Model

Pieter P

Torque caused by motor thrust

$$
\begin{aligned}
\vec{F}_i(n) &= C_T \rho n^2 D^4 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \\
&= \vec{F}_i(n_h) + \frac{d\vec{F}_i}{dn}(n_h)(n - n_h) + \ldots \\
&\approx \left(\frac{-mg}{4} + 2 C_T \rho n_h D^4 \left(n_i - n_h\right) \right) \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}
\end{aligned}
$$

$$
\begin{aligned}
\vec{M}_i &= \vec{r}_i \times \vec{F}_i \\
&= \vec{r}_i \times \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \left(\frac{-mg}{4} + 2 C_T \rho n_h D^4 \left(n_i - n_h\right) \right) \\
\vec{M}_1 &= \frac{L}{\sqrt{2}} \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \left(\frac{-mg}{4} + 2 C_T \rho n_h D^4 \left(n_1 - n_h\right) \right) \\
&= \frac{L}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \left(\frac{-mg}{4} + 2 C_T \rho n_h D^4 \left(n_1 - n_h\right) \right) \\
\vec{M}_2 &= \frac{L}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \left(\frac{-mg}{4} + 2 C_T \rho n_h D^4 \left(n_2 - n_h\right) \right) \\
&= \frac{L}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} \left(\frac{-mg}{4} + 2 C_T \rho n_h D^4 \left(n_2 - n_h\right) \right) \\
\vec{M}_3 &= \frac{L}{\sqrt{2}} \begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \left(\frac{-mg}{4} + 2 C_T \rho n_h D^4 \left(n_3 - n_h\right) \right) \\
&= \frac{L}{\sqrt{2}} \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} \left(\frac{-mg}{4} + 2 C_T \rho n_h D^4 \left(n_3 - n_h\right) \right) \\
\vec{M}_4 &= \frac{L}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \left(\frac{-mg}{4} + 2 C_T \rho n_h D^4 \left(n_4 - n_h\right) \right) \\
&= \frac{L}{\sqrt{2}} \begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix} \left(\frac{-mg}{4} + 2 C_T \rho n_h D^4 \left(n_4 - n_h\right) \right)
\end{aligned}
$$

$$
\begin{aligned}
\vec{M} &= \vec{M}_1 + \vec{M}_2 + \vec{M}_3 + \vec{M}_4 \\
&= \frac{L}{\sqrt{2}} \left(\frac{-mg}{4} - 2 C_T \rho n_h^2 \right) \left(\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} + \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} + \begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix} \right) \\
&\quad + \frac{L}{\sqrt{2}} 2 C_T \rho n_h D^4 n_1 \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \\
&\quad + \frac{L}{\sqrt{2}} 2 C_T \rho n_h D^4 n_2 \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} \\
&\quad + \frac{L}{\sqrt{2}} 2 C_T \rho n_h D^4 n_3 \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} \\
&\quad + \frac{L}{\sqrt{2}} 2 C_T \rho n_h D^4 n_4 \begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix} \\
&= \sqrt{2} L C_T \rho n_h D^4 \begin{pmatrix} n_1 + n_2 - n_3 - n_4 \\ n_1 - n_2 + n_3 - n_4 \\ 0 \end{pmatrix}
\end{aligned}
$$

$n_1 + n_2 - n_3 - n_4$

$$n_x \triangleq \frac{n_1 + n_2 - n_3 - n_4}{4}$$
$$n_y \triangleq \frac{n_1 - n_2 + n_3 - n_4}{4}$$
$$= 4\sqrt{2} L C_T \rho n_h D^4 \begin{pmatrix} n_x \\ n_y \\ 0 \end{pmatrix}$$

$$\begin{aligned} \vec{M} &= I\vec{\alpha} \\ \Leftrightarrow I^{-1}\vec{M} &= \vec{\alpha} \\ \Leftrightarrow \vec{\alpha} &= \begin{pmatrix} I_{xx}^{-1} & 0 & 0 \\ 0 & I_{yy}^{-1} & 0 \\ 0 & 0 & I_{xx}^{-1} \end{pmatrix} 4\sqrt{2} L C_T \rho n_h D^4 \begin{pmatrix} n_x \\ n_y \\ 0 \end{pmatrix} \\ \Leftrightarrow \vec{\alpha} &= 4\sqrt{2} L C_T \rho n_h D^4 \begin{pmatrix} \frac{n_x}{I_{xx}} \\ \frac{n_y}{I_{yy}} \\ 0 \end{pmatrix} \end{aligned}$$

$$k_3^x \triangleq \frac{4\sqrt{2} L C_T \rho n_h D^4}{I_{xx}} \tag{1}$$
$$k_3^y \triangleq \frac{4\sqrt{2} L C_T \rho n_h D^4}{I_{yy}} \tag{2}$$

Torque caused by ???

$$k_3^z \triangleq \frac{4 C_P \rho n_h D^5}{\pi I_{zz}} \tag{3}$$

Torque caused by inertia of the motors and propellers

$$M_{prop,m} = \left(I_{prop} + I_m\right)\left(-\dot{n}_1 + \dot{n}_2 + \dot{n}_3 - \dot{n}_4\right) \tag{4}$$
$$n_z \triangleq \frac{-n_1 + n_2 + n_3 - n_4}{4} \tag{5}$$
$$= 4\left(I_{prop} + I_m\right)\dot{n}_z \tag{6}$$
$$M_{prop,m} + M_z = 0 \tag{7}$$
$$M_z = -4\left(I_{prop} + I_m\right)\dot{n}_z \tag{8}$$
$$\alpha_z = -4\frac{I_{prop} + I_m}{I_{zz}}\dot{n}_z \tag{9}$$
$$??? \tag{10}$$
$$k_4^z \triangleq 8\pi\frac{I_{prop} + I_m}{I_{zz}} \tag{11}$$
$$\alpha_z = -k_4^z \dot{n}_z \tag{12}$$

Complete