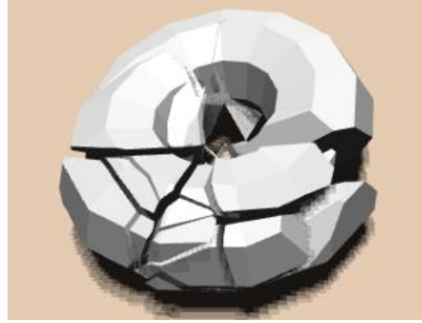

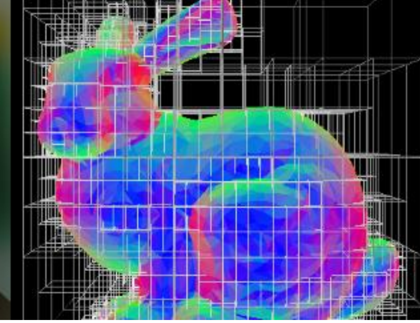

Parallel Algorithms

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CIS 565 - Fall 2015

Agenda

- Parallel Algorithms
  - Parallel Reduction
  - Scan (Naive and Work-Efficient)
  - Stream Compression
  - *S*ummed *A*rea *T*ables
  - Radix Sort

Parallel Reduction

- Given an array of numbers, design a parallel algorithm to find the sum.
- Consider:
  - *Arithmetic intensity*: compute to memory access ratio

Parallel Reduction

- Given an array of numbers, design a parallel algorithm to find:
  - The sum
  - The maximum value
  - The product of values
  - The average value
- How different are these algorithms?

Parallel Reduction

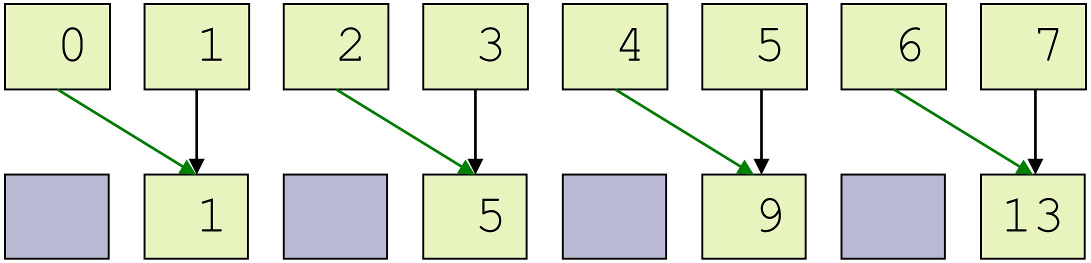
- *Reduction*: An operation that computes a single result from a set of data
- *Parallel Reduction*: Do it in parallel. Obviously

Parallel Reduction

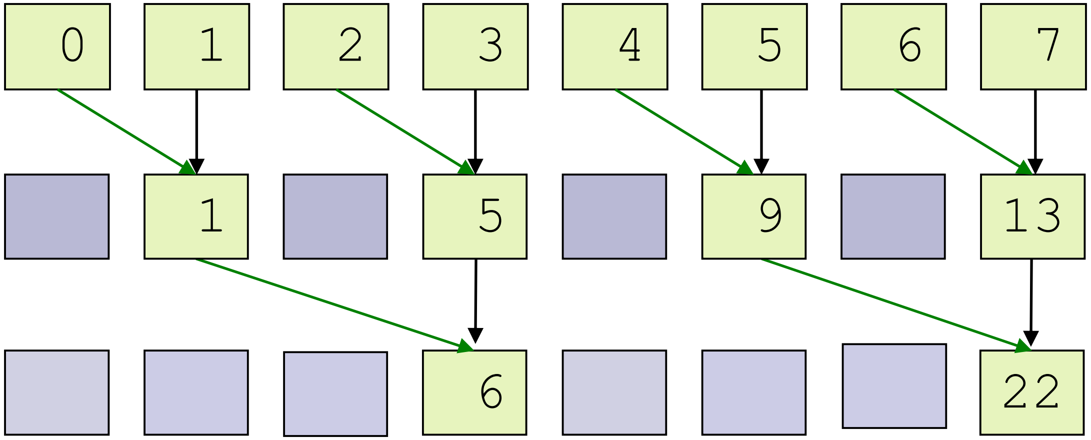
- Example. Find the sum:

0	1	2	3	4	5	6	7

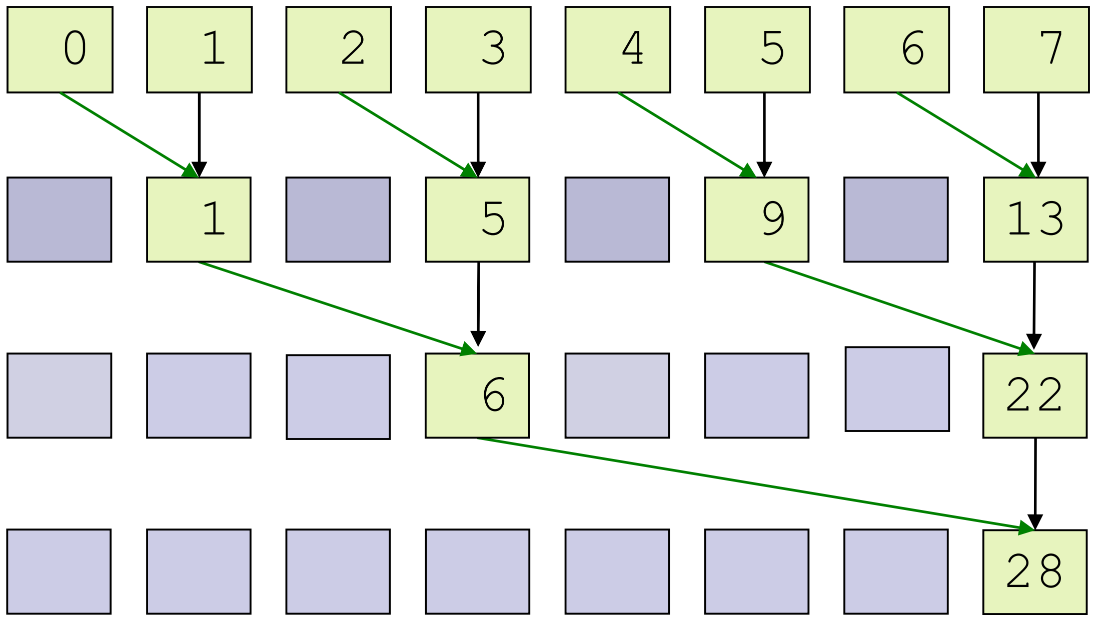
Parallel Reduction



Parallel Reduction

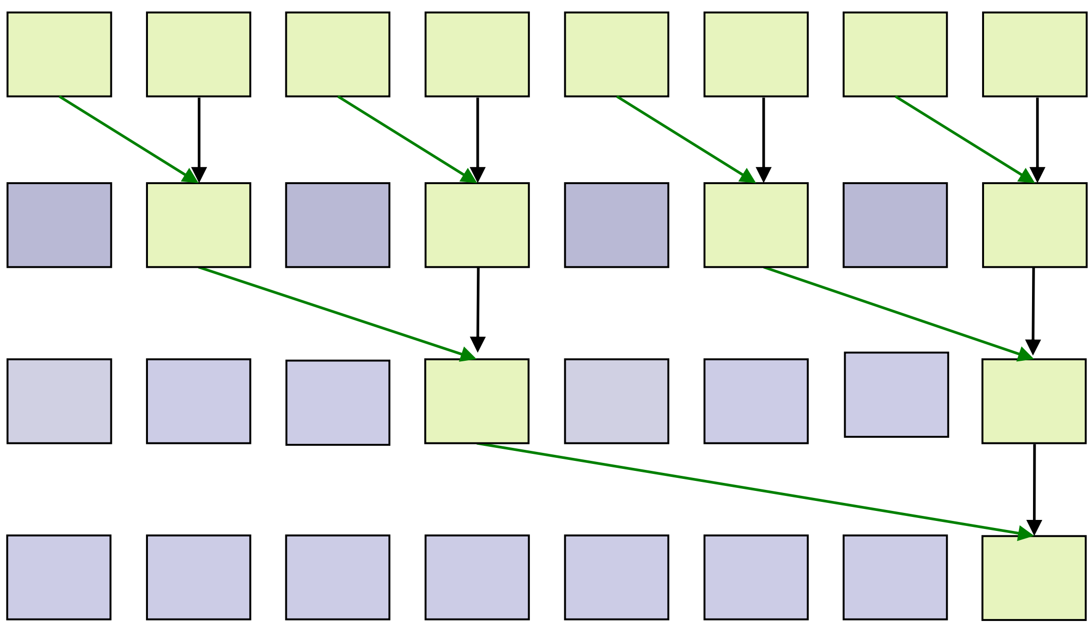


Parallel Reduction



Parallel Reduction

- Similar to brackets for a basketball tournament
- log(`n`) passes for `n` elements

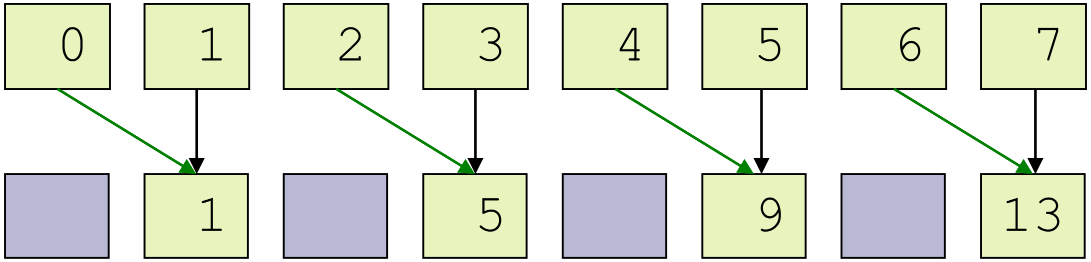


Parallel Reduction

- $d = 0, \ 2^{d+1} = 2$
- $2^{d+1} - 1 = 1$
- $2^d - 1 = 0$

```
for d = 0 to log2n - 1
  for all k = 0 to n – 1 by 2^(d+1) in parallel
    x[k + 2^(d+1) – 1] += x[k + 2^d – 1];

// In this pass, for k = (0, 2, 4, 6)
//   x[k + 1] += x[k];
```

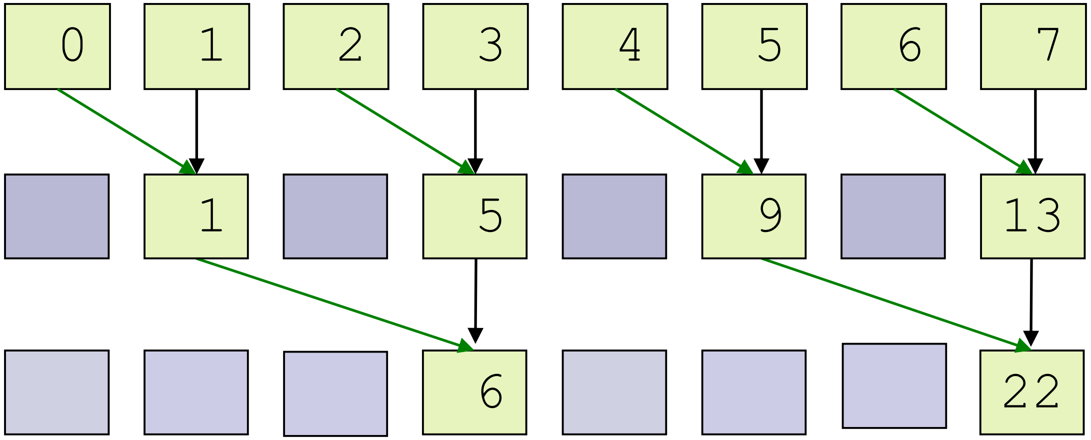


Parallel Reduction

- d = 1, 2^{d+1} = 4
- 2^{d+1} – 1 = 3
- 2^d – 1 = 1

```
for d = 0 to log2n - 1
  for all k = 0 to n – 1 by 2^(d+1) in parallel
    x[k + 2^(d+1) – 1] += x[k + 2^d – 1];

// In this pass, for k = (0, 4)
//   x[k + 3] += x[k + 1];
```

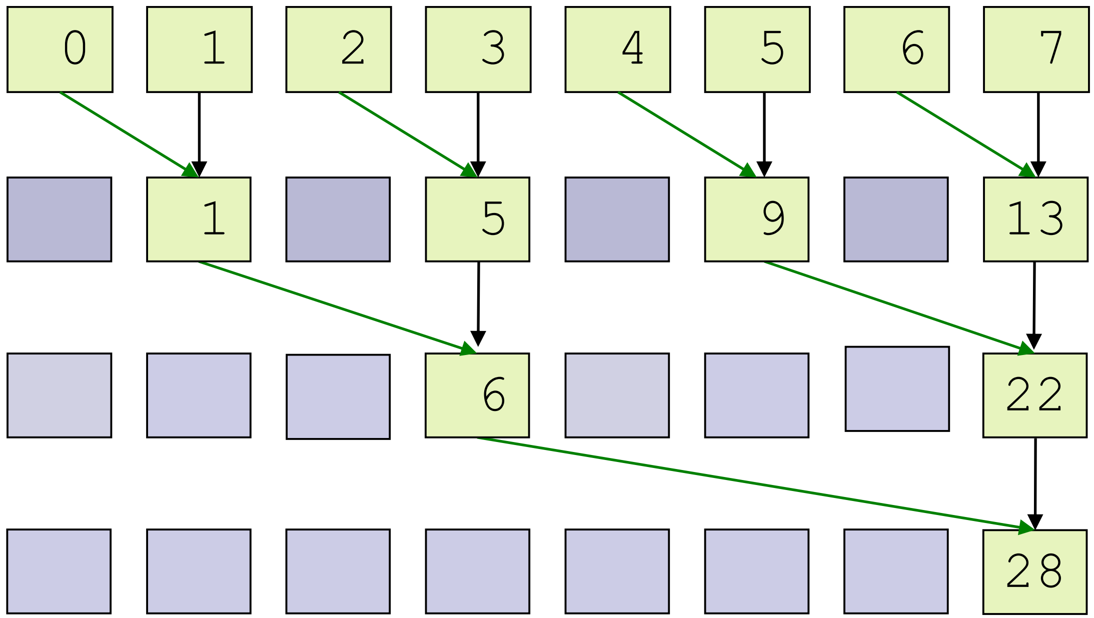


Parallel Reduction

- $d = 2,\ 2^{d+1} = 8$
- $2^{d+1} - 1 = 7$
- $2^{d} - 1 = 3$

```
for d = 0 to log2n - 1
  for all k = 0 to n – 1 by 2^(d+1) in parallel
    x[k + 2^(d+1) – 1] += x[k + 2^d – 1];

// In this pass, for k = (0)
//   x[k + 7] += x[k + 3];
```

