Formularium

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Basics

Symbols

- Absolute temperature: T
- \bullet Boltzmann constant: $k = 1.38064852 \times 10^{-23} \ \mathrm{m^2 \ kg \ s^{-2} \ K^{-1}}$
- Intrinsic free carrier concentration: n_i
- Electron concentration: n
- Hole concentration: p
- ullet Effective density of states in conduction band: N_c
- ullet Effective density of states in valence band: N_v
- Fermi level: ϕ_f
- Fermi energy: E_f
- Quasi Fermi energy for electrons (no equilibrium): E_{fn}
- Quasi Fermi energy for holes (no equilibrium): E_{fp}
- Intrinsic Fermi energy: E_i
- Valence band energy: E_v
- Conduction band energy: Ec
- Donor ionization energy: E_D
- Acceptor ionization energy: E_A
- ullet Band gap energy: $E_g riangleq E_c E_f$
- Number of ionized donor ions: N_D
- ullet Number of ionized acceptor ions: N_A
- Number of equivalent energy minima in conduction band: M_c
- Density-of-state effective electron mass: m_{de}
- Density-of-state effective hole mass: m_{dh}
- Recombination: R
- Thermal generation: G_{th}
- ullet External generation: G
- Net recombination rate of electrons and holes: $U riangleq R G_{th}$
- Equilibrium electron concentration: n_{po}
- ullet Equilibrium hole concentration: p_{no}
- Absorption coefficient: α
- $\bullet~$ Planck's constant: $6.62607015\times 10^{-34}~J~s$
- Photon frequency: ν
- ???: γ
- ullet Auger recombination rate: R_A
- ullet Auger coefficients: G_{An} , G_{Ap}
- ullet Impact ionization generation rate: G_i
- Electron/hole ionization rate: α_n , α_p
- ullet Free electron/hole velocity: v_n , v_p
- Impact ionization threshold energy: $E_{i,n}$, $E_{i,p}$
- Minority carrier effective lifetimes: τ_n , τ_p
- Electric field: ε
- ullet Electrostatic potential: Ψ
- Thermal voltage: $V_t = kT/q$
- Electron/hole mobility: μ_n , mu_p
- ullet Drift current density: $J_{
 m drift}$
- $\bullet~$ Diffusion current density: $J_{\rm Diffusion}$
- Diffusion constants: D_n , D_p

Concentration

$$n = N_c \exp\!\left(rac{E_f - E_c}{kT}
ight)
onumber \ p = N_v \exp\!\left(rac{E_v - E_f}{kT}
ight)$$

$$N_c=2igg(rac{2\pi m_{de}kT}{h^2}igg)^{3/2}M_c
onumber \ N_v=2igg(rac{2\pi m_{dh}kT}{h^2}igg)^{3/2}$$

Pure Semiconductor in Equilibrium

$$egin{aligned} n &= p = n_i \ n_i^2 &= np = N_c N_v \exp\left(rac{-E_g}{kT}
ight) \ E_f &= E_i = rac{E_c + E_v}{2} + rac{kT}{2} \mathrm{ln}\!\left(rac{N_v}{N_c}
ight) \end{aligned}$$

Doped Semiconductor in Equilibrium

Charge neutrality:

$$N_D + p = N_A + n$$

N-type

$$egin{aligned} N_D &> N_A \ n &= rac{1}{2}igg(N_D - N_A + \sqrt{ig(N_D - N_Aig)^2 + 4n_i^2}igg) \ &pprox N_D - N_A \ p &= rac{n_i^2}{n} \ &pprox rac{n_i^2}{N_D - N_A} \ E_f &pprox E_i + kT \lnigg(rac{N_D - N_A}{n_i}igg) > E_i \end{aligned}$$

P-type

$$egin{aligned} N_A &> N_D \ p &= rac{1}{2}igg(N_A - N_D + \sqrt{(N_A - N_D)^2 + 4n_i^2}igg) \ &pprox N_A - N_D \ n &= rac{n_i^2}{p} \ &pprox rac{n_i^2}{N_A - N_D} \ E_f &pprox E_i - kT \lnigg(rac{N_A - N_D}{n_i}igg) < E_i \end{aligned}$$

Semiconductor out of Equilibrium

$$pn > n_i^2$$
 $n = N_c \exp\left(rac{E_{fn} - E_c}{kT}
ight)$
 $= n_i \exp\left(rac{E_{fn} - E_i}{kT}
ight)$
 $p = N_v \exp\left(rac{E_v - E_{fp}}{kT}
ight)$
 $= n_i \exp\left(rac{E_i - E_{fp}}{kT}
ight)$
 $np = N_c N_v \exp\left(rac{(E_v - E_c) + (E_{fn} - E_{fp})}{kT}
ight)$
 $= n_i^2 \exp\left(rac{E_{fn} - E_{fp}}{kT}
ight)$

Carrier Recombination and Generation Mechanisms

$$egin{split} U_n &pprox rac{r_n}{ au_n} \ U_p &pprox rac{p-p_{no}}{ au_p} \ rac{dn}{dt} &= rac{dp}{dt} = -U + G = G_{th} + G - R \end{split}$$

Shockley-Read-Hall generation and recombination

Impurities with energy levels within the forbidden gap.

