

# Impulse and Step Response

Pieter P

Here, we'll discuss two important signals that are often used in signal processing, the delta function or unit impulse, and the unit step function.

## The Kronecker Delta Function

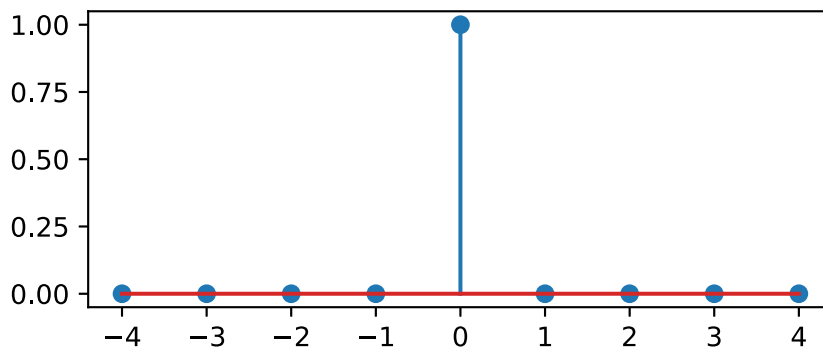
---

The Kronecker delta function or unit impulse  $\delta[n]$  is defined as a discrete function that is one when  $n$  is zero, and zero everywhere else:

$$\delta : \mathbb{Z} \rightarrow \mathbb{R} : n \mapsto \delta[n] \triangleq \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$

An alternative notation is  $\delta_{n,k}$ . This value is one if  $n = k$  and zero if  $n \neq k$ :

$$\delta_{n,k} \triangleq \delta[n - k]$$



### Impulse response

The impulse response  $h[n]$  of a DTLTI system  $T$  is defined as the output of the system when a Kronecker delta function is applied to its input:

$$h[n] \triangleq T(\delta[n])$$

The letter  $h$  will be used to refer to the impulse response of a system. As we'll see later, the impulse response can be used to define the system.

### Properties of the Kronecker Delta Function

The most important property of the Kronecker delta is its ability to select a single term from an (infinite) sum:

$$\begin{aligned} & \sum_{n=0}^{\infty} x[n] \cdot \delta[n - k] \\ &= x[0] \cdot \delta[0 - k] + x[1] \cdot \delta[1 - k] + \dots + x[k] \cdot \delta[k - k] + \dots \\ &= x[k] \end{aligned}$$

As you can see, all terms where  $n \neq k$  are zero, so only the  $k$ -th term remains.

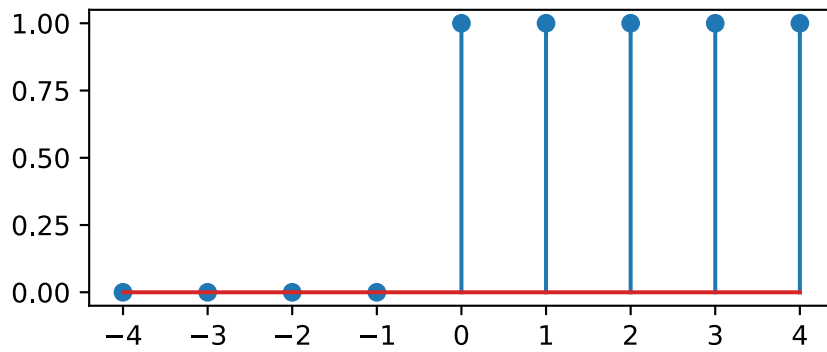
This is sometimes referred to as the **sifting property** of the delta function.

## The Heaviside Step Function

---

The (discrete) Heaviside step function  $U[n]$  is defined as a discrete function that is zero when  $n$  is negative, and one if  $n$  is zero or positive:

$$U : \mathbb{Z} \rightarrow \mathbb{R} : n \mapsto U[n] \triangleq \begin{cases} 0 & n < 0 \\ 1 & n \geq 0 \end{cases}$$



## Step Response

Just like the impulse response, we can define the step response as the output of the system when the Heaviside step function is applied to the input:

$$y_{\text{step}}[n] \triangleq T(U[n])$$

The step response is an important tool when investigating how a system responds to transients.

Unlike the impulse response, there is no specific symbol or letter for the step response.

## Properties of the Heaviside Step Function

The step function can also be written as the cumulative sum of the delta function:

$$U[n] = \sum_{k=-\infty}^n \delta[k]$$


---