

• **Equazione di Schrödinger**

$$i\hbar\frac{d}{dt}|\psi\rangle=H|\psi\rangle$$

• **Operatore P**

$$P=-i\hbar\frac{d}{dx}\qquad [X,P]=i\hbar\mathbb{1}$$

$$D_P=\{\psi\in L_2(a,b)\text{ ass. cont. }\wedge\psi'\in L_2(a,b)\}$$

$$[Q,P]=i\hbar\Rightarrow [Q,g(P)]=i\hbar g'(P)$$

• **Relazioni di indeterminazione**

$$(\Delta A)_{a,\psi}(\Delta B)_{b,\psi}\geqslant\frac{1}{2}\left|\left\langle [A,B]\right\rangle _{\psi}\right|$$

$$(\Delta X_i)_{\psi}(\Delta P_i)_{\psi}\geqslant\frac{\hbar}{2}$$

$$(\Delta t)_{\psi}(\Delta H)_{\psi}\geqslant\frac{\hbar}{2}$$

$$(\Delta t)_{\psi}:=\inf_A\frac{(\Delta A)_{\psi}}{\left|\frac{d}{dt}\langle A\rangle_{\psi}\right|}$$

• **Base generalizzata del momento**

$$\langle\vec{x}|\vec{p}\rangle=\frac{e^{\frac{i}{\hbar}\vec{p}\cdot\vec{x}}}{(2\pi\hbar)^{\frac{3}{2}}}$$

• **Equazione di continuità**

$$\vec{j}:=\frac{i\hbar}{2m}\left(\psi^*\vec{\nabla}\psi-\psi\vec{\nabla}\psi^*\right)$$

$$\frac{\partial}{\partial t}\rho(\vec{x},t)+\vec{\nabla}\cdot\vec{j}(\vec{x},t)=0$$

• **Operatore di evoluzione temporale**

$$U\left(t\right)=e^{-\frac{i}{\hbar}Ht}$$

• **Teo. di Wigner (trasformazione di simmetria)**

$$\begin{cases}\psi'=U\psi\\A'=UAU^{\dagger}\end{cases}$$

• **Generatore infinitesimo della simmetria**

$$Q:=i\hbar\frac{d}{ds}U(s)\Big|_{s=0}$$

$$U(s)=e^{-\frac{i}{\hbar}sQ}$$

• **Generatori delle rotazioni**

$$U(\varphi)=e^{i\varphi\frac{L_3}{\hbar}}$$

$$R=\begin{pmatrix}\cos\varphi&-\sin\varphi&0\\ \sin\varphi&\cos\varphi&0\\ 0&0&1\end{pmatrix}$$

$$[J_i,J_j]=i\hbar\varepsilon_{ijk}J_k$$

$$J_{\pm}:=J_x\pm iJ_y\qquad (J_{\pm})^{-1}=J_{\mp}$$

$$[J_z,J_{\pm}]=\pm\hbar J_{\pm}\qquad [J^2,J_{\pm}]=0$$

$$[J_+,J_-]=2\hbar J_z$$

$$\frac{J_{\pm}|j,m\rangle}{\hbar\sqrt{j(j+1)-m(m\pm1)}}=|j,m\pm1\rangle$$

$$\begin{cases}[J_i,X_j]=[L_i,X_j]=i\hbar\varepsilon_{ijk}X_k\\[J_i,P_j]=[L_i,P_j]=i\hbar\varepsilon_{ijk}P_k\end{cases}$$

• **Particella senza spin 3D in polari**

$$L^2\Psi_{l,m}(\vec{x})=\hbar^2l(l+1)\Psi_{l,m}(\vec{x})$$

$$L_z\Psi_{l,m}(\vec{x})=\hbar m\Psi_{l,m}(\vec{x})$$

$$L_z=-i\hbar\frac{\partial}{\partial\phi}$$

$$Y_l^m(\theta,\phi)=\sqrt{\frac{2l+1}{4\pi}}\sqrt{\frac{(l+|m|)!}{(l-|m|)!}}\frac{1}{2^l!}P_{l-m}(\theta)e^{im\phi}$$

Con $P_{l,m}$ polinomi di Legendre di grado l ; parità dell'armonica sferica uguale a quella di l :

$$Y_0^0=\frac{1}{2}\sqrt{\frac{1}{\pi}}$$

$$Y_1^{-1}=\frac{1}{2}\sqrt{\frac{3}{2\pi}}e^{-i\phi}\sin\theta$$

$$Y_1^0=\frac{1}{2}\sqrt{\frac{3}{\pi}}\cos\theta$$

$$Y_1^{+1}=-\frac{1}{2}\sqrt{\frac{3}{2\pi}}e^{i\phi}\sin\theta$$

• **Spin**

$$\vec{S}:=\vec{J}-\vec{L}$$

Ha stessa algebra di \vec{J} e \vec{L} . Non commuta con \vec{X} e \vec{P} .

