# Formularium

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## **Basics**

## Symbols

- ullet Absolute temperature: T
- Intrinsic free carrier concentration:  $n_i$
- Electron concentration: n
- Hole concentration: p
- Effective density of states in conduction band:  $N_c$
- ullet Effective density of states in valence band:  $N_v$
- Fermi level:  $\phi_f$
- Fermi energy:  $E_f$
- ullet Quasi Fermi energy for electrons (no equilibrium):  $E_{fn}$
- Quasi Fermi energy for holes (no equilibrium):  $E_{fp}$
- Intrinsic Fermi energy:  $E_i$
- Valence band energy:  $E_v$
- Conduction band energy:  $E_c$
- Donor ionization energy:  $E_D$
- ullet Acceptor ionization energy:  $E_A$
- ullet Band gap energy:  $E_g riangleq E_c E_f$
- ullet Number of ionized donor ions:  $N_D$
- Number of ionized acceptor ions:  $N_A$
- ullet Number of equivalent energy minima in conduction band:  $M_c$
- ullet Density-of-state effective electron mass:  $m_{de}$
- Density-of-state effective hole mass:  $m_{dh}$
- ullet Recombination: R
- ullet Thermal generation:  $G_{th}$
- ullet External generation: G
- ullet Net recombination rate of electrons and holes:  $U riangleq R G_{th}$
- ullet Equilibrium electron concentration:  $n_{po}$
- ullet Equilibrium hole concentration:  $p_{no}$
- Absorption coefficient:  $\alpha$
- $\bullet~$  Planck's constant:  $6.62607015\times 10^{-34}~J~s$
- Photon frequency:  $\nu$
- ???: γ
- Auger recombination rate:  $R_A$
- Auger coefficients:  $G_{An}$ ,  $G_{Ap}$
- ullet Impact ionization generation rate:  $G_i$
- Electron/hole ionization rate:  $\alpha_n$ ,  $\alpha_p$
- ullet Free electron/hole velocity:  $v_n$ ,  $v_p$
- ullet Impact ionization threshold energy:  $E_{i,n}$ ,  $E_{i,p}$
- Minority carrier effective lifetimes:  $au_n$ ,  $au_p$
- Electric field:  $\varepsilon$
- ullet Electrostatic potential:  $\Psi$
- Thermal voltage:  $V_t = kT/q$
- Electron/hole mobility:  $\mu_n$ ,  $mu_p$
- ullet Drift current density:  $J_{
  m drift}$
- ullet Diffusion current density:  $J_{
  m Diffusion}$
- Diffusion constants:  $D_n$ ,  $D_p$

## Concentration

$$n = N_c \exp\!\left(rac{E_f - E_c}{kT}
ight) 
onumber \ p = N_v \exp\!\left(rac{E_v - E_f}{kT}
ight)$$

$$N_c=2igg(rac{2\pi m_{de}kT}{h^2}igg)^{3/2}M_c 
onumber \ N_v=2igg(rac{2\pi m_{dh}kT}{h^2}igg)^{3/2}$$

Pure Semiconductor in Equilibrium

$$egin{aligned} n &= p = n_i \ n_i^2 &= np = N_c N_v \exp\left(rac{-E_g}{kT}
ight) \ E_f &= E_i = rac{E_c + E_v}{2} + rac{kT}{2} \ln\!\left(rac{N_v}{N_c}
ight) \end{aligned}$$

## Doped Semiconductor in Equilibrium

Charge neutrality:

$$N_D + p = N_A + n$$

N-type

$$egin{aligned} N_D > N_A \ n &= rac{1}{2}igg(N_D - N_A + \sqrt{\left(N_D - N_A
ight)^2 + 4n_i^2}igg) \ &pprox N_D - N_A \ p &= rac{n_i^2}{n} \ &pprox rac{n_i^2}{N_D - N_A} \ E_f pprox E_i + kT\lnigg(rac{N_D - N_A}{n_i}igg) > E_i \end{aligned}$$

P-type

$$egin{aligned} N_A &> N_D \ p &= rac{1}{2} igg( N_A - N_D + \sqrt{(N_A - N_D)^2 + 4 n_i^2} igg) \ &pprox N_A - N_D \ n &= rac{n_i^2}{p} \ &pprox rac{n_i^2}{N_A - N_D} \ E_f &pprox E_i - kT \ln igg( rac{N_A - N_D}{n_i} igg) < E_i \end{aligned}$$

## Semiconductor out of Equilibrium

$$pn > n_i^2$$
 $n = N_c \exp\left(rac{E_{fn} - E_c}{kT}
ight)$ 
 $= n_i \exp\left(rac{E_{fn} - E_i}{kT}
ight)$ 
 $p = N_v \exp\left(rac{E_v - E_{fp}}{kT}
ight)$ 
 $= n_i \exp\left(rac{E_i - E_{fp}}{kT}
ight)$ 
 $np = N_c N_v \exp\left(rac{(E_v - E_c) + (E_{fn} - E_{fp})}{kT}
ight)$ 
 $= n_i^2 \exp\left(rac{E_{fn} - E_{fp}}{kT}
ight)$ 

## Carrier Recombination and Generation Mechanisms

$$egin{aligned} U_n &pprox rac{n-n_{po}}{ au_n} \ U_p &pprox rac{p-p_{no}}{ au_p} \ rac{dn}{dt} &= rac{dp}{dt} = -U + G = G_{th} + G - R \end{aligned}$$

#### Shockley-Read-Hall generation and recombination

Impurities with energy levels within the forbidden gap.

