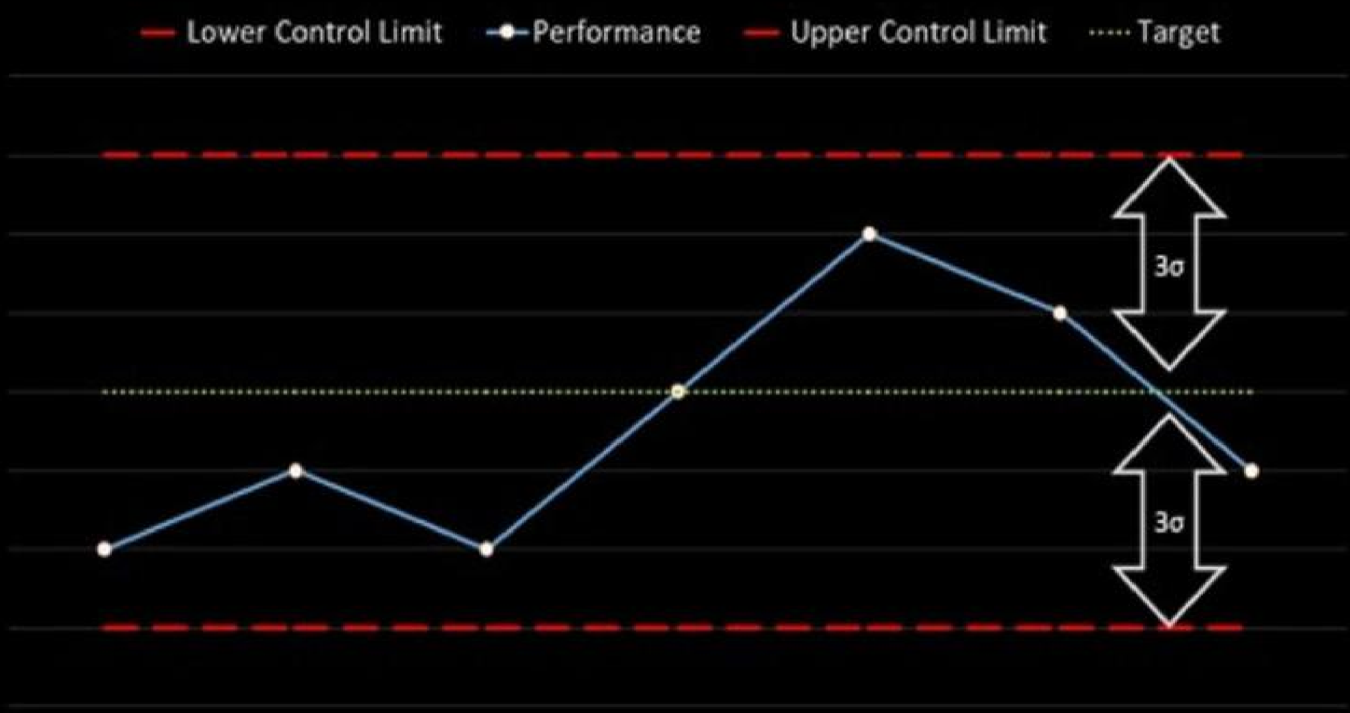
SUMEET SAVANT



CONTROL CHARTS
SIX SIGMA THINKING

THE ESSENTIAL BOOK FOR REDUCING VARIATION AND DEFECTS

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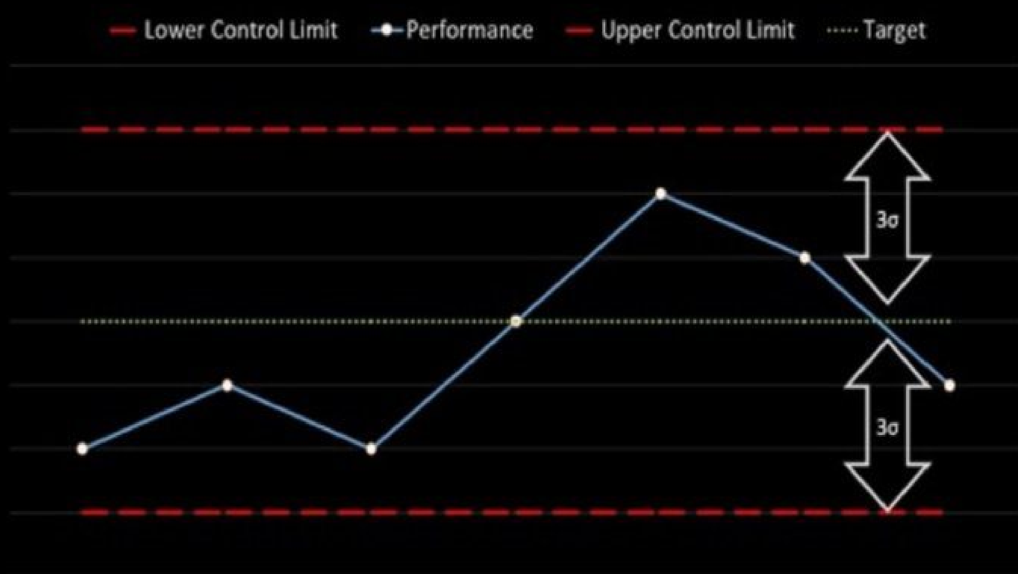

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Control Charts
Six Sigma Thinking Series

Sumeet Savant

ABOUT THE AUTHOR

Sumeet Savant is a Lean Six Sigma Master Black Belt Mentor and coach, with more than a decade of experience in executing, leading and mentoring Lean Six Sigma process improvement projects. He is a BTech, MBA, and Prince certified Practitioner. He has facilitated hundreds of process improvement projects, and coached hundreds of professionals, Yellow, Green, and Black Belts over the years. He lives in Mumbai, India with his family.

Control Charts

Control Charts

In simplest terms, **Control Charts** are,

• **Run Charts**.

• **Used in Statistical Control Process**.

• **To determine if your business processes are in control**.

• **Making use of Control Limits**.

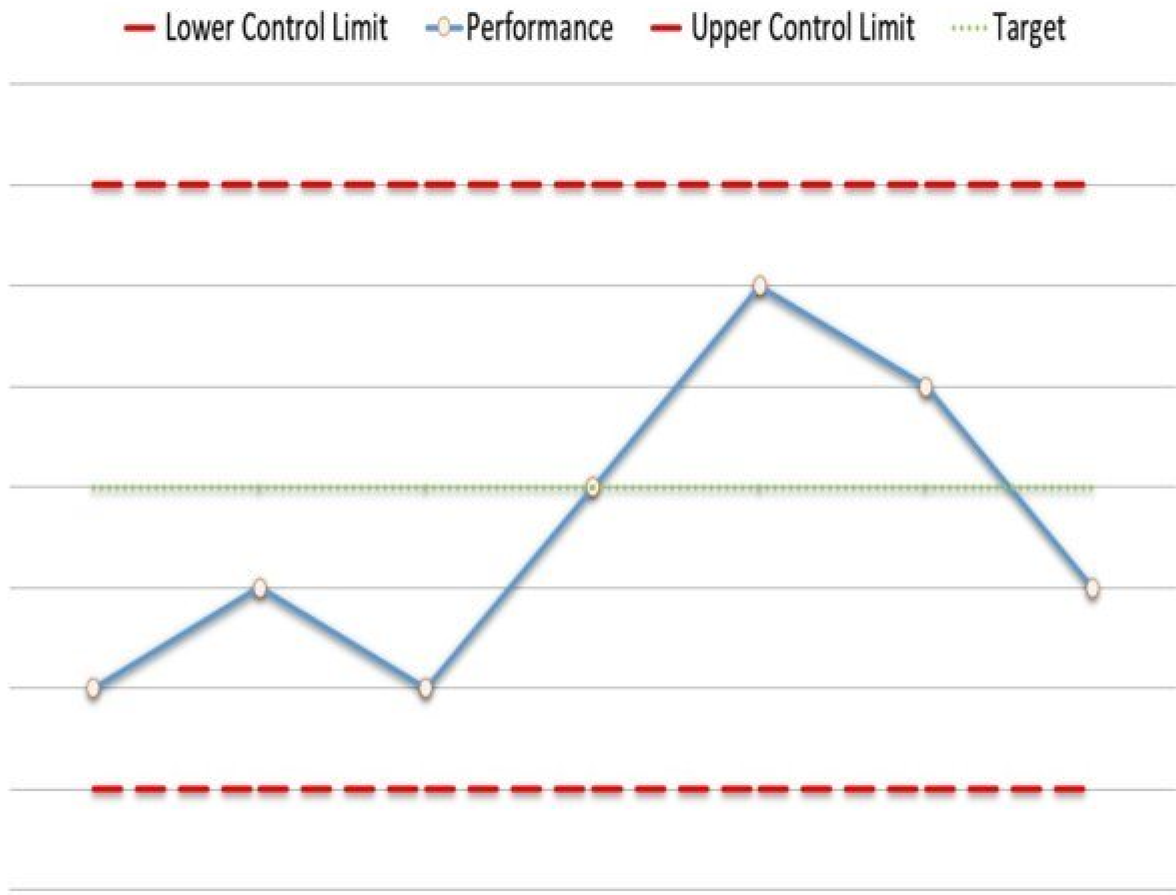
• **Over the performance of your business processes derived from the voice of processes**.

They are also known as **Shewhart Charts**.

After **Walter Andrew Shewhart**.

We will look at each of these terms to understand the concept in detail.

The following figure depicts a control chart,



Statistical Process Control

In simplest terms, **Statistical Process Control (SPC)** is,

• **A Quality Control Methodology**.

• **That makes use of statistical tools**.

• **To monitor a process**.

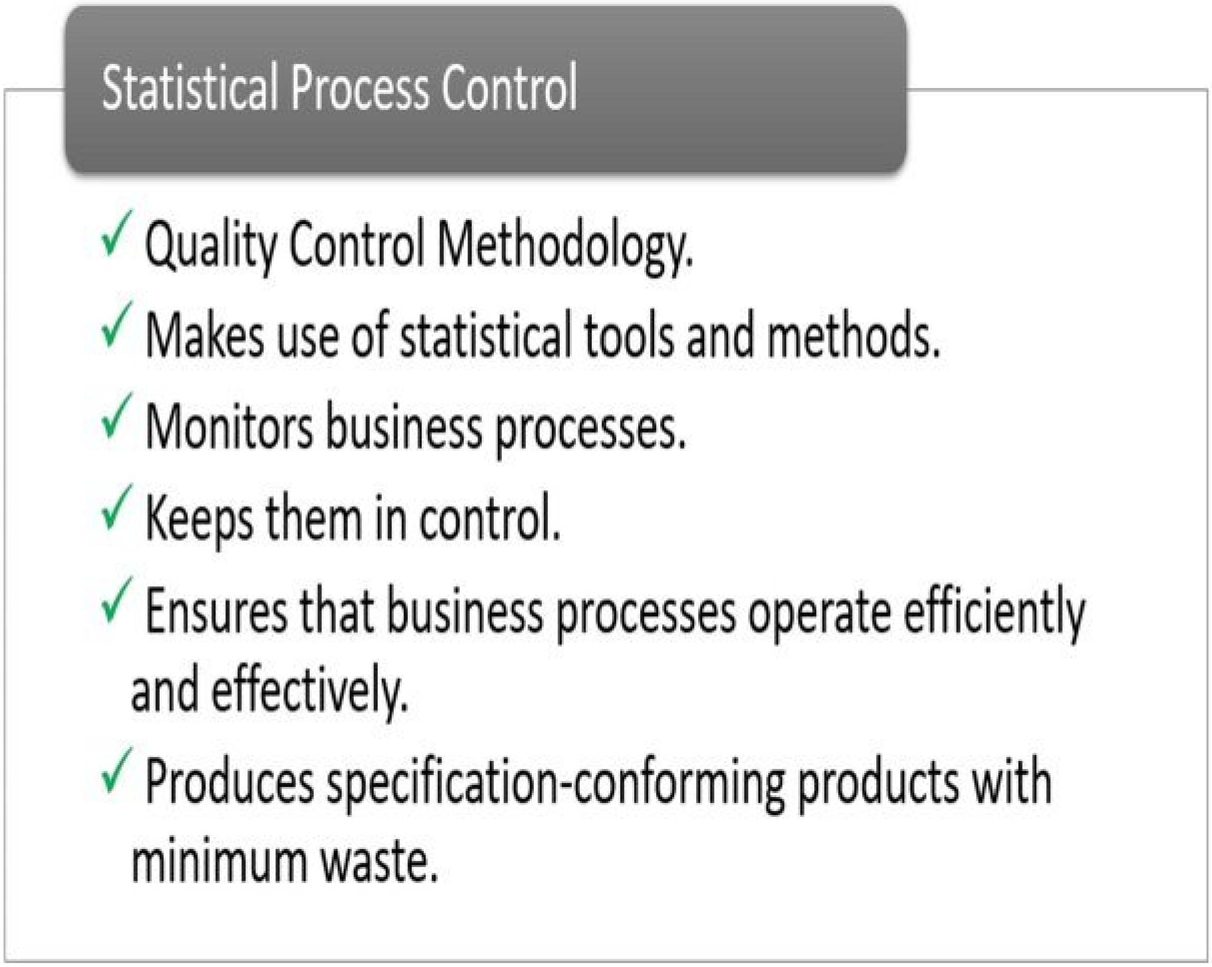
• **And keep them in control**.

• **To ensure that your business processes operate efficiently and effectively**.

• **And produces specification-conforming products with minimum waste**.

SPC makes extensive use of **Control Charts**.

The following figure depicts the concept of Statistical Process Control,



Voice of Process

Processes are gauged on their performance based on certain key metrics which indicate the overall health of the processes.

These measurements enable the business processes to 'talk' and give them their 'Voice'.

Hence, the **Voice of the Process (VOP)** is the means through which the process 'communicates' with the organization on its performance against customer needs and expectations.

The **Voice of the Process (VOP)** is captured through measurements of the key metrics and performance indicators and describe how the overall processes are performing in their current state.

Run Charts

Run Charts in the simplest terms are line graphs that are used to plot the performance of the processes over time.

They are also known as **Time Series Graphs** as the X-Axis of the chart or graphs is the time unit, which can be days or minutes, or months etc.

They tell you how the processes are **running or performing** over time.

The **Process Performance** can be gauged by the following from the Run Charts,

- **Trends**.
- **Patterns**.
- **Cycles**.
- **Shifts**.

The following figure depicts an example of a Run Chart used in SPC,

Stability and Control Charts

Stability in a process is the degree of its consistency, whereas **Capability** of a process is its ability to meet the customer specifications in terms of its performance.

It is very important to note that Stability and Capability are different concepts; a process can be stable but not capable, i.e. it can consistently perform badly.