$$S_x=\frac{\hbar}{2}\begin{pmatrix}0&1\\1&0\end{pmatrix}\qquad S_y=\frac{\hbar}{2}\begin{pmatrix}0&-i\\i&0\end{pmatrix}$$

$$S_z=\frac{\hbar}{2}\begin{pmatrix}1&0\\0&-1\end{pmatrix}$$

$$\vec{S}\cdot\hat{n}=\frac{\hbar}{2}\begin{pmatrix}\cos\theta&e^{-i\phi}\sin\theta\\e^{i\phi}\sin\theta&-\cos\theta\end{pmatrix}$$

$$|+\rangle_{\hat{n}}=\begin{pmatrix}\cos\frac{\theta}{2}\\e^{i\phi}\sin\frac{\theta}{2}\end{pmatrix}\qquad|-\rangle_{\hat{n}}=\begin{pmatrix}-\sin\frac{\theta}{2}\\e^{i\phi}\cos\frac{\theta}{2}\end{pmatrix}$$

$$\sigma_i\sigma_j=i\varepsilon_{ijk}\sigma_k$$

$$[\sigma_i,\sigma_j]=2i\varepsilon_{ijk}\sigma_k$$

Con $s=1$:

$$L_x=\frac{\hbar}{\sqrt{2}}\begin{pmatrix}0&1&0\\1&0&1\\0&1&0\end{pmatrix}\qquad L_y=\frac{\hbar}{\sqrt{2}}\begin{pmatrix}0&-i&0\\i&0&-i\\0&i&0\end{pmatrix}$$

$$L_z=\hbar\begin{pmatrix}1&0&0\\0&0&0\\0&0&-1\end{pmatrix}$$

• **Simmetrie discrete**

– Parità \mathbb{P} :

$$\begin{cases}\mathbb{P}\vec{X}\mathbb{P}^{\dagger}=-\vec{X}\\\mathbb{P}\vec{P}\mathbb{P}^{\dagger}=-\vec{P}\\\mathbb{P}\vec{L}\mathbb{P}^{\dagger}=\vec{L}\\\psi'(\vec{x})=\psi(-\vec{x})\end{cases}$$

$$\mathbb{P}=\mathbb{P}^{-1}=\mathbb{P}^{\dagger}$$

Unici autovalori possibili: ± 1

– Inversione temporale \mathbb{T} :

$$\begin{cases}\mathbb{T}\vec{X}\mathbb{T}^{\dagger}=\vec{X}\\\mathbb{T}\vec{P}\mathbb{T}^{\dagger}=-\vec{P}\\\mathbb{T}\vec{L}\mathbb{T}^{\dagger}=-\vec{L}\\\psi'(\vec{x},t)=\psi^*(\vec{x},-t)\end{cases}$$

• **Visuale di Heisenberg**

$$\begin{cases}|\psi_h(t)\rangle=U^{\dagger}|\psi_s(t)\rangle\\A_h(t)=U^{\dagger}A_s(t)U\end{cases}$$

Nota: U è l'operatore di evoluzione temporale.

$$\begin{cases}\frac{d}{dt}|\psi_h(t)\rangle=0\\\frac{dA_h(t)}{dt}=-\frac{i}{\hbar}[A_h,H_h]+\left(\frac{\partial A}{\partial t}\right)_h\end{cases}$$

• **Teo. di Ehrenfest**

$$\begin{cases}\frac{d}{dt}\langle Q\rangle_{\psi}=\frac{1}{m}\langle P\rangle_{\psi}\\\frac{d}{dt}\langle P\rangle_{\psi}=-\langle V'(Q)\rangle_{\psi}\end{cases}$$

• **Matrici densità**

$$\rho_{\psi}:=|\psi\rangle\langle\psi|\qquad\langle A\rangle_{\psi}=tr(\rho A)$$

Definizione di operatore densità: ρ tale che:

$$\begin{cases}\rho\text{ a.a.}\\\rho\geqslant 0\qquad\qquad tr(\rho^2)\leqslant 1\\tr(\rho)=1\end{cases}$$

