OFF-LATICE DIFFUSION LIMITED AGGREGATION ON CIRCLE, SIMULATION AND ANALYSIS

Student : Nguyen Duy Duc
Advisor : Denis Grebenkov
Laboratoire de Physique de la Matire Condense (PMC), Ecole Polytechnique, 91128 Palaiseau Cedex, France

30 August 2012

1 Abstract

In order to check if properties of 2D off-lattice Diffusion-limited Aggregation (DLA) grows on circle still true with DLA grows a seed, we investigate DLA grows on circle. To generate DLA clusters we use the method of hierarchical maps which enable us to grow cluster up to $6\text{x}10^6$ particles in a short time and to reduce statistical error in testing.

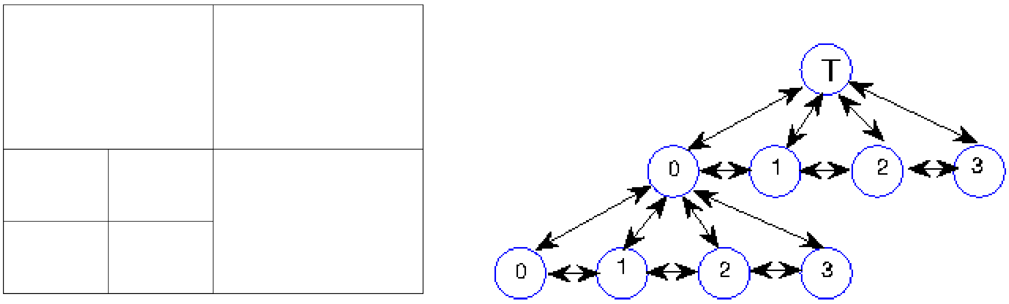
2 Introduction

Diffusion-limited aggregation (DLA) model introduced by Witten and Sander [1] in 1981 and there is still not complete theoretical understanding. The rules of the DLA model are based on an iterative stochastic process in which the particles, one at a time, follow Brownian trajectories until they touch and stick in an aggregate. In section 1, we present a detailed description of an algorithm that improves the basic process and produces these large clusters, in section 2, we present some testing results of big cluster, in section 3 the algorithm works well for 3D case.

3 Model and Method

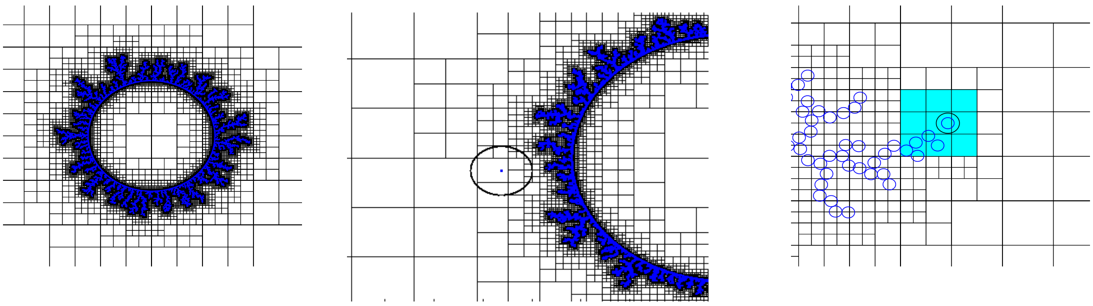
In the basic process [1] mobile particle takes random walk of xed step length until it hits the cluster and sticks to it, then another particle is launched and this process is repeated. It turn out that mobile particle takes much time running around empty regions between branches of the cluster and the algorithm is not very efficient.
The major improvement of the algorithm is to allow mobile particles take a large jump when particles are in the empty regions, but not cross a branch of the cluster. In the method of hierarchical maps [2,3,4], space is divided into regions of various sizes which help keep track of the nearest points on the cluster. Then the mobile particle can make large jumps in empty regions.
Besides, particles are launched from a randomly chosen position on a nite circle, that encloses the cluster, when mobile particles are not interested in cluster and start to drift away, we kill particles at some distance, and launch a new ones.
After this description, we now present the details of our algorithm. To define a square map, we control coordinate of one angle and size L, each map has a Flag will turn on if there are cluster particles whose centers are in its region or turn off if it is empty. If Flag turn on, the map will be divided into four sub-maps inside of size L/2 and controlled by four map pointers, on this level again a map containing turn off Flag indicates an