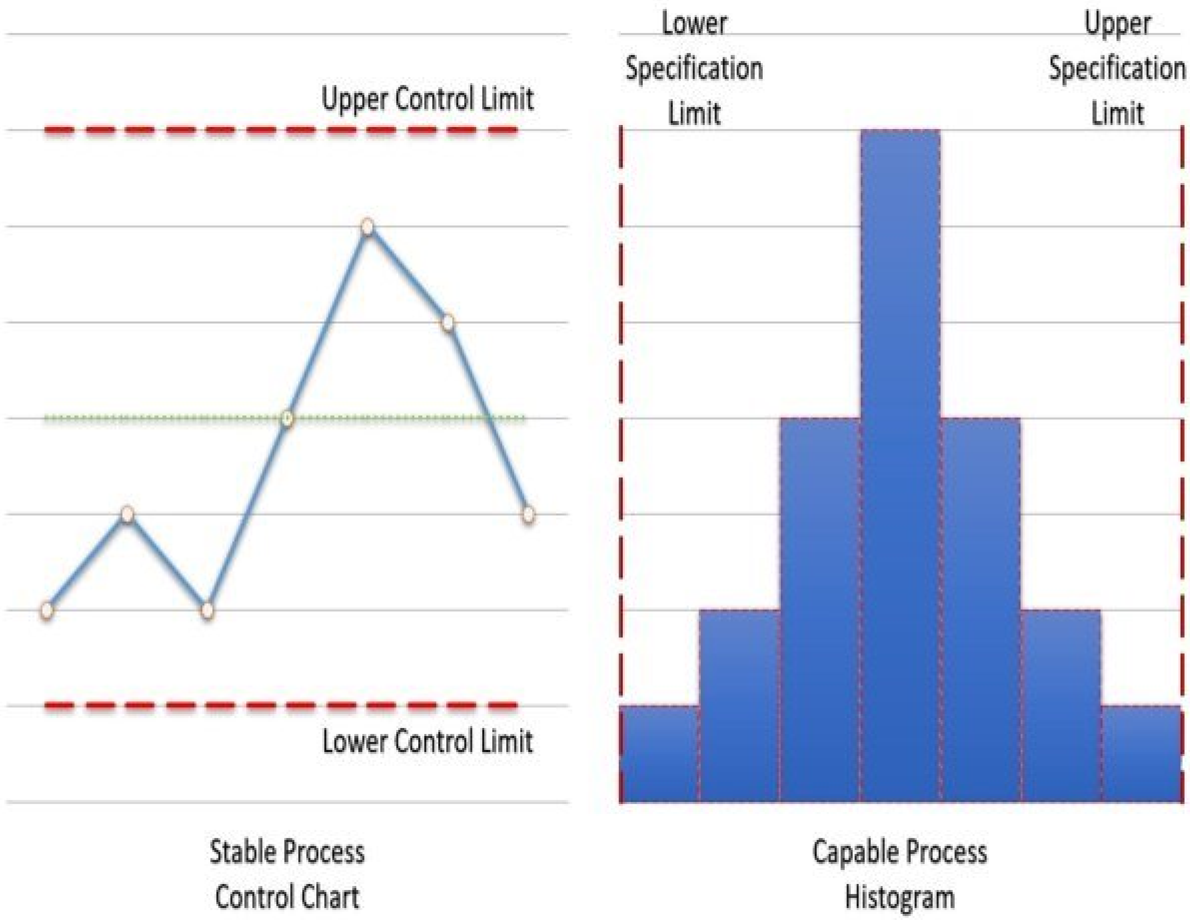
An ideal state of a process is that it should meet customer requirements (capable) consistently (stable).

An Histogram with Upper and Lower Specification limits along with the performance plotted helps visualize the Capability of a process.

However to visualize the stability of a process we need a run chart with Upper and Lower Control limits plotted along with its performance.

Such a chart is known as a Control Chart.

The following figure depicts an example of a Run Chart with the Control Limits to make it a Control Chart along with a Histogram, the process is both stable and capable,



Control Chart Elements

Elements of a Control Chart

A control chart is made up of a few important things that differentiate it from a run chart and these are,

• **X-Axes** The X Axis of a Control Chart is always time series, similar to the Run Chart.

• **Y-Axes** The Y Axis of a Control Chart is most often the performance of the process under study over the time.

• **Control Limits** This is the most important difference between the Run Chart and the Control Chart, as Control Charts have Control Limits at a distance of +/-3$\boldsymbol{\sigma}$ from the center line.

• **Median** This is the another important difference between the Run Chart and the Control Chart, as Control Charts have a central line which is often the median of the data.

Standard Deviation

Standard Deviation is a metric that expresses by how much the members of a group differ from the mean value for the group.

The Standard Deviation is a measure of how spread out the data points or numbers are, or a measure of the amount of variation or dispersion of a set of values.

Its symbol is σ (the greek letter sigma) and is calculated as,

$\sigma = \sqrt{} \sum [(Xi - \mu)^2 / (N-1)]$

Where, σ = Population Standard Deviation, N = the number of observations in the population, Xi = Observed values, μ = Population mean.

$s = \sqrt{} \sum [(Xi - X)^2 / (n-1)]$

Where, s = Sample Standard Deviation, n = the number of observations in the sample, Xi = Observed values, X = Sample mean.

Median

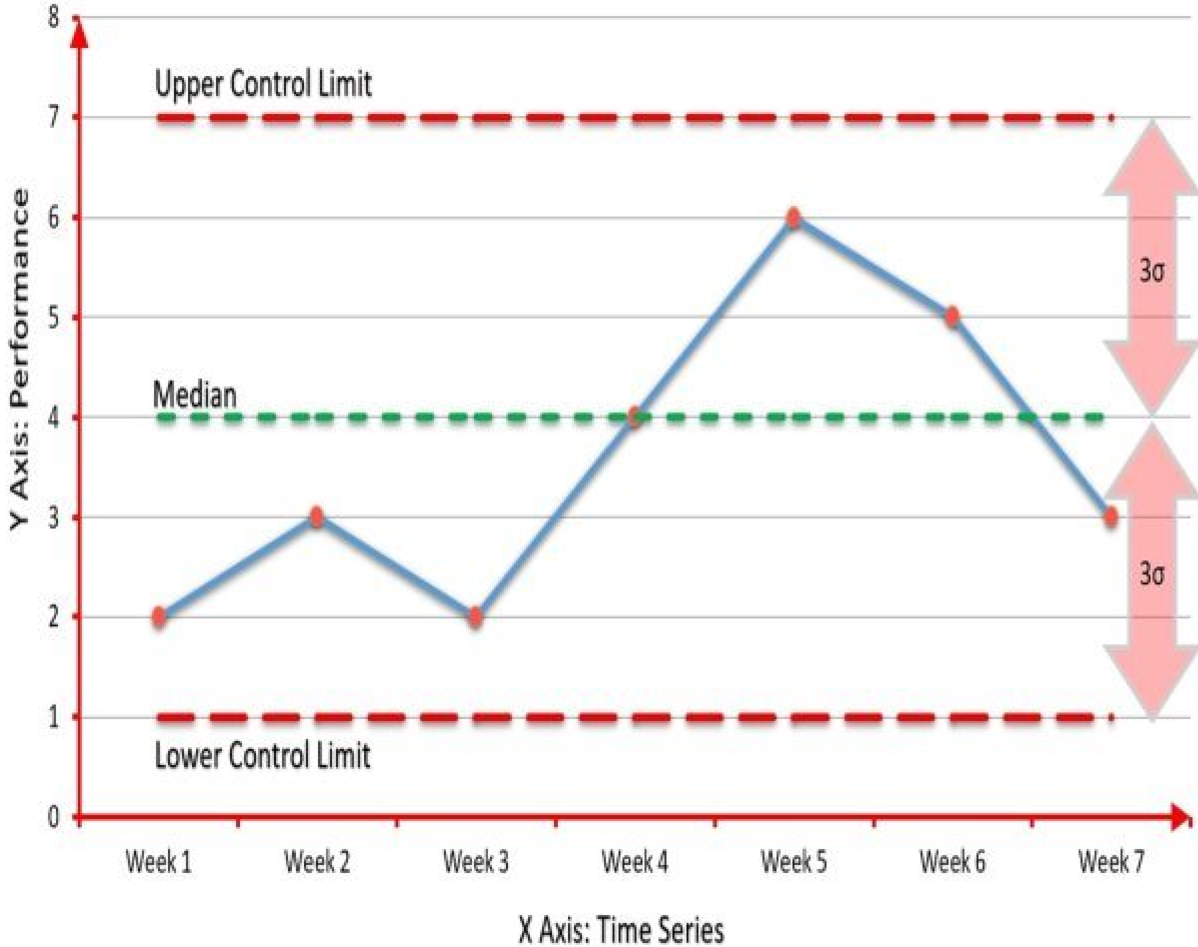
Median is a metric for measuring the central tendency of a data set, similar to the mean.

The Median is the middle number in a sorted, ascending or descending, data set or sample.

The median is the value that separates the higher half from the lower half of a data set.

The Median is preferred as it is more descriptive of the data set than the mean, especially when there are outliers in the sequence that might skew the mean of the values.

The following figure depicts the elements of a control chart,



Spotting Special Causes

Special Causes vs Common Causes

Six Sigma projects focus on reducing **variation**.

In Six Sigma terms and also on lines of Statistical Thinking, variation has its origin in either,

• **Common Causes** which are the usual, historical, quantifiable variation inherently present in a process.

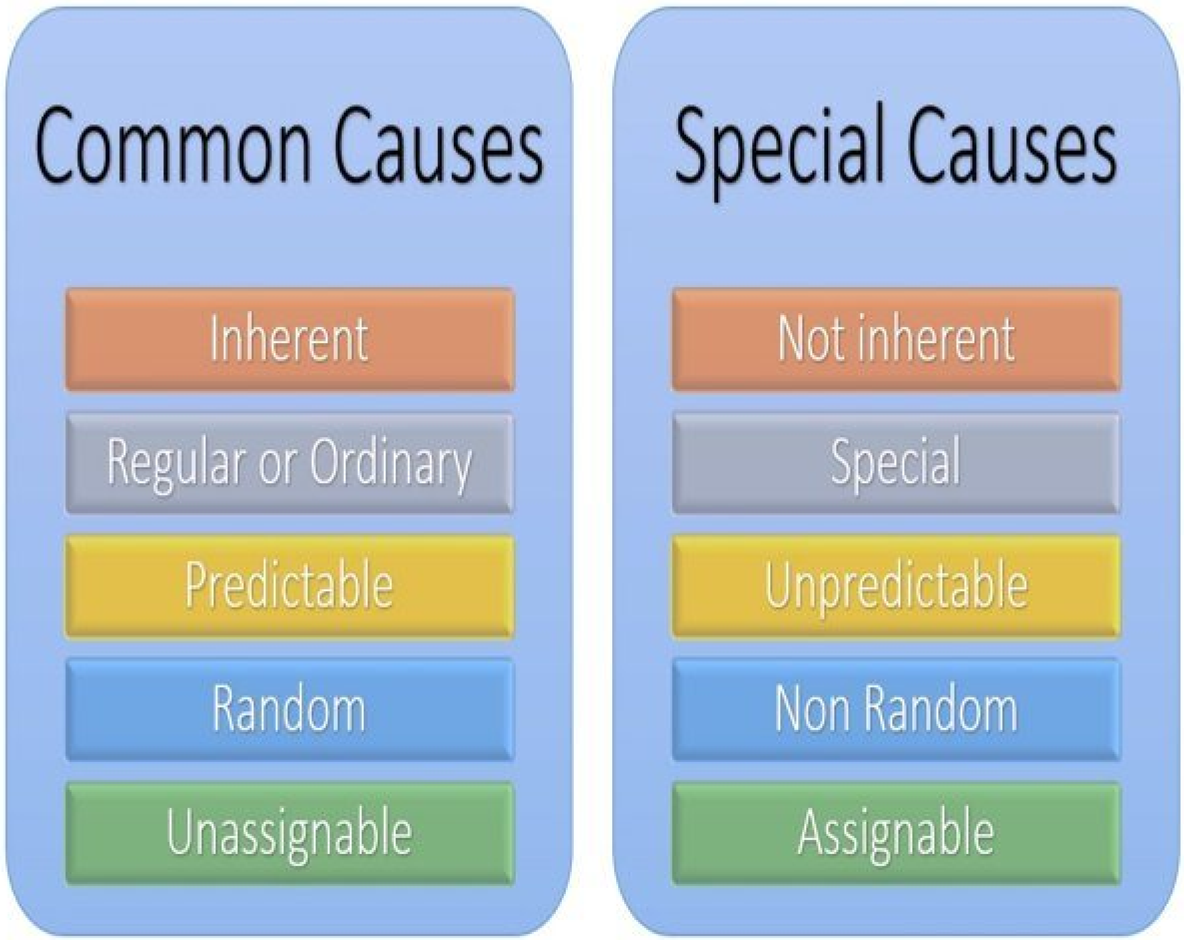
For example, substandard raw materials, poor design, and unclear procedures.

• **Special Causes** which are the unusual, not historically observed, non quantifiable variation not inherently present in the processes and hence assignable.

For example, machine breakdowns, sudden heavy loads or traffic, and power surges.

Control Charts and Run Charts are excellent tools to spot and separate special causes from common causes in process variation.

The following figure depicts the difference between common and special causes,



Over and Under reacting

It is highly essential to be able to separate the common and special causes.

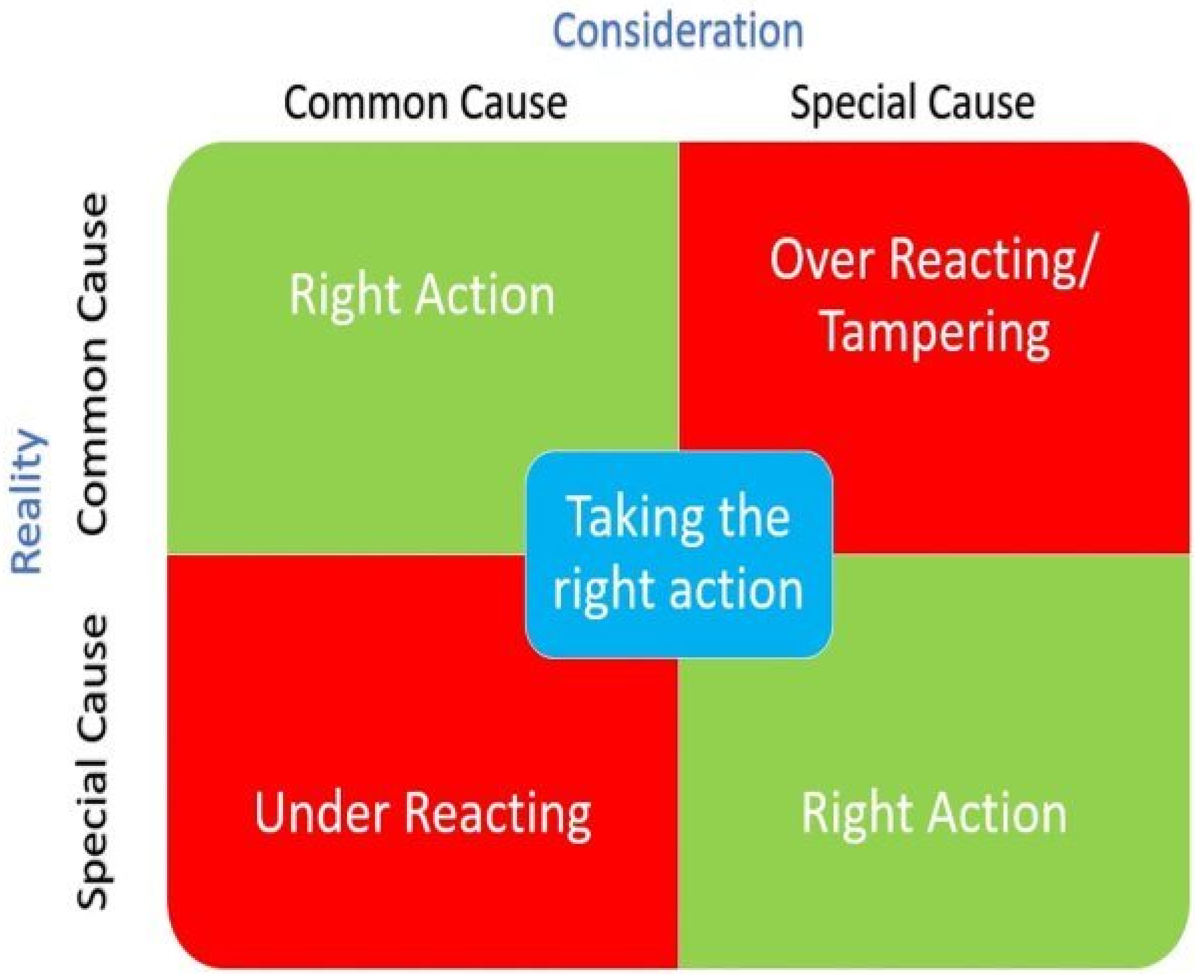
• **If we end up treating Special Causes as Common Causes** we most often end up Under Reacting which may result in increased variation due to corrective actions not being taken.

• **If we end up treating Common Causes as Special Causes** we most often end up Over Reacting or Tampering which may result in waste of effort and money.

Hence it is very important to be able to separate the special causes from common causes and decide the right amount and kind of action to be taken.

Which makes **Control Charts and Run Charts** such important tools, as we will next see how they can help us spot the special causes.

