Model

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Torque caused by motor thrust

$$\begin{split} \vec{F_i}(n) \;=\;& C_T \rho n^2 D^4 \begin{pmatrix}0 \\ 0 \\ 1\end{pmatrix} \\ \;=\;& \vec{F_i}(n_h) + \frac{d\vec{F_i}}{dn}(n_h) (n-n_h) + \dots \\ \;\approx\;& \left(\frac{-mg}{4} + 2 C_T \rho n_h D^4 \left(n_i-n_h\right)\right) \begin{pmatrix}0 \\ 0 \\ 1\end{pmatrix} \\ \\ \vec{M_i} \;=\;& \vec{r_i}\times\vec{F_i} \\ \;=\;& \vec{r_i} \times \begin{pmatrix}0 \\ 0 \\ 1\end{pmatrix} \left(\frac{-mg}{4} + 2 C_T \rho n_h D^4 \left(n_i-n_h\right)\right) \\ \vec{M_1} \;=\;& \frac{L}{\sqrt{2}} \begin{pmatrix}-1 \\ 1 \\ 0\end{pmatrix} \times \begin{pmatrix}0 \\ 0 \\ 1\end{pmatrix} \left(\frac{-mg}{4} + 2 C_T \rho n_h D^4 \left(n_1-n_h\right)\right) \\ \;=\;& \frac{L}{\sqrt{2}} \begin{pmatrix}1 \\ 1 \\ 0\end{pmatrix} \left(\frac{-mg}{4} + 2 C_T \rho n_h D^4 \left(n_1-n_h\right)\right) \\ \vec{M_2} \;=\;& \frac{L}{\sqrt{2}} \begin{pmatrix}1 \\ 1 \\ 0\end{pmatrix} \times \begin{pmatrix}0 \\ 0 \\ 1\end{pmatrix} \left(\frac{-mg}{4} + 2 C_T \rho n_h D^4 \left(n_2-n_h\right)\right) \\ \;=\;& \frac{L}{\sqrt{2}} \begin{pmatrix}1 \\ -1 \\ 0\end{pmatrix} \left(\frac{-mg}{4} + 2 C_T \rho n_h D^4 \left(n_2-n_h\right)\right) \\ \vec{M_3} \;=\;& \frac{L}{\sqrt{2}} \begin{pmatrix}-1 \\ -1 \\ 0\end{pmatrix} \times \begin{pmatrix}0 \\ 0 \\ 1\end{pmatrix} \left(\frac{-mg}{4} + 2 C_T \rho n_h D^4 \left(n_3-n_h\right)\right) \\ \;=\;& \frac{L}{\sqrt{2}} \begin{pmatrix}-1 \\ 1 \\ 0\end{pmatrix} \left(\frac{-mg}{4} + 2 C_T \rho n_h D^4 \left(n_3-n_h\right)\right) \\ \vec{M_4} \;=\;& \frac{L}{\sqrt{2}} \begin{pmatrix}1 \\ -1 \\ 0\end{pmatrix} \times \begin{pmatrix}0 \\ 0 \\ 1\end{pmatrix} \left(\frac{-mg}{4} + 2 C_T \rho n_h D^4 \left(n_4-n_h\right)\right) \\ \;=\;& \frac{L}{\sqrt{2}} \begin{pmatrix}-1 \\ -1 \\ 0\end{pmatrix} \left(\frac{-mg}{4} + 2 C_T \rho n_h D^4 \left(n_4-n_h\right)\right) \\ \\ \vec{M} \;=\;& \vec{M_1} + \vec{M_2} + \vec{M_3} + \vec{M_4} \\ \;=\;& \frac{L}{\sqrt{2}} \left(\frac{-mg}{4} - 2 C_T \rho n_h^2 \right) \left(\begin{pmatrix}1 \\ 1 \\ 0\end{pmatrix} + \begin{pmatrix}1 \\ -1 \\ 0\end{pmatrix} + \begin{pmatrix}-1 \\ 1 \\ 0\end{pmatrix} + \begin{pmatrix}-1 \\ -1 \\ 0\end{pmatrix} \right) \\ & + \frac{L}{\sqrt{2}} 2 C_T \rho n_h D^4 n_1 \begin{pmatrix}1 \\ 1 \\ 0\end{pmatrix} \\ & + \frac{L}{\sqrt{2}} 2 C_T \rho n_h D^4 n_2 \begin{pmatrix}1 \\ -1 \\ 0\end{pmatrix} \\ & + \frac{L}{\sqrt{2}} 2 C_T \rho n_h D^4 n_3 \begin{pmatrix}-1 \\ 1 \\ 0\end{pmatrix} \\ & + \frac{L}{\sqrt{2}} 2 C_T \rho n_h D^4 n_4 \begin{pmatrix}-1 \\ -1 \\ 0\end{pmatrix} \\ \;=\;& \sqrt{2} L C_T \rho n_h D^4 \begin{pmatrix} n_1 + n_2 - n_3 - n_4 \\ n_1 - n_2 + n_3 - n_4 \\ 0 \end{pmatrix} \\ n_x \triangleq \frac{n_1 + n_2 - n_3 - n_4} {4} \\ n_y \triangleq \frac{n_1 - n_2 + n_3 - n_4} {4} \\ \;=\;& 4\sqrt{2} L C_T \rho n_h D^4 \begin{pmatrix} n_x \\ n_y \\ 0 \end{pmatrix} \\ \\ \vec{M} \;=\;& I\vec{\alpha} \\ \Leftrightarrow\;&I^{-1}\vec{M} \;=\; \vec{\alpha} \\ \Leftrightarrow\;&\vec{\alpha}\;=\; \begin{pmatrix} I_{xx}^{-1} & 0 & 0 \\ 0 & I_{yy}^{-1} & 0 \\ 0 & 0 & I_{xx}^{-1} \end{pmatrix} 4\sqrt{2} L C_T \rho n_h D^4 \begin{pmatrix} n_x \\ n_y \\ 0 \end{pmatrix} \\ \Leftrightarrow\;&\vec{\alpha}\;=\; 4\sqrt{2} L C_T \rho n_h D^4 \begin{pmatrix} \frac{n_x}{I_{xx}} \\ \frac{n_y}{I_{yy}} \\ 0 \end{pmatrix} \end{split}$$

