# Formularium

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## Basics

### Symbols

- Absolute temperature: $T$
- Boltzmann constant: $k = 1.38064852 \times 10^{-23} \text{ m}^2 \text{ kg s}^{-2} \text{ K}^{-1}$
- Intrinsic free carrier concentration: $n_i$
- Electron concentration: $n$
- Hole concentration: $p$
- Effective density of states in conduction band: $N_c$
- Effective density of states in valence band: $N_v$
- Fermi level: $\phi_f$
- Fermi energy: $E_f$
- Quasi Fermi energy for electrons (no equilibrium): $E_{fn}$
- Quasi Fermi energy for holes (no equilibrium): $E_{fp}$
- Intrinsic Fermi energy: $E_i$
- Valence band energy: $E_v$
- Conduction band energy: $E_c$
- Donor ionization energy: $E_D$
- Acceptor ionization energy: $E_A$
- Band gap energy: $E_g \triangleq E_c - E_f$
- Number of ionized donor ions: $N_D$
- Number of ionized acceptor ions: $N_A$
- Number of equivalent energy minima in conduction band: $M_c$
- Density-of-state effective electron mass: $m_{de}$
- Density-of-state effective hole mass: $m_{dh}$
- Recombination: $R$
- Thermal generation: $G_{th}$
- External generation: $G$
- Net recombination rate of electrons and holes: $U \triangleq R - G_{th}$
- Equilibrium electron concentration: $n_{po}$
- Equilibrium hole concentration: $p_{no}$
- Absorption coefficient: $\alpha$
- Planck's constant: $6.62607015 \times 10^{-34} \text{ J s}$
- Photon frequency: $\nu$
- ???: $\gamma$
- Auger recombination rate: $R_A$
- Auger coefficients: $G_{An}, G_{Ap}$
- Impact ionization generation rate: $G_i$
- Electron/hole ionization rate: $\alpha_n, \alpha_p$
- Free electron/hole velocity: $v_n, v_p$
- Impact ionization threshold energy: $E_{i,n}, E_{i,p}$
- Minority carrier effective lifetimes: $\tau_n, \tau_p$
- Electric field: $\varepsilon$
- Electrostatic potential: $\Psi$
- Thermal voltage: $V_t = kT/q$
- Electron/hole mobility: $\mu_n, mu_p$
- Drift current density: $J_{\text{drift}}$
- Diffusion current density: $J_{\text{Diffusion}}$
- Diffusion constants: $D_n, D_p$

### Concentration

$$n = N_c \exp\left(\frac{E_f - E_c}{kT}\right)$$

$$p = N_v \exp\left(\frac{E_v - E_f}{kT}\right)$$

$$N_c = 2\left(\frac{2\pi m_{de}kT}{h^2}\right)^{3/2} M_c$$

$$N_v = 2\left(\frac{2\pi m_{dh}kT}{h^2}\right)^{3/2}$$

## Pure Semiconductor in Equilibrium

$$n = p = n_i$$

$$n_i^2 = np = N_c N_v \exp\left(\frac{-E_g}{kT}\right)$$

$$E_f = E_i = \frac{E_c + E_v}{2} + \frac{kT}{2}\ln\left(\frac{N_v}{N_c}\right)$$

## Doped Semiconductor in Equilibrium

Charge neutrality:

$$N_D + p = N_A + n$$

N-type

$$N_D > N_A$$

$$n = \frac{1}{2}\left(N_D - N_A + \sqrt{(N_D - N_A)^2 + 4n_i^2}\right)$$

$$\approx N_D - N_A$$

$$p = \frac{n_i^2}{n}$$

$$\approx \frac{n_i^2}{N_D - N_A}$$

$$E_f \approx E_i + kT\ln\left(\frac{N_D - N_A}{n_i}\right) > E_i$$

P-type

$$N_A > N_D$$

$$p = \frac{1}{2}\left(N_A - N_D + \sqrt{(N_A - N_D)^2 + 4n_i^2}\right)$$

$$\approx N_A - N_D$$

$$n = \frac{n_i^2}{p}$$

$$\approx \frac{n_i^2}{N_A - N_D}$$

$$E_f \approx E_i - kT\ln\left(\frac{N_A - N_D}{n_i}\right) < E_i$$

## Semiconductor out of Equilibrium

$$pn > n_i^2$$

$$n = N_c \exp\left(\frac{E_{fn} - E_c}{kT}\right)$$

$$= n_i \exp\left(\frac{E_{fn} - E_i}{kT}\right)$$

$$p = N_v \exp\left(\frac{E_v - E_{fp}}{kT}\right)$$

$$= n_i \exp\left(\frac{E_i - E_{fp}}{kT}\right)$$

$$np = N_c N_v \exp\left(\frac{(E_v - E_c) + (E_{fn} - E_{fp})}{kT}\right)$$

$$= n_i^2 \exp\left(\frac{E_{fn} - E_{fp}}{kT}\right)$$

## Carrier Recombination and Generation Mechanisms

$$U_n \approx \frac{n - n_{po}}{\tau_n}$$

$$U_p \approx \frac{p - p_{no}}{\tau_p}$$

$$\frac{dn}{dt} = \frac{dp}{dt} = -U + G = G_{th} + G - R$$

### Shockley-Read-Hall generation and recombination

Impurities with energy levels within the forbidden gap.

