Supplementary material for "A surrogate-assisted constrained optimization evolutionary algorithm by searching multiple kinds of global and local regions"

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Contents

I. Example illustration

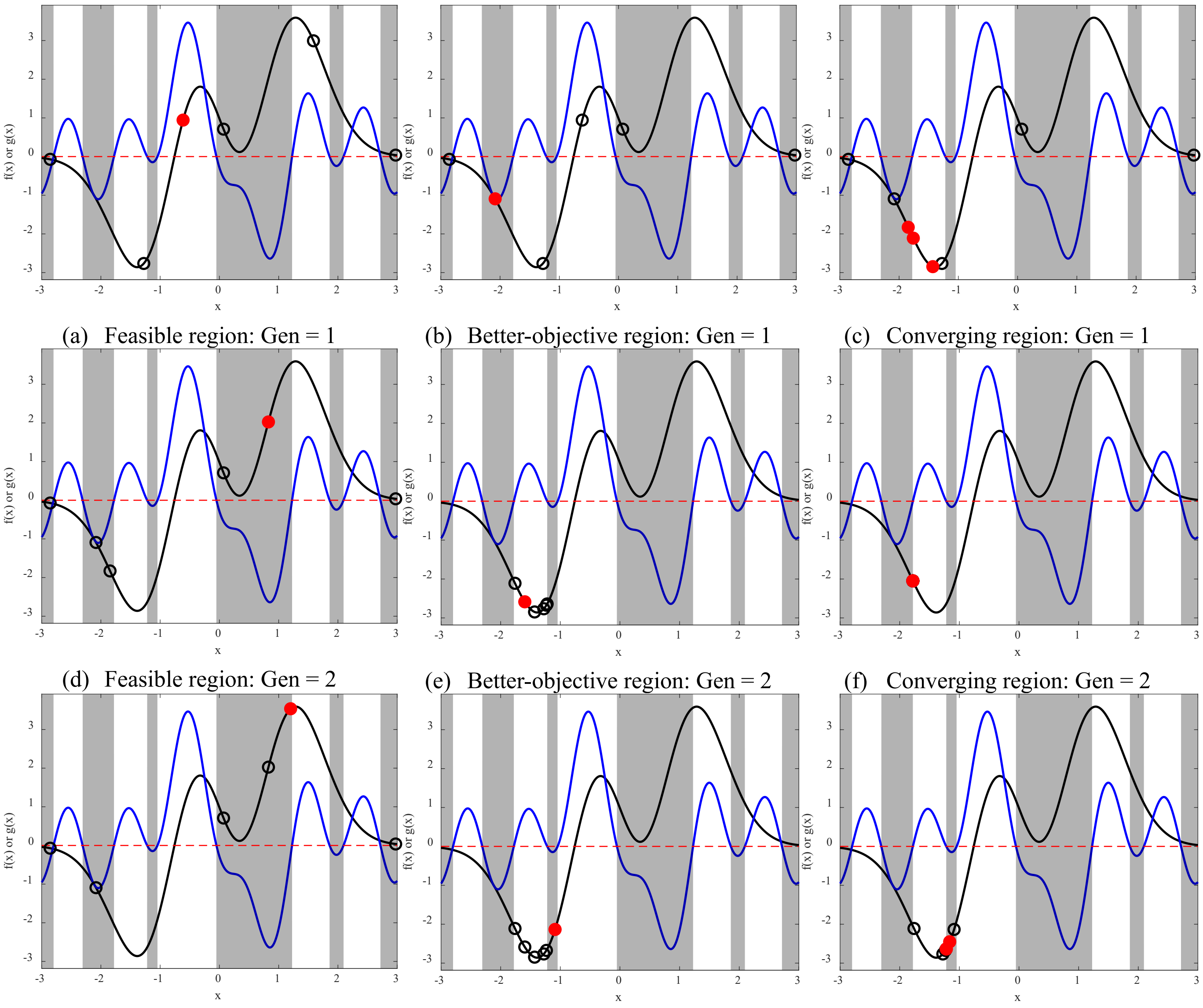
The one-dimensional artificial problem is formulated as in S-Eq. (1) - S-Eq. (2),

$$\begin{aligned} \min \quad & f(\mathbf{x}) = k(x,0) \\ s.t. \quad & g(\mathbf{x}) = k(x-0.2,-1) + \sin(2\pi(x-0.2)) \le 0 \end{aligned} \tag{1}$$

$$k(x,a) = 3(1-x)^2\left(e^{-x^2-(a+1)^2}\right) - 10\left(\frac{x}{5} - x^3 - a^5\right)\left(e^{-x^2-a^2}\right) - \frac{1}{3}e^{-(x+1)^2-a^2} \tag{2}$$

where a is a control parameter and x is the decision variable. The lower and the upper bounds of the decision variable are -3 and 3, respectively.

For MGRLR, the population size is set to 5 and the maximum number of function evaluations is set to 100. In each generation, the numbers of function evaluations for feasible region exploration, better objective exploration, and converging region exploration are 1, 1, and 3 respectively.



(a) Feasible region: Gen = 1 (b) Better-objective region: Gen = 1 (c) Converging region: Gen = 1

(d) Feasible region: Gen = 2 (e) Better-objective region: Gen = 2 (f) Converging region: Gen = 2

(g) Feasible region: Gen = 3 (h) Better-objective region: Gen = 3 (i) Converging region: Gen = 3

Fig S- I The diagram shows the global search process of MGRLR. The feasible region is shown by the gray background. The black and blue solid curves are the objective and constraint functions, respectively. The hollow circles are the parents for generating candidate offspring. The filled circles are the solutions selected for expensive evaluation. "Gen" is the number of generations in the search process. The solutions are shown on the objective function to demonstrate the search process.

Fig S- I (a), (d), and (g) suggest that the parents for exploring the feasible region are distributed in the feasible region. Therefore, diverse candidate offspring distributed in the feasible region can be obtained. When Gen = 1, the constraint surrogate is inaccurate. Therefore, the solution selected for expensive evaluation is predicted to be feasible but is infeasible in fact.

Fig S- I (b), (e), and (h) suggest that the parents for exploring the better-objective region are distributed in the region that contains objective values better than the current best feasible solution. As the best feasible solution becomes better, the better-objective region is adaptively reduced.

Fig S- I (c), (f), and (i) suggest that the parents for exploring the converging region are distributed around the best feasible solution on both the feasible and infeasible sides. When Gen = 2, the solutions are concentrated around a local optimum. However, after a solution with a better objective value is found, the solutions are distributed in the new promising region (Gen = 3).

In addition, the local search locations are the same as the parents in the converging region exploration step. Consequently, the search can benefit from both promising feasible and infeasible solutions.

To further illustrate the search process, the solutions evaluated in the global search are shown in Fig S- II. It can be observed that the solutions in the feasible region exploration step are mainly distributed in the feasible region. The solutions in the better-objective region exploration step are mainly distributed in the region containing promising objective values. The solutions in the converging region exploration step are mainly distributed around the best feasible solution found so far, thereby facilitating convergence. The convergence curve of the best feasible objective value is shown in Fig S- III, which indicates that the proposed MGRLR is efficient.

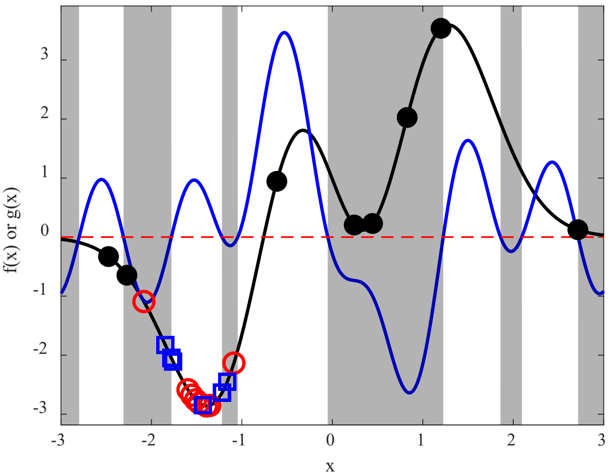


Fig S- II The distribution of the solutions that are expensively evaluated in the global search. The filled circles are the solutions in the feasible region exploration step. The hollow circles are the solutions in the better-objective region exploration step. The squares are the solutions in the converging region exploration step.

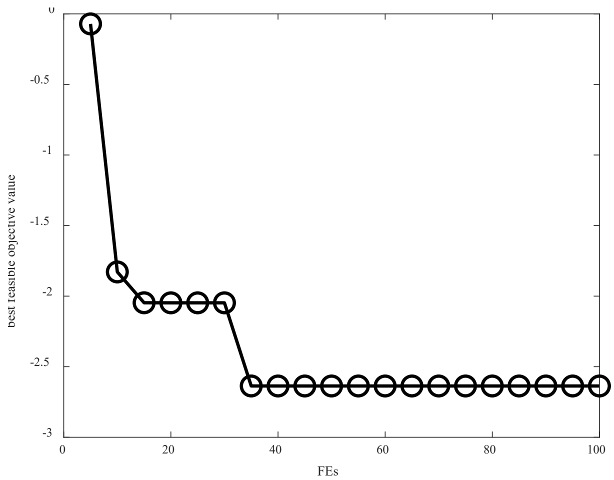


Fig S- III The convergence curve of the objective value of the best feasible solution.

II. Computational Complexity

When considering the same number of expensive function evaluations across different algorithms, the variation in computation time can be attributed to the time complexity of the operators excluding these evaluations. Therefore, our analysis focuses on analyzing the time complexity of the proposed MGRLR by disregarding the expensive function evaluations. To simplify the analysis, we assume the following parameters: the number of decision variable is D, the population size is N, the number of history samples in the archive is N_A, the numbers of samples for training global and local RBF models are respectively N_G and N_L, the number of candidate offspring is N_{pr}, and the number of iterations in SQP is q.

In the global exploration step, the time complexities of training global RBF models, calculating the Euclidean distance of each sample to other samples in the archive, and calculating the Euclidean distance of each candidate offspring to the samples in the archive are $O\left(DNN_G{}^3\right)$, $O\left(DNN_A{}^2\right)$, and $O\left(DNN_{pr}N_A\right)$, respectively. The time complexities of sorting the history samples and the candidate offspring depend on the specific sorting method adopted. The time complexity of the predictions based on the RBF model is $O\left(DNN_{pr}N_G\right)$.

In the local exploitation step, the time complexity of training local RBF models is $O\left(DNN_L{}^3\right)$. In each iteration of SQP, the time complexity is $O\left(D^3\right)$. Therefore, the time complexity of conducting SQP around all of the local search locations is $O\left(qND^3\right)$.

In summary, the total time complexity of MGRLR is $O\left(DN\left(N_G{}^3+N_A{}^2+N_{pr}N_A+N_{pr}N_G+N_L{}^3+qD^2\right)\right)$.

III. Comparison results

The Wilcoxon rank-sum test and Friedman test with a significance level of 0.05 are conducted to analyze the results statistically. The Hommel post-hoc procedure is adopted to adjust the p-value of the Friedman test.

The function error (the difference between the found best feasible objective value and the previously published optimum) is compared on 13 problems in CEC2006.

The feasible objective values found by the algorithms are compared on 6 problems in CEC2010 and 9 problems in CEC2017. For each problem in CEC2010 and CEC2017, the number of dimensions is set to 10 and 30 respectively to test the state-of-the-art algorithms.

The analysis of the components and the parameters of the proposed MGRLR is conducted based on 13 problems in CEC2006.

The best results are highlighted by the gray background.

A Component analysis

TABLE S-I Comparison results of the variants of MGRLR on 13 problems in CEC2006

Means and standard deviations of function errors are reported. If no feasible solution is found in at least one run of an algorithm for a particular problem, the number of runs in which at least one feasible solution was found is given. A large value is assigned to a run with no feasible solution to perform significance tests. The symbols '+ ', '- ', '= ' mean that the compared algorithm is significantly better than, worse than, and competitive with MGRLR.

