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**A Project Report**

**On**

**DENOISING SIGNALS AND IMAGES**

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**UNDER THE ESTEEMED GUIDANCE OF**

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## CERTIFICATE

This is to certify that the work which is being presented in the B.Tech. Project Report entitled **“Denoising Signals and Images”**, in partial fulfillment of the requirements for the award of the **Bachelor of Technology in Electronics & Communication Engineering** and submitted by Mr./Ms. **D. VENKATA GOUTHAM REDDY, O. SATYA SAI**, bearing Regd. No **190040127, 190040376**, to the Department of Electronics & Communication Engineering of KLEF, Vaddeswaram, Guntur is an authentic record of my own work carried out during a period from December 2021 to May 2022 under the supervision of DR.D.Bhavana.

**Professor, ECE Department.**

PROJECT SUPERVISOR

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# **ABSTRACT**

Thresholding is a technique used for signal and image denoising. The discrete wavelet transform uses two types of filters: (1) averaging filters, and (2) detail filters. When we decompose a signal using the wavelet transform, we are left with a set of wavelet coefficients that correlates to the high frequency sub-bands. These high frequency sub-bands consist of the details in the data set. If these details are small enough, they might be omitted without substantially affecting the main features of the data set. Additionally, these small details are often those associated with noise; therefore, by setting these coefficients to zero, we are essentially killing the noise. This becomes the basic concept behind thresholding-set all frequency sub-band coefficients that are less than a particular threshold to zero and use these coefficients in an inverse wavelet transformation to reconstruct the data set.

After implementing the double-density DWT, and double-density complex DWT for 1-D signals, we can develop two different methods using these DWTs to remove noise from an image. The double-density DWT method will be discussed first.

# INTRODUCTION

The general denoising procedure involves three steps. The basic version of the procedure follows the steps described below:

- Decompose: Choose a wavelet, choose a level  $N$ . Compute the wavelet decomposition of the signal at level  $N$ .
- Threshold detail coefficients: For each level from 1 to  $N$ , select a threshold and apply soft thresholding to the detail coefficients.
- Reconstruct: Compute wavelet reconstruction using the original approximation coefficients of level  $N$  and the modified detail coefficients of levels from 1 to  $N$ .

Two points must be addressed in particular:

- How to choose the threshold,
- How to perform the thresholding.

Signal and image denoising is an important application in fields concerned with image processing, signal processing, speech conversion and such where removing noisy elements is the prime task in extracting reliable information.

# THRESHOLDING

Thresholding is the simplest method of image segmentation. From a grayscale image, thresholding can be used to create binary images.

Thresholding can be done using the function `wthresh` which returns soft or hard thresholding of the input signal. Hard thresholding is the simplest method but soft thresholding has nice mathematical properties.

```
thr = 0.4;
```

## Hard thresholding:

Hard thresholding can be described as the usual process of setting to zero the elements whose absolute values are lower than the threshold. The hard threshold signal is  $x$  if  $x > thr$ , and is 0 if  $x \leq thr$ .

```
y = linspace(-1,1,100);  
ythard = wthresh(y, 'h', thr);
```

## Soft Thresholding:

Soft thresholding is an extension of hard thresholding, first setting to zero the elements whose absolute values are lower than the threshold, and then shrinking the nonzero coefficients towards 0. The soft threshold signal is  $\text{sign}(x)(x - thr)$  if  $x > thr$  and is 0 if  $x \leq thr$ .

```
ytsoft = wthresh(y, 's', thr);
```

The hard procedure creates discontinuities at  $x = \pm t$ , while the soft procedure does not.

