1st and 2nd order FD derivative

$$f'(x_{\ell+1}) = \frac{f(x_{\ell+1}) - f(x_\ell)}{\Delta x}$$

$$f(x_\ell) = \frac{f(x_{\ell+1}) - f(x_{\ell-1})}{2\Delta x}$$

$$f(x_{\ell+1}) = f(x_\ell + \Delta x) = f(x_\ell) + \Delta x \cdot f'(x_\ell) + \frac{\Delta x^2}{2} f''(x_\ell) + \frac{\Delta x^3}{6} f'''(x_\ell) + \mathcal{O}(\Delta x^4) \quad \Big| \cdot 1 \quad \Big| \cdot 1$$

$$f(x_{\ell-1}) = f(x_\ell - \Delta x) = f(x_\ell) - \Delta x \cdot f'(x_\ell) + \frac{\Delta x^2}{2} f''(x_\ell) - \frac{\Delta x^3}{6} f'''(x_\ell) + \mathcal{O}(\Delta x^4) \quad \Big| \cdot (-1) \quad \Big| \cdot 1$$

$$f(x_{\ell+1}) - f(x_{\ell-1}) = +2\Delta x\, f'(x_\ell) + \mathcal{O}(\Delta x^3) \rightsquigarrow f'(x_\ell) = \frac{f(x_{\ell+1}) - f(x_{\ell-1})}{2\Delta x} + \mathcal{O}(\Delta x^2)$$ first order derivative

$$f(x_{\ell+1}) + f(x_{\ell-1}) = 2f(x_\ell) + \Delta x^2 f''(x_\ell) + \mathcal{O}(\Delta x^4) \rightsquigarrow f''(x_\ell) = \frac{f(x_{\ell+1}) - 2f(x_\ell) + f(x_{\ell-1})}{\Delta x^2} + \mathcal{O}(\Delta x^2)$$ second order derivative

$$f'(x_\ell)=\frac{f(x_{\ell+1})-f(x_\ell)}{\Delta x}$$

$$f'(x_\ell)=\sum_{\ell=0}^{N-1} i q_\ell c_\ell e^{iq_\ell x} \; ; \; f(x_\ell)=\sum_{\ell=0}^{N-1} c_\ell e^{iq_\ell x_\ell} \; , \; f(x_{\ell+1})=\sum_{\ell=0}^{N-1} c_\ell e^{iq_\ell (x_\ell+\Delta x)}$$

$$f'(x_\ell)=\frac{f(x_{\ell+1})-f(x_\ell)}{\Delta x}=\frac{\sum_\ell \left(e^{iq_\ell\cdot(x_\ell+\Delta x)}-e^{iq_\ell x_\ell}\right)c_\ell}{\Delta x}=\sum_\ell \frac{e^{iq_\ell\cdot\Delta x}-1}{\Delta x}\cdot e^{iq_\ell x_\ell}\cdot c_\ell$$

Heat transport: $\frac{\partial T(x,t)}{\partial t}=\alpha\frac{\partial^2 T(x,t)}{\partial x^2}+\dot{q}_s(x,t)$ FT $\rightsquigarrow$ $\frac{\partial \sum_\ell c_\ell(t) e^{iq_\ell\cdot x}}{\partial t}=\alpha\sum_\ell -q_\ell^2\cdot c_\ell(t)\, e^{iq_\ell x}$

$$\sum_\ell \dot{c}_\ell\, e^{iq_\ell x}=\alpha\sum_\ell +q_\ell^2\, c_\ell\, e^{iq_\ell x} \rightsquigarrow \sum_\ell\left(\dot{c}_\ell+\alpha q_\ell^2 c_\ell\right)e^{iq_\ell x}=0$$

$$\dot{c}_\ell=-\alpha q_\ell^2\cdot c_\ell \rightsquigarrow c_\ell=c_\ell(0)\,e^{-\alpha q_\ell^2\cdot t}$$