# Gravity-driven fluid oscillations using straw Open-end Lab Report

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**Abstract**

The experiment is based on the phenomenon where a liquid column in a straw experience hydrostatic pressure which pushes it up and gravitational force which pulls it down, thus creating oscillations about the surface of the level. Ideally,the oscillations should last through eternity, but there are other factors which lead to damped oscillations. These factors make it interesting to study, and the experiment is build upon this study. Two different types of liquid with two types of straw are taken into account in this experiment, studying different cases, comparisons and then drawing conclusions.

## I. Objective

To model the gravity driven oscillations in a straw and study them:

- For different heights of the liquid
- For two different liquids - Water and Ethyl Alcohol

## II. Theory

The phenomenon of fluid oscillations can be modelled into two different mathematical formulations namely - Newton model[1] and Lorenceau model[2]. The Newton model, based on Newtonian laws, takes damping constant into account and Lorenceau model, based on energy loss equation, takes the contributing factors (for damping) singular pressure loss and Poiseuille friction into account. They will be discussed below in details.

### A. Newton Model

We consider a cylindrical straw, with radius of r, capped with finger is dipped in a beaker of liquid at depth h of the straw. The initial height of the water is $z_o$ and z is the height of the liquid at a time t. After the finger is lifted, the liquid starts oscillating about the level in beaker which decays over time.

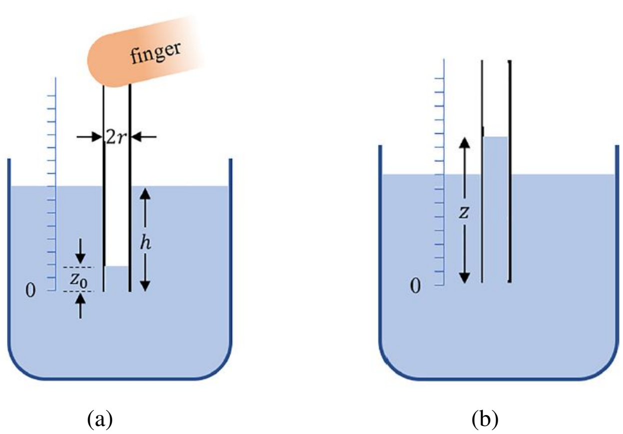



Figure 1. (a) Capped straw dipped in liquid, (b) After the finger is being removed [1]

In this model, we have to account for the Newtonian forces- Forces due to hydrostatic pressure ($F_p$), gravitational force ($F_g$), and the damping force ($F_d$). Hence, the force equation is given by:

$$\frac{dp}{dt} = F_p - F_g + F_d \tag{1}$$

where p is the momentum of the fluid which is $m\dot{z}$. Now the forces are,

$$F_p = \rho g h A$$

$$F_g = mg$$

$$F_d = b'\dot{z}$$

where A is the cross sectional area of the straw which is $\pi r^2$, $\rho$ is the density of the fluid, m is mass of liquid which is variable = $\rho z A$ and b' is the damping coefficient. So putting all the values in the equation,

$$\frac{d(m\dot{z})}{dt} = \rho g h A - mg + b'\dot{z} \tag{2}$$

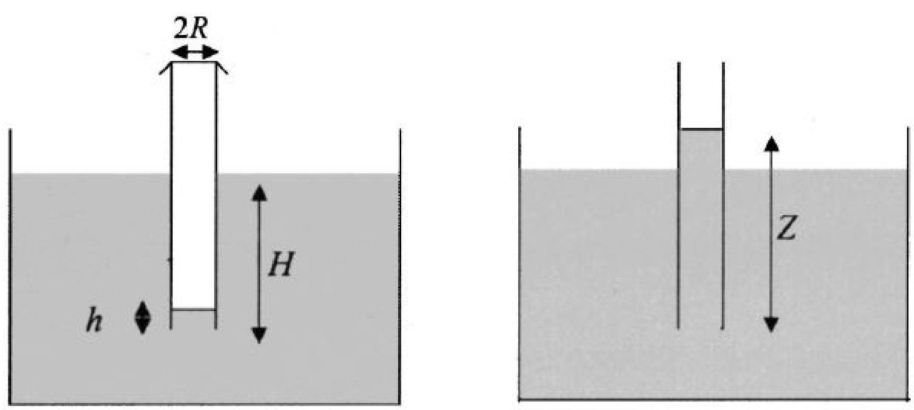
$$\Rightarrow \quad m\ddot{z} + \dot{m}\dot{z} = \rho g h A - mg + b'\dot{z} \tag{3}$$

$$\Rightarrow \quad \ddot{z} = -\frac{1}{z}(\dot{z}^2 + gz - gh + b\dot{z}) \tag{4}$$