Problem	MGRLR_FR	MGRLR_NoF	MGRLR_NoB	MGRLR_NoC	MGRLR_NoG	MGRLR_NoR	MGRLR_SL	MGRLR
1	2.77e-15(8.33e-15) =	4.26e-16(1.29e-15) =	7.11e-16(2.86e-15) =	1.42e-16(7.11e-16) =	0.00e+00(0.00e+00) =	6.54e-15(1.39e-14) -	1.18e-13(4.80e-13) -	7.11e-17(3.55e-16)
2	4.41e-01(8.36e-02) -	4.07e-01(9.62e-02) =	4.23e-01(1.22e-01) -	4.60e-01(4.67e-02) -	3.87e-01(6.48e-02) =	3.95e-01(9.66e-02) =	4.27e-01(9.03e-02) -	3.60e-01(8.64e-02)
4	3.49e-12(6.84e-12) -	-2.91e-13(1.46e-12) =	-5.82e-13(1.36e-12) =	2.62e-12(2.47e-12) -	8.73e-13(1.90e-12) -	0.00e+00(1.49e-12) =	1.08e-02(5.26e-02) -	-5.82e-13(2.01e-12)
6	-1.59e-11(2.18e-12) =	-1.64e-11(0.00e+00) =	-1.64e-11(0.00e+00) =	-1.64e-11(0.00e+00) =	(24) -	-1.64e-11(0.00e+00) =	-1.64e-11(0.00e+00) =	-1.64e-11(0.00e+00)
7	2.16e-10(7.31e-10) -	1.54e-06(7.68e-06) =	2.79e-02(1.40e-01) =	2.79e-01(3.49e-01) =	1.19e-14(1.65e-13) +	2.61e-09(8.94e-09) -	1.93e-03(9.64e-03) =	8.03e-13(1.04e-12)
8	4.33e-17(4.60e-18) +	1.10e-02(2.56e-02) -	5.05e-17(9.71e-18) =	5.05e-17(8.85e-18) =	3.26e-02(3.46e-02) -	5.11e-17(9.58e-18) =	5.82e-08(1.99e-07) -	4.72e-17(8.01e-18)
9	1.26e+00(5.54e+00) -	2.65e-08(6.18e-08) =	2.74e-07(1.06e-06) -	1.70e+01(5.26e+01) -	1.17e-09(3.70e-09) +	1.86e-08(5.38e-08) =	1.93e-01(6.06e-01) -	7.04e-09(1.61e-08)
10	3.35e+01(6.46e+01) -	1.51e-06(2.02e-06) -	3.96e-07(4.75e-07) +	2.05e-08(7.97e-08) +	4.58e-10(1.66e-09) +	5.25e-05(5.57e-05) -	4.41e-03(1.85e-02) =	7.02e-07(3.17e-06)
12	6.39e-16(6.79e-16) -	0.00e+00(0.00e+00) =	3.63e-02(3.12e-02) -	0.00e+00(0.00e+00) =	5.37e-03(8.64e-03) -	0.00e+00(0.00e+00) =	3.40e-15(1.70e-14) =	0.00e+00(0.00e+00)
16	9.44e-06(4.72e-05) -	4.63e-15(4.78e-16) -	1.95e-12(9.75e-12) -	4.81e-15(7.55e-16) -	4.17e-15(4.50e-16) =	5.05e-15(6.82e-16) -	2.82e-03(1.02e-02) -	4.17e-15(5.18e-16)
18	8.15e-02(1.41e-01) -	2.58e-01(1.49e-01) -	1.67e-01(1.36e-01) -	7.64e-03(3.82e-02) =	6.11e-02(9.10e-02) +	1.34e-01(1.01e-01) =	1.11e-01(1.50e-01) =	7.78e-02(1.35e-01)
19	1.90e-03(3.80e-03) -	2.05e-02(9.93e-02) -	2.01e-02(1.00e-01) =	1.11e-02(2.64e-02) -	1.73e-07(2.76e-07) +	2.00e-02(1.00e-01) =	1.46e-01(2.82e-01) -	1.72e-05(3.69e-05)
24	4.21e-14(1.66e-14) =	3.50e-14(3.11e-15) -	3.53e-14(5.35e-15) -	3.50e-14(3.36e-15) -	3.32e-14(4.35e-16) =	3.77e-14(7.84e-15) -	3.52e-13(1.29e-12) -	3.38e-14(1.90e-15)
+/-/=	1/9/3	0/6/7	1/6/6	1/6/6	5/4/4	0/5/8	0/8/5	NA
Rank	5.92	4.42	4.65	4.81	2.92	5.23	5.65	2.38
p-value	0.005	0.843	0.441	0.278	1	0.069	0.014	NA

B The influence of how the infeasible solutions with better objective values are selected

To investigate the influence of the strategy used to select promising infeasible solutions, four kinds of methods are implemented within MGRLR. First, the infeasible solutions with better objective values than the best feasible solution are archived in an external archive. Subsequently, one of the following methods is utilized to select a set of solutions as required in MGRLR (assume that the number of required infeasible solutions is K):

1) The infeasible solutions are clustered in the design space with the density-based spatial clustering of applications with noise (DBSCAN) [1] method. Before conducting DBSCAN, the solutions are normalized by using $\mathbf{x}_n = \frac{\mathbf{x} - \mathbf{x}_{\min}}{\mathbf{x}_{\max} - \mathbf{x}_{\min}}$. The elements in $\mathbf{x}_{\min}$ are the corresponding minimum values in each dimension of the solutions in the external archive. The elements in $\mathbf{x}_{\max}$ are the corresponding maximum values. In cases where the minimum value equals the maximum value in a given dimension d, let $\mathbf{x}_{\max}(d) = \mathbf{x}_{\min}(d) + 1$. After the clustering process is completed, all clusters are marked as unselected. Subsequently, the following steps are carried out until the desired number of solutions is selected: a) identifying the unselected cluster with the smallest constraint violation (CV) value, b) counting the number of solutions in this cluster as K_i, and c) all solutions in this cluster are selected and let $K = K - K_i$ if $K_i \le K$; otherwise, randomly selecting K solutions in this cluster. The MGRLR equipped with this strategy is abbreviated as MGRLR_DBC. (A threshold for a neighborhood search radius ε and a minimum number of neighbors *minpts* are required in DBSCAN. In the experiments, $\varepsilon = p \times \sqrt{D}$, *minpts* = 1. p is a parameter used to determine the neighborhood search radius and D is the number of design variables. A smaller p results in more clusters. $p = 0.05, 0.1, 0.2$ are respectively used in the experiments).
2) The infeasible solutions are first normalized using the same normalization method as described in "1)". Then, these solutions are clustered into K clusters using the K-means clustering [2] method in the design space. Following the clustering process, the solution with the smallest CV value is selected from each cluster. The MGRLR equipped with this strategy is abbreviated as MGRLR_KMC.
3) The infeasible solution with the smallest CV value is identified, recorded as $\mathbf{x}_{\min CV}$. Afterward, the infeasible solutions are normalized with the same normalization method as described in "1)". Then, K nearest solutions to $\mathbf{x}_{\min CV}$ are selected (including $\mathbf{x}_{\min CV}$ itself). The MGRLR equipped with this strategy is abbreviated as MGRLR_KN.
4) The infeasible solutions are sorted in ascending order based on CV values. Then, the solutions with K-smallest CV values are selected. The MGRLR equipped with this strategy is abbreviated as MGRLR, which is the algorithm proposed in this paper.

The MGRLR_DBCs with different p values are used to concentrate infeasible solutions around the most promising ones. First, the solution with the smallest CV value is identified, allowing the most promising infeasible solutions to be considered. Second, the solutions within the same cluster (obtained through clustering in the design space) are included by ignoring the CV values. Thus, this approach results in the selected solutions being more concentrated around the most promising solutions.

The MGRLR_KMC compulsorily classifies the infeasible solutions into K clusters, and the most promising infeasible solution in each cluster is selected. Therefore, the diversity of the infeasible solutions is maintained throughout the entire search process.

The MGRLR_KN identifies the solution with the smallest CV value first. The solutions closest to this solution are included. Thus, this approach results in the selected solutions being concentrated around the most promising solution. The diversity of the infeasible solutions is worse than the other three methods.

The MGRLR selects the infeasible solutions by only considering the CV values. In the early search phase, solutions with similar CV ranks may come from different regions of the search space. Hence, the infeasible solutions may exhibit good diversity, allowing for exploration in various regions. As the search progresses, a growing number of solutions with smaller CV values in each local region can be discovered. Consequently, the selected infeasible solutions are expected to converge towards the solution with the smallest CV value, particularly when the global feasible optimum is located on the boundary of the feasible region. In such instances, the search tends to exploit the local region that encompasses the solution with the smallest CV value.

In the experiments, each of the aforementioned methods is conducted in the following steps: 1) select a subset of the promising infeasible solutions as parents to generate candidate offspring during the constrained optimization step; 2) select a subset of the promising infeasible solutions as training samples for surrogate models in the constrained optimization step; 3) choose a subset of the promising infeasible solutions as local search locations.

The results are given in Table S-II.

TABLE S-II The comparison results of MGRLR with different methods for selecting the infeasible solutions on 13 problems in CEC2006

Problem	MGRLR_DBC (p = 0.05)	MGRLR_DBC (p = 0.1)	MGRLR_DBC (p = 0.2)	MGRLR_KMC	MGRLR_KN	MGRLR
1	1.42e-16(4.92e-16)=	5.68e-16(2.22e-15)=	2.13e-16(5.89e-16)=	1.21e-15(1.68e-15)-	4.97e-16(1.58e-15)=	7.11e-17(3.55e-16)
2	4.57e-01(7.22e-02)-	4.59e-01(6.72e-02)-	4.21e-01(7.81e-02)-	4.44e-01(8.90e-02)-	4.23e-01(9.33e-02)-	3.60e-01(8.64e-02)
4	-1.02e-12(1.97e-12)=	-8.73e-13(1.90e-12)=	-1.46e-13(1.28e-12)=	2.18e-12(9.22e-12)=	1.46e-13(1.65e-12)=	-5.82e-13(2.01e-12)
6	-1.29e-11(1.75e-11)=	-1.64e-11(0.00e+00)=	-1.64e-11(0.00e+00)=	-1.64e-11(0.00e+00)=	-1.64e-11(0.00e+00)=	-1.64e-11(0.00e+00)
7	3.82e-11(1.89e-10)=	1.16e-13(4.39e-13)+	1.91e-12(6.33e-12)=	2.76e-09(1.02e-08)=	4.49e-13(9.30e-13)=	8.03e-13(1.04e-12)
8	5.11e-17(1.04e-17)=	5.66e-17(9.75e-18)-	5.44e-17(6.85e-18)-	5.77e-17(1.31e-17)-	4.83e-17(1.14e-17)=	4.72e-17(8.01e-18)
9	1.01e-08(1.19e-08)-	1.20e-08(1.88e-08)-	6.76e-09(9.34e-09)+	2.67e-07(3.68e-07)-	1.01e-08(2.39e-08)=	7.04e-09(1.61e-08)
10	1.55e-07(4.67e-07)+	5.93e-08(1.30e-07)=	7.46e-08(1.13e-07)=	6.06e-03(2.93e-02)-	9.54e-08(2.50e-07)=	7.02e-07(3.17e-06)
12	0.00e+00(0.00e+00)=	0.00e+00(0.00e+00)=	0.00e+00(0.00e+00)=	0.00e+00(0.00e+00)=	0.00e+00(0.00e+00)=	0.00e+00(0.00e+00)
16	2.61e-14(1.08e-13)=	4.34e-15(5.66e-16)=	4.27e-15(5.18e-16)=	9.15e-10(3.16e-09)-	4.34e-15(8.06e-16)=	4.17e-15(5.18e-16)
18	6.05e-02(1.04e-01)=	1.06e-01(1.21e-01)=	1.55e-01(1.73e-01)=	7.86e-03(3.82e-02)=	1.59e-01(1.16e-01)-	7.78e-02(1.35e-01)
19	1.06e-04(4.46e-04)=	2.83e-05(5.20e-05)=	1.41e-05(2.54e-05)=	2.29e-05(3.32e-05)-	3.15e-05(5.69e-05)-	1.72e-05(3.69e-05)
24	3.35e-14(1.01e-15)=	3.38e-14(9.59e-16)-	3.44e-14(2.01e-15)-	4.95e-14(1.22e-14)-	3.61e-14(4.39e-15)-	3.38e-14(1.90e-15)
+/-/=	1/2/10	1/4/8	1/3/9	0/8/5	0/4/9	NA
Rank	3.12	3.38	3.42	5.12	3.65	2.31
p-value	0.849	0.620	0.583	6.26e-04	0.366	NA

C The influence of how the surrogates are constructed

The influence of the surrogate model utilized in MGRLR is investigated by employing four types of surrogate models. The employed surrogate models are as follows:

1) The Gaussian process model (also known as Kriging model) is adopted. The DACE toolbox [3] is used to obtain the surrogate models. The Gaussian correlation function is adopted. The MGRLR with this type of surrogate models is abbreviated as MGRLR_Kriging.
2) The Radial basis function network (RBF) with a cubic basis function is used as the surrogate technique (without a linear polynomial tail) [4]. The MGRLR with this type of surrogate models is abbreviated as MGRLR_ NoP.
3) The RBF with a thin plate spline basis function, $\varphi(r) = r^2 \ln r$, and a linear polynomial tail is used as the surrogate technique [4]. The MGRLR with this type of surrogate models is abbreviated as MGRLR_ TPS.
4) The RBF with a cubic basis function and a linear polynomial tail is used as the surrogate technique [5-6], which is used in this paper. The MGRLR with this type of surrogate models abbreviated as MGRLR.