# THRESHOLD SELECTION RULES

Recalling step 2 of the denoise procedure, the function `thselect` performs a threshold selection, and then each level is thresholded. This second step can be done using `wthcoeff`, directly handling the wavelet decomposition structure of the original signal. Four threshold selection rules are implemented in the function `thselect`. Typically it is interesting to show them in action when the input signal is a Gaussian white noise.

```
rng default
y = randn(1,1000);
```

Rule 1: Selection using principle of Stein's Unbiased Risk Estimate (SURE)

```
thr = thselect(y, 'rigrsure')
```

thr = 2.0518

Rule 2: Fixed form threshold equal to  $\sqrt{2 \cdot \log(\text{length}(y))}$

```
thr = thselect(y, 'sqtwoolog')
```

thr = 3.7169

Rule 3: Selection using a mixture of the first two options

```
thr = thselect(y, 'heursure')
```

thr = 3.7169

Rule 4: Selection using minimax principle

```
thr = thselect(y, 'minimaxi')
```

thr = 2.2163

Minimax and SURE threshold selection rules are more conservative and would be more convenient when small details of the signal lie near the noise range. The two other rules remove the noise more efficiently.

# DENOISING WHITE NOISE

Let us use the "blocks" test data credited to Donoho and Johnstone as the first example. Generate original signal `xref` and a noisy version `x` adding a standard Gaussian white noise.

```
sqrt_snr = 4;          % Set signal to noise ratio
init = 2055615866; % Set rand seed
[xref,x] = wnoise(1,11,sqrt_snr,init);
```

First denoise the signal using `wdenoise` with default settings. Compare the result with the original and noisy signals.

```
xd = wdenoise(x);
subplot(3,1,1)
plot(xref)
axis tight
title('Original Signal')
subplot(3,1,2)
plot(x)
axis tight
title('Noisy Signal')
subplot(3,1,3)
plot(xd)
axis tight
title('Denoised Signal - Signal to Noise Ratio = 4')
```

Denoise the noisy signal a second time, this time using soft heuristic SURE thresholding on detail coefficients obtained from the decomposition of `x`, at level 3 by `sym8` wavelet. Compare with the previous denoised signal.

```
xd2 = wdenoise(x,3,'Wavelet','sym8',...
    'DenoisingMethod','SURE',...
    'ThresholdRule','Soft');
figure
subplot(3,1,1)
plot(xref)
axis tight
title('Original Signal')
subplot(3,1,2)
plot(xd)
axis tight
title('First Denoised Signal: Default Settings')
subplot(3,1,3)
plot(xd2)
axis tight
title('Second Denoised Signal')
```

Since only a small number of large coefficients characterize the original signal, both denoised signals compare well with the original signal. You can use the [Wavelet Signal Denoiser](#) to explore the effects other denoising parameters have on the noisy signal.



## DEALING WITH NON-WHITE NOISE

When you suspect a non-white noise, thresholds must be rescaled by a level-dependent estimation of the level noise. As a second example, let us try the method on the highly perturbed part of an electrical signal. Let us use db3 wavelet and decompose at level 3. To deal with the composite noise nature, let us try a level-dependent noise size estimation.

```
load leleccum
indx = 2000:3450;
x = leleccum(indx); % Load electrical signal and select part of it.
```

Denoise the signal using soft fixed form thresholding and level-dependent noise size estimation.

```
xd = wdenoise(x,3,'Wavelet','db3',...
    'DenoisingMethod','UniversalThreshold',...
    'ThresholdRule','Soft',...
    'NoiseEstimate','LevelDependent');
Nx = length(x);
figure
subplot(2,1,1)
plot(indx,x)
axis tight
title('Original Signal')
subplot(2,1,2)
plot(indx,xd)
axis tight
title('Denoised Signal')
```

The result is quite good in spite of the time heterogeneity of the nature of the noise after and before the beginning of the sensor failure around time 2410.

# IMAGE DENOISING

The denoising method described for the one-dimensional case applies also to images and applies well to geometrical images. The two-dimensional denoising procedure has the same three steps and uses two-dimensional wavelet tools instead of one-dimensional ones. For the threshold selection, `prod(size(y))` is used instead of `length(y)` if the fixed form threshold is used.