empty region while a turn on Flag divided to the next ner map. This map structuring is continued to reach the finest map, level 1 map, whose size is equal to diameter of one particle. After mobile particle gets stuck we divide the map who contains that particle and also these maps near it by this trick: if the size of maps are larger than the distance from those maps to cluster particles, we divide those maps. This idea come from Whitney Decomposition idea. This property allow to take the step length for the next jump equal to the size of map where mobile particle is in. However, when mobile particle comes too close to cluster, it is necessary to check the neighbors and define the step length equal to the smallest distance from mobile particle to cluster partilces in neighbors to make sure particle do not step on each other.
Apply to our work, we fist define a map Root of level twenty, then set a circle of one thousand particles and let DLA grows on it, each particle is lunched form Rrelease and killed at Rkill. When the cluster is big enough we expand the map by easily adding a map of level twenty one to the tree structure.



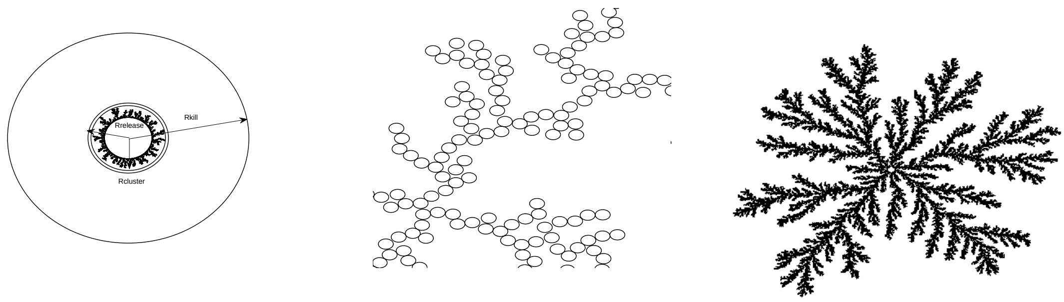
(a) (b)

Figure 1: (a)Map T.(b)Structure of map T, Map T has four sons named 0,1,2,3, each son has pointer point to each other, son named 0 has himself four sons.



(a) (b) (c)

Figure 2: (a)The structure of the hierarchical maps.(b) Mobile particle takes large step in empty region, make sure not to step on other particles and not to cross the branch.(c) Mobile particle takes small step when it comes close to cluster and the neighbors are checked



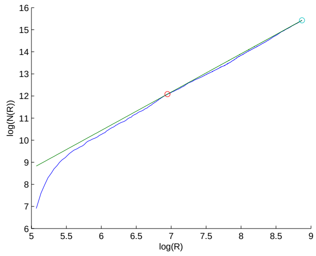
(a) (b) (c)

Figure 3: (a)Particle is lunched randomly on Rrelease which enclose Rcluster and killed if goes outside Rkill.(b)Particles stick together.(c)DLA of $6\text{x}10^6$ partilces built in 18 mins, it needs 6 millions of Nodes to contain particles coordinate, 39.046 millions of maps, 1.533 GB Ram.

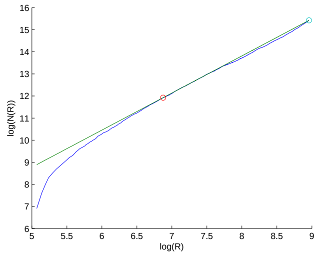
4 Analysis

4.1 Fractal dimention

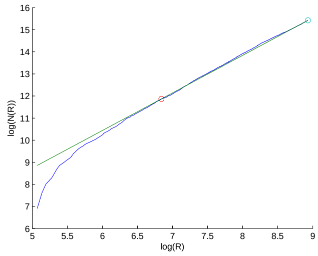
The fractal dimension Df is approximated by relation between radius R and the mass within that radius $N(R)$, as in equation $N(R) = R^{Df}$. In the figures, we grow six DLA of $5\text{x}10^5$ particles on circles, the mean valuse of Df is 1.7023 [5].



