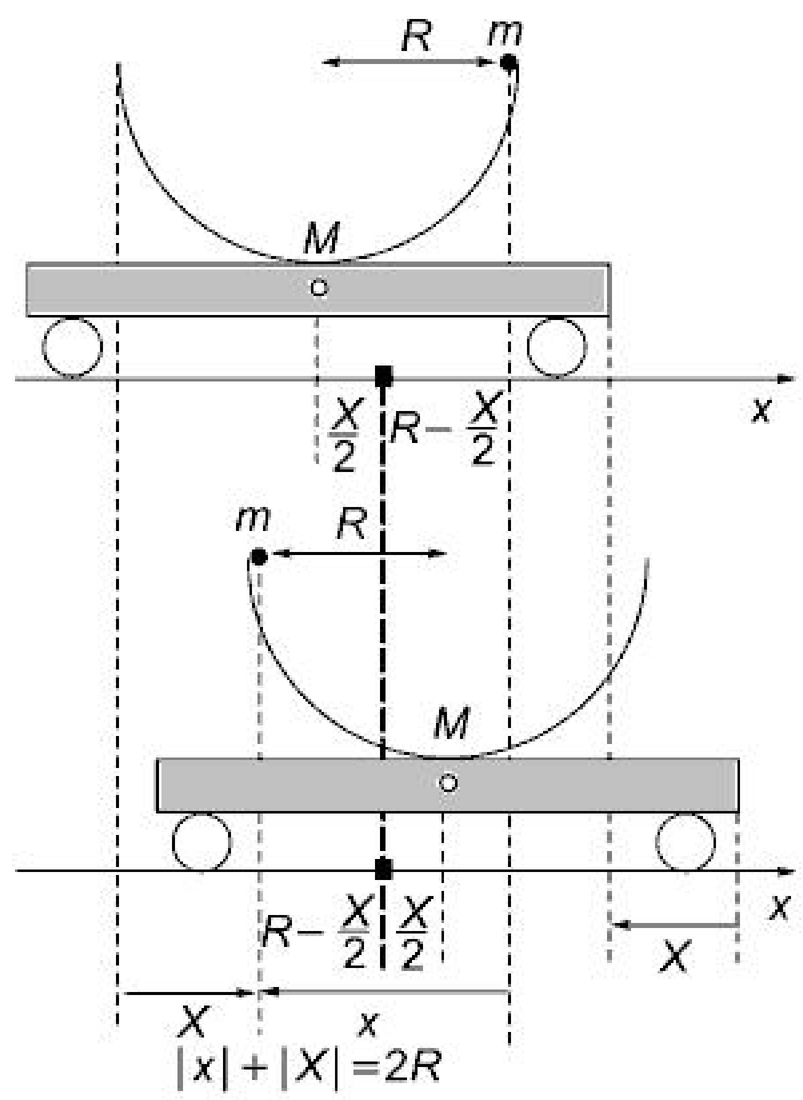
$$\tau = 3mgh\cdot\sin\alpha\cos\alpha = 3mgh\frac{\sin 2\alpha}{2}.$$

This torque is a maximum if $\sin 2\alpha$ is a maximum, that is, at $2\alpha = 90^\circ$, which means $\alpha = 45^\circ$. Thus

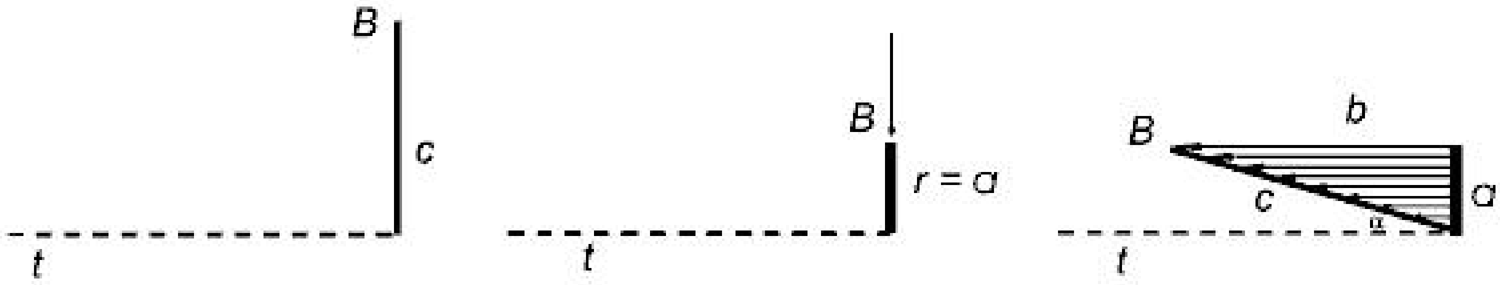
$$\tau_{\max} = 3mgh\sin 45^\circ\cos 45^\circ = \frac{3}{2}mgh = 9\ \mathrm{Nm}.$$

It is worth noting that the tension force and the torque are both independent of the length of the string.

Solution of Problem 55. a) Since the external forces acting on the system are all vertical, the centre of mass of the system will not move horizontally. When the ball has left the hemisphere, it must move vertically upwards: since the ball slides all the way along the hemisphere, it cannot have a horizontal velocity component at separation. Otherwise, its horizontal velocity would be equal to that of the cart, which would mean a displacement of the centre of mass of the system to the left or to the right. After separation, the motion of the ball is vertical projection, and the cart is brought to rest again. Thus, all the energy is concentrated on the ball again, and the ball will rise back to its initial height.



Let X denote the displacement of the cart and let x denote the horizontal component of the displacement of the ball. As shown in the figure, the distance of the centre of mass of



The moment of inertia of the compressed rod is $\Theta_c = \frac{1}{3} m_c r^2$, where $r = c \cdot \sin\alpha = a$ is the distance of point B from the axis. Finally, let us shear the rod in such a way that its length becomes again the original length, and every point of the rod moves parallel to the axis. This transformation does not change the moment of inertia, thus:

$$\Theta_c = \frac{1}{3} m_c c^2 \sin^2\alpha = \frac{1}{3} m_c a^2.$$

This result could have been obtained also by the definition of the moment of inertia:

$$\Theta_c = \sum m_i (l_i \sin\alpha)^2 = \sin^2\alpha \sum m_i l_i^2 = \sin^2\alpha \cdot \frac{1}{3} m_c c^2 = \frac{1}{3} m_c a^2.$$

Putting these results into the equation of energy conservation, and expressing the mass of each rod by the mass of rod c, we get that:

$$\left(\frac{3}{5} m_c + m_c\right) g \frac{1}{2} a = \frac{1}{2}\left(\frac{1}{3} \cdot \frac{3}{5} m_c + \frac{1}{3} m_c\right) a^2 \omega^2,$$

so

$$\frac{8}{5} m_c g a = \frac{8}{15} m_c a^2 \omega^2 \ \rightarrow \ 3g = a\omega^2$$

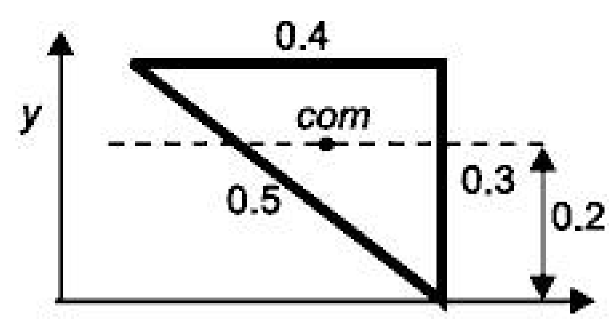
We have used the fact that $a = \frac{3}{5} c$, thus $m_a = \frac{3}{5} m_c$. Concluding, the value of the final velocity $v = a\omega$ of the vertex B is:

$$v = a\omega = \sqrt{3ga}\sqrt{3 \cdot 10\ \mathrm{ms^{-1}} \cdot 0.3\ \mathrm{m}} = 3\ \frac{\mathrm{m}}{\mathrm{s}}$$

and it is directed vertically downwards.

b) In this case the centre of mass of the triangle may move only in vertical direction, since all the external forces are vertical. The vertical forces cannot give rise to rotation in horizontal plane (i.e. about vertical axis) either, thus the rod b lying on the ground just translates parallelly. At the moment when the triangle hits the ground none of its points have horizontal velocity due to the conservation of horizontal momentum. Thus, according to the law of energy conservation, the angular velocity just before hitting the ground, as well as the velocity of point B at this instant coincide with the result obtained in case a).

The place where B hits the ground is now by $a/3 = 0.1$ m closer to the initial position of side b then in case a). To obtain this the position of the centre of mass has to be determined.



According to Pythagoras' theorem, side b of the triangle has a length 0.4 m. Let us use a coordinate

The instantaneous radius of rotation of point B is obtained from the Pythagorean theorem:

$$r_B = \sqrt{r_A^2 - 4R^2} = \sqrt{20R^2 - 4R^2} = 4R.$$

Finally, the instantaneous radius $r_S = \overline{CS}$ at the centre of mass is

$$r_S = \sqrt{\overline{CB}^2 + \overline{CS}^2} = \sqrt{16R^2 + R^2} = \sqrt{17}R.$$

Since the speed to be found is $v_B = r_B\omega$, the angular speed remains to be determined. The energy equation in terms of the unknown angular speed and the radius r_S obtained above is

$$mgR\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{5}}{5}\right) = \frac{1}{2}m17R^2\omega^2 + \frac{1}{2}\cdot\frac{1}{12}m4R^2\omega^2 = \frac{26}{3}mR^2\omega^2.$$

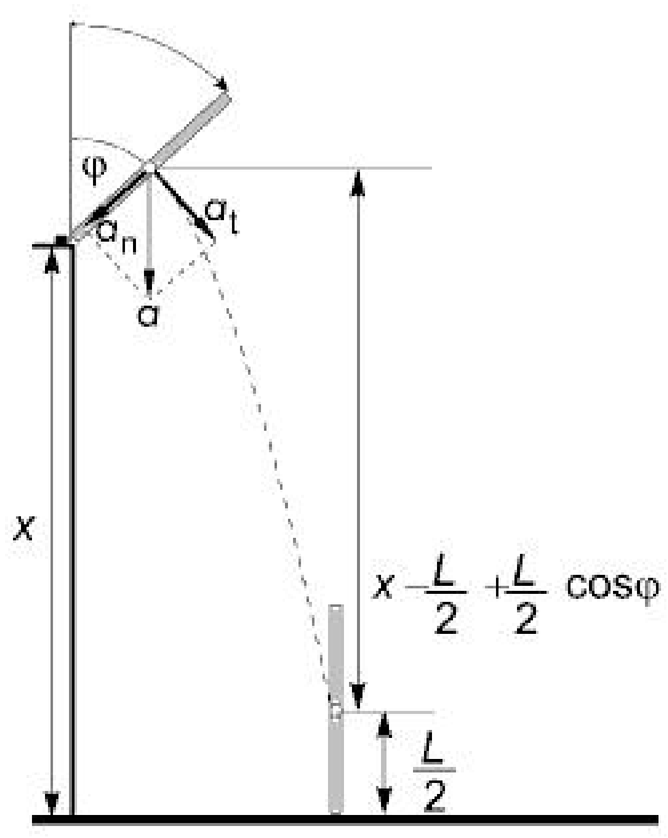
The solution of the equation for ω is

$$\omega = \sqrt{\frac{3}{26}\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{5}}{5}\right)\frac{g}{R}} = 0.5467\,\mathrm{s}^{-1}.$$

Thus the speed of the point B of the rod at the time instant in question is

$$v_B = 4R\omega = 2.19\,\frac{\mathrm{m}}{\mathrm{s}}.$$

Solution of Problem 60. Let us assume that the rod is thin and homogeneous and moves in a vertical plane perpendicular to the edge of the table. The peg makes the rod rotate about its bottom end without translation until it leaves the table. During this time, the centre of mass of the rod gains a horizontal velocity because of the force exerted by the peg, and maintains this velocity after leaving the table, since during that time there is no force acting horizontally. The rod loses contact with the peg (and due to its being slim also with the edge of the table) when the direction of the acceleration of its centre of mass becomes vertical. From that moment the horizontal component of the velocity of the centre of mass remains constant. The rod itself undergoes both translation and rotation. Its centre of mass moves on the path of a horizontal projection, while the rod itself rotates about its centre of mass with a constant angular velocity.



Let us find the angle rotated until the rod reaches the position of leaving the table and the velocity of the centre of mass at that position.

The angle the rod forms with the vertical as a function of time is:

$$\varphi=\varphi_0+\Delta\varphi=\varphi_0+\omega t=48.19^\circ+3.162\cdot\frac{180^\circ}{\pi}\cdot t,$$

where $\varphi_0=48.19^\circ$ is the angle at which the rod leaves the table.

Let us set up a table of values for the above quantities (which are measured in metre, second and degree.)

t	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.7275
x_s	0.1052	0.2104	0.3156	0.4208	0.526	0.6312	0.7364	0.766785
y_s	0.1678	0.4356	0.8034	1.1712	1.839	2.5068	3.2746	3.503
$\Delta\varphi$	18.1°	36.2°	54.35°	72.46°	90.58°	108.7°	126.8°	131.8°
φ	66.29°	84.39°	102.54°	120.65°	138.77°	156.89°	174.99°	180°

Solution of Problem 61. a) According to Newton's second law with impulse and momentum

$$F_1t_1+F_2t_2+F_3t_3=mv,$$

and

$$\bar{F}(t_1+t_2+t_3)=mv.$$

From these, the average force is the arithmetic mean of the magnitudes of the forces acting in the different phases weighted by their durations:

$$\bar{F}=\frac{F_1t_1+F_2t_2+F_3t_3}{t_1+t_2+t_3},$$

Its numerical value is

$$\bar{F}=\frac{10\text{ N}\cdot 4\text{ s}+4\text{ N}\cdot 14\text{ s}-15\text{ N}\cdot 2\text{ s}}{4\text{ s}+14\text{ s}+2\text{ s}}=3.3\text{ N}.$$

b) According to the work-kinetic energy theorem:

$$F_1s_1+F_2s_2+F_3s_3=\frac{1}{2}mv^2,$$

and

$$\bar{F}(s_1+s_2+s_3)=\frac{1}{2}mv^2.$$

From these

$$\bar{F}=\frac{F_1s_1+F_2s_2+F_3s_3}{s_1+s_2+s_3}.$$

The durations of the motion are given, but the distances belonging to the forces need to be determined. Calculating the distances:

$$s_1=\frac{1}{2}a_1t_1^2=\frac{F_1}{2m}t_1^2=\frac{10\text{ N}}{2m}\cdot 4^2\text{ s}^2=\frac{80\text{ kg}\cdot\text{m}}{m},$$

$$s_2=v_1t_2+\frac{1}{2}a_2t_2^2=a_1t_1t_2+\frac{1}{2}a_2t_2^2=\frac{F_1}{m}t_1t_2+\frac{F_2}{2m}t_2^2=$$

$$=\frac{10\text{ N}\cdot 4\text{ s}\cdot 14\text{ s}}{m}+\frac{4\text{ N}\cdot 14^2\text{ s}^2}{2m}=\frac{952\text{ kg}\cdot\text{m}'}{m}$$

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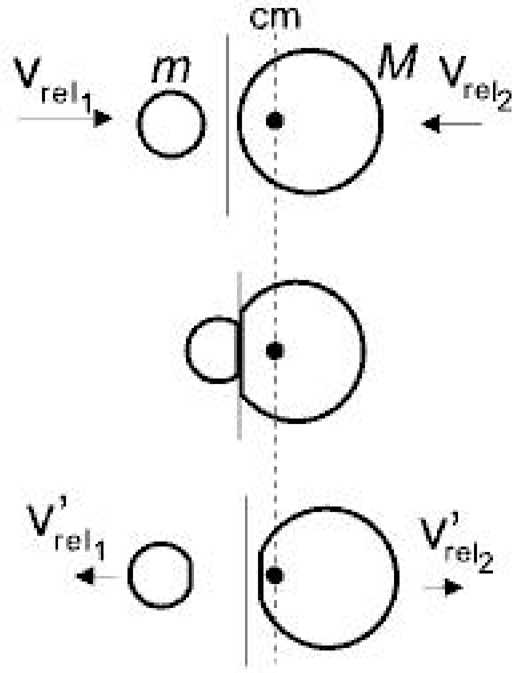
If the collision is not totally elastic, then the speeds after the collision are smaller than before the collision, the ratio of these speeds is $0 \leq k \leq 1$ thus for example for the ball of mass m_1:

$$k = \frac{c - u_1}{v_1 - c}.$$



(Of course the ratio $k = \dfrac{c - u_2}{v_2 - c}$ is the same for the two balls, because if the change in the linear momentum of one body decreases by a factor of k, then that of the other must decrease by the same factor according to the conservation of linear momentum.) Thus, the speeds of the balls after collision are:

$$u_1 = (k+1)c - kv_1,$$
$$u_2 = (k+1)c - kv_2.$$

Therefore, the number k (the so called collision number or coefficient) is suitable to characterize the collision from the aspect of elasticity. If we would like to describe how inelastic the colision is, we may use the number $\alpha = 1 - k$. In case of the totally elastic collisions $k = 1$, and $\alpha = 0$; in case of the totally inelastic collisions $k = 0$, and $\alpha = 1$.

The change in the kinetic energy of the system is:

$$\Delta E = \frac{1}{2}m_1 u_1^2 + \frac{1}{2}m_2 u_2^2 - \frac{1}{2}m_1 v_1^2 - \frac{1}{2}m_2 v_2^2,$$

Which, after substituting the velocities after the collision, can be written as:

$$\Delta E = \frac{m_1 m_2}{2(m_1 + m_2)}(v_1 - v_2)^2(k^2 - 1).$$

(Of course the lost energy is equal to $-\Delta E$.) In our case the ball of mass $m_1 = M$ was stationary thus using the notations $m_2 = m$ and $v_2 = v$ The result is:

$$\Delta E = \frac{mM}{2(m + M)}v^2(k^2 - 2).$$

Solution of Problem 65. Each object has a speed of $v_0 = \sqrt{2gh}$ when arriving at the rigid, horizontal ground. The lower ball of mass m_1, arriving first, rebounds with an upward velocity of the same magnitude v_0 since the collision is elastic. It then collides with the ball of mass m_2 still travelling downwards at a speed of v_0. The velocities of the objects after they collide with each other are given by the equation

$$u_i = (k+1)c - kv_i,$$

where the value of the coefficient of restitution k is $k = 1$ for a totally elastic collision, v_1 and v_2 are the velocities of the objects before the collision, and $c = \dfrac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$

Solution of Problem 69. With respect to the motion of the block, it does not matter whether the coefficients of static and kinetic friction are equal or not. In order for the block to slides back, it is necessary for the coefficient of static friction to be smaller then the tangent of the angle of inclination (so the angle of inclination must be greater then the 'critical angle' at which the block is just about to move). The turning of the block at the top of its path is momentary, thus this does not effect the time of the motion of the block. According to the problem, the interaction between the block and the bullet is momentary ('during the penetration the displacement of the block is negligible'), so during the interaction the system can be considered closed. (With respect to the internal forces the external ones are negligible.) Thus the total linear momentum of the system is conserved during the totally inelastic collision. The common initial speed is:

$$c=\frac{mv}{m+M}.$$

The block and the bullet embedded in it undergo uniformly decelerated motion, the acceleration of which has a magnitude of:

$$a_1=g(\sin\alpha+\mu\cos\alpha)$$

The time of the upward motion until the block stops is

$$t_1=\frac{c}{a_1}=\frac{mv}{2(m+M)g(\sin\alpha+\mu\cos\alpha)}.$$

The distance covered by the block during the upward motion is:

$$s=\frac{c^2}{2a_1}=\frac{m^2v^2}{2(m+M)^2g(\sin\alpha+\mu\cos\alpha)}.$$

After reaching the top, the block undergoes uniformly accelerated downward motion, covering the same distance as it covered when it moved up. The acceleration of this downward motion is:

$$a_2=g(\sin\alpha-\mu\cos\alpha).$$

The time while it moves down is:

$$t_2=\sqrt{\frac{2s}{a_2}}=\sqrt{\frac{m^2v^2}{(m+M)^2g(\sin\alpha+\mu\cos\alpha)g(\sin\alpha-\mu\cos\alpha)}}=$$

$$=\frac{mv}{(m+M)g}\cdot\frac{1}{\sqrt{\sin^2\alpha-\mu\cos^2\alpha}}.$$

The total time elapsed until the block reaches the buffer again is:

$$t_1+t_2=\frac{mv}{(m+M)g}\left(\frac{1}{\sin\alpha+\mu\cos\alpha}+\frac{1}{\sqrt{\sin^2\alpha-\mu^2\cos^2\alpha}}\right).$$

From the data of the problem v_1 and v_2 cannot be calculated, just $v_1 \sin\alpha$ and $v_2 \sin\alpha$. The endpoints of the possible v_1 and v_2 velocity vectors are on parallel horizontal lines.

Using the data of our problem: $k = 2$, $t_1 = 4\ \mathrm{s}$, $t_2 = 8\ \mathrm{s}$, $h = 196\ \mathrm{m}$, $v_1 \sin\alpha = 29.4\ \mathrm{m/s}$, $v_2 \sin\alpha = 14.7\ \mathrm{m/s}$.

Interesting results are gained if we calculate the speeds which belong to the different velocity-directions. For example there is no finite solution for v_1 if the angle is 0^0.

$10°$	$20°$	$30°$	$45°$	$60°$	$70°$	$80°$	$90°$	
169.3	85.95	58.8	36	33.95	31.28	29.85	29.4	(m/s)

Remark: If we would like to solve the problem for the special case when the velocities after the explosion are vertical, the result would not be unique. In this case the datum that the travelled horizontal distances are equal does not give any condition for the times of fall since the equation $v_1 \cos\alpha \cdot t_1 = v_2 \cos\alpha \cdot t_2$ is an identity because $\cos\alpha = \cos 90^0 = 0$. Thus the conditions of the problem can be satisfied for any height.

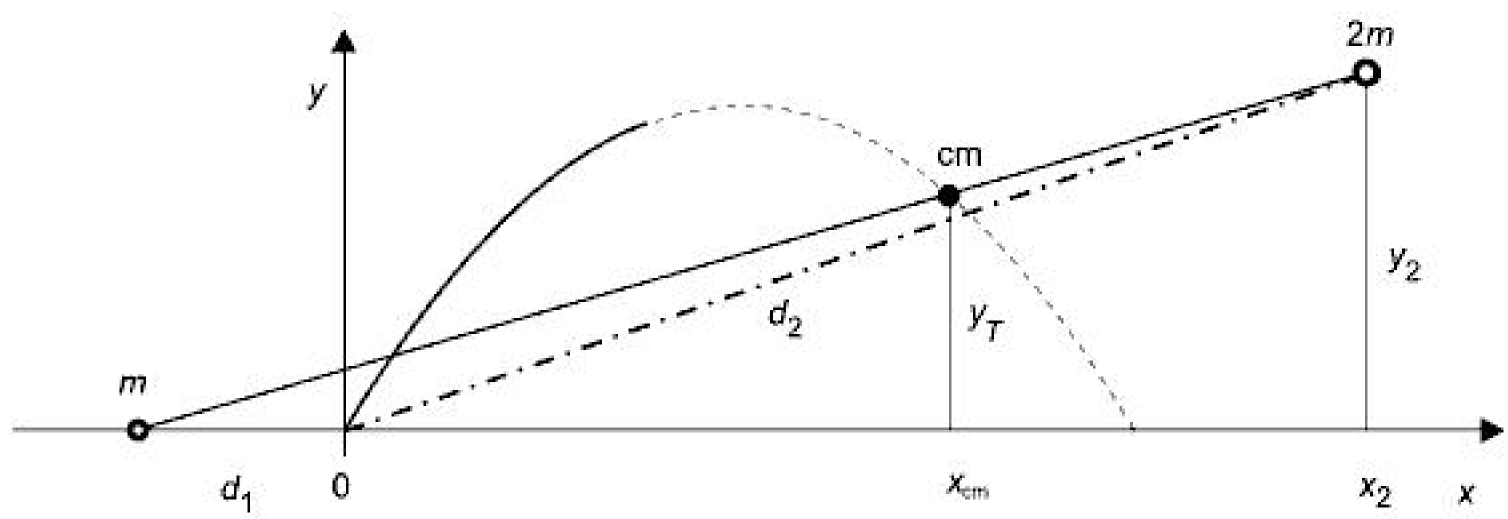
First solution of Problem 72. The simplest way to solve the problem is to apply the centre of mass theorem: the centre of mass of a system moves as if the total mass of the system was concentrated at the centre of the mass, and all the external forces acted upon it. In our case the centre of mass of the two pieces moves as if the projectile did not explode: it continues moving along the initial parabolic orbit of the projectile. (The explosion was caused by internal forces, and we neglect air resistance, as it is usual in these types of problems.) Let us follow the motion of both pieces and the centre of mass.

The position of the centre of mass is described by its coordinates. At the instant when the piece of mass $m_1 = m$ hits the ground, i.e. at time $t_2 = t_1 + \Delta t$ the coordinates of the centre of mass are:

$$x_{\mathrm{cm}} = v_0 \cos\alpha \cdot t_2 = 150\ \frac{\mathrm{m}}{\mathrm{s}} \cdot \cos 60° \cdot 20\ \mathrm{s} = 1500\ \mathrm{m},$$

$$y_{\mathrm{cm}} = v_0 \sin\alpha \cdot t_2 - \frac{1}{2} g t_2^2 = 150\ \frac{\mathrm{m}}{\mathrm{s}} \cdot \sin 60° \cdot 20\ \mathrm{s} - \frac{10}{2}\ \frac{\mathrm{m}}{\mathrm{s}^2} \cdot 400\ \mathrm{s}^2 = 598\ \mathrm{m}.$$

The centre of mass divides the section between the two pieces into two parts with lengths



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The speed of the centre of mass of the two balls at this moment is

$$v_{\rm cm} = \frac{m_1 u_1}{m_1 + m_2}.$$

The speed of the body with mass m_2 after the collision is

$$v_2 = 2v_{\rm cm} - u_2 = \frac{2m_1 u_1}{m_1 + m_2} = \frac{2m_1\sqrt{2gH}}{m_1 + m_2},$$

because $u_2 = 0$, and for the body with mass m_1

$$v_1 = 2v_{\rm cm} - u_1 = \frac{2m_1 u_1}{m_1 + m_2} - u_1 = \frac{m_1 - m_2}{m_1 + m_2}\sqrt{2gH}.$$

The heights to which the bodies rise

$$h_1 = \frac{v_1^2}{2g} = \left(\frac{m_1 - m_2}{m_1 + m_2}\sqrt{2gH}\right)^2 \cdot \frac{1}{2g} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 \cdot H,$$

and

$$h_2 = \frac{v_2^2}{2g} = \left(\frac{2m_1\sqrt{2gH}}{m_1 + m_2}\right)^2 \cdot \frac{1}{2g} = \frac{4m_1^2}{(m_1 + m_2)^2} \cdot H.$$

Making use of the condition $h_1 = h_2$, from the previous two equations the $(m_1 - m_2)^2 = 4m_1^2$ quadratic equation is acquired. After rearrangement

$$m_2^2 - 2m_1 m_2 - 3m_1^2 = 0,$$

whose solution is:

$$m_2 = \frac{2m_1 \pm \sqrt{4m_1^2 + 12m_1^2}}{2} = \frac{2m_1 \pm 4m_1}{2}$$

and the physically realistic value of the unknown mass is $m_2 = 3m_1 = 0.6\,\text{kg}$.

b) If the relationship $m_2 = 3m_1$ is substituted into any of the equations for the height to which the bodies rise, for the given ratio

$$h_1 = h_2 = \frac{4m_1^2}{(4m_1)^2} \cdot H = \frac{H}{4}$$

is acquired.

Solution of Problem 75. Since the friction is negligible, the forces between the two disks are radial, their tangential components are zero, so the disks do not start rotating after the collision.