The results are given in Table S-III.

TABLE S-III The comparison results of MGRLR with different surrogates on 13 problems in CEC2006

Problem	MGRLR_Kriging	MGRLR_NoP	MGRLR_TPS	MGRLR
1	1.86e-08(3.28e-08)-	2.34e-01(6.77e-01)-	4.26e-16(1.29e-15)=	7.11e-17(3.55e-16)
2	5.07e-01(6.93e-02)-	4.25e-01(9.33e-02)-	4.62e-01(7.51e-02)-	3.60e-01(8.64e-02)
4	1.17e-06(6.17e-07)-	-1.16e-12(2.03e-12)=	2.62e-12(3.86e-12)-	-5.82e-13(2.01e-12)
6	6.78e-06(2.44e-05)-	-1.64e-11(0.00e+00)=	-1.64e-11(0.00e+00)=	-1.64e-11(0.00e+00)
7	3.82e-03(3.48e-03)-	2.49e-02(8.68e-02)-	1.40e-01(2.76e-01)-	8.03e-13(1.04e-12)
8	8.83e-17(2.08e-17)-	5.05e-17(8.85e-18)=	5.94e-17(1.30e-17)-	4.72e-17(8.01e-18)
9	6.81e-03(8.28e-03)-	5.37e-04(5.88e-04)-	4.93e-03(1.46e-02)-	7.04e-09(1.61e-08)
10	1.84e+01(3.92e+01)-	5.24e+01(5.51e+01)-	9.19e-04(3.95e-03)-	7.02e-07(3.17e-06)
12	0.00e+00(0.00e+00)=	3.91e-15(1.96e-14)=	0.00e+00(0.00e+00)=	0.00e+00(0.00e+00)
16	2.12e-07(1.06e-06)-	1.31e-07(6.52e-07)-	4.45e-15(5.30e-16)=	4.17e-15(5.18e-16)
18	7.73e-03(3.82e-02)+	6.08e-02(1.04e-01)+	9.04e-02(1.22e-01)-	7.78e-02(1.35e-01)
19	3.26e+00(2.23e+00)-	3.18e-01(2.53e-01)-	7.44e-03(1.49e-02)-	1.72e-05(3.69e-05)
24	6.62e-11(1.57e-10)-	3.73e-14(6.15e-15)-	3.37e-14(1.04e-15)=	3.38e-14(1.90e-15)
+/-/=	1/11/1	1/8/4	0/8/5	NA
Rank	3.54	2.85	2.38	1.23
p-value	1.51e-05	0.005	0.087	NA

D Parameter analysis

TABLE S-IV Parameter analysis of MGRLR on 13 problems in CEC2006: population size

Means and standard deviations of function errors are reported. If no feasible solution is found in at least one run of an algorithm for a particular problem, the number of runs in which at least one feasible solution was found is given. A large value is assigned to a run with no feasible solution to perform significance tests. (N = 20, 40, 60, 80, 100).

Problem	N = 20	N = 40	N = 60	N = 80	N = 100
1	3.75e-01(5.58e-01)	0.00e+00(0.00e+00)	7.11e-17(3.55e-16)	3.55e-16(1.03e-15)	5.68e-16(1.60e-15)
2	3.49e-01(1.43e-01)	4.22e-01(1.12e-01)	3.60e-01(8.64e-02)	4.56e-01(7.92e-02)	4.46e-01(7.43e-02)
4	-1.46e-13(1.28e-12)	-4.37e-13(1.21e-12)	-5.82e-13(2.01e-12)	-4.37e-13(1.21e-12)	-2.91e-13(1.46e-12)
6	-1.64e-11(0.00e+00)	-1.64e-11(0.00e+00)	-1.64e-11(0.00e+00)	-1.64e-11(0.00e+00)	-1.64e-11(0.00e+00)
7	2.74e-02(1.37e-01)	8.43e-13(3.55e-12)	8.03e-13(1.04e-12)	2.51e-13(7.63e-13)	3.82e-13(4.30e-13)
8	4.77e-17(8.09e-18)	5.22e-17(6.05e-18)	4.72e-17(8.01e-18)	5.55e-17(9.81e-18)	5.16e-17(8.52e-18)
9	1.75e-09(4.63e-09)	1.43e-08(2.68e-08)	7.04e-09(1.61e-08)	4.84e-09(7.51e-09)	5.56e-09(1.67e-08)
10	1.89e-05(6.40e-05)	6.15e-07(2.04e-06)	7.02e-07(3.17e-06)	6.20e-06(3.02e-05)	3.74e-07(4.62e-07)
12	0.00e+00(0.00e+00)	0.00e+00(0.00e+00)	0.00e+00(0.00e+00)	0.00e+00(0.00e+00)	0.00e+00(0.00e+00)
16	5.02e-10(2.51e-09)	4.63e-15(6.00e-16)	4.17e-15(5.18e-16)	4.41e-15(6.02e-16)	4.88e-15(6.43e-16)
18	6.75e-02(1.18e-01)	1.40e-01(1.70e-01)	7.78e-02(1.35e-01)	6.05e-02(1.04e-01)	1.04e-01(1.42e-01)
19	1.11e-07(4.72e-07)	7.68e-06(1.41e-05)	1.72e-05(3.69e-05)	1.80e-04(6.03e-04)	1.70e-02(8.42e-02)
24	3.67e-14(8.00e-15)	3.52e-14(6.29e-15)	3.38e-14(1.90e-15)	3.60e-14(7.06e-15)	3.38e-14(1.06e-15)
Rank	2.88	3	2.27	3.23	3.62

TABLE S-V Parameter analysis of MGRLR on 13 problems in CEC2006: w

Means and standard deviations of function errors are reported. If no feasible solution is found in at least one run of an algorithm for a particular problem, the number of runs in which at least one feasible solution was found is given. A large value is assigned to a run with no feasible solution to perform significance tests. (w = 0.1, 0.2, 0.3).

Problem	w = 0.1	w = 0.2	w = 0.3
1	7.82e-16(2.24e-15)	7.11e-17(3.55e-16)	1.42e-16(4.92e-16)
2	4.01e-01(1.09e-01)	3.60e-01(8.64e-02)	4.26e-01(7.69e-02)
4	-7.28e-13(1.49e-12)	-5.82e-13(2.01e-12)	1.31e-12(2.32e-12)
6	-1.64e-11(0.00e+00)	-1.64e-11(0.00e+00)	-1.64e-11(0.00e+00)
7	1.05e-07(2.46e-07)	8.03e-13(1.04e-12)	-4.69e-15(3.89e-13)
8	5.50e-17(8.48e-18)	4.72e-17(8.01e-18)	5.22e-17(6.05e-18)
9	1.04e-04(5.18e-04)	7.04e-09(1.61e-08)	7.55e-10(9.28e-10)
10	2.09e-04(4.03e-04)	7.02e-07(3.17e-06)	5.33e-07(2.44e-06)
12	7.12e-03(2.26e-02)	0.00e+00(0.00e+00)	0.00e+00(0.00e+00)
16	3.95e-15(3.63e-16)	4.17e-15(5.18e-16)	4.81e-15(6.11e-16)
18	1.13e-01(1.31e-01)	7.78e-02(1.35e-01)	4.52e-02(9.77e-02)
19	7.04e-03(3.51e-02)	1.72e-05(3.69e-05)	2.52e-05(6.83e-05)
24	3.58e-14(6.97e-15)	3.38e-14(1.90e-15)	4.70e-14(4.67e-14)
Rank	2.27	1.73	2

TABLE S-VI Parameter analysis of MGRLR on 13 problems in CEC2006:N_G

Means and standard deviations of function errors are reported. If no feasible solution is found in at least one run of an algorithm for a particular problem, the number of runs in which at least one feasible solution was found is given. A large value is assigned to a run with no feasible solution to perform significance tests. (N_G = 50, 100, 300, 200, 400, 600, 800, 1000).

Problem	N_G = 50	N_G = 100	N_G = 200	N_G = 300	N_G = 400	N_G = 600	N_G = 800	N_G = 1000
1	9.24e-16 (4.62e-15)	4.26e-16 (1.29e-15)	7.11e-17 (3.55e-16)	1.42e-16 (7.11e-16)	1.42e-16 (7.11e-16)	7.11e-17 (3.55e-16)	1.28e-15 (4.43e-15)	2.84e-16 (8.39e-16)
2	4.74e-01 (6.89e-02)	4.23e-01 (9.12e-02)	3.60e-01 (8.64e-02)	4.28e-01 (1.11e-01)	3.81e-01 (7.99e-02)	4.16e-01 (1.16e-01)	4.71e-01 (6.37e-02)	4.38e-01 (6.65e-02)
4	0.00e+00 (0.00e+00)	0.00e+00 (1.49e-12)	-5.82e-13 (2.01e-12)	2.91e-13 (1.79e-12)	4.37e-13 (2.18e-12)	2.04e-12 (2.59e-12)	1.75e-12 (3.34e-12)	2.04e-12 (2.12e-12)
6	-1.64e-11 (0.00e+00)	-1.64e-11 (0.00e+00)	-1.64e-11 (0.00e+00)	-1.64e-11 (0.00e+00)	-1.64e-11 (0.00e+00)	-1.64e-11 (0.00e+00)	-1.64e-11 (0.00e+00)	-1.64e-11 (0.00e+00)
7	7.11e-14 (1.87e-13)	2.11e-13 (2.45e-13)	8.03e-13 (1.04e-12)	2.67e-05 (1.33e-04)	9.22e-14 (2.19e-13)	1.85e-12 (2.24e-12)	3.81e-10 (1.90e-09)	4.05e-13 (1.30e-12)
8	5.38e-17 (4.60e-18)	5.33e-17 (6.56e-18)	4.72e-17 (8.01e-18)	5.38e-17 (9.24e-18)	5.22e-17 (6.05e-18)	4.72e-17 (6.94e-18)	5.16e-17 (6.36e-18)	4.94e-17 (7.03e-18)
9	6.74e-08 (3.03e-07)	5.97e-09 (1.84e-08)	7.04e-09 (1.61e-08)	2.49e-04 (1.24e-03)	7.11e-05 (2.02e-04)	2.99e-07 (7.81e-07)	2.93e-01 (1.41e+00)	2.83e-06 (1.32e-05)
10	1.30e-06 (1.48e-06)	1.42e-07 (6.64e-07)	7.02e-07 (3.17e-06)	1.33e-07 (5.41e-07)	3.70e-06 (6.75e-06)	1.76e-06 (8.29e-06)	6.99e-07 (1.30e-06)	6.29e-07 (1.49e-06)
12	0.00e+00 (0.00e+00)	0.00e+00 (0.00e+00)	0.00e+00 (0.00e+00)	0.00e+00 (0.00e+00)	0.00e+00 (0.00e+00)	0.00e+00 (0.00e+00)	0.00e+00 (0.00e+00)	0.00e+00 (0.00e+00)
16	4.66e-15 (2.47e-15)	4.09e-15 (5.05e-16)	4.17e-15 (5.18e-16)	4.20e-15 (5.20e-16)	4.41e-15 (7.03e-16)	4.56e-15 (5.91e-16)	3.75e-08 (1.87e-07)	4.77e-15 (1.52e-15)
18	2.14e-01 (1.64e-01)	5.35e-02 (8.75e-02)	7.78e-02 (1.35e-01)	9.81e-02 (1.22e-01)	5.29e-02 (1.01e-01)	6.68e-02 (1.29e-01)	7.51e-02 (1.20e-01)	1.27e-01 (1.48e-01)
19	2.00e-06 (4.18e-06)	9.46e-06 (2.14e-05)	1.72e-05 (3.69e-05)	1.09e-05 (1.24e-05)	3.56e-05 (9.07e-05)	2.16e-03 (1.05e-02)	8.55e-06 (1.59e-05)	2.70e-06 (5.65e-06)
24	4.85e-14 (1.34e-14)	3.85e-14 (6.32e-15)	3.38e-14 (1.90e-15)	3.60e-14 (5.28e-15)	3.65e-14 (7.44e-15)	3.84e-14 (6.13e-15)	4.26e-14 (8.78e-15)	3.50e-14 (3.23e-15)
Rank	5.15	3.77	3	4.27	4.23	5	5.46	5.12