The following figure depicts the right action to be taken for the common and special causes and its effect,



Runs

Runs on a control or run chart are the groups of consecutive data points on either side of the center line of the chart.

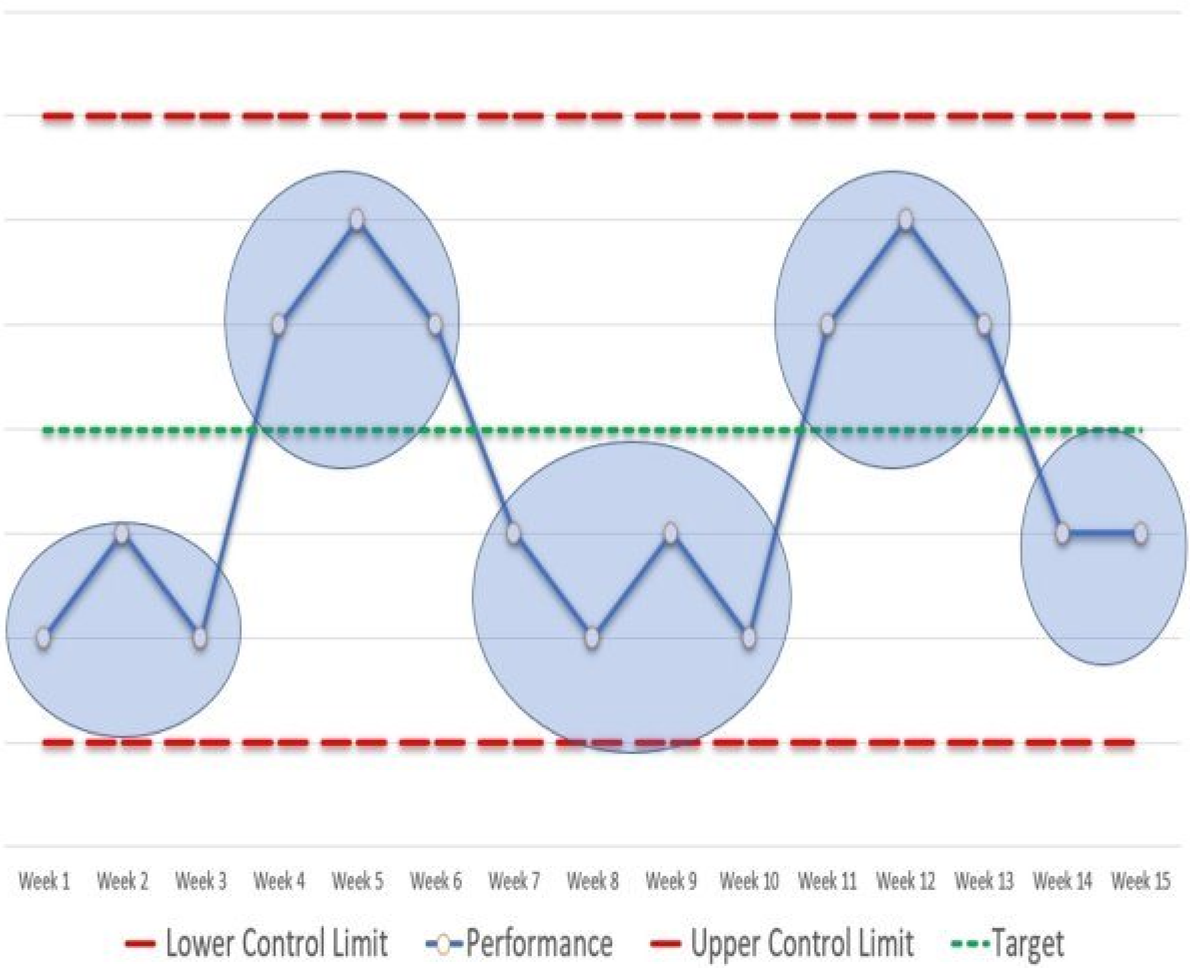
The patterns of the runs appearing on the control or run charts tell a lot about the process and its performance.

To study the patterns of the runs on the chart we need to do the following,

- Draw a circle around each run.
- Count the number of these circles.
- Study the pattern of these circles to identify any special causes.

For this to be effective we need at least 15 or more data points.

The following figure depicts a Control Chart with Runs identified,



Mixtures

Mixtures are a unique patterns of runs where either,

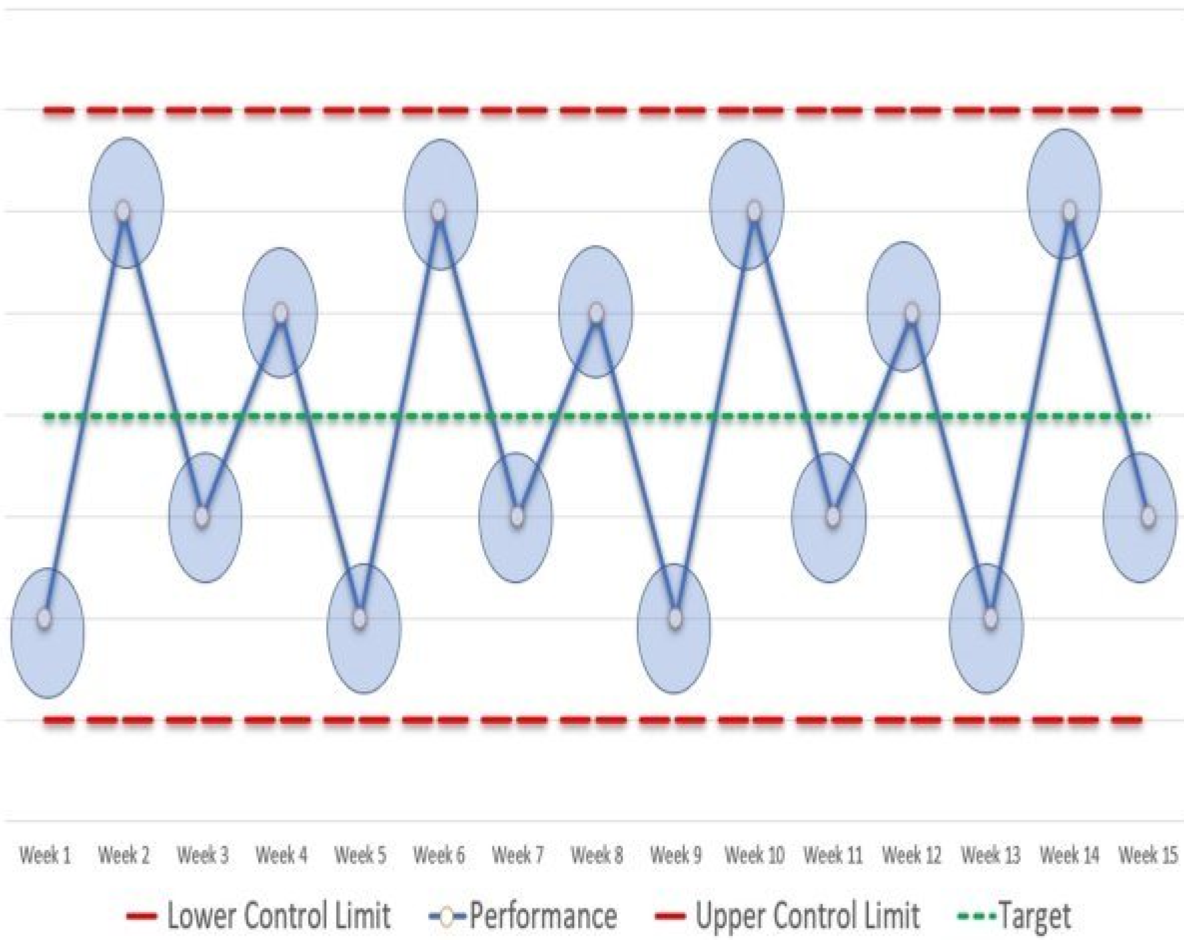
• There are large distances or gaps between the consecutive data points.

• Consecutive data points lie on the either side of the middle line.

The uniqueness about Mixtures is that they are characterized by frequent crossing of the center line and often characterized by an absence of points near the center line..

Mixtures most often are a result of combined data from two populations, or two processes operating at different levels.

The following figure depicts a Control Chart with Mixtures identified in the Runs,



Oscillations

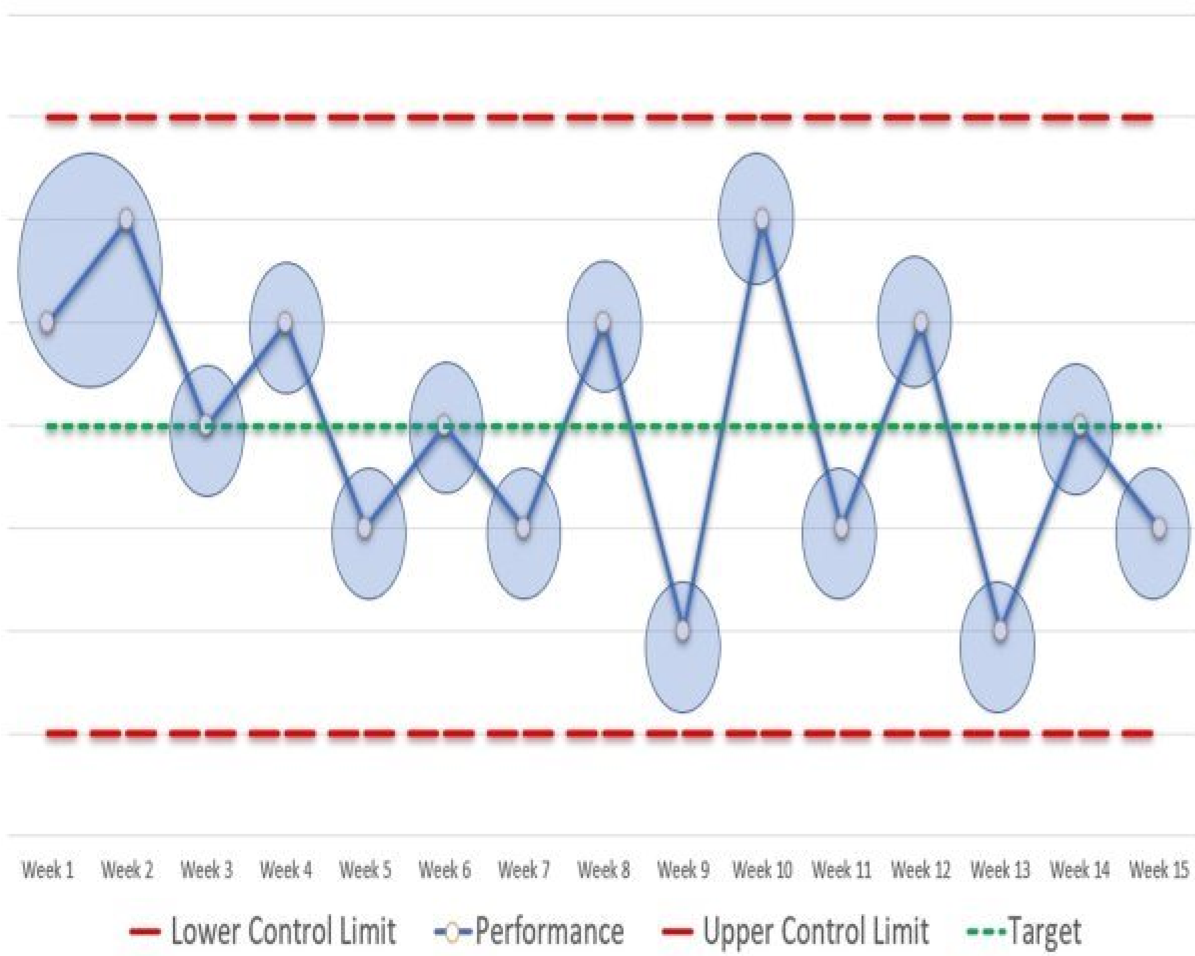
Oscillations are a unique patterns of runs where,

- The data fluctuates up and down.
- Indicating that the process is not steady.

Oscillations differ from Mixtures as in Oscillations the data points do not necessarily cross the center line.

However Mixtures are characterized by frequent crossings of the data points.

The following figure depicts a Control Chart with Oscillations identified in the Runs,



Clusters

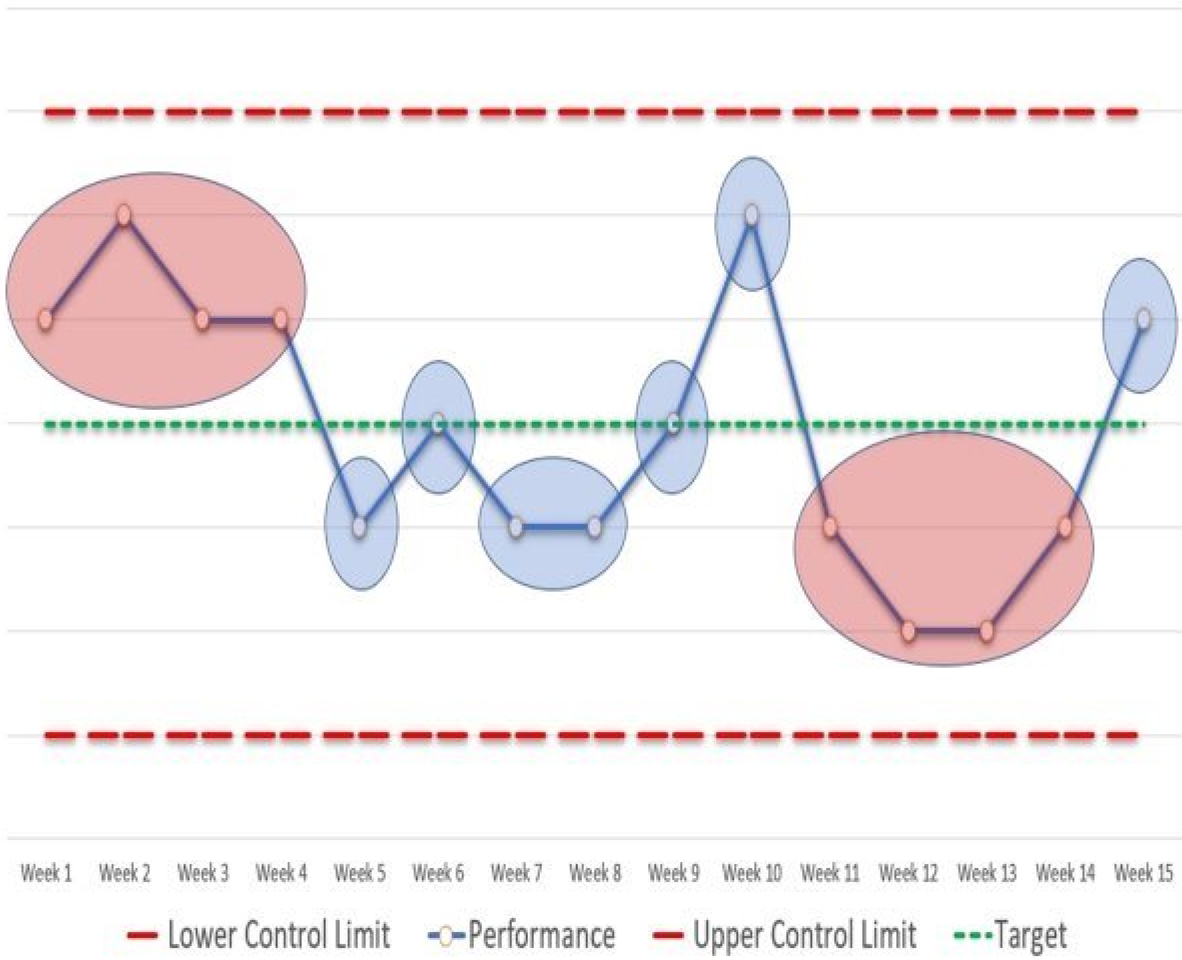
Clusters are a unique patterns of runs where,

- Data points are found in groups in one area of the chart.
- These data points appear as clusters on the chart.

Oscillations and Mixtures had data points away from each other whereas clusters are data points present in groups.

Clusters often indicate measurement problems, lot-to-lot or set-up variability, or sampling from a group of defective parts.