From geometric considerations the angle α of the force acting on the disk, which is initially at rest, satisfies the following equation:

$$\sin\alpha = \frac{R}{2R} = 0.5 \;\rightarrow\; \alpha = \arcsin 0.5 = 30^\circ.$$

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Initially the disks B and C are at rest, and the total angular momentum is carried by the (orbital) angular momentum of A. In the first collision this angular momentum is transferred to the disk B. In the second collision the impulse exerted by the thread is collinear with the thread, so it has no torque with respect to the midpoint of the thread and does not change the angular momenta of the disks. Formally:

$$mv\frac{l}{2}\cdot\frac{\sqrt{2}}{2}=mu_B\frac{l}{2}=m\frac{v}{2}\sqrt{2}\frac{l}{2}=mv\frac{l}{2}\cdot\frac{\sqrt{2}}{2}.$$

III. The total mechanical energy of the system is the sum of translational kinetic energies of its parts. The values of this sum before the first collision, and after the second collision are equal:

$$\frac{1}{2}m_Av_A^2=\frac{1}{2}m_Bu_B^2+\frac{1}{2}m_Cu_C^2=\frac{1}{2}m_B\left(u_{Bx}^2+u_{By}^2\right)+\frac{1}{2}m_C\left(u_{Cx}^2+u_{Cy}^2\right)=$$
$$=\frac{1}{2}m\left(\frac{v^2}{4}+\frac{v^2}{4}\right)+\frac{1}{2}m\left(\frac{v^2}{4}+\frac{v^2}{4}\right)=\frac{1}{2}mv^2=\frac{1}{2}m_Av_A^2,$$

since $m_A=m_B=m_C=m$.

Solution of Problem 77. In vertical direction the equation of motion of the released ball at the lowest point is:

$$K-mg=m\frac{v^2}{l}.$$

The speed v of the ball at this point is obtained from the energy conservation law:

$$mg\Delta h=\frac{1}{2}mv^2,$$

where Δh is the altitude loss of the ball, i.e., $l-l\cos\varphi_0=l(1-\cos\varphi_0)$. So the speed of the ball at its lowest position is:

$$v=\sqrt{2gl(1-\cos\varphi_0)}.$$

The threshold force in the thread is:

$$K_1=mg+m\frac{2gl(1-\cos\varphi_0)}{l}=mg(3-2\cdot\cos\varphi_0)=mg(3-2\cdot\cos60^\circ)=2mg.$$

Since the masses are equal, the two balls exchange their velocities in the totally elastic collision. The formula of the force in the second (shorter) thread is similar to that of the longer thread. The speed of the second ball is equal to the speed of the first ball, but in the centripetal acceleration $l/2$ has to be used for the radius. So

$$K_2=mg+m\frac{2gl(1-\cos\varphi)}{\frac{l}{2}}=mg\left(1+\frac{4gl(1-\cos\varphi)}{l}\right)=mg(5-4\cos\varphi),$$

where φ is the maximal initial angle of the longer pendulum, at which the threads do not break. This angle should be less than 60°, since otherwise the first thread would

which means that the acceleration of the weight is:

$$a = \frac{2m}{2m + M} g.$$

With numerical values:

$$a = \frac{2m}{2m + M} g = \frac{2 \cdot 3 \text{ kg}}{2 \cdot 3 \text{ kg} + 2 \text{ kg}} \cdot 9.81 \ \frac{\text{m}}{\text{s}^2} = 7.36 \ \frac{\text{m}}{\text{s}^2}.$$

The time needed for the first stage of the motion (until the collision) is:

$$t_1 = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2.4 \text{ m}}{9.81 \ \frac{\text{m}}{\text{s}^2}}} = 0.495 \text{ s}.$$

The time of the second stage of the motion can be determined from the following kinematic equation:

$$h = ut + \frac{1}{2} a t^2,$$

which has the more convenient form: $at^2 + 2ut - 2h = 0$.

The solution of this equation is:

$$t_2 = \frac{-u \mp \sqrt{u^2 + 2ah}}{a}.$$

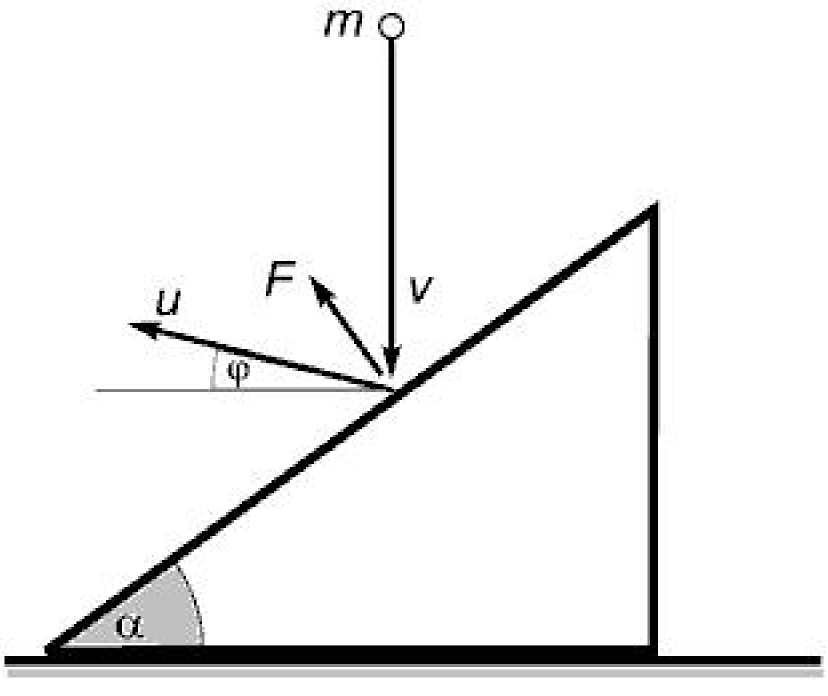
The numerical value of the physically reasonable root is:

$$t_2 = \frac{-3.64 \ \frac{\text{m}}{\text{s}} + \sqrt{13.25 \ \frac{\text{m}^2}{\text{s}^2} + 2 \cdot 7.36 \ \frac{\text{m}}{\text{s}^2} \cdot 1.2 \text{ m}}}{7.36 \ \frac{\text{m}}{\text{s}^2}} = 0.261 \text{ s}.$$

Thus the weight covers the distance $2h$ in

$$t = t_1 + t_2 = 0.495 \text{ s} + 0.261 \text{ s} = 0.756 \text{ s}.$$

Solution of Problem 81. As the inclined plane is frictionless, the force exerted on the object that collides elastically with it can only be perpendicular to the plane. Let us assume that the object is a small ball, because this way the collision can only be central. (Otherwise the normal force exerted by the plane might not go through the centre of mass of the object, which will result in the rotation of the object. In that case the conservation of kinetic energy for the translational motions will not hold.)



Let us also assume that the collision takes place in an instant, since then the gravitational force is negligible compared to the normal force, which makes our

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after the collision the chest continues its decelerating motion from the place where the collision occurred. Writing into the previous equation the expression for Δu, we get:

$$m\cot\alpha\sqrt{2gh}+M\left(v_1-\mu\frac{m\sqrt{2gh}}{M}\right)=(M+m)v_2.$$

Expressing the speed after the collision:

$$v_2=\frac{m\cot\alpha\sqrt{2gh}+Mv_1-\mu m\sqrt{2gh}}{m+M}.$$

Since in the problem $m=M$, the masses cancel out:

$$v_2=\frac{\cot\alpha\sqrt{2gh}+v_1-\mu\sqrt{2gh}}{2}=\frac{(\cot\alpha-\mu)\sqrt{2gh}+v_1}{2}.$$

The stopping distance does not depend on the mass, and its expression is:

$$s=\frac{v_2^2}{2\mu g}=\frac{v_1^2+2v_1(\cot\alpha-\mu)\sqrt{2gh}+(\cot\alpha-\mu)^2 2gh}{8\mu g}.$$

Numerically:

$$s=\frac{25+2\cdot 5\cdot(\cot 60^\circ-0.4)\cdot\sqrt{2\cdot 9.81\cdot 3}+(\cot 60^\circ-0.4)^2 2\cdot 9.81\cdot 3}{8\cdot 0.4\cdot 9.81}\ \mathrm{m}=1.29\ \mathrm{m}.$$

If the sand bag had not fallen into the chest, then from the place of the instantaneous speed 5 m/s the stopping distance would have been

$$s'=\frac{v_1^2}{2\mu g}=\frac{25}{2\cdot 0.4\cdot 9.81}\approx 3.18\ \mathrm{m},$$

so in this case the path covered by the chest would have been by $s'-s=3.18\ \mathrm{m}-$ $-\ 1.29\ \mathrm{m}=1.89\ \mathrm{m}$ more.

Second solution of Problem 82. In the previous solution we divided the whole collision into two successive processes, in a somewhat arbitrary way. First, only the vertical interaction was considered, the intermediate speed of the chest was determined, and then the horizontal interaction was regarded as a separate collision, which determined the final, common speed of the chest and the bag. Now, with the help of the centre-of-mass theorem, we describe the whole collision process as a single event. Let us apply the centre-of-mass theorem in horizontal direction. The horizontal component of the net external force acting on the system, consisting of the chest and the bag, is the friction force. Let V_1 denote the speed of the centre of mass of the system before the collision, and let V_2 be the speed after collision.

We use the result obtained in the previous solution for the 'increase' of the normal force of the ground:

$$K=mg+\frac{m\sqrt{2gh}}{\Delta t}.$$

While the two objects are in contact, the cart keeps on accelerating, with the single exception of the time instant when the small object reaches the height of $2R$ on the semicircle.

The force exerted by the small object on the cart is the smallest if it slides onto the cart at the minimum possible speed. Since the condition for reaching the top is that the normal force N exerted by the track should remain $N \geq 0$ all the way, the cart will gain the smallest possible final speed if the normal force N decreases to 0 exactly at the topmost point. Let us go through the conditions required.

Condition 1:

$$N_A = 0. \tag{1}$$

When the object has reached point A, the cart moves on with uniform motion in a straight line, thus it becomes an inertial reference frame. In the reference frame of the cart, the small object moves along a circular path of radius R until it reaches A-ig, and then continues on a parabolic path as a projectile with a horizontal initial velocity. (In the reference frame attached to the ground, it reaches the cart with free fall along a vertical line, with zero initial velocity.) The next condition is provided by the circular motion in the reference frame of the cart:

Condition 2:

$$mg + N_A = m\frac{v_{\mathrm{rel}}^2}{R},$$

and thus with (1),

$$g = \frac{v_{\mathrm{rel}}^2}{R}. \tag{2}$$

Since there is no friction, all forces are conservative, the total mechanical energy of the system is conserved. Let v_0 denote the speed of the small object when it hits the cart (same as its initial speed of sliding onto the cart).

Condition 3:

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}MV^2 + mgh,$$

where V is the speed of the cart and v_1 is the speed of the small object acquired by the time the small object rises to a height h above the surface of the cart. The small object needs to reach the point A at a height of $h = 2R$, and it needs to lose its speed there ($v_1 = 0$ is needed). Multiplied by two, the equation takes the form

$$mv_0^2 = MV^2 + 4mgR. \tag{3}$$

Since the small object is to stop at the topmost point of its path, its velocity relative to the cart at that point is the opposite of the velocity of the cart. In absolute value,

$$v_{rel} = V. \tag{4}$$

Since the external forces are all vertical, the sum of horizontal momenta is constant, that is,

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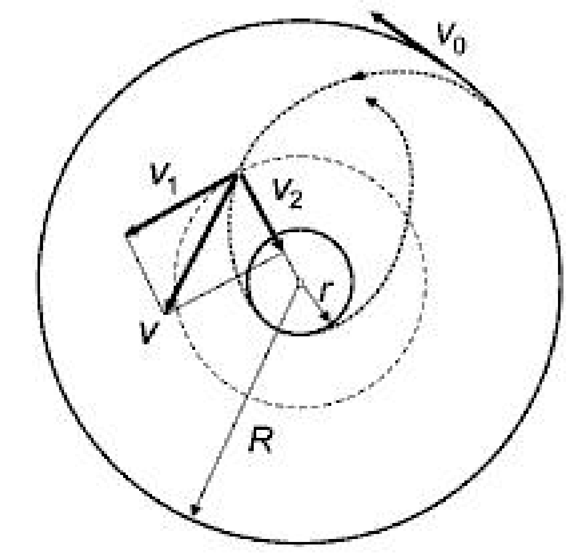


b) The conservation laws of angular momentum and mechanical energy can be used again, but this time the mass m has a velocity component v_1 normal to the string and a component v_2 parallel to the string. With these notations,

$$mv_0R = mv_1\frac{R}{2}, \tag{3}$$

$$Mg\frac{R}{2}+\frac{1}{2}mv_0^2 = \frac{1}{2}m(v_1^2+v_2^2)+\frac{1}{2}Mv_2^2. \tag{4}$$

The solution of the simultaneous equations is

$$v_2 = \sqrt{\frac{MgR-3mv_0^2}{m+M}} = \sqrt{\frac{2\,\mathrm{kg}\cdot 10\,\mathrm{m/s^2}\cdot 0.4\,\mathrm{m}-3\cdot 1\,\mathrm{kg}\cdot 0.89^2\,\mathrm{m^2/s^2}}{1\,\mathrm{kg}+2\,\mathrm{kg}}} = 1.368\,\frac{\mathrm{m}}{\mathrm{s}},$$

and from (3),

$$v_1 = 2v_0 = 1.78\,\mathrm{m/s}.$$

Therefore the mass m is moving at a speed of

$$v_{\frac{R}{2}} = \sqrt{v_1^2+v_2^2} = 2.246\,\mathrm{m/s} \approx 2.25\,\mathrm{m/s},$$

and the speed of the mass M is

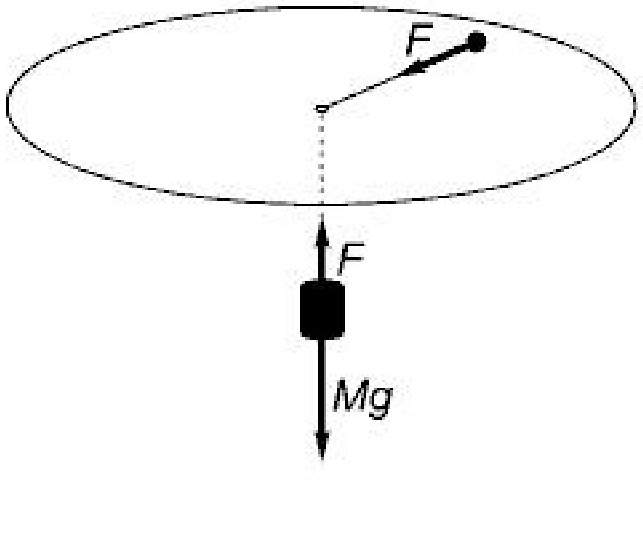
$$v_M = v_2 = 1.369\,\mathrm{m/s} \approx 1.37\,\mathrm{m/s}.$$

c) The accelerations can be determined by either of the two methods below: one using an inertial reference frame and the other using a rotating frame.

I. The acceleration of the mass m equals that of the end of the string. It has a centripetal component $a_n = r\omega^2 = v^2/r$ owing to the rotation of the horizontal segment of the string, and another component (a) owing to the decreasing length of the horizontal string segment. (At the time instants investigated, the instantaneous speed of the point of the string at the hole is 0.) Since the rotation of the string does not influence the motion of the mass M hanging from the string, the acceleration of the hanging object equals the acceleration component resulting from the change in length. Thus Newton's second law applied to the objects at the lowermost point gives the equations

$$Mg - F = Ma, \tag{5}$$

$$F = m\left(a+\frac{v^2}{r}\right). \tag{6}$$



The sum of (5) and (6) is $Mg = Ma + m\left(a+\frac{v^2}{r}\right)$,

and hence the acceleration a is

$$a = \frac{Mg - m\frac{v^2}{r}}{m+M}.$$

and thus

$$\omega_1 = 2.5\omega.$$

and

$$t = \frac{4\pi}{\omega_1 + 0.4\omega_1} = \frac{4\pi}{1.4\omega_1},$$

which can be combined with (7):

$$t = 4 \cdot \frac{1}{1.4}\sqrt{\frac{3.5R^2\pi^2 + 1.225h^2}{5gh}} = 4 \cdot \frac{1}{1.4}\sqrt{\frac{(3.5\pi^2 + 1.225)L}{5g}} = 1.08\,\mathrm{s}.$$

Remark. The time taken by the object to cover a certain segment of its descent can also be determined by using the vertical velocity component: Note that $h = v_v t/2$, and hence with the use of (8):

$$t = \frac{2h}{v_v} = \frac{2h}{0.7h\sqrt{\frac{5gh}{3.5R^2\pi^2+1.225h^2}}} = \frac{2}{0.7}\sqrt{\frac{(3.5\pi^2 + 1225) \cdot L}{5g}} = 1.08\,\mathrm{s}.$$

Solution of Problem 88. In the interaction between the ball and the disc angular momentum and mechanical energy are conserved. These result in two equations for the speed of the ball upon leaving and the angular speed of the disc.

The angular momentum of the ball after bowling is zero for the axis and the angular momentum of the originally stationary disc is also zero. Therefore, at the moment of detaching, the sum of the orbital angular momentum of the ball and the angular momentum of the disc are both zero:

$$mv_1R - \frac{1}{2}MR^2\omega = 0, \tag{1}$$

because the ball leaves the rim in the direction of the tangent. The total energy at this moment is:

$$\frac{1}{2}mv^2 = \frac{1}{2}mv_1^2 + \frac{1}{2} \cdot \frac{1}{2}MR^2\omega^2. \tag{2}$$

from (1) the angular speed of the disc is

$$\omega = \frac{2mv_1R}{MR^2} = 2\frac{m}{M} \cdot \frac{v_1}{R}.$$

If this is substituted into (2):

$$\frac{1}{2}mv^2 = \frac{1}{2}mv_1^2 + \frac{1}{2} \cdot \frac{1}{2}MR^2 \cdot 4\frac{m^2}{M^2} \cdot \frac{v_1^2}{R^2} = \left(\frac{m}{2} + \frac{m^2}{M}\right) \cdot v_1^2. \tag{3}$$

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Writing these into equation (2), we have:

$$v_1\sqrt{2Rh_1-h_1^2}=v_2\sqrt{2Rh_2-h_2^2}. \tag{3}$$

Equations (1) and (3) form a set of equations with two unknowns. Let us take the square of equation (3), isolate v_2^2 and substitute it into equation (1) to find:

$$v_2^2=v_1^2\frac{2Rh_1-h_1^2}{2Rh_2-h_2^2}$$

$$2g(h_2-h_1)=v_1^2-v_1^2\frac{2Rh_1-h_1^2}{2Rh_2-h_2^2}=\frac{2Rh_2-2Rh_1-h_2^2+h_1^2}{2Rh_2-h_2^2}v_1^2,$$

from which

$$v_1=\sqrt{\frac{2g(h_2-h_1)(2Rh_2-h_2^2)}{2Rh_2-h_2^2-2Rh_1+h_1^2}}=\sqrt{\frac{2g(h_2-h_1)(2R-h_2)h_2}{2R(h_2-h_1)+(h_1-h_2)(h_1+h_2)}}$$

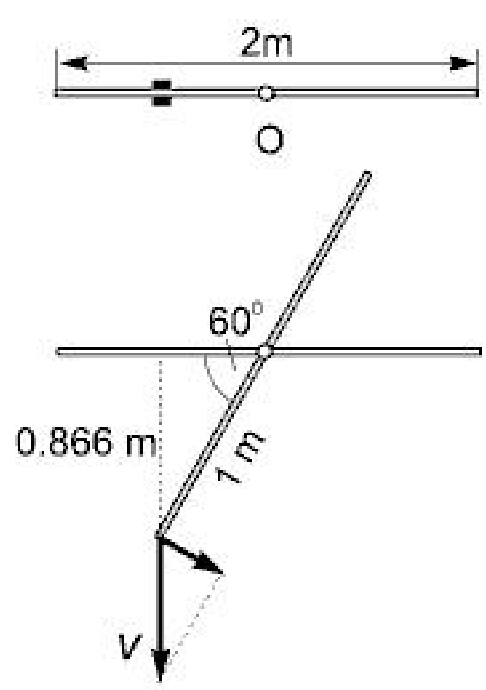
Simplifying the fraction under the square root by (h_2-h_1), we obtain:

$$v_1=\sqrt{2gh_2\frac{2R-h_2}{2R-(h_1+h_2)}}=2.5\,\frac{\text{m}}{\text{s}}$$

similarly (using the symmetry of the situation to exchange indices), we get:

$$v_2=\sqrt{2gh_1\frac{2R-h_1}{2R-(h_1+h_2)}}=1.5\,\frac{\text{m}}{\text{s}}.$$

Solution of Problem 90. Assume the 3 kg object to be a point so that its rotational inertia can be neglected. Since friction and the rod's mass are negligible, the only force acting on the object is gravitational force, therefore the object undergoes free-fall. Its path being vertical, the object moves a distance of:



$$h=\sqrt{\left(\frac{L}{2}\right)^2-\left(\frac{L}{4}\right)^2}=0.866\,\text{m}$$

until it drops off from the rod. Its velocity at that moment is $v=\sqrt{2gh}=4.16\,\text{m/s}$. The object's velocity can be resolved into components that are parallel and perpendicular to the rod. The latter equals the velocity of the rod's end at the moment when the object leaves the rod. After this moment the rod's end maintains its velocity, so its speed in the rod's vertical position is still:

$$v_P=v\cos\alpha,$$

After squaring:

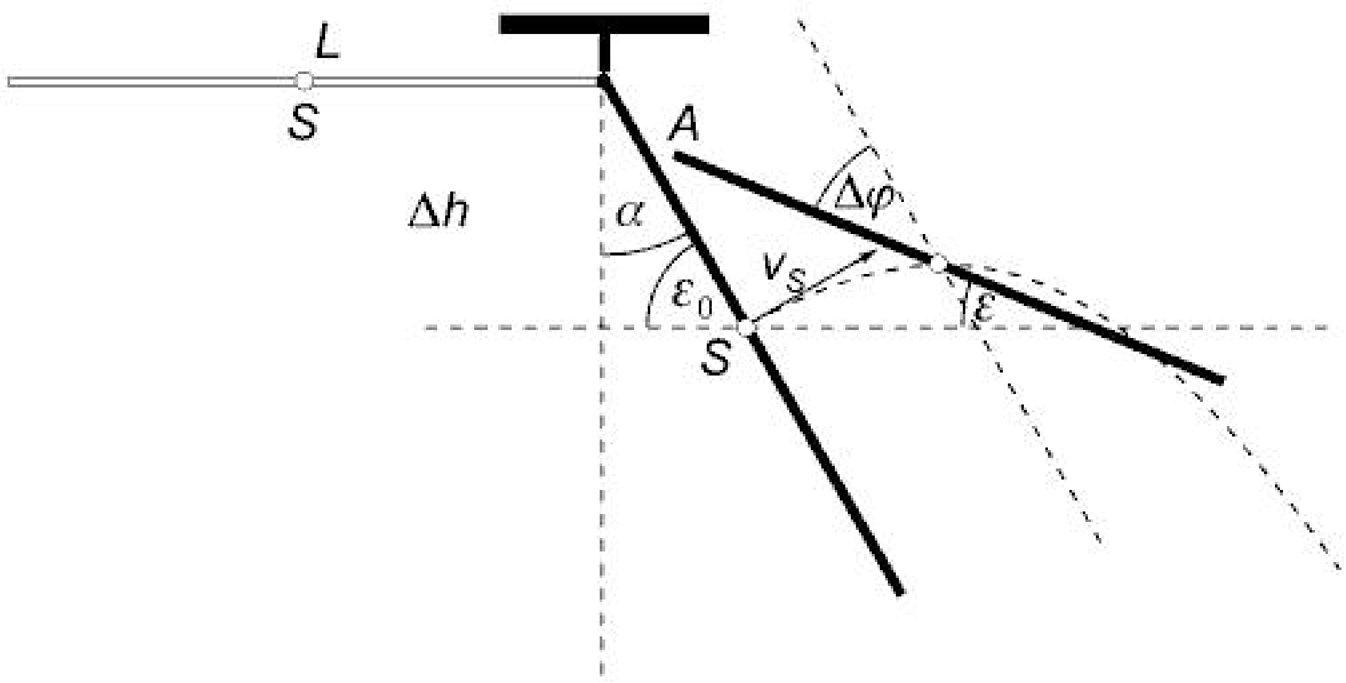
$$m^2 \cdot 9 \cdot 2gR = M^2 R^2 \cdot 6 \cdot \frac{m}{M}\frac{g}{R},$$

from which the requested ratio is

$$\frac{M}{m} = 3,$$

just as we acquired it from the first solution.

Solution of Problem 93. When the stick detaches from the hook, its centre of mass undergoes oblique projection. In the meantime, the stick rotates uniformly at the already acquired angular speed (the homogeneous gravitational force has no moment on the centre of mass). We determine the magnitude and the direction of the instantaneous velocity of the centre of mass (S) upon detachment and the angular speed of the stick, the rising time of the projection and from it the angular displacement of the stick and finally the angle enclosed by it and the horizontal.



From the work–energy theorem:

$$mg\Delta h = \frac{1}{2}\Theta_A \omega^2.$$

In detail:

$$mg\frac{L}{2}\cos\alpha = \frac{1}{2}\cdot\frac{1}{3}mL^2\omega^2,$$

from which the angular speed is

$$\omega = \sqrt{\frac{3g}{L}\cos\alpha}\ .$$

From this the magnitude of the velocity of the centre of mass is

$$v_{\mathrm{s}} = \frac{L}{2}\omega.$$

The angle of projection is also α. With it the vertical component of the velocity of the projection is

$$v_{\mathrm{s_y}} = v_{\mathrm{s}} \cdot \sin\alpha.$$

Rearranging this equation we get:

$$v_c^2 - u_c^2 = \frac{1}{12} L^2 \omega^2.$$

After factorising, and dividing the equation by (4), from here the solution is the same as solution I.

Third solution of Problem 94. (Using angular impulse–angular momentum theorem, with reference point at the edge of the table.) Since the line of action of the force F passes through the end of the rod at the edge of the table, its torque is zero with respect to this point. The weight force can be disregarded because the collision is instantaneous. Thus, the total change of the angular momentum of the rod (with respect to point A) is zero:

$$F \cdot r \cdot \Delta t = \Delta N = 0,$$

so

$$0 = m u_c \frac{L}{2} - m v_c \frac{L}{2} + \Theta_c \omega.$$

Due to the conservation of energy:

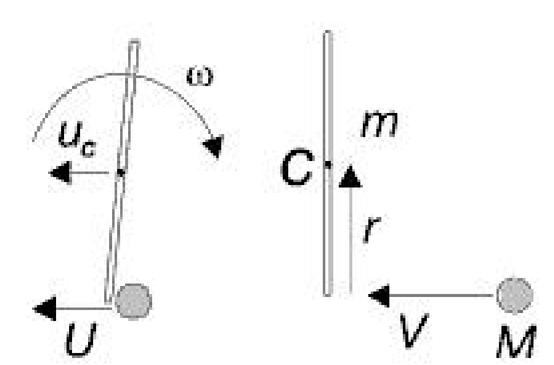
$$\frac{1}{2} m v_c^2 = \frac{1}{2} m u_c^2 + \frac{1}{2} \Theta_c \omega^2.$$

These equations are identical to the ones written down in solution I, and the rest of the solution is the same as there.

Fourth solution of Problem 94. (Using general collision theory.) At the moment of collision, the rod can be considered as a weightlessly floating object since during the instantaneous interaction the role of the gravitational force is negligible. The collision process can be regarded equivalently as if the rod, floating at rest in the space, was hit by an upward moving 'table of infinite mass' at one of its ends. This interaction determines the angular speed of the rod after the collision, and thus the period of its rotational motion.

This process is equivalent to the collision of a rod, initially lying at rest on a very smooth (e.g. air cushion) table, and a pointlike particle of 'infinite mass', sliding towards the rod. First, let us solve this problem in a general, parametric way. Let m be the mass of the rod, and let M denote the mass of the pointlike particle sliding towards the rod at a speed V.

Furthermore, let u_c be the speed of the centre of mass of the rod, and let U be the speed of the masspoint M after collision.



