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Thinning grayscale well-composed images

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Abstract

Usual approaches for constructing topological maps on discrete structures are based on cellular complexes topology. This paper aims to construct a coherent topological map defined on a square grid from a watershed transformation. The main idea behind the proposed approach is to impose some constraints on the original image in order to obtain good properties of the resulting watershed. We propose a definition of well-composed grayscale images based on the well-composed set theory and the cross-section topology. Properties of two different thinning algorithms are then studied and we show how to obtain a thin crest network. We derive an efficient algorithm that permits the construction of a meaningful topological map, resulting in a topological segmentation, i.e. a segmentation that describes in a coherent framework faces and contours. Finally, we demonstrate the usefulness of this algorithm for multilevel image segmentation.

1. Introduction

A digital image may be seen as the digitization of a piecewise continuous function. The discontinuities of this function are of primary importance in many shape recognition processes, as they usually describe the shapes of objects appearing on an image.

The notion of discontinuity is lost once a function is digitized. The structure describing domains where the underlying piecewise function is continuous is called a segmentation. Two main dual approaches to segmentation may be distinguished. The first approach consists of approximating the discrete function by a piecewise continuous function, and is usually referred to a region oriented segmentation. The second approach tries to directly catch the discontinuities of an underlying continuous function and is referred to a contour oriented segmentation. A topological segmentation may be viewed in this context as a process capturing both domains where an underlying piecewise function is continuous, and the set of points describing the discontinuities of the function.

A piecewise continuous function may be represented by a topological map, which can be viewed as a partition of the plane into three sets of points: a finite set S of points, a finite set A of disconnected Jordan arcs having elements of S as extremities, and a set of connected domains, the faces, whose boundaries are unions of elements of S and A. In this paper we aim to construct a topological segmentation that has a topological

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map structure consistent with a digital topological framework and with a topological map representing a continuous piecewise function.

Recent works (Braquelaire and Brun, 1998; Fiorio, 1996) aim to develop a coherent topological structure describing a digital image from the information provided by regions. The different structures described use a discrete topology based on the decomposition of the support domain of an image into three kinds of elements of different dimensions, i.e. surface elements, associated with the discrete points of the support, edge elements, which are the edges separating two surface elements, and vertices of the so defined grid (Fiorio, 1995; Kovalevsky, 1989). On one hand, such a partition has nice topological properties, but on the other hand, it suffers from many practical drawbacks, such as the amount of memory needed to store the entire partition, and difficulties faced when trying to construct the partition from the contour information.

Alternatively, a topological partition of an image may be directly defined by a digital topology involving only points on the square grid. In order to face the connectivity paradox, several neighborhood systems are usually used together. This is done either by considering different adjacency relations for points belonging to a set and its complement (Kong and Rosenfeld, 1989), or by assigning different neighborhoods to each point of $\mathbb{Z}^2$ in a data independent manner, which can be formally stated using the framework proposed in (Khalimsky et al., 1990) (this framework can also be used in the cellular complex approach (Kovalevsky, 1989)).

Watersheds or more generally graytone skeletons can be used in this context to retrieve a vertex/arc network (crest lines) and faces (catchment basins) from a discrete topographic surface such as the modulus of the gradient viewed as a relief, which is exactly the sought for partition of the image.

However, many consistency problems are encountered on a square grid. Approaches that work by suppressing points from a potential crest network (grayscale thinning) or by adding points to connected sets of points (Arcelli, 1981; Bertrand et al., 1997) do not usually guarantee that the extracted crest network is thin (Fig. 1). Thick configurations of crests pose obvious problems when one is trying to link points from the resulting crests network in order to obtain digital curves and vertices of the topological partition. Approaches that work by linking potential crest points (Meyer, 1989; Pierrot Deseilligny et al., 1998), constructing a raster graph, do not usually guarantee that the faces defined by the cycles of the graph are composed of a unique connected component (Fig. 2).

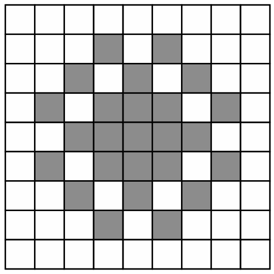


Fig. 1. Thick irreducible configuration of points (Arcelli, 1981).