 $U_{SRH} = \text{staat hopelijk in het formularium}$

Radiative generation and recombination

Usually negligible for indirect semiconductors

$$R_r = Bnp$$

N-type

$$egin{aligned} R_r &= Bpn_0 = rac{p}{ au_p} \ & au_p = rac{1}{Bn_0} pprox rac{1}{BN_D} \ & U_r = rac{p-p_0}{ au_p} \end{aligned}$$

P-type

$$egin{aligned} R_r &= Bnp_0 = rac{n}{ au_n} \ au_n &= rac{1}{Bp_0} pprox rac{1}{BN_A} \ U_r &= rac{n-n_0}{ au_n} \end{aligned}$$

Radiative generation

$$lpha = ig(h
u - E_qig)^{\gamma} \quad ig(h
u > E_qig)$$

Auger recombination and Impact Ionization

Auger recombination: Energy released by band-to-band recombination is given to another free hole or electron

$$egin{aligned} R_A &= G_{An} n^2 p + G_{Ap} p^2 n \ R_A &pprox G_{An} N_D^2 p = rac{p}{ au_p} \quad ext{n-type} \ R_A &pprox G_{Ap} N_A^2 n = rac{n}{ au_n} \quad ext{p-type} \end{aligned}$$

Impact ionization: Kinetic energy of an electorn or hole is released in a collision to a neutral atom and generates an electron-hole pair

$$egin{aligned} G_i &= lpha_n n v_n + lpha_p p v_p \ lpha_n(arepsilon) &= rac{qarepsilon}{E_{i,n}} \mathrm{exp}igg(-rac{b}{arepsilon}igg) \ lpha_p(arepsilon) &= rac{qarepsilon}{E_{i,p}} \mathrm{exp}igg(-rac{b}{arepsilon}igg) \end{aligned}$$

Effective minority carrier lifetime

$$rac{1}{ au} = rac{1}{ au_{
m SRH}} + rac{1}{ au_{
m rad}} + rac{1}{ au_{
m Auger}}$$

Carrier Transport

$$arepsilon = rac{1}{q}
abla E_i \ arepsilon = -
abla \Psi \ = -rac{E_i}{q}$$

 $\phi_f = -rac{E_f}{q}$

Equilibrium:

 $n = n_i \exp\!\left(rac{\Psi - \phi_f}{V_t}
ight)
onumber \ p = n_i \exp\!\left(rac{\phi_f - \Psi}{V_t}
ight)$

Out of equilibrium:

$$egin{aligned} n &= n_i \exp\!\left(rac{\Psi - \phi_{fn}}{V_t}
ight) \ p &= n_i \exp\!\left(rac{\phi_{fp} - \Psi}{V_t}
ight) \ pn &= n_i^2 \exp\!\left(rac{\phi_{fp} - \phi_{fn}}{V_t}
ight) \end{aligned}$$

Drift

$$egin{aligned} v_n &= -\mu_n arepsilon \ v_p &= -\mu_p arepsilon \end{aligned}$$

TODO: p. II-24 - II-26

$$egin{aligned} J_{ ext{drift},n} &= q \mu_n n arepsilon = \sigma_n arepsilon \ J_{ ext{drift},p} &= q \mu_p p arepsilon = \sigma_p arepsilon \end{aligned}$$

Diffusion

$$\begin{split} J_{\text{diffusion},n} &= -qD_n \left(-\nabla n \right) \\ J_{\text{diffusion},p} &= qD_p \left(-\nabla p \right) \\ D_n &= \mu_n \frac{kT}{q} \\ D_p &= \mu_p \frac{kT}{q} \end{split}$$

Current equations

$$egin{align} J_n &= q \mu_n \left(n arepsilon + rac{kT}{q}
abla n
ight) = - q \mu_n n
abla \phi_{fn} \ J_p &= q \mu_p \left(p arepsilon - rac{kT}{q}
abla p
ight) = - q \mu_p p
abla \phi_{fp} \ \end{split}$$

PN-junction

Symbols

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MOS-cap

MOSFET

N-FET

Fermi level

$$\phi_f = V_t \ln\!\left(rac{N_A}{n_i}
ight)$$

Threshold voltage

$$V_T = V_{FB} + 2\phi_f + rac{\sqrt{2q\epsilon_s N_A}}{C_{ox}} \sqrt{2\phi_f - V_{BS}}$$

Drain-Source current:

$$I_{DS, ext{lin}} = rac{\mu C_{ox} W}{L} \left(V_{GS} - V_T + rac{V_{DS}}{2}
ight) V_{DS} \ I_{DS, ext{sat}} = rac{\mu C_{ox} W}{2L} \left(V_{GS} - V_T
ight)^2$$

BJT

PV cell

$$egin{aligned} V_{oc} &= V_t \ln\!\left(rac{J_{sc}}{J_0}
ight) \ J_0 &= q \sqrt{rac{V_t \mu_p}{ au_p}} rac{n_i^2}{N_D} \ I_{np} &= I_{sc} - I_S \left[\exp\!\left(rac{V_{pn}}{V_t}
ight) - 1
ight] \end{aligned}$$