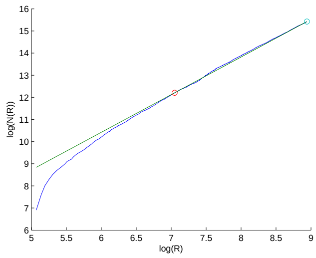
(a) $Df = 1.7356$



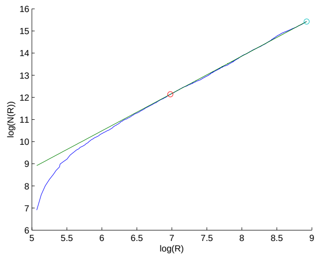
(b) $Df = 1.6799$



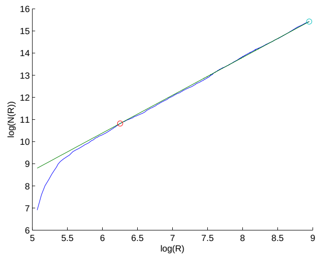
(c) $Df = 1.7012$



(d) $Df = 1.7011$

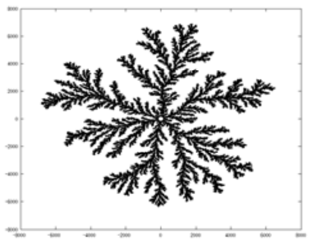


(e) $Df = 1.6884$

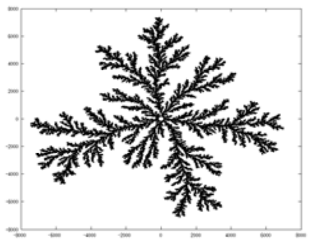


(f) $Df = 1.7073$

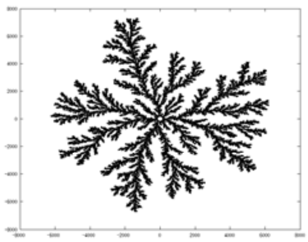
Figure 4: Log(R)-Log(N(R)) plot, and aproximation of Dimentions



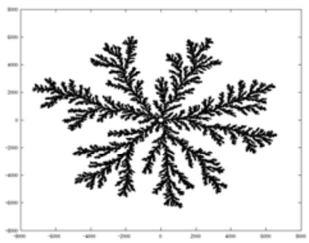
(a) 1



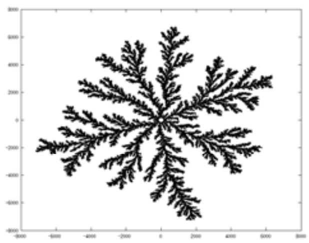
(b) 2



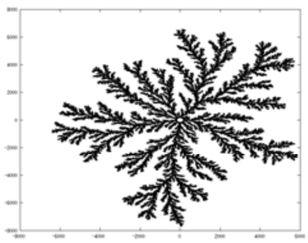
(c) 3



(d) 4



(e) 5



(f) 6

Figure 5: Plot of $5\text{x}10^6$ DLAs

Num	1	2	3	4	5	6
Df	1.7356	1.6799	1.7012	1.7011	1.6884	1.7073
Mean	1.7023					

4.2 Harmonic Measure

We compute the Harmonic measure of each branch of cluster using a biased mobile particles sampling technique, the probabilities of mobile particles hitting branches, here we grow a big DLA of 10^6 particles on circle, after growing-step of 5000 paricles we take Harmonic measure of 5x10^5 particles.
Figures 6,7 show that Harmonic measure is the same for each branch at the beginning, then only big branches "catch" paticles.