Generate a noisy image.

```
load woman
init = 2055615866;
rng(init);
x = X + 15*randn(size(X));
```

In this case fixed form threshold is used with estimation of level noise, thresholding mode is soft and the approximation coefficients are kept.

```
[thr,sorh,keepapp] = ddencmp('den','wv',x);
thr
```

```
thr = 107.9838
```

Denoise image using global thresholding option.

```
xd = wdencomp('gbl',x,'sym4',2,thr,sorh,keepapp);
figure('Color','white')
colormap(pink(255))
sm = size(map,1);
image(wcodemat(X,sm))
title('Original Image')
```

```
figure('Color','white')
colormap(pink(255))
image(wcodemat(x,sm))
title('Noisy Image')
```

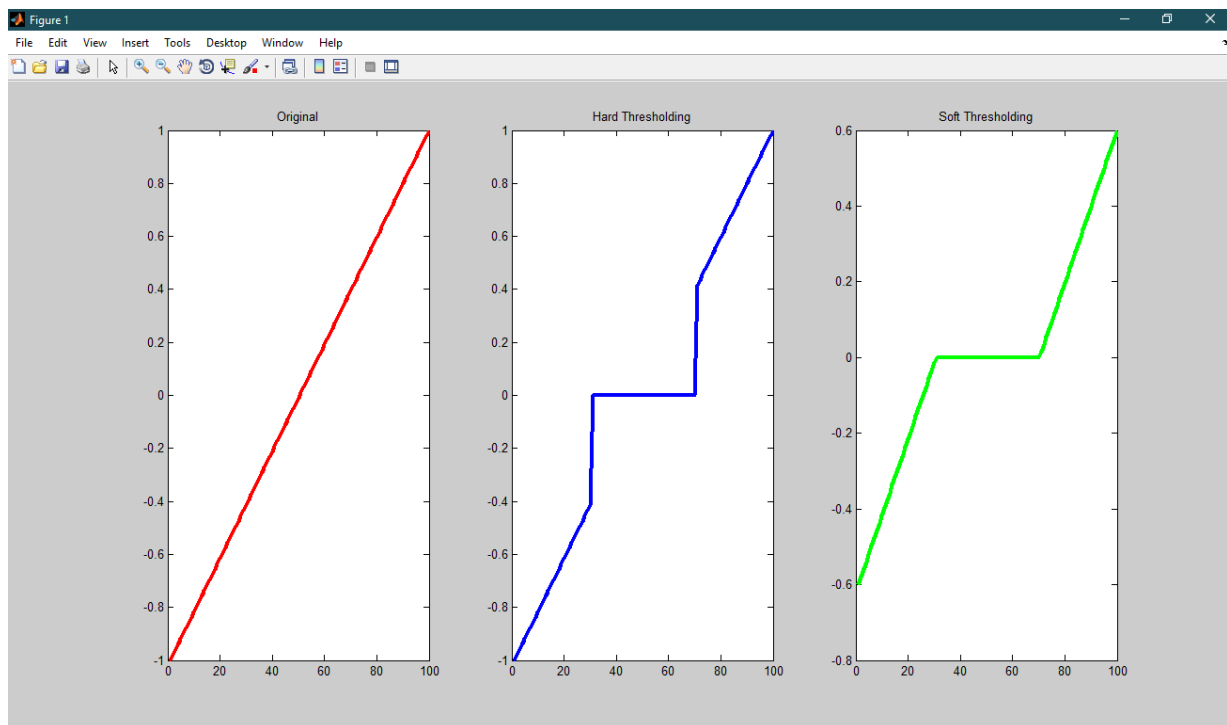
```
image(wcodemat(xd,sm))
title('Denoised Image')
```

# RESULTS AND DISCUSSIONS

Code and Outputs:

```
thr = 0.4;      % setting threshold

y = linspace(-1,1,100);
ythard = wthresh(y,'h',thr); % hard thresholding
ytsoft = wthresh(y,'s',thr); % soft thresholding
subplot(1,3,1)
plot(y)
title('Original')
subplot(1,3,2)
plot(ythard)
title('Hard Thresholding')
subplot(1,3,3)
plot(ytsoft)
title('Soft Thresholding')
```



**Comparison between Hard Thresholding and Soft Thresholding**

```
rng default
y = randn(1,1000);
thr = thselect(y,'heursure') % threshold selection using heursure

sqrt_snr = 4; % Set signal to noise ratio
init = 2055615866; % Set rand seed
[xref,x] = wnoise(1,11,sqrt_snr,init);

xd = wdenoise(x); % denoising signal
```