The following figure depicts a Control Chart with two Clusters identified in the Runs,



Trends

Trends are a unique patterns of runs where,

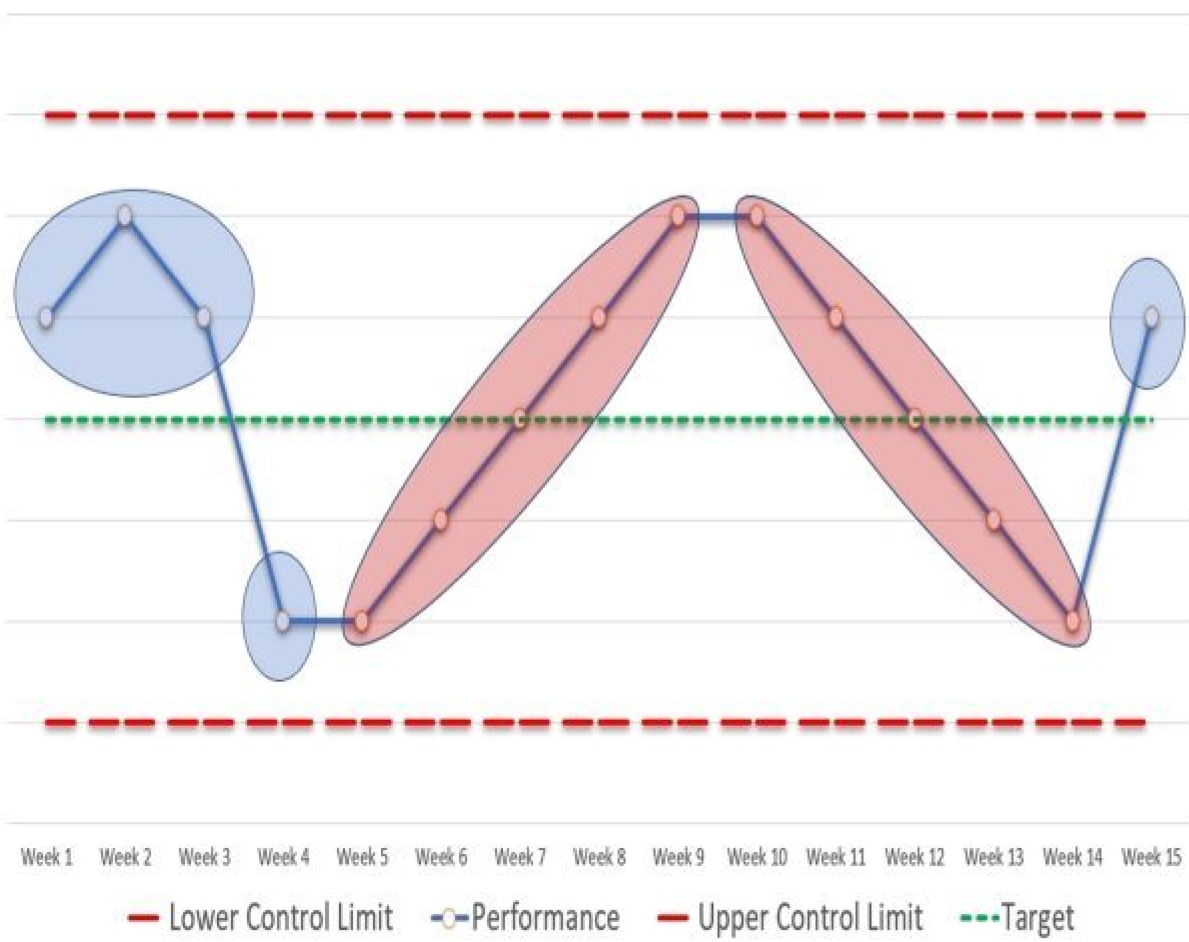
• Data points are following a sustained drift, either in an upward or downward direction.

• These data points appear to be following a trend in one particular direction.

Trends more often are characterized by 5 or more consecutive increasing or decreasing points.

Trends are easy to catch on the run and control charts and may warn that a process will soon go out of control and are often caused by worn tools, a machine that does not hold a setting, or periodic rotation of operators.

The following figure depicts a Control Chart with two Trends identified in the Runs,



Shifts

Shifts are a unique patterns of runs where,

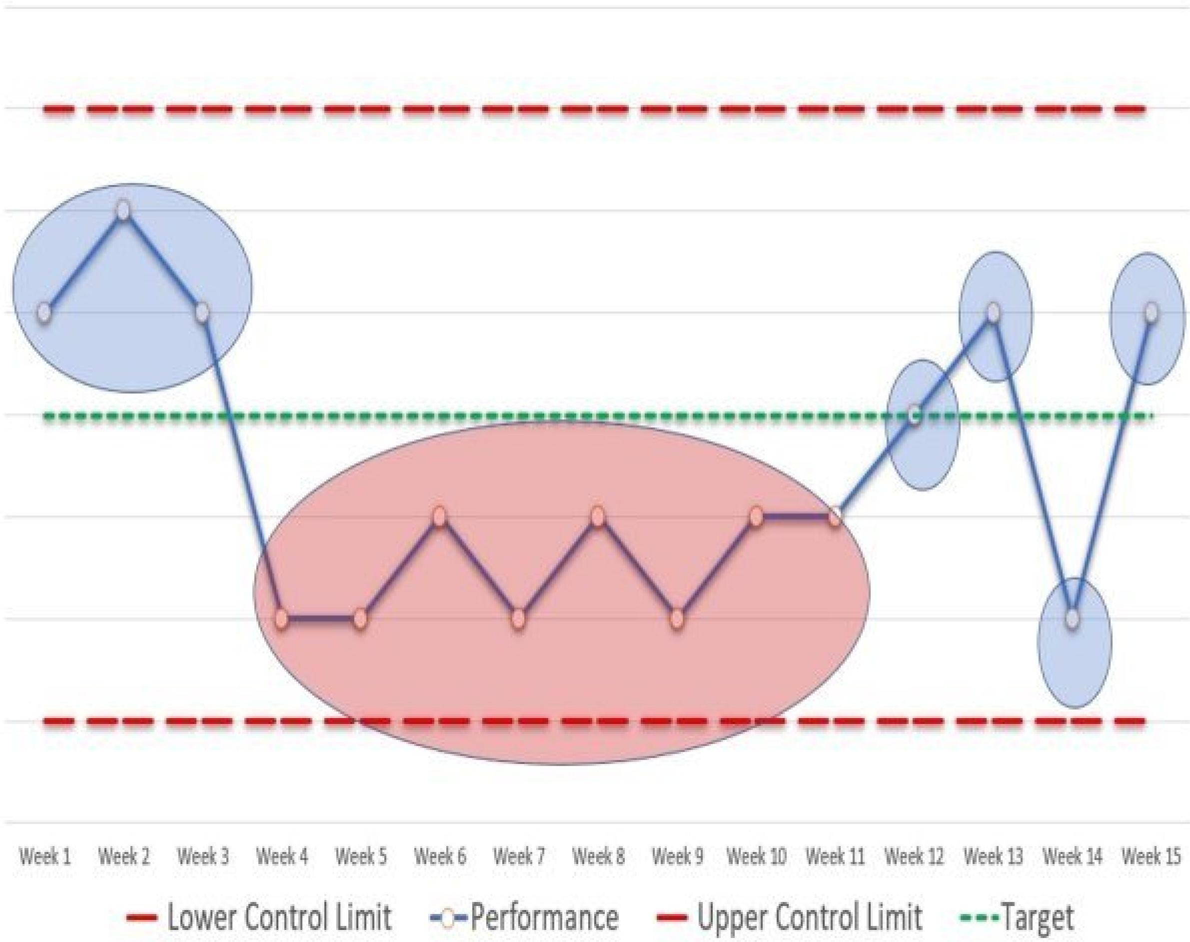
• Data points are shifted on either side of the center line.

• These data points appear to be literally shifted to one side of the center line and tend to stay there for some time.

Shifts more often are characterized by 6 or more consecutive data points lying on one side of the center.

Shifts are also easy to catch on the run and control charts and may warn that a process will soon go out of control.

The following figure depicts a Control Chart with a Shift identified in the Runs,



Astronomical Points

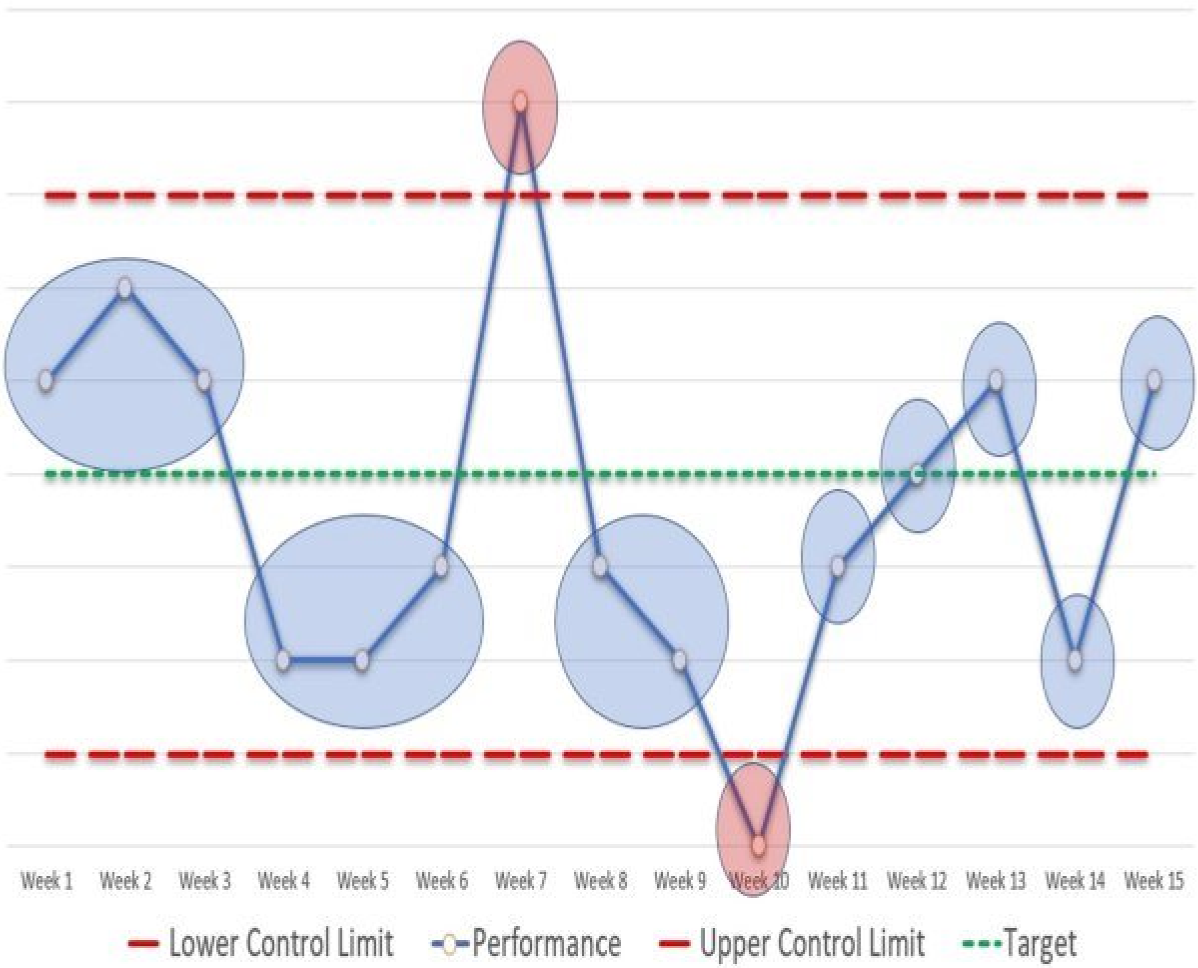
Astronomical Points are a unique patterns of runs where,

- Data points have dramatically different values.
- Most often beyond the control limits.

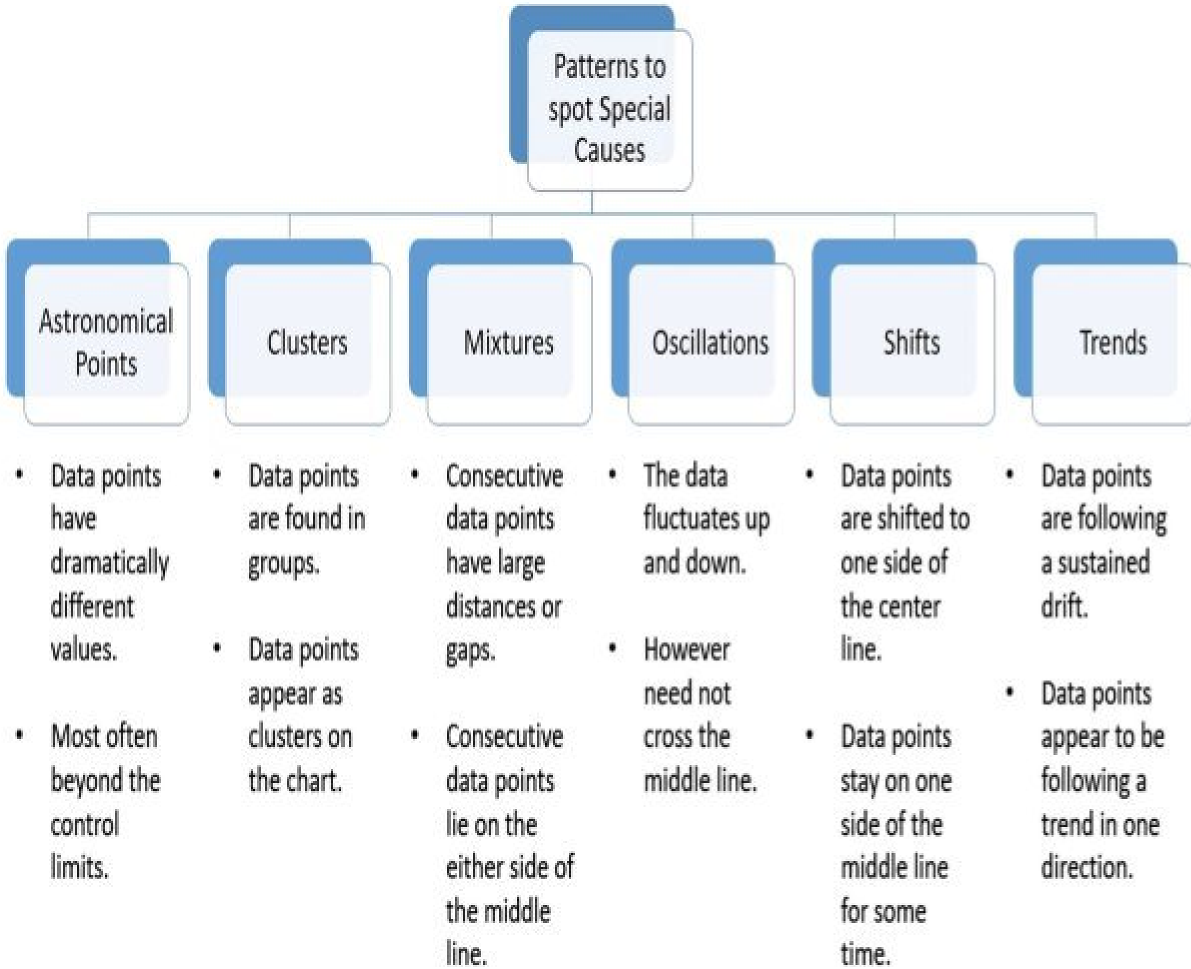
These scenarios are clearly unstable where certain data points may be above and over the upper control limit.

Or below and under the lower control limit.

The following figure depicts a Control Chart with two Astronomical data points identified in the Runs,



The following figure depicts a summary of patterns to spot the special causes using control charts,



Classification of Control Charts

Types of Data

Control Charts can be broadly classified based on the two **Data Types**.

• **Attribute Data Type** is the type of numerical data that can be counted.

This data type follows the Discrete Distribution.

For example, defects, defectives.

• **Continuous Data Type** is the type of numerical data that can be measured.