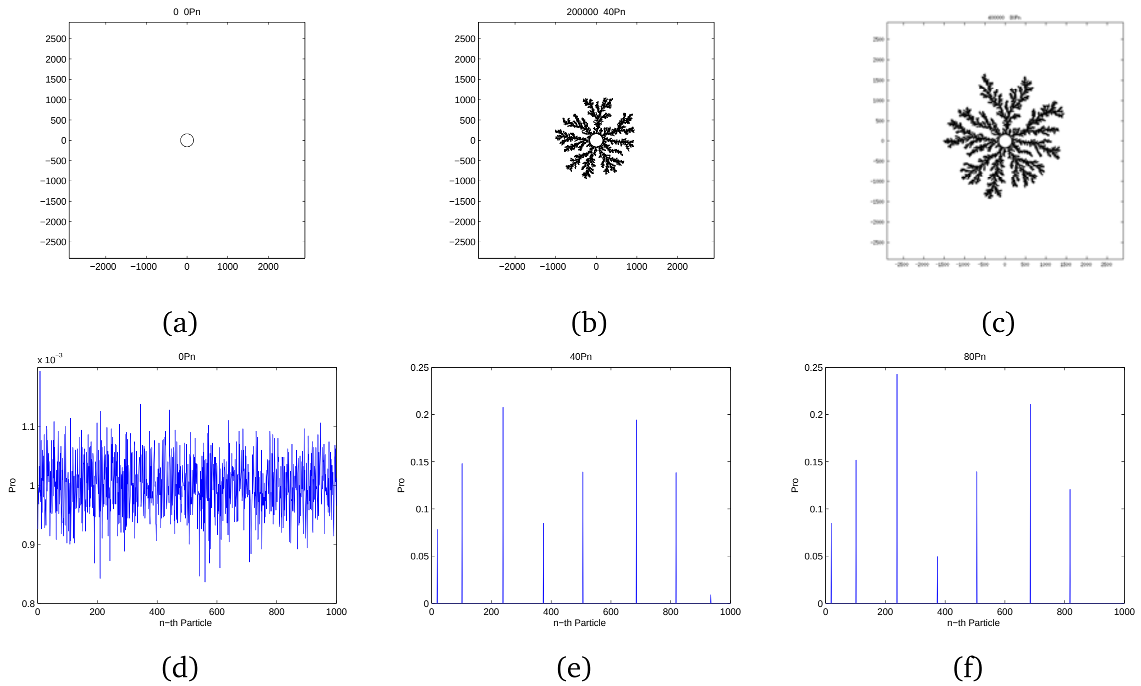


Figure 6: (a)(b)(c) Picture of DLA at 0,40,60 step.(d)(e)(f) Harmonic measure of branches at these step.

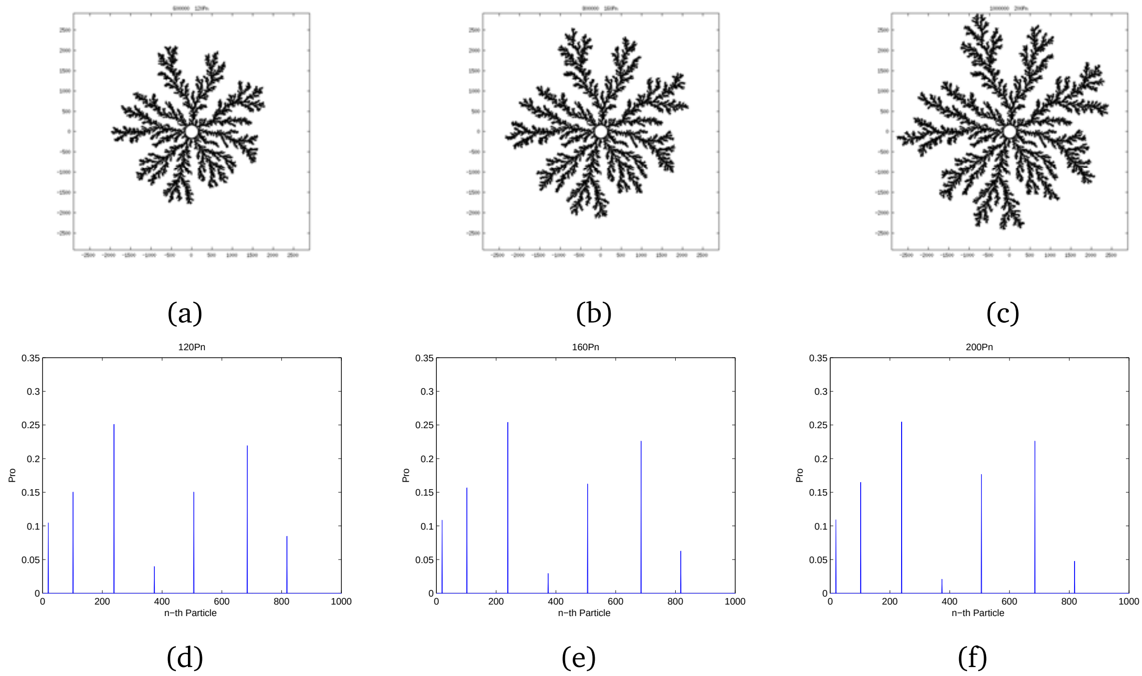


Figure 7: (a)(b)(c) Picture of DLA at 80,160,200 step.(d)(e)(f) Harmonic measure of branches at these steps.