Here b' is taken as $b\rho A$ to simplify the equation. So finally we have a differential equation (4), which we would be using to generate data for our Newton model. So we see here that Newton model only considers a damping constant, not the individual factors contributing to the damping.

### B. *Lorenceau model*

As given in the figure below, let the radius of straw be R and depth of straw below water be H. The initial liquid level is h when the straw was capped and Z is the height of the liquid at a time t . The fluid is assumed to have laminar flow.




(a) (b)

Figure 2. (a) Capped straw dipped in liquid, (b) After the finger is being removed [2]

The radius of the straw is much larger than the capillary length, so the capillary forces can be neglected. So the main driving force is the hydrostatic pressure,

$$F = \rho g \pi R^2 (H - Z)$$

Integrating the force will give the potential energy,

$$U = \frac{1}{2}\rho g \pi R^2 - \rho g H \pi R^2 Z$$

Hence the total energy,

$$E = \text{Kin. energy} + U$$

$$E = \frac{1}{2}\rho \pi R^2 Z \dot{Z}^2 + \frac{1}{2}\rho g \pi R^2 - \rho g H \pi R^2 Z \tag{5}$$

In dimensionless variables and scaling the mass by $\rho \pi R^2 H$, the energy equation simplifies to,

$$e = \frac{1}{2} z \dot{z}^2 + \frac{1}{2} z^2 - z \tag{6}$$

Now coming to the causes of energy dissipation, we can neglect viscous dissipation for a short period of time. But for a longer period, we cannot neglect it. But first let's discuss singular pressure loss which occurs at the entrance of the straw. This happens due to difference in radius of the straw and the beaker, which causes eddies to occur at the entrance and thus dissipating energy. Its expression is given by:

$$\Delta P = \frac{1}{2}\rho \dot{Z}^2$$

The pressure loss is always positive, so the associated energy is always negative. So we land with two cases with the energy loss equation,
When the liquid is rising (dz > 0),

$$de = \frac{1}{2}\dot{z}^2 dz$$

and when liquid is falling (dz < 0),

$$de = -\frac{1}{2}\dot{z}^2 dz$$

Hence comparing them with equation (6) gives,
For dz > 0

$$z\ddot{z} + \dot{z} = 1 - z \tag{7}$$

and, for dz < 0

$$z\ddot{z} = 1 - z \tag{8}$$

But our experiments are conducted over a larger time scale, so we have to consider the Poiseuille friction $\Omega$ given by,

$$\Omega = \frac{16 \eta H^{1/2}}{\rho R^2 g^{1/2}} \tag{9}$$

which is a constant.$\eta$ is the viscosity of fluid. The equation (7) and (8) is modified with a factor,
For dz > 0

$$z\ddot{z} + \dot{z} = 1 - z - \Omega z \dot{z} \tag{10}$$

and, for dz < 0

$$z\ddot{z} = 1 - z - \Omega z \dot{z} \tag{11}$$

These are are the equations we will use to model Lorenceau data. So we see here unlike Newton model, Lorenceau model considers the factors which dissipates energy leading to damping. Hence this model can be considered more accurate or closer to the experimental data.

### C. Analysing Data

Using the above models, the experimental data are fitted. The plots will be in time domain. But what information we can get from the plots or from the dataset? As it is a damping oscillation we can get to know about the frequencies involved in the oscillation, and to do that we have to convert the time domain to frequency domain, using fast fourier transform.

*1) Fourier transform:* Fourier transform is just a mathematical function which represents a signal (basically a function) as series of sinusoidal functions. In this context, it takes a dataset (say frequency in x domain) and gives the amount of a given frequency present in that dataset.