• **Oscillatore armonico 1D**

$$H=\frac{P^2}{2m}+\frac{1}{2}m\omega^2X^2$$

$$\hat{X}:=\sqrt{\frac{m\omega}{\hbar}}X\qquad\hat{P}:=\frac{P}{\sqrt{m\omega\hbar}}$$

$$\left[\hat{X},\hat{P}\right]=i$$

$$a:=\frac{\hat{X}+i\hat{P}}{\sqrt{2}}\qquad a^{\dagger}=\frac{\hat{X}-i\hat{P}}{\sqrt{2}}$$

$$H=\hbar\omega\left(N+\frac{1}{2}\right)$$

$$N:=a^{\dagger}a\qquad aa^{\dagger}-a^{\dagger}a=1$$

$$aa^{\dagger}=N+1$$

$$u_0(x)=\left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}}e^{-\frac{m\omega}{\hbar}\frac{x^2}{2}}$$

$$|\nu\rangle=\frac{(a^{\dagger})^{\nu}|0\rangle}{\sqrt{\nu!}}$$

$$\begin{cases}a|\nu\rangle=\sqrt{\nu}|\nu-1\rangle\\a^{\dagger}|\nu\rangle=\sqrt{\nu+1}|\nu+1\rangle\end{cases}$$

• **Atomo H**

$$H_r=\frac{\vec{P}_{\mu}^2}{2\mu}-\frac{e^2}{r}$$

$$E=-\frac{E_I}{n^2}$$

$$\Psi_{n,l,m}=\frac{N_{n,l}}{a_0^{\frac{3}{2}}}e^{-\frac{r}{a_0n}}\left(\frac{2r}{na_0}\right)^lL\left(\frac{2r}{na_0}\right)Y_l^m(\theta,\phi)$$

$$e^2:=2,3\cdot 10^{-28}\text{ J}\cdot\text{m}$$

$$a_0:=\frac{\hbar^2}{e^2\mu}\simeq 0,5\cdot 10^{-10}\text{ m}$$

$$E_I:=\frac{e^2}{2a_0}=\frac{\hbar^2}{2\mu a_0}=13,6\text{ eV}=1\text{ Ry}$$

• **Perturbazioni stazionarie**

Problema:

$$H=H_0+W=H_0+\lambda\hat{W}$$

$$\begin{cases}E_a^{\lambda}=\mathcal{E}_0+\lambda\mathcal{E}_1+\lambda^2\mathcal{E}_2+O\left(\lambda^3\right)\\|\lambda\rangle=|0\rangle+\lambda|1\rangle+\lambda^2|2\rangle+O\left(\lambda^3\right)\end{cases}$$

Convenzione:

$$\begin{cases}|0\rangle=|E_a^0\rangle\\\langle\lambda|\lambda\rangle=1\quad\forall\lambda\in\mathbb{R}\\\langle 0|\lambda\rangle\in\mathbb{R}\quad\forall\lambda\in\mathbb{R}\end{cases}$$

Soluzioni:

$$\mathcal{E}_1=\left\langle E_a^0\left|\hat{W}\right|E_a^0\right\rangle$$

$$\left\langle E_b^0\right|1\rangle=\frac{\left\langle E_b^0\left|\hat{W}\right|E_a^0\right\rangle}{-E_b^0+E_a^0}$$

$$\mathcal{E}_n^k=\left\langle \mathcal{E}_n^0\left|\hat{W}\right|\mathcal{E}_n^{k-1}\right\rangle$$

$$\mathcal{E}_2=\sum_{b\neq a}\frac{\left|\left\langle E_a^0\left|\hat{W}\right|E_b^0\right\rangle\right|^2}{E_a^0-E_b^0}$$

• **Perturbazioni: caso degenero**

Sia $\{|\phi_r\rangle\}_{r=1,\dots,d}$ BON di \mathcal{H}_0

Ordine λ^q :

$$\begin{aligned}(H_0-\mathcal{E}_0)|q,k\rangle+\left(\hat{W}-\mathcal{E}_1^k\right)|q-1,k\rangle\\-\mathcal{E}_2^k|q-2,k\rangle-\dots-\mathcal{E}_q^k|0,k\rangle=0\end{aligned}$$

Ordine λ :

$$\sum_{t=1}^d\left\langle\phi_s\left|\hat{W}\right|\phi_t\right\rangle\langle\phi_t|0,r\rangle=\mathcal{E}_1^{(r)}\langle\phi_s|0,k\rangle$$

che è equazione ad autovalori/autovettori da risolvere $\forall\phi_s$

• **EM**

$$H=\frac{1}{2m}\left(\vec{P}-\frac{q}{c}\vec{A}(\vec{X},t)\right)^2+q\varphi(\vec{X},t)$$