With these notations the momentum conservation law is:

$$MV = MU + m u_C. \tag{1}$$

Let the reference point of the angular momentum be the geometric point (at rest), where the collision takes place.

and the time of fall obtained from the relationship $t=\Delta v_y/g$ is

$$t=\frac{\sqrt{v_{0y}^2+2gs_y}-v_{0y}}{g}=0.4685\,\mathrm{s}\approx 0.47\,\mathrm{s}.$$

The total horizontal displacement of the lower one of the two small objects is

$$\Delta x=\Delta s_{x1}+\Delta s_{x2}=L(1-\cos\alpha)+v_{0x}t=$$

$$=5.48\cdot 10^{-3}\,\mathrm{m}+0.183\,\frac{\mathrm{m}}{\mathrm{s}}\cdot 0.47\,\mathrm{s}=9.15\cdot 10^{-2}\,\mathrm{m}=9.15\,\mathrm{cm}.$$

This is equal to the distance between the impact points of the two objects.

Second solution of Problem 95. This is a general solution for the case when the masses of the small objects are not negligible. The equations are set up for the small object and for the rod separately, and finally for the whole system:

$$S-mg\sin\alpha=mL\omega^2 \qquad (=ma_n \text{ for the normal direction}) \qquad (1)$$

$$K+mg\cos\alpha=mL\beta \qquad (=ma_t \text{ for the tangential direction}) \qquad (2)$$

$$S=\mu K \qquad (\text{for the time instant of separation}) \qquad (3)$$

$$Mg\frac{L}{2}\cos\alpha-KL=\frac{1}{3}ML^2\beta \qquad (\text{for the rod}) \qquad (4)$$

$$Mg\frac{l}{2}\sin\alpha+mgL\sin\alpha=\frac{1}{2}\cdot\frac{1}{3}ML^2\omega^2+\frac{1}{2}m(L\omega)^2 \qquad (\text{for the system}) \qquad (5)$$

The solution is

$$\omega=\sqrt{\frac{3g(M+2m)\sin\alpha}{L(M+3m)}}, \qquad L\beta=\frac{3M+6m}{2M+6m}\cdot g\cos\alpha,$$

and hence the tangent of the angle at which separation occurs is

$$\tan\alpha=\frac{M(M+3m)}{8M^2+42Mm+54m^2}\cdot\mu=\frac{M}{8M+18m}\cdot\mu.$$

If $m=0$, then $\tan\alpha=\frac{\mu}{8}$ and $\alpha=6^\circ$. The solution can be finished in the same way as solution 1.

Solution of Problem 96. The constraining force exerted by the half-rod (that really acts at the point of contact of the two halves) can be determined by using Newton's law of motion for the centre of mass of the lower half-rod. It is worth setting up the equation of motion for two components, the tangential one and the radial one.

Remarks on the figure:

1. Force K actually acts on the lower half-rod at the point of contact of the two half-rods. However, according to the law of motion of the centre of mass, the centre of mass moves as if the total mass of the body were concentrated there and all external

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hence the magnitude of centrifugal force acting on the whole rod can be calculated as:

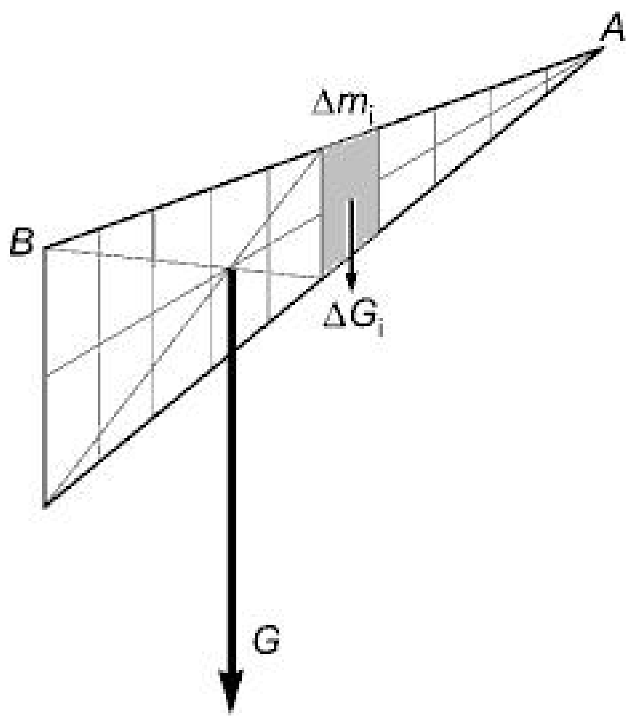
$$F_{cf} = \sum \Delta \overline{F}_i = \frac{r_{\max}\omega^2}{2} \sum \Delta m_i = \frac{1}{2} m r_{\max}\omega^2.$$

More precisely the centrifugal force is the sum of an arithmetic sequence:

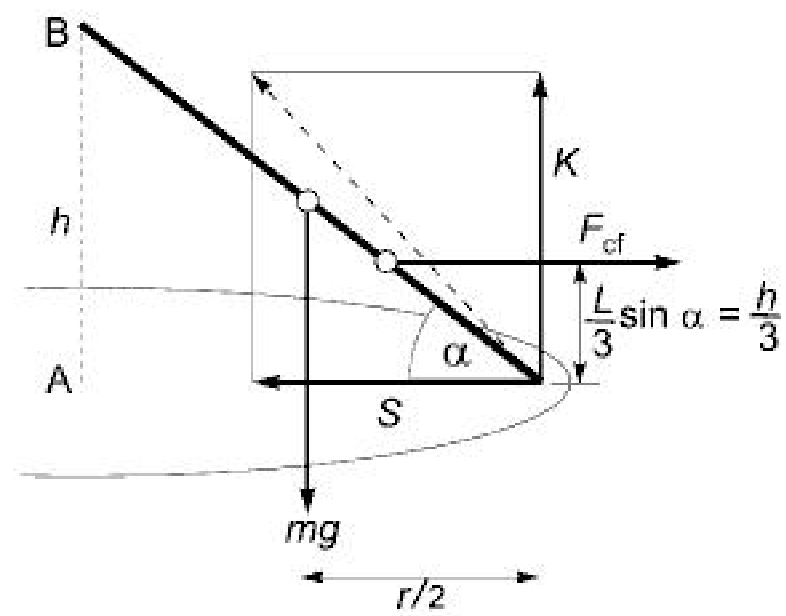
$$F = \sum_{i=1}^{n} \Delta F_i = \sum_{i=1}^{n} \frac{m}{n} \cdot i \cdot \frac{r_{\max}}{n} \cdot \omega^2 = \frac{m}{n^2} r_{\max}\omega^2 \sum_{i=1}^{n} i = \frac{m r_{\max}\omega^2}{n^2} \cdot \frac{1+n}{2} \cdot n$$
$$= m r_{\max}\omega^2 \left(\frac{1}{2n} + \frac{1}{2}\right).$$

If $n \to \infty$ then $1/(2n) \to 0$ and the magnitude of centrifugal force is: $F_{cf} = \frac{1}{2} m r_{\max}\omega^2$.

The second step is to determine the point of action of the centrifugal force, which is the resultant of parallel forces F_i. To do this, let us set up a model.



A vertical triangle in a uniform gravitational field provides the same distribution of forces if set as shown. Side AB is divided into n equal segments and vertical lines drawn from each point of division cut the triangle into n parts, whose weights (ΔG_i) show the same pattern as the centrifugal forces acting on the parts of the rod. We know that the point of action of the gravitational force acting on this triangle is at its centroid, which is at the 2/3 of its median. Therefore the line of action of the gravitational force divides side AB in the ratio $1:2$. Similarly the point of action of the centrifugal force divides the rod in the ratio $1:2$, so it is at a distance of $2L/3$ from the end that is above the axis or $L/3$ from the end that touches the disk.



Equations that express the conditions for translational and rotational equilibrium can now be set up. The condition for translational equilibrium in the vertical direction is:

$$mg - K = 0, \qquad (1)$$

while in the horizontal direction we have:

$$F_{cf} - S = 0, \qquad (2)$$

where K and S are the normal and frictional forces exerted by the disk respectively, and F_{cf} is the centrifugal force calculated above. As $K = mg$ from equation (1) and

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After rearrangement, we have:

$$\omega^2\left(\frac{r}{2}-\frac{r}{3}\right)\cdot\tan\alpha=\frac{g}{2}.$$

From which the required angular velocity of the disk is:

$$\omega=\sqrt{\frac{3g}{r\cdot\tan\alpha}}=\sqrt{\frac{3g}{h}}=7.07\,\mathrm{s}^{-1}.$$

The normal force exerted by the disk on the rod is $K=mg$, while for the horizontal static friction that points towards the axis of rotation, we get:

$$S=mr\omega^2/2=mr\frac{3g}{h}=\frac{3}{2}\frac{r}{h}mg=\frac{3}{2}\cdot\frac{0.8}{0.6}mg=2mg$$

The angle formed by the net force exerted by the disk and the horizontal can be calculated as:

$$\tan\varepsilon=\frac{mg}{m\frac{r}{2}\omega^2}=\frac{2g}{r\omega^2}=\frac{2g}{r\cdot\frac{3g}{(r\cdot\tan\alpha)}}=\frac{2\cdot\tan\alpha}{3}=\frac{2h}{3r}=\frac{2\cdot 0.6}{3\cdot 0.8}=0.5,$$

thus

$$\varepsilon=\mathrm{arc\ tan}\,0.5=26.57^\circ,$$

while the magnitude of the net force exerted by the disk is:

$$T=\frac{mg}{\sin\alpha}=\frac{mg}{0.4472}=2.236\ mg,$$

or:

$$T=\sqrt{S^2+K^2}=\sqrt{4m^2g^2+m^2g^2}=\sqrt{5}\cdot mg=2.236\ mg,$$

The angle formed by the rod and the horizontal is:

$$\alpha=\mathrm{arc\ tan}\frac{h}{r}=\mathrm{arc\ tan}\frac{0.6}{0.8}=36.87^\circ$$

Note that the line of action of the resultant force exerted by the disk does not pass through the centre of mass of the rod (although in case of an equilibrium or simple translation it would), therefore the resultant torque with respect to the centre of mass is not zero, which causes the angular momentum of the rod to change throughout the motion.

Solution of Problem 98. a) First we find the relation between the acceleration and the angular acceleration. Let a be the acceleration of the centre of the rod, a_1 the acceleration of its endpoint to which the cord is attached, and let β denote the angular acceleration of the rod. Since the geometric centre of the rod coincides with its centre of mass, $a_1=a+l\beta/2$.

The equation of motion of the weight is:

$$m_1g-K=m_1a_1. \qquad (1)$$

external torques is zero, the angular momentum is conserved. In this case, the two disks rotate in opposite direction at the same angular speed while the rod stays at rest.

$$E_{\text{spring}} = E_{\text{rot}},$$
$$N = 0.$$

In details:

$$\frac{1}{2}D(\Delta l)^2 = 2\cdot\frac{1}{2}\cdot\Theta\omega_a^2,$$

and inserting the moment of inertia $\Theta = \frac{1}{2}mR^2$ of the disks, and simplifying by 2 we get that:

$$D(\Delta l)^2 = mR^2\omega_a^2.$$

Finally, the angular speed of the disks is:

$$\omega_a = \frac{\Delta l}{R}\sqrt{\frac{D}{m}} = \frac{0.05}{0.1}\sqrt{\frac{1800\ \text{N/m}}{4\ \text{kg}}} = 10.607\ 1/\text{s}.$$

Case b). In this case, the two disks rotate in the same direction, but because the sum of the external torques is zero, as in the previous case, the total angular momentum of the system must remain zero. This is possible only if the rod (of negligible mass), along with the two disks, begins rotating in the opposite direction. Thus, the sum of the two (equal) spin angular momenta of the disks and the orbital angular momentum of the whole system have the same size but opposite direction. (We remark that if the rod were fixed, then this constrain would exert an external torque and the Earth would take over the angular momentum.)

The mechanical energy is conserved:

$$\frac{1}{2}D(\Delta l)^2 = 2\cdot\frac{1}{2}\cdot\Theta\omega_b^2 + 2\cdot\frac{1}{2}mv_c^2, \tag{1}$$

and the angular momentum is conserved as well:

$$2\cdot\Theta\omega_b - 2\cdot mv_c\frac{d}{2} = 0, \tag{2}$$

where v_c is the speed of the centre of the disks:

$$v_c = \frac{d}{2}\cdot\Omega, \tag{3}$$

and Ω is the angular speed of the rotating rod. Inserting (3) into equation (2):

$$2\cdot\frac{1}{2}mR^2\omega_b = 2\cdot m\frac{d}{2}\cdot\Omega\cdot\frac{d}{2},$$

from which the angular speed of the rod is:

$$\Omega = 2\frac{R^2}{d^2}\omega_b, \quad \text{thus} \quad v_c = \frac{d}{2}\cdot\Omega = \frac{R^2}{d}\omega_b.$$

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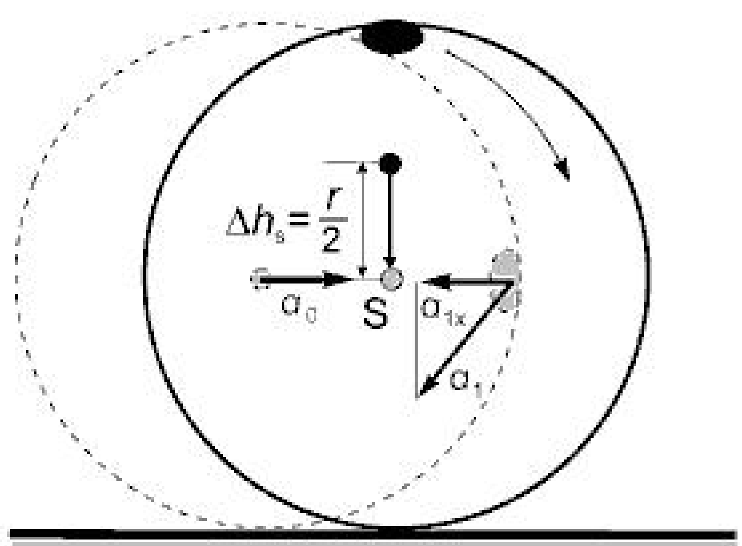
thus the ball hits the ground with a velocity of:

$$V=\omega\sqrt{R^2-r^2}=(R+r)\sqrt{\frac{2g(R-r)}{R^2+r^2(3m/2M-1)}}=1.3\,\frac{\mathrm{m}}{\mathrm{s}}.$$

First solution of Problem 105. Assuming that the hoop stays vertical, we are dealing with motion in a plane. Since the system starts from rest, in the absence of horizontal forces the common centre of mass S of hoop and weight will descend (with a non-uniform acceleration) along a vertical line. During the fall, if the centre of the hoop is displaced to the left of the vertical line drawn through the centre of mass, the weight will be displaced to the right. Since the hoop and the weight have equal masses, the centre of mass S is at the midpoint of the radius connecting the centre of the hoop to the weight, that is, at a distance of $r/2$ from each. Thus their horizontal motions (displacement, instantaneous velocity, the x component of acceleration) are symmetrical. The centre of the hoop does not accelerate vertically. Its horizontal acceleration vector first points to the left and then to the right.



a) In the position investigated by the problem, the centre of the hoop is at rest, its acceleration $\vec{a}_0$ is equal and opposite to the component $\vec{a}_{1x}$ of the acceleration of the weight. Therefore it is enough to determine the horizontal component of the acceleration of the weight.
Since the centre of the hoop is at rest (instantaneous axis of rotation), the speed of the centre of mass S is

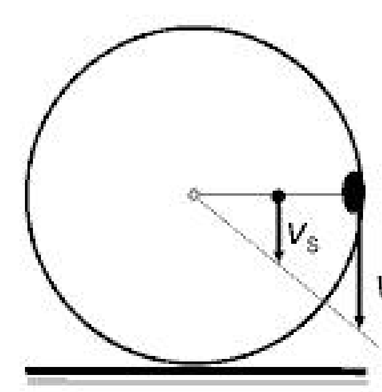
$$v_S=\frac{r}{2}\omega,$$

and the speed of the weight is

$$v_1=r\omega=2v_S, \tag{1}$$

both in the vertical direction. ω denotes the instantaneous angular velocity of the hoop. At the same time instant, the horizontal component of the acceleration of the weight is equal to the centripetal acceleration of its rotation about the centre of mass (since the horizontal acceleration of point S stays 0 throughout), that is,

$$a_{1x}=\frac{v_{\mathrm{rel}}^2}{\frac{r}{2}}=\frac{2v_{\mathrm{rel}}^2}{r}, \tag{2}$$



where, as obtained from (1), the speed of the weight relative to the centre of mass is

$$v_{\mathrm{rel}}=v_1-v_S=v_1-\frac{v_1}{2}=\frac{v_1}{2}. \tag{3}$$

The speed v_1 of the weight needs to be determined. The work-energy theorem can be used:

$$mg\Delta h=\frac{1}{2}mv_1^2+\frac{1}{2}\Theta_0\omega^2,$$

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The solution of the simultaneous equations provides the acceleration a_0 and the force K in question. The acceleration in $(9')$ is substituted in $(8')$:

$$K = 2mg - 2m\cdot\frac{r}{2}\cdot\omega^2\cdot\cos\alpha - 2m\cdot\frac{r}{2}\beta\cdot\sin\alpha.$$

The angular velocity ω is taken from $(6')$ and the angular acceleration β is taken from $(7')$:

$$K = 2mg - 2m\cdot\frac{r}{2}\cdot\frac{4g(1-\cos\alpha)}{r(4-\cos^2\alpha)}\cdot\cos\alpha - 2m\cdot\frac{r}{2}\cdot\frac{K\sin\alpha}{3mr}\cdot\sin\alpha.$$

r is cancelled in the second term, and $2mr$ is cancelled in the last term of the right-hand side. The terms containing K are transferred to the left-hand side and the common factors are pulled out on each side:

$$K\left(1+\frac{\sin^2\alpha}{3}\right) = 2mg\left(1-\frac{2(1-\cos\alpha)\cdot\cos\alpha}{4-\cos^2\alpha}\right).$$

$\sin^2\alpha$ is replaced with $1-\cos^2\alpha$, and common denominators are applied:

$$K\cdot\frac{4-\cos^2\alpha}{3} = 2mg\cdot\frac{4-\cos^2\alpha-2(1-\cos\alpha)\cos\alpha}{4-\cos^2\alpha}.$$

Hence the normal force in question is

$$K = 2mg\frac{3(4+\cos^2\alpha-2\cos\alpha)}{(4-\cos^2\alpha)^2}. \qquad (10')$$

The acceleration of the centre of the hoop is given by the horizontal component of its acceleration relative to the centre of mass, since the centre is not accelerating vertically:

$$a_0 = \frac{r}{2}\cdot\beta\cdot\sin\alpha - \frac{r}{2}\cdot\omega^2\cdot\sin\alpha = \frac{K\sin\alpha}{3mr}\cdot\frac{r}{2}\cdot\cos\alpha - \frac{4g(1-\cos\alpha)}{r(4-\cos^2\alpha)}\cdot\frac{r}{2}\cdot\sin\alpha.$$

With the substitution of the value of K from $(10')$, rearrangement and the use of a common denominator:

$$a_0 = \frac{[(4+\cos^2\alpha-2\cos\alpha)\cos\alpha-2(1-\cos\alpha)(4-\cos^2\alpha)]\sin\alpha}{(4-\cos^2\alpha)^2}\cdot g.$$

The answers to the problem's questions are obtained by substituting 90^0 for α:

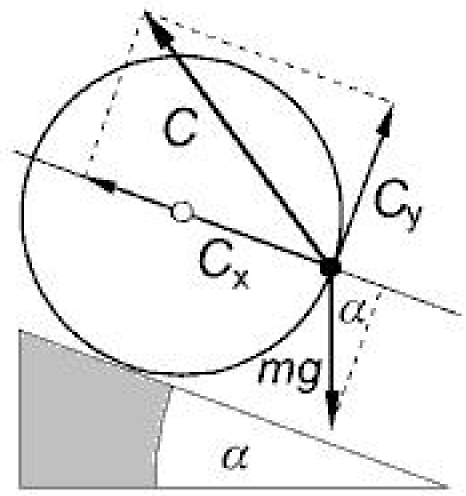
$$a_0 = -\frac{8}{16}g = -\frac{g}{2}, \qquad \text{and} \qquad K = \frac{24}{16}mg = \frac{3}{2}mg = 1.5\cdot 50\,\mathrm{N} = 75\,\mathrm{N}.$$

Third solution of Problem 105. The first question of the problem can be answered in an unusual way. The acceleration of the centre of the hoop at the time instant when the angular displacement of the weight is 90° is obtained directly as the normal acceleration calculated at the point where the tangent to its trajectory is vertical: $a_{1x} = v_1^2/\varrho$. The magnitude of the acceleration is determined as in Solution 1, and the centre of curvature ϱ of the trajectory is easily obtained by noticing that the motion of the common centre

a coordinate system that moves with the ring but does not rotate). The direction of the latter is opposite to the acceleration of the centre in the direction of the slope, so at this moment the acceleration of the beetle in the direction of the centre (that is, along the slope) is

$$a_{\mathrm{r}} = \frac{v^2}{R} - a,$$

and the acceleration in the direction of the tangent of the ring (that is, perpendicular to the slope) is a in the inertial reference frame. Let us take our coordinate system fixed to the slope with axis x being parallel with it and axis y being perpendicular to it. The acceleration of the beetle is caused by the gravitational force and the clinging force C, whose components have magnitudes C_x and C_y.



The equations of the motion of the beetle in the x and y directions are:

$$C_x - mg\sin\alpha = m\left(\frac{v^2}{R} - a\right), \quad (3)$$

$$mg\cos\alpha - C_y = ma. \quad (4)$$

Substituting (1) and (2) into (3) gives

$$C_x = mg\sin\alpha + m\frac{5}{2}\pi g\sin\alpha - m\frac{g\sin\alpha}{2} = \frac{1}{2}mg(1+5\pi)\sin\alpha,$$

with numerical values

$$C_x = 0.5\cdot 10^{-3}\,\mathrm{kg}\cdot 9.81\frac{\mathrm{m}}{\mathrm{s}^2}\cdot(1+5\pi)\sin 20^\circ = 0.028\,\mathrm{N},$$

and (4) gives

$$C_y = mg\left(\cos\alpha - \frac{\sin\alpha}{2}\right) = 10^{-3}\,\mathrm{kg}\cdot 9.81\frac{\mathrm{m}}{\mathrm{s}^2}\cdot(\cos 20^\circ - 0.5\cdot\sin 20^\circ) = 0.00754\,\mathrm{N}.$$

The magnitude of the resultant clinging force is

$$C = \sqrt{C_x^2 + C_y^2} = \sqrt{(28\cdot 10^{-3})^2\,\mathrm{N}^2 + (7.54\cdot 10^{-3})^2\,\mathrm{N}^2} =$$
$$= 28.997\cdot 10^{-3}\,\mathrm{N} \approx 29\cdot 10^{-3}\,\mathrm{N}.$$

(It can be seen that at this speed the radial acceleration is already dominant.)
The direction of the resultant force relative to the surface of the slope is

$$\tan\gamma = \frac{C_y}{C_x} = \frac{7.54}{28} = 0.2693 \quad \rightarrow \quad \gamma = \arctan 0.2693 = 15^\circ,$$

and relative to the ground, in a direction upwards along the slope it is $\gamma' = 20^\circ + 15^\circ = 35^\circ$.

Multiplying the expression in the bracket by Δt, we obtain:

$$mv_x = \mu(mg\Delta t + 2mv_y).$$

Since the collision occurs in a very short time ($\Delta t \to 0$), the first term in the bracket can be neglected. Thus the horizontal velocity (v_x) of the ball is found to be:

$$v_x = 2\mu v_y. \tag{3}$$

The time needed to decelerate the rotation of the ball equals the time of the deceleration of the horizontal translation, therefore the average angular deceleration is:

$$\beta = \frac{\omega}{\Delta\tau}.$$

This means that the net torque acting on the ball should be:

$$M = \Theta\beta,$$

substituting the average quantities, we obtain:

$$Sr = \Theta\frac{\omega}{\Delta\tau},$$

which yields:

$$\omega = \frac{Sr}{\Theta}\cdot\Delta\tau.$$

Substituting the friction from equation (2), we get:

$$\omega = \frac{\mu\left(mg + \frac{2mv_y}{\Delta t}\right)R}{\Theta}\cdot\Delta\tau.$$

If $\Delta t = \Delta\tau$ and $\Delta t \to 0$, the initial angular velocity of the ball is:

$$\omega = \frac{2\mu m v_y R}{\Theta}. \tag{4}$$

Using equations (3) and (4), we can now calculate the loss in the mechanical energy of the ball:

$$-\Delta E = \frac{mv_x^2}{2} + \frac{\Theta\omega^2}{2} = 2m\mu^2 v_y^2 + \frac{2m^2\mu^2 v_y^2 R^2}{\Theta}.$$

Substituting the expressions for the vertical component of velocity ($\sqrt{2gh}$) and the rotational inertia ($2mR^2/3$), we get that the maximum heat produced by the collision is:

$$Q = -\Delta E = 4\mu^2 mgh + 6\mu^2 mgh = 10\mu^2 mgh.$$

Inserting given data, the quantities used in the solution are: $v_y = 2\ \mathrm{m/s}$, $v_x = 1\ \mathrm{m/s}$, $\omega = 150\ \mathrm{s}^{-1}$, while the loss of mechanical energy or the maximum heat produced is:

$$Q_{\max} = 10\cdot 0.25^2\cdot 0.003\,\mathrm{kg}\cdot 10\,\frac{\mathrm{m}}{\mathrm{s}^2}\cdot 0.2\,\mathrm{m} = 3.75\cdot 10^{-3}\,\mathrm{J}.$$

Note that our result gives the upper limit of the heat produced in the collision. It is possible that the time of deceleration of the horizontal translation and rotation is less

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The angular acceleration of the wheel of fortune is

$$\beta_{\mathrm{wf}} = \frac{5g}{2R}\sin\alpha.$$

This is caused by the signed sum of the torques acting on it:

$$M - M_{\mathrm{sf}} = \Theta_{\mathrm{wf}}\beta_{\mathrm{wf}},$$

where $M_{\mathrm{sf}} = F_f R$. For the torque produced

$$M = F_f R + \Theta_{\mathrm{wf}} \cdot \frac{a_t}{R} = mg\sin\alpha \cdot R + \Theta_{\mathrm{wf}} \cdot \frac{5}{2}\frac{g}{R}\sin\alpha.$$

Finally, the rotational inertia of the wheel of fortune should be determined. Let us determine the mass of a unit surface area. The total surface area of the wheel of fortune is the sum of the area of the nappe, which is $\frac{R}{2} \cdot 2R\pi$, and the area of the two bases, which is $2 \cdot R^2\pi$. With this the mass of a unit surface area is

$$\varrho = \frac{m}{A} = \frac{m}{2R^2\pi + \frac{R}{2}2R\pi} = \frac{m}{3R^2\pi}.$$

The rotational inertia is additive:

$$\Theta_{\mathrm{nappe}} = m_n R^2 = \varrho A_n R^2 = \frac{m}{3R^2\pi} \cdot 2R\pi\frac{R}{2} \cdot R^2 = \frac{m}{3}R^2,$$

$$\Theta_{\mathrm{base}} = \frac{1}{2}m_b R^2 = \frac{1}{2}2\varrho \cdot A_b R^2 = \cdot\varrho R^2\pi \cdot R^2 = \frac{m}{3R^2\pi} \cdot R^2\pi R^2 = \frac{m}{3}R^2.$$

(The two are accidentally equal.)