For discrete dataset,

$$X_k = \sum_{n=0}^{N-1} X_n \cdot e^{-\frac{i2\pi kn}{N}} \tag{12}$$

We take $\omega_n = (\frac{i2\pi kn}{N})$, then we can write the equation for discrete fourier transform as

$$X_k = X_0 \cdot e^{-\omega_0 i} + X_1 \cdot e^{-\omega_1 i} + \cdots + X_n \cdot e^{-\omega_n i} \tag{13}$$

We can represent the above equation in the form of matrix,

$$\begin{bmatrix} F_0 \\ F_1 \\ . \\ . \\ . \\ F_{n-1} \end{bmatrix} = \begin{bmatrix} 1 & 1 & ..... & 1 \\ 1 & \omega_n^2 & ..... & \omega_n^{n-1} \\ 1 & \omega_n^2 & ..... & \omega_n^{2(n-1)} \\ . & . & ..... & . \\ . & . & ..... & . \\ 1 & \omega_n^{n-1} & ..... & \omega_n^{(n-1)^2} \end{bmatrix} \begin{bmatrix} f_0 \\ f_1 \\ . \\ . \\ . \\ f_{n-1} \end{bmatrix} \tag{14}$$

$$F = X.f$$

where f is the dataset and F is the transform. This is computationally very slow and its complexity is in order $N^2$, hence more number of data will lead to slow computation. **Fast Fourier transform** is way of making the computation faster as its complexity is in order $Nlog(N)$. It basically splits the matrices into multiple matrices by exploiting its symmetry, making the multiplication computationally simpler. We will be using **fft** function of the **numpy** package while analysing the data, which will use the above method.

*2) Lorentzian Fit:* Now that we got the data in frequency domain, we can analyse it and find the frequency for which the power curve peaks.

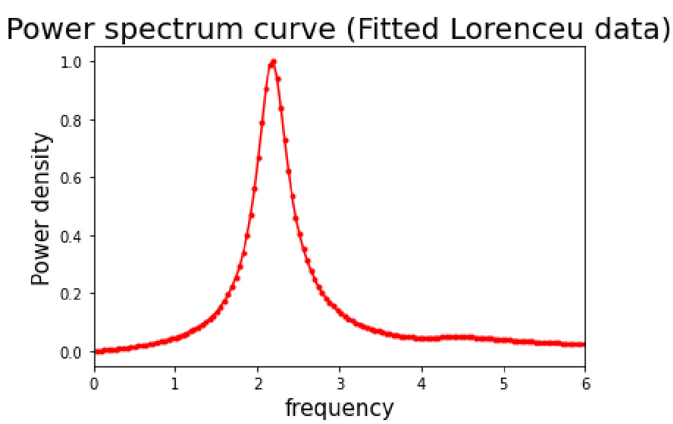



Figure 3. Power spectrum obtained from Fourier transform

As it is a discrete data set, we cannot pin point the peak. Hence, we do a lorentzian fit to the curve. The Lorentzian function is given by:

$$y = \frac{A}{1 + \left(\frac{x-x_0}{\gamma}\right)^2} \tag{15}$$

where A = amplitude (height) of the peak, $x_0$ = position of the peak, and $\gamma$ = peak half width at half maximum level.

*3) Frequency for small oscillations:* In limit of small oscillations, we can really simplify the equation (4). First, we take y=z-h (oscillation about the surface level). So our assumption is y < < h. The equation simplifies to :

$$\ddot{y} = -\frac{g}{h}.y \tag{16}$$

which is basically Hooke's law. So we can get the natural frequency as

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi}\left(\frac{g}{h}\right)^{1/2} \tag{17}$$

We can compare the natural frequency with the frequency we get from the peak of the Lorentzian fit.