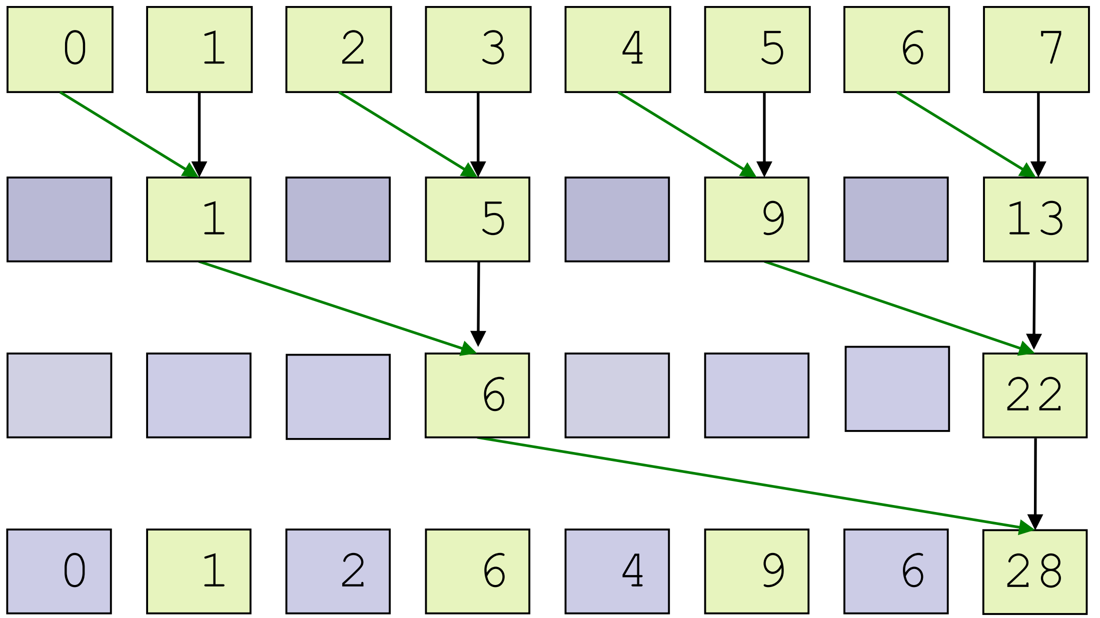


Parallel Reduction

- Note the `+=`
- The array is modified in place

```
for d = 0 to log2n - 1
  for all k = 0 to n - 1 by 2^(d+1) in parallel
    x[k + 2^(d+1) - 1] += x[k + 2^d - 1];
```

$$\text{for } d = 0 \text{ to } \log_2 n - 1$$
$$\quad \text{for all } k = 0 \text{ to } n - 1 \text{ by } 2^{d+1} \text{ in parallel}$$
$$\quad\quad x[k + 2^{d+1} - 1] \mathrel{+}= x[k + 2^d - 1];$$



All-Prefix-Sums

- *All-Prefix-Sums*
  - Input
    - Array of *n* elements: $[a_0, a_1, \ldots, a_{n-1}]$
    - Binary associate operator: $\oplus$
    - Identity: *I*
  - Outputs the array: $[I, a_0, (a_0 \oplus a_1), \ldots, (a_0 \oplus a_1 \oplus \ldots \oplus a_{n-2})]$

Images from http://http.developer.nvidia.com/GPUGems3/gpugems3_ch39.html

All-Prefix-Sums

- Example
  - If $\oplus$ is addition, the array
    - [3 1 7 0 4 1 6 3]
  - is transformed to
    - [0 3 4 11 11 15 16 22]
- Seems sequential, but there is an efficient parallel solution

Scan

- *Exclusive* *Scan*: Element *j* of the result does not include element *j* of the input:
  - `In:  [3 1 7  0  4  1  6  3]`
  - `Out: [0 3 4 11 11 15 16 22]`
- *Inclusive* *Scan* (*Prescan*): All elements including *j* are summed
  - `In:  [3 1 7  0  4  1  6  3]`
  - `Out: [3 4 11 11 15 16 22 25]`

Scan

- How do you generate an *exclusive scan* from an *inclusive scan*?
  - `Input:     [3 1 7  0  4  1  6  3]`
  - `Inclusive: [3 4 11 11 15 16 22 25]`
  - `Exclusive: [0 3 4  11 11 15 16 22]`
    - `// Shift right, insert identity`
- How do you go in the opposite direction?

Scan

- Use cases
  - *Stream compaction*
  - *Summed-area tables* for variable width image processing
  - *Radix sort*
  - …

Scan

- Used to convert certain sequential computation into equivalent parallel computation

Sequential	Parallel

`out[0] = 0;`
`for j from 1 to n do`
`  out[j] = out[j-1] + f(in[j-1]);`
	`forall j in parallel do`
	`  temp[j] = f(in[j]);`
	`all_prefix_sums(out, temp);`

Image from http://http.developer.nvidia.com/GPUGems3/gpugems3_ch39.html

Scan

- Design a parallel algorithm for inclusive scan
  - `In:   [3 1 7  0  4  1  6  3]`
  - `Out: [3 4 11 11 15 16 22 25]`
- Consider:
  - Total number of additions

Scan

- Single thread (*Sequential Scan*) is trivial:

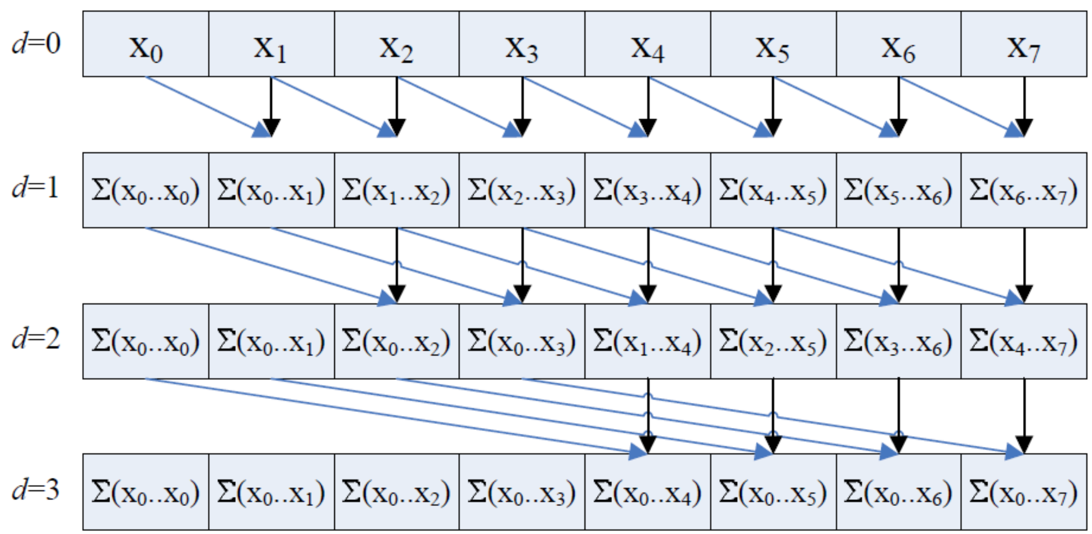
```
out[0] := 0
for k := 1 to n do
  out[k] := in[k-1] + out[k-1]
```

- *n* adds for an array of length *n*
- How many adds will our parallel version have?

Image from http://http.developer.nvidia.com/GPUGems3/gpugems3_ch39.html

Scan

- *Naive Parallel Scan*



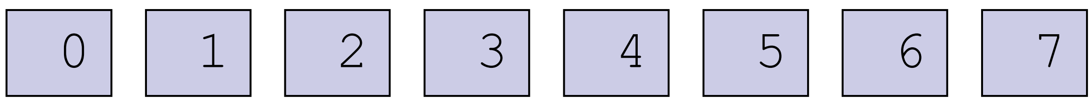

```
for d = 1 to log2n
  for all k in parallel
    if (k >= 2^(d-1))
      x[k] = x[k – 2^(d-1)] + x[k];
```

- Is this exclusive or inclusive?
- Each thread
  - Writes one sum
  - Reads two values

Image from http://developer.download.nvidia.com/compute/cuda/1_1/Website/projects/scan/doc/scan.pdf

Scan

- *Naive Parallel Scan*: Input



Scan

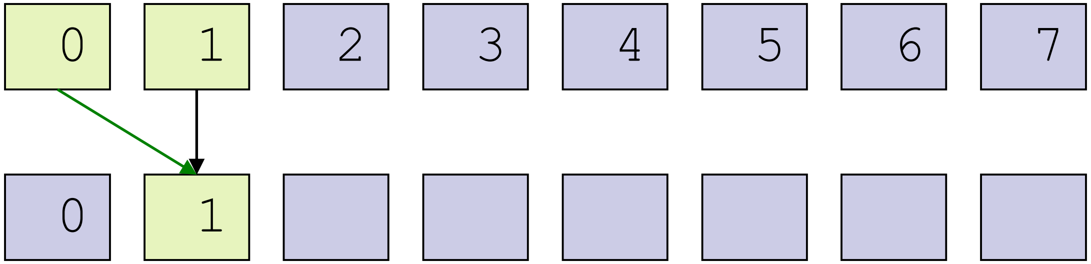
- *Naive Parallel Scan*: $d = 1,\ 2^{d-1} = 1$

0	1	2	3	4	5	6	7
0							

```
for d = 1 to log2n
  for all k in parallel
    if (k >= 2^(d-1))
      x[k] = x[k – 2^(d-1)] + x[k];
```

Scan

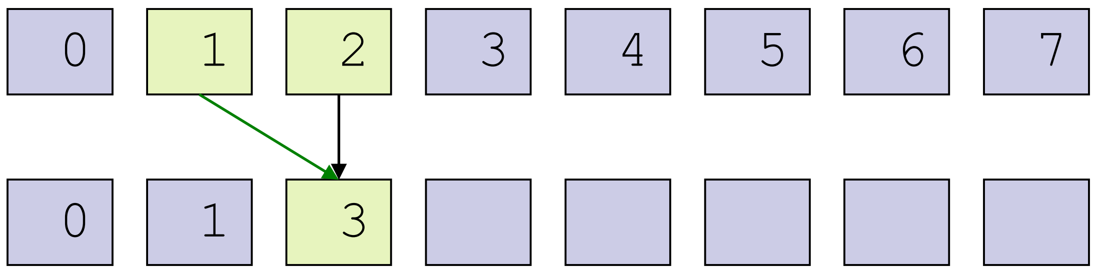
- *Naive Parallel Scan*: `d = 1, 2^(d-1) = 1` ($d = 1,\ 2^{d-1} = 1$)




```
for d = 1 to log2n
  for all k in parallel
    if (k >= 2^(d-1))
      x[k] = x[k – 2^(d-1)] + x[k];
```

Scan

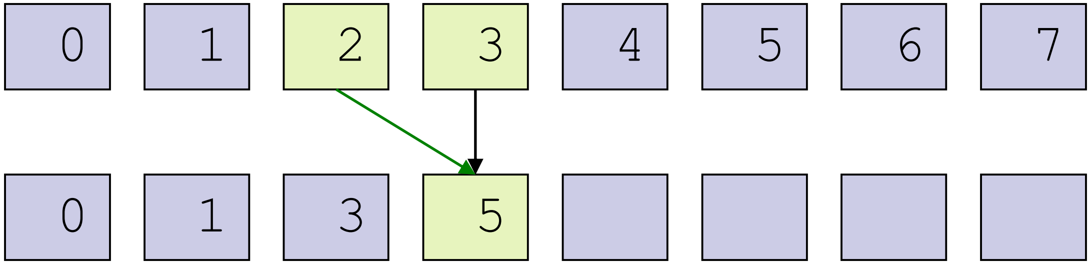
- *Naive Parallel Scan*: `d = 1, 2^(d-1) = 1` ($d = 1,\ 2^{d-1} = 1$)



for d = 1 to $\log_2 n$
  for all k in parallel
    if (k >= 2^{d-1})
      x[k] = x[k – 2^{d-1}] + x[k];

Scan

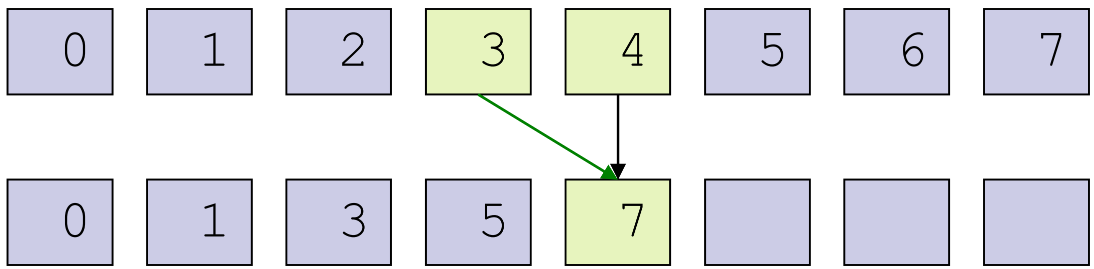
- *Naive Parallel Scan*: $d = 1,\ 2^{d-1} = 1$




```
for d = 1 to log2n
  for all k in parallel
    if (k >= 2^(d-1))
      x[k] = x[k – 2^(d-1)] + x[k];
```

Scan

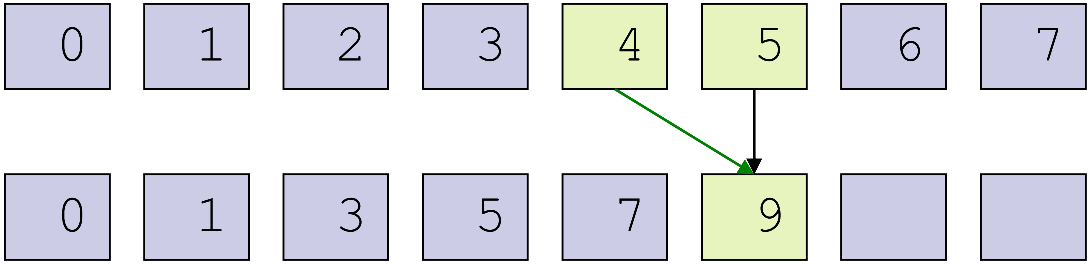
- *Naive Parallel Scan*: $d = 1,\ 2^{d-1} = 1$




```
for d = 1 to log2n
  for all k in parallel
    if (k >= 2^(d-1))
      x[k] = x[k – 2^(d-1)] + x[k];
```

Scan

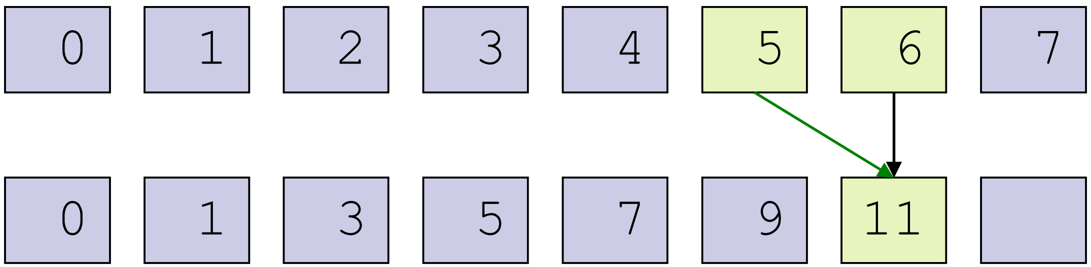
- *Naive Parallel Scan*: `d = 1, 2^(d-1) = 1`




```
for d = 1 to log2n
  for all k in parallel
    if (k >= 2^(d-1))
      x[k] = x[k – 2^(d-1)] + x[k];
```