Latecki proposed to face the problem of thickness of skeletons on digital binary images by forbidding some configurations of points (Latecki et al., 1995). He proposed a thinning operator that preserves the properties of the so called well-composed binary images, resulting in a thin skeleton. He also demonstrated a Jordan theorem that is verified on well-composed sets of points. He extended the property of well-composedness to multicolor images.

In this contribution, we first recall some classical notions of digital topology (Section 2) and some properties of well-composed sets (Section 3). We redefine grayscale well-composed images using the cross section topology formalism (Bertrand et al., 1996, 1997; Meyer, 1989), adapt a grayscale

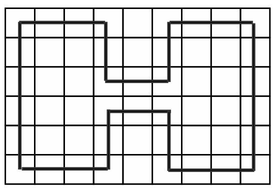


Fig. 2. Connected components associated with a cycle of a raster graph.

thinning algorithm to well-composed graylevel images, and prove some of its properties (Section 4). We derive an algorithm which constructs a topological partition from an irreductible thin well-composed image, and finally present an application (Section 5).

2. Digital topology: basic notions

A discrete image I is a function from $\mathbb{Z}^2$ to a set E. If $E = \{0, 1\}$, I is said to be a binary digital image. If $E = \{0, \ldots, k\}$, I is said to be a grayscale digital image. A point of a digital image is a couple $p = (x, y) \in \mathbb{Z}^2$. Two points $p_1 = (x_1, y_1) \in \mathbb{Z}^2$ and $p_2 = (x_2, y_2) \in \mathbb{Z}^2$ are

- 4-adjacent if and only if $d_4(p_1, p_2) = |x_1 - x_2| + |y_1 - y_2| = 1$.
- 8-adjacent if and only if $d_8(p_1, p_2) = \max(|x_1 - x_2|, |y_1 - y_2|) = 1$.

The n-neighborhood $\Gamma_n(p)$ of a point p is the set of all the points n-adjacent to p, with $n = 4$ or $n = 8$. A point p is said to be diagonally adjacent to a point p' iff p is 8-adjacent to p' but not 4-adjacent to p'. A n-connected path is an ordered set $C = \{p_1, \ldots, p_m\}$ such that for all the points $p_{i,1<i\leqslant n}$, p_i is n-adjacent to p_{i-1}. A n-connected simple curve C is a n-connected path such that all the points of C have at most two n-connected neighbors in C. A n-connected simple closed curve $C = \{p_1, \ldots, p_m\}$ is a n-connected simple curve such that $p_1 \in \Gamma_n(p_m)$. A set $S \subset \mathbb{Z}^2$ is n-connected iff $\forall(p_1, p_2) \in S^2, \exists C \subset S^2, C$ is a n-connected path and $(p_1, p_2) \in C$. A n-connected component of a set $S \subset \mathbb{Z}^2$ is a subset C of S such that $\forall p_1 \in C$, $\forall p_2 \in S$, $p_2 \in \Gamma_n(p_1) \Rightarrow p_2 \in C$.

3. Well-composed sets of points

A coherent topological structure of a digital image should respect an equivalent of the Jordan curve theorem by which the complement of a simple closed curve is a set composed of two connected components. One of the major drawbacks of digital topology is known as the connectivity paradox. If the same neighborhood system is used for studying the connectivity of a set and its complement, the Jordan curve theorem does not have its digital counterpart on a square grid. In order to solve this problem, two neighborhood systems (8 and 4-neighborhood) are used together, defining the connectivity of respectively a set and its complement (Kong and Rosenfeld, 1989). Latecki et al. (1995) proposed a different approach. By forbidding some local configurations of the studied sets of points, one can use a coherent 4-neighborhood system. In this section, we present the results that he obtained which are useful for the rest of this article. Readers interested in the proofs of those results may refer to (Latecki, 1999; Latecki et al., 1995).

A subset S of $\mathbb{Z}^2$ is weakly well-composed iff each 8-connected component of S is a 4-connected component of S. A subset S of $\mathbb{Z}^2$ is well-composed iff S and $\overline{S}$ are weakly well-composed. The Fig. 3 illustrates the concept of well-composedness. On the left hand side of the drawing, the set of gray points form a weakly well-composed set of points, while the right hand side drawing shows a set of points which is well-composed. In order for a set of points to be well-composed, configurations like the one surrounded by a bold square are forbidden. Thus, well-composed sets can be characterized by local configurations.