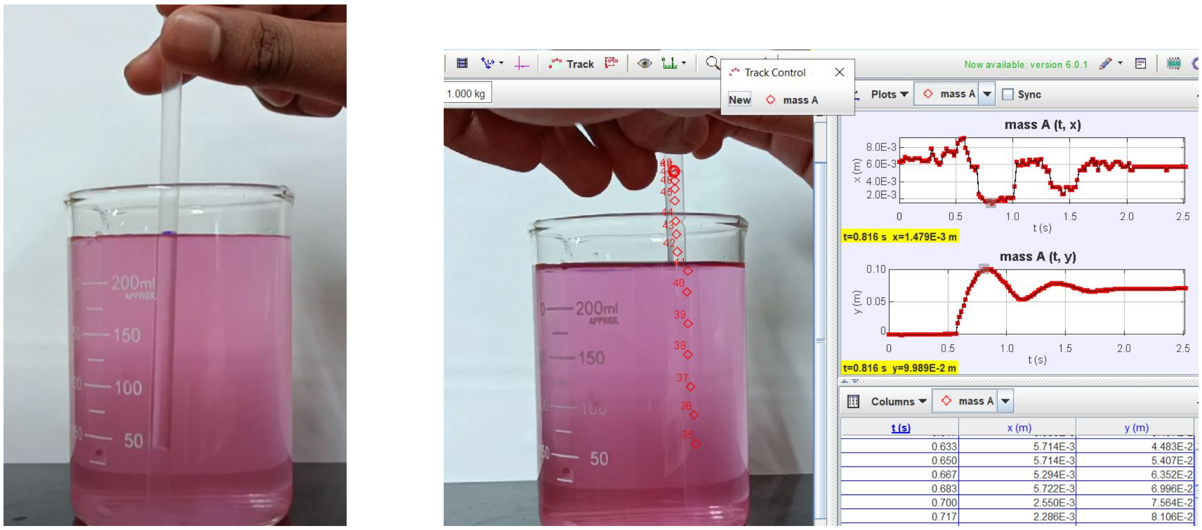
## III. Apparatus used

1) Plastic straw and glass tube (straw for the exp.)
2) Glass beaker
3) Red colour
4) Water and ethyl alcohol (95%)
5) Camera for recording
6) Camera stand
7) Installed tracker software

## IV. Method

### A. The experiment

We take a transparent straw and beaker full of liquid. Then we colourize the liquid with red water color so that we can track the liquid level with ease. The straw is hand held and capped, submerged into the beaker of liquid at a measured depth (h). The finger is then lifted and the fluid oscillations are then recorded by camera which is kept approximately along the liquid level. The radius of the straw was calculated with a vernier calipers.




(a) (b)

Figure 4. (a) Experimental Setup, (b) Collecting data points using tracker

### B. Tracker software

The video recording of the oscillations is imported in to the tracker software. The calibration was done using height of the straw submerged. The level of the liquid was traced by marking points frame by frame (Figure 3b). The initial level of the fluid was considered to be at 0 cm. We had the data points for the fluid level vs time, which we exported as data file.

### C. Coding

I did the coding in Python for analysing the data we collected (in .txt format) from the tracker software. This code is based on work by r31415smith/AdvLab [3].

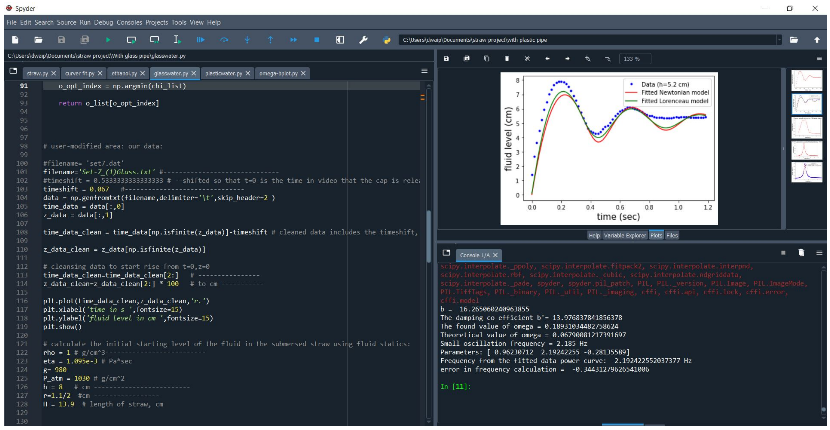



Figure 5. Workspace in spyder

Here are some of the highlights of the code:

*1) Extracting the data:* The .txt file was used to extract the data. Some changes where made such that the rise starts at t=0.

