# The Z-transform

Pieter l

### Signals as a Sum of Delta Functions

Any discrete signal can be written as an infinite sum of scaled Kronecker delta functions.

$$x[n] = \sum_{k=0}^{+\infty} x[k] \delta[n-k]$$

You can easily see that all terms where  $n \neq k$  are zero, because the Kronecker delta is zero in that case. Only the term for n = k is non-zero, in which case the Kronecker delta is one, so the result is just x[k]. This is a consequence of the sifting property of the delta function, covered in the <u>previous page</u>.

#### DTLTI Transformations as Convolutions

You can express the output of any discrete-time linear time-invariant system T as the convolution of the input with the impulse response of the system, h[n]:

$$T(x[n]) = x[n] * h[n]$$

#### Proof

The proof itself is very simple: We just decompose the input as a sum of delta functions, as described in the previous paragraph, and then we use the linearity and time-invariance to bring the T operator inside of the summation.

$$egin{aligned} y[n] &= T\left(x[n]
ight) \ &= T\left(\sum_{k=0}^{\infty}x[k]\delta[n-k]
ight) \ &= \sum_{k=0}^{\infty}T\left(x[k]\delta[n-k]
ight) \ &= \sum_{k=0}^{\infty}x[k]T\left(\delta[n-k]
ight) \ &= \sum_{k=0}^{\infty}x[k]h[n-k] \ &\triangleq x[n]*h[n] \end{aligned}$$

The \* symbol in the last step is called the convolution operator, and it is defined as the sum in the step before it.  $\Box$ 

## Eigenfunctions - Munder Construction M