This leads to the statement of a new Jordan theorem on a well-composed set:

Theorem 3.1. *The complement of a well-composed simple closed curve is formed by two well-composed connected components.*

Classical thinning algorithms do not usually conserve the property of well-composedness.

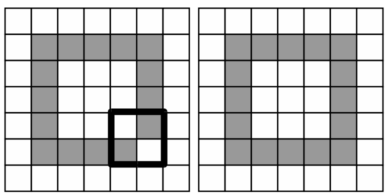


Fig. 3. A non well-composed set of points (on the left) and a well-composed set of points (on the right).

However, by slightly modifying the definition of simple points, we obtain a thinning operator that conserves well-composedness.

A point p is simple for a well-composed set $S \subset \mathbb{Z}^2$ iff there exists a bijection between 4-connected components of $S \cup \{p\}$ and 4-connected components of $S \setminus \{p\}$, and between 4-connected components of $\overline{S} \cup \{p\}$ and 4-connected components of $\overline{S} \setminus \{p\}$. Simple points for well-composed sets can be characterized by a few local configurations. One can show that a point p is simple for a well-composed set S iff there exists exactly one 4-connected component of S 4-adjacent to p and one 4-connected component of $\overline{S}$ 4-adjacent to p.

Thinning a well-composed set S consists in iteratively deleting simple points of S. By construction, the thinning operation preserves well-composedness, i.e. a set S' obtained by thinning a well-composed set S is a well-composed set.

A set S is irreducible iff no point of S is simple. Let the n-interior of a set S of points be the set $n - \mathrm{int}(S) = \{p \in S, \Gamma_n(p) \subset S\}$.

An irreducible well-composed set is thin, which can be formally stated as:

Theorem 3.2. *The* 8-*connected components of the* 4-*interior of an irreducible well-composed set are reduced to one point.*

The proof of this theorem found in (Latecki, 1999) holds for the earlier definition of simple points. This can be used to enumerate all the irreducible local configurations (Fig. 4). On this figure, white lines show how to link points from an irreducible well-composed set of points in order to obtain a vertex/arc network defining a coherent topological partition.

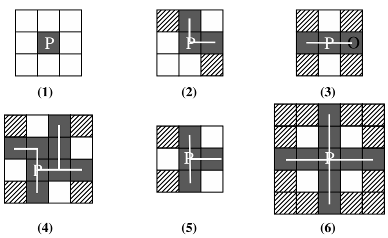


Fig. 4. Local irreducible configurations for well-composed sets. The hashed points are not considered in the configurations.

4. Well-composed grayscale images

Meyer (1989) proposed to characterize crest points on a grayscale image by studying a family of binary images, which are obtained by thresholding the original image by all possible threshold values. Crest points are the points that change the homotopy of one of the binary images. More recently, cross-section topology has been introduced by Bertrand et al. (1996, 1997). It has led to the development of an efficient watershed algorithm (Couprie and Bertrand, 1997). A definition of well-composed multicolor images has also been proposed (Latecki, 1995).

In this section, we introduce well-composed grayscale images (Latecki, 2000), which is compatible with the cross section topology, and we extend the notion of simple points in such a framework. We then study some new properties of thinning operators applied on well-composed grayscale images. Finally, we derive a thinning operator that enables the construction of a topological map from a relief.

4.1. Basic definitions

Let I be a grayscale digital image, defined as a function from $\mathbb{Z}^2$ to $E = \{1, \ldots, m\}$.

A c-section of I is the set of points $F_c = \{(x, y) \in \mathbb{Z}^2, I(x, y) \geqslant c\}$.

A c-cut of I is the set of points $E_c = \{(x, y) \in \mathbb{Z}^2, I(x, y) = c\}$.

A grayscale digital image I is said to be well-composed iff all its c-sections are well-composed sets. The local configuration depicted on Fig. 5 is

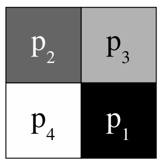


Fig. 5. Forbidden configuration for well-composed grayscale images. $I(p_1) > I(p_2) > I(p_3) > I(p_4)$.

forbidden for well-composed grayscale images, since the two points of higher values are diagonally adjacent. Then, there exists c such that the c-section of I is not a well-composed set. The other forbidden configurations may be constructed by rotations and symmetries.