TABLE S-VII Parameter analysis of MGRLR on 13 problems in CEC2006:N_G (time)

Means and standard deviations of consumed time (s) are reported. (N_G = 50, 100, 300, 200, 400, 600, 800, 1000).

Problem	N_G = 50	N_G = 100	N_G = 200	N_G = 300	N_G = 400	N_G = 600	N_G = 800	N_G = 1000
1	4.48e+01 (3.95e+00)	3.22e+01 (9.25e-01)	5.76e+01 (3.34e+00)	9.88e+01 (1.01e+01)	1.08e+02 (1.31e+01)	1.39e+02 (1.45e+01)	1.98e+02 (2.67e+01)	2.71e+02 (5.95e+01)
2	6.02e+01 (7.32e+00)	7.07e+01 (8.32e+00)	8.30e+01 (7.61e+00)	1.14e+02 (7.23e+00)	1.33e+02 (8.59e+00)	1.91e+02 (1.51e+01)	2.51e+02 (2.69e+01)	3.10e+02 (6.22e+01)
4	3.63e+01 (2.54e+00)	4.09e+01 (2.84e+00)	4.76e+01 (2.22e+00)	6.94e+01 (1.48e+00)	7.95e+01 (5.22e+00)	1.03e+02 (3.60e+00)	1.38e+02 (1.90e+01)	1.65e+02 (2.97e+01)
6	2.62e+01 (2.13e+00)	2.98e+01 (1.78e+00)	3.75e+01 (1.92e+00)	5.73e+01 (5.05e+00)	6.88e+01 (4.60e+00)	9.15e+01 (6.63e+00)	1.20e+02 (5.82e+00)	1.73e+02 (3.47e+01)
7	3.77e+01 (1.90e+00)	4.36e+01 (2.31e+00)	5.25e+01 (2.83e+00)	7.86e+01 (5.17e+00)	8.85e+01 (6.17e+00)	1.15e+02 (6.01e+00)	1.59e+02 (1.04e+01)	1.98e+02 (3.63e+01)
8	2.66e+01 (1.44e+00)	3.17e+01 (1.82e+00)	4.13e+01 (1.58e+00)	6.53e+01 (5.60e+00)	7.88e+01 (5.85e+00)	1.11e+02 (6.42e+00)	1.69e+02 (3.01e+01)	2.38e+02 (5.41e+01)
9	3.57e+01 (1.57e+00)	4.23e+01 (2.75e+00)	5.23e+01 (2.27e+00)	8.11e+01 (6.53e+00)	9.64e+01 (5.41e+00)	1.31e+02 (9.97e+00)	1.79e+02 (2.27e+01)	2.65e+02 (5.79e+01)
10	3.45e+01 (1.86e+00)	3.99e+01 (2.44e+00)	4.81e+01 (2.21e+00)	7.42e+01 (3.56e+00)	8.61e+01 (4.53e+00)	1.17e+02 (6.96e+00)	1.62e+02 (1.71e+01)	1.88e+02 (3.13e+01)
12	3.61e+01 (1.50e+00)	4.01e+01 (2.67e+00)	4.67e+01 (2.15e+00)	6.84e+01 (3.27e+00)	7.93e+01 (2.83e+00)	1.05e+02 (8.21e+00)	1.35e+02 (1.34e+01)	1.70e+02 (3.61e+01)
16	4.14e+01 (2.32e+00)	4.68e+01 (3.12e+00)	5.43e+01 (2.52e+00)	7.93e+01 (3.06e+00)	9.09e+01 (4.87e+00)	1.19e+02 (7.42e+00)	1.67e+02 (2.18e+01)	2.03e+02 (4.31e+01)
18	4.30e+01 (4.09e+00)	4.48e+01 (3.79e+00)	5.31e+01 (4.29e+00)	7.97e+01 (5.47e+00)	9.08e+01 (4.34e+00)	1.30e+02 (1.46e+01)	1.79e+02 (2.24e+01)	2.04e+02 (4.96e+01)
19	5.71e+01 (2.49e+00)	6.42e+01 (3.97e+00)	7.53e+01 (3.49e+00)	1.09e+02 (4.30e+00)	1.05e+02 (5.90e+00)	1.49e+02 (1.88e+01)	2.24e+02 (3.19e+01)	2.48e+02 (4.37e+01)
24	2.75e+01 (2.19e+00)	3.03e+01 (1.90e+00)	3.78e+01 (1.47e+00)	5.39e+01 (1.41e+00)	5.58e+01 (1.98e+00)	7.52e+01 (4.20e+00)	9.25e+01 (5.55e+00)	1.62e+02 (5.40e+01)
Rank	1.15	1.85	3	4.08	4.92	6	7	8

TABLE S-VIII Parameter analysis of MGRLR on 13 problems in CEC2006: (LRA_1, LRA_2)

Means and standard deviations of function errors are reported. If no feasible solution is found in at least one run of an algorithm for a particular problem, the number of runs in which at least one feasible solution was found is given. A large value is assigned to a run with no feasible solution to perform significance tests. $\left(LRA_1 = 0, 0.05, 0.1; LRA_2 = 0.5, 1, 2\right)$

Problem	(LRA_1, LRA_2) = (0, 0.5)	(LRA_1, LRA_2) = (0, 1)	(LRA_1, LRA_2) = (0, 2)	(LRA_1, LRA_2) = (0.05, 0.5)	(LRA_1, LRA_2) = (0.05, 1)
1	2.71e-13(1.21e-12)	1.90e-12(5.40e-12)	9.84e-12(4.14e-11)	2.20e-15(7.36e-15)	3.55e-16(1.03e-15)
2	4.67e-01(9.25e-02)	4.15e-01(1.03e-01)	4.40e-01(9.74e-02)	4.16e-01(1.10e-01)	4.01e-01(1.02e-01)
4	1.53e-02(7.65e-02)	2.59e-02(1.29e-01)	6.49e-02(3.24e-01)	0.00e+00(1.05e-12)	-7.28e-13(1.49e-12)
6	-1.64e-11(0.00e+00)	-1.29e-11(1.75e-11)	-1.64e-11(0.00e+00)	-1.64e-11(0.00e+00)	-1.54e-11(4.73e-12)
7	2.87e-05(5.18e-05)	1.87e-09(9.35e-09)	2.84e-02(1.37e-01)	8.27e-13(1.95e-12)	1.25e-13(2.72e-13)
8	3.23e-08(6.88e-08)	3.81e-05(1.68e-04)	3.15e-07(1.54e-06)	5.50e-17(7.47e-18)	5.72e-17(1.01e-17)
9	1.35e-04(4.91e-04)	1.04e-02(2.88e-02)	6.36e-03(1.76e-02)	3.05e-08(1.19e-07)	1.78e-04(8.90e-04)
10	3.82e-02(1.79e-01)	1.79e-02(8.51e-02)	8.53e-06(2.10e-05)	1.75e-06(1.98e-06)	3.46e-07(7.58e-07)
12	2.20e-14(1.04e-13)	2.36e-14(6.52e-14)	2.25e-04(1.13e-03)	0.00e+00(0.00e+00)	0.00e+00(0.00e+00)
16	5.71e-03(2.30e-02)	6.96e-04(2.37e-03)	6.92e-04(1.22e-03)	5.80e-15(1.53e-15)	4.38e-15(6.13e-16)
18	1.03e-01(1.20e-01)	1.49e-01(1.59e-01)	1.41e-01(1.61e-01)	1.13e-01(1.31e-01)	6.05e-02(1.04e-01)
19	1.69e-02(3.28e-02)	3.18e-02(6.43e-02)	9.91e-02(1.95e-01)	1.76e-02(8.82e-02)	1.93e-03(6.24e-03)
24	3.52e-14(5.33e-15)	3.82e-14(8.44e-15)	3.47e-14(3.08e-15)	3.88e-14(6.27e-15)	3.55e-14(4.42e-15)
Rank	6.85	6.69	5.81	5.46	4.12

Problem	(LRA_1, LRA_2) = (0.05, 2)	(LRA_1, LRA_2) = (0.1, 0.5)	(LRA_1, LRA_2) = (0.1, 1)	(LRA_1, LRA_2) = (0.1, 2)
1	2.13e-16(5.89e-16)	7.11e-17(3.55e-16)	4.26e-16(1.48e-15)	7.82e-16(2.72e-15)
2	4.58e-01(1.02e-01)	3.60e-01(8.64e-02)	4.24e-01(7.05e-02)	4.77e-01(4.93e-02)
4	-5.82e-13(1.36e-12)	-5.82e-13(2.01e-12)	2.91e-13(2.08e-12)	-1.46e-13(1.28e-12)
6	-1.64e-11(0.00e+00)	-1.64e-11(0.00e+00)	-1.64e-11(0.00e+00)	-1.64e-11(0.00e+00)
7	6.97e-13(8.33e-13)	8.03e-13(1.04e-12)	4.04e-13(6.60e-13)	2.23e-12(9.33e-12)
8	5.16e-17(7.52e-18)	4.72e-17(8.01e-18)	5.44e-17(9.75e-18)	5.33e-17(9.55e-18)
9	2.75e-06(1.31e-05)	7.04e-09(1.61e-08)	1.98e-04(6.90e-04)	1.57e-04(6.22e-04)
10	1.97e-06(7.07e-06)	7.02e-07(3.17e-06)	1.12e-06(2.75e-06)	1.29e-06(4.63e-06)
12	0.00e+00(0.00e+00)	0.00e+00(0.00e+00)	0.00e+00(0.00e+00)	0.00e+00(0.00e+00)
16	4.59e-15(1.05e-15)	4.17e-15(5.18e-16)	4.34e-15(6.73e-16)	4.20e-15(4.53e-16)
18	9.68e-02(1.42e-01)	7.78e-02(1.35e-01)	1.14e-01(1.15e-01)	1.27e-01(1.48e-01)
19	4.92e-06(7.51e-06)	1.72e-05(3.69e-05)	5.45e-02(2.73e-01)	7.36e-06(2.28e-05)
24	3.51e-14(5.40e-15)	3.38e-14(1.90e-15)	3.46e-14(6.04e-15)	3.60e-14(5.84e-15)
Rank	4.96	2.42	4.04	4.65

TABLE S-IX Parameter analysis of MGRLR on 13 problems in CEC2006: number of iterations in SQP: q

Means and standard deviations of function errors are reported. If no feasible solution is found in at least one run of an algorithm for a particular problem, the number of runs in which at least one feasible solution was found is given. A large value is assigned to a run with no feasible solution to perform significance tests. (q = 100, 200, 300, 400, 500).