4.3 Diameter of Branch

Because we use 1000 particles for the circle, so they will grow to 1000 branches of DLA, even use much efficient algorithm [6] to compute the diameter of a point set it takes us days for 10^5 DLA and weeks for 10^6 DLA grows on circle. Here are results we have for 10^5 DLA.
Set $S_0 = 1$ we have $\tau = q - 1$. The idea now is to set $S_t(q, \tau) = 1$ so we can consider τ is a function of q or q is a function of τ.

$$r_n^t = \max_{i,j \in branch} |i - j| \tag{1}$$

$$S_t(q, \tau) = \sum_{n=1}^{N} (p_n^t)^q (r_n^t / N)^{-\tau} \tag{2}$$

$$S_0(q, \tau) = \sum_{n=1}^{N} (p_n^0)^q (r_n^0 / N)^{-\tau} = N^{1-q+\tau} \tag{3}$$

$$D_q^t = \frac{\tau(q)}{q - 1}, q \neq 1 \tag{4}$$

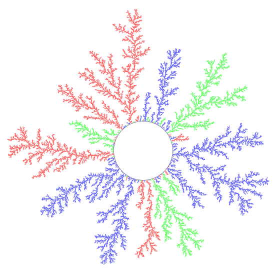
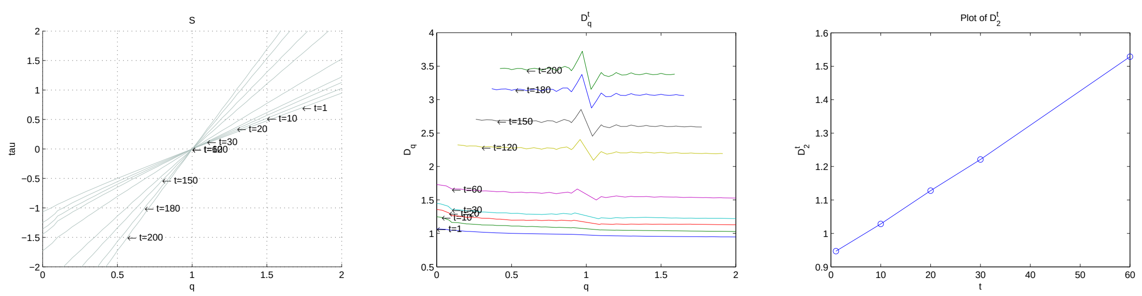


Figure 8: Picture of 10^5 DLA and its branches



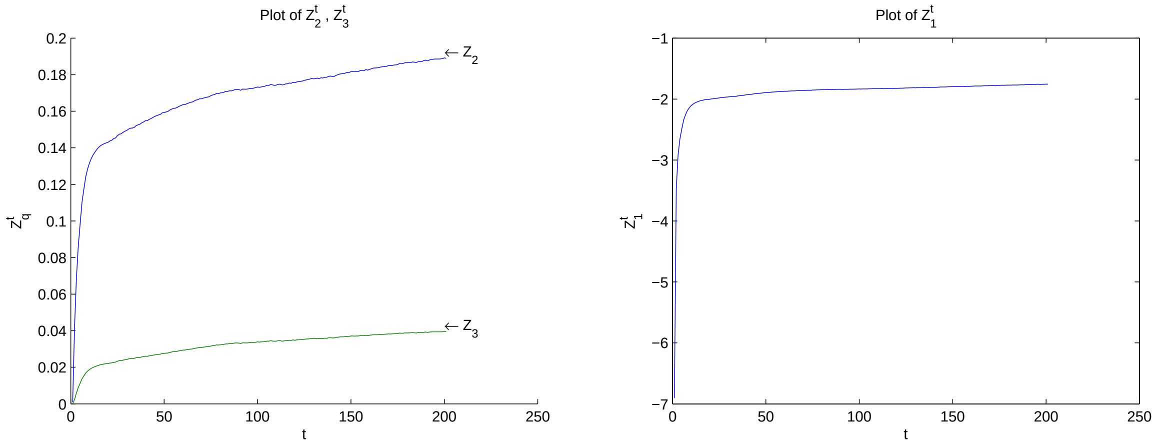
(a) (b) (c)

Figure 9: (a) Plot of $\tau(q)$ for some steps t.(b) Plot of D_q^t for some steps t.(c) Plot of D_2^t for all steps t.

4.4 $Z_q^t, \tilde{Z}_1^t$

$$Z_q^t = \sum_{n=1}^{N} (p_n^t)^q \tag{5}$$

$$\tilde{Z}_1^t = \sum_{n=1}^{N} p_n^t ln(p_n^t) p_n^t \text{Harmonic measure at n-th branch} \tag{6}$$



(a) (b)

Figure 10: (a) Plot of Z_q^t.(b) Plot of $\tilde{Z}_1^t$

4.5 Group

Divide 1000 branches into succession groups of 2,4,8,20,40,50,100 and see the behavior of Z_2^t. In the figure Gn is the graph of Z_2^t computed from group of n branches. Randomly make groups of 2,4,8,20,40,50,100 of branches from 1000 branches and see the behavior of Z_2^t . In the figure RGn is the graph of Z_2^t computed from random group of n branches. Beside, it is necessary create several times, here we creat 4 times.