With these the rotational inertia of the wheel of fortune is

$$\Theta_{\mathrm{wf}} = \frac{2}{3}mR^2.$$

The torque that should be produced by us is

$$M = mg\sin\alpha R + \frac{2}{3}mR^2 \cdot \frac{5}{2}\frac{g}{R}\sin\alpha = mgR\sin\alpha\left(1 + \frac{5}{3}\right) = \frac{8}{3}mgR\sin\alpha =$$

$$= \frac{8}{3}20 \cdot 0.54 \cdot \frac{\sqrt{7}}{4}\ \mathrm{Nm} \approx 19.1\ \mathrm{Nm}.$$

b) The work done by us is

$$W = \Delta E_{\mathrm{wf}} + \Delta E_{\mathrm{ball}} = \frac{1}{2}\Theta_{\mathrm{wf}}\omega_{\mathrm{szk}}^2 + \frac{1}{2}\Theta_{\mathrm{ball}}\omega_{\mathrm{ball}}^2 = \frac{1}{2} \cdot \frac{2}{3}mR^2\omega_{\mathrm{wf}}^2 + \frac{1}{2} \cdot \frac{2}{5}mr^2\omega_{\mathrm{ball}}^2.$$

Because of the constraining condition $R^2\omega_{\mathrm{wf}}^2 = r^2\omega_{\mathrm{ball}}^2$, that is,

$$W = \frac{1}{3}mR^2\omega_{\mathrm{wf}}^2 + \frac{1}{5}mR^2\omega_{\mathrm{wf}}^2 = mR^2\omega_{\mathrm{wf}}^2\left(\frac{1}{3} + \frac{1}{5}\right) = \frac{8}{15}mR^2 \cdot \omega_{\mathrm{wf}}^2.$$

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a) The rolling times are equal, if the accelerations are the same. From equation (I.) we can see that the accelerations depend only on the moments of inertia, so these latter quantities have to be calculated.

The moment of inertia of the solid sphere is:

$$\Theta_S = \frac{2}{5} m R^2,$$

while that of the hollow sphere (or spherical shell) is (from data table):

$$\Theta_H = \frac{2}{5} m \frac{R^5 - r^5}{R^3 - r^3},$$

where R is the outer, r is the inner radius. In our case $r = R/2$, which means that:

$$\Theta_H = \frac{2}{5} m \frac{R^5 - \left(\frac{R}{2}\right)^5}{R^3 - \left(\frac{R}{2}\right)^3} = \frac{2}{5} m R^2 \frac{\frac{32-1}{32}}{\frac{8-1}{8}} = \frac{2}{5} m R^2 \frac{31 \cdot 8}{32 \cdot 7} = \frac{2}{5} m R^2 \frac{31}{28} = \frac{31}{28} \Theta_S.$$

Expressing Θ_H with the mass and the radius,

$$\Theta_H = \frac{31}{28} \Theta_T = \frac{31}{28} \cdot \frac{2}{5} m R^2 = \frac{31}{70} m R^2.$$

Now substituting the values Θ_S and Θ_H into equation (I.), a relationship can be derived between the slopes α_T and α_H of the incline in the two cases:

$$a_S = \frac{m g \sin\alpha_S}{m + \frac{\Theta_S}{R^2}} = \frac{m g \sin\alpha_H}{m + \frac{\Theta_S}{R^2}} = a_H.$$

Inserting here the moments of inertia:

$$\frac{m g \sin\alpha_S}{m + \frac{\frac{2}{5} m R^2}{R^2}} = \frac{m g \sin\alpha_H}{m + \frac{\frac{31}{70} m R^2}{R^2}}.$$

After cancellation, we get that:

$$\frac{\sin\alpha_S}{1 + \frac{2}{5}} = \frac{\sin\alpha_H}{1 + \frac{31}{70}} \quad \rightarrow \quad \frac{5 \sin\alpha_S}{7} = \frac{70 \sin\alpha_H}{101},$$

which means that the slope of the incline in the second experiment is:

$$\sin\alpha_H = \frac{505}{490} \sin\alpha_S = \frac{505}{490} \sin 30^\circ = 0.5153,$$

so

$$\alpha_H = \arcsin 0.5153 = 31.8^\circ.$$

b) The threshold values for the coefficients of static friction are obtained by writing the moments of inertia into the equation (III.). We obtain that the solid sphere is rolling without sliding if:

$$\mu_{S0} \geq \frac{\tan\alpha_S}{\frac{mR^2}{\Theta_S} + 1} = \frac{\tan\alpha_S}{\frac{mR^2}{\frac{2}{5} m R^2} + 1} = \frac{2 \tan\alpha_S}{7} = \frac{2 \tan 30^\circ}{7} = 0.165.$$

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from which we get that the minimum coefficient of friction needed for the disk to roll without slipping is:

$$\mu = \frac{M}{3M+m} \cdot \frac{F}{mg} = \frac{5\,\mathrm{kg}}{25\,\mathrm{kg}} \cdot \frac{100\,\mathrm{N}}{100\,\mathrm{N}} = \frac{1}{5} = 0.2 > 0.1.$$

This means that in our case the disk slips on the cart. After having ascertained this, let us investigate the motion of the system.

a) As the disk slips, one equation (the restraining condition) is 'lost', but at the same time a new one is gained, since the kinetic friction can be written in the form of $S = \mu mg$. Therefore, our equations will be:

$$F + \mu mg = ma \tag{5}$$

$$\mu mg = MA \tag{6}$$

$$Fr - \mu mgr = \frac{1}{2} mr^2 \beta. \tag{7}$$

Our three unknowns (a, A and β) can easily be calculated from the above equations one by one:

$$a = \frac{F}{m} + \mu g = \frac{100\,\mathrm{N}}{10\,\mathrm{kg}} + 0.1 \cdot 10\,\frac{\mathrm{m}}{\mathrm{s}^2} = 11\,\frac{\mathrm{m}}{\mathrm{s}^2} \text{ to the right} \tag{8}$$

$$A = \mu \frac{m}{M} g = 0.1 \cdot \frac{10\,\mathrm{kg}}{5\,\mathrm{kg}} \cdot 10\,\frac{\mathrm{m}}{\mathrm{s}^2} = 2\,\frac{\mathrm{m}}{\mathrm{s}^2} \text{ to the left} \tag{9}$$

$$\beta = \frac{2F}{mr} - \mu \frac{2g}{r} = \frac{200\,\mathrm{N}}{10\,\mathrm{kg} \cdot 0.2\,\mathrm{m}} - 0.1 \cdot \frac{20\,\mathrm{m/s}^2}{0.2\,\mathrm{m}} = 90 \frac{1}{\mathrm{s}^2}. \tag{10}$$

b) The kinetic energy of the system is the sum of the kinetic energies of the two objects. The energy of the disk is the sum of its translational and rotational energy:

$$E_{\text{disk}} = \frac{1}{2} mv^2 + \frac{1}{2} \Theta \omega^2, \tag{11}$$

while the energy of the cart is:

$$E_{\text{cart}} = \frac{1}{2} MV^2. \tag{12}$$

Let us therefore calculate the speed of the cart and the angular velocity of the disk at the moment when the length of the unwound string becomes $L = 2\,\mathrm{m}$.

Let t be the time needed for the string to reach an unwound length of L. During this time the disk rotates through an angle of:

$$\varphi = \frac{1}{2} \beta t^2,$$

where $\varphi = L/r$, which yields

$$t = \sqrt{\frac{2\varphi}{\beta}} = \sqrt{\frac{2L}{r\beta}}.$$

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Newton's second law for the rotational motion of the ball (R is the radius of the ball, β is the angular acceleration of the ball):

$$SR = \frac{2}{5} m R^2 \beta, \tag{3}$$

The constraints for rolling without sliding are:

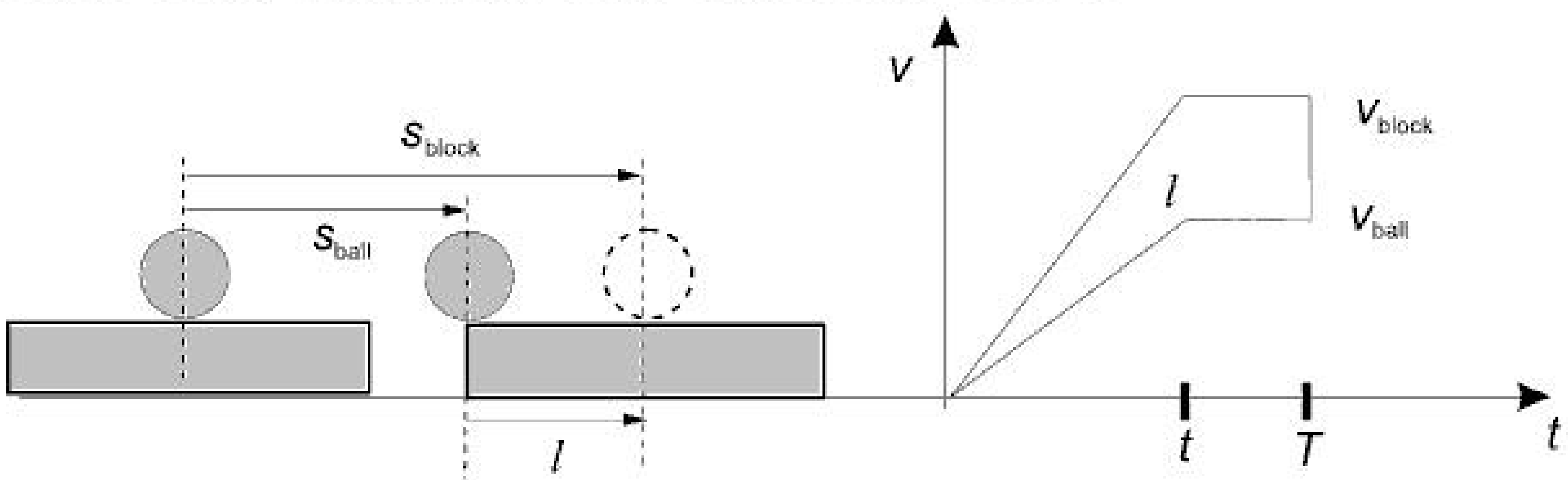
$$A = a + R\beta, \qquad \text{and} \qquad \beta = \frac{A-a}{R}. \tag{4}$$

substituting (4) into (3), cancelling R, and then adding (1) and (2) and expressing S from (2) and substituting it into (3), the acceleration of the ball and the block will be:

$$a = \frac{2F}{2m+7M}$$

$$A = \frac{7}{2}a = \frac{7F}{2m+7M}.$$

The displacements of the ball and the block are shown in the figure. The area of the shaded region of the velocity–time graph is proportional to the distance covered by the ball on the block, which is half of the length of the block l.



The speeds of the block and the ball with respect to the ground at the end of the accelerating period are:

$$V = At, \qquad \text{and} \qquad v = at.$$

the distances covered by the block and the ball during the time T are:

$$s_{\text{block}} = \frac{1}{2}At^2 + At(T-t),$$

$$s_{\text{ball}} = \frac{1}{2}at^2 + at(T-t),$$

where t is the time during which the force is exerted on the block, $T-t$ is the time of uniform motion until the ball falls off. The ball falls off if $l = s_{block} - s_{ball}$, so the equation for the asked time T is:

$$l = \frac{1}{2}(A-a)t^2 + (A-a)t(T-t),$$

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Substituting equations (1), (3) and (4) into the above equation, we find:

$$F = 2m\frac{\mu}{\mu_0}\cdot\frac{1-\mu_0}{1-\mu}g + \frac{\mu}{1-\mu}mg + \mu\frac{1-2\mu}{1-\mu}mg.$$

After rearranging the expression, we get:

$$F = \frac{2\mu(1-\mu_0\mu)}{\mu_0(1-\mu)}mg = \frac{2\cdot 0.2(1-0.6\cdot 0.2)}{0.6(1-0.2)}\cdot 12\,\mathrm{kg}\cdot 10\,\frac{\mathrm{m}}{\mathrm{s}^2} = 88\,\mathrm{N}.$$

So the motion of the cylinder will be purely translational if the pushing force is greater than $88\,\mathrm{N}$. Note that the system requires a minimum of $F_0 = 2mg\mu_0 = 144\,\mathrm{N}$ to start moving, therefore if the cube and cylinder begin moving, the pushing force can be decreased to $88\,\mathrm{N}$ to keep the cylinder's motion purely rotational. If the magnitude of the pushing force is decreased further, the cylinder will start rolling, rubbing the side of the cube. If the force becomes less than $F = 2mg\mu = 48\,\mathrm{N}$, the system stops. The magnitudes of other quantities when the force is $F = 88\,\mathrm{N}$ are: $a = \frac{1}{6}g = 1.67\,\frac{\mathrm{m}}{\mathrm{s}^2}$, $S_1 = 30\,\mathrm{N}$, $S_2 = 18\,\mathrm{N}$, $K_1 = 150\,\mathrm{N}$, $K_2 = 50\,\mathrm{N}$, $K_3 = 90\,\mathrm{N}$, $\sum F = ma = 20\,\mathrm{N}$.

Solution of Problem 119. Let us find out what happens. Each of the three objects will remain at rest if both $m_3 g < \mu_2 m_2 g$ and $m_3 g < \mu_1(m_1+m_2)g$ are true. Since in our case none of the above inequalities hold, this is not the solution of the problem. The board will start to move if

$$\mu_2 m_2 g > \mu_1(m_1+m_2)g.$$

Since the data given satisfies this inequality, we can state that the board starts to move. The next step is to decide whether the brick slips on the board or moves together with it.

Let us assume that the brick slips on the board. In this case both frictions take their maximum values, thus applying Newton's second law to each of the three objects and eliminating the tension, we get that the accelerations of the board and brick are:

$$a_1 = \frac{\mu_2 m_2 g - \mu_1(m_1+m_2)g}{m_2} = 0.15g,$$

$$a_2 = \frac{m_3 g - \mu_2 m_2 g}{m_2+m_3} = 0.1g.$$

This means that the acceleration of the board would be greater than that of the brick, which is impossible. Therefore, the only solution of the problem is that the board and brick move together with the same acceleration. In this case the magnitude of friction between the board and brick can be anything between zero and its maximum value. From the laws of motion of the two objects, we get that their acceleration is:

$$a = \frac{m_3 - \mu_1(m_1+m_2)}{m_1+m_2+m_3}\cdot g = 0.12g = 1.18\,\frac{\mathrm{m}}{\mathrm{s}^2}.$$

The magnitude of static friction between the board and brick is:

$$S = \frac{m_1 m_3 + \mu_1(m_1+m_2)(m_2+m_3)}{m_1+m_2+m_3} = 6.28\,\mathrm{N},$$

After multiplying by the denominator and expanding the numerator on the right-hand side, we obtain:

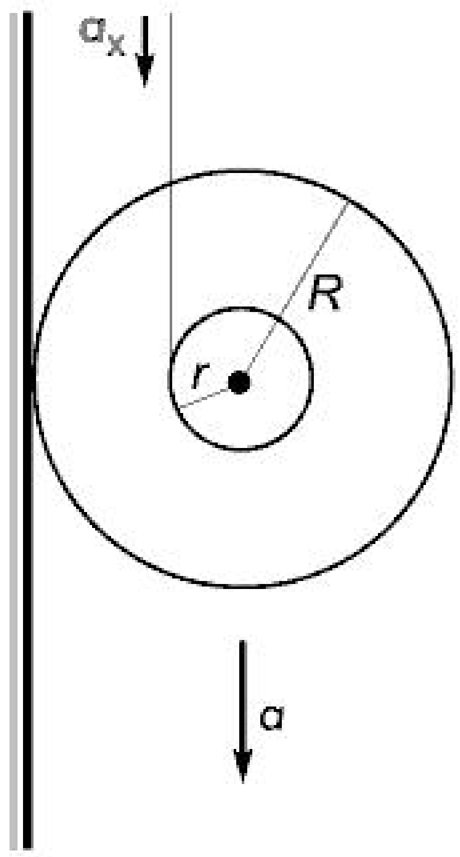
$$m_2 = m_1 \frac{\Theta}{\Theta + m_1(r_1 - r_2)(r_1 + r_2)} = m_1 \frac{\Theta}{\Theta + m_1(r_1^2 - r_2^2)}.$$

Substituting known values, we find:

$$m_2 = 5\,\mathrm{kg} \frac{0.25}{0.25 + 5(0.09 - 0.04)} = 2.5\,\mathrm{kg}.$$

(According to equation (4), the angular acceleration of the disks is: $\beta = 12.5\ \mathrm{s}^{-1}$. The tension in the cords can be calculated from equation (1): $F = 31.25\,\mathrm{N}$.)

Solution of Problem 124. The printed letters will be clear if the cylinder rolls on the wall without slipping. (In reality printing obviously also requires the cylinder to be pushed onto the wall, but in our case this is impossible as the cords are vertical. If there was a normal force acting to push the cylinder onto the wall, the problem would become inexplicit because of the presence of static friction, therefore the solution for the acceleration would be an interval instead of a single value.)



If the cords were wrapped around the cylindrical surface, the cylinder could be made to roll without slipping by simply keeping the ends of the cords in our stationary hands. In this case, however, the cords run down from disks of smaller radius than R, and therefore the tangential acceleration of an arbitrary point on the cylindrical surface in a reference frame attached to the axis of the cylinder is greater than the acceleration of the axis of the cylinder in a reference frame attached to the wall. Therefore if the ends of the cords were held stationary, the letters on the cylindrical surface would move upwards relative to the wall, which would make the text printed onto the wall smudgy. This means that the ends of the cords should be accelerated downwards to achieve clear printing (rolling without slipping).

Let a be the acceleration of the axis (or centre of mass) of the cylinder, a_x be the acceleration of the ends of the cords and K be the tension in the cords. Applying Newton's second law to the cylinder, we have:

$$mg - K = ma. \tag{1}$$

The cylinder rotates with a constant angular acceleration. Writing Newton's second law in angular form (torques are taken about the centre of mass), we obtain:

$$Kr = \Theta\beta, \tag{2}$$

where β is the angular acceleration of the cylinder. If the cylinder rolls on the wall without slipping, the connection between its acceleration and angular acceleration is given by:

$$a = R\beta. \tag{3}$$

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be calculated from the equation: $mg - D\Delta l_0 = 0$, thus: $\Delta l_0 = mg/D$. (D denotes the spring constant.) Therefore our equations are:

$$K_2 - mg = ma \tag{1}$$

$$(K_1 - K_2)R = \Theta \cdot \beta \tag{2}$$

$$K_1 - D\Delta l_1 = 0 \tag{3}$$

$$\Delta l_0 - \frac{mg}{D} = 0. \tag{4}$$

The thread does not slide on the disc thus $\beta = a/R$, so equation (2) can be written in the form:

$$K_1 - K_2 = \frac{\Theta}{R^2} a. \tag{2'}$$

Adding equation (1) and (2) and substituting K_1 from equation (3) we gain:

$$D\Delta l_1 - mg = ma + \Theta \frac{a}{R^2} = \left(m + \frac{\Theta}{R^2}\right) a. \tag{5}$$

Expressing Δl_1 in terms of the extension with respect to the initial elongation equation (5) will be the following:

$$D\Delta l_0 + D\Delta l - mg = a\left(m + \frac{\Theta}{R^2}\right).$$

Considering equation (4):

$$mg + D\Delta l - mg = D\Delta l = a\left(m + \frac{\Theta}{R^2}\right).$$

In this equation a is the acceleration of the small body. This can be expressed with the displacement of the small body $\Delta h = -\Delta l$ and finding the common denominator the following is gained:

$$-D\Delta h = a \cdot \frac{mR^2 + \Theta}{R^2}$$

from this the acceleration of the small body is:

$$a = -\frac{DR^2}{mR^2 + \Theta} \cdot \Delta h,$$

so the acceleration of the small body is proportional to the displacement and oppositely directed, which means that the system undergoes simple harmonic motion, applying Newton's second law:

$$\sum F = ma = -\frac{mDR^2}{mR^2 + \Theta} \cdot \Delta h.$$

The spring constant which may characterize this motion is:

$$D^* = \frac{mDR^2}{mR^2 + \Theta}$$

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following way: by setting up the work–kinetic energy theorem between the lowest (that is, stationary) position and position 4:

$$\Sigma W = \Delta E_{\text{kin}}.$$

The left side of the equation contains the sum of the works done by the spring and by the gravitational force. The work done by the spring is positive until it becomes loose and then it is negative, the work done by the gravitational force is negative throughout the process.

$$\frac{1}{2}k(\Delta l_0 + \Delta l_1)^2 - \frac{1}{2}k(\Delta l_0)^2 - mg(\Delta l_1 + 2\Delta l_0) = \frac{1}{2}mv^2 - 0.$$

After squaring and multiplying by 2:

$$k(\Delta l_0)^2 + 2k\Delta l_0 \Delta l_1 + k(\Delta l_1)^2 - k(\Delta l_0)^2 - 2mg\Delta l_1 - 4mg\Delta l_0 = mv^2.$$

After combining the like terms and substituting $\Delta l_0 = mg/k$:

$$2k\frac{mg}{k}\Delta l_1 + k(\Delta l_1)^2 - 2mg\Delta l_1 - 4mg\frac{mg}{k} = mv^2.$$

The sum of the first and third terms is 0, dividing by m and taking the square root, the requested velocity is

$$v = \sqrt{\frac{k}{m}(\Delta l_1)^2 - 4\frac{mg^2}{k}} = \sqrt{\frac{k}{m}[(\Delta l_1)^2 - 4(\Delta l_0)^2]} =$$

$$= \sqrt{\frac{20}{0.1}\cdot(0.15^2 - 4\cdot 0.05^2)\ \frac{\text{m}^2}{\text{s}^2}} = \sqrt{2.5}\ \frac{\text{m}}{\text{s}} = 1.58\ \frac{\text{m}}{\text{s}},$$

the same as the previous result.

Solution of Problem 129. As the hung body causes an elongation of Δl of the spring in equilibrium:

$$m \cdot g = D \cdot \Delta l. \tag{1}$$

While moving, the body undergoes forced oscillation. The frequency of the external shocks is equal to the natural frequency of the elastic system because of the 'high amplitude' (resonance). The shocks follow each other with a time interval

$$T_g = \frac{x}{v}$$

due to the small shocks received at the fittings, when the first wheels of the truck arrive at the fittings.

In the case of resonance the natural period of the system is equal to the period of the external shocks:

$$T_g = 2\pi\sqrt{\frac{m}{D}} = \frac{x}{v}, \tag{2}$$

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The extension of the spring at that time instant is

$$\Delta L_1 = \frac{mg}{D} = \frac{20\,\mathrm{N}}{80\,\frac{\mathrm{N}}{\mathrm{m}}} = 0.25\,\mathrm{m}.$$

The distance of the upper end of the spring above ground as a function of time is

$$h = L_0 + v_0 t = 0.6\,\mathrm{m} + 0.5\,\frac{\mathrm{m}}{\mathrm{s}} \cdot t.$$

To be able to see what happens next, let us temporarily attach the reference frame to the uniformly moving upper end of the spring. From that point of view, the object is seen to recede at a uniform speed of $-v_0$ until equilibrium is reached and then to perform a simple harmonic oscillation with a maximum speed of $-v_0$. Therefore, the situation is analogous to that of an object hanging in equilibrium on a spring stretched by $\Delta L_1 = mg/D$ and suddenly given a downward speed of v_0 to make it oscillate harmonically.

Since the maximum speed and angular frequency of the oscillation are determined by the given information, the amplitude can be calculated. The angular frequency is

$$\omega = \sqrt{D/m} = \sqrt{80\,\mathrm{N\,m^{-1}}/2\,\mathrm{kg}} = \sqrt{40}\,\mathrm{s^{-1}} \approx 6.32\,\mathrm{s^{-1}}.$$

Given the maximum speed and the angular fequency, the amplitude is obtained from the equation

$$v_0 = v_{\max} = A\omega = A\sqrt{D/m}$$

as follows:

$$A = v_0\sqrt{\tfrac{m}{D}} = 0.5\,\tfrac{\mathrm{m}}{\mathrm{s}}\sqrt{\tfrac{2\,\mathrm{kg}}{80\,\mathrm{N/m}}} = 7.91\cdot 10^{-2}\,\mathrm{m}.$$

The length of the spring at the time instant when the object starts to rise is

$$L_0 + \Delta L_1 = L_0 + \frac{mg}{D} = 0.6\,\mathrm{m} + \frac{20}{80}\,\mathrm{m} = 0.85\,\mathrm{m}.$$

The period is

$$T = 2\pi\sqrt{\tfrac{m}{D}} = \tfrac{2\pi}{\omega} = \tfrac{2\pi}{\sqrt{40}}\,\mathrm{s} \approx 1\,\mathrm{s}.$$

After the time instant t_1 when the oscillation starts, the instantaneous height y of the object, the length L of the spring, the extension ΔL of the spring and the speed v of the object are given by the functions (1), (2), (3) and (4), respectively:

$$y = v_0(t-t_1) - v_0\sqrt{\tfrac{m}{D}}\cdot\sin\left[\sqrt{\tfrac{D}{m}}(t-t_1)\right].$$

With the factor v_0 taken out:

$$y = v_0\left\{t - t_1 - \sqrt{\tfrac{m}{D}}\cdot\sin\left[\sqrt{\tfrac{D}{m}}(t-t_1)\right]\right\}. \tag{1}$$

$$L = h - y = L_0 + v_0 t - v_0(t-t_1) + v_0\sqrt{\tfrac{m}{D}}\cdot\sin\left[\sqrt{\tfrac{D}{m}}(t-t_1)\right].$$

With the factor v_0 taken out:

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so the amplitude of the oscillation is

$$A=\sqrt{\frac{mv_0^2}{D}}=\sqrt{\frac{1\ \text{kg}\cdot 1\ \frac{\text{m}^2}{\text{s}^2}}{100\ \frac{\text{N}}{\text{m}}}}=0.1\ \text{m}.$$

It means that the longest distance between the points A and B is

$$L_0+A=0.5\ \text{m}+0.1\ \text{m}=0.6\ \text{m}=60\ \text{cm}.$$

b) Until the body catches up to point B, it performs a half period of the oscillation, and then—when the rubber thread gets loose—it covers the distance L_0 to the point B at a constant speed v_0, so the unknown time is:

$$t=\frac{T}{2}+\frac{L_0}{v_0}=\pi\sqrt{\frac{m}{D}}+\frac{L_0}{v_0}=\pi\sqrt{\frac{1\ \text{kg}}{100\ \frac{\text{N}}{\text{m}}}}+\frac{0.5\ \text{m}}{1\ \frac{\text{m}}{\text{s}}}=0.814\ \text{s}.$$

Solution of Problem 134. a) The length of the pipe determines the wavelength of the standing wave in it. Thus the wavelengths are the same in the two cases. From this point of view it is not important whether the pipe is open or closed at the end. The frequencies of the sounds differ because the speed of sound is not the same in air and in helium.

The speed of sound can be obtained from data tables. For example at $0°\text{C}$ it is:

$$c_{\text{air}}=331.8\ \text{m/s}$$
$$c_{He}=970\ \text{m/s}.$$

The relation between the sound speed and wavelength is $c=\nu\lambda$, which means that the ratio of the two frequencies is:

$$\frac{c_{He}}{c_{\text{air}}}=\frac{\nu_{He}\lambda}{\nu_{\text{air}}\lambda}=\frac{\nu_{He}}{\nu_{\text{air}}}=\frac{970}{331.8}=2.923.$$

This ratio is approximately $3:1$, so the sound in helium is by a tritave, i.e., by an octave and a fifth higher than the normal a' tone. It is close to the e''' tone, do–do–sol. (More precisely, by half of a half tone, i.e., by $\sqrt[24]{2}$ lower than this: $\sqrt[24]{2}\cdot 2.923\approx 3.00$. Only a few people would notice this difference.)

The frequency of the sound in helium is $\nu_{He}=1286$ Hz.

b) In the case of the open pipe half of the wavelength of the fundamental mode is equal to the length of the pipe, while in the case of the closed pipe the quarter of the wavelength gives the pipe length. So the length of the pipe depends only on whether or not it is open or closed, and not on the gas we blow into it.

