PDE: $u'' + f(x) = 0 \qquad x \in \Omega = [0, L]$

Residual: $R(x; c_1, c_2, \ldots, c_N) = u_h'' + f(x)$

Residual orthogonality: $(R, v) = 0 \quad \forall\, v \in V$

$$\Rightarrow \left(u_h'' + f(x), v\right) = 0$$

$$\Rightarrow \left(u_h'', v\right) = -(f, v) \quad \forall\, v \in V$$

Thus, trial soln in V: $u_h = \sum_{j=1}^{N} c_j \psi_j(x)$ & $u_h'' = \sum_{j=1}^{N} c_j \psi_j''(x)$

$$\Rightarrow \left(\sum c_j \psi_j'', v\right) = -(f, v) \quad \forall\, v \in V$$

$$\Rightarrow \left(\sum c_j \psi_j'', \psi_i\right) = -\left(f(x), \psi_i\right) \quad \text{where } V = \text{span}\{\psi_i\},\ i = 1, 2, \ldots, N$$

$$\Rightarrow \sum (\psi_j'', \psi_i) c_j = -(f(x), \psi_i)$$

$$\rightarrow \boxed{\sum A_{ij} \cdot c_j = b_i} \quad \text{where } A_{ij} = (\psi_j'', \psi_i),\ b_i = -(f, \psi_i)$$

Example

$\psi_i = x^i (x - L) \qquad \psi_i(0) = \psi_i(L) = 0$

$A_{ij} = (\psi_j'', \psi_i) \qquad b_i = \int_0^L f(x)\, x^i (x - L) \cdot dx$

$$\text{or, } \boxed{Ac = b}$$

⊛ we solve the integral by gauss-quadrature.

⊛ we compute the ψ_j'' using spectral?