6/23/2019 Model

Model

Pieter F

Torque caused by motor thrust

$$\begin{split} \vec{F}_l(n) &= C_T \rho n^2 D^l \begin{pmatrix} 0 \\ 1 \\ 1 \\ &= \vec{F}_i(n_h) + \frac{d\vec{F}_l}{dn}(n_h)(n-n_h) + \dots \\ &\approx \left(\frac{-mg}{4} + 2C_T \rho n_h D^l \left(n_i - n_h \right) \right) \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \\ \end{split}$$

$$\vec{M}_l &= \vec{r}_l \times \vec{F}_l \\ &= \vec{r}_l \times \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \\ \end{pmatrix} \left(\frac{-mg}{4} + 2C_T \rho n_h D^l \left(n_l - n_h \right) \right) \\ \vec{M}_1 &= \frac{L}{\sqrt{2}} \begin{pmatrix} -1 \\ 1 \\ 0 \\ 1 \\ \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ 1 \\ \end{pmatrix} \left(\frac{-mg}{4} + 2C_T \rho n_h D^l \left(n_1 - n_h \right) \right) \\ &= \frac{L}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \\ 0 \\ \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ 1 \\ \end{pmatrix} \left(\frac{-mg}{4} + 2C_T \rho n_h D^l \left(n_1 - n_h \right) \right) \\ \vec{M}_2 &= \frac{L}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \\ 0 \\ \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ 1 \\ \end{pmatrix} \left(\frac{-mg}{4} + 2C_T \rho n_h D^l \left(n_2 - n_h \right) \right) \\ &= \frac{L}{\sqrt{2}} \begin{pmatrix} -1 \\ -1 \\ 0 \\ \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ 1 \\ \end{pmatrix} \left(\frac{-mg}{4} + 2C_T \rho n_h D^l \left(n_2 - n_h \right) \right) \\ \vec{M}_3 &= \frac{L}{\sqrt{2}} \begin{pmatrix} -1 \\ -1 \\ 0 \\ \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ 1 \\ \end{pmatrix} \left(\frac{-mg}{4} + 2C_T \rho n_h D^l \left(n_3 - n_h \right) \right) \\ &= \frac{L}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \\ 0 \\ \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ 1 \\ \end{pmatrix} \left(\frac{-mg}{4} + 2C_T \rho n_h D^l \left(n_4 - n_h \right) \right) \\ \vec{M}_4 &= \frac{L}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \\ 0 \\ \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ 1 \\ \end{pmatrix} \left(\frac{-mg}{4} + 2C_T \rho n_h D^l \left(n_4 - n_h \right) \right) \\ \vec{M} &= \vec{M}_1 + \vec{M}_2 + \vec{M}_3 + \vec{M}_4 \\ &= \frac{L}{\sqrt{2}} \left(\frac{-mg}{4} - 2C_T \rho n_h^2 \right) \left(\begin{pmatrix} 1 \\ 1 \\ 0 \\ \end{pmatrix} + \begin{pmatrix} 1 \\ -1 \\ 0 \\ \end{pmatrix} + \begin{pmatrix} -1 \\ -1 \\ 0 \\ \end{pmatrix} \right) \\ &+ \frac{L}{\sqrt{2}} 2C_T \rho n_h D^l n_2 \begin{pmatrix} 1 \\ -1 \\ 0 \\ \end{pmatrix} \\ &+ \frac{L}{\sqrt{2}} 2C_T \rho n_h D^l n_3 \begin{pmatrix} 1 \\ 1 \\ 0 \\ \end{pmatrix} \\ &+ \frac{L}{\sqrt{2}} 2C_T \rho n_h D^l n_3 \begin{pmatrix} 1 \\ 1 \\ 0 \\ \end{pmatrix} \\ &= \sqrt{2} L C_T \rho n_h D^l n_4 \begin{pmatrix} n_1 + n_2 - n_3 - n_4 \\ n_1 - n_2 + n_3 - n_4 \\ \end{pmatrix} \\ &= \sqrt{2} L C_T \rho n_h D^l \begin{pmatrix} n_1 + n_2 - n_3 - n_4 \\ n_1 - n_2 + n_3 - n_4 \\ \end{pmatrix}$$

6/23/2019 Model

$$egin{aligned} n_x & riangleq rac{n_1+n_2-n_3-n_4}{4} \ n_y & riangleq rac{n_1-n_2+n_3-n_4}{4} \ &= 4\sqrt{2}LC_T
ho n_h D^4 \left(egin{aligned} n_x \ n_y \ 0 \end{array}
ight) \end{aligned}$$

$$egin{aligned} ec{M} &= I ec{lpha} \ \Leftrightarrow I^{-1} ec{M} &= ec{lpha} \ \Leftrightarrow ec{lpha} &= egin{pmatrix} I_{xx}^{-1} & 0 & 0 \ 0 & I_{yy}^{-1} & 0 \ 0 & 0 & I_{xx}^{-1} \end{pmatrix} 4 \sqrt{2} L C_T
ho n_h D^4 egin{pmatrix} n_x \ n_y \ 0 \end{pmatrix} \ \Leftrightarrow ec{lpha} &= 4 \sqrt{2} L C_T
ho n_h D^4 egin{pmatrix} rac{n_x}{I_{xx}} \ rac{n_y}{I_{yy}} \ 0 \end{pmatrix} \end{aligned}$$

$$k_3^x \triangleq \frac{4\sqrt{2}LC_T\rho n_h D^4}{I_{xx}} \tag{1}$$

$$k_3^x \triangleq \frac{4\sqrt{2}LC_T\rho n_h D^4}{I_{xx}}$$

$$k_3^y \triangleq \frac{4\sqrt{2}LC_T\rho n_h D^4}{I_{yy}}$$

$$(2)$$

Torque caused by ???

$$k_3^z \triangleq \frac{4C_P \rho n_h D^5}{\pi I_{zz}} \tag{3}$$

Torque caused by inertia of the motors and propellers

$$M_{prop,m} = (I_{prop} + I_m)(-\dot{n}_1 + \dot{n}_2 + \dot{n}_3 - \dot{n}_4)$$
 (4)

$$M_{prop,m} = (I_{prop} + I_m) \left(-\dot{n}_1 + \dot{n}_2 + \dot{n}_3 - \dot{n}_4 \right)$$

$$n_z \triangleq \frac{-n_1 + n_2 + n_3 - n_4}{4}$$

$$= 4 \left(I_{prop} + I_m \right) \dot{n}_z$$

$$M_{prop,m} + M_z = 0$$

$$M_z = -4 \left(I_{prop} + I_m \right) \dot{n}_z$$

$$\alpha_z = -4 \frac{I_{prop} + I_m}{I_{zz}} \dot{n}_z$$

$$(9)$$

$$=4\left(I_{prop}+I_{m}\right)\dot{n}_{z}\tag{6}$$

$$M_{prop,m} + M_z = 0 (7)$$

$$M_z = -4 (I_{prop} + I_m) \dot{n}_z \tag{8}$$

$$I_{prop} + I_m$$
 . (2)

$$\alpha_z = -4 \frac{I_{prop} + I_m}{I_{zz}} \dot{n}_z \tag{9}$$

$$??? (10)$$

$$\alpha_z = -4 \frac{I_{zz}}{I_{zz}} n_z$$

$$????$$

$$k_4^z \triangleq 8\pi \frac{I_{prop} + I_m}{I_{zz}}$$

$$\alpha_z = -k_4^z \dot{n}_z$$

$$(10)$$

$$(11)$$

$$\alpha_z = -k_4^z \dot{n}_z \tag{12}$$

Complete