Performing the calculations with air, the lengths of the different pipes are:

$$l_{\text{open}}=\frac{\lambda}{2}=\frac{c_{\text{air}}}{2\nu_{\text{air}}}=\frac{331.8\ \text{m/s}}{2\cdot 440\ 1/\text{s}}=0.377\ \text{m}=37.7\ \text{cm}.$$

$$l_{\text{closed}}=\frac{\lambda}{4}=\frac{c_{\text{air}}}{4\nu_{\text{air}}}=\frac{l_{\text{open}}}{2}=18.85\ \text{cm}.$$

According to equation (1), the kinetic energy of the spaceship in a circular orbit of radius $2r_1$ is:

$$E_{\text{kin}_2} = G \cdot \frac{Mm}{4r_1}, \tag{3}$$

while its potential energy is:

$$E_{\text{pot}_2} = -G\frac{Mm}{2r_1}. \tag{4}$$

The total energy in the circular orbit of radius $r_2 = 2r_1$ is $E_{total_2} = E_{\text{total}_1} + \Delta E_{\text{kin}}$, since in point A, where the magnitude of velocity changes instantaneously, the potential energy does not change yet, therefore:

$$E_{\text{kin}_2} + E_{\text{pot}_2} = E_{\text{kin}_1} + E_{\text{pot}_1} + \Delta E_{\text{kin}}.$$

Substituting equations (1), (2), (3) and (4), we obtain:

$$G\frac{Mm}{4r_1} - G\frac{Mm}{2r_1} = G\frac{Mm}{2r_1} - G\frac{Mm}{r_1} + \Delta E_{\text{kin}}.$$

Hence the increase in the kinetic energy of the spaceship in point A is:

$$\Delta E_{\text{kin}} = \frac{1}{4}G\frac{Mm}{r_1} = \frac{1}{2}\cdot G\frac{Mm}{2r_1} = \frac{1}{2}E_{\text{kin}_1},$$

which means that during the first course correction in point A the kinetic energy of the spaceship should be increased by 50%.

b) After the first course correction the spaceship moves in an elliptical orbit in which its total mechanical energy remains constant. The second task is to change the elliptical orbit into a circular orbit. Since during the second correction only the direction of the velocity is changed, the total mechanical energy of the spaceship is the same in its new circular orbit as it was in the transfer ellipse.

According to Kepler's laws, the total mechanical energy of a spaceship orbiting the Earth depends on only the semimajor axis of its orbit. Since the mechanical energies of the spaceship in the transfer ellipse (between points A and C) and in the outer circle (of radius r_2) are the same, the semimajor axes of the two orbits should also be equal. In the case of the outer circle the semimajor axis is $a_{\text{circ}} = 2r_1$, therefore the semimajor axis of the transfer ellipse is also $a_{\text{ell}} = 2r_1$. By measuring distance $2r_1$ starting from point A onto the line going through point F, we get the centre of the transfer ellipse, which is point O on the inner circle. Point C, which is the point where the elliptical orbit intersects the line perpendicular to AF and going through point O, is the end of the semiminor axis of the transfer ellipse. This is the point where the second course correction should be carried out. (The first appropriate moment mentioned in the problem is therefore the moment when the spaceship reaches point C.) Since $FC = 2r_1$ and $FO = r_1$, the angle enclosed by them is $\varphi = 30^0$. This is the angle by which the direction of the velocity should be changed. During the second course correction the total mechanical energy of the spaceship remains constant in a translating (approximately

speed of the Earth and the radius of the orbit of the spaceship according to Newton's second law:

$$G \cdot \frac{Mm}{r^2} = mr\omega^2,$$

where $G = 6.672 \cdot 10^{-11}\ \frac{\mathrm{N \cdot m^2}}{\mathrm{kg^2}}$ is the gravitational constant, $M = 5.974 \cdot 10^{24}\,\mathrm{kg}$ is the mass of the Earth, m is the mass of the spacehip, and r is the asked distance between the spaceship and the centre of the Earth. Using the above equation the value of this radius is:

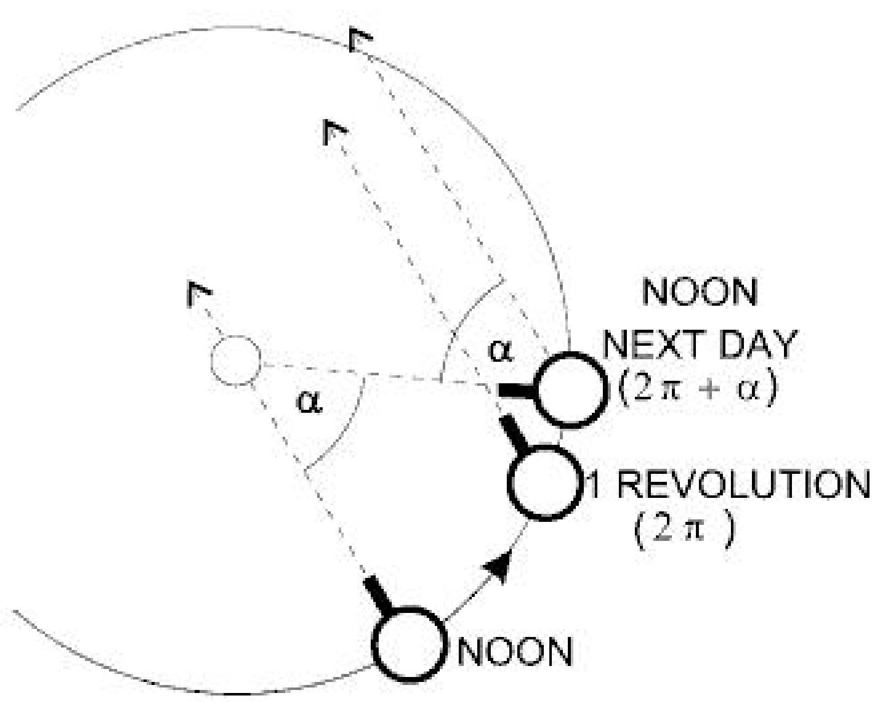
$$r = \sqrt[3]{\frac{GM}{\omega^2}},$$

and thus the speed of the spaceship with respect to the reference frame which is moving with the Earth (but not rotating) is $v = r\omega$.

ω is equal to the angular speed of the Earth. Because $\omega = 2\pi/T$, where T is the period of the rotation of the Earth. If this is known, the speed of the spaceship can be calculated:

$$v = \omega \sqrt[3]{\frac{GM}{\omega^2}} = \sqrt[3]{GM\omega} = \sqrt[3]{\frac{G2\pi M}{T}}.$$

What is the value of T? It can be calculated easily if it is not found in a table. The time between two consecutive solar noons is known: it is 24 hours, which is 86400 seconds. This is called a solar day. This is longer than the amount of time it takes the Earth to complete one revolution about its axis, because while it revolves 360° with respect to the inertial frame of reference, its centre moves forward along its orbit about the Sun, thus the Earth has to rotate more than 360° in order that the Sun should be above the same point again. Let us determine the period of the rotation in the inertial frame of reference (sidereal day) in solar seconds.



Let α be the angle subtended by the arc along which the Earth moves during one solar day to the Sun. During this time, the Earth turns an angle of

$$\varphi = 2\pi + \alpha = \omega T_{So},$$

where ω is the angular speed of the rotation of the Earth in an inertial frame of reference, and T_{So} is a solar day.

The angle turned during one day α, counting 365.26 solar days in one year is

$$\alpha = \frac{2\pi}{365.26},$$

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from which the acceleration of the blocks is: $a=\frac{m_0-m}{m_0+m}\cdot g$.

Either of the first two equations can be used to determine the tensions in the string:

$$K=mg+ma=mg\left(1+\frac{m_0-m}{m_0+m}\right)=\frac{2m_0m}{m_0+m}g.$$

Taking the double of this, we get the resultant of the two tensions, which is equal to the gravitational force m_0g acting on the other end of the lever, so:

$$\frac{4mm_0}{m_0+m}g=m_0g.$$

Thus $m=\frac{m_0}{3}$.

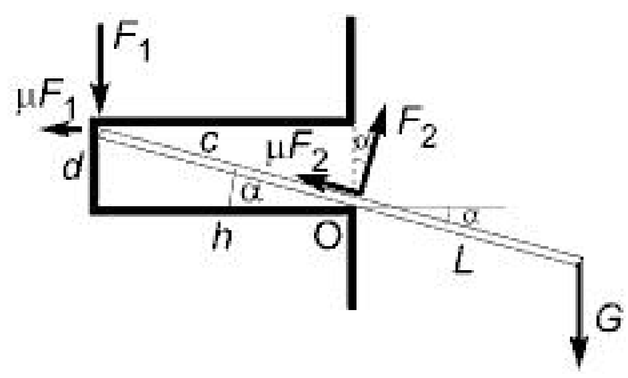
The acceleration of the two blocks is:

$$a=\frac{m_0-m_0/3}{m_0+m_0/3}g=\frac{2}{3}g.$$

Solution of Problem 150. The shape of the hole can be described by angle α, for which:

$$\tan\alpha=\frac{d}{h},$$

where d is the diameter and h is the depth of the hole. Let c and L be the lengths of the parts of the rod that are inside and outside the hole respectively, and G be the weight of the coat hanged onto the end of the rod. In the extreme case the maximum frictional forces act at both contact points. (Coefficients of kinetic and static friction are assumed to be the same.)



The forces exerted by the wall (as shown in the figure) are: F_1, μF_1, F_2, μF_2. The rod is in equilibrium if the resultant force and resultant torque on it are zero. Let us write the condition for translational equilibrium separately for the horizontal and vertical components of forces:

$$G+F_1-F_2\cos\alpha-\mu F_2\sin\alpha=0, \qquad (1)$$

$$F_2\sin\alpha-\mu F_2\cos\alpha-\mu F_1=0. \qquad (2)$$

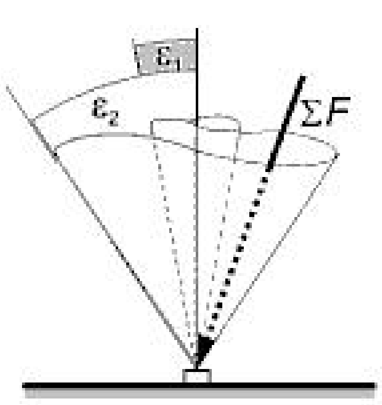
The sum of torques about point O is:

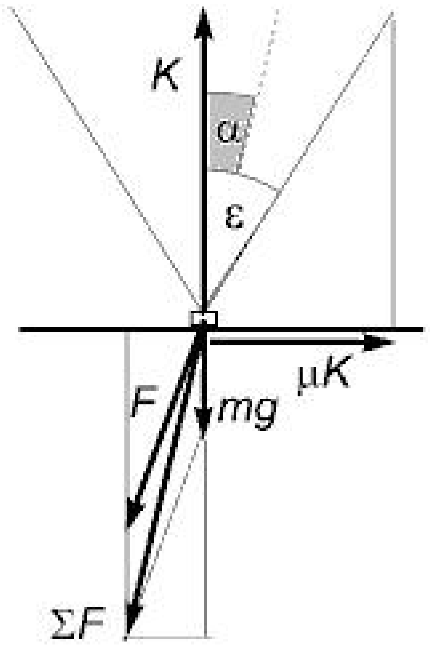
$$GL\cos\alpha-F_1c\ \cos\alpha-\mu F_1c\ \sin\alpha=0. \qquad (3)$$

Thus we have a system of equations with three unknowns that can easily be determined. Let us isolate F_2 from equation (2):

$$F_2=\frac{\mu F_1}{\sin\alpha-\mu\cos\alpha}. \qquad (4)$$

However, if the line of action of the above force encloses an angle less than ε with the normal, the body will not slip, no matter how great the magnitude of the resultant is.





In other words, if the line of action of the above force is inside a cone, whose axis is the normal and whose cone angle is 2ε (see figure), the body will not slip. If it isn't the case, it will. So, the body will not slip if:

$$\mu K \geq S,$$

if $\sum F$ is the resultant of all forces except for the normal force and friction, then the above inequality takes the form of:

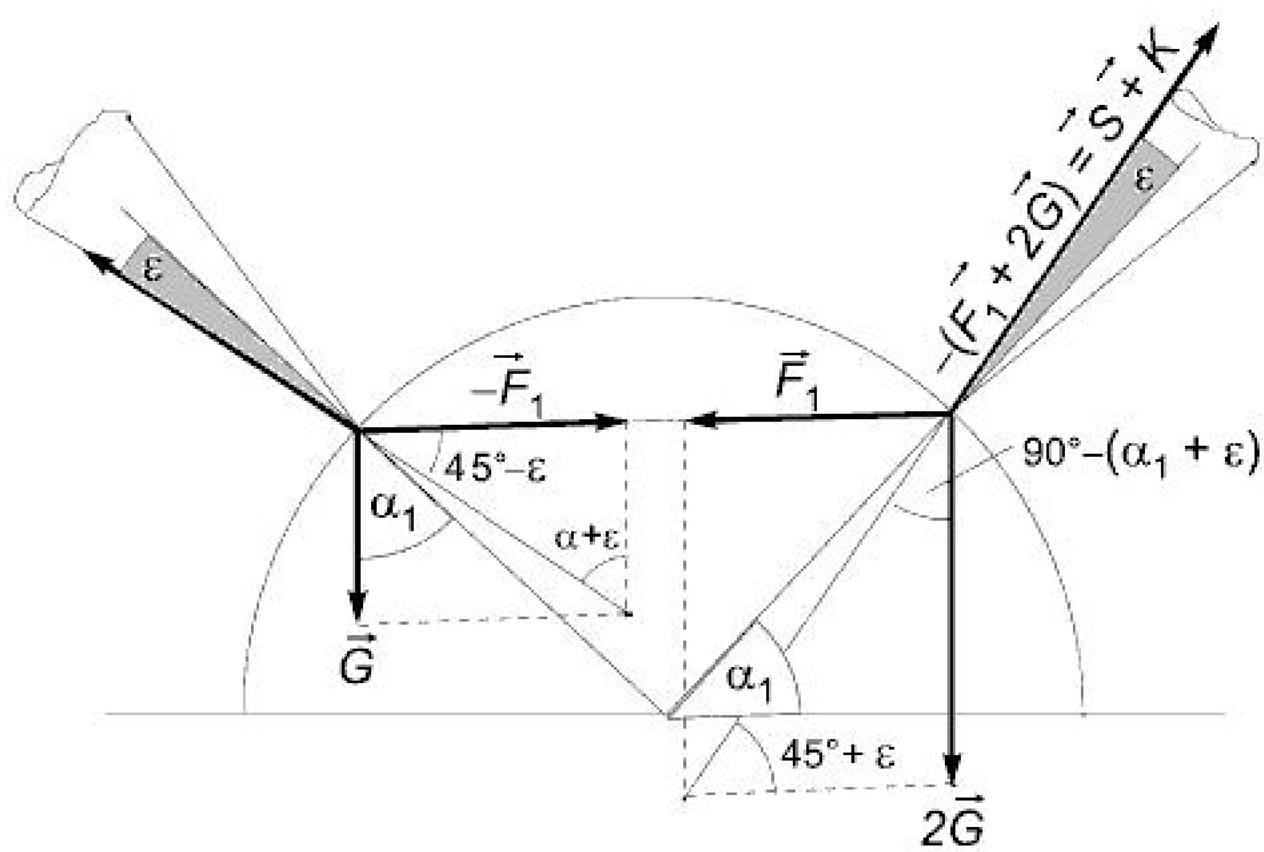
$$\mu \cdot \sum F \cos\alpha \geq \sum F \sin\alpha,$$

in the extreme case, using that $\alpha_{\max} = \varepsilon$, we find:

$$\mu \cos\varepsilon = \sin\varepsilon,$$

thus: $\tan\varepsilon = \mu$, as it was stated.

Let us now investigate the extreme cases of the beads finding triangles in which the angle formed by the horizontal and the radius drawn to the greater bead can be expressed with the help of angle ε.



In the situation when the greater bead is in its right extreme position, let us apply the law of sines to the triangles formed by the forces shown:

$$\frac{2G}{F} = \frac{\sin(45° + \varepsilon)}{\cos(\alpha + \varepsilon)}, \qquad \text{and} \qquad \frac{G}{F} = \frac{\sin(45° - \varepsilon)}{\sin(\alpha + \varepsilon)},$$

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Substituting values in case a) gives:

$$\cos\alpha = \frac{0.8}{1}\sqrt{\frac{1}{0.5}} = 1.1313 > 1,$$

which is impossible, so in case a) there are only the two trivial equilibrium positions: at $\alpha_1 = 0^\circ$ the equilibrium is stable and at $\alpha_2 = 180^\circ$ the equilibrium is unstable. For every other angle $0 < \alpha < 180^\circ$ the torque of the buoyant force is greater than that of the gravitational force.

In case b), we get three equilibrium positions:

$$\cos\alpha = \frac{0.8}{1}\sqrt{\frac{1}{0.853}} = 0.866,$$

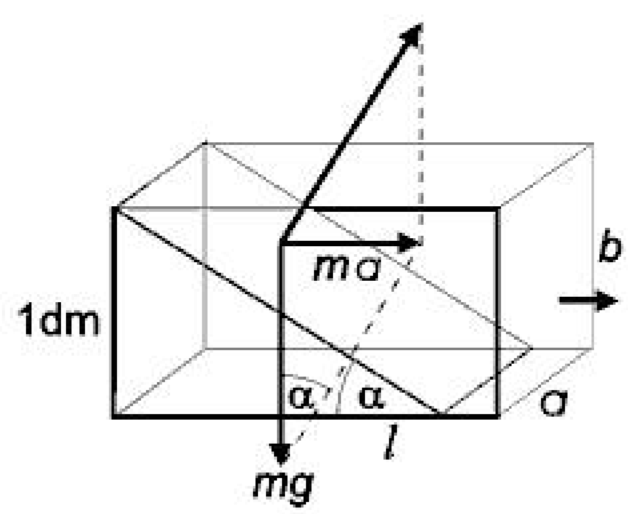
so the two torques will cancel once they reach the angle $\alpha = 30^\circ$. The equilibrium positions are therefore: $\alpha_1 = 0^\circ$ and $\alpha_2 = 180^\circ$ both being unstable and $\alpha_3 = 30^\circ$, which is stable.

Solution of Problem 161. If the cart is released and left on its own, its motion will be rather complicated. When the cart suddenly starts to move, some water usually spills out, and the water that remains in the cart starts to oscillate. This means that instead of the cart and the hanging object having a constant acceleration, they will undergo a very complicated motion that cannot be described using basic mathematics. Let us therefore assume that the string is very long, the table is very high and the cart is not released suddenly, but keeping one hand on it, we increase its acceleration slowly to the final constant value. Our calculations below will all concern this final state of the system.

The volume of water in the cart is $V = ahl = 1\,\mathrm{dm}\cdot 0.9\,\mathrm{dm}\cdot 2\,\mathrm{dm} = 1.8\,\mathrm{dm}^3$ initially, therefore the initial mass of water is $m_1 = 1.8\,\mathrm{kg}$. Thus the mass of the cart itself is $m_3 = M - m_1 = 0.2\,\mathrm{kg}$.

When the cart moves with constant acceleration, the surface of the water is not horizontal, but forms an angle α with the horizontal. Angle α is determined by the equation:

$$\tan\alpha = \frac{a}{g}.$$



Since the level of water was quite high initially, it is almost guaranteed that some water will spill out. In order to check this, let us assume that no water spills out and see what follows from that. In this case, the acceleration of the cart and water can be determined by applying Newton's second law to the cart (adding the total mass of water to the cart's own mass) and the hanging object, from which we obtain:

$$a = \frac{m_2}{M + m_2}\cdot g,$$

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Solution of Problem 171. The initial separation between the pistons is

$$d=\frac{V_1}{A}=\frac{2000\,\mathrm{cm}^3}{100\,\mathrm{cm}^2}=20\,\mathrm{cm}.$$

After the compression, the air forms a cylinder of length x cm, and accordingly, the spring is also compressed by x cm. The piston on the left is held in equilibrium by the combined effect of the external air pressure, the pressure of the enclosed gas and the force of the spring, therefore the pressure of the enclosed gas needs to grow to a value of

$$p_2=p_1+\frac{Dx}{A}=10^5\,\mathrm{Pa}+\frac{1000\,\mathrm{N/m}}{10^{-2}\,\mathrm{m}^2}\cdot x,$$

where D is the spring constant. Then the volume of the enclosed gas is

$$V_2=A\cdot x=10^{-2}\,\mathrm{m}^2\cdot x.$$

Since temperature is constant, Boyle's law can be applied:

$$p_1V_1=p_2V_2=\left(p_1+\frac{Dx}{A}\right)Ax.$$

Rearranged by the powers of x:

$$Dx^2+p_1Ax-p_1V_1=0, \tag{1}$$

and the solution of the equation is

$$x=\frac{-p_1A\pm\sqrt{p_1^2A^2+4Dp_1V_1}}{2D}.$$

Numerically, the equation (1) is

$$10^3\,\frac{\mathrm{N}}{\mathrm{m}}\cdot x^2+10^3\,\mathrm{N}\cdot x-2\cdot 10^2\,\mathrm{N\,m}=0,$$

which simplifies to $5x^2+5x-1=0$, and the solution is

$$x=\frac{-5\pm\sqrt{25+20}}{10}=0.1708\,\mathrm{m}\approx 0.171\,\mathrm{m}.$$

Hence the volume of the air in the final position is

$$V_2=A\cdot x=10^{-2}\,\mathrm{m}^2\cdot 0.171\,\mathrm{m}=1.71\cdot 10^{-3}\,\mathrm{m}^3=1.71\,\mathrm{dm}^3.$$

Solution of Problem 172. The equation of the line passing through the points 2 and 4 is

$$p=\frac{p_2-p_1}{T_1-T_4}\cdot T-p_1, \tag{1}$$

where T and p denote the abcissa and ordinate of any point of the line. Since the point 3 also lies on this line, its coordinates T_3 and p_3 satisfy the equation:

$$p_3=\frac{p_2-p_1}{T_1-T_4}\cdot T_3-p_1. \tag{2}$$

If the result is substituted into (6), the following value is obtained for the height of the mercury column in the third tube:

$$y = \frac{p_1 + h - p}{2} = \frac{232.8 + 152 - 106.4}{2} \cdot \text{cmHg} = 139.2 \text{ cmHg},$$

that is, the height of the mercury column in the third tube is $139.2\,\text{cm}$. The values of p and y are now substituted into (3):

$$x = p + y - p_1 = 106.4 \text{ cmHg} + 139.2 \text{ cmHg} - 232.8 \text{ cmHg} = 12.8 \text{ cmHg},$$

that is, the height of the mercury column in the fourth tube is $x = 12.8\,\text{cm}$.

The value of z is obtained from (7) by substituting the numerical value of p:

$$z = \frac{p_0 + h - p}{2} = \frac{76 + 152 - 106.4}{2} \cdot \text{cmHg} = 60.8 \text{ cmHg}.$$

Finally, the value of z is substituted in (4) to give v:

$$v = h - z = 152 \text{ Hgcm} - 60.8 \text{ cmHg} = 91.2 \text{ cmHg}.$$

The results are therefore as follows: $p = 106.4$ cmHg $= 1.4 \cdot 10^5$ Pa, $x = 12.8\,\text{cm}$, $y = 139.2\,\text{cm}$, $z = 60.8\,\text{cm}$, $v = 91.2\,\text{cm}$.

Solution of Problem 176. The boiling point of ether being $35\ ^\circ\text{C}$ means that ether boils at that temperature at normal atmospheric pressure. Since there is a mercury column lying over the ether in the tube, the pressure at the surface of the ether is greater than normal atmospheric pressure, meaning that the ether is initially in the liquid state. (The pressure of $19\,\text{cm}$ of mercury is equal to $1/4$ of the atmospheric pressure, thus the total pressure on the ether is $5/4$ of the atmospheric pressure, $\approx 1.25 \cdot 10^5$ Pa.) If the tube is inverted, the pressure p_e of the ether and the pressure of the mercury will balance the atmospheric pressure, that is,

$$p_e + 19 \text{ cmHg} = 76 \text{ cmHg},$$

and hence

$$p_e = 57 \text{ cmHg} = \frac{3}{4} p_0$$

At $3/4$ of the atmospheric pressure the ether at the boiling-point temperature will evaporate to form an unsaturated vapour.

In the liquid state, the volume of the enclosed ether is

$$V_e = A l_e = 0.2\,\text{cm}^2 \cdot 0.25\,\text{cm} = 0.05\,\text{cm}^3,$$

where A is the cross-sectional area of the tube and l_e is the length of the liquid ether column. The mass of the ether is

$$m = \varrho \cdot V_e = 0.7\,\frac{\text{g}}{\text{cm}^3} \cdot 0.05\,\text{cm}^3 = 0.0035\,\text{g}.$$

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from which the volume swept by the piston is:

$$\Delta V = \frac{N_2V_1 - N_1V_2}{N_1 + N_2}. \tag{3}$$

The relationship between the numbers of particles and the initial data of the gases is given by the state equation:

$$p_1V_1 = N_1kT_1, \qquad p_2V_2 = N_2kT_2.$$

From these, the numbers of particles can be substituted into (3):

$$\Delta V = \frac{\frac{p_2V_2}{kT_2}V_1 - \frac{p_1V_1}{kT_1}V_2}{\frac{p_1V_1}{kT_1} + \frac{p_2V_2}{kT_2}}.$$

After simplifying and factoring out:

$$\Delta V = \frac{(p_2T_1 - p_1T_2)V_1V_2}{V_1p_1T_2 + p_2V_2T_1} = \frac{(3\cdot 10^5\cdot 350 - 2\cdot 10^5\cdot 280)\cdot 4\cdot 5}{2\cdot 10^5\cdot 4\cdot 280 + 3\cdot 10^5\cdot 5\cdot 350}\ \mathrm{dm}^3 = 1.31\ \mathrm{dm}^3.$$

The displacement of the piston is

$$\Delta s = \frac{\Delta V}{A} = \frac{1.31\ \mathrm{m}^3}{1\ \mathrm{dm}^2} = 1.31\ \mathrm{dm}.$$

(Those who start with the gas law written for the changes in the state of the gases enclosed in the two parts, end up with the following:

$$\frac{p_1V_1}{T_1} = \frac{p_c(V_1 - \Delta V)}{T_c}, \tag{1'}$$

$$\frac{p_2V_2}{T_2} = \frac{p_c(V_2 + \Delta V)}{T_c}. \tag{2'}$$

$(1')$ divided by $(2')$ gives

$$\frac{p_1V_1T_2}{p_2V_2T_1} = \frac{V_1 - \Delta V}{V_2 + \Delta V}.$$

After rearrangement the change in volume is acquired immediately.)