$$U_{SRH} = \text{staat hopelijk in het formularium}$$

### Radiative generation and recombination

Usually negligible for indirect semiconductors

$$R_r = Bnp$$

N-type

$$R_r = Bpn_0 = \frac{p}{\tau_p}$$
$$\tau_p = \frac{1}{Bn_0} \approx \frac{1}{BN_D}$$
$$U_r = \frac{p - p_0}{\tau_p}$$

P-type

$$R_r = Bnp_0 = \frac{n}{\tau_n}$$
$$\tau_n = \frac{1}{Bp_0} \approx \frac{1}{BN_A}$$
$$U_r = \frac{n - n_0}{\tau_n}$$

Radiative generation

$$\alpha = \left(h\nu - E_g\right)^\gamma \quad \left(h\nu > E_g\right)$$

### Auger recombination and Impact Ionization

Auger recombination: Energy released by band-to-band recombination is given to another free hole or electron

$$R_A = G_{An}n^2p + G_{Ap}p^2n$$
$$R_A \approx G_{An}N_D^2 p = \frac{p}{\tau_p} \quad \text{n-type}$$
$$R_A \approx G_{Ap}N_A^2 n = \frac{n}{\tau_n} \quad \text{p-type}$$

Impact ionization: Kinetic energy of an electorn or hole is released in a collision to a neutral atom and generates an electron-hole pair

$$G_i = \alpha_n n v_n + \alpha_p p v_p$$
$$\alpha_n(\varepsilon) = \frac{q\varepsilon}{E_{i,n}}\exp\left(-\frac{b}{\varepsilon}\right)$$
$$\alpha_p(\varepsilon) = \frac{q\varepsilon}{E_{i,p}}\exp\left(-\frac{b}{\varepsilon}\right)$$

### Effective minority carrier lifetime

$$\frac{1}{\tau} = \frac{1}{\tau_{\text{SRH}}} + \frac{1}{\tau_{\text{rad}}} + \frac{1}{\tau_{\text{Auger}}}$$

## Carrier Transport

$$\varepsilon = \frac{1}{q}\nabla E_i$$
$$\varepsilon = -\nabla\Psi$$
$$\Psi = -\frac{E_i}{q}$$
$$\phi_f = -\frac{E_f}{q}$$

Equilibrium:

$$n = n_i \exp\left(\frac{\Psi - \phi_f}{V_t}\right)$$
$$p = n_i \exp\left(\frac{\phi_f - \Psi}{V_t}\right)$$

Out of equilibrium:

$$n = n_i \exp\left(\frac{\Psi - \phi_{fn}}{V_t}\right)$$

$$p = n_i \exp\left(\frac{\phi_{fp} - \Psi}{V_t}\right)$$

$$pn = n_i^2 \exp\left(\frac{\phi_{fp} - \phi_{fn}}{V_t}\right)$$

Drift

$$v_n = -\mu_n \varepsilon$$
$$v_p = -\mu_p \varepsilon$$

TODO: p. II-24 - II-26

$$J_{\text{drift},n} = q\mu_n n\varepsilon = \sigma_n \varepsilon$$
$$J_{\text{drift},p} = q\mu_p p\varepsilon = \sigma_p \varepsilon$$

Diffusion

$$J_{\text{diffusion},n} = -qD_n\left(-\nabla n\right)$$
$$J_{\text{diffusion},p} = qD_p\left(-\nabla p\right)$$
$$D_n = \mu_n \frac{kT}{q}$$
$$D_p = \mu_p \frac{kT}{q}$$

Current equations

$$J_n = q\mu_n\left(n\varepsilon + \frac{kT}{q}\nabla n\right) = -q\mu_n n\nabla\phi_{fn}$$
$$J_p = q\mu_p\left(p\varepsilon - \frac{kT}{q}\nabla p\right) = -q\mu_p p\nabla\phi_{fp}$$

## PN-junction

### Symbols

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## MOS-cap

## MOSFET

### N-FET

Fermi level

$$\phi_f = V_t \ln\left(\frac{N_A}{n_i}\right)$$

Threshold voltage

$$V_T = V_{FB} + 2\phi_f + \frac{\sqrt{2q\epsilon_s N_A}}{C_{ox}}\sqrt{2\phi_f - V_{BS}}$$

Drain-Source current:

$$I_{DS,\text{lin}} = \frac{\mu C_{ox} W}{L}\left(V_{GS} - V_T + \frac{V_{DS}}{2}\right)V_{DS}$$
$$I_{DS,\text{sat}} = \frac{\mu C_{ox} W}{2L}(V_{GS} - V_T)^2$$

## BJT

## PV cell

$$V_{oc} = V_t \ln\left(\frac{J_{sc}}{J_0}\right)$$

$$J_0 = q\sqrt{\frac{V_t \mu_p}{\tau_p}} \frac{n_i^2}{N_D}$$

$$I_{np} = I_{sc} - I_S \left[ \exp\left(\frac{V_{pn}}{V_t}\right) - 1 \right]$$