Trasformazione di Gauge:

$$\begin{cases} \vec{A}'=\vec{A}+\vec{\nabla}\Lambda(\vec{x},t) \\ \varphi'=\varphi-\frac{1}{c}\frac{\partial}{\partial t}\Lambda(\vec{x},t) \\ \psi'(\vec{x},t)=e^{i\frac{q}{\hbar c}\Lambda(\vec{x},t)}\psi(\vec{x},t)=:U_{\Lambda}\psi(\vec{x},t) \end{cases}$$

Trasformazione di Gauge è trasformazione di simmetria: solo osservabili covarianti sono fisici: bisogna imporre:

$$f_{\vec{A}',\varphi'}=U_{\Lambda}f_{\vec{A},\varphi}U_{\Lambda}^{\dagger}$$

Gauge di Landau (con $\vec{E}=0$, $\vec{B}=-B_0\hat{u}_z$):

$$\begin{cases} \varphi\equiv 0 \\ \vec{A}=(B_0y,0,0) \\ H=\frac{1}{2m}\left(P_x-\frac{q}{c}B_0Y\right)^2+\frac{P_y^2}{2m}+\frac{P_z^2}{2m} \end{cases}$$

Asse z è indipendente ed è onda 1D libera.

Autostati di onda xy (u_n sono le autofunzioni di oscillatore armonico):

$$E_n=\hbar\omega\left(n+\frac{1}{2}\right)=\frac{\hbar qB_0}{mc}\left(n+\frac{1}{2}\right)$$

$$\Psi_{n,p_x}(x,y)=e^{i\frac{p_x}{\hbar}x}u_n(y-y_0)$$

$$\omega:=\frac{qB_0}{mc}\qquad y_0:=\frac{cp_x}{qB_0}$$

Gauge simmetrica:

$$\vec{A}=\left(\frac{B_0y}{2},-\frac{B_0x}{2},0\right)$$

• **Effetto Aharonov-Bohm**

$$\frac{1}{2m}\left(P_s-\frac{q}{c}\frac{\Phi}{2\pi R}\right)^2$$

Doppia fenditura con solenoide acceso:

$$\begin{aligned} |\psi(x)|^2&\propto\left|\psi_1(x)+e^{\frac{iq}{\hbar c}\int_{\gamma_2-\gamma_1}\vec{A}\cdot d\vec{l}}\psi_2(x)\right|^2 \\ &=\left|\psi_1(x)+e^{\frac{iq}{\hbar c}\Phi}\psi_2(x)\right|^2 \end{aligned}$$

• **Scattering**

Sezione d'urto differenziale generico:

$$f_K(\theta,\phi)=-\frac{1}{4\pi}\frac{2\mu}{\hbar^2}\int d^3r'e^{-i\vec{x}'\cdot\vec{K}}V(\vec{x})$$

$$\frac{d\sigma}{d\Omega}(\theta,\phi)=\frac{\mu^2}{4\pi^2\hbar^4}\left|\int d^3x'e^{-i\vec{K}\cdot\vec{x}'}V(\vec{x}')\right|^2$$

Con $\vec{K}:=k(\hat{u}_{\theta,\phi}-\hat{u}_z)$

Potenziale centrale: $K:=\left|\vec{K}\right|=2\sin\frac{\theta}{2}k$

Con potenziale centrale di Yukawa:

$$V(r)=\beta\frac{e^{-\alpha r}}{r}$$

$$f_K^B(\theta,\phi)=-\frac{2\mu\beta}{\hbar^2(K^2+\alpha^2)}$$

$$\frac{d\sigma_B}{d\Omega}(\theta)=\frac{4\mu^2\beta^2}{\hbar^4}\frac{1}{\left(\alpha^2+4k^2\sin^2\frac{\theta}{2}\right)^2}$$

$$\frac{d\sigma_B}{d\Omega}\xrightarrow{\alpha\rightarrow 0}\frac{\mu^2\beta^2}{4\hbar^4k^4\sin^4\frac{\theta}{2}}$$

• **Buca di potenziale**

Con potenziale $V(x)=+\infty\mathbb{1}_{[0,a]c}$:

$$E_n=\frac{\pi^2\hbar^2}{2ma^2}n^2$$

$$\psi_n(x)=\sqrt{\frac{2}{a}}\sin\left(\frac{n\pi}{a}x\right)$$

Con potenziale $V(x)=+\infty\mathbb{1}_{\{x>|a|/2\}}$

$$E_n=\frac{\hbar^2n^2\pi^2}{2ma^2}$$

$$\psi_n(x)=\begin{cases}\sqrt{\frac{2}{a}}\cos\left(\frac{n\pi x}{a}\right) & n\text{ dispari} \\ \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi x}{a}\right) & n\text{ pari} \end{cases}$$