Those who start with the conservation of the total energy of the gases enclosed in the cylinder can write up the following:

$$E = \frac{3}{2}p_1V_1 + \frac{5}{2}p_2V_2 = \frac{3}{2}\cdot 2\cdot 10^5\frac{\mathrm{N}}{\mathrm{m}^2}\cdot 4\cdot 10^{-3}\ \mathrm{m}^3 + \frac{5}{2}\cdot 3\cdot 10^5\ \frac{\mathrm{N}}{\mathrm{m}^2}\cdot 5\cdot 10^{-3}\ \mathrm{m}^3 =$$
$$= 1200\mathrm{J} + 3750\mathrm{J} = 4950\mathrm{J}.$$

The energy in the new equilibrium expressed with the common temperature is

$$\frac{3}{2}N_1kT_c + \frac{5}{2}N_2kT_c = \left(\frac{3}{2}\frac{p_1V_1}{T_1} + \frac{5}{2}\frac{p_2V_2}{T_2}\right)T_c.$$

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Thus, with the substitution of (8) and (9) in (7), the function $Q(V)$ is

$$Q(V)=p_0V_0\left\{\left[\frac{5}{2}\left(\frac{V}{V_0}\right)-\frac{5}{2}\left(\frac{V}{V_0}\right)^2-\frac{5}{9}\right]+\left[\left(\frac{V}{V_0}\right)-\frac{1}{2}\left(\frac{V}{V_0}\right)^2-\frac{4}{9}\right]\right\}.$$

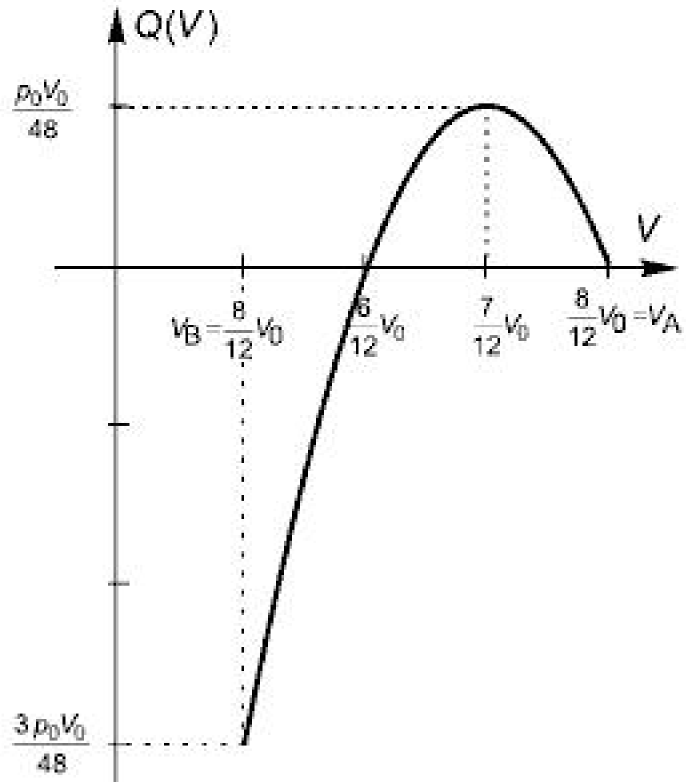
Simplified and rearranged:

$$Q(V)=p_0V_0\left[\frac{7}{2}\left(\frac{V}{V_0}\right)-3\left(\frac{V}{V_0}\right)^2-1\right]. \tag{10}$$

The analysis of the function $Q(V)$ provides the following information. The net heat absorbed increases until the gas is gradually compressed from the initial volume of V_A to $7V_0/12$. The net heat absorbed so far is

$$Q_{\text{absorbed}}=Q\left(\frac{7}{12}V_0\right)=p_0V_0\left[\frac{7}{2}\cdot\frac{7V_0}{12V_0}-3\cdot\left(\frac{7V_0}{12V_0}\right)^2-1\right]=$$

$$=p_0V_0\left(\frac{49}{24}-\frac{3\cdot 49}{144}-1\right)=\frac{p_0V_0}{48}=\frac{1.2\cdot 10^5\,\mathrm{Pa}\cdot 12\cdot 10^{-3}\,\mathrm{m}^3}{48}=30\,\mathrm{J}.$$



As the gas is compressed further from the volume $7V_0/12$, it gives off heat. That heat is obtained as the difference of the total heat transfer during the whole process and the heat transfer during the compression to $7V_0/12$, since

$$Q_{(V_A\to V_B)}=Q_{\left(V_A\to\frac{7}{12}V_0\right)}+Q_{\left(\frac{7}{12}V_0\to V_B\right)},$$

where

$$Q_{\left(V_A\to\frac{7}{12}V_0\right)}=Q_{\text{absorbed}}=\frac{p_0V_0}{48}=30\,\mathrm{J}.$$

With the substitution of $V_B=5V_0/12$ in the function $Q(V)$ (10), the total heat transfer during the whole process is

$$Q_{(V_A\to V_B)}=p_0V_0\left(-3\cdot\frac{25}{144}+\frac{7}{2}\cdot\frac{5}{12}-1\right)=-p_0V_0\frac{9}{144}=-\frac{3p_0V_0}{48}.$$

Thus the heat absorbed by the gas is

$$Q_{\left(\frac{7}{12}V_0\to V_B\right)}=Q_{(V_A\to V_B)}-Q_{\left(V_A\to\frac{7}{12}V_0\right)}=-\frac{3p_0V_0}{48}-\frac{p_0V_0}{48}=-\frac{p_0V_0}{12}.$$

With numerical data:

$$Q_{\left(\frac{7}{12}V_0\to V_B\right)}=-\frac{1}{12}\cdot 1.2\cdot 10^5\,\mathrm{Pa}\cdot 12\cdot 10^{-3}\,\mathrm{m}^3=-120\,\mathrm{J},$$

therefore the heat given off during the process is $Q_{\text{off}}=120\,\mathrm{J}$.

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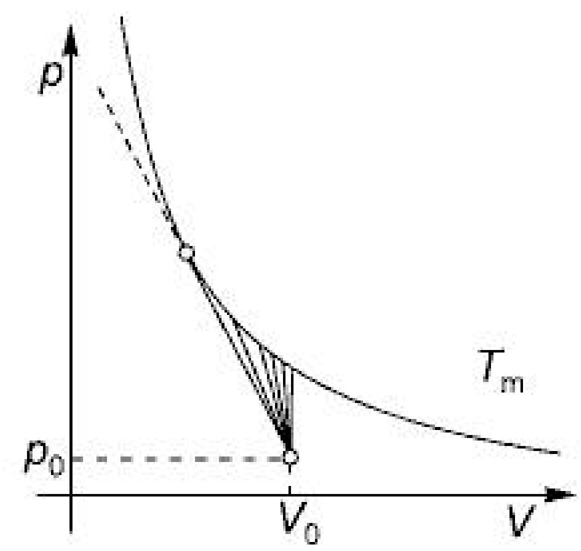
Therefore the ratio of energies is:

$$\frac{U_2}{U_1}=\frac{\frac{f_1}{2}\cdot 0.8N_1kT_2+\frac{f_2}{2}\cdot 0.4N_1kT_2}{\frac{f_1}{2}\cdot N_1kT_1}=$$

$$=\frac{0.8f_1+0.4f_2}{f_1}\cdot\frac{T_2}{T_1}=\frac{4+1.2}{5}\frac{T_2}{T_1}=\frac{2.6}{2.5}\cdot\frac{T_2}{T_1}=1.04\frac{T_2}{T_1}.$$

First solution of Problem 194. $\Delta p/\Delta V$ is positive, because if it was negative, then the maximum temperature of the gas would be 5 °C during the process. Let T_0 and V_0 stand for the initial data and let $\left|\frac{\Delta p}{\Delta V}\right|$ be x.



Our task can be rephrased as follows: we are looking for the gradient of all straight lines passing through point $(V_0,\ p_0)$ of the state plane that are between the vertical direction and the gradient of the tangent of the isothermal curve belonging to temperature $T_{\max}$ and belong to the compression of the gas. These fall into an interval closed from the left and open from the right. From the right the limiting value of the gradient $-\infty$. Knowing this, we only have to find the left boundary of the interval, that is, the gradient of the line with the highest gradient (its absolute value is the smallest) that has a common point with the isothermal curve belonging to temperature $T_{\max}$. From the property of the hyperbola it can be seen that every straight line passing through point $(V_0,\ p_{0)})$ has two points of intersection with it except for the 'vertical' line, the 'horizontal' line and the lines belonging to the points of tangency (in the case of the latter we can speak about two coinciding points of intersection). See the graph of Solution 2 as well.

Let us use the universal gas law.
For the initial data of an ideal gas

$$p_0V_0=NkT_0,$$

the relationship between the relevant data is

$$pV=NkT.$$

According to the condition for compression

$$\frac{p-p_0}{V-V_0}=-x.$$

From here, the pressure at any time is

$$p=-xV+xV_0+p_0.$$

Substituting this into the gas law rearranged for T gives

$$T=\frac{1}{Nk}pV=\frac{1}{Nk}\left[-xV^2+(p_0+xV_0)V\right]=-\frac{x}{Nk}V^2+\frac{p_0+xV_0}{Nk}V.$$

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So the pressure above the piston is:

$$p_1 = \frac{mgn_1T_1}{\left(1-\frac{n_1T_1}{n_2T_2}\right)An_2T_2} = \frac{mg}{A}\cdot\frac{n_1T_1}{(n_2T_2-n_1T_1)} =$$
$$= \frac{0.5\ \text{kg}\cdot 9.81\ \text{m/s}^2}{0.01\ \text{m}^2}\cdot\frac{0.05\cdot 183}{0.03\cdot 319-0.05\cdot 183} = 10685.9\ \text{Pa}.$$

And the pressure under the piston is:

$$p_2 = p_1 + \frac{mg}{A} = 10685.9\ \text{Pa} + \frac{0.5\ \text{kg}\cdot 9.81\ \text{m/s}^2}{0.01\ \text{m}^2} = 11176.4\ \text{Pa}.$$

From the first equation the common volume of the two gases is:

$$V_0 = \frac{n_1RT_1}{p_1} = \frac{A}{mg}\cdot R(n_2T_2-n_1T_1).$$

Numerically:

$$V_0 = \frac{0.05\ \text{mol}\cdot 8.31\ \frac{\text{J}}{\text{mol}\cdot\text{K}}\cdot 183\ \text{K}}{10685.9\frac{\text{N}}{\text{m}^2}} = 0.00712\ \text{m}^3 = 7.12\ \text{dm}^3.$$

To determine the final temperature T of the system and the displacement Δx of the piston, we use the combined gas law and the law of energy conservation.

The combined gas law, written for the two gases separately between their initial and final states, gives the following two equations:

$$\frac{p_1V_1}{T_1} = \frac{p(V_0+xA)}{T}, \tag{1}$$
$$\frac{p_2V_0}{T_2} = \frac{(p+\frac{mg}{A})\cdot(V_0-xA)}{T}. \tag{2}$$

The law of energy conservation for the whole process in the isolated cylinder has the form:

$$\Delta E_1 + \Delta E_2 + \Delta E_3 + \Delta E_4 = 0.$$

Detailing the terms, we get:

$$\frac{f_1}{2}n_1R(T-T_1) + \frac{f_2}{2}n_2R(T-T_2) + cm(T-T_0) + mg\Delta x = 0. \tag{3}$$

The three unknowns are T, p and x. The above three equations unambiguously determine their values. The solution of the problem can be considerably simplified by noticing that the pressure arising from the weight of the piston is much less than the pressure of the gases, and thus it is negligible. (The pressure in the upper part of the cylinder is 10685.9 Pa, in the lower part it is 11176.4 Pa, while the pressure due to the weight of the piston is

$$p_P = \frac{mg}{A} = \frac{0.5\ \text{kg}\cdot 9.81\ \text{m/s}^2}{0.01\ \text{m}^2} = 490.5\ \text{Pa},$$

which is less than 4.6%.)

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After determining the number of particles, the intensity of particle flow can be determined:

$$I' = \frac{1}{2}10^{-7}\ \mathrm{m}^2 \cdot 277\ \frac{\mathrm{m}}{\mathrm{s}} \cdot \frac{9.826 \cdot 10^{26}}{80\ \mathrm{m}^3} = 1.7 \cdot 10^{20}\ \frac{1}{\mathrm{s}}.$$

The estimated time with this train of thought is

$$t' = \frac{\Delta N}{I'} = \frac{0.01 \cdot N}{I'} = \frac{9.826 \cdot 10^{24}}{1.7 \cdot 10^{20}} = 57800\ \mathrm{s} = 16\ \mathrm{hours}.$$

(The calculation that takes speed distribution into consideration gives a value of

$$I'' = \frac{1}{\sqrt{2\pi}} A \frac{N}{V} \sqrt{\frac{RT}{M}} = \sqrt{\frac{2}{\pi}} I' = 0.8 I' = 1.36 \cdot 10^{20}\ \frac{1}{\mathrm{s}}$$

for the intensity of particle flow, with which the requested time is

$$t'' = \frac{\Delta N}{I''} = \frac{9.826 \cdot 10^{24}}{1.36 \cdot 10^{20}\ \mathrm{s}^{-1}} = 72250\ \mathrm{s} = 20.1\ \mathrm{hours}.$$

The elementary solutions given through both trains of thought are around the correct value, their order of magnitude is the same.)

Solution of Problem 198. Since the volume and temperature of the helium gas are given, we need to calulate its pressure to find its mass. According to the equation of state:

$$pV = \frac{m_{\mathrm{He}}}{M_{\mathrm{He}}} RT,$$

solving for the mass of the helium, we get:

$$m_{\mathrm{He}} = \frac{pVM_{\mathrm{He}}}{RT}. \tag{1}$$

In order to determine the pressure of the helium gas, let us apply Newton's second law to the cylinder-piston system and then to the piston itself. Let the masses of piston and cylinder be m and M respectively, and let K be the tension in the string. (The mass of helium gas can be neglected.) If the system is considered, the internal forces cancel out:

$$(m+M)g - K = (m+M)a, \tag{2}$$

Forces acting on the cylinder are gravitational force (mg) and the force (p_0A) due to the atmospheric pressure both pointing downwards and the tension and pA exerted by the helium both pointing upwards.

$$mg + p_0A - K - pA = ma. \tag{3}$$

As we have three unknowns (K,a,p) one more equation is needed. This will be Newton's second law in angular form applied to the pulley. Using that $\beta = a/r$, we obtain:

$$Kr = \Theta \frac{a}{r}. \tag{4}$$

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while in its final state it is:

$$p_3 = p_0 + \frac{mg}{A} = p_1 = 13.6\,\frac{\mathrm{N}}{\mathrm{cm}^2}.$$

The mass of air can be determined using the initial values of its density and volume:

$$m_{\mathrm{air}} = \varrho_1 Ah = 1.8\,\frac{\mathrm{g}}{\mathrm{dm}^3}\cdot 20\,\mathrm{cm}^2\cdot 33\,\mathrm{cm} = 1.118\,\mathrm{g}.$$

Assuming that the pressure of the air is the same ($p_1 = p_3 = p_0 + mg/A$) in its first and third (final) state, the change in its temperature can easily be determined (the same way as in the case of an isobaric process):

$$\frac{V_1}{V_3} = \frac{T_1}{T_3},$$

Isolating for T_3 gives:

$$T_3 = T_1\frac{V_3}{V_1}.$$

Let us substitute the expressions for the volumes:

$$T_3 = T_1\frac{(h+h_1)A}{hA} = T_1\frac{h+h_1}{h} = 273\,\mathrm{K}\cdot\frac{40\,\mathrm{cm}}{33\,\mathrm{cm}} = 330.91\,\mathrm{K},$$

so the change in the temperature is:

$$\Delta T = T_3 - T_1 = 57.91\,\mathrm{K} \approx 58\,\mathrm{K}.$$

Using equations (1), (2) and (3) and substituting known values, we get that the heat transmitted to the air is:

$$Q = 0.7\,\frac{\mathrm{J}}{\mathrm{g\,K}}\cdot 1.188\,\mathrm{g}\cdot 58\,\mathrm{K} + \frac{13.6+14.96}{2}\,\frac{\mathrm{N}}{\mathrm{cm}^2}\cdot 200\,\mathrm{cm}^3\cdot 10^{-2}\,\frac{\mathrm{m}}{\mathrm{cm}} =$$
$$= 48.23\,\mathrm{J} + 28.65\,\mathrm{J} = 76.79\,\mathrm{J}.$$

In this process the internal energy of the gas is increased by $48.23\,\mathrm{J}$, while $28.65\,\mathrm{J}$ is the work done by the gas. The former energy remains in the system, while the latter increases the energy of the environment.

Solution of Problem 202. The cylinder is rising slowly, that is, it remains in equilibrium. Therefore, the pressure of the gas in the cylinder is constant, which means that we are dealing with an isobaric process.

The heat absorbed at constant pressure is

$$Q = C_p n \Delta T, \tag{1}$$

where C_p is the molar heat at constant pressure, n is the number of moles, and ΔT is the change in temperature.

The relationship between the two molar heats is

$$C_p - C_v = R,$$

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If expression

$$T_2 - T_1 = \frac{p_2 V_2}{Nk} - \frac{p_1 V_1}{Nk} \tag{10}$$

is substituted in place of ΔT and

$$\frac{c_p}{c_v} = \frac{f+2}{f} = 1 + \frac{2}{f},$$

is also considered, these give

$$\frac{f}{2} = \frac{c_v}{c_p - c_v} = \frac{1}{\gamma - 1}.$$

If this and the change in temperature acquired from (10) are substituted into (9), (8) is acquired.)

With this, equation (5) gives the heat delivered by the heating filament:

$$Q = 1200\ \text{J} + 300\ \text{J} + 10\ \text{J} = 1510\ \text{J}.$$

(Those who determined mass with Method 1 received $1192\ \text{J} + 310\ \text{J} = 1502\ \text{J}$ for the energy delivered.)

Solution of Problem 206. Our data are $p_0 = 10^5$ Pa; $V_0 = 1\ \text{m}^3$; $A = 0.1\ \text{m}^2$; $v = 1\ \text{cm/s}$.

If the temperature of the gas is constant, then its energy does not change either. According to the first law of thermodynamics

$$Q = -W.$$

Let us apply this to a short time of the process:

$$P\Delta t = pAv\Delta t,$$

where P is the heating power. From here

$$P = pAv.$$

According to the problem, the heating power should be determined as a function of time. Let us apply the gas law

$$pV = p_0 V_0,$$

where in our case

$$V = V_0 + Avt,$$

and so the pressure expressed with the variables of the process is

$$p = \frac{p_0 V_0}{V_0 + Avt},$$

and with it the power as a function of time is

$$P = \frac{p_0 V_0 A v}{V_0 + Avt}.$$

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and

$$V_0=(10\,\mathrm{dm}^2+40\,\mathrm{dm}^2)\cdot 1.5\,\mathrm{dm}=75\,\mathrm{dm}^3=7.5\cdot 10^{-2}\,\mathrm{m}^3.$$

At constant pressure,

$$\frac{T_1-T_0}{T_0}=\frac{V_1-V_0}{V_0}=\frac{(A_2-A_1)x}{V_0}.$$

Hence

$$T_1-T_0=T_0\frac{A_2-A_1}{V_0}x. \tag{2}$$

(2) can be substituted into (1):

$$c_v\varrho_{300}V_0T_0\frac{A_2-A_1}{V_0}x=Q-p_0(A_2-A_1)x.$$

With the substitution of $Q=Pt$ and rearrangement to express the displacement in question:

$$x=\frac{Pt}{(A_2-A_1)(p_0+c_v\varrho_{300}T_0)}=$$

$$=\frac{4320\,\mathrm{J}}{(4\cdot 10^{-2}\,\mathrm{m}^2-1\cdot 10^{-2}\,\mathrm{m}^2)\left(10^5\,\frac{\mathrm{N}}{\mathrm{m}^2}+712\,\frac{\mathrm{J}}{\mathrm{kg\cdot K}}\cdot 1.176\,\frac{\mathrm{kg}}{\mathrm{m}^3}\cdot 300\,\mathrm{K}\right)}=4.1\,\mathrm{cm}.$$

b) The temperature of the air in the final state is obtained from (2):

$$T_1=T_0\frac{(A_2-A_1)x}{V_0}+T_0=T_0\frac{A_2-A_1}{A_1+A_2}\cdot\frac{x}{l}+T_0=$$

$$=300\,\mathrm{K}\cdot\frac{40\,\mathrm{dm}^2-10\,\mathrm{dm}^2}{10\,\mathrm{dm}^2+40\,\mathrm{dm}^2}\cdot\frac{4.1\,\mathrm{cm}}{15\,\mathrm{cm}}+300\,\mathrm{K}=349.2\,\mathrm{K}.$$

Second solution of Problem 210. From the universal gas equation:

$$p_0V_0=nRT_0.$$

The heat absorbed is

$$Q=c_pm\Delta T=\frac{f+2}{2}\frac{k}{m_0}m\Delta T=\frac{f+2}{2}\frac{k}{m_0}(nN_Am_0)\Delta T=\frac{f+2}{2}Rn\Delta T=$$

$$=\frac{7}{2}Rn\Delta T=\frac{7}{2}\frac{p_0V_0}{T_0}\Delta T,$$

and hence the change in temperature and the final temperature are

$$\Delta T=\frac{2}{7}\frac{QT_0}{p_0V_0}=\frac{2\cdot 4320\,\mathrm{J}\cdot 300\,\mathrm{K}}{7\cdot 10^5\,\frac{\mathrm{N}}{\mathrm{m}^2}\cdot 7.5\cdot 10^{-2}\,\mathrm{m}^2}=49.4\ ^\circ\mathrm{C},$$

and

$$T=T_0+\Delta T=27\ ^\circ\mathrm{C}+49.4\ ^\circ\mathrm{C}=76.4\ ^\circ\mathrm{C}=349\,\mathrm{K}.$$

(The same result can also be obtained without using the concept of the excess pressure of a curved liquid film. Let F denote the total force exerted by the tube on the membrane. In the situations shown by figures a) and c), $F < 4R\pi\alpha$, since F is the resultant of the lengthwise components of the forces acting on elementary arcs.

In the case b), $F = 4R\pi\alpha$, this is the maximum value of F. Since the membrane is in equilibrium,

$$pR^2\pi = F + p_0R^2\pi, \tag{1}$$

and hence

$$p = p_0 + \frac{F}{R^2\pi}. \tag{2}$$

With the substitution of the maximum value of F in (1), (2) gives the same result for $p_{\max}$ as obtained above.)

The temperature is calculated from the universal gas equation:

$$\frac{p_0R^2\pi h}{T_0} = \frac{p_{\max}\left[R^2\pi h + (2/3)R^3\pi\right]}{T},$$

where $R^2\pi h$ is the volume of the cylinder and $\frac{2}{3}R^3\pi$ is the volume of the hemisphere. Hence,

$$T = \frac{p_{\max}\left[h + (2/3)R\right]}{p_0 h}\cdot T_0 = \frac{1040\,\mathrm{Pa}\cdot\left[25\cdot10^{-3}\,\mathrm{m} + (2/3)\cdot5\cdot10^{-3}\,\mathrm{m}\right]}{1000\,\mathrm{Pa}\cdot25\cdot10^{-3}\,\mathrm{m}}\cdot250\,\mathrm{K} = 295\,\mathrm{K}.$$

b) The first law of thermodynamics states $Q = \Delta U + W_{gas}$. The change in internal energy is $\Delta U = \frac{f}{2}Nk\Delta T$, where $Nk = \frac{p_0R^2h}{T_0}$ from the universal gas equation applied to the initial state. Therefore,

$$\Delta U = \frac{f}{2}\frac{p_0R^2\pi h}{T_0}\cdot\Delta T = \frac{5}{2}\cdot\frac{1000\,\mathrm{Pa}\cdot25\cdot10^{-6}\,\mathrm{m}^2\pi\cdot25\cdot10^{-5}\,\mathrm{m}}{250\,\mathrm{K}\cdot(295\,\mathrm{K} - 250\,\mathrm{K})} = 8.8\cdot10^{-4}\,\mathrm{J}.$$

The work done by the gas is used for increasing the energy of the membrane and displacing the external air:

$$W_g = 2\alpha\Delta A + p_0\Delta V = 2\alpha(2R^2\pi - R^2\pi) + p_0\frac{2}{3}R^3\pi = 2.7\cdot10^{-4}\,\mathrm{J}.$$

Thus, from the first law of thermodynamics, the heat absorbed is $Q = 8.8\cdot10^{-4}\,\mathrm{J} +$ $+\,2.7\cdot10^{-4}\,\mathrm{J} = 11.5\cdot10^{-4}\,\mathrm{J}$.

Solution of Problem 216. Let p_1, V_1, T_1. stand for the initial state variables of the enclosed gas. Furthermore, let us consider the state when the piston has moved furthest from its initial position and therefore stops for a moment. Let the state variables of the gas be p_2, V_2, T_2 then. According to the condition set in the problem:

$$V_2 = 2V_1. \tag{1}$$

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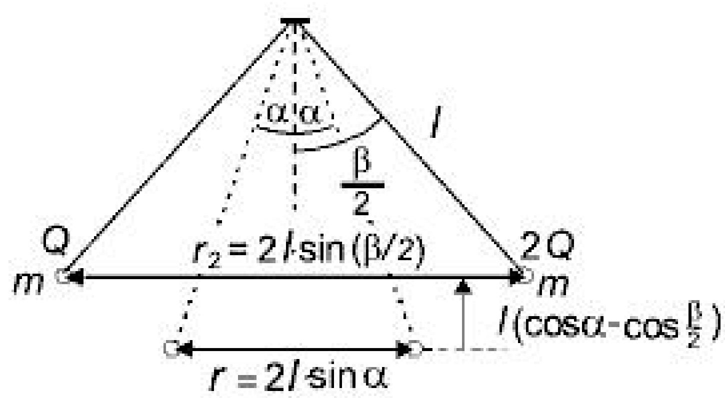
Chapter 8

Electrostatics Solutions

8.1 Electrostatics

Solution of Problem 223. Because of the equal masses of the balls and the symmetry of the opposite forces, the displacements of the balls are also symmetric. Thus after releasing the balls at their extreme positions both threads make an angle $\beta/2$ with the vertical, where β is the angle between the two threads.



According to the work-energy theorem, the sum of the works done by the forces acting on the balls is equal to the change of the kinetic energy of the system. Since at the initial and final (extreme) position the balls are at rest, their kinetic energy is zero, thus

$$k\cdot 2Q^2\left(\frac{1}{2l\sin\alpha}-\frac{1}{2l\sin\frac{\beta}{2}}\right)-2mgl\left(\cos\alpha-\cos\frac{\beta}{2}\right)=0.$$

From here the smaller charge is:

$$Q=\sqrt{\frac{2mgl\left(\cos\alpha-\cos\frac{\beta}{2}\right)}{2Q^2\left(\frac{1}{2l\sin\alpha}-\frac{1}{2l\sin\frac{\beta}{2}}\right)}}=\sqrt{\frac{2mgl^2\left(\cos\alpha-\cos\frac{\beta}{2}\right)\cdot\sin\frac{\beta}{2}\sin\alpha}{k\cdot\left(\sin\frac{\beta}{2}-\sin\alpha\right)}}.$$

Inserting the numerical values, we get:

$$Q=\sqrt{\frac{2\cdot 10^{-4}\ \mathrm{kg}\cdot 9.81\frac{\mathrm{m}}{\mathrm{s}^2}\cdot 0.09\ \mathrm{m}^3\,(\cos 20^\circ-\cos 42^\circ)\cdot\sin 42^\circ\cdot\sin 20^\circ}{9\cdot 10^9\,\frac{\mathrm{Nm}^2}{\mathrm{C}^2}\,(\sin 42^\circ-\sin 20^\circ)}}=5.2\cdot 10^{-8}\mathrm{C}.$$

The bigger charge is $2Q=10.4\cdot 10^{-8}\mathrm{C}$, of course.

Solution of Problem 224. Let us use an inertial reference frame attached to the centre of mass of the two particles. The above described motion is possible if in this reference frame the two particles perform a uniform circular motion around their centre of mass. (Viewing this motion from other, 'moving' inertial reference frames, the speeds of the particles would not be constant.) The centripetal force needed to maintain the uniform circular motion is produced by the Coulomb force. (The gravitational force is by many

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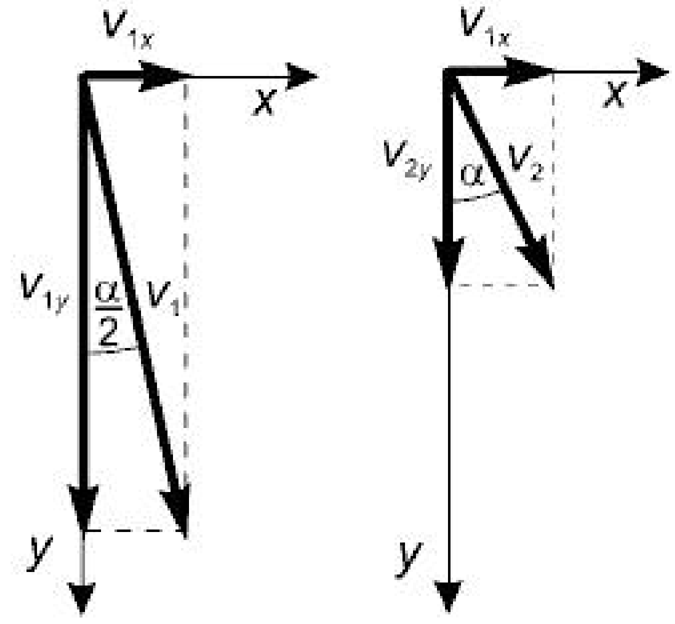
of direction of the velocity changes from $\alpha/2$ to α. The equations describing the change of the angle are:

$$\sin\alpha = \frac{v_{1x}}{v_2}, \qquad \sin\frac{\alpha}{2} = \frac{v_{1x}}{v_1},$$

hence

$$\frac{v_2}{v_1} = \frac{\sin\frac{\alpha}{2}}{\sin\alpha} = \frac{\sin\frac{\alpha}{2}}{2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}} = \frac{1}{2\cos\frac{\alpha}{2}}.$$



The potential difference between the lattices should decrease the velocity from v_1 to v_2. According to the work-kinetic energy theorem:

$$eV = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2,$$

from which the potential difference between the lattices is:

$$V = \frac{m}{2e}(v_2^2 - v_1^2) = \frac{mv_1^2}{2e}\left[\left(\frac{v_2}{v_1}\right)^2 - 1\right] = V_0\left(\frac{1}{4\cos^2\frac{\alpha}{2}} - 1\right) =$$

$$= 60000\,\mathrm{V}\left(\frac{1}{4\cos^2 15^\circ} - 1\right) = -43923\,\mathrm{V}$$

This is the potential of the second lattice relative to the first (the charge of the electron is negative).