Scan

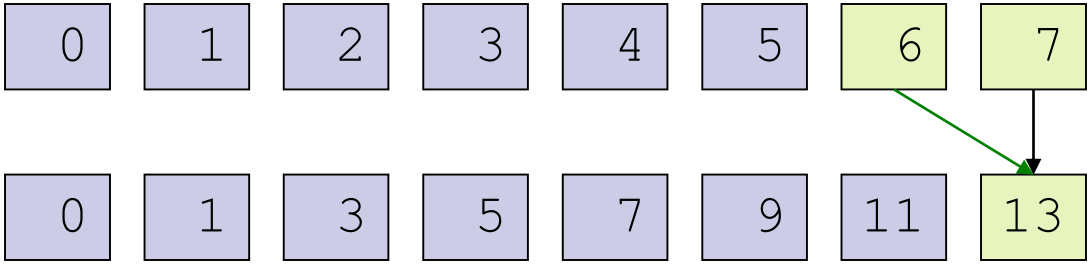
- *Naive Parallel Scan*: `d = 1, 2`$^{d-1}$ `= 1`




```
for d = 1 to log2n
  for all k in parallel
    if (k >= 2^(d-1))
      x[k] = x[k – 2^(d-1)] + x[k];
```

Scan

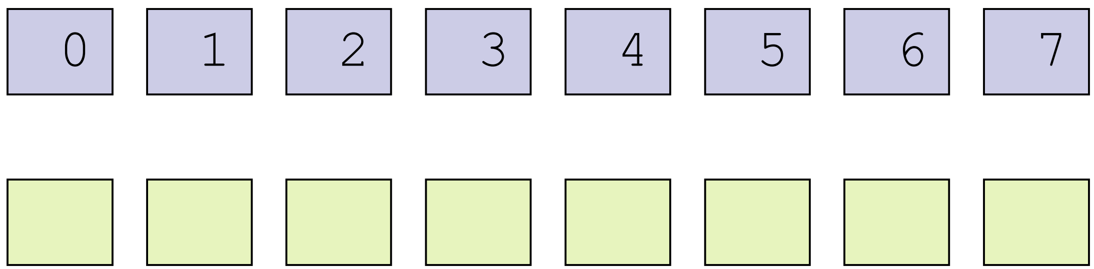
- *Naive Parallel Scan*: $d = 1,\ 2^{d-1} = 1$




```
for d = 1 to log2n
  for all k in parallel
    if (k >= 2^(d-1))
      x[k] = x[k – 2^(d-1)] + x[k];
```

Scan

- *Naive Parallel Scan*: `d = 1, 2^(d-1) = 1` ($d = 1,\ 2^{d-1} = 1$)

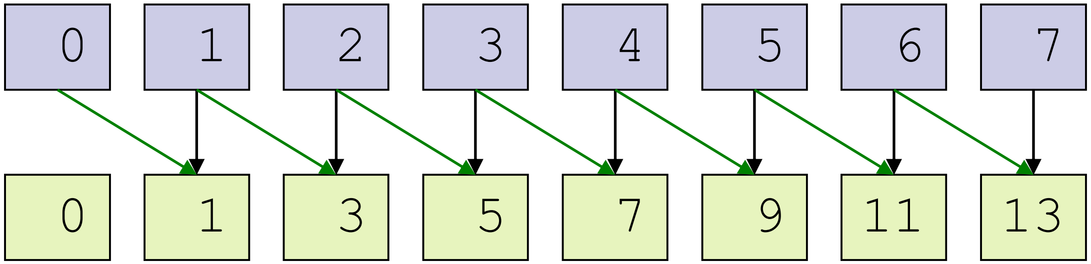


- Recall, it runs in parallel!

```
for d = 1 to log2n
  for all k in parallel
    if (k >= 2^(d-1))
      x[k] = x[k – 2^(d-1)] + x[k];
```

Scan

- *Naive Parallel Scan*: `d = 1, 2^(d-1) = 1` ($d = 1$, $2^{d-1} = 1$)

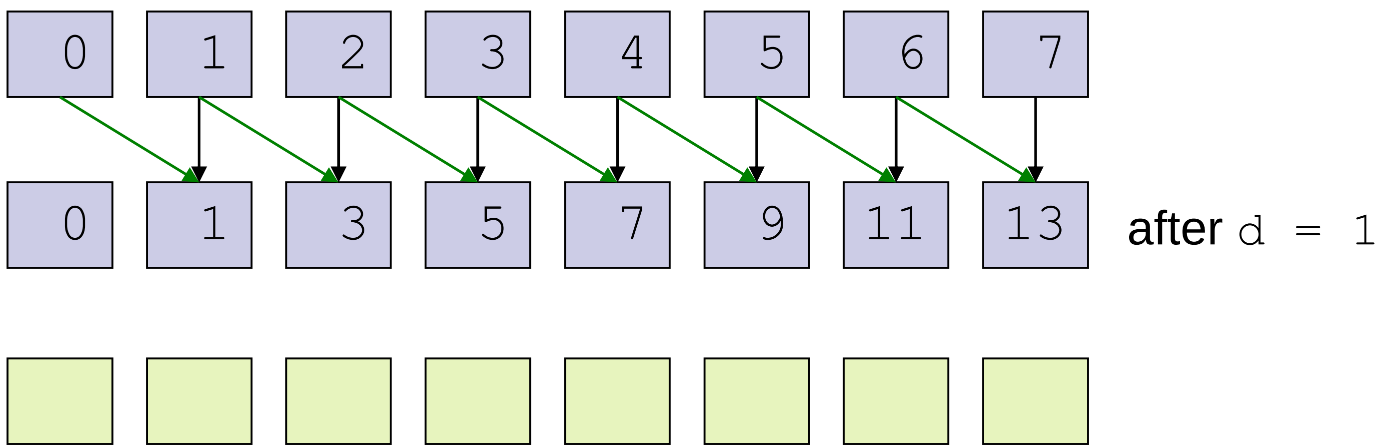


- Recall, it runs in parallel!

```
for d = 1 to log2n
  for all k in parallel
    if (k >= 2^(d-1))
      x[k] = x[k – 2^(d-1)] + x[k];
```

Scan

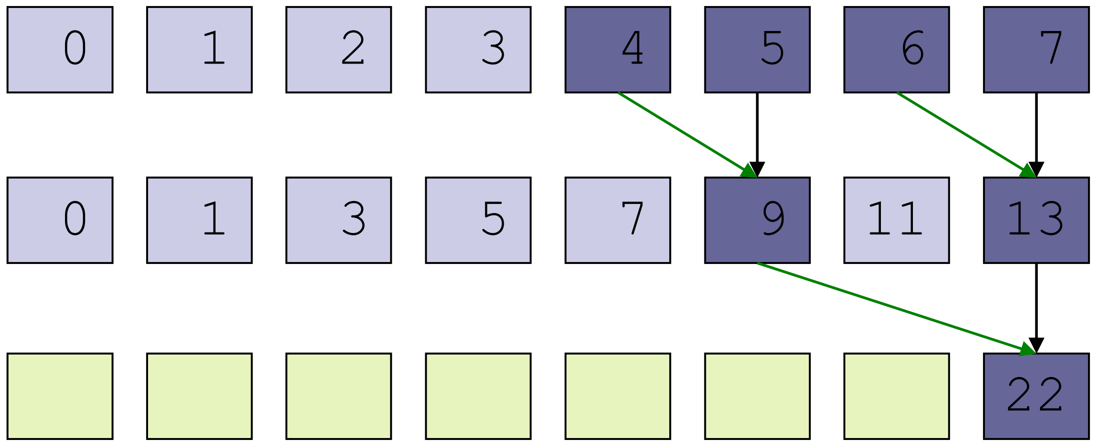
- *Naive Parallel Scan*: `d = 2, 2^(d-1) = 2` ($d = 2$, $2^{d-1} = 2$)




```
for d = 1 to log2n
  for all k in parallel
    if (k >= 2^(d-1))
      x[k] = x[k – 2^(d-1)] + x[k];
```

Scan

- *Naive Parallel Scan*: d = 2, $2^{d-1} = 2$



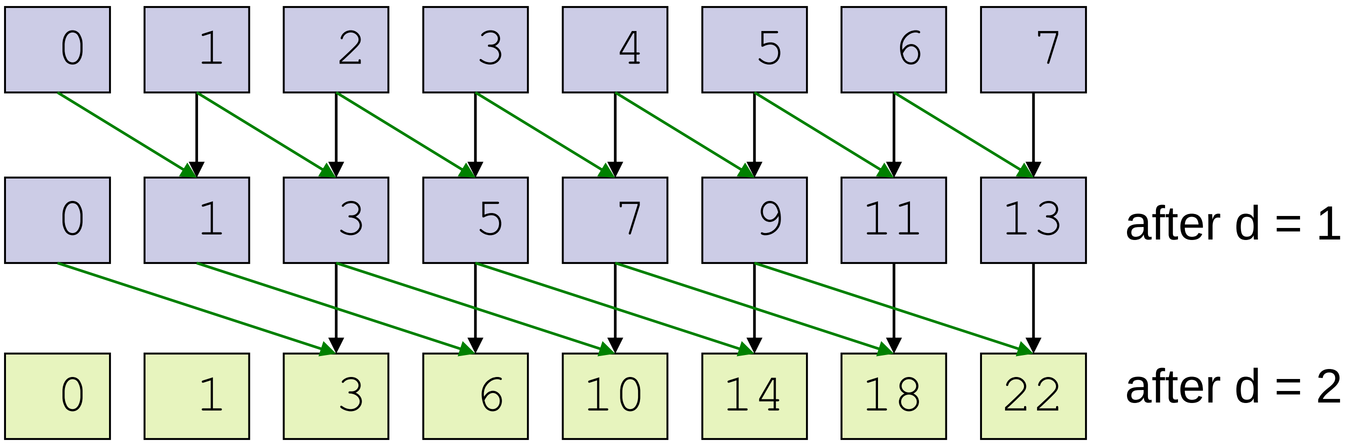
after `d = 1`

- Consider only `k = 7`

```
for d = 1 to log2n
  for all k in parallel
    if (k >= 2^(d-1))
      x[k] = x[k – 2^(d-1)] + x[k];
```

Scan

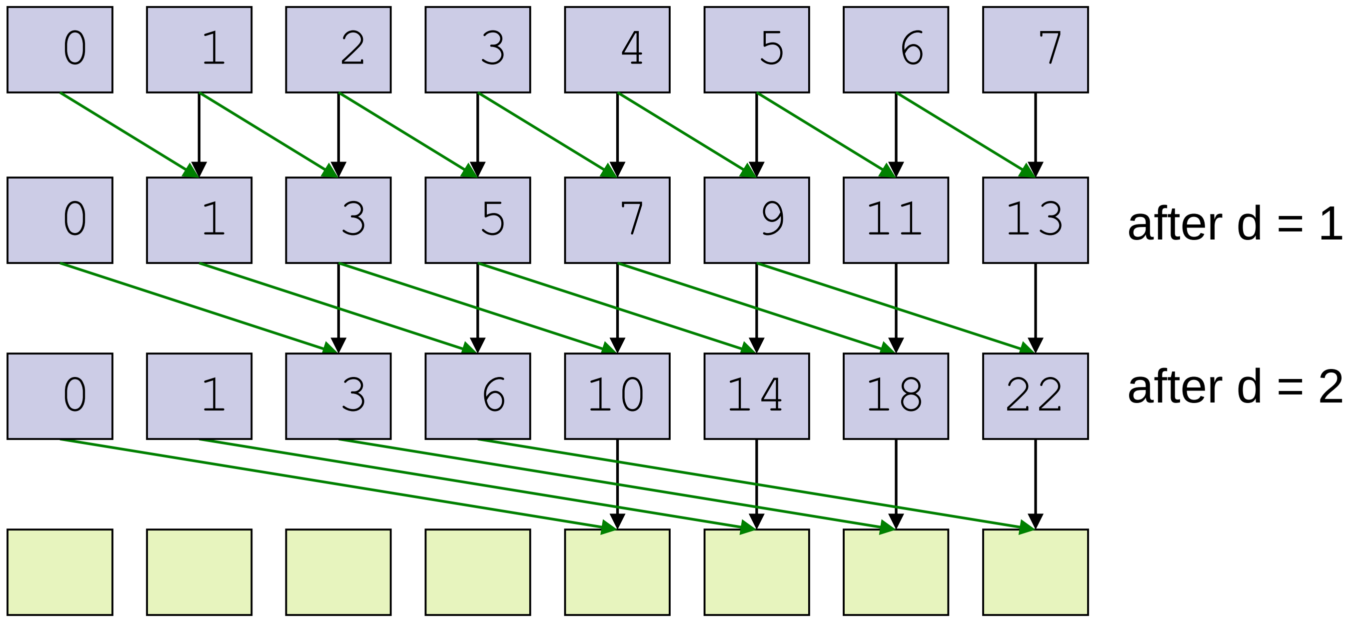
- *Naive Parallel Scan*: d = 2, $2^{d-1} = 2$




```
for d = 1 to log2n
  for all k in parallel
    if (k >= 2^(d-1))
      x[k] = x[k – 2^(d-1)] + x[k];
```

Scan

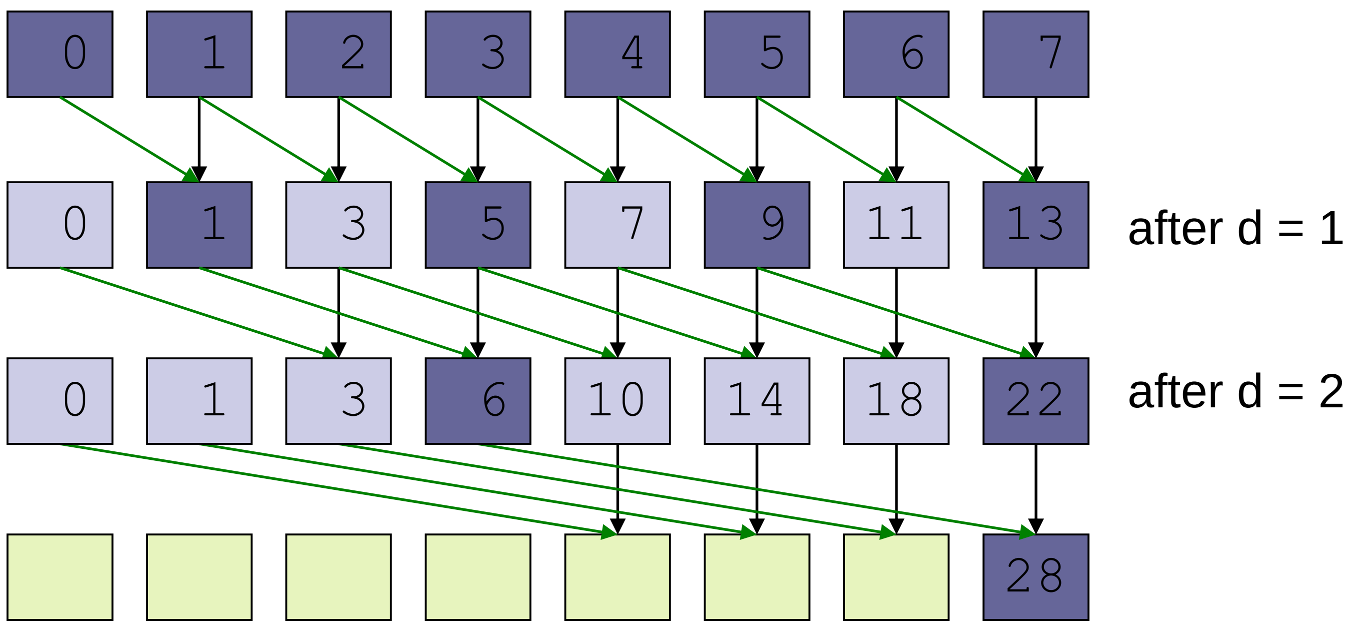
- *Naive Parallel Scan*: d = 3, $2^{d-1} = 4$




```
for d = 1 to log2n
  for all k in parallel
    if (k >= 2^(d-1))
      x[k] = x[k – 2^(d-1)] + x[k];
```