A point $p \in \mathbb{Z}^2$ is said to be destructible iff it is simple for the section F_c, with $c = I(p)$.

We define the two sets $\Gamma^+(p) = \{p' \in \Gamma_8(p), I(p') \geqslant I(p)\}$ and $\Gamma^-(p) = \{p' \in \Gamma_8(p), I(p') < I(p)\}$.

A destructible point p verifies that both $\Gamma^+(p)$ and $\Gamma^-(p)$ are composed of a unique 4-connected component, with a point 4-adjacent to p.

The destruction of a point p is the operation consisting in subtracting 1 from $I(p)$. Inversely, the construction operation is the operation that consists in adding 1 to $I(p)$.

Theorem 4.1. *Let I be a well-composed grayscale image, and p be a destructible point of I. Then p is simple for the sections $F_{I(p)}, \ldots, F_{\alpha(I,p)}$ with $\alpha(I,p) = \max_{y \in \{x \in \Gamma_4(p), I(x) < I(p)\}} (I(y))$. Moreover, the sets $F_{I(p)} \setminus \{p\}, \ldots, F_{\alpha(I,p)} \setminus \{p\}$ are well-composed.*

The preceding result can be derived by observing the three configurations depicted on Fig. 6. We impose that $I(p) \geqslant I(p_1)$, $I(p) \geqslant I(p_2)$, $I(p) \geqslant I(p_3)$ and that $I(p) \leqslant I(p_4)$. The preceding result can be demonstrated by observing that p is simple for the k-sections, with $k > \alpha(I,p)$.

The preceding property indicates the consistency of the destruction operation.

Any graylevel image may be converted into a well-composed grayscale image with a single pass algorithm that considers only local configurations.

If there exist points p, p', p_1 and p_2 such that p' is diagonal adjacent to p, p_1 and p_2 are both 4-adjacent to p and p', and such that $I(p) \leqslant I(p')$, $I(p_1) > I(p)$ and $I(p_2) > I(p)$, then the image is not well-composed (Fig. 5). By raising the value of p to the minimum of $I(p_1)$ and $I(p_2)$, this 2×2 configuration is not a forbidden configuration anymore. Other local forbidden configurations may be created by raising the value of a point, and can be treated in a straightforward way by propagation. By iteratively applying the preceding algorithm to all the 2×2 local configurations that satisfy these constraints, the resulting image is a well-composed image. The result is not unique. Some other algorithms (derived from the parallel reconstruction algorithm already proposed in (Latecki, 2000)) may raise the value of a minimum number of points. In practical cases, the reconstruction algorithm can recover important structures, such as the structure depicted in Fig. 3.

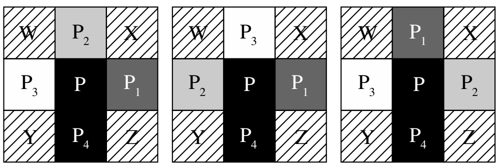


Fig. 6. Configurations of destructible point.

4.2. Thinning well-composed grayscale images

We will now derive the properties of a thinning operator applied on a well-composed grayscale image. Thinning a grayscale well-composed image I consists in iteratively destroying destructible points from I.

Theorem 4.2. *Let I be a well-composed grayscale image, and I' be obtained by thinning I. Then I' is a well-composed grayscale image.*

This can be demonstrated by observing that if I' is obtained by thinning I, then a c-section of I' can be obtained, due to the definition of a destructible point, by thinning the c-section of I, independently of the value of c. All the c-sections of I are well-composed. As thinning conserves well-composedness, all the c-sections of I' are well-composed, thus I' is well-composed.

A graylevel well-composed image I is irreducible iff no point of I is destructible.

We define a plateau of a well-composed image I as a 4-connected set S such that $\forall (p,p') \in S^2$, $I(p) = I(p')$. A regional minimum S is a plateau such that $\forall p \in S$, $\Gamma_4(p) \cap \overline{S} \neq \emptyset$, $\forall p' \in \Gamma_4(p) \cap \overline{S}$, $I(p') > I(p)$.

Theorem 4.3. *The set composed of all the points belonging to regional minima of a well-composed image I is well-composed.*

Let p and p' be two 8-adjacent but not 4-adjacent points in regional minima of I. Suppose that $\Gamma_4(p) \cap \Gamma_4(p')$ is not included in a regional minimum of I. Then I is not well-composed, since this construction is forbidden for well-composed images. This leads to a contradiction.