This data type follows the Continuous Distribution.

For example, length, area, volume etc.

Attribute Data Control Charts

Attribute Data Control Charts can be further classified based on whether the control charts cater to the **Defects or Defectives**.

• **Defects** are the flaws, issues, errors and hence essentially a non conformity to customer requirements, specifications and standards; for example, bugs in the code.

Further the chart that caters to a constant number of opportunities is the c Chart, and for variable number of opportunities is the u Chart.

• **Defectives** are the non conforming items that have the defects; for example, the code with the defects.

Further the chart that caters to a fixed sample size is the np Chart, and for variable sample size is the p Chart.

Continuous Data Control Charts

Continuous Data Control Charts can be further classified based on the **subgroup size** the control charts cater to.

- **n = 1** where the subgroup size is 1.

For example, the IMR Chart.

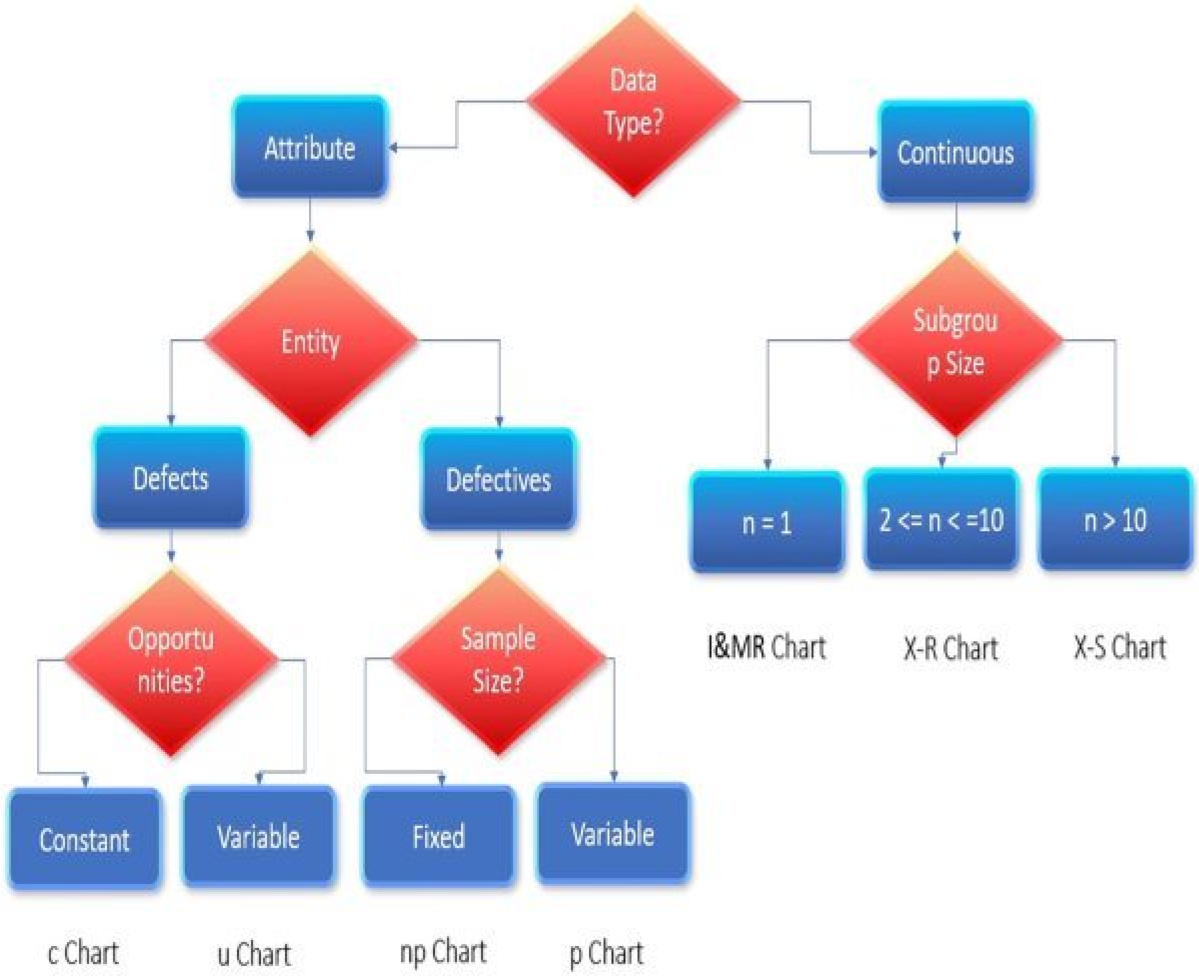
- **2 ≤ n ≤ 10** where the subgroup size is between 2 to 10.

For example, the XR Chart.

- **n > 10** where the subgroup size is greater than 10.

For example, the XS Control Chart.

The following figure depicts the classification of Control Charts and in which scenario each is used,



Control Charts for Attribute Data

Control Chart for Attribute Data

As we have seen earlier, **Attribute Data Type** is the type of numerical data that can be counted and follows the Discrete Distribution.

We can used control charts for the Attribute Data like the **Defects** and **Defectives**.

There are 4 types Attribute Data Control Charts,

- **c Chart**
- **u Chart**
- **np Chart**
- **p Chart**

Control Charts for Defects

As we have seen earlier, **Defects** are the flaws, errors, issues that act as a non conformity to customer requirements, specifications and standards.

Of the 4 types of control charts for Attribute Data 2 are used for Defects and they are,

• **c Chart**, used when the lot size or the number of the defects or defect opportunities is a constant.

• **u Chart**, used when the lot size or the number of the defects or defect opportunities is variable.

Control Charts for Defectives

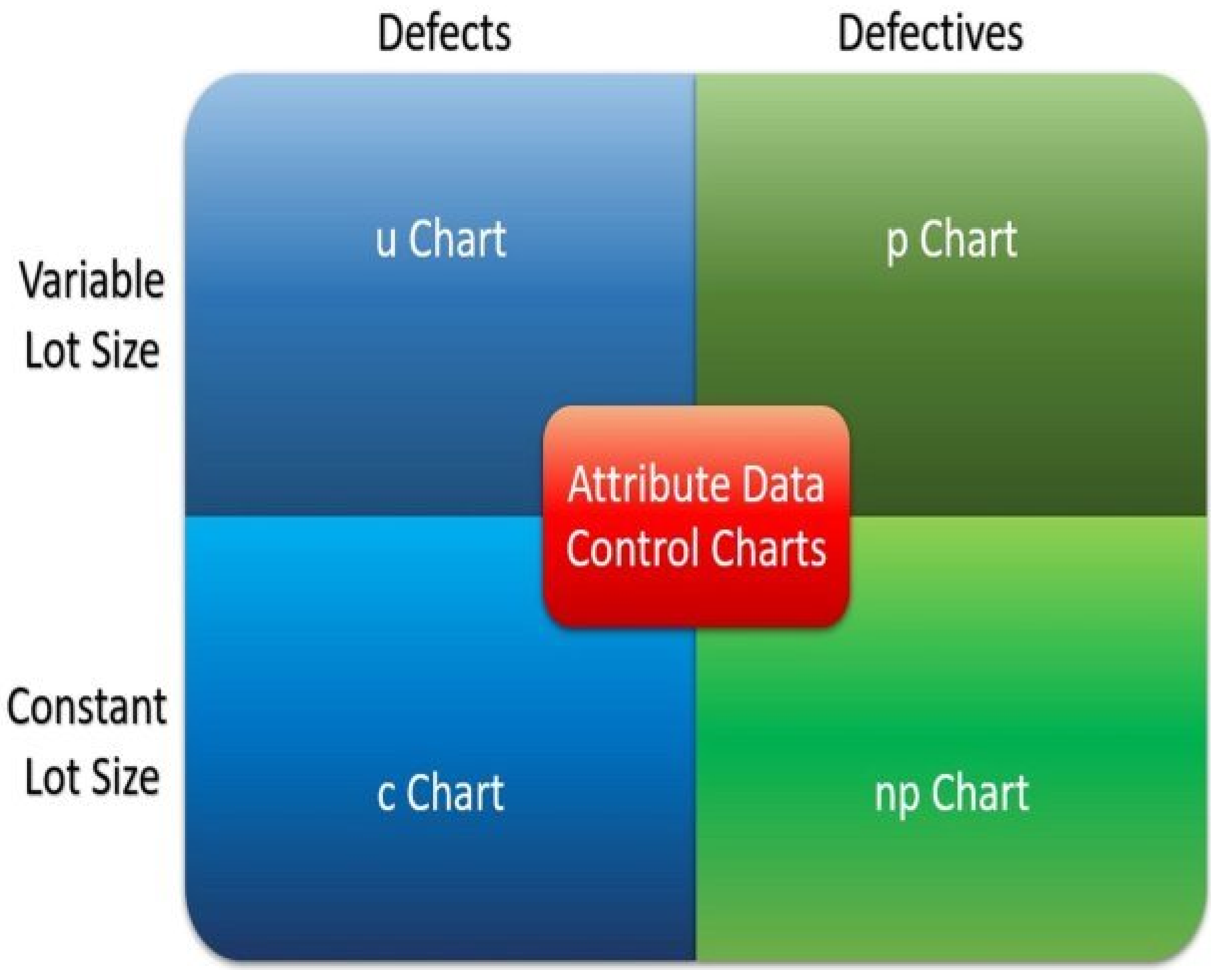
As we have seen earlier, **Defectives** are the non conforming items that have the defects in them.

Of the 4 types of control charts for Attribute Data, 2 are used for Defectives and they are,

• **np Chart**, used when the lot size or the number of the defectives is a constant.

• **p Chart**, used when the lot size or the number of the defectives or is variable.

The following figure depicts the types of Attribute Data Control Charts and in which scenario each is used,



u Chart

u Chart

u Chart is the control chart that can be typically used to count the number of defects per unit.

With the possibility that the number or size of inspection units for which defects are to be counted may vary.

U charts show the number of defects per single unit on the y-axis.

It is important to note that the Y-Axis shows the number of defects per single unit while the X-axis shows the sample group.

The u Chart can be used in scenarios similar to below,

- Monitoring the count of defects per lot of shipped products where the lot size varies.
- Monitoring the count of failed operations in a hospital per day.
- Monitoring the count of delivery slippages per day for an e-commerce business.

Poisson distribution is the basis for the chart.

The formula for the center line on the u Chart is calculated as,

$CL = \sum X_i / \sum n_i$

Where,

X_i = Count of defects in subgroup i.

n_i = Count of items in subgroup i.

The formula for the Upper Control Limit line on the u Chart is calculated as,

$UCL = u + k \sqrt{[u / n_i]}$

The formula for the Lower Control Limit line on the u Chart is calculated as,

$LCL = u - k \sqrt{[u / n_i]}$

Where,

u = Process Mean.

k = Parameter for Test 1; default is 3.

n_i = Size of subgroup i.

The following figure depicts the formulas for u Chart,

Centre Line:

$CL = \sum Xi / \sum ni$

Where,

Xi = Count of defects in subgroup i.

ni = Count of items in subgroup i.

Upper Control Limit:

$UCL = u + k \sqrt{[u / ni]}$

Lower Control Limit:

$LCL = u - k \sqrt{[u / ni]}$

Where,

u = Process Mean.

k = Parameter for Test 1; default is 3.

ni = Size of subgroup i.

Formulae for u Chart

u Chart Case Study

Below table talks about the number of code modules tested per week and the number of defects found. Is the process in control?

The calculations are included in the image using the formulas mentioned earlier.

Week	Modules Tested	Defects Found
1	10	4
2	12	6
3	8	2
4	11	4
5	13	4
6	9	2
7	7	0
8	10	6
9	11	4
10	13	2
11	8	0
12	9	2
13	13	6
14	10	4
15	9	2

Centre Line:

$CL = \sum Xi / \sum ni$

$= (10+12+8+11+13+9+7+10+11+13+8+9+13+10+9) /$

$(4+6+2+4+4+2+0+6+4+2+0+2+6+4+2)$

$= 153 / 48$

$= 3.1875$

Upper Control Limit:

$UCL = u + k \sqrt{[u / ni]}$

$= 3.1875 + 3 \sqrt{[3.1875 / 10]}$

$= 3.1875 + 3*0.56$

$= 4.8675$

Lower Control Limit:

$LCL = u - k \sqrt{[u / ni]}$

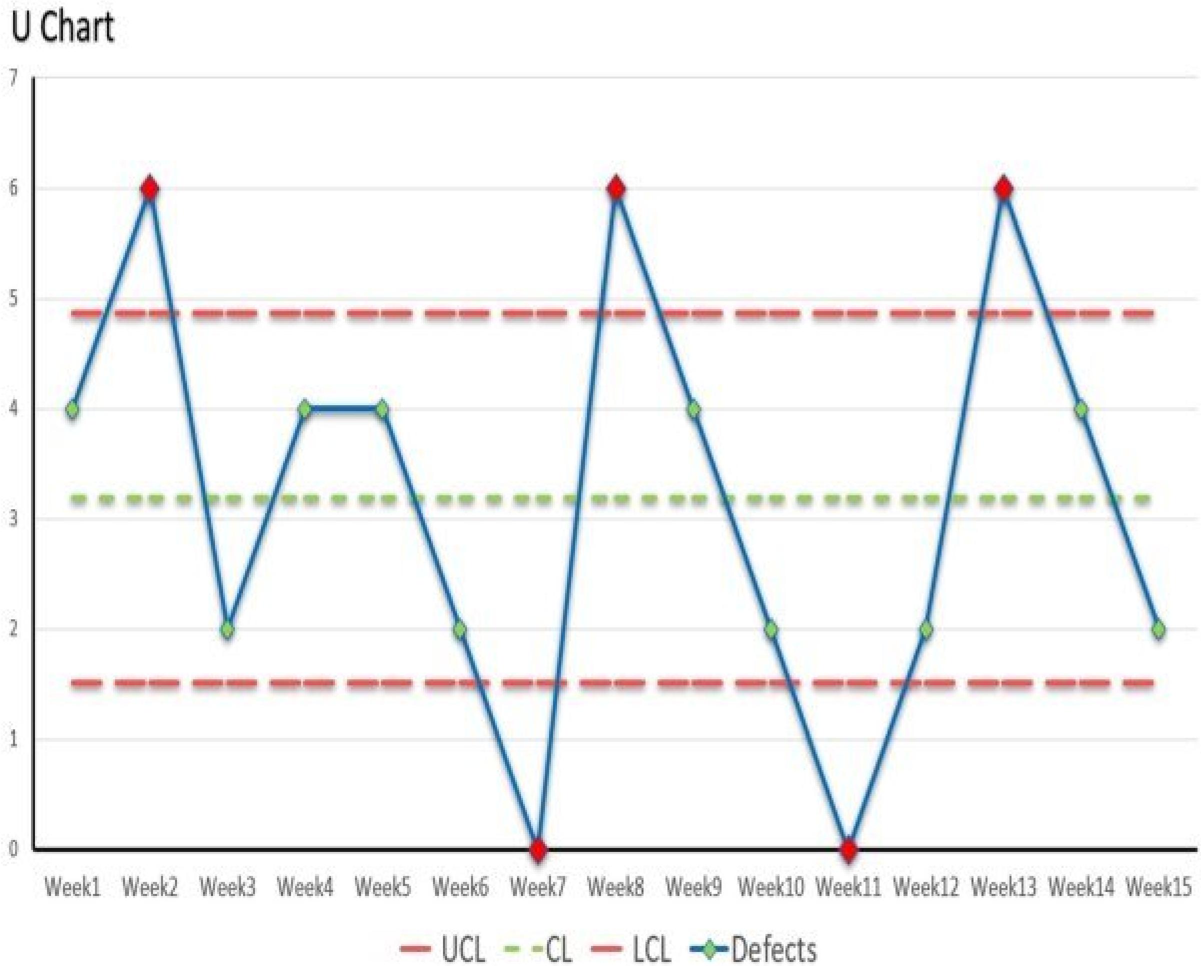
$= 3.1875 - 3 \sqrt{[3.1875 / 10]}$

$= 3.1875 - 3*0.56$

$= 1.5075$

Note:
ni is taken as 10 here
for calculating
LCL and UCL
For simplicity
It is size of first subgroup
Ideally
It should be calculated
for each subgroup

The following figure depicts the u Chart for the case study, and clearly the process is out of control,



c Chart

c Chart

c Chart is the control chart that can be typically used to count the number of defects per unit.