Solution of Problem 229. a) Due to the nature of force (central and repulsive) acting on the moving charge, its path can only be a hyperbola.

According to the work-kinetic energy theorem, the work done by the electric field is equal to the change in the particle's kinetic energy. As the particle's initial position is at infinity, the work done by the electric field is the negative of the particle's potential energy at the point of its closest approach, thus:

$$-k\cdot\frac{qQ}{r} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2, \tag{1}$$

where r is the smallest separation and $k = 9\cdot 10^9\ \mathrm{Nm^2/C}$.

another one with area $c(c-x)$ and capacitance C_2, which is filled with air. Their capacitances are

$$C_1 = \frac{\varepsilon_0 \varepsilon_{\mathrm{r}} c x}{d} \qquad \text{and} \qquad C_2 = \frac{\varepsilon_0 c(c-x)}{d}.$$

The equivalent capacitance and the charge collected on the plates as function of time are

$$C = C_1 + C_2 = \frac{\varepsilon_0 c}{d}[c + (\varepsilon_{\mathrm{r}} - 1)x] = \frac{\varepsilon_0 c}{d}\left[c + (\varepsilon_{\mathrm{r}} - 1)\frac{a_0}{2}t^2\right],$$

and

$$Q = \frac{\varepsilon_0 c}{d}\left[c + (\varepsilon_{\mathrm{r}} - 1)\frac{a_0}{2}t^2\right]U = \frac{\varepsilon_0 c^2}{d}U + \frac{\varepsilon_0(\varepsilon_{\mathrm{r}} - 1)ca_0 U}{2d} \cdot t^2,$$

respectively. From the same train of thought it can be seen that as the insulator penetrates between the plates by Δx, the charge of the capacitor changes by

$$\Delta Q = \Delta C U = \left(\frac{\varepsilon_0 \varepsilon_{\mathrm{r}} c \Delta x}{d} - \frac{\varepsilon_0 c \Delta x}{d}\right)U = \frac{\varepsilon_0(\varepsilon_{\mathrm{r}} - 1)cU}{d}\Delta x.$$

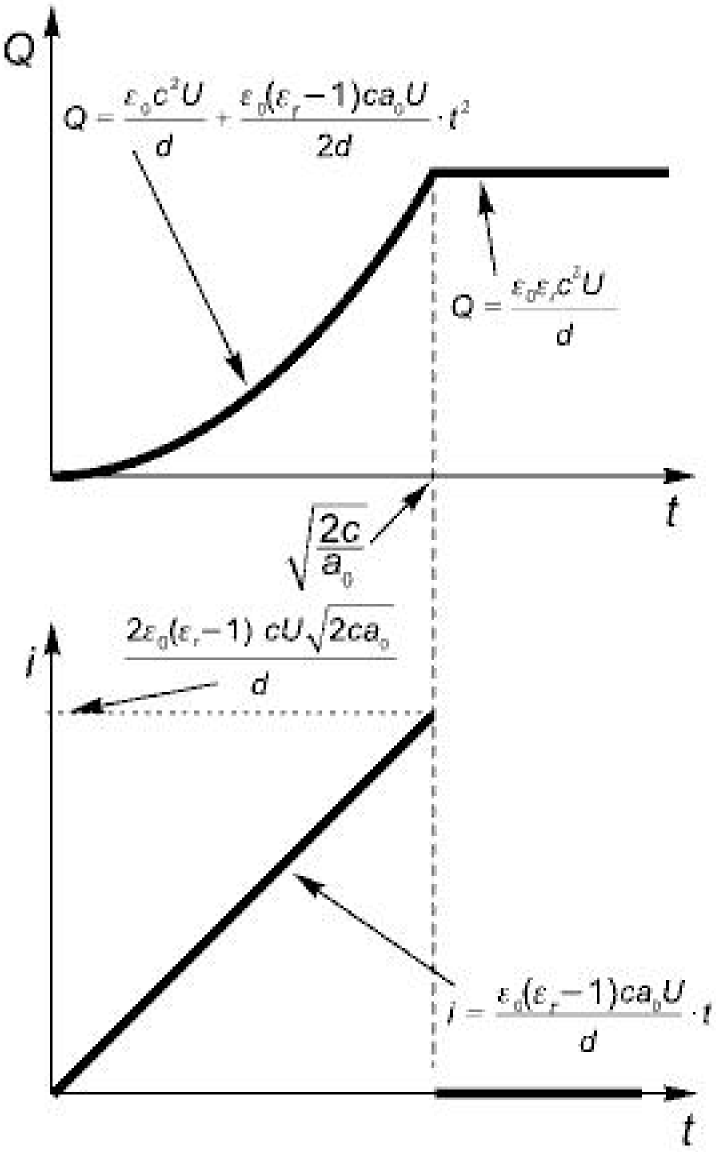
This charge is delivered by the emf source, so the current in the wires leading to the capacitor is

$$i = \frac{\Delta Q}{\Delta t} = \frac{\varepsilon_0(\varepsilon_{\mathrm{r}} - 1)cU}{d} \cdot \frac{\Delta x}{\Delta t} = \frac{\varepsilon_0(\varepsilon_{\mathrm{r}} - 1)cU}{d} \cdot v = \frac{\varepsilon_0(\varepsilon_{\mathrm{r}} - 1)cU}{d}a_0 t,$$

with numerical values

$$i = \frac{8.85 \cdot 10^{-12}\frac{\mathrm{A\,s}}{\mathrm{V\,m}} \cdot (101-1) \cdot 200\,\mathrm{mm} \cdot 2\frac{\mathrm{m}}{\mathrm{s}^2} \cdot 100\,\mathrm{V}}{2\,\mathrm{mm}} \cdot t = 1.77 \cdot 10^{-5}\frac{\mathrm{A}}{\mathrm{s}} \cdot t,$$

that is, the current increases linearly from zero.



The work that was required to increase the separation of the plates of the capacitor was done at the expense of this energy.

[Remark: This can be understood by considering the following:

The work done by us is the work done by a changing force, because in the case of capacitor B both the charge and the potential difference across it change. So the work is given by an integral.

The work done by us is

$$W=\int_{d}^{2d} F\,\mathrm{d}x,$$

where F is the magnitude of the force between the plates and x is the relevant separation of the plates, which changes from d to $2d$. The electrostatic force that acts on the plates of the capacitors – as we know – is half of the product of the resultant electric field and the charge on the disc:

$$F=\frac{1}{2}QE,$$

where based on Gauss' law, the magnitude of the electric field is

$$E=\frac{1}{\varepsilon_0}\cdot\frac{Q}{A}.$$

(Due to the constant velocity) the force exerted by us in the direction of displacement on the plate of capacitor B that is pulled by us has the same magnitude as the electrostatic force that acts on it:

$$F_B=\frac{1}{2}Q_B\cdot\frac{1}{\varepsilon_0}\frac{Q_B}{A}=\frac{1}{2}Q_B^2\cdot\frac{1}{\varepsilon_0 A}=\frac{1}{2}\frac{C_B^2V_B^2}{\varepsilon_0 A} \qquad \text{(I.)}$$

When the separation between the plates of capacitor B is increased, both its capacity and the potential difference across it change. Let us describe this two-variable function as a one-variable function of the distance between the plates. Let x stand for the changing distance between the plates of the capacitor. With it, the capacitance of capacitor B is:

$$C_B=\frac{\varepsilon_0 A}{x}.$$

The (changing) charges on the two capacitors connected in series are equal, so the potential differences across them are

$$V_A=\frac{Q}{C_A} \qquad \text{and} \qquad V_B=\frac{Q}{C_B}.$$

Their ratio is

$$\frac{V_A}{V_B}=\frac{C_B}{C_A}, \qquad \text{that is,} \qquad V_A=V_B\frac{C_B}{C_A}.$$

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Let x be the charge on the dropped capacitor when it has connected to the free ends of the other two capacitors. The total charge of each of the three conductors containing the points marked A, B and C in the figure stays constant since they are insulated from one another. Thus the charge on the upper plate of capacitor 1 is $Q_1 - x$, the charge on its lower plate is $-Q_1 + x$, the charge on the upper plate of capacitor 2 is $-Q_2 + x$, and the charge on its lower plate is $Q_2 - x$.

The potential difference of point B relative to point A, as calculated across the capacitor (1) on the left, is

$$V_{BA} = V_1 = \frac{Q_1'}{C_1} = \frac{3 \cdot 10^{-4} - x}{2 \cdot 10^{-6}}\,\mathrm{V},$$

and the same potential difference calculated through the other two capacitors, that is, along the path BCA is

$$V_{BCA} = V_{BC} + V_{CA} = V + V_2 = \frac{x}{C} + \frac{-Q_2}{C_2} = \frac{x}{1.5 \cdot 10^{-6}\ \mathrm{F}} - \frac{3.6 \cdot 10^{-4}\,\mathrm{C} - x}{3 \cdot 10^{-6}\ \mathrm{F}}.$$

Since the electrostatic field is conservative, these two voltages are equal:

$$\frac{3 \cdot 10^{-4}\,\mathrm{C} - x}{2 \cdot 10^{-6}\ \mathrm{F}} = \frac{x}{1.5 \cdot 10^{-6}\ \mathrm{F}} - \frac{3.6 \cdot 10^{-4}\,\mathrm{C} - x}{3 \cdot 10^{-6}\ \mathrm{F}}.$$

Multiplied by the common denominator $6 \cdot 10^{-6}\mathrm{F}$:

$$9 \cdot 10^{-4}\,\mathrm{C} - 3x = 4x - 7.2 \cdot 10^{-4}\,\mathrm{C} - 2x,$$

and hence the charge of the dropped capacitor is

$$x = \frac{16.2\,\mathrm{C}}{9} = 1.8 \cdot 10^{-4}\,\mathrm{C}.$$

x can be used to determine the potential difference across each capacitor:

$$V_1 = V_{AB} = \frac{3 \cdot 10^{-4}\,\mathrm{C} - 1.8 \cdot 10^{-4}\,\mathrm{C}}{2 \cdot 10^{-6}\ \mathrm{F}} = +60\,\mathrm{V}.$$

$$V_2 = V_{CA} = \frac{-3.6 \cdot 10^{-4}\,\mathrm{C} + 1.8 \cdot 10^{-4}\,\mathrm{C}}{3 \cdot 10^{-6}\ \mathrm{F}} = -60\,\mathrm{V},$$

and the potential difference of the capacitor dropped onto them is

$$V = V_{BC} = \frac{1.8 \cdot 10^{-4}\,\mathrm{C}}{1.5 \cdot 10^{-6}\ \mathrm{F}} = +120\,\mathrm{V}.$$

b) It is easy to see that the amount of charge passing through the point A is the charge lost by capacitor 2 and gained by capacitor 1, that is,

$$\Delta Q = x = 1.8 \cdot 10^{-4}\ \text{coulombs}$$

passed from 2 to 1, which means towards the left.

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a) In general the loop rule states that for any loop:

$$\sum_{\text{loop}} V = 0.$$

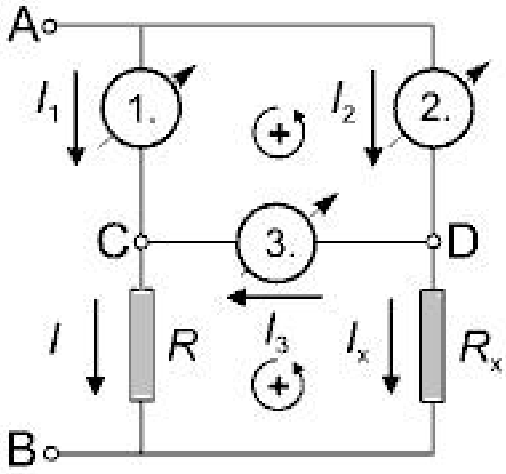
Traversing the upper loop in the counterclockwise direction (using the directions of currents shown in the figure), we find:

$$I_1 R_0 + I_2 R_0 + I_3 R_0 = 0,$$

which yields

$$I_3 = -I_1 - I_2 = -2.5\,\mathrm{A} - (-1.5\,\mathrm{A}) = -1\,\mathrm{A},$$

Thus the third ammeter reads $1\,\mathrm{A}$, and the direction of the current in that branch is opposite the direction of traverse, so it flows from point D to C.



b) In this part our task is to express I_3 as a function of resistance R_x. We have 6 unknowns (the currents in the 5 branches and resistance R), which should be determined using the potential difference between points A and B and the four known resistances. Let us use Kirchoff's laws again: equations (1) and (2) are junction rules ($\sum_{junction} I = 0$) applied at points D and C respectively, while equations (3), (4) and (5) are loop rules ($\sum_{loop} V = 0$) applied to the upper and lower loops and finally to loop $ADBA$. Resistance R can easily be calculated using the data given in part a), but let us work with it now as if it was a parameter. Using the directions shown in the figure and working with the magnitudes of currents, we obtain:

$$I_2 = I_3 + I_x \tag{1}$$

$$I = I_1 + I_3 \tag{2}$$

$$I_1 R_0 = I_2 R_0 + I_3 R_0 \tag{3}$$

$$I_x R_x = I_3 R_0 + IR \tag{4}$$

$$V = I_2 R_0 + I_x R_x. \tag{5}$$

Let us solve the system of equations step by step. Let us simplify equation (3) by R_0 and substitute I_2 from equation (1) into equations (3) and (5):

$$I = I_1 + I_3 \tag{2}$$

$$I_1 = I_3 + I_x + I_3 \tag{3'}$$

$$I_x R_x = I_3 R_0 + IR \tag{4}$$

$$V = I_3 R_0 + I_x R_0 + I_x R_x \tag{5'}$$

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Using these, the following relationships are acquired:

$$I_2 = I_1 \frac{R_2}{R_2 + R_x},$$

$$I_3 = I_2 \frac{R_2}{R_2 + R_x} = I_1 \cdot \left(\frac{R_2}{R_2 + R_x} \right)^2$$

$$\vdots$$

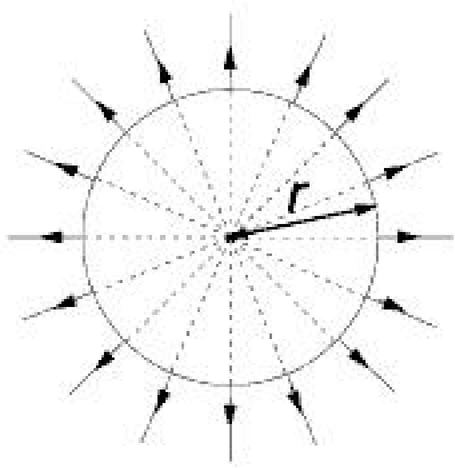
$$I_n = I_1 \cdot \left(\frac{R_2}{R_2 + R_x} \right)^{n-1}$$

$$I_x = I_1 \cdot \left(\frac{R_2}{R_2 + R_x} \right)^n = 1\ \text{A} \cdot \left(\frac{2}{3} \right)^{21} = 2.00485 \cdot 10^{-4}\ \text{A}.$$

With this, the potential difference across the terminating resistor R_x is

$$U_x = R_x I_x = 3\Omega \cdot 2.00485 \cdot 10^{-4}\ \text{A} = 0.6015\ \text{mV}$$

Solution of Problem 252. The capacitance of the metal sphere with respect to infinity, where the potential is zero, can be calculated such that the charge on the sphere, Q, is divided by the potential at the surface of the sphere, having a charge of Q. Due to the symmetry of the sphere, the distribution of the charge on the sphere is uniform, which creates a central electric field whose field lines are perpendicular to the surface of the sphere, thus outside the sphere they are drawn as if they came from the centre of the sphere. Therefore the potential at the surface of the sphere is the same as the potential at a distance of r from a pointlike charge of Q placed into the centre of the sphere, which is:



$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r}.$$

thus the capacitance of the sphere of radius r is

$$C = 4\pi\varepsilon_0 r.$$

If the potential difference between the ends of the wires is V, then at each touch the sphere carries a charge of $Q_1 = CV = 4\pi\varepsilon_0 r V$, thus during a time t the amount of charge carried between the ends of the wires is $Q = ntQ_1$, where according to the data $n = 54\ 1/\text{min} = 0.9\ 1/\text{s}$. Thus the average current is

$$I = \frac{Q}{t} = nQ_1 = n4\pi\varepsilon_0 r V,$$

and applying Ohm's law the resistance can be calculated as:

$$R = \frac{V}{I} = \frac{1}{n4\pi\varepsilon_0 r} = \frac{1}{0.9\ \text{s}^{-1} 4\pi 8.85 \cdot 10^{-12}\ \frac{\text{As}}{\text{Vm}} \cdot 0.01\ \text{m}} \approx 10^{12}\Omega.$$

The electric field is homogeneous, and in the arrangement of the figure it is directed horizontally from left to right.

Solution of Problem 255. The speed of the electron is obtained from the work-energy theorem:

$$e\Delta V = \frac{1}{2}mv^2.$$

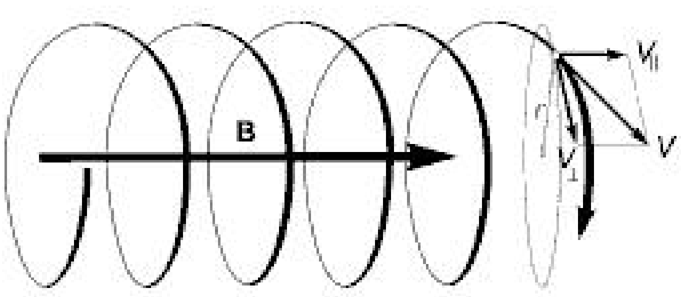
Hence

$$v = \sqrt{2\frac{e}{m}\Delta V} = \sqrt{2\cdot\frac{1.6\cdot 10^{-19}\,\mathrm{C}}{9\cdot 10^{-31}\,\mathrm{kg}}\cdot 800\,\mathrm{V}} = 1.686\cdot 10^7\,\frac{\mathrm{m}}{\mathrm{s}}.$$

The electron is acted on by the magnetic Lorentz force:

$$\vec{F} = e\vec{v}\times\vec{B}.$$

(Since the lines of magnetic induction and the lines of magnetic field intensity have the same direction in vacuum, we can consider field lines the same as lines of magnetic induction.) The vectorial product is perpendicular to both $\vec{v}$ and $\vec{B}$, so the force will not change the magnitude of the velocity, and therefore the magnitude of the force vector also stays constant: $F = evB\sin\alpha$. Since the force has no component parallel to $\vec{B}$, the motion of the electron in the direction of the field lines is uniform, while the projection of its motion on a plane perpendicular to the field lines is uniform circular. The radius of its orbit can be determined by setting the Lorentz force equal to the centripetal force:



$$evB\sin\alpha = \frac{mv^2\sin^2\alpha}{r}.$$

Hence

$$r = \frac{mv\sin\alpha}{eB}.$$

Thus the trajectory of the electron is a helix with its axis parallel to the field lines. In a reference frame moving along with the electron at a parallel speed of $v_{\|} = v\cos\alpha$, the electron is moving in a circular orbit at a tangential speed of $v_\perp = v\sin\alpha$, and its orbital period is

$$T = \frac{2r\pi}{v_\perp} = \frac{2\pi mv\sin\alpha}{eBv\sin\alpha} = \frac{2\pi m}{eB} =$$
$$= \frac{2\pi\cdot 9\cdot 10^{-31}\,\mathrm{kg}}{1.6\cdot 10^{-19}\,\mathrm{C}\cdot 0.02\,\mathrm{T}} = 1.767\cdot 10^{-9}\,\mathrm{s},$$

independently of the initial direction of the velocity of the electron.

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As in our case $\vec{v}$ is perpendicular to $\vec{B}$, the magnitude of the magnetic force will be $F = QvB$, which is now substituted into the previous equation to get:

$$QvB\sin\alpha = ma_x. \tag{3}$$

Note that in equation (3) $v\sin\alpha = v_y$, which is the y coordinate of velocity. Assuming that acceleration is the time derivative of velocity and velocity is the time derivative of displacement, we obtain:

$$QB \cdot \frac{\mathrm{d}y}{\mathrm{d}t} = m \cdot \frac{\mathrm{d}v_x}{\mathrm{d}t}.$$

Integrating this leads us to:

$$QB\int_0^h \mathrm{d}y = m \cdot \int_0^v \mathrm{d}v_x, \tag{4}$$

where the integration limits of depth are chosen to be 0 and h and the corresponding values 0 and $v_x = v$ are the limits of velocity. We assumed that the x coordinate of velocity reaches its maximum at the deepest point of the path, since that is the point where the velocity of the ball becomes horizontal. After integration equation (4) becomes:

$$QBh = mv.$$

Substituting v from equation (2), we find:

$$QBh = m\sqrt{2gh}.$$

From which the deepest point's y coordinate is ($y_{\max} = h$):

$$h = \frac{2m^2g}{Q^2B^2} = \frac{2\cdot(0.003\cdot 10^{-3}\,\mathrm{kg})^2\cdot 9.81\,\mathrm{m/s^2}}{(5\cdot 10^{-5}\,\mathrm{C})^2\cdot(0.4\ \mathrm{T})^2} = 0.441\,\mathrm{m}.$$

This is the depth in which the velocity of the ball reaches its maximum, so:

$$v_{\max} = \sqrt{2gh} = \frac{2mg}{QB} = \frac{2\cdot 0.003\cdot 10^{-3}\,\mathrm{kg}\cdot 9.81\,\frac{\mathrm{m}}{\mathrm{s}^2}}{5\cdot 10^{-5}\,\mathrm{C}\cdot 0.4\ \mathrm{T}} = 2.94\,\frac{\mathrm{m}}{\mathrm{s}}.$$

Remarks: 1. The data given in this problem are unreal. The radius of a ball of mass 0.003 g cannot be less than a few millimeters, because if it was charged to $5\cdot 10^{-5}$ C, the electric potential near to the surface of the ball would be approximately of magnitude

$$V = 9\cdot 10^9\frac{5\cdot 10^{-5}}{10^{-6}}\,\mathrm{V} = 4.5\cdot 10^{11}\,\mathrm{V}$$

Therefore it is impossible to charge this ball to the given magnitude, as electric sparks would be emitted (that discharge the ball) well before reaching the required charge.

2. The path of the falling ball is a cycloid. Integrating equation (4) to an upper limit of y gives:

$$QBy = mv_x,$$

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Remark. The extreme value of the function can be found using derivative as well. The place of the minimum is given by the solution of equation $\frac{\mathrm{d}M}{\mathrm{d}\varphi}=0$. The equation leads to the quadratic equation

$$2\cdot\cos^2\varphi-\cos\varphi-1=0,$$

whose solution is

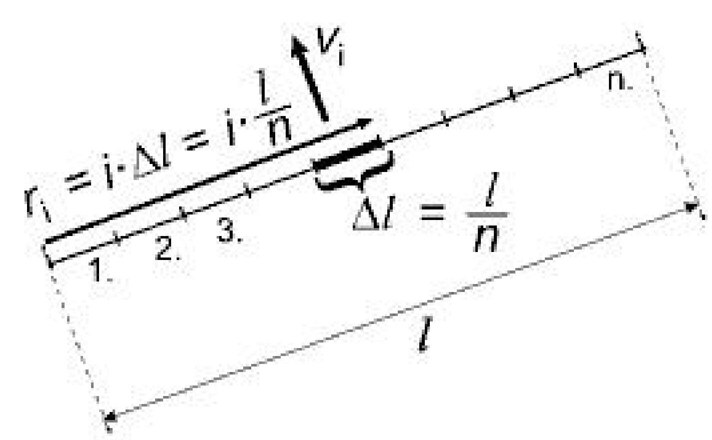
$$\cos\varphi=-\frac{1}{2},\qquad \text{that is,}\qquad \varphi=\frac{2\pi}{3}\ \mathrm{rad}=120^\circ.$$

Solution of Problem 263. The system described above is the model of a unipolar induction generator, i.e. a generator which produces DC current by electromagnetic induction.

The conduction electrons in the disc move on a circular path due to the rotation, therefore the magnetic deflecting force (or Lorentz force) makes them move in the radial direction building up a potential difference between the shaft and the perimeter of the disc. If resistor R is connected across the shaft and the perimeter of the disc, a current will flow in the circuit. The conduction electrons will drift outwards, and since now their velocities have a radial component as well, the Lorentz force will act on them in a direction opposite the rotation, which makes the disc decelerate (Lenz's law). The deceleration caused by the torque of this Lorentz force is proportional to the current flowing in the radial direction, which is proportional to the potential difference existing between the centre and the perimeter of the disc. Since this potential difference depends upon the angular velocity of the rotation, the disc will accelerate until the magnitude of the torque of Lorentz force reaches the magnitude of the torque caused by mass m.

Let us first determine the potential difference induced between the shaft and the perimeter of the disc, and then express the current flowing in the circuit. The second step is then to determine the net torque of the forces acting on the current flowing in the radial direction, which then should be made equal to the torque of mass m.

To find the potential difference between the centre and the perimeter of the disc, let us consider a metal rod rotating about one of its ends in a plane perpendicular to the uniform magnetic field. The potential difference induced between the ends of the rod can be written as the sum of the potential differences $\Delta V=B\Delta lr\omega$ induced in the elements of the rod. If the rod is divided into n equal parts of length $\Delta l=l/n$, the sum can be written as:



$$V=\sum_{i=1}^{n}\Delta V_i=\sum_{i=1}^{n}B\Delta lv_i=\sum_{i=1}^{n}B\cdot\frac{l}{n}\cdot r_i\omega=\sum_{i=1}^{n}B\cdot\frac{l}{n}\cdot i\cdot\frac{l}{n}\cdot\omega,$$

where $r_i=i\cdot\frac{l}{n}$ is the distance of the i^{th} element from the rotation axis, or in other words it is the radius of its circular path. If this radius is multiplied by the angular

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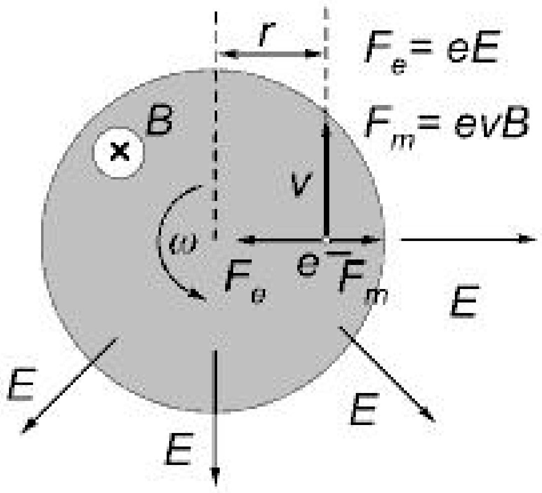
In our case we have two wires of length $l/2$, which are connected in parallel and across which the voltage is the same as the induced voltage in only one of them:

$$V=\frac{B\omega l^2}{8}.$$

Substituting the data: $V=3.16\cdot 10^{-3}\,\mathrm{V}$.

