C++ Implementation

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Simplifying the Difference Equation

Recall the Simple Moving Average difference equation:

$$y[n] = \frac{1}{N} \sum_{i=0}^{N-1} x[n-i] \tag{1}$$

A naive approach would be to implement the difference equation directly: keeping the last $N-1$ inputs, and calculate the sum on each iteration, calculating $N-1$ additions at each time step.
However, we can do much better if we notice how only two terms of the sum change each time:

$$\begin{aligned}
y[n+1] &= \frac{1}{N} \sum_{i=0}^{N-1} x[n+1-i] \\
&= \frac{1}{N} \left(x[n+1] + \sum_{i=1}^{N-1} x[n+1-i] \right) \\
&= \frac{1}{N} \left(x[n+1] + \sum_{i=1}^{N-1} x[n+1-i] + x[n+1-N] - x[n+1-N] \right) \\
&= \frac{1}{N} \left(x[n+1] + \sum_{i=1}^{N} x[n+1-i] - x[n+1-N] \right) \\
&= \frac{1}{N} \left(x[n+1] + \sum_{i=0}^{N-1} x[n-i] - x[n+1-N] \right) \\
&= y[n] + \frac{1}{N} \Big(x[n+1] - x[n+1-N] \Big)
\end{aligned}$$

We can now define the sum $S[n]$ as follows:

$$\begin{aligned}
S[n] &\triangleq N \cdot y[n] \\
&= \sum_{i=0}^{N-1} x[n-i] \\
\Leftrightarrow \;\; y[n] &= S[n]/N
\end{aligned}$$

The difference equation then becomes:

$$S[n+1] = S[n] + x[n+1] - x[n+1-N] \tag{2}$$

To update the sum, each iteration now requires only one addition and one subtraction, as well as some housekeeping to remember the previous inputs. To get the output $y[n]$, a division by N is needed.

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We can now implement Equation 2 directly, and we'll use a rounding division instead of truncating the quotient. Note that this rounding operation is valid for unsigned integer types only.
The previous inputs $x[n-i]$ are kept in a circular buffer.

```
#include <stdint.h>

template <uint8_t N, class input_t = uint16_t, class sum_t = uint32_t>
class SMA {
  public:
    input_t operator()(input_t input) {
        sum -= previousInputs[index];
        sum += input;
        previousInputs[index] = input;
        if (++index == N)
            index = 0;
        return (sum + (N / 2)) / N;
        static_assert(
            sum_t(0) < sum_t(-1),  // Check that `sum_t` is an unsigned type
            "Error: sum data type should be an unsigned integer, otherwise, "
            "the rounding operation in the return statement is invalid.");
    }

  private:
    uint8_t index             = 0;
    input_t previousInputs[N] = {};
    sum_t sum                 = 0;
};
```