1/7/2019 Model

## Model

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## Torque caused by motor thrust

$$\begin{split} \vec{F}_{l}(n) &= C_{T}\rho n^{2}D^{4}\begin{pmatrix} 0\\ 1\\ 1\\ \end{array} \\ &= \vec{F}_{l}(n_{h}) + \frac{d\vec{F}_{l}}{dn}(n_{h})(n-n_{h}) + \dots \\ &\approx \left(\frac{-mg}{4} + 2C_{T}\rho n_{h}D^{4}(n_{l}-n_{h})\right)\begin{pmatrix} 0\\ 0\\ 1\\ \end{pmatrix} \\ \vec{M}_{l} &= \vec{r}_{l} \times \vec{F}_{l} \\ &= \vec{r}_{l} \times \begin{pmatrix} 0\\ 0\\ 1\\ 1\\ \end{pmatrix} \left(\frac{-mg}{4} + 2C_{T}\rho n_{h}D^{4}(n_{l}-n_{h})\right) \\ \vec{M}_{1} &= \frac{L}{\sqrt{2}}\begin{pmatrix} -1\\ 1\\ 0\\ \end{pmatrix} \times \begin{pmatrix} 0\\ 0\\ 1\\ \end{pmatrix} \left(\frac{-mg}{4} + 2C_{T}\rho n_{h}D^{4}(n_{1}-n_{h})\right) \\ &= \frac{L}{\sqrt{2}}\begin{pmatrix} 1\\ 1\\ 0\\ \end{pmatrix} \times \begin{pmatrix} 0\\ 0\\ 1\\ \end{pmatrix} \left(\frac{-mg}{4} + 2C_{T}\rho n_{h}D^{4}(n_{2}-n_{h})\right) \\ \vec{M}_{2} &= \frac{L}{\sqrt{2}}\begin{pmatrix} 1\\ 1\\ 0\\ \end{pmatrix} \times \begin{pmatrix} 0\\ 0\\ 1\\ \end{pmatrix} \left(\frac{-mg}{4} + 2C_{T}\rho n_{h}D^{4}(n_{2}-n_{h})\right) \\ &= \frac{L}{\sqrt{2}}\begin{pmatrix} -1\\ -1\\ 0\\ \end{pmatrix} \times \begin{pmatrix} 0\\ 0\\ 1\\ \end{pmatrix} \left(\frac{-mg}{4} + 2C_{T}\rho n_{h}D^{4}(n_{2}-n_{h})\right) \\ \vec{M}_{3} &= \frac{L}{\sqrt{2}}\begin{pmatrix} -1\\ -1\\ 0\\ \end{pmatrix} \times \begin{pmatrix} 0\\ 0\\ 1\\ \end{pmatrix} \left(\frac{-mg}{4} + 2C_{T}\rho n_{h}D^{4}(n_{3}-n_{h})\right) \\ &= \frac{L}{\sqrt{2}}\begin{pmatrix} -1\\ 1\\ 0\\ \end{pmatrix} \times \begin{pmatrix} 0\\ 0\\ 1\\ \end{pmatrix} \left(\frac{-mg}{4} + 2C_{T}\rho n_{h}D^{4}(n_{3}-n_{h})\right) \\ \vec{M}_{4} &= \frac{L}{\sqrt{2}}\begin{pmatrix} -1\\ -1\\ 0\\ \end{pmatrix} \times \begin{pmatrix} 0\\ 0\\ 1\\ \end{pmatrix} \left(\frac{-mg}{4} + 2C_{T}\rho n_{h}D^{4}(n_{4}-n_{h})\right) \\ \vec{M} &= \vec{M}_{1} + \vec{M}_{2} + \vec{M}_{3} + \vec{M}_{4} \\ &= \frac{L}{\sqrt{2}}\begin{pmatrix} -mg}{4} - 2C_{T}\rho n_{h}^{2} \right) \left(\begin{pmatrix} 1\\ 1\\ 0\\ \end{pmatrix} + \begin{pmatrix} 1\\ -1\\ 0\\ \end{pmatrix} + \begin{pmatrix} -1\\ 1\\ 0\\ \end{pmatrix} \right) + \begin{pmatrix} -1\\ -1\\ 0\\ \end{pmatrix} \right) \\ &+ \frac{L}{\sqrt{2}}2C_{T}\rho n_{h}D^{4}n_{1} \begin{pmatrix} 1\\ 1\\ 0\\ \end{pmatrix} \\ &+ \frac{L}{\sqrt{2}}2C_{T}\rho n_{h}D^{4}n_{4} \begin{pmatrix} -1\\ -1\\ 0\\ \end{pmatrix} \\ &= \sqrt{2}LC_{T}\rho n_{h}D^{4}n_{4} \begin{pmatrix} 1\\ -1\\ 0\\ \end{pmatrix} \\ &= \sqrt{2}LC_{T}\rho n_{h}D^{4}n_{4} \begin{pmatrix} n_{1}+n_{2}-n_{3}-n_{4}\\ 0\\ \end{pmatrix} \\ &= \sqrt{2}LC_{T}\rho n_{h}D^{4} \begin{pmatrix}$$