Scan

- *Naive Parallel Scan*: d = 3, $2^{d-1} = 4$

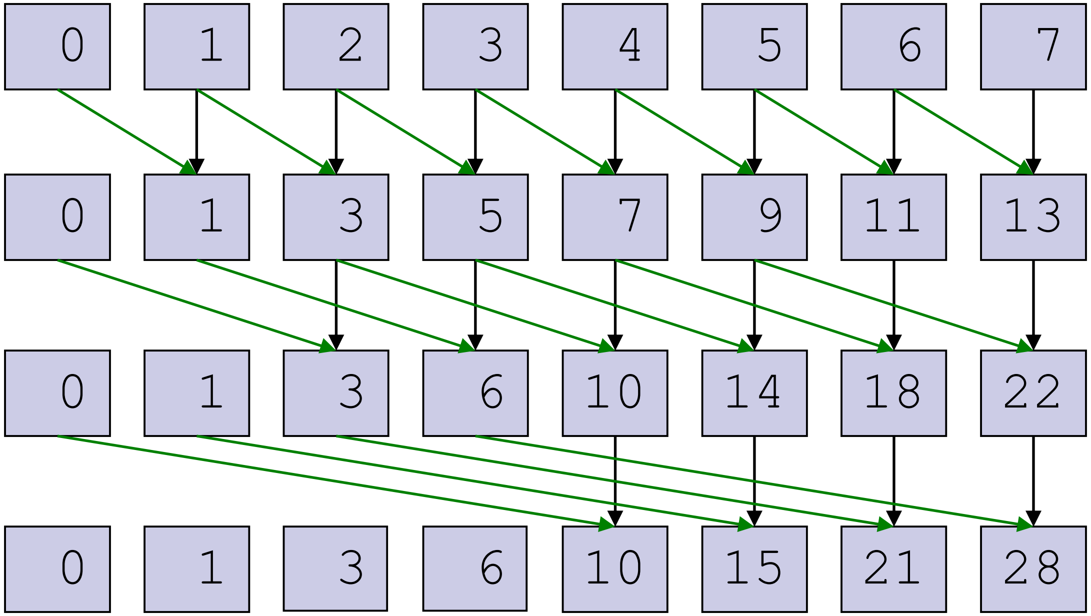


- Consider only `k = 7`

```
for d = 1 to log2n
  for all k in parallel
    if (k >= 2^(d-1))
      x[k] = x[k – 2^(d-1)] + x[k];
```

Scan

- *Naive Parallel Scan*: Final

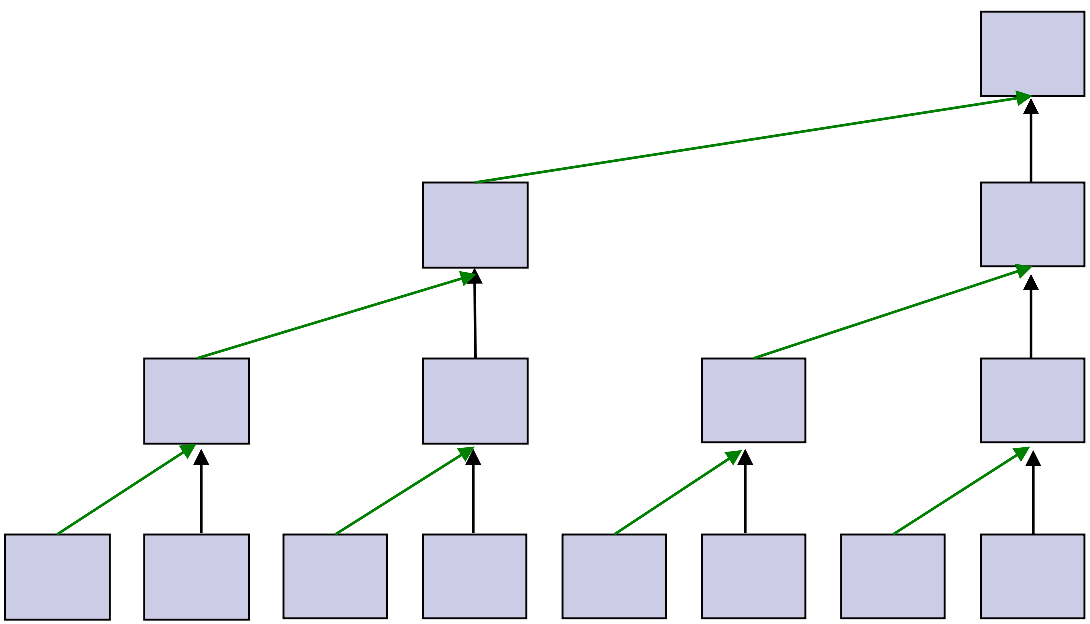


Work-Efficient Parallel Scan

- Number of adds
  - *Sequential Scan*: `O(n)`
  - *Naive Parallel Scan*: $O(n\log_2(n))$
- How can we make it faster?

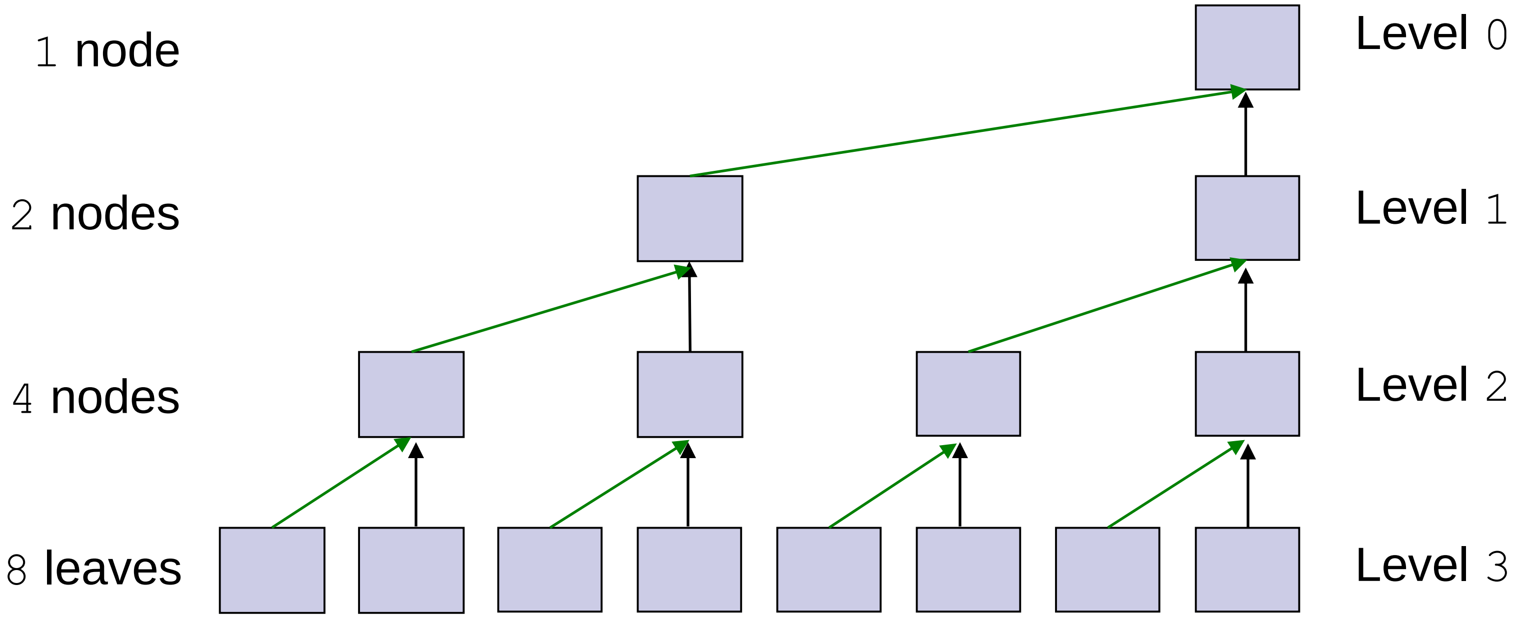
Work-Efficient Parallel Scan

- *Balanced binary tree*
  - n leaves = $\log_2 n$ levels
  - Each level, d, has 2^d nodes



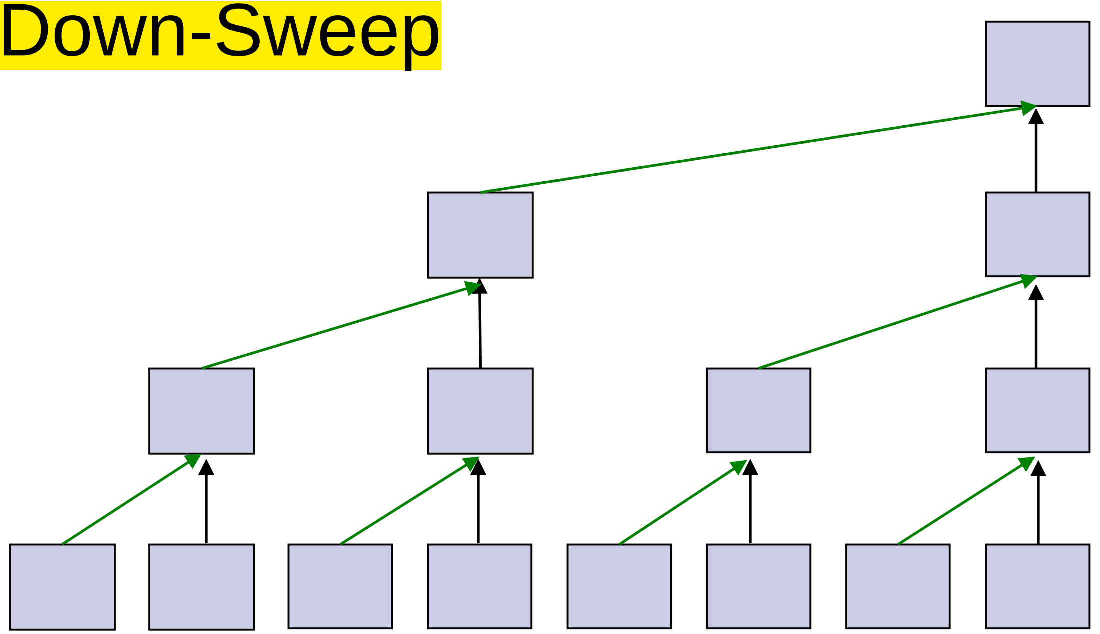
Work-Efficient Parallel Scan

- *Balanced binary tree*
  - n leaves = $\log_2 n$ levels
  - Each level, d, has 2^d nodes



Work-Efficient Parallel Scan

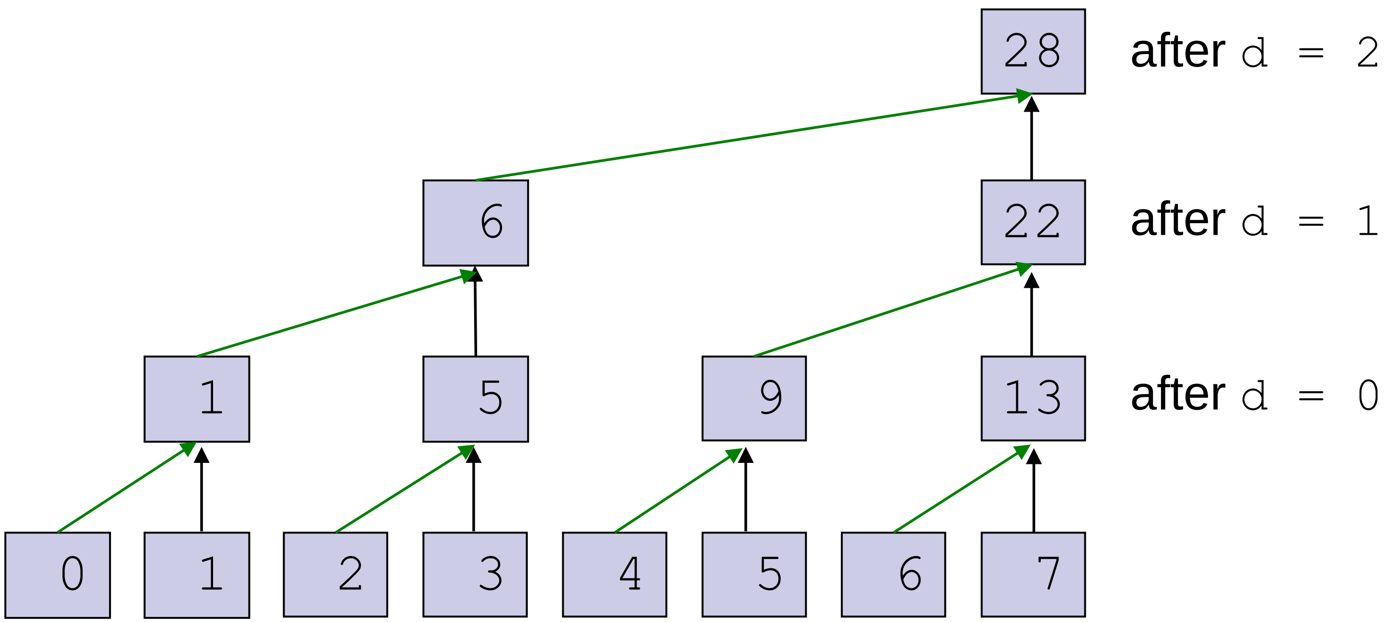
- Use a *balanced binary tree* (in concept) to perform Scan in two phases:
  - Up-Sweep (Parallel Reduction)
  - Down-Sweep



Work-Efficient Parallel Scan

- Up-Sweep

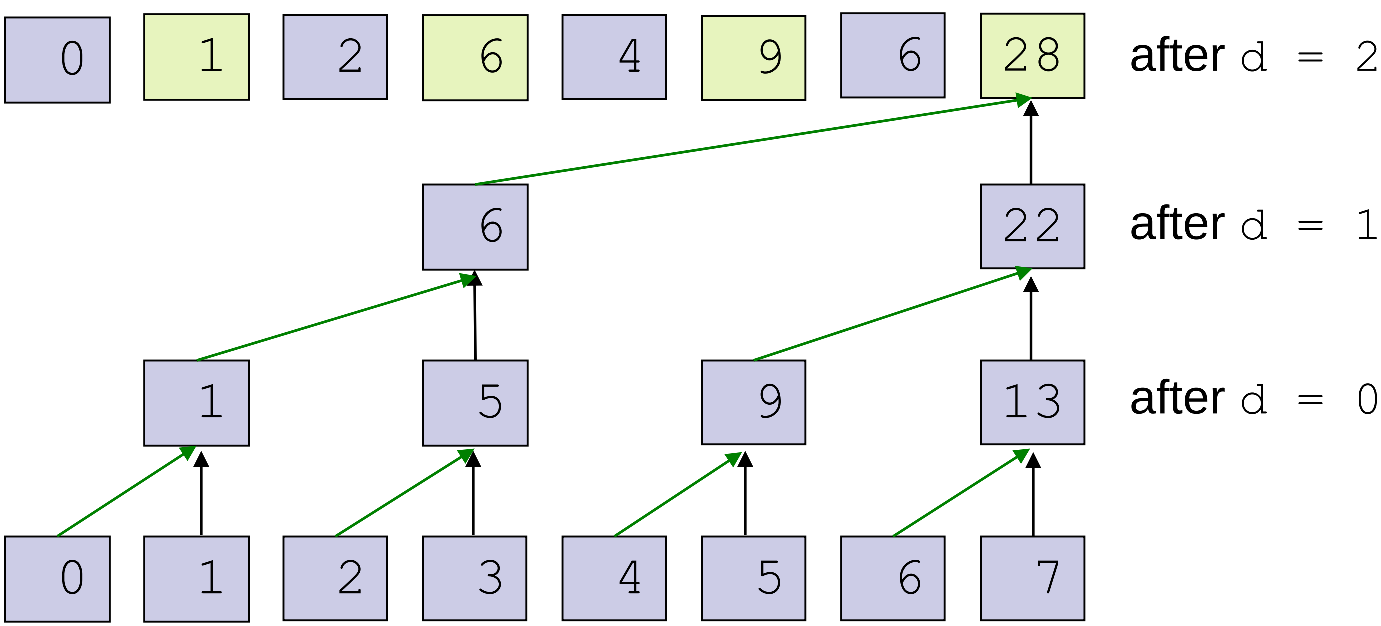
```
// Same code as our Parallel Reduction
for d = 0 to log₂n - 1
  for all k = 0 to n – 1 by 2^(d+1) in parallel
    x[k + 2^(d+1) – 1] += x[k + 2^d – 1];
```