 $U_{SRH}={
m staat}$  hopelijk in het formularium

#### Radiative generation and recombination

Usually negligible for indirect semiconductors

$$R_r = Bnp$$

N-type

$$R_r = Bpn_0 = rac{p}{ au_p}$$
  $au_p = rac{1}{Bn_0} pprox rac{1}{BN_D}$   $U_r = rac{p-p_0}{ au_p}$ 

P-type

$$R_r = Bnp_0 = rac{n}{ au_n}$$
  $au_n = rac{1}{Bp_0} pprox rac{1}{BN_A}$   $U_r = rac{n-n_0}{ au_n}$ 

Radiative generation

$$lpha = ig(h
u - E_gig)^{\gamma} \quad ig(h
u > E_gig)$$

# Auger recombination and Impact Ionization

Auger recombination: Energy released by band-to-band recombination is given to another free hole or electron

$$egin{aligned} R_A &= G_{An} n^2 p + G_{Ap} p^2 n \ R_A &pprox G_{An} N_D^2 p = rac{p}{ au_p} \quad ext{n-type} \ R_A &pprox G_{Ap} N_A^2 n = rac{n}{ au_n} \quad ext{p-type} \end{aligned}$$

Impact ionization: Kinetic energy of an electorn or hole is released in a collision to a neutral atom and generates an electron-hole pair

$$egin{aligned} G_i &= lpha_n n v_n + lpha_p p v_p \ lpha_n(arepsilon) &= rac{qarepsilon}{E_{i,n}} \exp\left(-rac{b}{arepsilon}
ight) \ lpha_p(arepsilon) &= rac{qarepsilon}{E_{i,p}} \exp\left(-rac{b}{arepsilon}
ight) \end{aligned}$$

Effective minority carrier lifetime

$$\frac{1}{\tau} = \frac{1}{\tau_{\text{SRH}}} + \frac{1}{\tau_{\text{rad}}} + \frac{1}{\tau_{\text{Auger}}}$$

## **Carrier Transport**

$$arepsilon = rac{1}{q} 
abla E_i$$
 $arepsilon = -
abla \Psi$ 
 $\Psi = -rac{E_i}{q}$ 

$$\phi_f = -rac{E_f}{q}$$

Equilibrium:

$$n = n_i \exp\!\left(rac{\Psi - \phi_f}{V_t}
ight) 
onumber \ p = n_i \exp\!\left(rac{\phi_f - \Psi}{V_t}
ight)$$

Out of equilibrium:

$$egin{aligned} n &= n_i \exp\!\left(rac{\Psi - \phi_{fn}}{V_t}
ight) \ p &= n_i \exp\!\left(rac{\phi_{fp} - \Psi}{V_t}
ight) \ pn &= n_i^2 \exp\!\left(rac{\phi_{fp} - \phi_{fn}}{V_t}
ight) \end{aligned}$$

Drift

$$egin{aligned} v_n &= -\mu_n arepsilon \ v_p &= -\mu_p arepsilon \end{aligned}$$

TODO: p. II-24 - II-26

$$egin{aligned} J_{ ext{drift},n} &= q \mu_n n arepsilon &= \sigma_n arepsilon \ J_{ ext{drift},p} &= q \mu_p p arepsilon &= \sigma_p arepsilon \end{aligned}$$

Diffusion

$$egin{align*} J_{ ext{diffusion},n} &= -qD_n\left(-
abla n
ight) \ J_{ ext{diffusion},p} &= qD_p\left(-
abla p
ight) \ D_n &= \mu_n rac{kT}{q} \ D_p &= \mu_p rac{kT}{q} \ \end{array}$$

Current equations

$$egin{align} J_n &= q \mu_n \left( n arepsilon + rac{kT}{q} 
abla n 
ight) = -q \mu_n n 
abla \phi_{fn} \ J_p &= q \mu_p \left( p arepsilon - rac{kT}{q} 
abla p 
ight) = -q \mu_p p 
abla \phi_{fp} \ \end{split}$$

## PN-junction

Symbols

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# MOS-cap

# **MOSFET**

N-FET

Fermi level

$$\phi_f = V_t \ln\!\left(rac{N_A}{n_i}
ight)$$

Threshold voltage

$$V_T = V_{FB} + 2\phi_f + rac{\sqrt{2q\epsilon_s N_A}}{C_{ox}} \sqrt{2\phi_f - V_{BS}}$$

Drain-Source current:

$$egin{aligned} I_{DS, ext{lin}} &= rac{\mu C_{ox}W}{L} igg(V_{GS} - V_T + rac{V_{DS}}{2}igg) V_{DS} \ I_{DS, ext{sat}} &= rac{\mu C_{ox}W}{2L} ig(V_{GS} - V_Tig)^2 \end{aligned}$$

BJT

PV cell

$$egin{aligned} V_{oc} &= V_t \ln\!\left(rac{J_{sc}}{J_0}
ight) \ & J_0 &= q \sqrt{rac{V_t \mu_p}{ au_p}} rac{n_i^2}{N_D} \ & I_{np} &= I_{sc} - I_S \left[\exp\!\left(rac{V_{pn}}{V_t}
ight) - 1
ight] \end{aligned}$$