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$$n_x riangleq rac{n_1 + n_2 - n_3 - n_4}{4}$$
 $n_y riangleq rac{n_1 - n_2 + n_3 - n_4}{4}$ 
 $= 4\sqrt{2}LC_T 
ho n_h D^4 \left(egin{array}{c} n_x \\ n_y \\ 0 \end{array}
ight)$ 

$$\begin{split} \vec{M} &= I \vec{\alpha} \\ \Leftrightarrow I^{-1} \vec{M} &= \vec{\alpha} \\ \Leftrightarrow \vec{\alpha} &= \begin{pmatrix} I_{xx}^{-1} & 0 & 0 \\ 0 & I_{yy}^{-1} & 0 \\ 0 & 0 & I_{xx}^{-1} \end{pmatrix} 4 \sqrt{2} L C_T \rho n_h D^4 \begin{pmatrix} n_x \\ n_y \\ 0 \end{pmatrix} \\ \Leftrightarrow \vec{\alpha} &= 4 \sqrt{2} L C_T \rho n_h D^4 \begin{pmatrix} \frac{n_x}{I_{xx}} \\ \frac{n_y}{I_{yy}} \\ 0 \end{pmatrix} \end{split}$$

$$k_3^x \triangleq \frac{4\sqrt{2}LC_T\rho n_h D^4}{I_{xx}} \tag{1}$$

$$k_3^x \triangleq \frac{4\sqrt{2}LC_T\rho n_h D^4}{I_{xx}}$$

$$k_3^y \triangleq \frac{4\sqrt{2}LC_T\rho n_h D^4}{I_{yy}}$$

$$(2)$$

Torque caused by ???

$$k_3^z \triangleq \frac{4C_P \rho n_h D^5}{\pi I_{zz}} \tag{3}$$

## Torque caused by inertia of the motors and propellers

$$M_{prop,m} = (I_{prop} + I_m)(-\dot{n}_1 + \dot{n}_2 + \dot{n}_3 - \dot{n}_4)$$
(4)

$$M_{prop,m} = (I_{prop} + I_m) \left( -\dot{n}_1 + \dot{n}_2 + \dot{n}_3 - \dot{n}_4 \right)$$

$$n_z \triangleq \frac{-n_1 + n_2 + n_3 - n_4}{4}$$

$$= 4 \left( I_{prop} + I_m \right) \dot{n}_z$$

$$M_{prop,m} + M_z = 0$$

$$M_z = -4 \left( I_{prop} + I_m \right) \dot{n}_z$$

$$\alpha_z = -4 \frac{I_{prop} + I_m}{I_{zz}} \dot{n}_z$$

$$(9)$$

$$=4\left(I_{prop}+I_{m}\right)\dot{n}_{z}\tag{6}$$

$$M_{prop,m} + M_z = 0 (7)$$

$$M_z = -4 \left( I_{prop} + I_m \right) \dot{n}_z \tag{8}$$

$$\alpha_z = -4 \frac{I_{prop} + I_m}{I_m} \dot{n}_z \tag{9}$$

$$\alpha_z = -4 \frac{I_{zz}}{I_{zz}} n_z$$

$$????$$

$$k_4^z \triangleq 8\pi \frac{I_{prop} + I_m}{I_{zz}}$$

$$\alpha_z = -k_4^z \dot{n}_z$$

$$(10)$$

$$(11)$$

$$\alpha_z = -k_4^z \dot{n}_z \tag{12}$$

## Complete