The Z-transform

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Signals as a Sum of Delta Functions

Any discrete signal can be written as an infinite sum of scaled Kronecker delta functions.

$$x[n] = \sum_{k=0}^{+\infty} x[k]\delta[n-k]$$

You can easily see that all terms where $n \neq k$ are zero, because the Kronecker delta is zero in that case. Only the term for $n = k$ is non-zero, in which case the Kronecker delta is one, so the result is just $x[k]$. This is a consequence of the sifting property of the delta function, covered in the previous page.

DTLTI Transformations as Convolutions

You can express the output of any discrete-time linear time-invariant system T as the convolution of the input with the impulse response of the system, $h[n]$:

$$T\left(x[n]\right) = x[n] * h[n]$$

Proof

The proof itself is very simple: We just decompose the input as a sum of delta functions, as described in the previous paragraph, and then we use the linearity and time-invariance to bring the T operator inside of the summation.

$$\begin{aligned} y[n] &= T\left(x[n]\right) \\ &= T\left(\sum_{k=0}^{\infty} x[k]\delta[n-k]\right) \\ &= \sum_{k=0}^{\infty} T\left(x[k]\delta[n-k]\right) \\ &= \sum_{k=0}^{\infty} x[k]T\left(\delta[n-k]\right) \\ &= \sum_{k=0}^{\infty} x[k]h[n-k] \\ &\triangleq x[n] * h[n] \end{aligned}$$

The $*$ symbol in the last step is called the convolution operator, and it is defined as the sum in the step before it. $\square$

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