Problem	q = 100	q = 200	q = 300	q = 400	q = 500
1	3.62e-15(1.23e-14)	4.41e-15(9.93e-15)	7.11e-17(3.55e-16)	1.42e-16(7.11e-16)	0.00e+00(0.00e+00)
2	4.56e-01(1.10e-01)	4.46e-01(7.96e-02)	3.60e-01(8.64e-02)	4.74e-01(5.32e-02)	4.16e-01(7.49e-02)
4	-4.37e-13(1.91e-12)	-1.60e-12(1.84e-12)	-5.82e-13(2.01e-12)	-2.91e-13(1.01e-12)	4.37e-13(1.91e-12)
6	-1.64e-11(0.00e+00)	-1.64e-11(0.00e+00)	-1.64e-11(0.00e+00)	-1.64e-11(0.00e+00)	-1.64e-11(0.00e+00)
7	3.72e-13(1.69e-12)	2.86e-12(1.38e-11)	8.03e-13(1.04e-12)	5.63e-13(2.03e-12)	3.51e-14(2.35e-13)
8	5.27e-17(6.94e-18)	5.22e-17(9.21e-18)	4.72e-17(8.01e-18)	4.94e-17(9.03e-18)	4.66e-17(8.85e-18)
9	1.14e-08(1.55e-08)	2.25e-08(3.98e-08)	7.04e-09(1.61e-08)	2.34e-08(6.39e-08)	3.25e-08(7.51e-08)
10	1.34e-03(3.30e-03)	4.67e-06(1.87e-05)	7.02e-07(3.17e-06)	9.31e-07(1.03e-06)	6.55e-08(1.06e-07)
12	0.00e+00(0.00e+00)	0.00e+00(0.00e+00)	0.00e+00(0.00e+00)	0.00e+00(0.00e+00)	0.00e+00(0.00e+00)
16	4.31e-15(7.69e-16)	4.41e-15(8.32e-16)	4.17e-15(5.18e-16)	4.06e-15(4.23e-16)	4.41e-15(7.48e-16)
18	5.22e-02(1.14e-01)	1.59e-01(1.27e-01)	7.78e-02(1.35e-01)	2.29e-02(6.34e-02)	1.43e-01(1.23e-01)
19	1.90e-05(5.32e-05)	3.94e-06(5.54e-06)	1.72e-05(3.69e-05)	1.68e-04(7.41e-04)	8.00e-06(1.79e-05)
24	4.82e-14(6.08e-14)	3.59e-14(4.21e-15)	3.38e-14(1.90e-15)	3.77e-14(6.26e-15)	4.17e-14(1.01e-14)
Rank	3.27	3.27	2.35	3.19	2.92

TABLE S-X Parameter analysis of MGRLR on 13 problems in CEC2006: number of candidate offspring: NPr

Means and standard deviations of function errors are reported. If no feasible solution is found in at least one run of an algorithm for a particular problem, the number of runs in which at least one feasible solution was found is given. A large value is assigned to a run with no feasible solution to perform significance tests. (NPr = 100, 200, 300, 400, 500).

Problem	NPr = 100	NPr = 200	NPr = 300	NPr = 400	NPr = 500
1	9.95e-16(2.35e-15)	9.95e-16(2.41e-15)	2.84e-16(8.39e-16)	1.63e-15(2.29e-15)	7.11e-17(3.55e-16)
2	4.39e-01(1.05e-01)	4.33e-01(1.04e-01)	3.76e-01(1.35e-01)	4.15e-01(1.13e-01)	3.60e-01(8.64e-02)
4	2.91e-13(1.01e-12)	2.91e-13(1.79e-12)	-4.37e-13(1.21e-12)	-5.82e-13(1.36e-12)	-5.82e-13(2.01e-12)
6	-1.64e-11(0.00e+00)	-1.64e-11(0.00e+00)	-1.64e-11(0.00e+00)	-1.64e-11(0.00e+00)	-1.64e-11(0.00e+00)
7	2.55e-12(4.52e-12)	6.43e-13(8.77e-13)	9.08e-14(5.74e-13)	9.80e-13(4.00e-12)	8.03e-13(1.04e-12)
8	5.50e-17(1.10e-17)	5.22e-17(9.21e-18)	5.38e-17(8.33e-18)	4.94e-17(7.03e-18)	4.72e-17(8.01e-18)
9	8.98e-08(4.06e-07)	4.91e-09(4.73e-09)	3.98e-08(1.56e-07)	2.10e-08(4.51e-08)	7.04e-09(1.61e-08)
10	4.74e-08(5.56e-08)	5.55e-07(9.87e-07)	6.51e-08(1.88e-07)	9.14e-08(4.26e-07)	7.02e-07(3.17e-06)
12	5.89e-15(2.94e-14)	0.00e+00(0.00e+00)	0.00e+00(0.00e+00)	4.00e-17(1.10e-16)	0.00e+00(0.00e+00)
16	5.02e-15(1.31e-15)	5.48e-15(1.20e-15)	4.73e-15(6.24e-16)	4.17e-15(6.32e-16)	4.17e-15(5.18e-16)
18	1.27e-01(1.48e-01)	1.57e-01(1.37e-01)	8.21e-02(1.32e-01)	2.13e-01(1.73e-01)	7.78e-02(1.35e-01)
19	6.88e-05(3.23e-04)	2.50e-05(4.40e-05)	3.78e-04(1.86e-03)	2.65e-04(3.68e-04)	1.72e-05(3.69e-05)
24	5.11e-14(5.56e-14)	1.95e-13(2.09e-13)	3.67e-14(4.04e-15)	3.37e-14(1.74e-15)	3.38e-14(1.90e-15)
Rank	3.73	3.73	2.69	2.77	2.08

TABLE S-XI Parameter analysis of MGRLR on 13 problems in CEC2006:N_L

Means and standard deviations of function errors are reported. If no feasible solution is found in at least one run of an algorithm for a particular problem, the number of runs in which at least one feasible solution was found is given. A large value is assigned to a run with no feasible solution to perform significance tests. (N_L = 1D, 2D, 3D, 4D, 5D, 6D, 7D).

Problem	$N_L = 1D$	$N_L = 2D$	$N_L = 3D$	$N_L = 4D$	$N_L = 5D$
1	3.84e-15(1.88e-14)	2.84e-16(6.65e-16)	1.42e-16(7.11e-16)	4.97e-16(1.09e-15)	7.11e-17(3.55e-16)
2	3.92e-01(1.13e-01)	4.38e-01(7.40e-02)	3.95e-01(9.94e-02)	4.14e-01(1.05e-01)	3.60e-01(8.64e-02)
4	-1.46e-12(3.32e-12)	-1.31e-12(2.07e-12)	-5.82e-13(1.36e-12)	-1.31e-12(1.78e-12)	-5.82e-13(2.01e-12)
6	4.42e-10(2.29e-09)	-1.64e-11(0.00e+00)	-1.64e-11(0.00e+00)	-1.64e-11(0.00e+00)	-1.64e-11(0.00e+00)
7	4.50e-01(4.20e-01)	3.44e-03(1.00e-02)	5.88e-05(2.48e-04)	3.83e-07(1.92e-06)	8.03e-13(1.04e-12)
8	5.88e-17(1.62e-17)	6.22e-17(2.44e-17)	5.16e-17(1.02e-17)	5.61e-17(1.52e-17)	4.72e-17(8.01e-18)
9	3.58e-03(4.12e-03)	5.90e-04(1.87e-03)	2.07e-05(8.41e-05)	7.47e-08(1.23e-07)	7.04e-09(1.61e-08)
10	1.95e+02(2.79e+02)	1.66e+01(4.31e+01)	1.41e-03(4.96e-03)	1.03e-05(1.59e-05)	7.02e-07(3.17e-06)
12	4.09e-16(1.21e-15)	2.86e-15(1.26e-14)	7.99e-17(3.37e-16)	2.25e-04(1.13e-03)	0.00e+00(0.00e+00)
16	1.42e-04(2.63e-04)	1.93e-07(9.65e-07)	1.35e-11(6.74e-11)	4.41e-15(4.81e-16)	4.17e-15(5.18e-16)
18	1.33e-01(9.31e-02)	2.46e-02(6.28e-02)	9.93e-02(9.74e-02)	2.19e-01(1.35e-01)	7.78e-02(1.35e-01)
19	1.05e+00(9.11e-01)	4.89e-03(1.35e-02)	2.82e-04(5.88e-04)	1.00e-01(2.04e-01)	1.72e-05(3.69e-05)
24	4.46e-14(2.08e-14)	1.92e-13(7.74e-13)	9.55e-14(1.98e-13)	4.18e-14(1.27e-14)	3.38e-14(1.90e-15)
Rank	5.65	4.85	3.96	4.54	2.12

Problem	$N_L = 6D$	$N_L = 7D$
1	1.42e-16(4.92e-16)	2.84e-16(8.39e-16)
2	4.58e-01(1.04e-01)	3.94e-01(1.13e-01)
4	4.37e-13(4.11e-12)	0.00e+00(1.49e-12)
6	-1.54e-11(4.73e-12)	-1.64e-11(0.00e+00)
7	7.69e-14(5.05e-13)	5.90e-14(2.77e-13)
8	5.16e-17(7.52e-18)	4.72e-17(6.94e-18)
9	6.55e-09(2.15e-08)	1.21e-09(3.87e-09)
10	4.80e-07(1.61e-06)	4.37e-08(1.37e-07)
12	0.00e+00(0.00e+00)	0.00e+00(0.00e+00)
16	4.38e-15(4.23e-16)	4.27e-15(5.78e-16)
18	1.13e-01(1.20e-01)	1.87e-01(1.40e-01)
19	1.55e-05(5.80e-05)	5.89e-05(1.66e-04)
24	3.74e-14(5.54e-15)	3.43e-14(1.45e-15)
Rank	3.73	3.15

E Comparison with state-of-the-art algorithms

TABLE S-XII Comparison results of MGRLR with 4 state-of-the-art algorithms on 13 problems in CEC2006

Means and standard deviations of function errors are reported. If no feasible solution is found in at least one run of an algorithm for a particular problem, the number of runs in which at least one feasible solution was found is given. A large value is assigned to a run with no feasible solution to perform significance tests. The symbols '+ ', '- ', '= ' mean that the compared algorithm is significantly better than, worse than, and competitive with MGRLR.