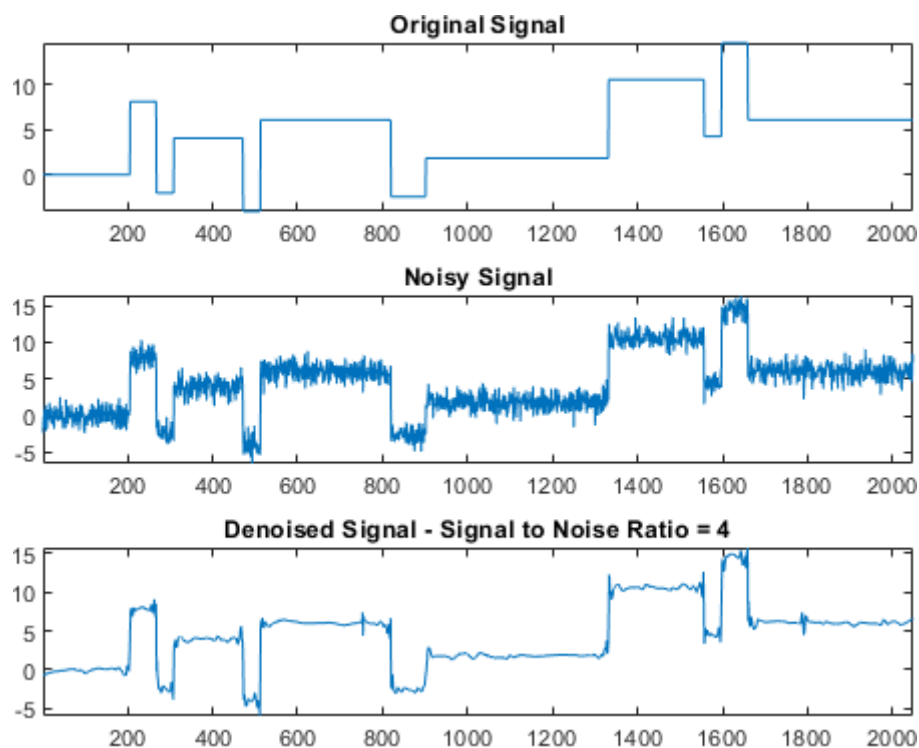
```
subplot(3,1,1)  
plot(xref)
```

---

```

axis tight
title('Original Signal')
subplot(3,1,2)
plot(x)
axis tight
title('Noisy Signal')
subplot(3,1,3)
plot(xd)
axis tight
title('Denoised Signal - Signal to Noise Ratio = 4')

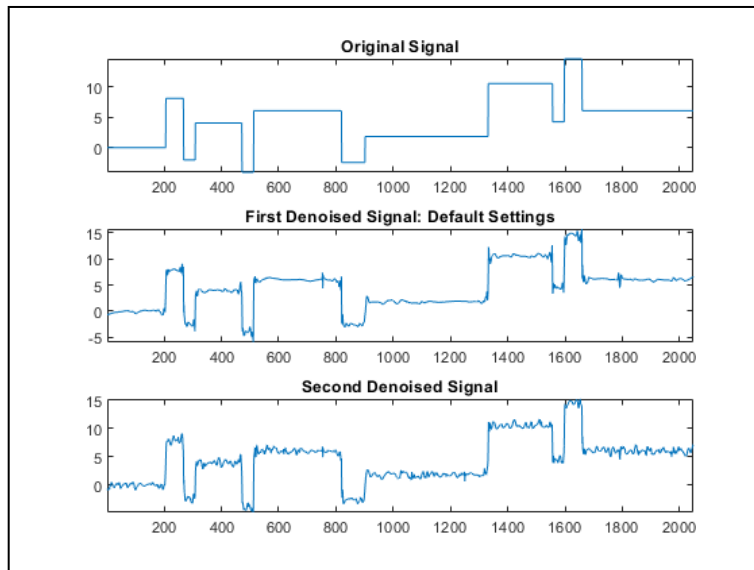
```



```

xd2 = wdenoise(x,3,'Wavelet','sym8',...
'DenoisingMethod','SURE',...
'ThresholdRule','Soft');
figure
subplot(3,1,1)
plot(xref)
axis tight
title('Original Signal')
subplot(3,1,2)
plot(xd)
axis tight
title('First Denoised Signal: Default Settings')
subplot(3,1,3)
plot(xd2)
axis tight
title('Second Denoised Signal')

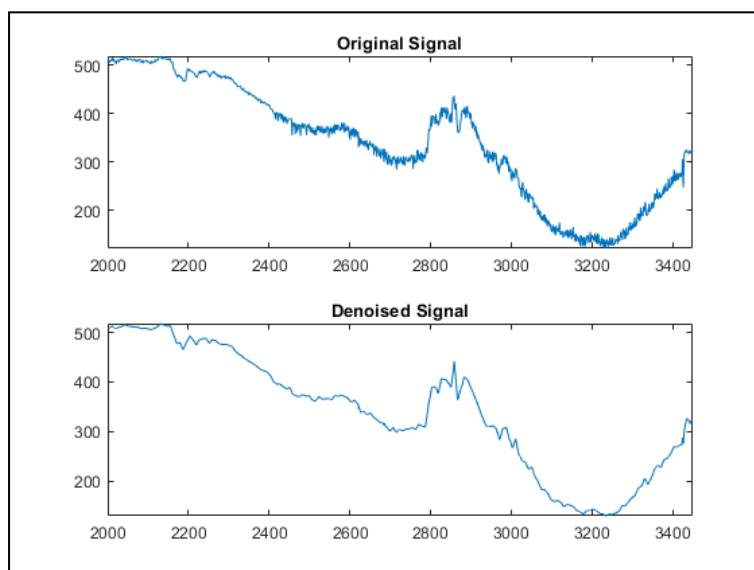
```



```

load leleccum % Dealing with non-white noise