However, it requires a fixed sample size and hence the number or size of inspection units for which defects are to be counted may not vary.

C charts show the number of defects per sample, which often times include more than one unit on the y-axis.

It is important to note that the Y-Axis shows the number of defects per sample while the X-axis shows the sample group.

The c Chart can be used in scenarios similar to below,

- Monitoring the count of defects per lot of shipped products where the lot size remains fixed.
- Monitoring the count of code bugs that must be reworked per delivered code module in a week where the number of delivered modules remain fixed.
- Monitoring the count of faulty measurements of a batch of fixed number of products produced per day.

Similar to the u Chart, Poisson distribution is the basis for the chart.

The formula for the center line on the c Chart is calculated as,

$CL = \sum X_i / m$

Where,

X_i = Count of defects in subgroup i.

m = Count of subgroups.

The formula for the Upper Control Limit line on the c Chart is calculated as,

$UCL = c + k \sqrt{c}$

The formula for the Lower Control Limit line on the c Chart is calculated as,

$LCL = c - k \sqrt{c}$

Where,

c = Process Mean.

k = Parameter for Test 1; default is 3.

The following figure depicts the formulas for c Chart,

Centre Line:

$CL = \sum X_i / m$

Where,

X_i = Count of defects in subgroup i.

m = Count of subgroups.

Upper Control Limit:

$UCL = c + k \sqrt{c}$

Lower Control Limit:

$LCL = c - k \sqrt{c}$

Where,

c = Process Mean.

k = Parameter for Test 1; default is 3.

Formulae for c Chart

c Chart Case Study

Below table talks about the number of defects caught by the testing team against the code developed per week. Is the process in control?

The calculations are included in the image using the formulas mentioned earlier.

Week	Defects
1	4
2	6
3	2
4	4
5	4
6	2
7	0
8	6
9	4
10	2
11	0
12	2
13	6
14	4
15	2

Centre Line:

CL = ∑Xi / m

= (4+6+2+4+4+2+0+6+4+2+0+2+6+4+2) /15

= 48 / 15

= 2.53

Upper Control Limit:

UCL = c + k √ c

= 2.53 + 3 √ 2.53

= 2.53 + 4.77

= 7.3

Lower Control Limit:

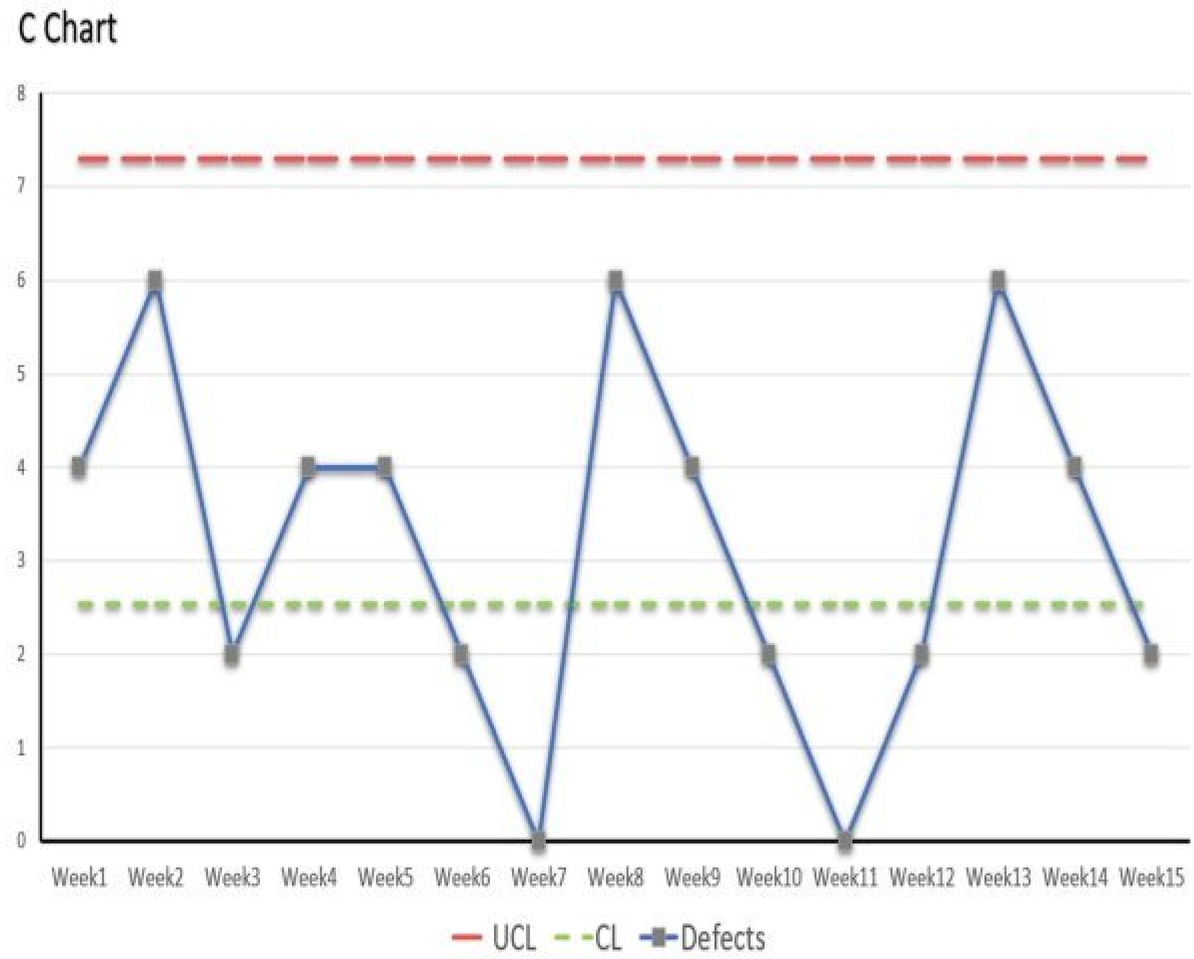
LCL = c - k √ c

= 2.53 - 3 √ 2.53

= 2.53 - 4.77

= -2.24 (Can be ignored since negative)

The following figure depicts the c Chart for the case study, and clearly the process is in control,



p Chart

p Chart

p Chart is the control chart that can be typically used to monitor proportion of defectives.

With the possibility that the sample size and hence the number or size of inspection units of which defectives are to be counted may vary.

p charts show the number of defects per single unit on the y-axis.

It is important to note that the Y-Axis shows the number of defectives per sample while the X-axis shows the sample group.

The p Chart can be used in scenarios similar to below,

- Monitoring the count of defective products returned per lot of shipped products where the lot size varies.
- Monitoring the count of defective modules developed per day out of varying number of modules developed.
- Monitoring the count of reopened cases with the help desk where number of cases raised varies per day.

Binomial distribution is the basis for the chart.

The formula for the center line on the p Chart is,

$CL = \sum X_i / \sum n_i$

Where,

X_i = Count of defectives in subgroup i.

n_i = Count of items in subgroup i.

The formula for the Upper Control Limit line is,

$UCL = p + k \sqrt{[\ p(1-p) / n_i\ ]}$

The formula for the Lower Control Limit line is,

$LCL = p - k \sqrt{[\ p(1-p) / n_i\ ]}$

Where,

p = Process Proportion.

k = Parameter for Test 1; default is 3.

n_i = Count of items in subgroup i.

The following figure depicts the formulas for p Chart,

Centre Line:

$CL = \sum X_i / \sum n_i$

Where,

X_i = Count of defectives in subgroup i.

n_i = Count of items in subgroup i.

Upper Control Limit:

$UCL = p + k \sqrt{[\, p(1-p) / n_i \,]}$

Lower Control Limit:

$LCL = p - k \sqrt{[\, p(1-p) / n_i \,]}$

Where,

p = Process Proportion.

k = Parameter for Test 1; default is 3.

n_i = Count of items in subgroup i.

Formulae for p Chart

p Chart Case Study

Below table talks about the number of products shipped per week by an e-commerce business, and the proportion of products returned due to defects. Is the process in control?

The calculations are included in the image using the formulas mentioned earlier.

Week	Products Shipped	Proportion Returned
1	100	0.22
2	100	0.33
3	100	0.24
4	100	0.20
5	100	0.18
6	100	0.24
7	100	0.24
8	100	0.29
9	100	0.18
10	100	0.27
11	100	0.31
12	100	0.26
13	100	0.31
14	100	0.24
15	100	0.22

Centre Line:

$CL = \sum Xi / \sum ni$

$= (22 + 33 + 24 + 20 + 18 + 24 + 24 + 29 + 18 + 27 + 31 + 26 + 31 + 24 + 22) / (100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100)$

$= 373 / 1500$

$= 0.2487 = 25\%$

Upper Control Limit:

$UCL = p + k \sqrt{[\, p(1-p) / ni \,]}$

$= 0.2487 + 3\sqrt{[0.2487(1 - 0.2487) / 100]}$

$= 0.2487 + 3\sqrt{[0.2487 * 0.7513/100]}$

$= 0.2487 + 3 * 0.043 = 0.3777 = 38\%$

Lower Control Limit:

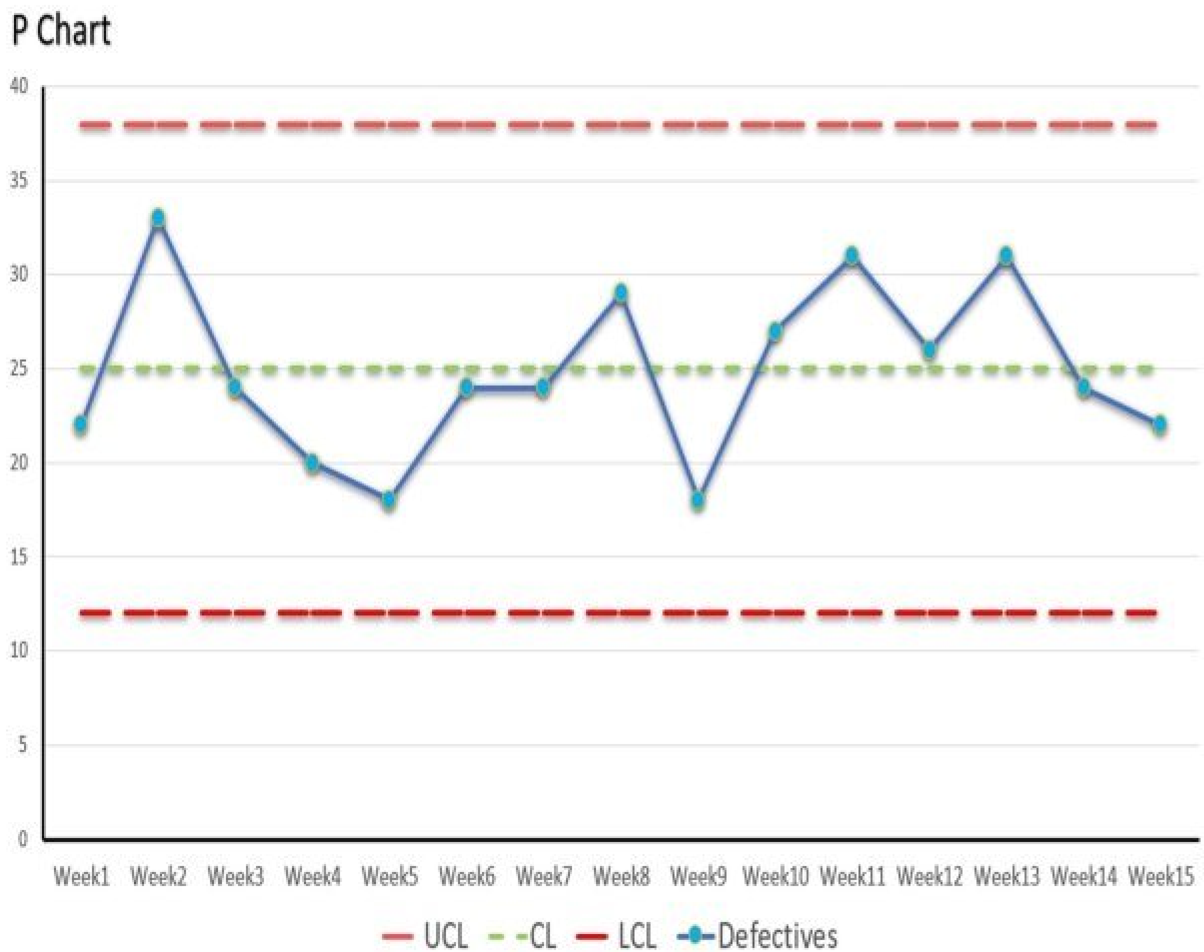
$LCL = u - k \sqrt{[\, u / ni \,]}$

$= 0.2487 - 3\sqrt{[0.2487(1 - 0.2487) / 100]}$

$= 0.2487 - 3\sqrt{[0.2487 * 0.7513/100]}$

$= 0.2487 - 3 * 0.043 = 0.1183 = 12\%$

The following figure depicts the p Chart for the case study, and clearly the process is in control,



np Chart

np Chart

np Chart is the control chart that can be typically used to monitor proportion of defectives.

However, it requires a fixed sample size and hence the number or size of inspection units of which defectives are to be counted may not vary.

np charts show the number of defectives per sample, which often times include more than one unit on the y-axis.

It is important to note that the Y-Axis shows the number of defectives per sample while the X-axis shows the sample group.

The np Chart can be used in scenarios similar to below,

- Monitoring the count of defective products returned per lot of shipped products where the lot size remains fixed.
- Monitoring the count of defective modules developed per day out of fixed number of modules delivered.
- Monitoring the count of reopened cases with the help desk where number of cases raised per day remains fixed.

Binomial distribution is the basis for the chart.

The formula for the center line on the np Chart is calculated as,

CL = n_ip

Where,

n_i = Count of items in subgroup i.

p = Process proportion.

The formula for the Upper Control Limit line is,

UCL = nip + k √ [nip(1-nip)]

The formula for the Lower Control Limit line is,

LCL = nip - k √ [nip(1-nip)]

Where,

p = Process Proportion.

k = Parameter for Test 1; default is 3.

ni = Count of items in subgroup i.

The following figure depicts the formulas for np Chart,

Centre Line:

$CL = n_i p$

Where,

n_i = Count of items in subgroup i.

p = Process proportion.

Upper Control Limit:

$UCL = n_i p + k \sqrt{[\, n_i p(1-p) \,]}$

Lower Control Limit:

$LCL = n_i p - k \sqrt{[\, n_i p(1-p) \,]}$

Where,

p = Process Proportion.

k = Parameter for Test 1; default is 3.

n_i = Count of items in subgroup i.

Formulae for np Chart

np Chart Case Study

Below table talks about the number of products shipped and returned due to defects per week. Is the process in control?

The calculations are included in the image using the formulas mentioned earlier.