Problem	GloSADE	SACCDE	MPMLS	DSI	MGRLR
1	5.86e-07(3.15e-07)-	9.61e-14(4.69e-14)-	2.98e-02(6.20e-03)-	4.22e-01(6.66e-01)-	7.11e-17(3.55e-16)
2	5.05e-01(2.97e-02)-	4.46e-01(8.32e-02)-	3.98e-01(5.68e-02)-	4.88e-01(8.76e-02)-	3.60e-01(8.64e-02)
4	1.37e-06(2.81e-06)-	7.98e-07(5.35e-07)-	2.00e-03(1.19e-03)-	2.32e-10(4.33e-10)-	-5.82e-13(2.01e-12)
6	8.90e-05(3.64e-04)-	7.85e-06(1.69e-05)-	1.70e-06(8.36e-06)-	1.60e-05(3.41e-05)-	-1.64e-11(0.00e+00)
7	1.03e-05(1.93e-05)-	1.44e-04(1.42e-04)-	1.13e-01(6.43e-02)-	2.79e-02(1.40e-01)-	8.03e-13(1.04e-12)
8	1.20e-11(2.76e-11)-	2.80e-03(1.40e-02)-	2.92e-15(1.41e-14)-	3.23e-02(3.44e-02)-	4.72e-17(8.01e-18)
9	2.06e+02(8.98e+01)-	1.85e+02(1.46e+02)-	1.53e-02(3.90e-02)-	1.66e-02(5.41e-02)-	7.04e-09(1.61e-08)
10	3.53e+01(5.85e+01)-	2.63e-03(6.07e-03)-	2.49e+01(1.81e+01)-	8.02e-04(2.00e-03)-	7.02e-07(3.17e-06)
12	2.49e-04(1.22e-03)-	8.20e-02(6.56e-02)-	0.00e+00(0.00e+00)=	5.25e-02(4.89e-02)-	0.00e+00(0.00e+00)
16	3.88e-02(4.25e-02)-	3.78e-06(6.85e-06)-	3.34e-05(3.14e-05)-	7.80e-05(3.90e-04)-	4.17e-15(5.18e-16)
18	1.17e-01(1.06e-01)-	1.53e-02(5.29e-02)+	7.65e-02(9.25e-02)+	1.14e-01(1.44e-01)-	7.78e-02(1.35e-01)
19	4.10e+00(3.77e+00)-	9.09e-01(1.41e+00)-	5.37e+00(4.04e+00)-	1.26e-01(1.65e-01)-	1.72e-05(3.69e-05)
24	5.94e-08(5.36e-08)-	2.94e-06(2.64e-06)-	6.64e-13(5.15e-13)-	6.91e-10(2.89e-09)-	3.38e-14(1.90e-15)
+/-/=	0/13/0	1/12/0	1/11/1	0/13/0	NA
Rank	3.92	3.62	3.5	2.92	1.04
p-value	3.16E-05	3.14E-04	6.99E-04	0.023	NA

TABLE S-XIII Comparison results of MGRLR with 4 state-of-the-art algorithms on 6 problems in CEC2010

Means and standard deviations of objective values are reported. If no feasible solution is found in at least one run of an algorithm for a particular problem, the number of runs in which at least one feasible solution was found is given. A large value is assigned to a run with no feasible solution to perform significance tests. The symbols '+ ', '- ', '= ' mean that the compared algorithm is significantly better than, worse than, and competitive with MGRLR.

Problem(10*D*)	GloSADE	SACCDE	MPMLS	DSI	MGRLR
1	-3.32e-01(3.93e-02)-	-4.44e-01(1.60e-01)=	-4.62e-01(6.08e-02)=	-4.37e-01(1.03e-01)=	-4.36e-01(1.07e-01)
7	2.02e+05(1.55e+05)-	2.63e+05(1.41e+05)-	1.17e+03(9.60e+02)-	1.03e+03(1.80e+03)-	2.78e+02(4.17e+02)
8	1.27e+06(1.19e+06)-	1.05e+06(2.57e+06)-	1.05e+04(8.58e+03)-	3.96e+04(1.37e+05)-	2.40e+02(2.74e+02)
13	-5.32e+01(4.44e+00)-	-5.74e+01(3.52e+00)=	-5.84e+01(3.78e+00)=	-5.15e+01(6.93e+00)-	-5.64e+01(4.45e+00)
14	2.28e+11(5.16e+11)-	7.20e+13(5.68e+13)-	2.91e+09(4.42e+09)-	(24)-	6.94e+04(4.24e+04)
15	(2)-	(2)-	(4)-	(23)-	8.80e+10(4.21e+11)
+/-/=	0/6/0	0/4/2	0/4/2	0/5/1	NA
Rank	4.25	4.08	2	3.17	1.5
p-value	0.025	0.045	1	0.667	NA
Problem(30*D*)	GloSADE	SACCDE	MPMLS	DSI	MGRLR
1	-2.41e-01(2.64e-02)-	-3.60e-01(6.52e-02)+	-3.42e-01(4.34e-02)=	-3.21e-01(6.49e-02)=	-3.27e-01(8.18e-02)
7	2.80e+07(2.07e+07)-	1.93e+07(4.99e+06)-	2.02e+06(1.80e+06)-	4.42e+05(3.75e+05)-	2.10e+04(1.58e+04)
8	1.40e+08(9.02e+07)-	4.46e+09(1.83e+09)-	2.11e+08(3.41e+08)-	7.46e+06(1.30e+07)-	2.36e+05(2.04e+05)
13	-2.95e+01(5.02e+00)-	-5.69e+01(2.55e+00)+	-4.41e+01(6.40e+00)-	-5.19e+01(4.01e+00)=	-5.27e+01(2.89e+00)
14	3.44e+11(3.90e+11)-	3.66e+14(8.90e+13)-	1.97e+12(1.40e+12)-	1.98e+14(2.44e+14)-	7.14e+08(6.63e+08)
15	(17)=	(9)-	(15)=	(22)=	(12)
+/-/=	0/5/1	2/4/0	0/4/2	0/3/3	NA
Rank	3.5	3.58	3.25	2.83	1.83
p-value	0.667	0.542	1	1	NA

TABLE S-XIV Comparison results of MGRLR with 4 state-of-the-art algorithms on 9 problems in CEC2017

Means and standard deviations of objective values are reported. If no feasible solution is found in at least one run of an algorithm for a particular problem, the number of runs in which at least one feasible solution was found is given. A large value is assigned to a run with no feasible solution to perform significance tests. The symbols '+ ', '- ', '= ' mean that the compared algorithm is significantly better than, worse than, and competitive with MGRLR.

Problem(10*D*)	GloSADE	SACCDE	MPMLS	DSI	MGRLR
1	2.39e+03(6.29e+02)-	2.85e+00(3.29e+00)-	2.28e+03(1.04e+03)-	1.60e-01(6.42e-01)+	1.56e+00(2.88e+00)
2	1.03e+03(3.19e+02)-	4.34e+00(6.87e+00)-	5.40e+02(3.15e+02)-	3.44e-01(6.53e-01)+	4.74e-01(6.14e-01)
4	1.07e+02(1.11e+01)-	6.07e+01(1.22e+01)-	9.37e+01(9.87e+00)-	6.40e+01(1.52e+01)-	5.19e+01(9.98e+00)
5	4.44e+01(1.74e+01)-	9.37e+01(5.03e+01)-	7.55e+00(1.00e+00)-	9.44e+00(1.24e+01)=	6.52e+00(1.21e+00)
13	2.06e+02(1.30e+02)-	3.44e+02(2.79e+02)-	3.10e+01(2.30e+01)=	(24)=	2.71e+01(2.36e+01)
19	(0)=	(0)=	(0)=	(0)=	(0)=
20	2.48e+00(4.01e-01)-	2.20e+00(3.72e-01)=	2.62e+00(3.73e-01)-	3.16e+00(6.33e-01)-	2.20e+00(3.21e-01)
22	7.88e+02(4.70e+02)-	4.35e+03(1.31e+04)-	5.20e+01(3.11e+01)-	(20)-	2.67e+01(3.01e+01)
28	(0)=	(0)=	(0)=	(0)=	(0)=
+/-/=	0/7/2	0/6/3	0/6/3	2/3/4	NA
Rank	3.89	3.44	3.33	2.67	1.67
p-value	0.007	0.068	0.112	1	NA
Problem(30*D*)	GloSADE	SACCDE	MPMLS	DSI	MGRLR
1	3.45e+04(6.90e+03)-	1.47e+04(4.70e+03)+	5.01e+04(7.65e+03)-	6.10e+03(1.92e+03)+	1.89e+04(6.75e+03)
2	2.74e+04(4.70e+03)-	7.65e+03(2.88e+03)=	1.34e+04(2.02e+03)-	3.90e+03(1.28e+03)+	7.04e+03(1.83e+03)
4	4.43e+02(2.87e+01)-	1.67e+02(1.79e+01)+	3.67e+02(3.46e+01)-	2.28e+02(5.21e+01)-	1.84e+02(2.28e+01)
5	8.67e+03(4.00e+03)-	1.00e+03(4.53e+02)-	1.50e+02(4.81e+01)-	1.73e+02(2.56e+02)-	6.30e+01(3.23e+01)
13	(0)=	(3)=	(0)=	(0)-	(7)=
19	(0)=	(0)=	(0)=	(0)=	(0)=
20	1.04e+01(6.32e-01)=	1.04e+01(4.12e-01)=	1.05e+01(4.64e-01)-	1.16e+01(1.10e+00)-	1.03e+01(5.67e-01)
22	(0)=	(0)=	(0)=	(0)=	(0)=
28	(0)=	(0)=	(0)=	(0)=	(0)=
+/-/=	0/5/4	2/1/6	0/6/3	2/4/3	NA
Rank	3.83	2.61	3.67	2.78	2.11
p-value	0.038	1	0.090	1	NA

TABLE S-XV Comparison results of MGRLR with 4 state-of-the-art algorithms on 13 problems in CEC2006 (time)

Means and standard deviations of consumed time (s) are reported. The symbols '+ ', '- ', '= ' mean that the compared algorithm is significantly better than, worse than, and competitive with MGRLR.

Problem	GloSADE	SACCDE	MPMLS	DSI	MGRLR
1	1.38e+02(1.15e+01)-	5.04e+02(3.26e+01)-	2.50e+01(1.07e+00)+	3.52e+02(3.40e+01)-	5.76e+01(3.34e+00)
2	3.11e+02(4.38e+01)-	7.26e+02(4.00e+01)-	3.58e+01(1.18e+00)+	4.68e+02(2.01e+02)-	8.30e+01(7.61e+00)
4	2.08e+02(1.93e+01)-	2.46e+02(3.51e+01)-	1.29e+01(6.38e-01)+	2.86e+02(1.02e+02)-	4.76e+01(2.22e+00)
6	1.49e+02(1.77e+01)-	1.48e+02(9.80e+00)-	9.80e+00(6.95e-01)+	2.38e+02(1.12e+02)-	3.75e+01(1.92e+00)
7	2.23e+02(2.18e+01)-	1.61e+03(1.77e+02)-	2.00e+01(7.79e-01)+	1.30e+02(8.01e+01)-	5.25e+01(2.83e+00)
8	1.86e+02(1.71e+01)-	1.82e+02(1.85e+01)-	9.66e+00(5.32e-01)+	1.62e+02(1.03e+02)-	4.13e+01(1.58e+00)
9	2.09e+02(2.84e+01)-	1.05e+03(8.30e+01)-	1.59e+01(6.51e-01)+	3.94e+02(9.06e+01)-	5.23e+01(2.27e+00)
10	2.48e+02(2.35e+01)-	1.45e+03(1.71e+02)-	1.79e+01(1.12e+00)+	1.19e+02(4.43e+01)-	4.81e+01(2.21e+00)
12	2.04e+02(2.29e+01)-	1.70e+02(1.44e+01)-	1.24e+01(1.16e+00)+	1.57e+02(1.27e+02)-	4.67e+01(2.15e+00)
16	2.43e+02(1.03e+01)-	1.79e+03(1.04e+03)-	1.95e+01(2.42e+00)+	2.52e+02(2.16e+02)-	5.43e+01(2.52e+00)
18	2.52e+02(1.78e+01)-	1.95e+03(1.91e+02)-	2.35e+01(1.46e+00)+	1.06e+02(6.70e+01)-	5.31e+01(4.29e+00)
19	2.48e+02(3.91e+01)-	9.75e+02(5.85e+01)-	3.29e+01(2.17e+00)+	4.21e+02(1.33e+02)-	7.53e+01(3.49e+00)
24	1.68e+02(1.38e+01)-	1.08e+02(6.34e+00)-	1.41e+01(1.10e+00)+	2.19e+02(1.01e+02)-	3.78e+01(1.47e+00)
+/-/=	0/13/0	0/13/0	13/0/0	0/13/0	NA
Rank	3.65	4.58	1	3.77	2
p-value	0.075	3.14E-4	1	0.043	NA

TABLE S-XVI Comparison results of MGRLR with 4 state-of-the-art algorithms on 6 problems in CEC2010 (time)

Means and standard deviations of consumed time (s) are reported. The symbols '+ ', '- ', '= ' mean that the compared algorithm is significantly better than, worse than, and competitive with MGRLR.

