Formularium

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Basics

Symbols

- Absolute temperature: T
- Boltzmann constant: $k = 1.38064852 \times 10^{-23}\ \mathrm{m^2\ kg\ s^{-2}\ K^{-1}}$
- Intrinsic free carrier concentration: n_i
- Electron concentration: n
- Hole concentration: p
- Effective density of states in conduction band: N_c
- Effective density of states in valence band: N_v
- Fermi level: ϕ_f
- Fermi energy: E_f
- Quasi Fermi energy for electrons (no equilibrium): E_{fn}
- Quasi Fermi energy for holes (no equilibrium): E_{fp}
- Intrinsic Fermi energy: E_i
- Valence band energy: E_v
- Conduction band energy: E_c
- Donor ionization energy: E_D
- Acceptor ionization energy: E_A
- Band gap energy: $E_g \triangleq E_c - E_f$
- Number of ionized donor ions: N_D
- Number of ionized acceptor ions: N_A
- Number of equivalent energy minima in conduction band: M_c
- Density-of-state effective electron mass: m_{de}
- Density-of-state effective hole mass: m_{dh}
- Recombination: R
- Thermal generation: G_{th}
- External generation: G
- Net recombination rate of electrons and holes: $U \triangleq R - G_{th}$
- Equilibrium electron concentration: n_{po}
- Equilibrium hole concentration: p_{no}
- Absorption coefficient: α
- Planck's constant: $6.62607015 \times 10^{-34}\ \mathrm{J\ s}$
- Photon frequency: ν
- ???: γ
- Auger recombination rate: R_A
- Auger coefficients: G_{An}, G_{Ap}
- Impact ionization generation rate: G_i
- Electron/hole ionization rate: α_n, α_p
- Free electron/hole velocity: v_n, v_p
- Impact ionization threshold energy: $E_{i,n}$, $E_{i,p}$
- Minority carrier effective lifetimes: τ_n, τ_p
- Electric field: ε
- Electrostatic potential: Ψ
- Thermal voltage: $V_t = kT/q$
- Electron/hole mobility: μ_n, mu_p
- Drift current density: J_{drift}
- Diffusion current density: $J_{\mathrm{Diffusion}}$
- Diffusion constants: D_n, D_p

Concentration

$$n = N_c \exp\left(\frac{E_f - E_c}{kT}\right)$$
$$p = N_v \exp\left(\frac{E_v - E_f}{kT}\right)$$

$$N_c = 2\left(\frac{2\pi m_{de} kT}{h^2}\right)^{3/2} M_c$$
$$N_v = 2\left(\frac{2\pi m_{dh} kT}{h^2}\right)^{3/2}$$

Pure Semiconductor in Equilibrium

$$
\begin{aligned}
n &= p = n_i \\
n_i^2 &= np = N_c N_v \exp\left(\frac{-E_g}{kT}\right) \\
E_f &= E_i = \frac{E_c + E_v}{2} + \frac{kT}{2}\ln\left(\frac{N_v}{N_c}\right)
\end{aligned}
$$

Doped Semiconductor in Equilibrium

Charge neutrality:

$$N_D + p = N_A + n$$

N-type

$$
\begin{aligned}
N_D &> N_A \\
n &= \frac{1}{2}\left(N_D - N_A + \sqrt{(N_D - N_A)^2 + 4n_i^2}\right) \\
&\approx N_D - N_A \\
p &= \frac{n_i^2}{n} \\
&\approx \frac{n_i^2}{N_D - N_A} \\
E_f &\approx E_i + kT\ln\left(\frac{N_D - N_A}{n_i}\right) > E_i
\end{aligned}
$$

P-type

$$
\begin{aligned}
N_A &> N_D \\
p &= \frac{1}{2}\left(N_A - N_D + \sqrt{(N_A - N_D)^2 + 4n_i^2}\right) \\
&\approx N_A - N_D \\
n &= \frac{n_i^2}{p} \\
&\approx \frac{n_i^2}{N_A - N_D} \\
E_f &\approx E_i - kT\ln\left(\frac{N_A - N_D}{n_i}\right) < E_i
\end{aligned}
$$

Semiconductor out of Equilibrium

$$pn > n_i^2$$

$$
\begin{aligned}
n &= N_c \exp\left(\frac{E_{fn} - E_c}{kT}\right) \\
&= n_i \exp\left(\frac{E_{fn} - E_i}{kT}\right) \\
p &= N_v \exp\left(\frac{E_v - E_{fp}}{kT}\right) \\
&= n_i \exp\left(\frac{E_i - E_{fp}}{kT}\right) \\
np &= N_c N_v \exp\left(\frac{(E_v - E_c) + (E_{fn} - E_{fp})}{kT}\right) \\
&= n_i^2 \exp\left(\frac{E_{fn} - E_{fp}}{kT}\right)
\end{aligned}
$$

Carrier Recombination and Generation Mechanisms

$$
\begin{aligned}
U_n &\approx \frac{n - n_{po}}{\tau_n} \\
U_p &\approx \frac{p - p_{no}}{\tau_p} \\
\frac{dn}{dt} &= \frac{dp}{dt} = -U + G = G_{th} + G - R
\end{aligned}
$$