Theorem 4.4. *There exists a bijection between regional minima of a well-composed image I and regional minima of an image obtained by thinning I.*

We consider an image I' obtained by thinning a well-composed image I. The points belonging to a regional minimum of a well-composed set I are not destructible. If a point of a regional minimum of I is destructible, then it is 4-adjacent to at least one point with a lower value of I, and thus, it is not a point of a regional minimum. The regional minima of I are then subsets of regional minima of I' of I.

Let us suppose that a regional minimum S' of I' contains several regional minima of I. Let c be the value of the highest regional minimum S of I such that $S \subset S'$. The set $\overline{F_{c+1}} \cap S'$ contains at least two 4-connected components bounded by points adjacent to S (Jordan theorem). Thus, $\overline{F_{c+1}}$ contains more connected components than the set $\overline{F'_{c+1}}$, where F'_{c+1} is the $(c+1)$-section of I'. I' is not obtained by thinning I, which contradicts the hypotheses.

By considering the two preceding properties, we can observe that an algorithm computing an irreducible well-composed image from a well-composed image I can be accelerated by first enlarging regional minima of I. This kind of algorithm uses a two pass strategy. During the first pass, all 4-connected regions composed of points of the same value are labeled. Then plateaus are treated in the increasing order of there value. At each iteration the considered plateau is enlarged. This is done by destroying destructible points (replacing the value of the point p by $\alpha(I,p)$ while the point is destructible) that are adjacent to the plateau and for which the new value is equal to the value of the plateau.

The complementary sets of regional minima are arbitrarily thick. This is partly due to configurations like the one depicted on Fig. 7. In this figure, the set composed of the points marked by a circle can be used to define an interesting face of a topological partition.

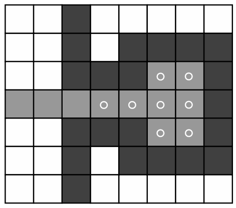


Fig. 7. Regional quasi-minimum.

We define a regional quasi-minimum as a 4-connected component of the set $Q = \{p \in \mathbb{Z}^2, \Gamma^-(p) = \emptyset\}$ defined on I. Note that a regional minimum is a regional quasi-minimum.

The set Q defined on an irreducible well-composed image is not generally well-composed. For example, on Fig. 8, Q is composed of all the points marked by a circle, and Q is not well-composed. This is due to the presence of peaks, i.e. points without upper neighbors. This leads to the following developments.

4.3. Leveling well-composed grayscale images

Leveling an image I (Bertrand et al., 1997) consists in iteratively destroying destructible points and peaks from I.

Theorem 4.5. *Let I be a graylevel well-composed image, and let I' be obtained by leveling I. Then I' is well-composed.*

The destruction of a destructible point from a graylevel well-composed image I preserves well-composedness (c.f. Theorem 4.2). Now consider a peak p from a well-composed image I. Let p' be one of its neighbors such that $\forall p'' \in \Gamma_8(p)$

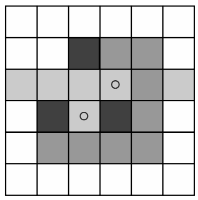


Fig. 8. Non well-composed quasi-minimum.

$I(p'') \leqslant I(p')$. As p is a peak, we have $I(p) > I(p')$. Let I' the image obtained by destroying p from I. Then $I'(p) \geqslant I'(p')$, and the ordering of the other points from $\Gamma_8(p)$ is not changed. Thus, we do not create a forbidden configuration for well-composed graylevel images, and I' is well-composed.

As the two operations are conserving well-composedness, their iterative application on a well-composed image will yield in a well-composed image.

By applying the leveling transformation on a well-composed graylevel image until stability, we obtain a well-composed irreducible graylevel image such that none of its points is a peak. On such images, regional quasi-minima are well-composed.