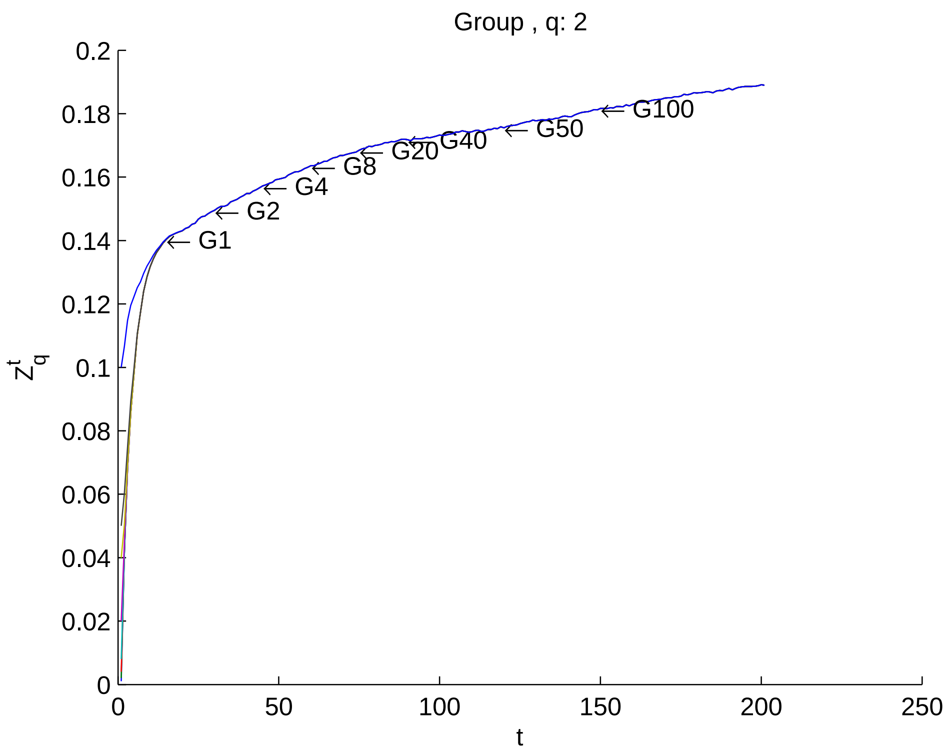
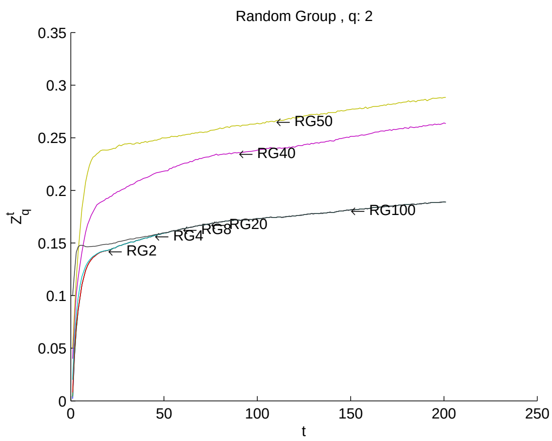
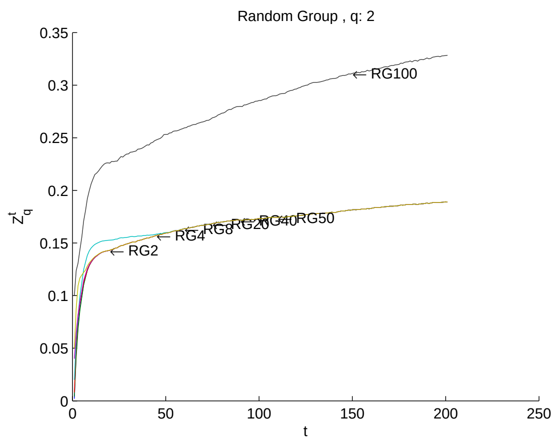