Work-Efficient Parallel Scan

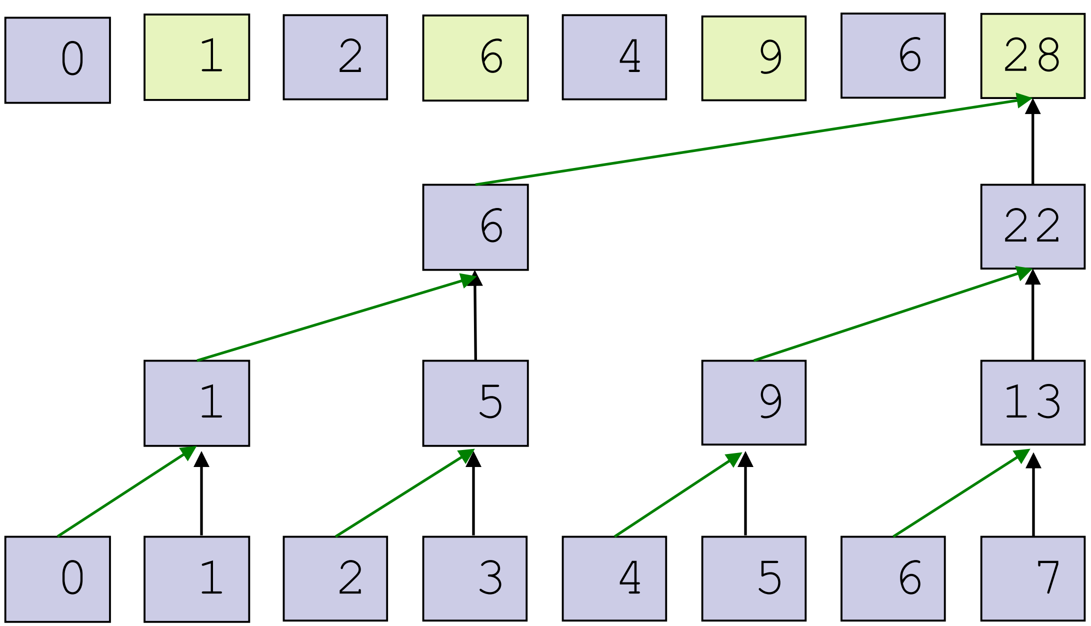
- Up-Sweep

```
// Same code as our Parallel Reduction
for d = 0 to log2n - 1
  for all k = 0 to n – 1 by 2^(d+1) in parallel
    x[k + 2^(d+1) – 1] += x[k + 2^d – 1];
```



Work-Efficient Parallel Scan

- Down-Sweep
  - "Traverse" back down tree using partial sums to build the scan in place.
    - Set root to zero
    - At each pass, a node passes its value to its left child, and sets the right child to the sum of the previous left child's value and its value



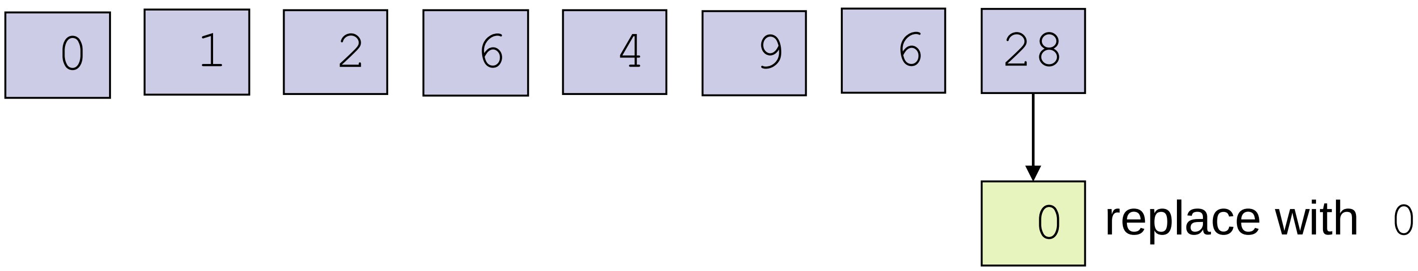
Work-Efficient Parallel Scan

- Down-Sweep
  - “Traverse” back down tree using partial sums to build the scan in place.
    - Set root to zero
    - At each pass, a node passes its value to its left child, and sets the right child to the sum of the previous left child’s value and its value

```
x[n - 1] = 0
for d = log2n – 1 to 0
  for all k = 0 to n – 1 by 2^(d+1) in parallel
    t = x[k + 2^d – 1];                // Save left child
    x[k + 2^d – 1] = x[k + 2^(d+1) – 1];  // Set left child to this node’s value
    x[k + 2^(d+1) – 1] += t;            // Set right child to old left value +
                                        // this node’s value
```

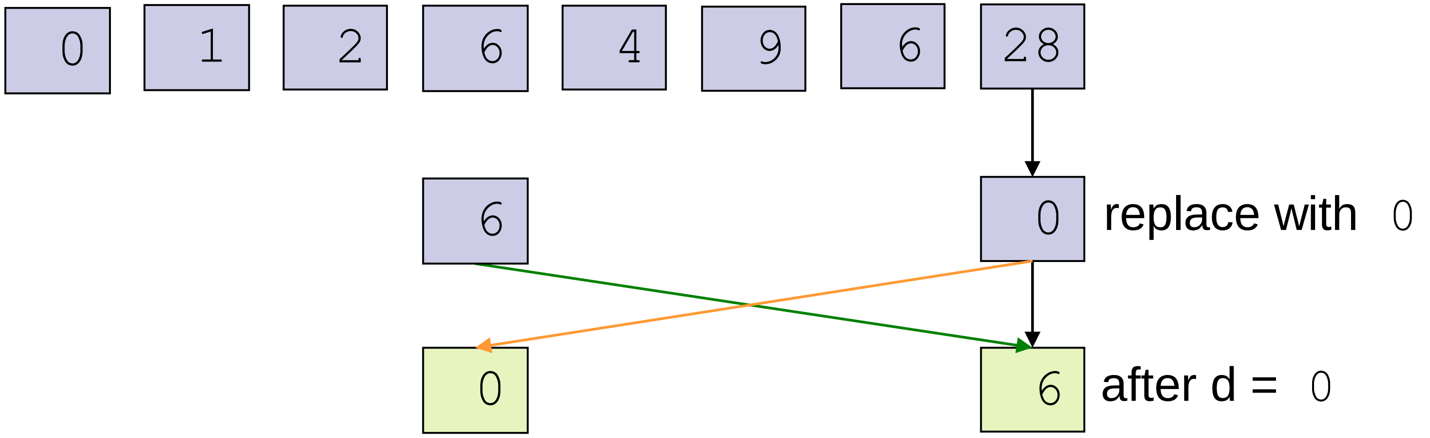
Work-Efficient Parallel Scan

- Down-Sweep



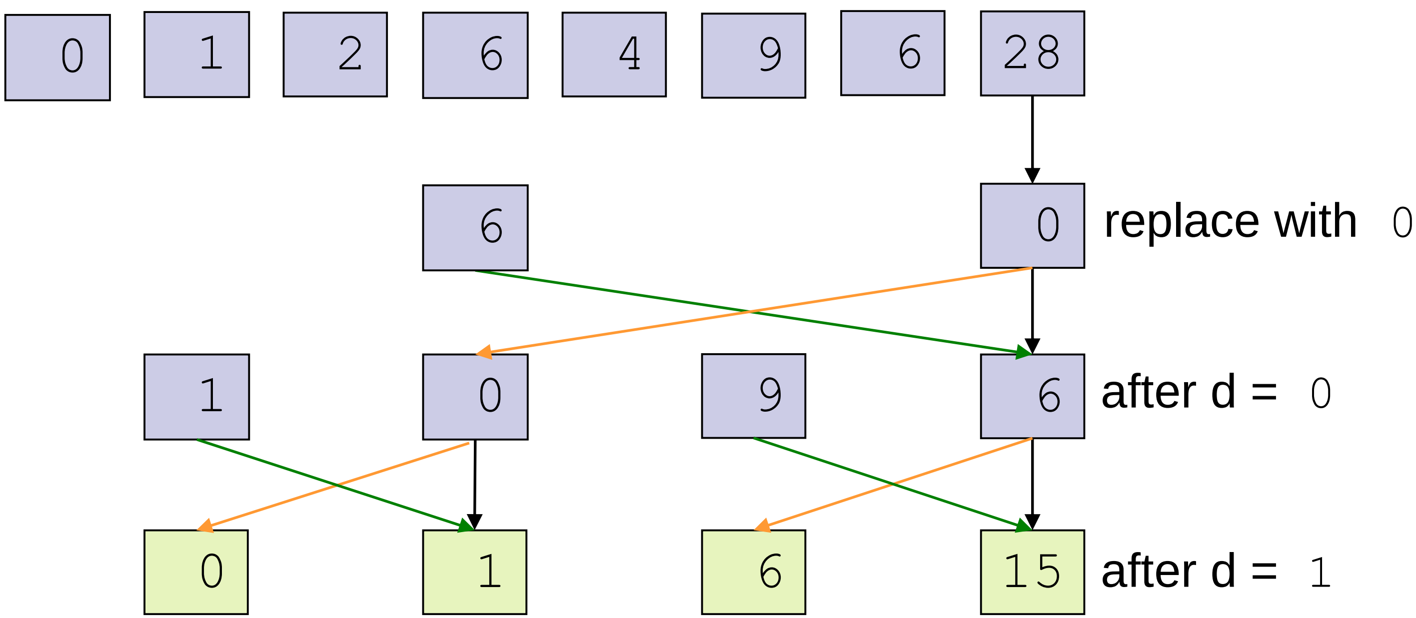
Work-Efficient Parallel Scan

- Down-Sweep



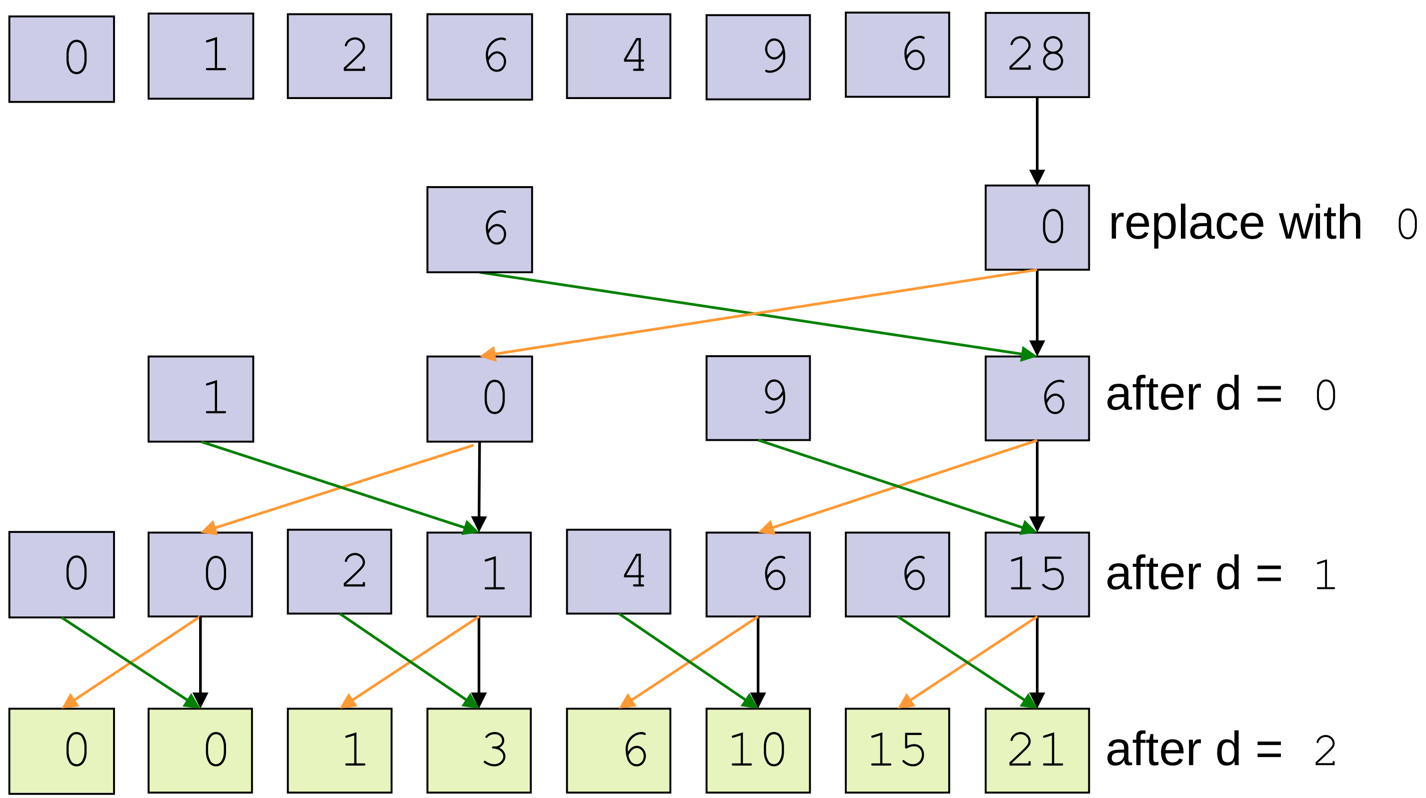
Work-Efficient Parallel Scan

- Down-Sweep



Work-Efficient Parallel Scan

- Down-Sweep

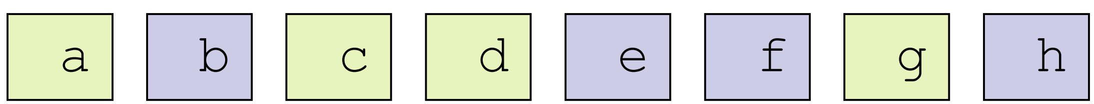


Work-Efficient Parallel Scan

- Up-Sweep
  - O(n) adds
- Down-Sweep
  - O(n) adds
  - O(n) swaps

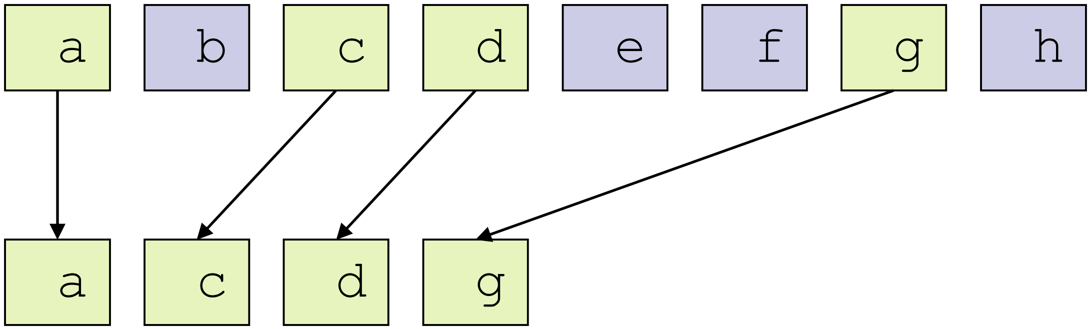
Stream Compaction

- *Stream Compaction*
  - Given an array of elements
    - Create a new array with elements that meet a certain criteria, e.g. non null
    - Preserve order