Problem(10*D*)	GloSADE	SACCDE	MPMLS	DSI	MGRLR
1	2.12e+02(5.37e+01)-	3.73e+02(1.07e+02)-	2.72e+01(2.65e+00)+	1.85e+02(8.02e+01)-	6.47e+01(5.50e+00)
7	2.35e+02(2.24e+01)-	3.09e+02(2.23e+01)-	2.54e+01(1.68e+00)+	2.45e+02(2.61e+01)-	5.31e+01(1.90e+00)
8	2.00e+02(6.82e+00)-	3.06e+02(1.92e+01)-	2.60e+01(2.57e+00)+	2.45e+02(3.91e+01)-	6.07e+01(2.24e+00)
13	2.32e+02(2.07e+01)-	4.44e+02(1.40e+02)-	2.37e+01(2.68e+00)+	1.39e+02(1.27e+02)-	5.66e+01(1.91e+00)
14	2.18e+02(4.85e+01)-	6.22e+02(5.02e+01)-	2.05e+01(5.38e-01)+	1.03e+02(4.85e+01)-	6.13e+01(3.06e+00)
15	1.64e+02(1.52e+01)-	6.09e+02(3.88e+01)-	2.06e+01(4.68e-01)+	1.43e+02(2.40e+01)-	6.58e+01(2.61e+00)
+/-/=	0/6/0	0/6/0	6/0/0	0/6/0	NA
Rank	3.67	5	1	3.33	2
p-value	0.679	0.010	1	1	NA
Problem(30*D*)	GloSADE	SACCDE	MPMLS	DSI	MGRLR
1	6.52e+02(7.12e+01)-	6.75e+02(5.48e+01)-	6.30e+01(3.10e+00)+	2.71e+02(1.53e+02)-	8.50e+01(5.60e+00)
7	6.21e+02(5.59e+01)-	5.86e+02(5.70e+01)-	6.47e+01(2.91e+00)+	8.57e+02(1.84e+02)-	6.99e+01(3.53e+00)
8	6.29e+02(5.47e+01)-	5.24e+02(5.58e+01)-	6.42e+01(2.43e+00)+	6.54e+02(9.43e+01)-	8.55e+01(5.95e+00)
13	6.07e+02(5.44e+01)-	8.03e+02(1.05e+02)-	6.37e+01(4.81e+00)+	6.64e+02(1.98e+02)-	8.63e+01(3.81e+00)
14	4.97e+02(1.02e+02)-	7.04e+02(5.72e+01)-	5.54e+01(8.88e-01)+	2.23e+02(1.66e+02)=	8.87e+01(5.65e+00)
15	4.16e+02(5.11e+01)-	6.97e+02(5.40e+01)-	6.27e+01(2.63e+00)+	2.85e+02(9.64e+01)-	9.08e+01(4.05e+00)
+/-/=	0/6/0	0/6/0	6/0/0	0/5/1	NA
Rank	3.83	4.33	1	3.67	2.17
p-value	0.679	0.176	1	1	NA

TABLE S-XVII Comparison results of MGRLR with 4 state-of-the-art algorithms on 9 problems in CEC2017 (time)

Means and standard deviations of consumed time (s) are reported. The symbols '+ ', '- ', '= ' mean that the compared algorithm is significantly better than, worse than, and competitive with MGRLR.

Problem(10*D*)	GloSADE	SACCDE	MPMLS	DSI	MGRLR
1	1.99e+02(1.09e+01)-	4.83e+02(3.77e+01)-	2.69e+01(2.04e+00)+	3.02e+02(2.65e+01)-	5.34e+01(3.24e+00)
2	2.19e+02(3.24e+01)-	4.41e+02(2.60e+01)-	2.42e+01(1.63e+00)+	2.95e+02(2.94e+01)-	5.15e+01(2.59e+00)
4	2.07e+02(1.05e+01)-	3.61e+02(3.67e+01)-	2.61e+01(1.37e+00)+	2.00e+02(1.72e+02)-	5.50e+01(5.11e+00)
5	2.00e+02(2.06e+01)-	6.41e+02(8.46e+01)-	2.74e+01(2.62e+00)+	2.95e+02(2.33e+01)-	5.48e+01(2.58e+00)
13	1.80e+02(1.19e+01)-	8.79e+02(1.12e+02)-	2.51e+01(7.48e-01)+	2.41e+02(4.67e+01)-	6.28e+01(3.49e+00)
19	2.45e+02(1.87e+01)-	2.94e+02(5.46e+01)-	2.66e+01(1.25e+00)+	1.51e+02(1.13e+02)-	5.22e+01(2.00e+00)
20	2.27e+02(3.02e+01)-	7.41e+02(6.62e+01)-	2.35e+01(1.61e+00)+	4.85e+01(2.04e+01)+	6.75e+01(4.96e+00)
22	1.93e+02(1.55e+01)-	8.38e+02(3.53e+01)-	2.68e+01(1.23e+00)+	2.11e+02(5.06e+01)-	6.56e+01(3.23e+00)
28	2.11e+02(1.53e+01)-	4.78e+02(8.95e+01)-	2.56e+01(2.11e+00)+	1.33e+02(4.53e+01)-	5.14e+01(2.20e+00)
+/-/=	0/9/0	0/9/0	9/0/0	1/8/0	NA
Rank	3.44	5	1	3.44	2.11
p-value	0.736	0.001	1	0.736	NA
Problem(30*D*)	GloSADE	SACCDE	MPMLS	DSI	MGRLR
1	6.47e+02(7.45e+01)-	4.63e+02(5.87e+01)-	6.43e+01(3.23e+00)+	7.01e+02(8.77e+01)-	7.34e+01(4.90e+00)
2	6.11e+02(6.15e+01)-	5.14e+02(2.53e+01)-	6.50e+01(2.98e+00)+	5.64e+02(8.28e+01)-	7.27e+01(5.25e+00)
4	6.23e+02(6.03e+01)-	7.31e+02(6.47e+01)-	6.44e+01(3.05e+00)+	7.77e+02(1.33e+02)-	8.40e+01(1.06e+01)
5	6.25e+02(6.49e+01)-	7.97e+02(5.65e+01)-	6.59e+01(3.09e+00)+	8.24e+02(1.03e+02)-	6.84e+01(2.75e+00)
13	4.78e+02(1.07e+02)-	9.58e+02(1.39e+02)-	6.53e+01(3.08e+00)+	6.79e+02(1.66e+02)-	8.87e+01(3.61e+00)
19	4.07e+02(6.73e+01)-	7.98e+02(5.56e+01)-	6.52e+01(2.56e+00)+	3.88e+02(5.91e+01)-	6.79e+01(2.51e+00)
20	2.90e+02(5.66e+00)-	7.40e+02(2.10e+01)-	8.48e+01(1.93e+01)=	6.87e+01(3.48e+01)+	8.50e+01(4.05e+00)
22	5.65e+02(4.72e+01)-	1.01e+03(1.40e+02)-	9.12e+01(1.12e+01)=	6.36e+02(7.63e+01)-	8.85e+01(2.12e+00)
28	5.51e+02(5.71e+01)-	7.74e+02(5.29e+01)-	1.54e+02(2.83e+01)-	3.53e+02(1.20e+02)-	7.55e+01(4.75e+00)
+/-/=	0/9/0	0/9/0	6/1/2	1/8/0	NA
Rank	3.67	4.33	1.33	3.78	1.89
p-value	0.171	0.010	1	0.113	NA

F Convergence Curves

Fig S-IV Convergence curves of GloSADE, SACCDE, MPMLS, DSI, and MGRLR on 13 problems in CEC2006

The average function error is adopted as the performance indicator.

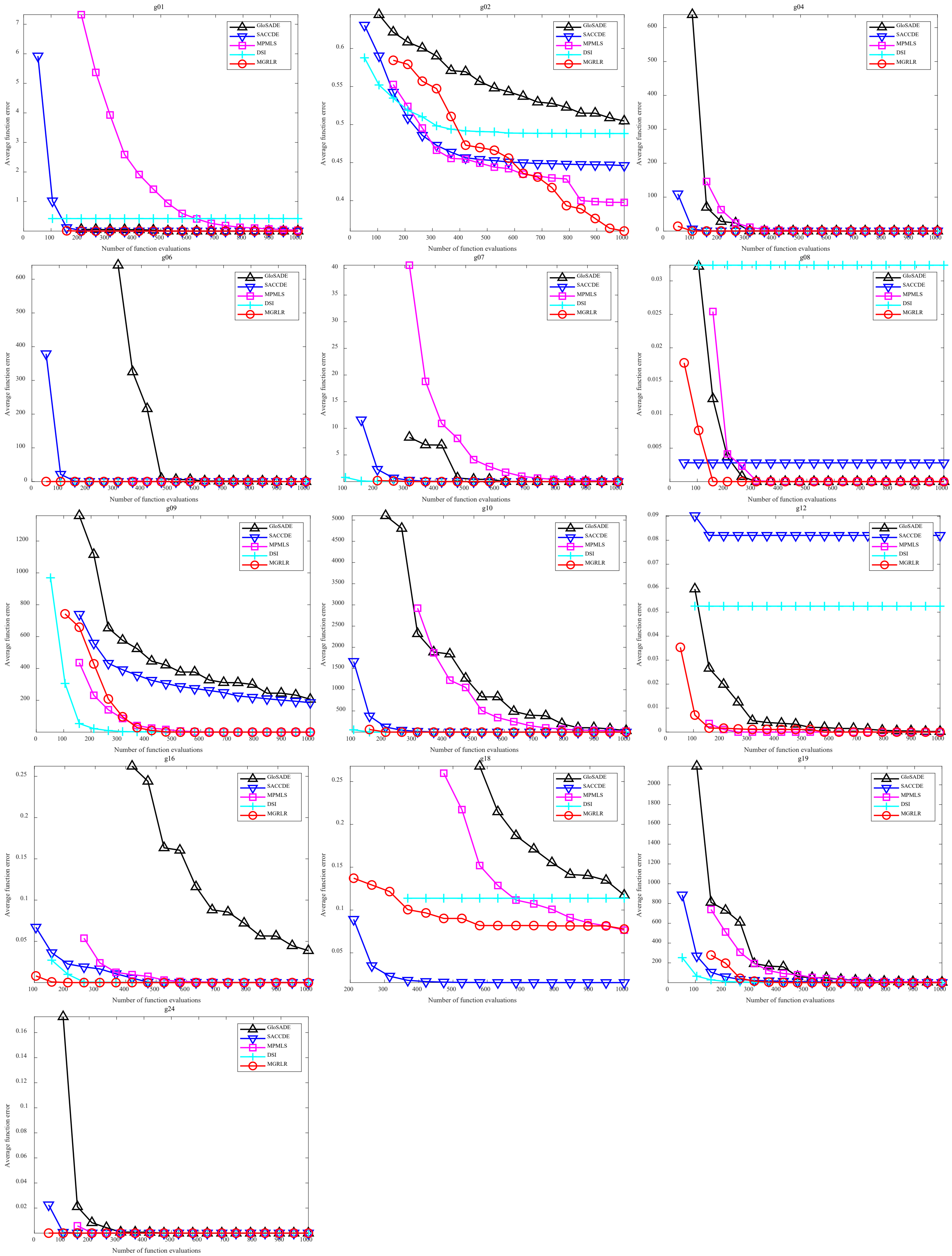


Fig S-V Convergence curves of GloSADE, SACCDE, MPMLS, DSI, and MGRLR on 6 problems in CEC2010(10D)

The average best feasible objective value is adopted as the performance indicator. The number of dimensions is 10.

