Formularium

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Basics

Symbols

- ullet Absolute temperature: T
- Intrinsic free carrier concentration: n_i
- Electron concentration: n
- Hole concentration: p
- Effective density of states in conduction band: N_c
- ullet Effective density of states in valence band: N_v
- Fermi level: ϕ_f
- Fermi energy: E_f
- Quasi Fermi energy for electrons (no equilibrium): E_{fn}
- ullet Quasi Fermi energy for holes (no equilibrium): E_{fp}
- Intrinsic Fermi energy: E_i
- Valence band energy: E_v
- Conduction band energy: E_c
- Donor ionization energy: E_D
- Acceptor ionization energy: E_A
- ullet Band gap energy: $E_g riangleq E_c E_f$
- ullet Number of ionized donor ions: N_D
- Number of ionized acceptor ions: N_A
- Number of equivalent energy minima in conduction band: M_c
- Density-of-state effective electron mass: m_{de}
- ullet Density-of-state effective hole mass: m_{dh}
- ullet Recombination: R
- Thermal generation: G_{th}
- External generation: G
- Net recombination rate of electrons and holes: $U \triangleq R G_{th}$
- Equilibrium electron concentration: n_{po}
- Equilibrium hole concentration: p_{no}
- Absorption coefficient: α
- ullet Planck's constant: $6.62607015 imes 10^{-34}~J~s$
- Photon frequency: ν
- ???: γ
- Auger recombination rate: R_A
- Auger coefficients: G_{An} , G_{Ap}
- Impact ionization generation rate: G_i
- Electron/hole ionization rate: α_n , α_p
- ullet Free electron/hole velocity: v_n , v_p
- ullet Impact ionization threshold energy: $E_{i,n}$, $E_{i,p}$
- Minority carrier effective lifetimes: τ_n , τ_p
- Electric field: ε
- Electrostatic potential: Ψ
- Thermal voltage: $V_t = kT/q$
- ullet Electron/hole mobility: μ_n , mu_p
- ullet Drift current density: $J_{
 m drift}$
- ullet Diffusion current density: $J_{
 m Diffusion}$
- Diffusion constants: D_n , D_p

Concentration

$$egin{aligned} n &= N_c \expigg(rac{E_f - E_c}{kT}igg) \ p &= N_v \expigg(rac{E_v - E_f}{kT}igg) \ N_c &= 2igg(rac{2\pi m_{de}kT}{h^2}igg)^{3/2}M_c \ N_v &= 2igg(rac{2\pi m_{dh}kT}{h^2}igg)^{3/2} \end{aligned}$$

Pure Semiconductor in Equilibrium

$$egin{aligned} n &= p = n_i \ n_i^2 &= np = N_c N_v \exp\left(rac{-E_g}{kT}
ight) \ E_f &= E_i = rac{E_c + E_v}{2} + rac{kT}{2} \mathrm{ln}\left(rac{N_v}{N_c}
ight) \end{aligned}$$

Doped Semiconductor in Equilibrium

Charge neutrality:

$$N_D + p = N_A + n$$

N-type

$$egin{aligned} N_D > N_A \ n &= rac{1}{2}igg(N_D - N_A + \sqrt{ig(N_D - N_A)^2 + 4n_i^2}igg) \ &pprox N_D - N_A \ p &= rac{n_i^2}{n} \ &pprox rac{n_i^2}{N_D - N_A} \ E_f &pprox E_i + kT \lnigg(rac{N_D - N_A}{n_i}igg) > E_i \end{aligned}$$

P-type

$$egin{aligned} N_A &> N_D \ p &= rac{1}{2}igg(N_A - N_D + \sqrt{\left(N_A - N_D
ight)^2 + 4n_i^2}igg) \ &pprox N_A - N_D \ n &= rac{n_i^2}{p} \ &pprox rac{n_i^2}{N_A - N_D} \ E_f &pprox E_i - kT \lnigg(rac{N_A - N_D}{n_i}igg) < E_i \end{aligned}$$

Semiconductor out of Equilibrium

$$pn > n_i^2$$
 $n = N_c \exp\left(rac{E_{fn} - E_c}{kT}
ight)$
 $= n_i \exp\left(rac{E_{fn} - E_i}{kT}
ight)$
 $p = N_v \exp\left(rac{E_v - E_{fp}}{kT}
ight)$
 $= n_i \exp\left(rac{E_i - E_{fp}}{kT}
ight)$
 $np = N_c N_v \exp\left(rac{(E_v - E_c) + (E_{fn} - E_{fp})}{kT}
ight)$
 $= n_i^2 \exp\left(rac{E_{fn} - E_{fp}}{kT}
ight)$

Carrier Recombination and Generation Mechanisms

$$U_n pprox rac{n-n_{po}}{ au_n}$$
 $U_p pprox rac{p-p_{no}}{ au_p}$
 $rac{dn}{dt} = rac{dp}{dt} = -U + G = G_{th} + G - R$

Shockley-Read-Hall generation and recombination

Impurities with energy levels within the forbidden gap.

