Model

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Torque caused by motor thrust

$$\vec{F}_i(n) = C_T \rho n^2 D^4 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$= \vec{F}_i(n_h) + \frac{d\vec{F}_i}{dn}(n_h)(n - n_h) + \dots$$

$$\approx \left(\frac{-mg}{4} + 2C_T \rho n_h D^4 (n_i - n_h) \right) \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\vec{M}_i = \vec{r}_i \times \vec{F}_i$$

$$= \vec{r}_i \times \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \left(\frac{-mg}{4} + 2C_T \rho n_h D^4 (n_i - n_h) \right)$$

$$\vec{M}_1 = \frac{L}{\sqrt{2}} \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \left(\frac{-mg}{4} + 2C_T \rho n_h D^4 (n_1 - n_h) \right)$$

$$= \frac{L}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \left(\frac{-mg}{4} + 2C_T \rho n_h D^4 (n_1 - n_h) \right)$$

$$\vec{M}_2 = \frac{L}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \left(\frac{-mg}{4} + 2C_T \rho n_h D^4 (n_2 - n_h) \right)$$

$$= \frac{L}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} \left(\frac{-mg}{4} + 2C_T \rho n_h D^4 (n_2 - n_h) \right)$$

$$\vec{M}_3 = \frac{L}{\sqrt{2}} \begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \left(\frac{-mg}{4} + 2C_T \rho n_h D^4 (n_3 - n_h) \right)$$

$$= \frac{L}{\sqrt{2}} \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} \left(\frac{-mg}{4} + 2C_T \rho n_h D^4 (n_3 - n_h) \right)$$

$$\vec{M}_4 = \frac{L}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \left(\frac{-mg}{4} + 2C_T \rho n_h D^4 (n_4 - n_h) \right)$$

$$= \frac{L}{\sqrt{2}} \begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix} \left(\frac{-mg}{4} + 2C_T \rho n_h D^4 (n_4 - n_h) \right)$$

$$\vec{M} = \vec{M}_1 + \vec{M}_2 + \vec{M}_3 + \vec{M}_4$$

$$= \frac{L}{\sqrt{2}} \left(\frac{-mg}{4} - 2C_T \rho n_h^2 \right) \left(\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} + \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} + \begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix} \right)$$

$$+ \frac{L}{\sqrt{2}} 2C_T \rho n_h D^4 n_1 \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

$$+ \frac{L}{\sqrt{2}} 2C_T \rho n_h D^4 n_2 \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$$

$$+ \frac{L}{\sqrt{2}} 2C_T \rho n_h D^4 n_3 \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$$

$$+ \frac{L}{\sqrt{2}} 2C_T \rho n_h D^4 n_4 \begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix}$$

$$= \sqrt{2} L C_T \rho n_h D^4 \begin{pmatrix} n_1 + n_2 - n_3 - n_4 \\ n_1 - n_2 + n_3 - n_4 \\ 0 \end{pmatrix}$$

$$n_x \triangleq \frac{n_1 + n_2 - n_3 - n_4}{4}$$

$$n_y \triangleq \frac{n_1 - n_2 + n_3 - n_4}{4}$$

$$= 4\sqrt{2}LC_T\rho n_h D^4 \begin{pmatrix} n_x \\ n_y \\ 0 \end{pmatrix}$$

$$\vec{M} = I\vec{\alpha}$$

$$\Leftrightarrow I^{-1}\vec{M} = \vec{\alpha}$$

$$\Leftrightarrow \vec{\alpha} = \begin{pmatrix} I_{xx}^{-1} & 0 & 0 \\ 0 & I_{yy}^{-1} & 0 \\ 0 & 0 & I_{xx}^{-1} \end{pmatrix} 4\sqrt{2}LC_T\rho n_h D^4 \begin{pmatrix} n_x \\ n_y \\ 0 \end{pmatrix}$$

$$\Leftrightarrow \vec{\alpha} = 4\sqrt{2}LC_T\rho n_h D^4 \begin{pmatrix} \frac{n_x}{I_{xx}} \\ \frac{n_y}{I_{yy}} \\ 0 \end{pmatrix}$$

$$k_3^x \triangleq \frac{4\sqrt{2}LC_T\rho n_h D^4}{I_{xx}} \tag{1}$$

$$k_3^y \triangleq \frac{4\sqrt{2}LC_T\rho n_h D^4}{I_{yy}} \tag{2}$$

Torque caused by ???

$$k_3^z \triangleq \frac{4C_P\rho n_h D^5}{\pi I_{zz}} \tag{3}$$

Torque caused by inertia of the motors and propellers

$$M_{prop,m} = (I_{prop} + I_m)(-\dot{n}_1 + \dot{n}_2 + \dot{n}_3 - \dot{n}_4) \tag{4}$$

$$n_z \triangleq \frac{-n_1 + n_2 + n_3 - n_4}{4} \tag{5}$$

$$= 4(I_{prop} + I_m)\dot{n}_z \tag{6}$$

$$M_{prop,m} + M_z = 0 \tag{7}$$

$$M_z = -4(I_{prop} + I_m)\dot{n}_z \tag{8}$$

$$\alpha_z = -4\frac{I_{prop} + I_m}{I_{zz}}\dot{n}_z \tag{9}$$

$$??? \tag{10}$$

$$k_4^z \triangleq 8\pi\frac{I_{prop} + I_m}{I_{zz}} \tag{11}$$

$$\alpha_z = -k_4^z\dot{n}_z \tag{12}$$

Complete