Theorem 4.6. *Let I be an irreducible well-composed graylevel image such that no point of I is a peak. The set $Q = \{p \in \mathbb{Z}^2, \Gamma^-(p) = \emptyset\}$ defined on I is well-composed.*

Let I be a well-composed grayscale image. Let p and p' be two points 8-adjacent, but not 4-adjacent, belonging to the set $Q = \{p \in \mathbb{Z}^2, \Gamma^-(p) = \emptyset\}$, such that the two points p_1 and p_2 4-adjacent to p and p' are not belonging to Q. Necessarily, $I(p) = I(p')$, $I(p_1) \geqslant I(p)$ and $I(p_2) \geqslant I(p)$. Moreover, as I is well-composed, $I(p_1) = I(p)$ or $I(p_2) = I(p)$. Let's suppose, without loss of generality, that $I(p_1) = I(p)$, which corresponds to Fig. 9.

$I(p_3) \geqslant I(p)$ by hypothesis. $I(p_3) = I(p)$ otherwise p_3 is either destructible or I is not well-composed. $I(p_5) \geqslant I(p_3)$ and $I(p_6) < I(p_3)$ otherwise p_3 is destructible again. Then, p_4 is destructible, which leads to a contradiction.

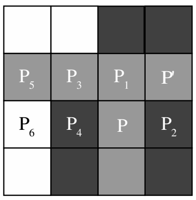


Fig. 9. Proof of Theorem 4.6.

We can now expose the following result, which states that each cut of an irreducible graylevel image without peaks is thin.

Theorem 4.7. *Let I be an irreducible well-composed graylevel image from $\mathbb{Z}^2$ to a set E, such that no point of I is a peak, and the set $Q = \{p \in \mathbb{Z}^2, \Gamma^-(p) = \emptyset\}$ defined on I. For all $c \in E$, the 8-connected components of the 4-interior of the set $E_c \setminus Q$ are composed of at most one point, where E_c is the c-cut of I.*

Let $c \in E$. Consider a point p with $\Gamma^-(p) \neq \emptyset$ and $I(p) = c$. If $\{p' \in \Gamma_8(p), I(p') > I(p)\} = \emptyset$, then the c-cut of I respects one of the local configurations of the irreducible well-composed sets depicted on Fig. 4. We now consider two points p and p' belonging to the same c-cut, 8-adjacent to each other, such that p is adjacent to at least one point of lower value and one point of upper value. Thus, one of the local configurations depicted on Fig. 10 holds.

The point noted D is destructible on these configurations and consequently I is not irreducible.

5. Constructing a topological partition and application

The topological map defined on the digital plane is composed of:

- a set S of points which are vertices of the map,
- a set A of arcs, which are 4-connected disconnected digital curves, of which the extremities are elements of S,

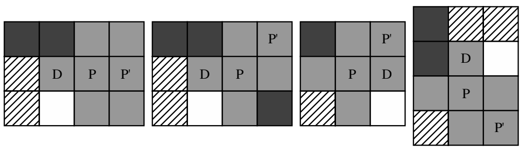


Fig. 10. Proof of Theorem 4.7.

- a set F of faces, which are 4-connected sets of points, of which the boundaries are elements of S and A.

Such a structure can be built from an irreducible well-composed grayscale image without peak, obtained by a leveling transformation of a well-composed grayscale image. The faces of the topological partition are the regional quasi-minima. The vertex/arc network can be retrieved by linking points that have at least one lower neighbor in the following way:

- Points belonging to the same c-cut are linked together using one of the configurations of Fig. 4.
- Points belonging to different c-cuts are linked according to one of the configurations of Fig. 11.

We can prove, by reasoning on local configurations, that the cycles of the graph constructed correspond to regional quasi-minima which contain at least one point. The key idea is to remark that on Fig. 11, an arc separates two components of lower value containing at least one regional quasi-minimum. Moreover, all the points not belonging to a quasi-minimum are linked and correspond to the edges or the vertices of the map. No extra memory is then needed to store information on edges and vertices of the topological map.

Fig. 12 represents an irreducible well-composed image without peaks and its constructed vertex/arc network.

Different structures can be used to store this topological map, such as a combinatorial map (Fiorio, 1996) or a raster graph (Pierrot Deseilligny et al., 1998). Note that both structures can be obtained from an irreducible well-composed image without peak with a single pass linear algorithm.

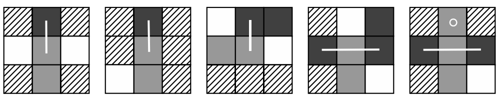


Fig. 11. Arcs between two cuts.