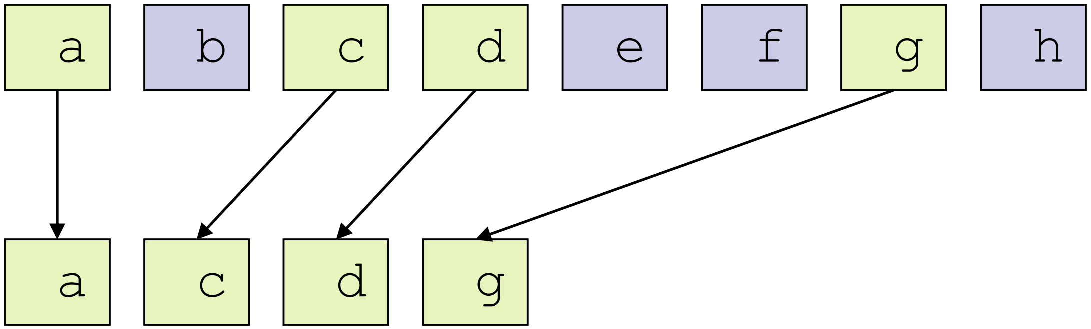
Stream Compaction

- Stream Compaction
  - Given an array of elements
    - Create a new array with elements that meet a certain criteria, e.g. non null
    - Preserve order



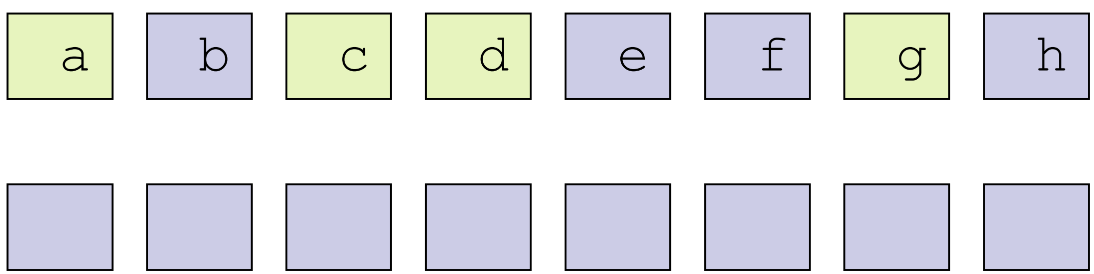
Stream Compaction

- Stream Compaction
  - Used in path tracing, collision detection, sparse matrix compression, etc.
  - Can reduce bandwidth from GPU to CPU



Stream Compaction

- Stream Compaction
  - *Step 1*: Compute temporary array containing
    - 1 if corresponding element meets criteria
    - 0 if element does not meet criteria



Stream Compaction

- Stream Compaction
  - *Step 1*: Compute temporary array

a	b	c	d	e	f	g	h
1							

Stream Compaction

- Stream Compaction
  - *Step 1*: Compute temporary array

a	b	c	d	e	f	g	h
1	0						

Stream Compaction

- Stream Compaction
  - *Step 1*: Compute temporary array

a	b	c	d	e	f	g	h
1	0	1					

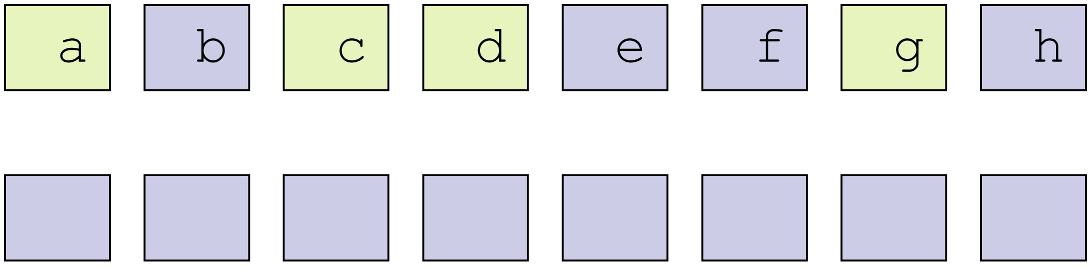
Stream Compaction

- Stream Compaction
  - *Step 1*: Compute temporary array

a	b	c	d	e	f	g	h
1	0	1	1	0	0	1	0

Stream Compaction

- Stream Compaction
  - *Step 1*: Compute temporary array



- It runs in parallel!

Stream Compaction

- Stream Compaction
  - *Step 1*: Compute temporary array

a	b	c	d	e	f	g	h
1	0	1	1	0	0	1	0

- It runs in parallel!

Stream Compaction

- Stream Compaction
  - *Step 2*: Run exclusive scan on temporary array

	a	b	c	d	e	f	g	h
	1	0	1	1	0	0	1	0
Scan result:								

Stream Compaction

- Stream Compaction
  - *Step 2*: Run exclusive scan on temporary array

	a	b	c	d	e	f	g	h
	1	0	1	1	0	0	1	0
Scan result:	0	1	1	2	3	3	3	4

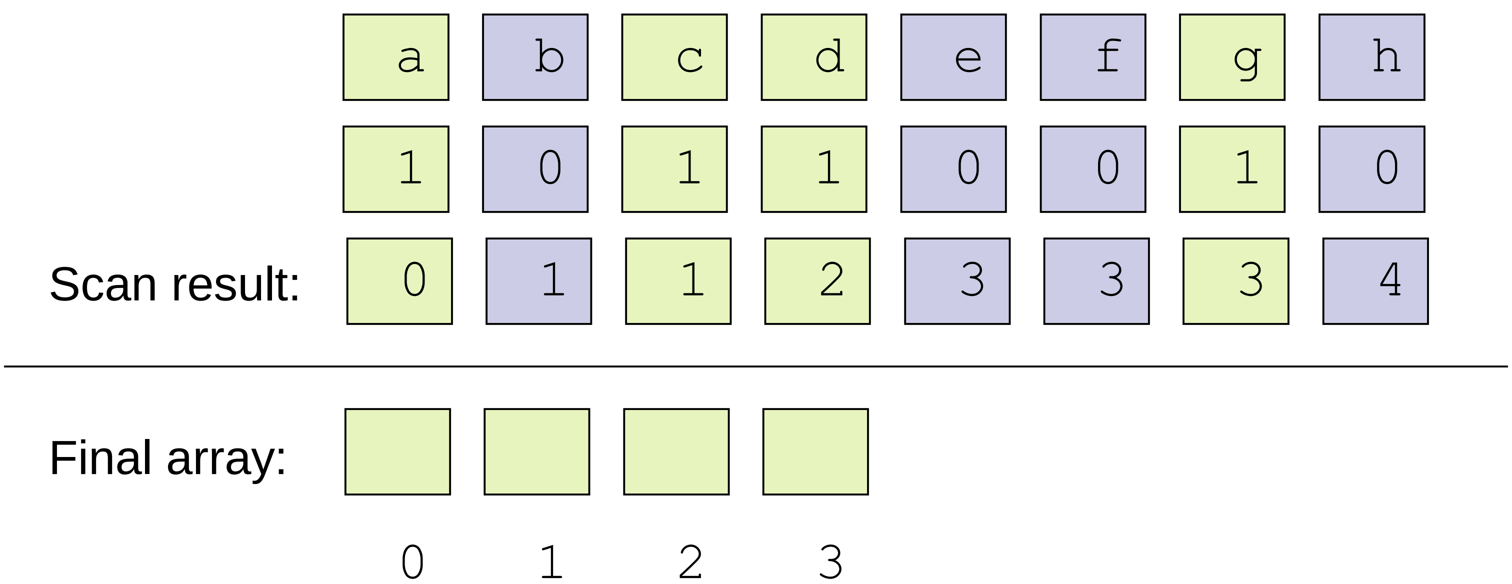
  - Scan runs in parallel
  - What can we do with the results?

Stream Compaction

- Stream Compaction
  - *Step 3*: Scatter
    - Result of scan is index into final array
    - Only write an element if temporary array has a 1

Stream Compaction

- Stream Compaction
  - *Step 3*: Scatter



Stream Compaction

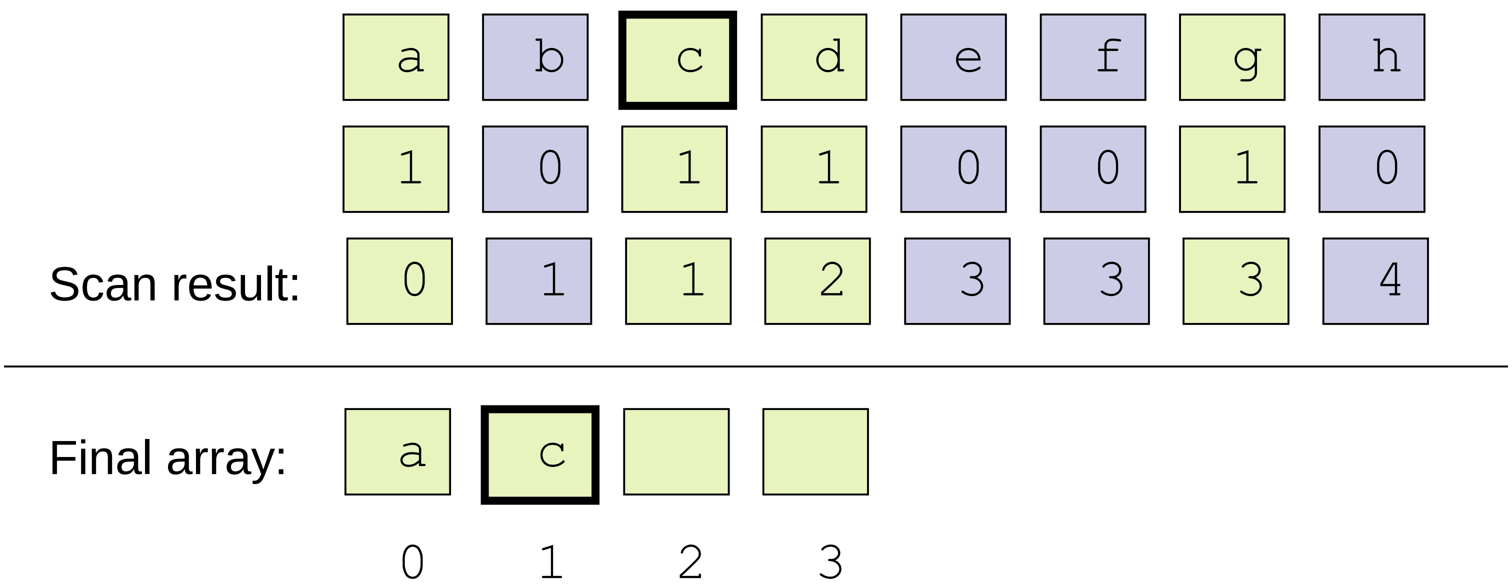
- Stream Compaction
  - *Step 3*: Scatter