Week	Products Shipped	Products Returned
1	100	8
2	100	7
3	100	12
4	100	5
5	100	18
6	100	2
7	100	10
8	100	16
9	100	14
10	100	6
11	100	7
12	100	8
13	100	6
14	100	10
15	100	16

Centre Line:

CL = nip

= (8 + 7 + 12 + 5 + 18 + 2 + 10 + 16 + 14 + 6 + 7 + 8 + 6 + 10 + 16) / 15

= 145 / 15

= 9.67

Upper Control Limit:

UCL = nip + k √ [nip(1-p)]

= 9.67 + 3√ [9.67(1- 0.097)]

= 9.67 + 3√8.73201

= 9.67 + 8.86 = 18.53

Lower Control Limit:

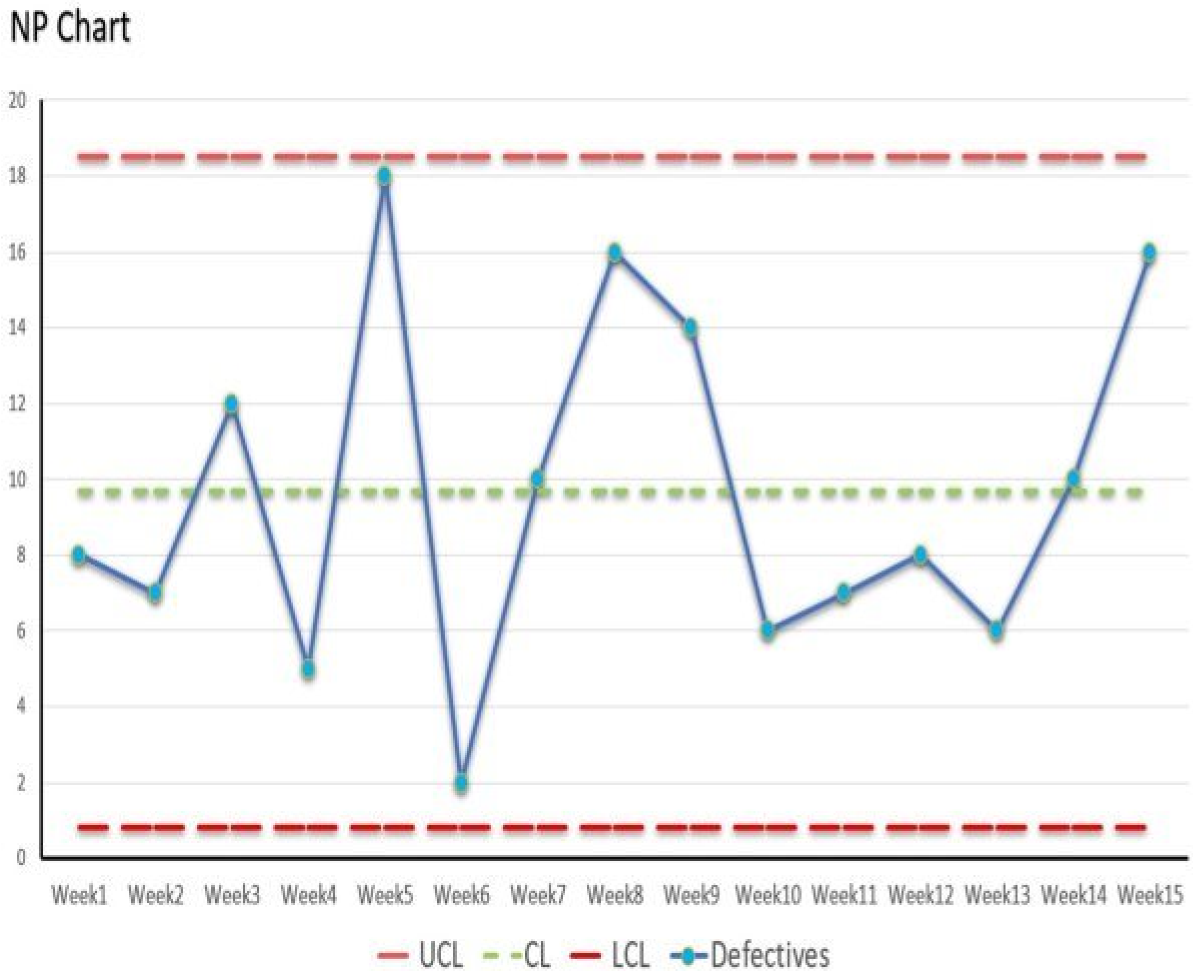
LCL = nip - k √ [nip(1-p)]

= 9.67 - 3√ [9.67(1- 0.097)]

= 9.67 - 3√8.73201

= 9.67 - 8.86 = 0.81

The following figure depicts the np Chart for the case study, and clearly the process is in control,



Control Charts for Continuous Data

Control Chart for Continuous Data

As we have seen earlier, **Continuous Data Type** is the type of numerical data that can be measured and follows the Continuous Distribution.

We can use control charts for the Continuous Data based on the size of the **Subgroup**.

There are 3 types Continuous Data Control Charts,

- **IMR Chart**
- **XR Chart**
- **XS Chart**

Subgroup and Subgroup Size

Subgroups in the simplest terms are subsets of a group, which can themselves stand as an independent group.

Subgroup Size in the simplest terms are number of subsets of a group, which can themselves stand as an independent group.

Statistically **Subgroup Size** is the number of outcomes in each period of data collection.

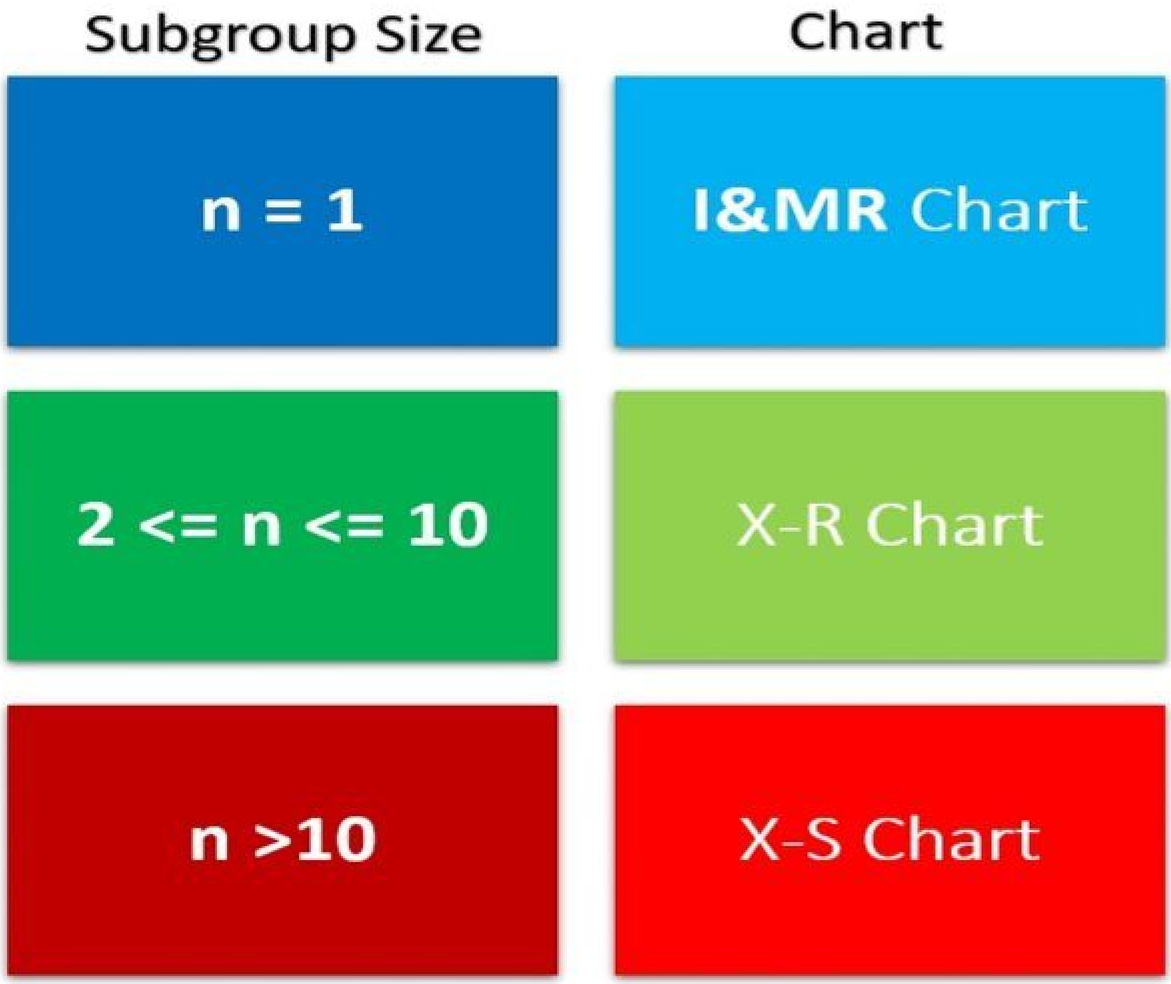
For example, if data is randomly collected for 5 employees from each shift, i.e. Morning, Afternoon, and Night, on a particular day, then the number of subgroups is 3(the shifts Morning, Afternoon, Night), each with a Subgroup size of 5(the 5 employees in each shift).

Control Chart based on Subgroup Size

Of the 3 types of Control Charts used for Continuous Data,

- **IMR Chart** is used for subgroup size equal to 1.
- **XR Chart** is used for subgroup size between to 2 and 10.
- **XS Chart** is used for subgroup size greater than 10.

The following figure depicts the types of Continuous Data Control Charts and in which scenario each is used,



IMR Control Chart

Individual and Moving Range Chart

Individual and Moving Range Chart is the control chart that can be typically used to monitor performance of a process in terms of its individual performance readings and its moving range.

Hence, it requires a subgroup size is 1.

The IMR consists of two charts,

- **Individuals Chart**, the control chart for single series individual readings.

- **Moving Range Chart**, the control chart for the moving range which is the absolute difference between two successive data values of the series.

The following figure depicts the formulas for Individuals Chart,

Centre Line:

$CLx = \bar{X} = \sum Xi/n$

Where,

n = Count of items.

Xi = Value of individual items.

Upper Control Limit:

$UCLx = \bar{X} + D\,\bar{R}$

Lower Control Limit:

$LCLx = \bar{X} - D\,\bar{R}$

Where,

$\bar{R}$ = Moving Range Average.

D = Parameter; use 2.66.

Formulae for Individuals Chart

The following figure depicts the formulas for Moving Range Chart,

Centre Line:

$CLr = \bar{R} = \sum R_{i+1}/n-1$

Where,

n = Count of items.

R_{i+1} = Value of moving range, calculated as $X_{i+1} - X_i$.

Upper Control Limit:

$UCLr = E\bar{R}$

Lower Control Limit:

$LCLr = 0$

Where,

$\bar{R}$ = Moving Range Average.

E = Parameter; use 3.27.

Formulae for Moving Range Chart

IMR Chart Case Study

Below table talks about the times taken for download of media files from an internet movies and entertainment site. Is the process in control?

The calculations are included in the image using the formulas mentioned earlier.

Week	Download Time	Moving Range
1	3.5	
2	2.4	0.9
3	4.1	1.7
4	2.8	1.3
5	3	0.2
6	4.7	1.7
7	1.2	2.5
8	0.9	0.3
9	2.5	1.6
10	3.1	0.6
11	3.6	0.5
12	4.1	0.5
13	3.8	0.3
14	2.5	1.3
15	2.8	0.3

Individuals:

Centre Line:

$\bar{X}$= ∑Xi/n = (3.5 + 2.4 + 4.1 + 2.8 + 3 + 4.7 + 1.2 + 0.9 + 2.5 + 3.1 + 3.6 + 4.1 + 3.8 + 2.5 + 2.8) / 15

= 3

Control Limits:

UCLx = $\bar{X}$ + D $\bar{R}$

= 3 + 2.66*0.98

= 5.6

LCLx = $\bar{X}$ - D $\bar{R}$

= 3 - 2.66*0.98

= 0.4

Moving Range:

Centre Line:

$\bar{R}$= ∑Ri+1/n-1 = (0.9 + 1.7 + 1.3 + 0.2 + 1.7 + 2.5 + 0.3 + 1.6 + 0.6 + 0.5 + 0.5 + 0.3 + 1.3 + 0.3) / 14

= 0.98

Control Limits:

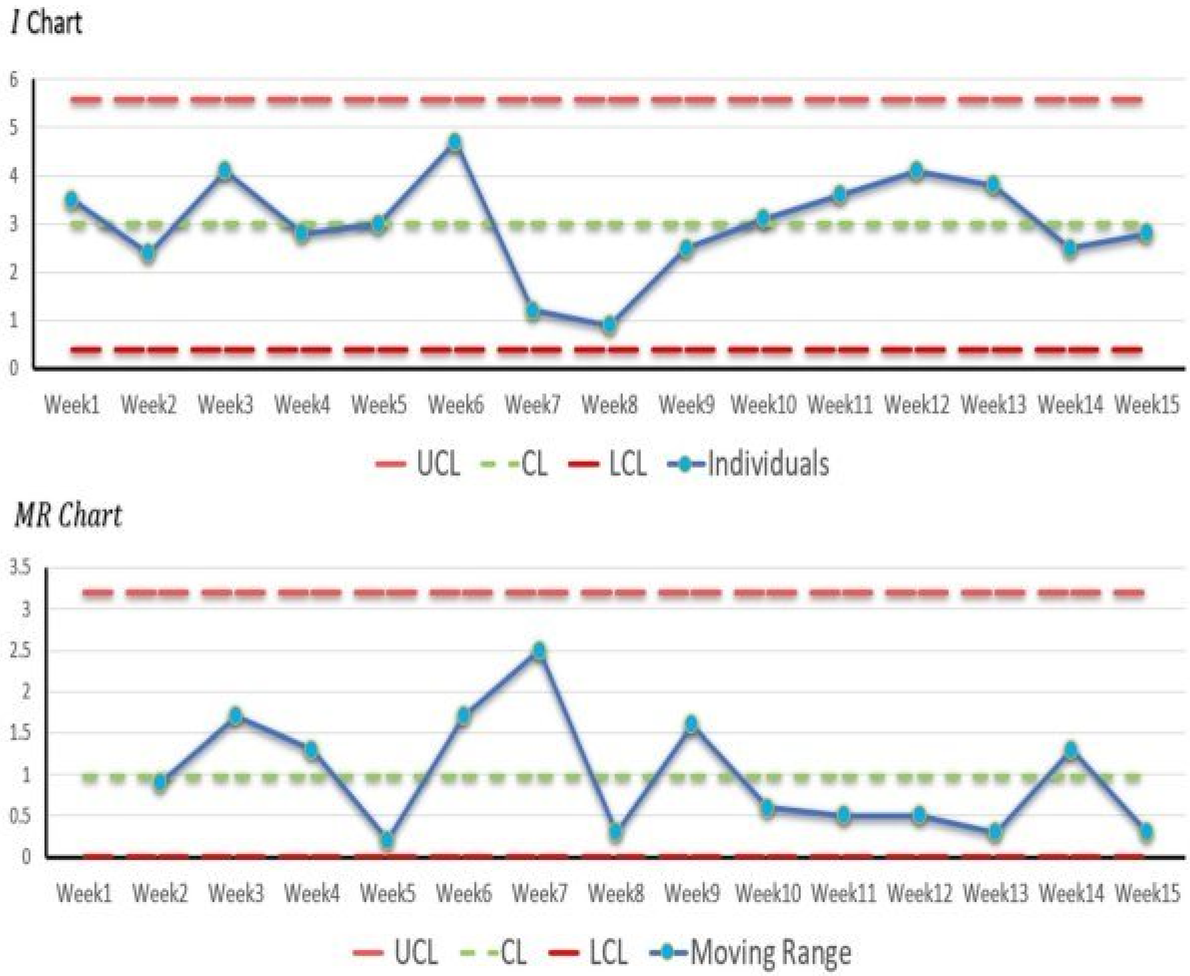
UCLr = E $\bar{R}$

= 3.27*0.98

= 3.2046

LCLr = 0

The following figure depicts the IMR Charts for the case study, and clearly the process is in control,



XR Control Chart

XR Chart

XR Chart is the control chart that can be typically used to monitor performance of a process in terms of means and range of the readings taken in subgroups.

Typically subgroups are of size between 2 and 10.

It consists of two charts,

• **X Chart**, the control chart for means of the items of the subgroups.

• **R Chart**, the control chart for the range of the items in the subgroups.

The following figure depicts the formulas for X Chart,

Data Line:

Subgroup Average:

$\overline{X} = \sum Xi/n$

Centre Line:

Overall Process Average

$\overline{\overline{X}} = \sum \overline{X}i/m$

Where,

n = Count of items in subgroups.

m = Count of Subgroups.

Xi = ith measure in subgroup.

$\overline{X}$i= Average of ith subgroup.

Upper Control Limit:

$UCLx = \overline{\overline{X}} + A_2\,\overline{R}$

Lower Control Limit:

$LCL\ x = \overline{\overline{X}} - A_2\,\overline{R}$

Where,

$\overline{R}$= Average Range.

A_2 = Parameter depending on size of Subgroup.