Radiative generation and recombination

Usually negligible for indirect semiconductors

$$R_r = Bnp$$

N-type

$$egin{aligned} R_r &= Bpn_0 = rac{p}{ au_p} \ & au_p = rac{1}{Bn_0} pprox rac{1}{BN_D} \ & U_r = rac{p-p_0}{ au_p} \end{aligned}$$

P-type

$$R_r = Bnp_0 = rac{n}{ au_n}$$
 $au_n = rac{1}{Bp_0} pprox rac{1}{BN_A}$ $U_r = rac{n-n_0}{ au_n}$

Radiative generation

$$lpha = ig(h
u - E_gig)^\gamma \quad ig(h
u > E_gig)$$

Auger recombination and Impact Ionization

Auger recombination: Energy released by band-to-band recombination is given to another free hole or electron

$$egin{aligned} R_A &= G_{An} n^2 p + G_{Ap} p^2 n \ R_A &pprox G_{An} N_D^2 p = rac{p}{ au_p} \quad ext{n-type} \ R_A &pprox G_{Ap} N_A^2 n = rac{n}{ au_n} \quad ext{p-type} \end{aligned}$$

Impact ionization: Kinetic energy of an electorn or hole is released in a collision to a neutral atom and generates an electron-hole pair

$$egin{aligned} G_i &= lpha_n n v_n + lpha_p p v_p \ lpha_n(arepsilon) &= rac{qarepsilon}{E_{i,n}} \exp\left(-rac{b}{arepsilon}
ight) \ lpha_p(arepsilon) &= rac{qarepsilon}{E_{i,p}} \exp\left(-rac{b}{arepsilon}
ight) \end{aligned}$$

Effective minority carrier lifetime

$$rac{1}{ au} = rac{1}{ au_{
m SRH}} + rac{1}{ au_{
m rad}} + rac{1}{ au_{
m Auger}}$$

Carrier Transport

$$arepsilon = rac{1}{q}
abla E_i$$
 $arepsilon = -
abla \Psi$
 $\Psi = -rac{E_i}{q}$
 $\phi_f = -rac{E_f}{q}$

Equilibrium:

$$n=n_i \exp\!\left(rac{\Psi-\phi_f}{V_t}
ight)
onumber \ p=n_i \exp\!\left(rac{\phi_f-\Psi}{V_t}
ight)$$

Out of equilibrium:

$$egin{aligned} n &= n_i \expigg(rac{\Psi - \phi_{fn}}{V_t}igg) \ p &= n_i \expigg(rac{\phi_{fp} - \Psi}{V_t}igg) \ pn &= n_i^2 \expigg(rac{\phi_{fp} - \phi_{fn}}{V_t}igg) \end{aligned}$$

Drift

$$egin{aligned} v_n &= -\mu_n arepsilon \ v_p &= -\mu_p arepsilon \end{aligned}$$

TODO: p. II-24 - II-26

$$egin{aligned} J_{ ext{drift},n} &= q \mu_n n arepsilon &= \sigma_n arepsilon \ J_{ ext{drift},p} &= q \mu_p p arepsilon &= \sigma_p arepsilon \end{aligned}$$

Diffusion

$$egin{align*} J_{ ext{diffusion},n} &= -qD_n \left(-
abla n
ight) \ J_{ ext{diffusion},p} &= qD_p \left(-
abla p
ight) \ D_n &= \mu_n rac{kT}{q} \ D_p &= \mu_p rac{kT}{q} \ \end{array}$$

Current equations

$$egin{align} J_n &= q \mu_n \left(n arepsilon + rac{kT}{q}
abla n
ight) = - q \mu_n n
abla \phi_{fn} \ J_p &= q \mu_p \left(p arepsilon - rac{kT}{q}
abla p
ight) = - q \mu_p p
abla \phi_{fp} \ \end{split}$$

PN-junction

Symbols

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MOS-cap

MOSFET

N-FET

Fermi level

$$\phi_f = V_t \ln\!\left(rac{N_A}{n_i}
ight)$$

Threshold voltage

$$V_T = V_{FB} + 2\phi_f + rac{\sqrt{2q\epsilon_s N_A}}{C_{ox}}\sqrt{2\phi_f - V_{BS}}$$

Drain-Source current:

$$egin{aligned} I_{DS, ext{lin}} &= rac{\mu C_{ox}W}{L} igg(V_{GS} - V_T + rac{V_{DS}}{2}igg) V_{DS} \ I_{DS, ext{sat}} &= rac{\mu C_{ox}W}{2L} ig(V_{GS} - V_Tig)^2 \end{aligned}$$

BJT

PV cell

$$V_{oc} = V_t \ln\!\left(rac{J_{sc}}{J_0}
ight)$$

$$J_0 = q \sqrt{rac{V_t \mu_p}{ au_p}} rac{n_i^2}{N_D}
onumber$$
 $I_{np} = I_{sc} - I_S \left[\exp\!\left(rac{V_{pn}}{V_t}
ight) - 1
ight]$