indx = 2000:3450;
x = leleccum(indx); % Load electrical signal and select part of it.
xd = wdenoise(x,3,'Wavelet','db3',...
    'DenoisingMethod','UniversalThreshold',...
    'ThresholdRule','Soft',...
    'NoiseEstimate','LevelDependent');
Nx = length(x);
figure
subplot(2,1,1)
plot(indx,x)
axis tight
title('Original Signal')
subplot(2,1,2)
plot(indx,xd)
axis tight
title('Denoised Signal')

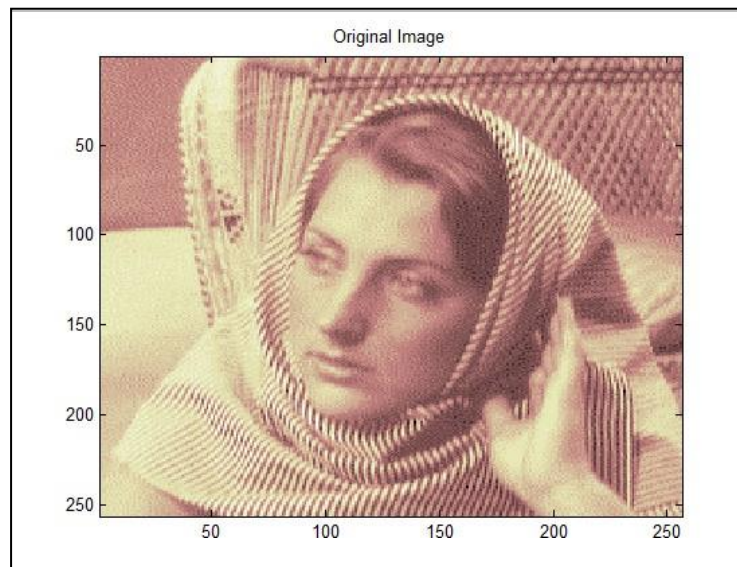
```



```

load woman
init = 2055615866;
rng(init);
x = X + 15*randn(size(X));
[thr,sorh,keepapp] = ddencomp('den','wv',x);
xd = wdencomp('gbl',x,'sym4',2,thr,sorh,keepapp);
figure('Color','white')
colormap(pink(255))
sm = size(map,1);
image(wcodemat(X,sm))
title('Original Image')