Formulae for $\overline{X}$ Chart

The following figure depicts the formulas for R Chart,

Data Line:

Subgroup Range:

$\overline{R}$ = Xmax - Xmin

Centre Line:

Overall Process Range

$\overline{\overline{R}} = \sum \overline{R}i/m$

Where,

m = Count of Subgroups.

Xmax = max measure in subgroup.

Xmin = min measure in subgroup.

$\overline{R}$i= Range of ith subgroup.

Upper Control Limit:

UCLr = $D_4 \overline{R}$

Lower Control Limit:

LCLr = $D_3 \overline{R}$

Where,

$\overline{R}$= Average Range.

D_3, D_4 = Parameters depending on size of Subgroup.

Formulae for R Chart

XR Chart Case Study

Below table talks about the times taken for download of media files from an internet movies and entertainment site once in every six hour for 15 days. Is the process in control?

The calculations are included in the image using the formulas mentioned earlier.

Sub group	1	2	3	4	$\bar{X}$	$\bar{R}$
1	3.5	2.8	2.4	2.8	2.9	1.1
2	2.4	4.1	3.5	2.5	3.1	1.7
3	4.1	3.5	2.8	3.8	3.6	1.3
4	2.8	2.4	4.1	4.1	3.4	1.7
5	3	4.7	3.8	3.6	3.8	1.7
6	4.7	3	3	3.1	3.5	1.7
7	1.2	0.9	4.1	2.5	2.2	3.2
8	0.9	1.2	2.5	0.9	1.4	1.6
9	2.5	3.1	2.8	1.2	2.4	1.9
10	3.1	2.5	2.6	4.7	3.2	2.2
11	3.6	2.8	3.1	3	3.1	0.8
12	4.1	2.5	1.2	2.8	2.7	2.9
13	3.8	4.1	0.9	4.1	3.2	3.2
14	2.5	3.8	4.1	2.4	3.2	1.7
15	2.8	3	2.4	3.5	2.9	1.1

$\bar{X}$ Chart:

Centre Line:

$\bar{\bar{X}} = \sum \bar{X}i/m = (2.9 + 3.1 + 3.6 + 3.4 + 3.8 + 3.5 + 2.2 + 1.4 + 2.4 + 3.2 + 3.1 + 2.7 + 3.2 + 3.2 + 2.9) / 15$

$= 2.9$

Control Limits:

$UCLx = \bar{\bar{X}} + A2\,\bar{R}$

$= 2.9 + 0.729*1.9$

$= 2.9 + 1.4 = 4.3$

$UCLx = \bar{\bar{X}} - A2\,\bar{R}$

$= 2.9 - 0.729*1.9$

$= 2.9 - 1.4 = 1.5$

$\bar{R}$ Chart:

Centre Line:

$\bar{\bar{R}} = \sum \bar{R}i/m = (1.1 + 1.7 + 1.3 + 1.7 + 1.7 + 1.7 + 3.2 + 1.6 + 1.9 + 2.2 + 0.8 + 2.9 + 3.2 + 1.7 + 1.1) / 15$

$= 1.9$

Control Limits:

$UCLr = D4\,\bar{R}$

$= 2.282*1.9$

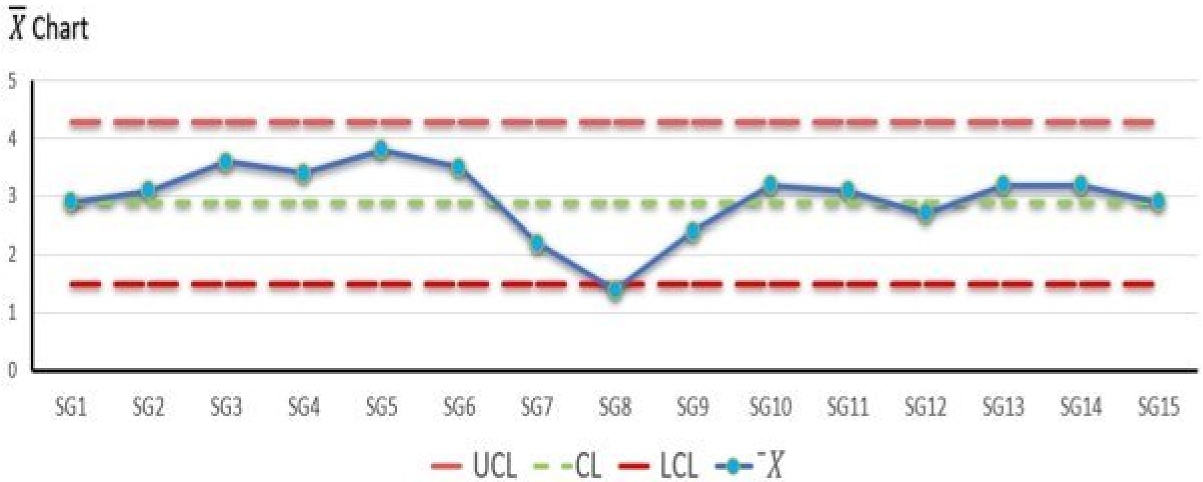
$= 4.3$

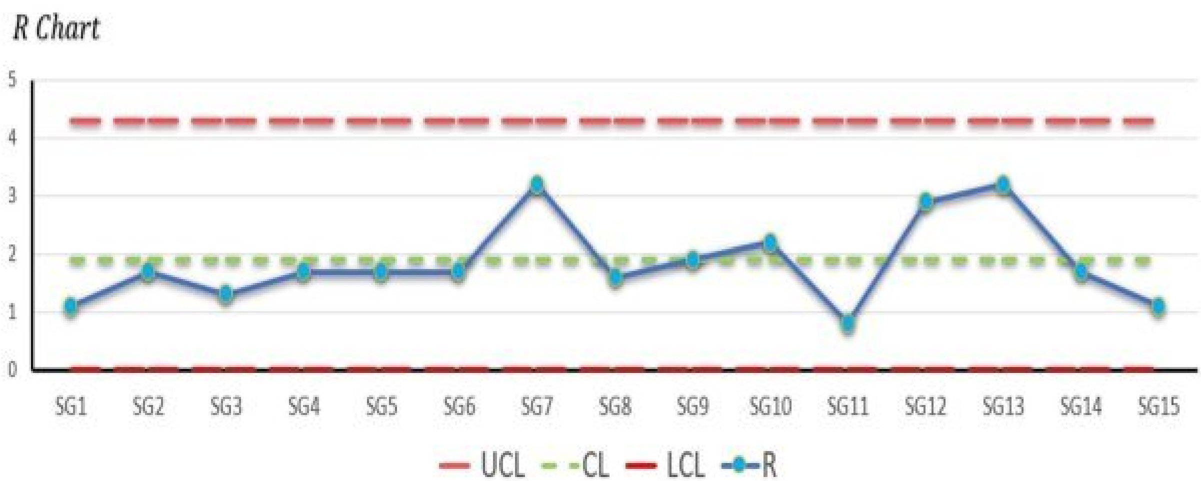
$LCLr = D3\,\bar{R}$

$= 0*1.9$

$=0$

The following figure depicts the XR Charts for the case study, and clearly the process is in control,





XS Control Chart

XS Chart

XS Chart is the control chart that can be typically used to monitor performance of a process in terms of means and Standard Deviation of the readings taken in subgroups.

Typically subgroups are of size more than 10; the reason being Standard Deviation is a better estimate of the variation in large subgroups than the range.

It consists of two charts,

- **X Chart**, the control chart for means of the items of the subgroups.

- **S Chart**, the control chart for the standard deviation of the items in the subgroups.

The following figure depicts the formulas for X Chart,

Data Line:

Subgroup Average:

$\overline{X} = \sum Xi/n$

Centre Line:

Overall Process Average

$\overline{\overline{X}} = \sum \overline{X}i/m$

Where,

n = Count of items in Subgroups.

m = Count of Subgroups.

Xi = ith measure in Subgroup.

$\overline{X}$i= Average of ith Subgroup.

Upper Control Limit:

$UCLx = \overline{\overline{X}} + A_3 \overline{S}$

Lower Control Limit:

$LCLx = \overline{\overline{X}} - A_3 \overline{S}$

Where,

$\overline{S}$= Average Range.

A3 = Parameter depending on size of Subgroup.

Formulae for $\overline{X}$ Chart

The following figure depicts the formulas for S Chart,

Data Line:

Subgroup Range:

$S = \sqrt{[(X_i - \bar{X})^2 / n-1]}$

Centre Line:

Overall Process Range

$\bar{S} = \sum \bar{S}i/m$

Where,

n = Count of items in Subgroups.

m = Count of Subgroups.

Xi = ith measure in Subgroup.

$\bar{X}$ = Average of items in Subgroup.

$\bar{S}$i = Std Dev of ith Subgroup.

Upper Control Limit:

$UCL_s = B_4 \bar{S}$

Lower Control Limit:

$LCL_s = B_3 \bar{S}$

Where,

$\bar{S}$ = Overall Standard Deviation.

B_3, B_4 = Parameters depending on size of Subgroup.

Formulae for S Chart

XS Chart Case Study

Below table talks about the times taken for download of media files from an internet movies and entertainment site once in every six hour for 15 days. Is the process in control?

The calculations are included in the image using the formulas mentioned earlier.

Sub group	1	2	3	4	$\overline{X}$	S
1	3.5	2.8	2.4	2.8	2.9	0.5
2	2.4	4.1	3.5	2.5	3.1	0.8
3	4.1	3.5	2.8	3.8	3.6	0.6
4	2.8	2.4	4.1	4.1	3.4	0.9
5	3	4.7	3.8	3.6	3.8	0.7
6	4.7	3	3	3.1	3.5	0.8
7	1.2	0.9	4.1	2.5	2.2	1.5
8	0.9	1.2	2.5	0.9	1.4	0.8
9	2.5	3.1	2.8	1.2	2.4	0.8
10	3.1	2.5	2.6	4.7	3.2	1
11	3.6	2.8	3.1	3	3.1	0.3
12	4.1	2.5	1.2	2.8	2.7	1.2
13	3.8	4.1	0.9	4.1	3.2	1.6
14	2.5	3.8	4.1	2.4	3.2	0.9
15	2.8	3	2.4	3.5	2.9	0.5

$\overline{X}$ Chart:

Centre Line:

$\overline{\overline{X}} = \sum \overline{X}i/m$ = (2.9 + 3.1 + 3.6 + 3.4 + 3.8 + 3.5 + 2.2 + 1.4 + 2.4 + 3.2 + 3.1 + 2.7 + 3.2 + 3.2 + 2.9) / 15

= 2.9

Control Limits:

UCLx = $\overline{\overline{X}}$ + A3 $\overline{S}$

= 2.9 + 1.628 * 0.9

= 2.9 + 1.5 = 4.4

UCLx = $\overline{\overline{X}}$ - A3 $\overline{S}$

= 2.9 - 1.628 * 0.9

= 2.9 - 1.5 = 1.4

S Chart:

Centre Line:

$\overline{S} = \sum \overline{S}i/m$ = (0.5 + 0.8 + 0.6 + 0.9 + 0.7 + 0.8 + 1.5 + 0.8 + 0.8 + 1 + 0.3 + 1.2 + 1.6 + 0.9 + 0.5) / 15

= 0.9

Control Limits:

UCLs = B4 $\overline{S}$

= 2.266*0.9

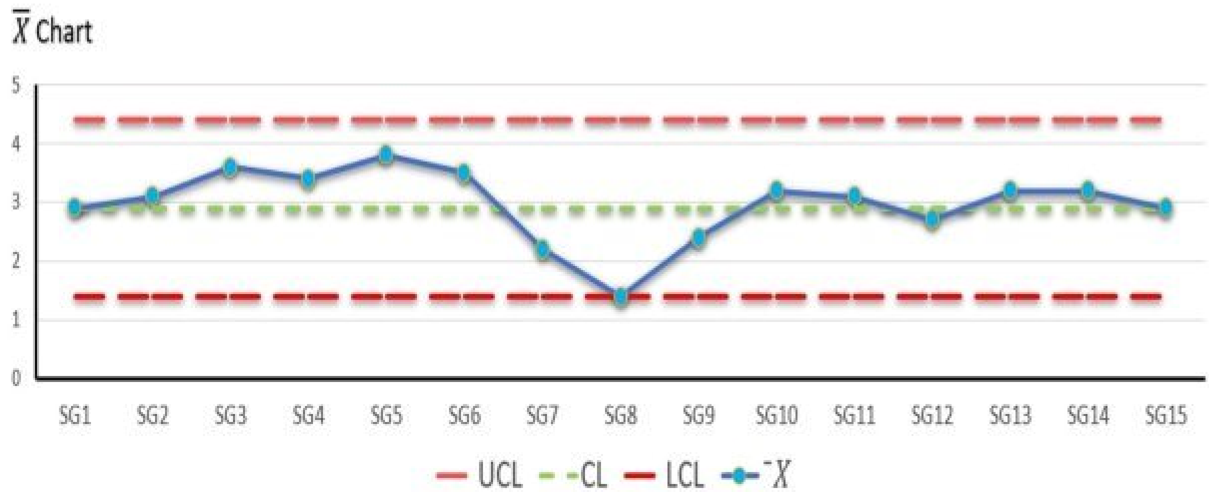
= 2.0

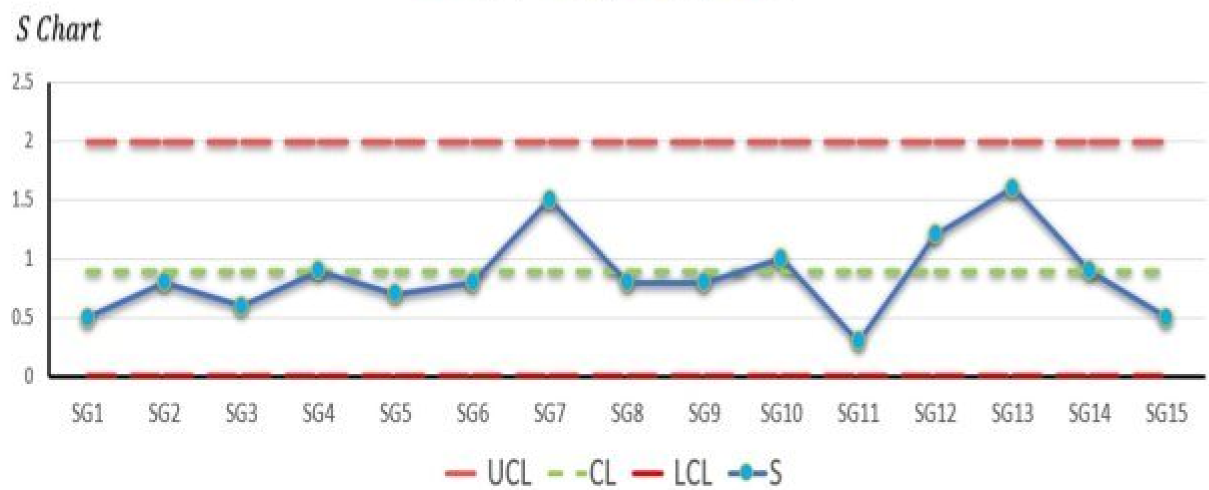
LCLs = B3 $\overline{S}$

= 0*0.9

= 0

The following figure depicts the XS Charts for the case study, and clearly the process is in control,





Author's Note

I thank you for choosing the book, I have presented to you a detailed concept of Control Charts.

We covered all major control charts like c Chart, u Chart, p Chart, np Chart, IMR Chart, XR Chart, and the XS Chart and how to choose a particular chart depending on a given scenario.

We saw the elements of a control chart, and the inferences we can draw from them like Mixtures, Oscillations, Clusters, Trends, Shifts, and Astronomical Points in order to identify special causes.

We also saw concepts of Stability and Capability, Special and Common Causes and the required actions.

And to end the discussion we also saw the relevant case studies for all the charts.

I hope this adds value to you and helps you in your path to achieve success in your Six Sigma journey.

Please leave a review wherever you bought the book, and it will help me in my quest to provide good useful products to you on Lean Six Sigma.

All the very best,

Sumeet Savant

Lean Six Sigma Master Black Belt and Coach