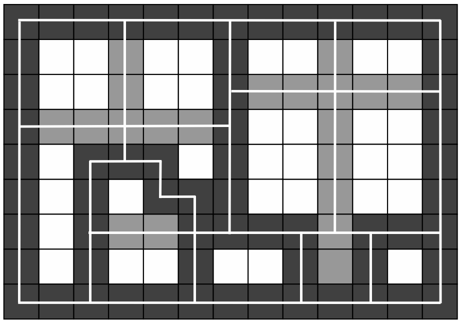


Fig. 12. An irreducible well-composed grayscale image without peaks and its crest network.

Moreover, the raster graph structure can be obtained with a parallel constant time algorithm, as the linking process involves only local configurations.

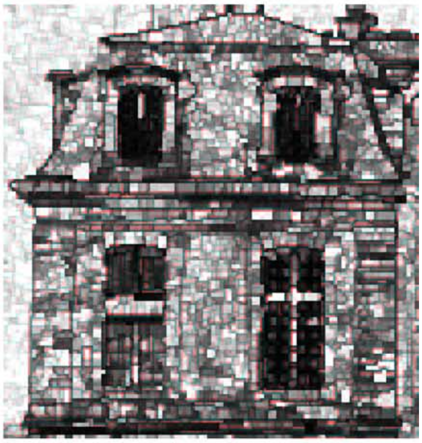
The correspondence between the so defined map and the map that can be constructed from the linear discontinuities of the underlying piecewise continuous function is achieved when the topology of the interesting set of points is preserved under digitization. Note also that the digitization of a continuous curve on the square grid is a digital well-composed curve (Latecki, 1999). That motivates the use of the 4-neighborhood connectivity.

Fig. 13 demonstrates the usefulness of this result for image segmentation. The first image of the figure is the original image. The second image is the irreducible well-composed image without peaks that has been constructed using the following operations.

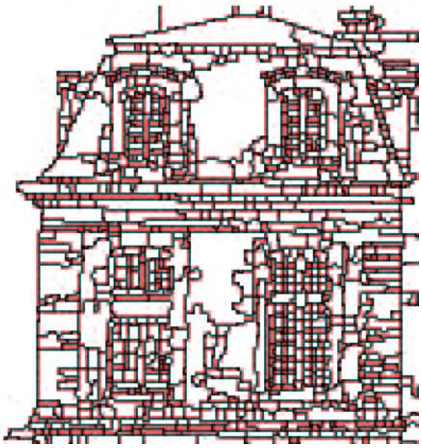
The modulus of the gradient of the original image has first been computed. The obtained graylevel image has then be turned into a well-composed grayscale image. Finally, the shown result has been obtained by applying the leveling transform until stability.

The third image is the extracted crest network, where some arcs have been removed using the following strategy. The curves consisting in points of the same cut are linked. Then the curves containing a point such as the modulus of the gradient at this point is lower than a first threshold and such as none of its point has a modulus above a

(a)



(b)



(c)

Fig. 13. (a) Original image, (b) irreducible image without peaks and (c) extracted crest network.

second threshold value are suppressed from the map. Other segmentation processes can be used to further simplify the map, including region oriented segmentation algorithms, or segmentation processes that fully use the duality contour/region.

Note also that relevant points for computer vision algorithms (such as corners) are correctly retrieved by the proposed approach.

The definition of destructible points presented in this article is not suited for non well-composed images. For such images, destructible points are defined with the classical 4/8-neighborhood system (Bertrand et al., 1997). The thinning and leveling transformations defined with that system results in images with drawbacks such as thick configurations of points (when the 8-neighborhood system is used to characterize crest points) or 8-connected isolated points.

6. Conclusion

In this contribution, we have used the cross section topology formalism in order to define a thinning operator which conserves well-composedness on gray level images. Moreover, we have proposed a way to construct a topological map from the resulting thin image, and have shown that the obtained map is coherent in the sense that:

- a bijection exists between cycles of the map and 4-connected regions of the thin image,
- each Jordan arc of the map is a 4-connected digital curve.

The map can be constructed very efficiently from a gray level image. The application to topological segmentation has also been demonstrated. Some features still remain, such as thin portions of the crest network. Some other restrictions may be studied in order to face this problem. Other work may extend the definition of well-composed grayscale images into three dimensions.

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