COMPUTATIONAL ASTROPHYSICS

Observatorio
Astronómico
Nacional

Computational Astrophysics

05. Energy Conservation

Eduard Larrañaga
Observatorio Astronómico Nacional
Universidad Nacional de Colombia

Heat contributions

Heat contributions

We consider the first law of thermodynamics,

$$dQ = pd\left(\frac{1}{\rho}\right) + de$$

where

dQ : Change in heat per unit (mass)

de : Change in energy (per unit mass)

Heat contributions

The heat term may have many contributions such as

$$\rho \frac{dQ}{dt} = \overrightarrow{v} \cdot \vec{f} - \overrightarrow{\nabla} \cdot \overrightarrow{F}_{rad} - \overrightarrow{\nabla} \cdot \overrightarrow{q} + Q^{(vis)} + \eta_e J^2$$

where

$\overrightarrow{v} \cdot \vec{f}$: work done by the force $\vec{f}$

$-\overrightarrow{\nabla} \cdot \overrightarrow{F}_{rad}$: rate at which radiation energy is lost by emission (or augmented by absorption) per unit volume.

$\overrightarrow{F}_{rad} = \int d\nu \int d\Omega \hat{n} I_\nu(\hat{n}, \vec{r})$: radiative flux vector

$I_\nu(\hat{n}, \vec{r})$: specific intensity evaluated at $\vec{r}$ in the direction $\hat{n}$

Heat contributions

The heat term may have many contributions such as

$$\rho \frac{dQ}{dt} = \overrightarrow{v} \cdot \vec{f} - \overrightarrow{\nabla} \cdot \overrightarrow{F}_{rad} - \overrightarrow{\nabla} \cdot \overrightarrow{q} + Q^{(vis)} + \eta_e J^2$$

where

$-\overrightarrow{\nabla} \cdot \overrightarrow{q}$: rate at which random motion (of electrons principally) transport thermic energy in the fluid.

$\overrightarrow{q}$: (conductive) flux of heat through the boundaries of an element

Heat contributions

The heat term may have many contributions such as

$$\rho \frac{dQ}{dt} = \overrightarrow{v} \cdot \vec{f} - \overrightarrow{\nabla} \cdot \overrightarrow{F}_{rad} - \overrightarrow{\nabla} \cdot \overrightarrow{q} + Q^{(vis)} + \eta_e J^2$$

where

$Q^{(vis)} = \boldsymbol{\sigma} : \overrightarrow{\nabla}\,\overrightarrow{v}$: rate of volumetric heating due to viscosity

$\eta_e J^2$: rate of Ohmic (volumetric) heating.

This term is usually $\overrightarrow{J} \cdot \overrightarrow{E}$ and in the Lagrangain frame we have $\overrightarrow{E} = \eta_e \overrightarrow{J}$.

η_e : electric resistivity of the fluid

Energy Conservation Equation

Energy Conservation Equation

Using all the heating contributions, we write the first law of thermodynamics as

$$\rho \frac{dQ}{dt} = \rho \frac{de}{dt} + \rho p \frac{d}{dt}\left(\frac{1}{\rho}\right) = \overrightarrow{v} \cdot \vec{f} - \overrightarrow{\nabla} \cdot \overrightarrow{F}_{rad} - \overrightarrow{\nabla} \cdot \overrightarrow{q} + Q^{(vis)} + \eta_e J^2$$

Using the Lagrangian form of the continuity equation we can write

$$\frac{d}{dt}\left(\frac{1}{\rho}\right) = \frac{1}{\rho} \overrightarrow{\nabla} \cdot \overrightarrow{v}.$$

Energy Conservation Equation

Therefore we obtain the energy conservation equation in the Lagrangian frame,

$$\rho \frac{de}{dt} = -p\overrightarrow{\nabla} \cdot \overrightarrow{v} + \overrightarrow{v} \cdot \vec{f} - \overrightarrow{\nabla} \cdot \overrightarrow{F}_{rad} - \overrightarrow{\nabla} \cdot \overrightarrow{q} + Q^{(vis)} + \eta_e J^2$$

where the term $-p\overrightarrow{\nabla} \cdot \overrightarrow{v}$ represents the work done by pressure.

Energy Conservation Equation

Using the continuity equation in the Eulerian frame we obtain

$$\rho \frac{de}{dt} = \frac{d(\rho e)}{dt} - e\frac{d\rho}{dt} = \frac{d(\rho e)}{dt} + e\rho \overrightarrow{\nabla} \cdot \overrightarrow{v}$$

and using the Lagrangian derivative, we get the energy conservation equation in the Eulerian frame,

$$\frac{\partial(\rho e)}{\partial t} + \overrightarrow{\nabla} \cdot (e\rho \overrightarrow{v}) = -p\overrightarrow{\nabla} \cdot \overrightarrow{v} + \overrightarrow{v} \cdot \vec{f} - \overrightarrow{\nabla} \cdot \overrightarrow{F}_{rad} - \overrightarrow{\nabla} \cdot \overrightarrow{q} + Q^{(vis)} + \eta_e J^2$$