$$\begin{align} k_3^x \;\triangleq\;& \frac{4\sqrt{2} L C_T \rho n_h D^4} {I_{xx}} \\ k_3^y \;\triangleq\;& \frac{4\sqrt{2} L C_T \rho n_h D^4} {I_{yy}} \end{align}$$

Torque caused by ???

$$\begin{align} k_3^z \;\triangleq\;& \frac{4 C_P \rho n_h D^5} {\pi I_{zz}} \end{align}$$

Torque caused by inertia of the motors and propellers

$$\begin{align} M_{prop, m} \;=\;& \left(I_{prop} + I_m\right) \left(-\dot{n}_1 + \dot{n}_2 + \dot{n}_3 - \dot{n}_4 \right) \\ n_z \triangleq \frac{-n_1 + n_2 + n_3 - n_4} {4} \\ \;=\;& 4 \left(I_{prop} + I_m\right) \dot{n}_z \\ M_{prop, m} + M_z \;=\;& 0 \\ M_z \;=\;& -4 \left(I_{prop} + I_m\right) \dot{n}_z \\ \alpha_z \;=\;& -4 \frac{I_{prop} + I_m} {I_{zz}} \dot{n}_z \\ ??? \\ k_4^z \;\triangleq\; 8\pi \frac{I_{prop} + I_m} {I_{zz}} \\ \alpha_z \;=\;& -k_4^z \dot{n}_z \end{align}$$

Complete