Solution of Problem 265. Based on the direction of rotation, there are two cases to distinguish: 1. $\vec{B}$ and $\vec{\omega}$ point in opposite directions, 2. $\vec{B}$ and $\vec{\omega}$ point in the same direction.

Case 1: $\vec{B}$ and $\vec{\omega}$ are oppositely directed:



In the figure, the induction vector points towards the page and the angular velocity vector points away from the page. The figure shows the (relevant) forces acting on a particular electron when the constant electron density has already set. (There may be fluctuations.) These forces are the magnetic Lorentz-force and the electric force owing to the field of the separated charges. The Lorentz-force now points radially outwards. Since the electron accelerates towards the centre of the circular path, the electric force must act towards the centre. Thus the electric field vector points radially outwards.

The net force (expressed in terms of the magnitudes of the vectors) provides the centripetal force:

$$Ee-er\omega B=mr\omega^2. \tag{1}$$

Hence the magnitude of the electric field is

$$E=\frac{(e\omega B+m\omega^2)}{e}\cdot r. \tag{2}$$

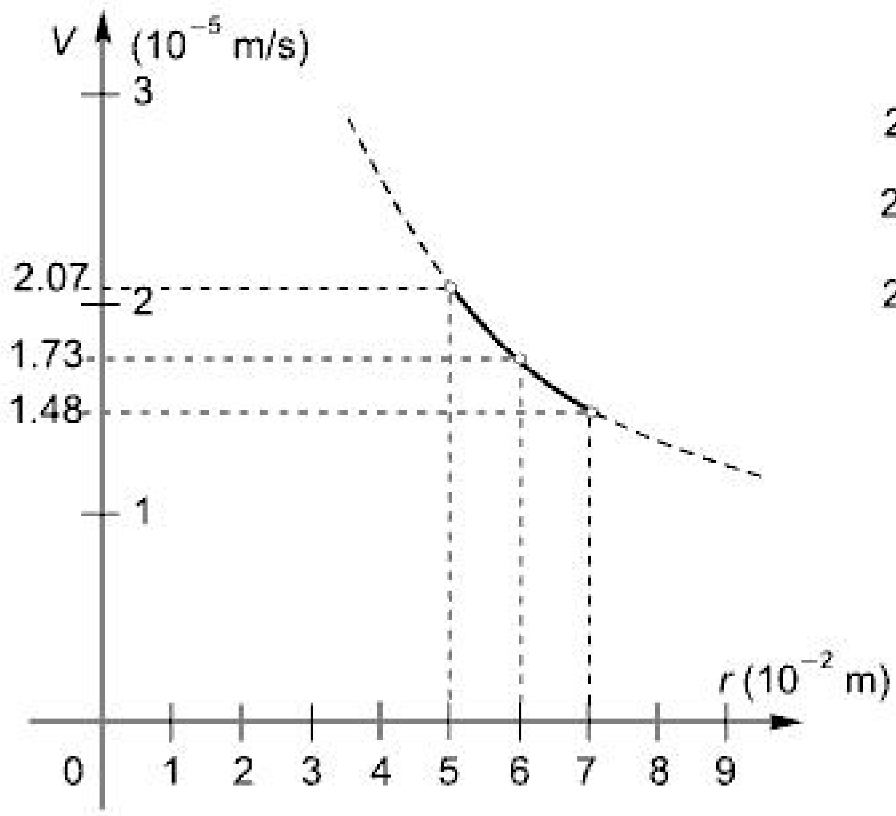
What charge distribution is responsible for the electric field expressed in (2)?

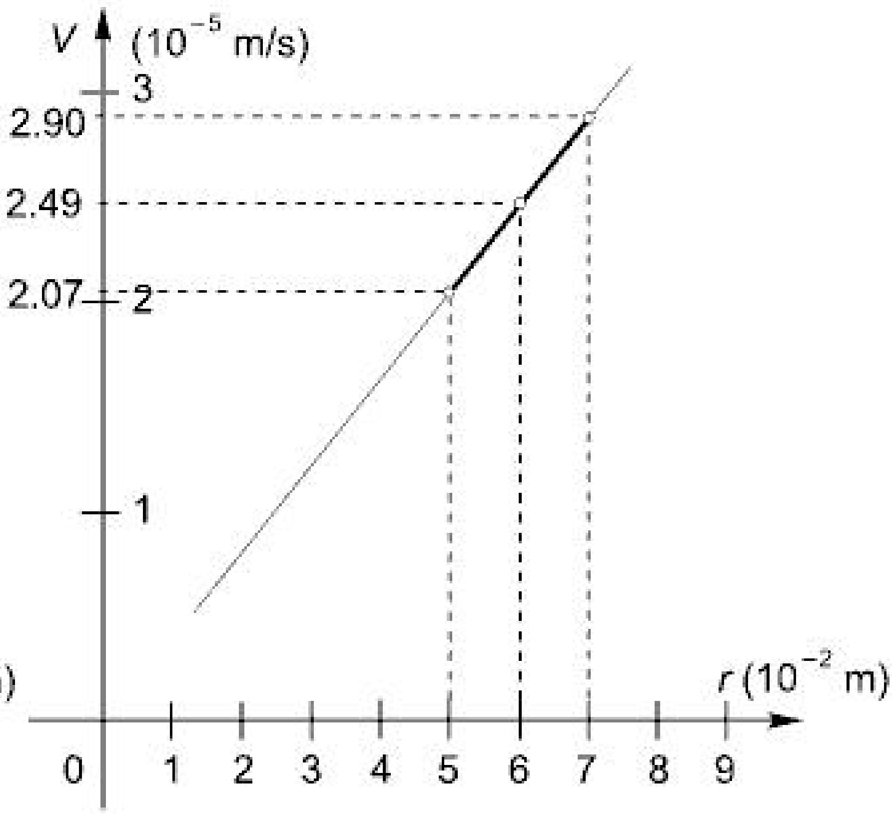
It is known that the magnitude of the electric field in the interior of a long straight cylinder of uniform volume charge density is:

$$E=\frac{\varrho}{2\varepsilon_0}\cdot r, \tag{3}$$

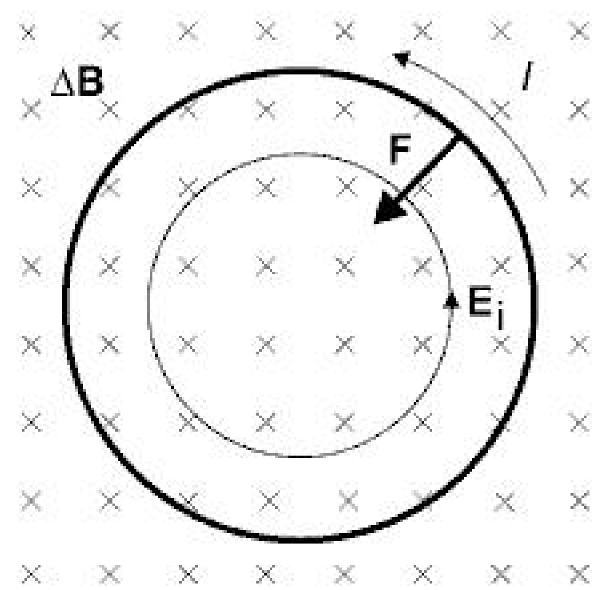
and in the case of positive charge density it points radially outwards, that is, it has the same structure as the electric field established in our rotating cylinder. (With a difference worth mentioning: in the rotating metal cylinder, the result will be accurate for a short cylinder too, since that is the only distribution of the electric field that can provide the force distribution required by the stationary state of rigid rotation). This leads to the conclusion that the charge density inside the rotating cylinder is uniform. The magnitude of that charge distribution is obtained from (2) and (3):

$$\varrho=\frac{2\varepsilon_0\omega(eB+m\omega)}{e}. \tag{4}$$





First solution of Problem 269. The magnetic field changing in time induces an electric field that enters the metallic ring and starts a current in it. On this already current-bearing conductor (whose induction increases in time), a magnetic Lorentz force acts from the same field, which is perpendicular to both the induction lines and the conducting ring, and because of the direction of the induced current points in the direction of the centre of the ring, so it strives to crash the ring. Therefore it creates tensile stress in the ring. (If the ring were in a magnetic field whose induction decreases in time, the force would strive to burst the ring.) We have to determine this force.



As the field changes uniformly in time, the induced electric field and so the loop emf is constant in time, self induction has no part in the process.

The magnitude of the current induced in the conducting ring of resistance r is (applying Ohm's law to our case)

$$I = \frac{U_0}{r} = \frac{\Delta\Phi}{\Delta t \cdot r} = \frac{\Delta B R^2 \pi}{\Delta t \cdot \varrho \frac{2R\pi}{A}} = \frac{\Delta B}{\Delta t} \cdot \frac{RA}{2\varrho}.$$

The simplest way to calculate the created tensile stress is by determining the resultant magnetic field acting on a semicircle and dividing this by twice the cross-sectional area of the wire.

Applied to the Ohmic loops I. and II.:

$$\sum_{I.}^{\bigcirc} V = \varepsilon_1, \qquad \text{and} \qquad \sum_{II.}^{\bigcirc} V = \varepsilon_2.$$

In detail, using the assumed directions of currents:

$$I_1 R_1 - I R_m = \frac{\Delta B}{\Delta t} \cdot A_1, \qquad \text{and} \qquad I_2 R_2 + I R_m = \frac{\Delta B}{\Delta t} \cdot A_2.$$

Hence I_1 and I_2 are

$$I_1 = \frac{\Delta B}{\Delta t} \cdot \frac{A_1}{R_1} + I \cdot \frac{R_m}{R_1}, \tag{1}$$

$$I_2 = \frac{\Delta B}{\Delta t} \cdot \frac{A_2}{R_2} - I \cdot \frac{R_m}{R_2}. \tag{2}$$

According to Kirchhoff's junction rule applied to point A,

$$I_2 = I + I_1. \tag{3}$$

If (1) and (2) are substituted in (3), an equation is obtained with the single unknown being the current flowing through the meter:

$$\frac{\Delta B}{\Delta t} \cdot \frac{A_2}{R_2} - I \cdot \frac{R_m}{R_2} = I + \frac{\Delta B}{\Delta t} \cdot \frac{A_1}{R_1} + I \cdot \frac{R_m}{R_1}.$$

Rearranged:

$$\frac{\Delta B}{\Delta t} \cdot \left(\frac{A_2}{R_2} - \frac{A_1}{R_1} \right) = I \left(\frac{R_m}{R_2} + 1 + \frac{R_m}{R_1} \right).$$

Hence, by finding the common denominator and simplifying, the following expression is obtained for the current in question:

$$I = \frac{\Delta B}{\Delta t} \cdot \frac{A_2 R_1 - A_1 R_2}{R_m (R_1 + R_2) + R_1 R_2}. \tag{4}$$

Numerically, expressed in terms of the resistance R_m of the meter:

$$I = 0.4 \frac{\mathrm{V}}{\mathrm{m}^2} \cdot \frac{0.2\,\mathrm{m}^2 \cdot 5\,\Omega - 0.1\,\mathrm{m}^2 \cdot 2\,\Omega}{R_m \cdot 7\,\Omega + 10\,\Omega^2} = \frac{0.32\,\mathrm{V}}{7 \cdot R_m + 10\Omega}.$$

Question a) can be answered if the given value of $0.5\,\Omega$ is substituted for R_m. To answer question b), the voltage across the meter needs to be expressed and the value of $R_m = \infty$ substituted. Thus, in the a) the meter reads a current of

$$I = \frac{0.32}{7 \cdot 0.5 + 10}\,\mathrm{A} = 0.0237\,\mathrm{A}.$$

It is instructive to determine the currents I_1 and I_2 flowing in the arcs $A1B$ and $A2B$, too. If the value (4) of I is substituted in (1) and (2) and a common denominator is applied, the following expressions are obtained:

$$I_1 = \frac{\Delta B}{\Delta t} \cdot \frac{A_1 [R_m (R_1 + R_2) + R_1 R_2] + A_2 R_1 R_m - A_1 R_2 R_m}{[R_m (R_1 + R_2) + R_1 R_2] R_1}.$$

Solution of Problem 274. A moving electron is affected by both magnetic and electric fields. We should investigate the fields that are present in the centre of the circle at the time instants in question and the direction of the electric field and magnetic induction vectors that characterise these.

When no magnetic field is present ($I_a = 0$), only the changing magnetic flux creates (a stationary) electric field, which accelerates the electron with a force of $\vec{F} = e\vec{E}$ (regardless of its velocity condition). When there is current in the coil again (e.g. at time instant $t_1 = 0.6$ s), a magnetic field is also present in the centre of the circle, which acts with a force of $\vec{F} = e\vec{v} \times \vec{B}$ on the electron; this force is perpendicular to the previous force. Therefore we should determine the magnitude of the electric field and the induction of the magnetic field in the centre of the central circle of the coil at the given moment.

a) When the current is zero, only electric field is present. Within the thin iron wire a so-called toroidal magnetic field is created, whose flux density can be considered constant for the whole cross-section, because the 3.6-mm diameter of the iron wire can be neglected relative to the 200-mm diameter of the circle. This thin magnetic flux tube acts like the thin current-bearing wire in the creation of the magnetic field, therefore the (non-stationary) electric field created by the magnetic flux element can be described in the same way as the magnetic field created by the current element, with a law that corresponds to the Biot–Savart law. All one needs to do is determine which quantities correspond to each other in the two processes. The corresponding quantities are given by the circuital law and the law of induction.

In the case of a straight conductor for one induction line

$$B2r\pi = \mu_0 I \qquad \rightarrow \qquad B = \frac{\mu_0}{2\pi}\frac{I}{r},$$

in the case of a straight thin coil (flux tube)

$$E2r\pi = -\frac{\Delta\Phi}{\Delta t} \qquad \rightarrow \qquad E = -\frac{1}{2r\pi}\frac{\Delta\Phi}{\Delta t}.$$

From these, it is obvious that the quantity corresponding to $\mu_0 I$ in the Biot–Savart law is $\dfrac{\Delta\Phi}{\Delta t}$. With this, the two types of Biot–Savart law are

$$\Delta B = \frac{\mu_0}{4\pi}\frac{I\Delta l}{r^2}\sin\alpha, \qquad \text{and} \qquad \Delta E = \frac{1}{4\pi}\frac{\Delta\Phi}{\Delta t}\frac{\Delta l}{r^2}\sin\alpha.$$

In the centre of the circle the resultant electric field can be determined through the known summation:

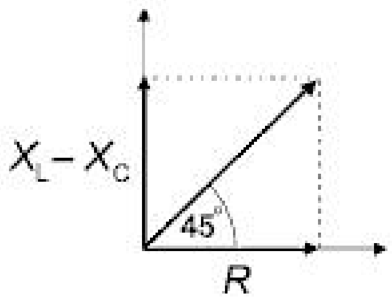
$$E = \sum \Delta E = \sum \frac{1}{4\pi}\frac{\Delta\Phi}{\Delta t}\frac{\Delta l}{r^2}\sin\alpha = \frac{1}{4\pi}\frac{\Delta\Phi}{\Delta t}\frac{1}{r^2}\sum \Delta l \sin 90^\circ,$$

where $\sum \Delta l = 2r\pi$ is the circumference of the circle. From this, after simplifying by $2r\pi$ the following expression is acquired for the electric field in the centre of the circle:

$$E = \frac{1}{2r}\frac{\Delta\Phi}{\Delta t},$$

which is constant due to the condition given in the problem.

Solution of Problem 277. a) As the current lags behind the voltage, the circuit is of inductive nature. The phase angle is $\varphi = \pi/4 \mathrm{rad} = 45°$.



The impedance is

$$Z = \frac{V_{\mathrm{rms}}}{I_{\mathrm{rms}}} = \frac{V_{\max}}{I_{\max}} = \frac{200\,\mathrm{V}}{7.07\,\mathrm{A}} = 28.29\,\Omega.$$

The ohmic resistance is:

$$R = Z \cdot \cos 45° = 28.29\,\Omega \cdot \cos 45° = 20\,\Omega.$$

From the isosceles right-angled triangle:

$$X_L - X_C = R, \qquad \text{,thatis,} \qquad \omega L - \frac{1}{\omega C} = R.$$

From this

$$C = \frac{1}{\omega(\omega L - R)} = \frac{1}{628\,\mathrm{s}^{-1} \cdot (628 \cdot 0.143 - 20)\,\Omega} = 22.81\,\mu\mathrm{F}.$$

b) The maximum potential difference across the coil is $V_{L_{\max}} = I_{\max} X_L$, so the maximum potential difference across the coil is

$$V_{L_{\max}} = I_{\max} \cdot \omega L = 7.07\,\mathrm{A} \cdot 628\,\mathrm{s}^{-1} \cdot 0.143\,\mathrm{H} = 634.91\,\mathrm{V}.$$

As the potential difference across the coil is ahead of the voltage V in the circuit by $\pi/4$, the potential difference across the coil as function of time is

$$V_L = 634.91\,\mathrm{V} \sin\left(628\ \frac{1}{\mathrm{s}} \cdot t + \frac{\pi}{4}\right).$$

The maximum potential difference across the capacitor is

$$V_{C_{\max}} = I_{\max} X_C = I_{\max} \frac{1}{\omega C} = \frac{7.07\,\mathrm{A}}{628\,\mathrm{s}^{-1} \cdot 22.8 \cdot 10^{-6}\ \mathrm{F}} = 493.77\,\mathrm{V},$$

and as the potential difference across the capacitor lags behind the terminal voltage by $3\pi/4$, the potential difference as function of time is

$$V_C = 493.77\,\mathrm{V} \cdot \sin\left(628 \frac{1}{\mathrm{s}} \cdot t + 3\frac{\pi}{4}\right).$$

Solution of Problem 278. According to the statement of the problem, the impedance of the circuit is independent of the resistance of the Ohmic resistor. Let us find out the necessary condition of this.

We illustrate the currents and voltages through and across the individual electric components by rotating vectors. To draw an adequate diagram, we use the following facts:

1. The part RC and the coil are connected in parallel, thus the voltage across the coil and the voltage of the generator are the same. ($U_{RC} = U_L = U$.)

2. The current of the coil (I_L) has a delay of $90°$ with respect to the voltage.

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while the inductive reactance of the coil is:

$$X_L = \omega L = \frac{1}{2\omega C} = 6\,\mathrm{k\Omega}. \tag{3}$$

Let us now find out what happens if the angular frequency is changed to $\omega/\sqrt{2}$. The inductive reactance should by divided by $\sqrt{2}$ while the capacitive reactance should be multiplied by $\sqrt{2}$ to get their new values. Thus, according to equation (1), the net impedance of the parallel LC circuit between points c and d will be:

$$\frac{(6\,\mathrm{k\Omega}/\sqrt{2})\cdot 12\,\mathrm{k\Omega}\cdot\sqrt{2}}{12\,\mathrm{k\Omega}\cdot\sqrt{2}-(6\,\mathrm{k\Omega}/\sqrt{2})} = 4\sqrt{2}\,\mathrm{k\Omega},$$

being an inductive type of impedance. The net impedance between points b and d is:

$$X_{bd} = X_C - X_{cd} = 12\sqrt{2}\,\mathrm{k\Omega} - 4\sqrt{2}\,\mathrm{k\Omega} = 8\sqrt{2}\,\mathrm{k\Omega}$$

being a capacitive type of impedance.

The net impedance between points a and d can be calculated as:

$$Z_{ad} = \sqrt{R^2 + X_{bd}^2} = \sqrt{(5\,\mathrm{k\Omega})^2 + (8\sqrt{2}\,\mathrm{k\Omega})^2} = \sqrt{153}\,\mathrm{k\Omega} = 12.37\,\mathrm{k\Omega}.$$

Therefore the ammeter in the second case reads:

$$I = \frac{V_{gen}}{Z} = \frac{5\,\mathrm{V}}{12.37\,\mathrm{k\Omega}} = 0.404\ \mathrm{mA}.$$

The net impedance between points a and c is:

$$X_{ac} = \sqrt{R^2 + X_C^2} = \sqrt{(5\,\mathrm{k\Omega})^2 + (12\sqrt{2}\,\mathrm{k\Omega})^2} = \sqrt{313}\,\mathrm{k\Omega} = 17.69\,\mathrm{k\Omega}.$$

Therefore in the second case the reading of the voltmeter (which is the RMS voltage across these two points) will be:

$$V_{ac} = X_{ac}\cdot I = 17.68\,\mathrm{k\Omega}\cdot 0.404\ \mathrm{mA} = 7.15\,\mathrm{V}.$$

Solution of Problem 282. The parts in the two branches that are above points P and Q should have the same net impedances and phase differences. In the left branch, where elements are connected in parallel, the current leads the potential difference by φ, whose tangent is:

$$\tan\varphi = \frac{R}{X_{C_e}} = \frac{R\omega C}{2},$$

in the right branch, where elements are connected in series:

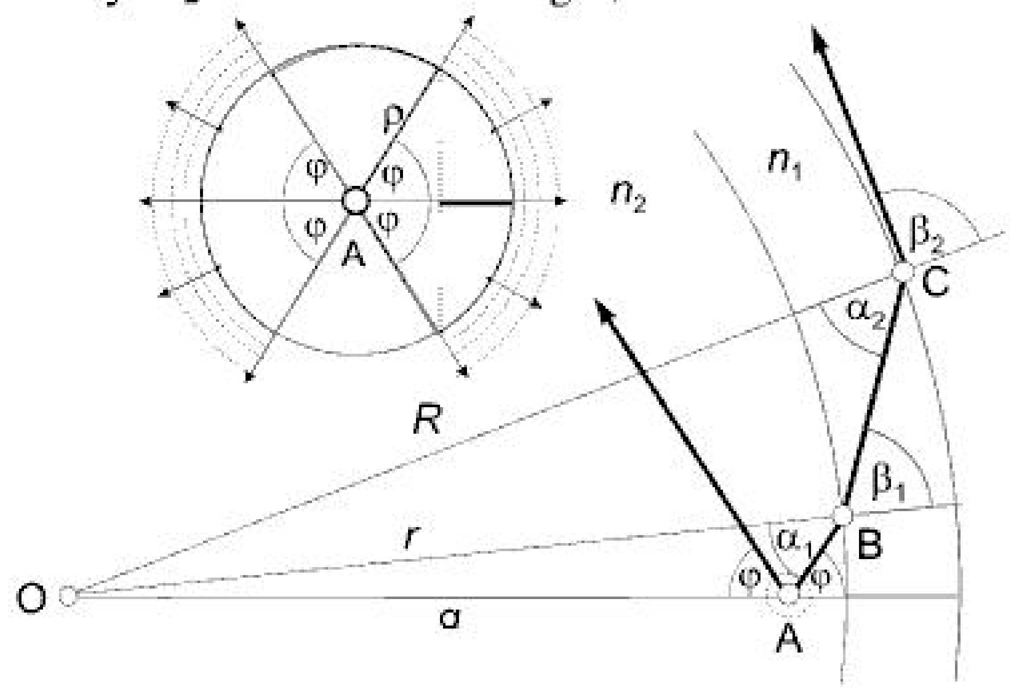
$$\tan\varphi = \frac{X_C}{R_x} = \frac{1/\omega C}{R_x}.$$

Assuming that the right-hand sides of the equations are equal, we obtain:

$$\frac{R\omega C}{2} = \frac{1}{\omega C R_x}.$$

boundary for the first time will sooner or later leave the system. These light rays form a double cone in the inside of the shell, whose cone angle 2φ is determined only by the critical angle at the glass-air boundary. Note that this reasoning neglects the phenomenon of absorption loss that happens during reflections even in materials that are permeable to light and which causes the warming-up of the system. After stating these assumptions, let us start solving the problem.

Let us follow back the way of the light ray that leaves the outer surface of the glass shell at point C with an angle of refraction $\beta_2 = 90°$. In this case the angle of incidence at the glass-air boundary α_2 is the critical angle, so:



$$\sin\alpha_2 = \frac{1}{n_2}$$

The angle of refraction at the carbon disulphide-glass boundary β_1 can be determined applying the Sine-law to triangle OBC:

$$\frac{\sin\beta_1}{\sin\alpha_2} = \frac{R}{r},$$

so:

$$\sin\beta_1 = \frac{R}{r}\sin\alpha_2 = \frac{1}{n_2}\cdot\frac{R}{r}.$$

Applying the law of refraction to point B, we find:

$$\frac{\sin\alpha_1}{\sin\beta_1} = \frac{n_2}{n_1},$$

hence

$$\sin\alpha_1 = \frac{n_2}{n_1}\cdot\sin\beta_2 = \frac{1}{n_1}\cdot\frac{R}{r}.$$

Finally, the angle formed by the light ray and the radius at point A is given by the Sine-law applied to triangle OAB:

$$\frac{\sin\varphi}{\sin\alpha_1} = \frac{r}{a},$$

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$$\frac{1}{-(2-x)}+\frac{1}{16-x}=\frac{1}{-2}.$$

After ordering the equation:

$$x^2-18x+4=0.$$

The solutions are: $x=9\pm\sqrt{77}$, so seemingly, we have discovered two solutions again. $x_1=17.77$ cm and $x_2=0.22$ cm. But the solution of the problem is only the $x=$ $=0.22$ cm, because $17.77\text{ cm}>16\text{ cm}$, thus the lens should have been placed behind the screen, which would make it impossible to form an image.

Solution of Problem 290. A so-called Galilean telescope is to be built, which would create an upright image. This is why the middle lens is needed in order to turn the upside-down image back. The first image of the object, which is at infinity, is formed at the focus of the objective of focal length f_1. The next image is formed in front of the eyepiece at a distance of f_3 from it, which is its focal length. Since the length of the tube is d, the distance which remains for the sum of the object distance and the image distance of the middle lens, $o+i$ is equal to $d-f_1-f_3$. So

$$o+i=d-f_1-f_3.$$

The lens equation for the middle lens is:

$$\frac{1}{o}+\frac{1}{k}=\frac{1}{f_2}.$$

The equation system leads to a quadratic equation for o and i. The solutions will be:

$$o=\frac{d-f_1-f_3-\sqrt{(d-f_1-f_3)(d-f_1-f_3-4f_2)}}{2},$$

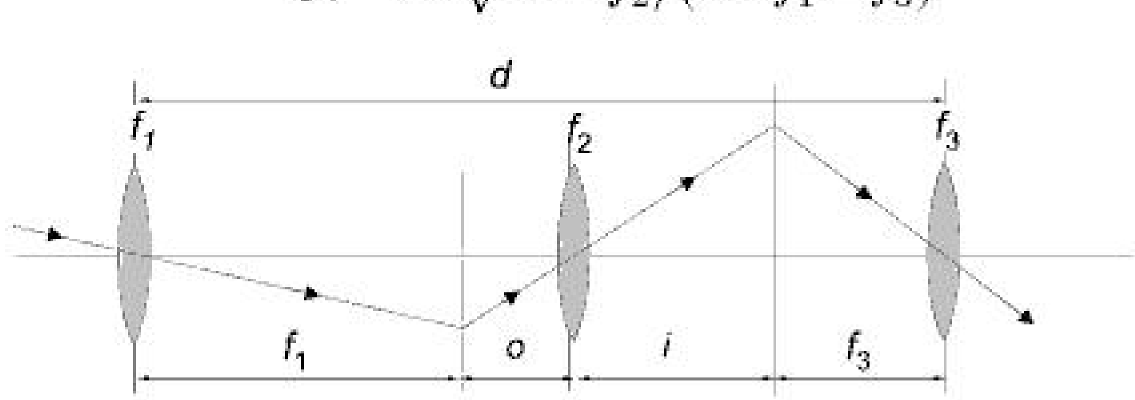
$$i=\frac{d-f_1-f_3+\sqrt{(d-f_1-f_3)(d-f_1-f_3-4f_2)}}{2}.$$

The angular magnification of the middle lens is f_1/o, the angular magnification of the eyepiece is i/f_3, thus the total angular magnification is:

$$N=\frac{f_1}{o}\cdot\frac{i}{f_3}=\frac{f_1}{f_3}o.$$

using the values of i and o:

$$N=\frac{f_1}{f_3}\cdot\frac{1+\sqrt{1-4f_2/(d-f_1-f_3)}}{1-\sqrt{1-4f_2/(d-f_1-f_3)}}.$$



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