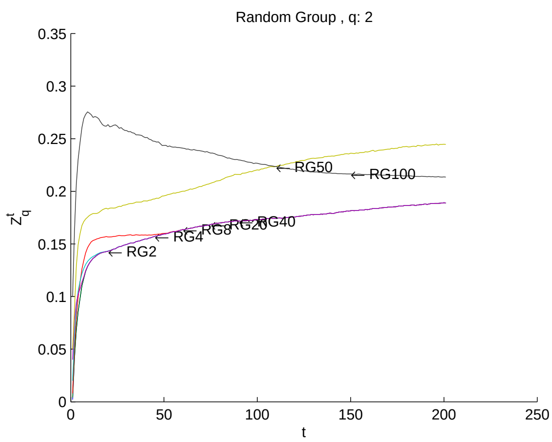
Figure 11: Plot of Z_2^t of succession groups



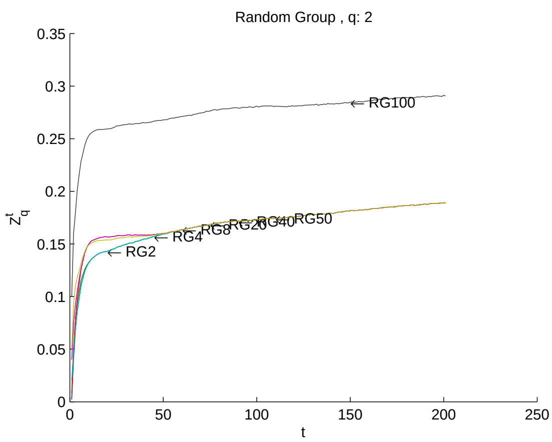
(a)



(b)



(c)



(d)

Figure 12: Plot of Z_2^t of random groups for four times.

4.6 Correlations

$$
\begin{aligned}
C_m^t &= \sum_k^N p_k^t p_{k+m}^t, \text{If } k+m > N \text{ we take } k+m-N && (7)\\
C_m^0 &= \sum_k^N \left(p_k^0 p_{k+m}^0\right) = \sum_k^N \left(\frac{1}{N}\right)^2 = \frac{1}{N} && (8)\\
C_0^t &= \sum_k^N \left(p_k^t\right)^2 = Z_2^t && (9)\\
CC_m^t &= \frac{1}{100}\sum_{l=1}^{100} C_{m,l}^t && (10)
\end{aligned}
$$

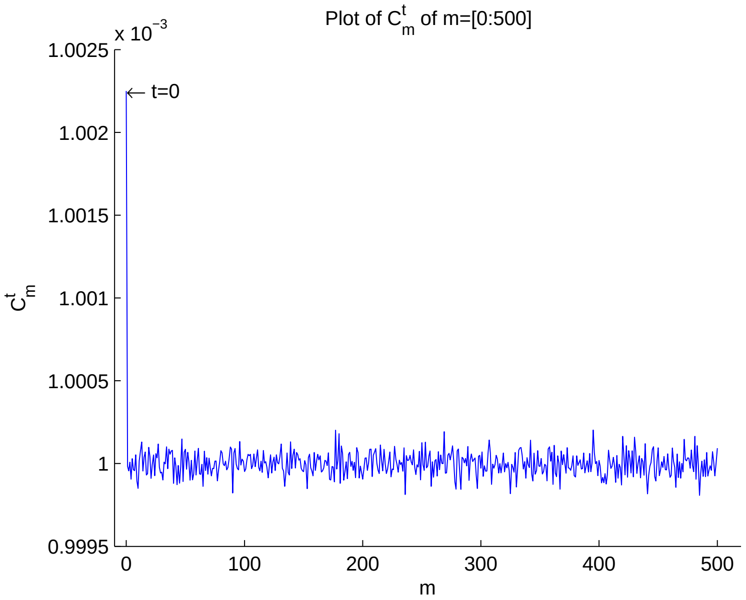
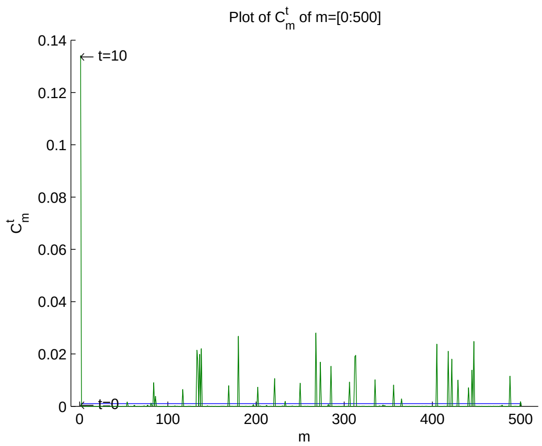
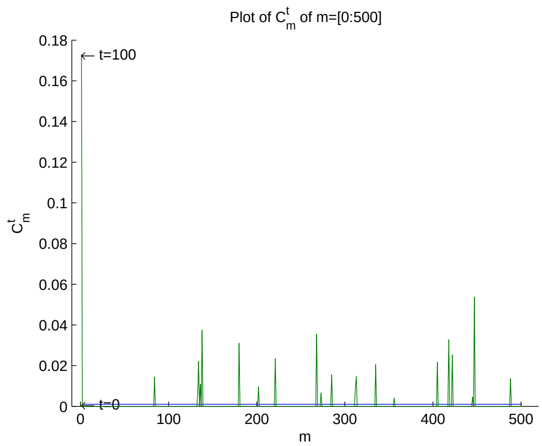