	a	b	c	d	e	f	g	h
	1	0	1	1	0	0	1	0
Scan result:	0	1	1	2	3	3	3	4

Final array:	a			
	0	1	2	3

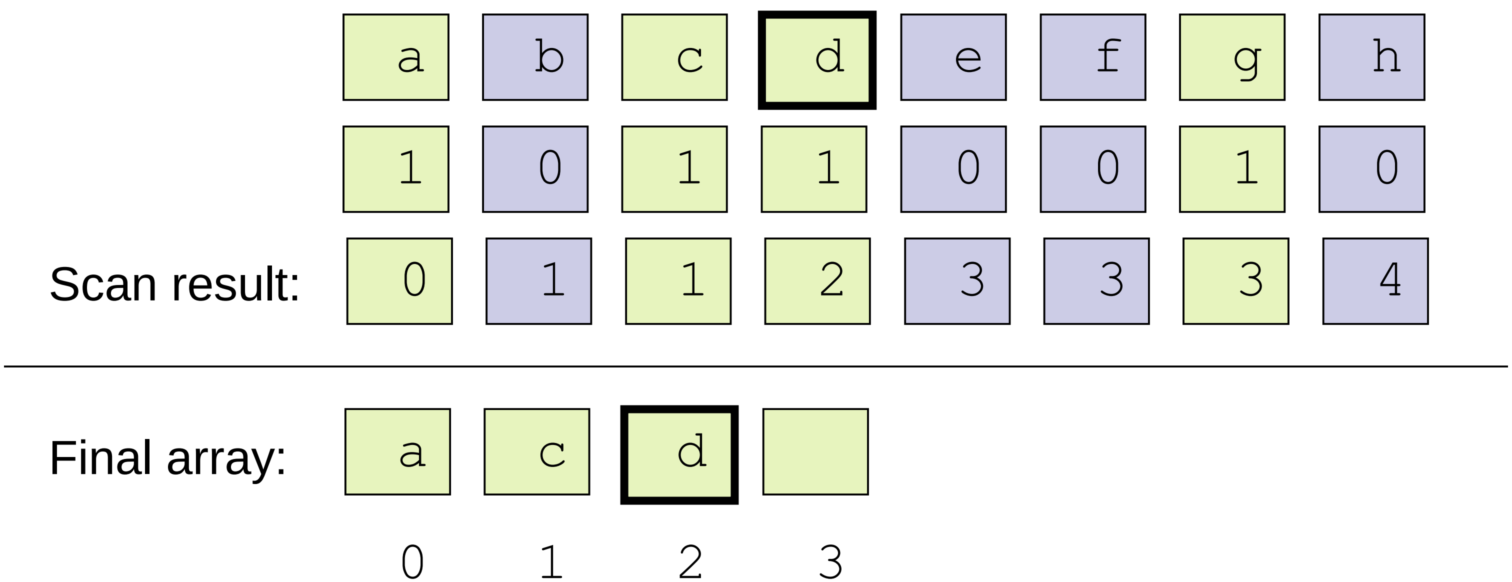
Stream Compaction

- Stream Compaction
  - *Step 3*: Scatter



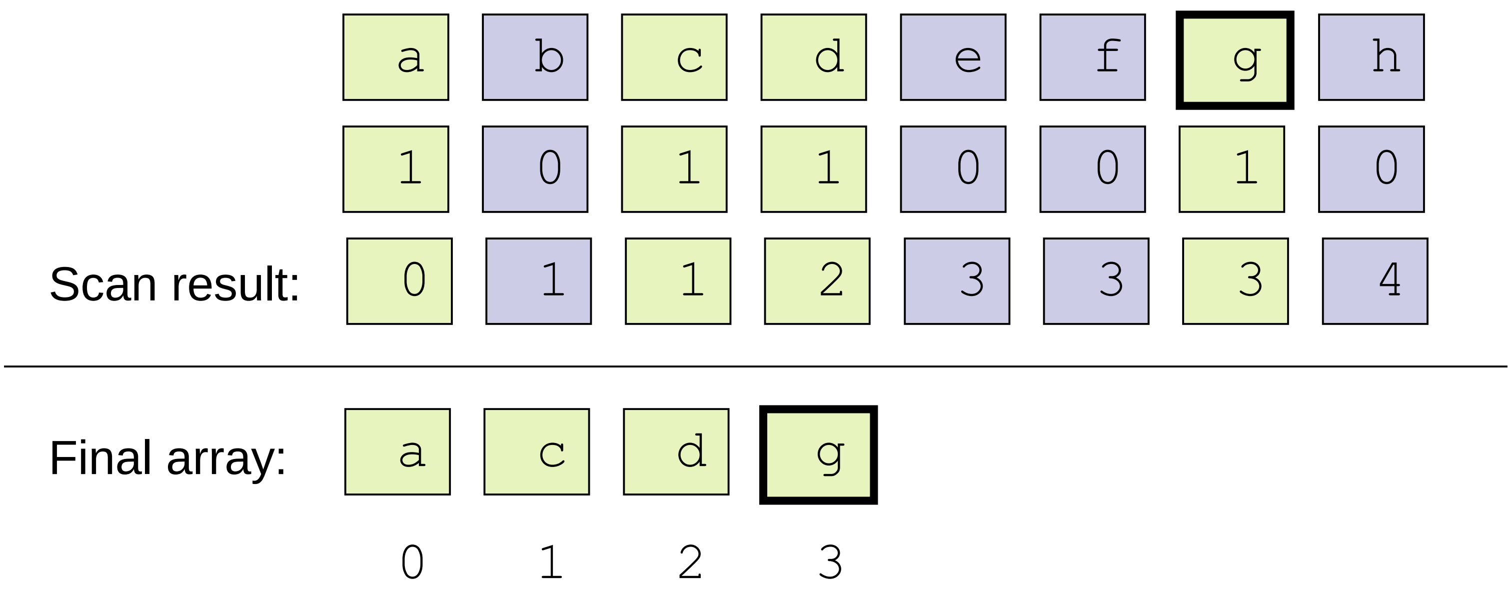
Stream Compaction

- Stream Compaction
  - *Step 3*: Scatter



Stream Compaction

- Stream Compaction
  - *Step 3*: Scatter



Stream Compaction

- Stream Compaction
  - *Step 3*: Scatter

	a	b	c	d	e	f	g	h
	1	0	1	1	0	0	1	0
Scan result:	0	1	1	2	3	3	3	4

Final array:				
	0	1	2	3

- Scatter runs in parallel!

Stream Compaction

- Stream Compaction
  - *Step 3*: Scatter

	a	b	c	d	e	f	g	h
	1	0	1	1	0	0	1	0
Scan result:	0	1	1	2	3	3	3	4

Final array:	a	c	d	g
	0	1	2	3

- Scatter runs in parallel!

Summed Area Table

- *S*ummed *A*rea *T*able (*SAT*): 2D table where each element stores the sum of all elements in an input image between the lower left corner and the entry location.

Summed Area Table

- Example:

Input image

2	1	0	0
0	1	2	0
1	2	1	0
1	1	0	2

SAT

4	9	12	14
2	6	9	11
2	5	6	8
1	2	2	4

$$(1 + 1 + 0) + (1 + 2 + 1) + (0 + 1 + 2) = 9$$

Summed Area Table

- Benefit
  - Used to perform different width filters at every pixel in the image in constant time per pixel
  - Just sample four pixels in SAT:

$$s_{filter} = \frac{s_{ur} - s_{ul} - s_{lr} + s_{ll}}{w \times h},$$

Image from http://http.developer.nvidia.com/GPUGems3/gpugems3_ch39.html

Summed Area Table

- Uses
  - Approximate depth of field
  - Glossy environment reflections and refractions

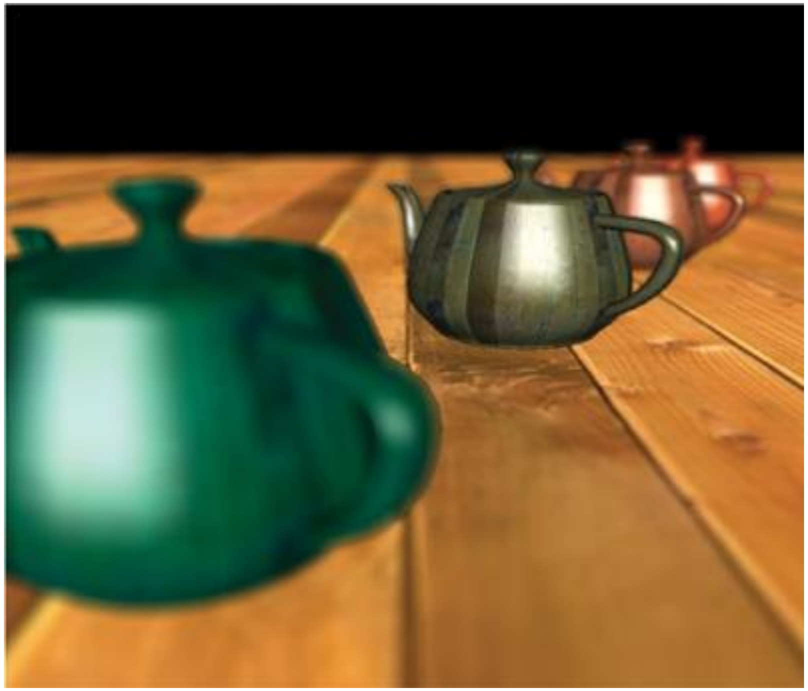


Image from http://http.developer.nvidia.com/GPUGems3/gpugems3_ch39.html

Summed Area Table

Input image

2	1	0	0
0	1	2	0
1	2	1	0
1	1	0	2

SAT

Summed Area Table

Input image

2	1	0	0
0	1	2	0
1	2	1	0
1	1	0	2

SAT

1			

Summed Area Table

Input image

2	1	0	0
0	1	2	0
1	2	1	0
1	1	0	2

SAT

1	2		

Summed Area Table

Input image

2	1	0	0
0	1	2	0
1	2	1	0
1	1	0	2

SAT

1	2	2	

Summed Area Table

Input image

2	1	0	0
0	1	2	0
1	2	1	0
1	1	0	2

SAT

1	2	2	4

Summed Area Table

Input image

2	1	0	0
0	1	2	0
1	2	1	0
1	1	0	2

SAT

2			
1	2	2	4

Summed Area Table

Input image

2	1	0	0
0	1	2	0
1	2	1	0
1	1	0	2

SAT

2	5		
1	2	2	4

Summed Area Table

…

Summed Area Table

Input image

2	1	0	0
0	1	2	0
1	2	1	0
1	1	0	2

SAT

4	9		
2	6	9	11
2	5	6	8
1	2	2	4

Summed Area Table

Input image

2	1	0	0
0	1	2	0
1	2	1	0
1	1	0	2

SAT

4	9	12	
2	6	9	11
2	5	6	8
1	2	2	4

Summed Area Table

Input image

2	1	0	0
0	1	2	0
1	2	1	0
1	1	0	2

SAT

4	9	12	14
2	6	9	11
2	5	6	8
1	2	2	4

Summed Area Table

How would implement this on the GPU?

Summed Area Table

How would compute a SAT on the GPU using inclusive scan?

Summed Area Table

- Step 1 of 2:

Input image

2	1	0	0
0	1	2	0
1	2	1	0
1	1	0	2

Partial SAT

2	3	3	3
0	1	3	3
1	3	4	4
1	2	2	4

One inclusive scan for each row

Summed Area Table

- Step 2 of 2:

Partial SAT

2	3	3	3
0	1	3	3
1	3	4	4
1	2	2	4

Final SAT

4	9	12	14
2	6	9	11
2	5	6	8
1	2	2	4

One inclusive scan for each column, bottom to top

Radix Sort

- Efficient for small sort keys
  - k-bit keys require k passes

Radix Sort

- Each radix sort pass partitions its input based on one bit
- First pass starts with the *l*east *s*ignificant *b*it (*LSB*). Subsequent passes move towards the *m*ost *s*ignificant *b*it (*MSB*)

MSB `010` *LSB*

Radix Sort

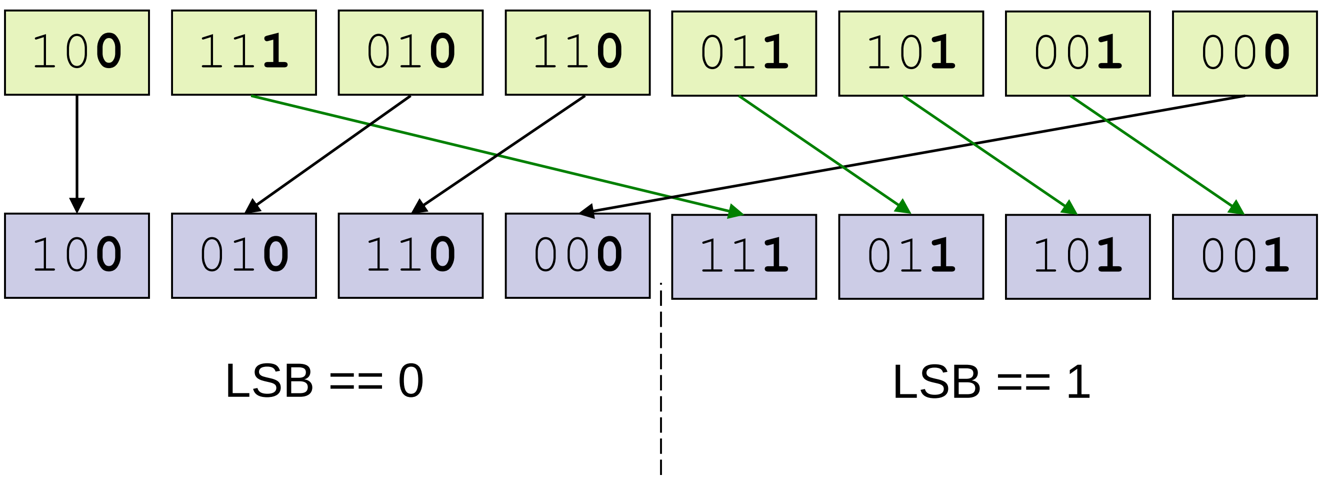
- Example input:

100	111	010	110	011	101	001	000

Example from http://http.developer.nvidia.com/GPUGems3/gpugems3_ch39.html

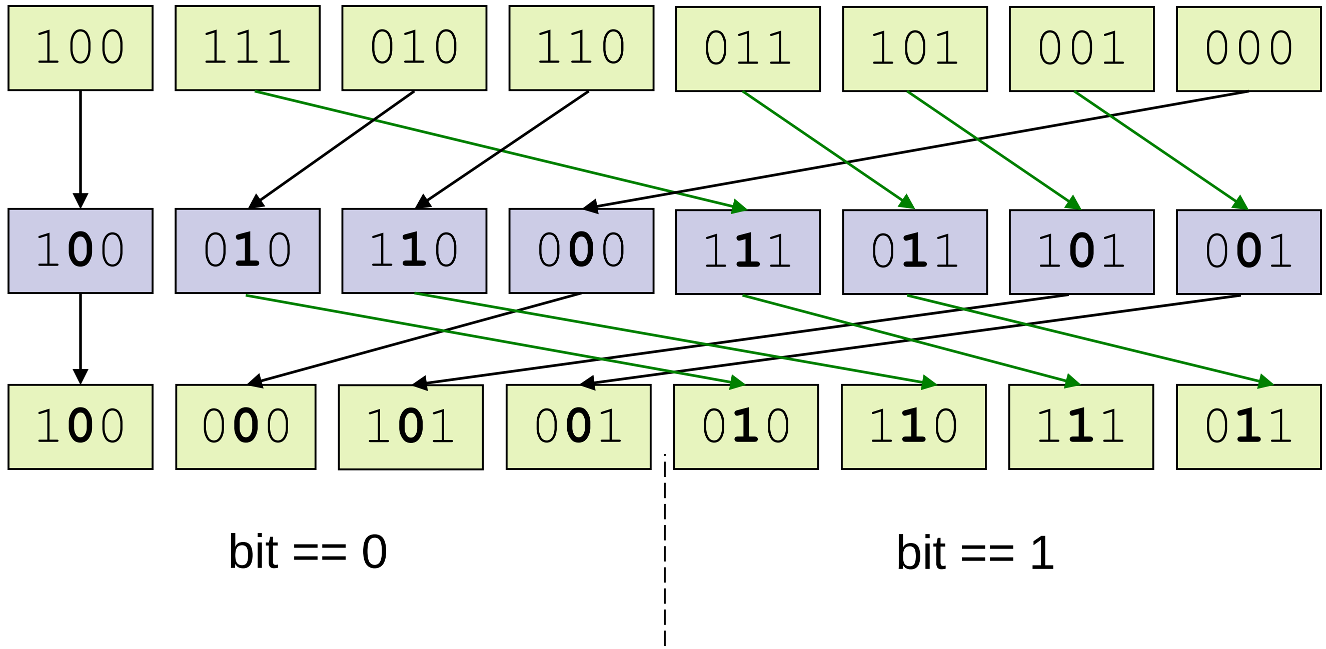
Radix Sort

- First pass: partition based on LSB



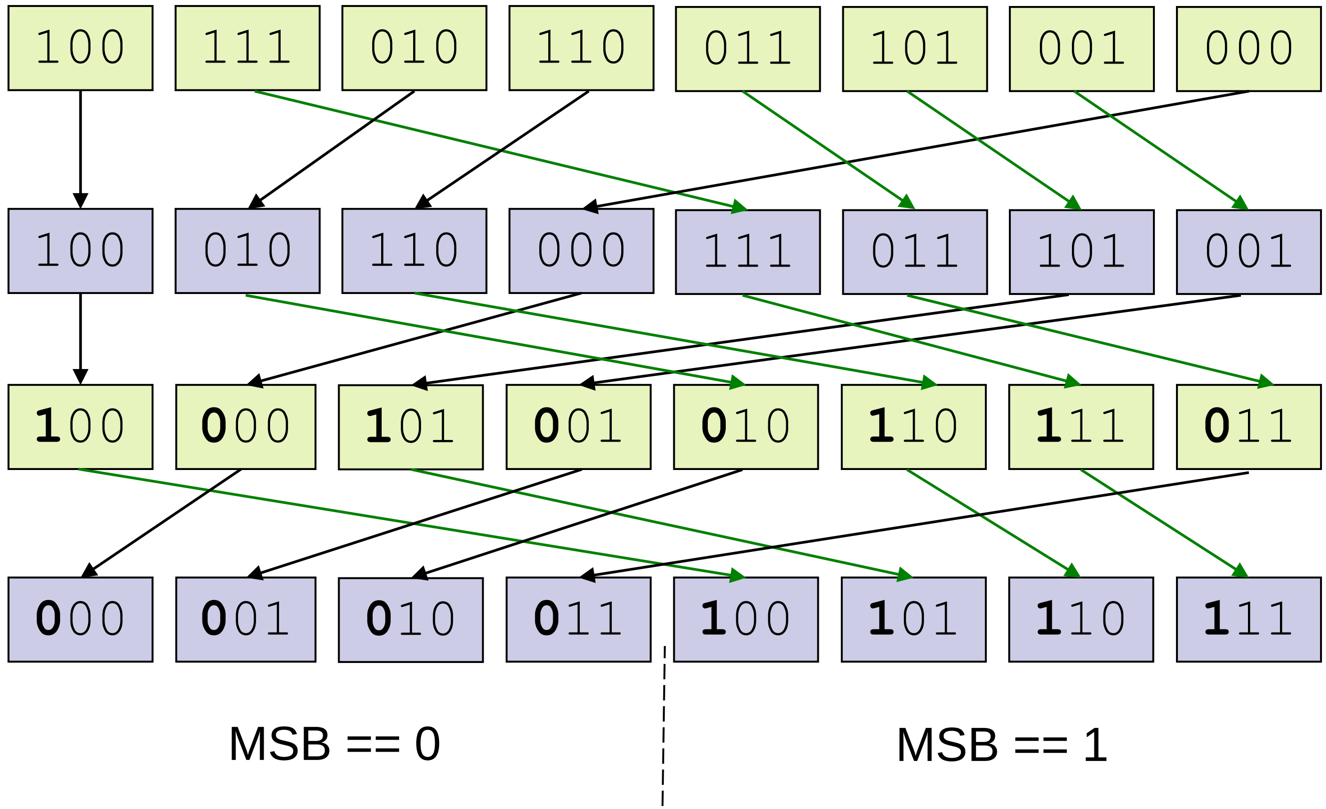
Radix Sort

- Second pass: partition based on middle bit



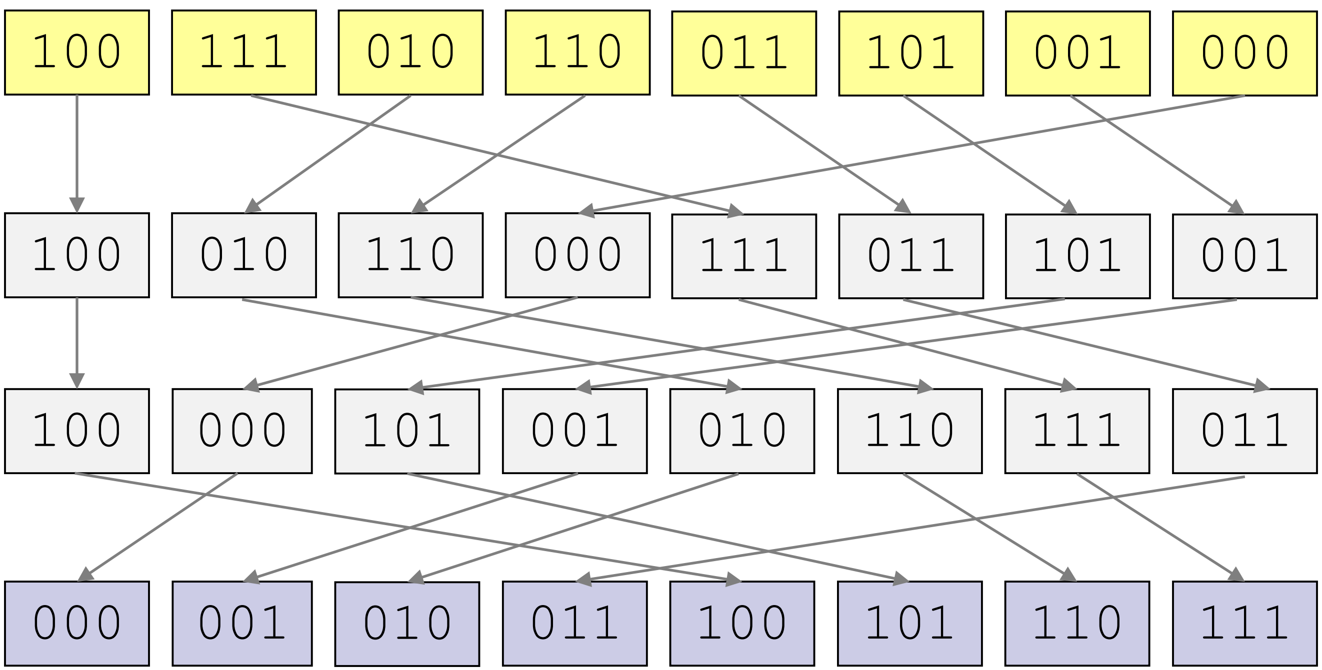
Radix Sort

- Final pass: partition based on MSB



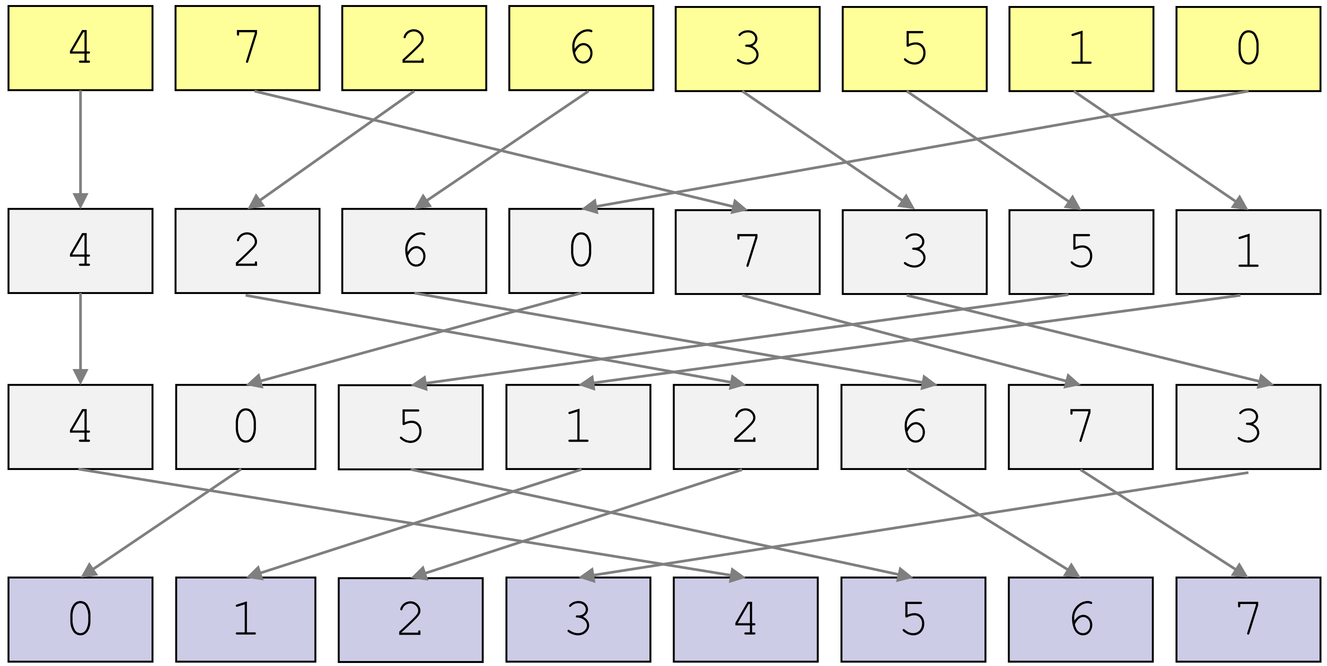
Radix Sort

- Completed:



Radix Sort

- Completed:



Parallel Radix Sort

- Where is the parallelism?

Parallel Radix Sort

1. Break input arrays into tiles
   - Each tile fits into shared memory for an SM
2. Sort tiles in *parallel* with *radix sort*
3. Merge pairs of tiles using a *parallel bitonic merge* until all tiles are merged.

Our focus is on Step 2

Parallel Radix Sort

- Where is the parallelism?
  - Each tile is sorted in parallel
  - Where is the parallelism within a tile?

Parallel Radix Sort

- Where is the parallelism?
  - Each tile is sorted in parallel
  - Where is the parallelism within a tile?
    - Each pass is done in sequence after the previous pass. No parallelism
    - Can we parallelize an individual pass? How?
  - Merge also has parallelism

Parallel Radix Sort

- Implement *spilt*. Given:
  - Array, *i*, at pass *n*:

100	111	010	110	011	101	001	000

  - Array, *b*, which is true/false for bit *n*:

0	1	0	0	1	1	1	0

- Output array with false keys before true keys:

100	010	110	000	111	011	101	001

Parallel Radix Sort

- Step 1: Compute *e* array

100	111	010	110	011	101	001	000	i array
0	1	0	0	1	1	1	0	b array
1	0	1	1	0	0	0	1	e array

Parallel Radix Sort

- Step 2: Exclusive Scan *e*

100	111	010	110	011	101	001	000	i array
0	1	0	0	1	1	1	0	b array
1	0	1	1	0	0	0	1	e array
0	1	1	2	3	3	3	3	f array

Parallel Radix Sort

- Step 3: Compute *totalFalses*

100	111	010	110	011	101	001	000	i array
0	1	0	0	1	1	1	0	b array
1	0	1	1	0	0	0	1	e array
0	1	1	2	3	3	3	3	f array

Chuyển bit 0 có sign = 1
suy ra lúc scan lại thì sẽ biết phía trước
bao nhiêu bit 0

totalFalses = e[n – 1] + f[n – 1]
totalFalses = 1 + 3
totalFalses = 4

Parallel Radix Sort

- Step 4: Compute *t* array

100	111	010	110	011	101	001	000	i array
0	1	0	0	1	1	1	0	b array
1	0	1	1	0	0	0	1	e array
0	1	1	2	3	3	3	3	f array
								t array

t[i] = i – f[i] + totalFalses

totalFalses = 4

Parallel Radix Sort

- Step 4: Compute *t* array

100	111	010	110	011	101	001	000	i array
0	1	0	0	1	1	1	0	b array
1	0	1	1	0	0	0	1	e array
0	1	1	2	3	3	3	3	f array
4								t array

t[0] = 0 – f[0] + totalFalses
t[0] = 0 – 0 + 4
t[0] = 4

0: i
f[0]: số thằng 0 phía trước
4: tổng số thằng 0

totalFalses = 4

Parallel Radix Sort

- Step 4: Compute *t* array

100	111	010	110	011	101	001	000	i array
0	1	0	0	1	1	1	0	b array
1	0	1	1	0	0	0	1	e array
0	1	1	2	3	3	3	3	f array
4	4							t array

t[1] = 1 – f[1] + totalFalses
t[1] = 1 – 1 + 4
t[1] = 4

totalFalses = 4

Parallel Radix Sort

- Step 4: Compute *t* array

100	111	010	110	011	101	001	000	i array
0	1	0	0	1	1	1	0	b array
1	0	1	1	0	0	0	1	e array
0	1	$\boxed{1}$	2	3	3	3	3	f array
4	4	$\boxed{5}$						t array

t[2] = 2 – f[2] + totalFalses
t[2] = 2 – 1 + 4
t[2] = 5

totalFalses = 4

Parallel Radix Sort

- Step 4: Compute *t* array

100	111	010	110	011	101	001	000	i array
0	1	0	0	1	1	1	0	b array
1	0	1	1	0	0	0	1	e array
0	1	1	2	3	3	3	3	f array
4	4	5	5	5	6	7	8	t array

t[i] = i – f[i] + totalFalses

totalFalses = 4

Parallel Radix Sort

- Step 5: Scatter based on address *d*

100	111	010	110	011	101	001	000	i array
0	1	0	0	1	1	1	0	b array
1	0	1	1	0	0	0	1	e array
0	1	1	2	3	3	3	3	f array
4	4	5	5	5	6	7	8	t array
0								d[i] = b[i] ? t[i] : f[i]

Parallel Radix Sort

- Step 5: Scatter based on address *d*

100	111	010	110	011	101	001	000	i array
0	1	0	0	1	1	1	0	b array
1	0	1	1	0	0	0	1	e array
0	1	1	2	3	3	3	3	f array
4	4	5	5	5	6	7	8	t array
0	4							d[i] = b[i] ? t[i] : f[i]

Parallel Radix Sort

- Step 5: Scatter based on address *d*

100	111	010	110	011	101	001	000	i array
0	1	0	0	1	1	1	0	b array
1	0	1	1	0	0	0	1	e array
0	1	1	2	3	3	3	3	f array
4	4	5	5	5	6	7	8	t array
0	4	1						d[i] = b[i] ? t[i] : f[i]

Parallel Radix Sort

- Step 5: Scatter based on address *d*

100	111	010	110	011	101	001	000	i array
0	1	0	0	1	1	1	0	b array
1	0	1	1	0	0	0	1	e array
0	1	1	2	3	3	3	3	f array
4	4	5	5	5	6	7	8	t array
0	4	1	2	5	6	7	3	d[i] = b[i] ? t[i] : f[i]

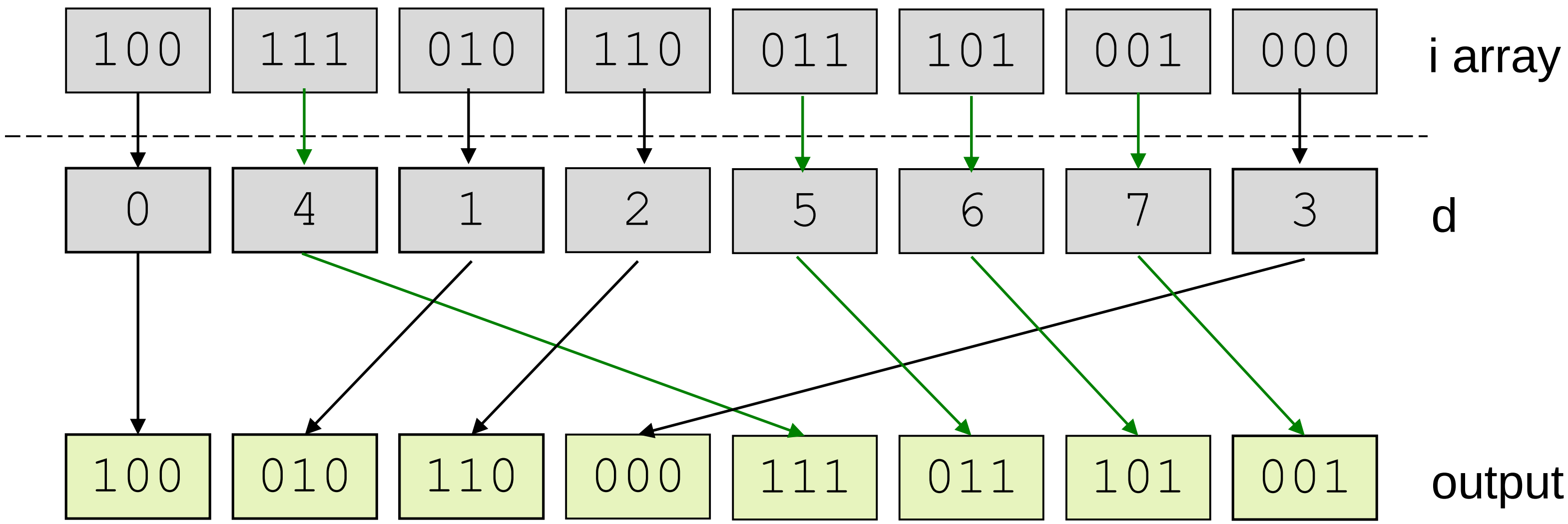
Parallel Radix Sort

- Step 5: Scatter based on address *d*

100	111	010	110	011	101	001	000	i array
0	4	1	2	5	6	7	3	d
								output

Parallel Radix Sort

- Step 5: Scatter based on address *d*



Parallel Radix Sort

- Given k-bit keys, how do we sort using our new *split* function?

- Once each tile is sorted, how do we merge tiles to provide the final sorted array?

Summary

- Parallel reduction, scan, and sort are building blocks for many algorithms
- An understanding of parallel programming and GPU architecture yields efficient GPU implementations