• **Polari**

$$\begin{cases} x=r\sin\theta\cos\phi \\ y=r\sin\theta\sin\phi \\ z=r\cos\theta \end{cases}$$

$$\vec{\nabla}f=\left(\frac{\partial f}{\partial r},\frac{1}{r}\frac{\partial f}{\partial\theta},\frac{1}{r\sin\theta}\frac{\partial f}{\partial\phi}\right)$$

$$\vec{\nabla}\cdot\vec{F}=\frac{1}{r^2}\frac{\partial r^2F_r}{\partial r}+\frac{1}{r\sin\theta}\frac{\partial F_{\theta}\sin\theta}{\partial\theta}+\frac{1}{r\sin\theta}\frac{\partial F_{\phi}}{\partial\phi}$$

$$\int_{\mathbb{R}^3}d^3x=\int_0^{\infty}dr\int_0^{\pi}d\theta\int_0^{2\pi}d\phi\,r^2\sin\theta$$

• **Formule goniometriche**

$$\sin^2(\theta/2)=(1-\cos\theta)/2$$

$$\cos^2(\theta/2)=(1+\cos\theta)/2$$

$$\tan^2(\theta/2)=(1-\cos\theta)/(1+\cos\theta)$$

$$\tan(2\theta)=2\tan\theta/(1-\tan^2\theta)$$

$$\sin\alpha\cos\beta=[\sin(\alpha+\beta)+\sin(\alpha-\beta)]/2$$

$$\cos\alpha\cos\beta=[\cos(\alpha+\beta)+\cos(\alpha-\beta)]/2$$

$$\sin\alpha\sin\beta=[\cos(\alpha-\beta)+\cos(\alpha+\beta)]/2$$

Con $t=\tan(\theta/2)$, con $\theta\neq(2k+1)\pi$

$$\sin\theta=\frac{2t}{1+t^2}\qquad\cos\theta=\frac{1-t^2}{1+t^2}$$

$$\sin\alpha+\sin\beta=2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\cos\alpha+\cos\beta=2\cos\left(\frac{\alpha-\beta}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right)$$

• **Integrali**

$$\int\sin^2x\,dx=\frac{1}{2}(x-\sin x\cos x)$$

$$\int\cos^2x\,dx=\frac{1}{2}(x+\sin x\cos x)$$

$$\int\cos^3x\,dx=\sin x-\frac{\sin^3x}{3}$$

$$\int\sin^3x\,dx=-\cos x+\frac{\cos^3x}{3}$$

$$\int_{-\infty}^{+\infty}e^{-ax^2+bx}\,dx=\sqrt{\frac{\pi}{a}}\exp\left(\frac{b^2}{4a}\right)$$

$$\int_0^{+\infty}x^2e^{-x^2}\,dx=\frac{\sqrt{\pi}}{4}$$

$$\int_0^{\infty}x^ne^{-x}\,dx=n!$$

• **Taylor**

$$\sin x=\sum_{n=0}^{+\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)!}$$

$$\cos x=\sum_{n=0}^{+\infty}(-1)^n\frac{x^{2n}}{(2n)!}$$

$$\tan x=x+\frac{x^3}{3}+2\frac{x^5}{15}+O(x^7)$$

• **Eq. differenziale**

$$x''(t)-\omega^2x(t)=0:$$

$$x(t)=Ae^{\omega t}+Be^{-\omega t}$$

$$\dot{y}^t+a(t)y(t)=b(t):$$

$$y(t)=e^{-A(t)}\left[c+\int b(t)e^{A(t)}\,dt\right]$$

con $A(t)$ una primitiva di $a(t)$.

$$\ddot{y}+a\dot{y}+by=0\quad a,b\in\mathbb{R}$$

In base a Δ di eq. di secondo grado associata:

$$y(t)=c_1e^{\lambda_1t}+c_2e^{\lambda_2t}\quad\Delta>0$$

$$y(t)=c_1e^{\lambda t}+tc_2e^{\lambda t}\quad\Delta=0$$

$$y(t)=c_1e^{\alpha t}\cos(\beta t)+c_2e^{\alpha t}\sin(\beta t)\quad\Delta<0$$

Con $\alpha:=\Re\mathfrak{e}(\lambda)$, $\beta:=\Im\mathfrak{m}(\lambda)$

• **Coefficienti di Clebsch-Gordan**

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