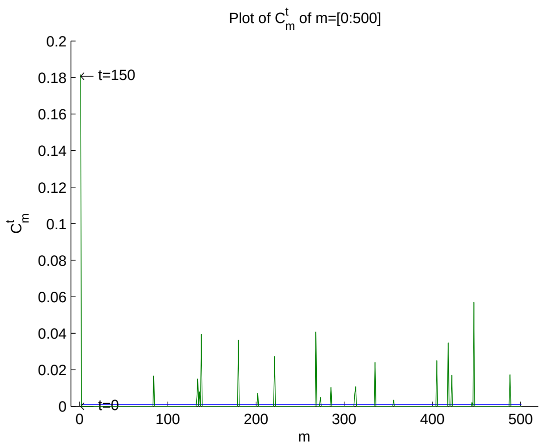
Figure 13: Plot of C_m^0



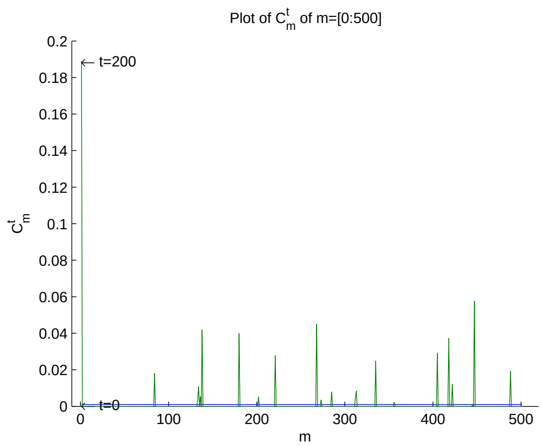
(a)



(b)



(c)

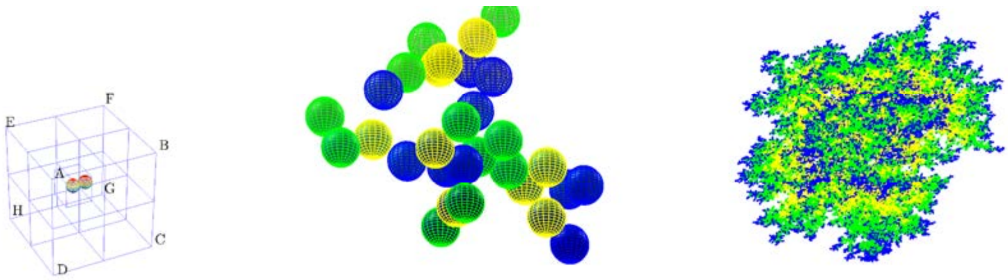


(d)

Figure 14: Plot of C_m^0 with its compare (a) C_m^{10} (b) C_m^{100} (c) C_m^{150} (d) C_m^{200}

5 3D DLA

The algorithm works well for 3D case.



(a) (b) (c)

Figure 15: (a) Structure of maps (b) 3D DLA grows on seed.(c) 10^5 3D DLA grows in 3 mins

6 Out look

Next step we want to analyze huge 2D, 3D DLA of mass up to $10\text{x}10^6$.

References

[1] T.A. Witten and L.M. Sander, Diffusion-limited aggregation, a kinetic critical phenomenon, Physical Review Letters 47, 14001403 (1981)
[2] Paul Meakin,The structure of two-dimensional Witten-Sander aggregates, J.Phys. A: Math. Gen. 18 (1985) L661-L666
[3] Peter OSSADNIK, Multiscaling analysis of large-scale off-latice DLA, Physica A 176 (1991) 454-462
[4] R. C. Ball and R. M. Brady, J. Phys. A 18, L809 (1985).
[5] T.A Witten and L.M. Sander, Diffusion-limited aggregation, Physical Review B 27,9 (1983)
[6] Gregoire Malandain, Jean-Daniel Boissonnat, Computing the Diameter of a Point Set