```

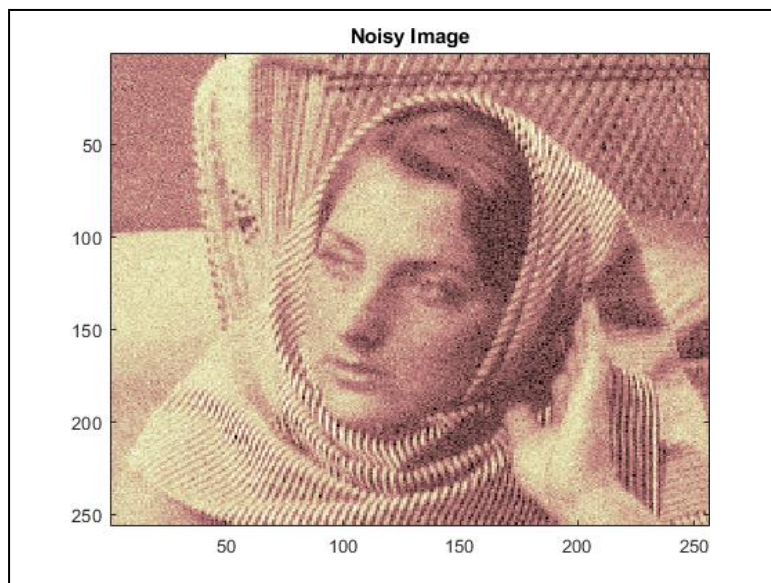


**Original Loaded Image**

```

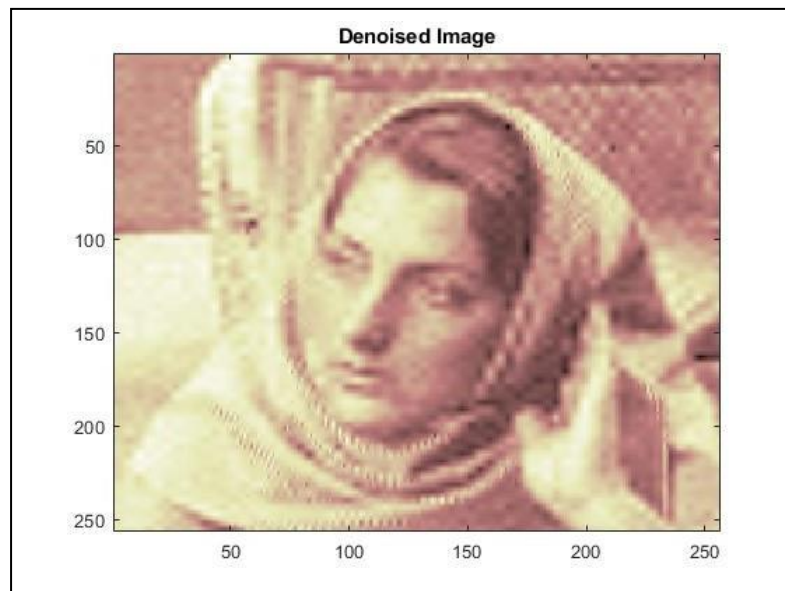
figure('Color','white')
colormap(pink(255))
image(wcodemat(x,sm))
title('Noisy Image')

```



**Noisy Image**

```
image(wcodemat(xd,sm))  
title('Denoised Image')
```



**Denoised Image**



## CONCLUSIONS

Denoising methods based on wavelet decomposition is one of the most significant applications of wavelets. Because wavelets localize features in your data to different scales, you can preserve important signal or image features while removing noise. The basic idea behind wavelet denoising, or wavelet thresholding, is that the wavelet transform leads to a sparse representation for many real-world signals and images. What this means is that the wavelet transform concentrates signal and image features in a few large-magnitude wavelet coefficients. Wavelet decompositions provide a useful basis for localized approximation of functions with any degree of regularity at different scales and with a desired accuracy.

## REFERENCES

<https://www.sciencedirect.com/topics/engineering/wavelet-decomposition>

<https://in.mathworks.com/help/wavelet/ug/denoising-signals-and-images.html>

<https://in.mathworks.com/help/wavelet/ref/wdenoise.html>

<https://in.mathworks.com/help/wavelet/ref/waveletsignaldenoiser-app.html>