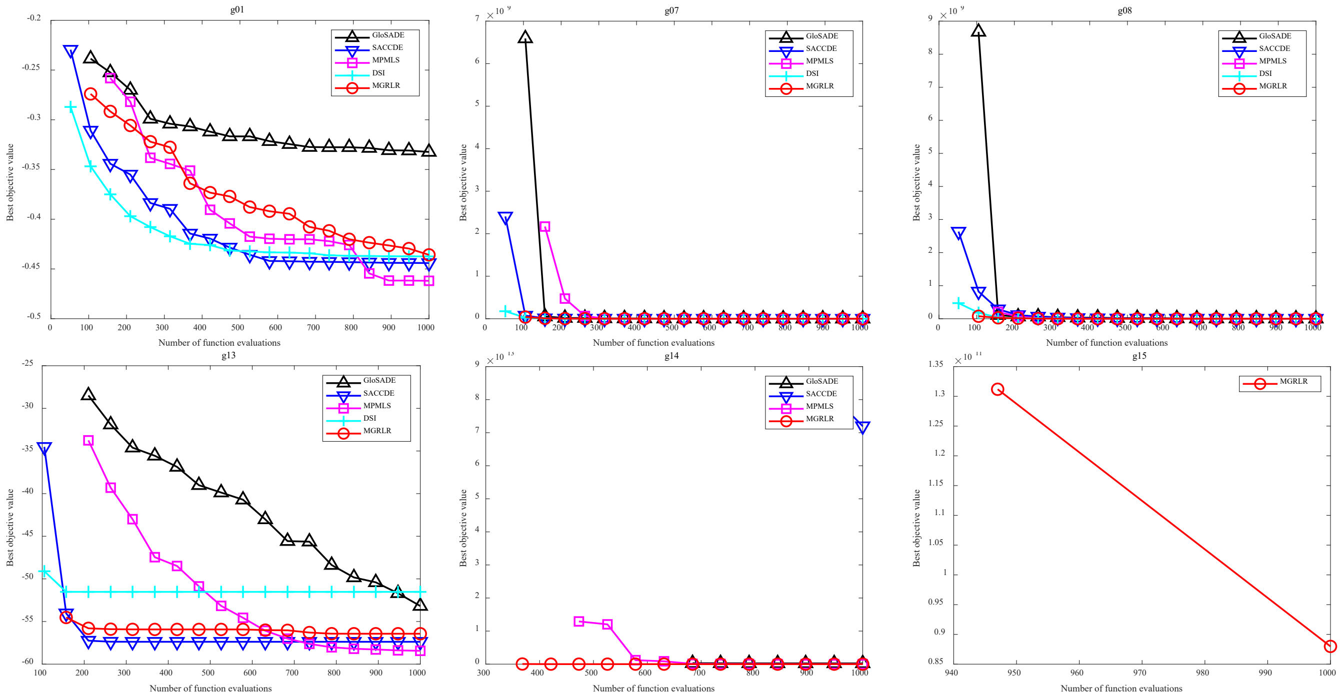


Fig S-VI Convergence curves of GloSADE, SACCDE, MPMLS, DSI, and MGRLR on 5 problems in CEC2010(30D)

The average best feasible objective value is adopted as the performance indicator. The number of dimensions is 30.

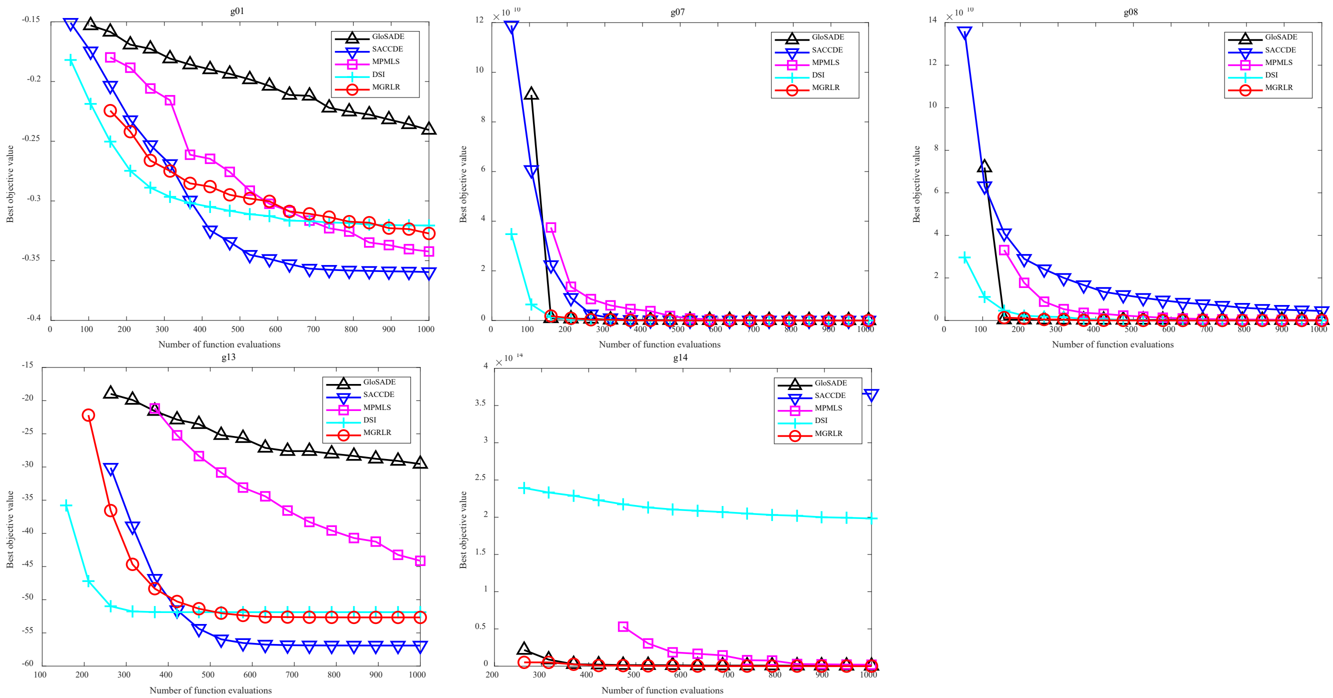


Fig S-VII Convergence curves of GloSADE, SACCDE, MPMLS, DSI, and MGRLR on 7 problems in CEC2017(10D)

The average best feasible objective value is adopted as the performance indicator. The number of dimensions is 10.

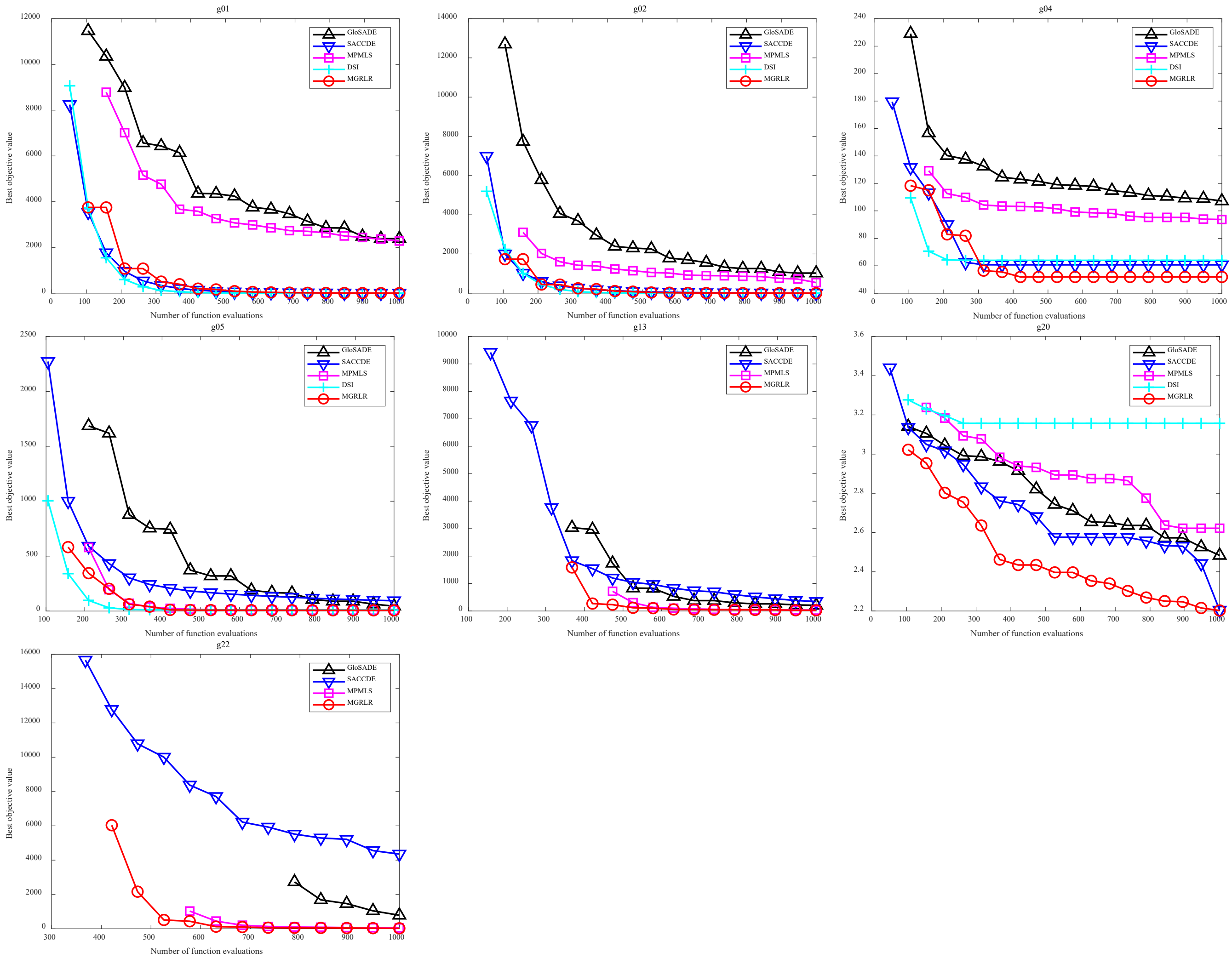
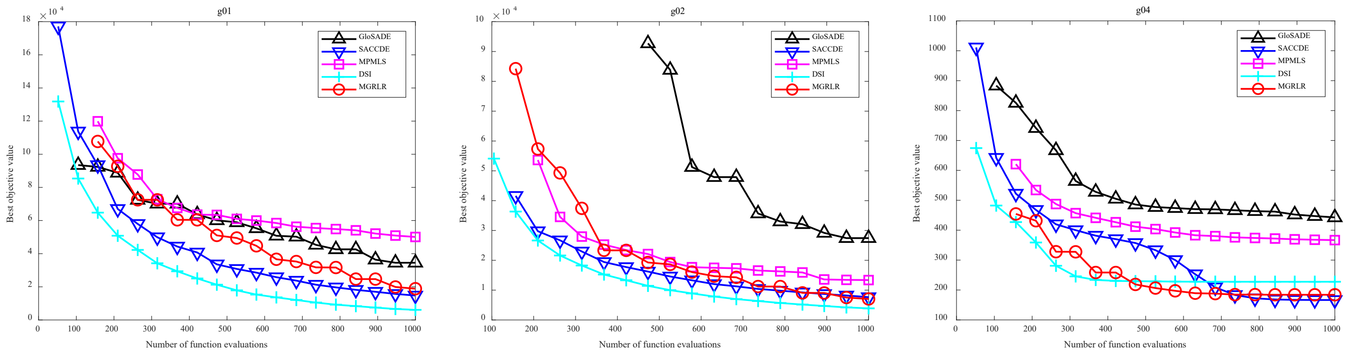
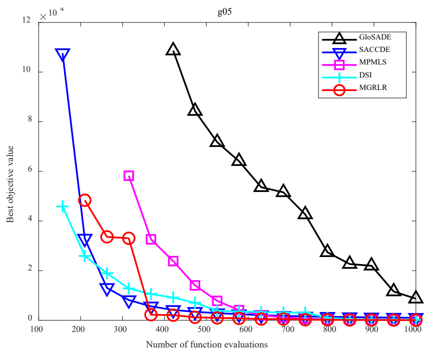
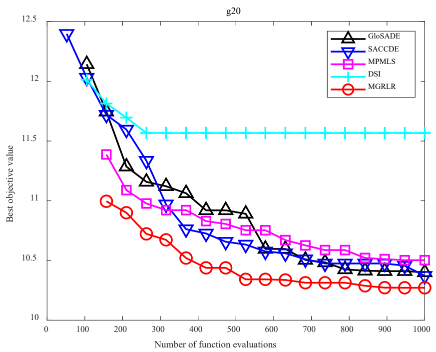


Fig S-VIII Convergence curves of GloSADE, SACCDE, MPMLS, DSI, and MGRLR on 5 problems in CEC2017(30D)

The average best feasible objective value is adopted as the performance indicator. The number of dimensions is 30.







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