Shockley-Read-Hall generation and recombination

Impurities with energy levels within the forbidden gap.

$$U_{SRH} = \text{staat hopelijk in het formularium}$$

Radiative generation and recombination

Usually negligible for indirect semiconductors

$$R_r = Bnp$$

N-type

$$\begin{aligned} R_r &= Bpn_0 = \frac{p}{\tau_p} \\ \tau_p &= \frac{1}{Bn_0} \approx \frac{1}{BN_D} \\ U_r &= \frac{p - p_0}{\tau_p} \end{aligned}$$

P-type

$$\begin{aligned} R_r &= Bnp_0 = \frac{n}{\tau_n} \\ \tau_n &= \frac{1}{Bp_0} \approx \frac{1}{BN_A} \\ U_r &= \frac{n - n_0}{\tau_n} \end{aligned}$$

Radiative generation

$$\alpha = \left(h\nu - E_g\right)^\gamma \quad \left(h\nu > E_g\right)$$

Auger recombination and Impact Ionization

Auger recombination: Energy released by band-to-band recombination is given to another free hole or electron

$$\begin{aligned} R_A &= G_{An}n^2p + G_{Ap}p^2n \\ R_A &\approx G_{An}N_D^2p = \frac{p}{\tau_p} \quad \text{n-type} \\ R_A &\approx G_{Ap}N_A^2n = \frac{n}{\tau_n} \quad \text{p-type} \end{aligned}$$

Impact ionization: Kinetic energy of an electorn or hole is released in a collision to a neutral atom and generates an electron-hole pair

$$\begin{aligned} G_i &= \alpha_n n v_n + \alpha_p p v_p \\ \alpha_n(\varepsilon) &= \frac{q\varepsilon}{E_{i,n}} \exp\left(-\frac{b}{\varepsilon}\right) \\ \alpha_p(\varepsilon) &= \frac{q\varepsilon}{E_{i,p}} \exp\left(-\frac{b}{\varepsilon}\right) \end{aligned}$$

Effective minority carrier lifetime

$$\frac{1}{\tau} = \frac{1}{\tau_{\text{SRH}}} + \frac{1}{\tau_{\text{rad}}} + \frac{1}{\tau_{\text{Auger}}}$$

Carrier Transport

$$\begin{aligned} \varepsilon &= \frac{1}{q} \nabla E_i \\ \varepsilon &= -\nabla \Psi \\ \Psi &= -\frac{E_i}{q} \\ \\ \phi_f &= -\frac{E_f}{q} \end{aligned}$$

Equilibrium:

$$\begin{aligned} n &= n_i \exp\left(\frac{\Psi - \phi_f}{V_t}\right) \\ p &= n_i \exp\left(\frac{\phi_f - \Psi}{V_t}\right) \end{aligned}$$

Out of equilibrium:

$$n = n_i \exp\left(\frac{\Psi - \phi_{fn}}{V_t}\right)$$
$$p = n_i \exp\left(\frac{\phi_{fp} - \Psi}{V_t}\right)$$
$$pn = n_i^2 \exp\left(\frac{\phi_{fp} - \phi_{fn}}{V_t}\right)$$

Drift

$$v_n = -\mu_n \varepsilon$$
$$v_p = -\mu_p \varepsilon$$

TODO: p. II-24 - II-26

$$J_{\text{drift},n} = q\mu_n n\varepsilon = \sigma_n \varepsilon$$
$$J_{\text{drift},p} = q\mu_p p\varepsilon = \sigma_p \varepsilon$$

Diffusion

$$J_{\text{diffusion},n} = -qD_n\left(-\nabla n\right)$$
$$J_{\text{diffusion},p} = qD_p\left(-\nabla p\right)$$
$$D_n = \mu_n \frac{kT}{q}$$
$$D_p = \mu_p \frac{kT}{q}$$

Current equations

$$J_n = q\mu_n \left(n\varepsilon + \frac{kT}{q}\nabla n\right) = -q\mu_n n \nabla \phi_{fn}$$
$$J_p = q\mu_p \left(p\varepsilon - \frac{kT}{q}\nabla p\right) = -q\mu_p p \nabla \phi_{fp}$$

PN-junction

Symbols

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MOS-cap

MOSFET

N-FET

Fermi level

$$\phi_f = V_t \ln\left(\frac{N_A}{n_i}\right)$$

Threshold voltage

$$V_T = V_{FB} + 2\phi_f + \frac{\sqrt{2q\epsilon_s N_A}}{C_{ox}}\sqrt{2\phi_f - V_{BS}}$$

Drain-Source current:

$$I_{DS,\text{lin}} = \frac{\mu C_{ox} W}{L}\left(V_{GS} - V_T + \frac{V_{DS}}{2}\right) V_{DS}$$
$$I_{DS,\text{sat}} = \frac{\mu C_{ox} W}{2L}\left(V_{GS} - V_T\right)^2$$

BJT

PV cell

$$V_{oc} = V_t \ln\left(\frac{J_{sc}}{J_0}\right)$$

$$J_0 = q\sqrt{\frac{V_t \mu_p}{\tau_p}}\frac{n_i^2}{N_D}$$

$$I_{np} = I_{sc} - I_S \left[\exp\left(\frac{V_{pn}}{V_t} \right) - 1 \right]$$