*2) Newton and Lorenceau model:* We use the Newton and Lorenceau model to study out data, for that we need to solve the differential equations (4), (10) and (11). I used the **odeint** from the package **scipy.integrate** to solve the equations. Odeint is based LSODA algorithm[4].

*3) Chi-square fit:* Now we have to fit the data with the two models. I looked up many algorithms for curve-fit but they looked complicated. I decided to go for chi-square fit test. Chi-square is given by the formula:

$$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i}$$

where $O_i$ is the experimental value and $E_i$ is the expected value or the value we get by solving the model. Chi-square fit test says that the plot with minimum value of chi-square is the best plot. But it has a drawback. We have to guess a value of b' or $\Omega$ every time. For that I used a for loop which feeds guesses into the chi-square test, and we got the best fit hence the best parameters.

*4) Fast Fourier Transform:* As discussed above, I used the **numpy.fft.fft** to get the fourier transform (Section II c). Then I calculated the power density and from the sample frequency from **numpy.fft.fftfreq**, we get the power spectrum which gives the frequency information about the oscillation.

*5) Lorentzian Fit:* After I got the power spectrum, it had to be fitted to Lorentzian function (Section II c). So I used **curve_fit** from the package **scipy.optimize** to fit with the functions and got the required parameters.

*6) Output:* In summary, the code gives the following outputs:

- The damping co-efficient b' from the Newton model fit
- The Poiseuille friction $\Omega$ from Lorenceau model fit
- Chi-square value from Newton model fit and Lorenceau fit to compare
- The theoretical natural frequency and the natural frequency from the Lorentzian fit. Also the error in the frequency calculation.

They are tabulated in table I, II, and III. All the data and code are uploaded here **Data - codes**.

## V. Observations

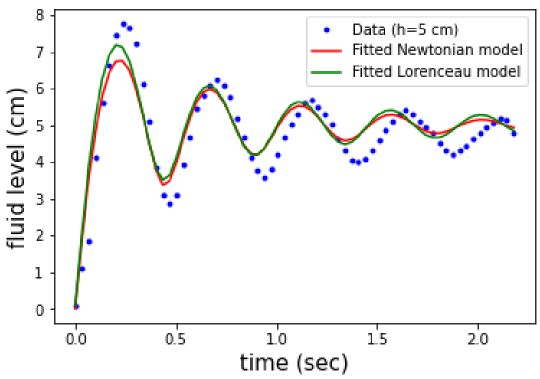
Two types of liquid are used -

- Water, $\rho$=1$g/cm^3$
- Diluted ethanol (1:1 ratio of water and 95% ethanol), $\rho$=0.894$g/cm^3$

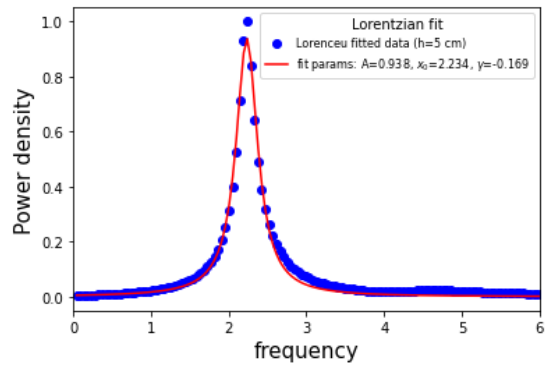
### *A. Diluted ethanol with glass tube (r=0.55 cm, $\rho$=0.894 $g/cm^3$)*

From equation (9),
Calculated value of omega = 0.0759

**For h=5cm,**



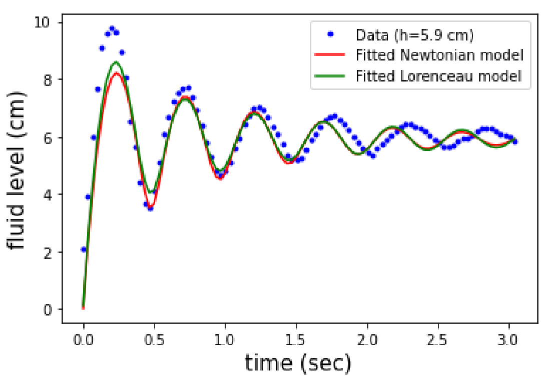

(a) Data fitted with Newton and Lorenceau model



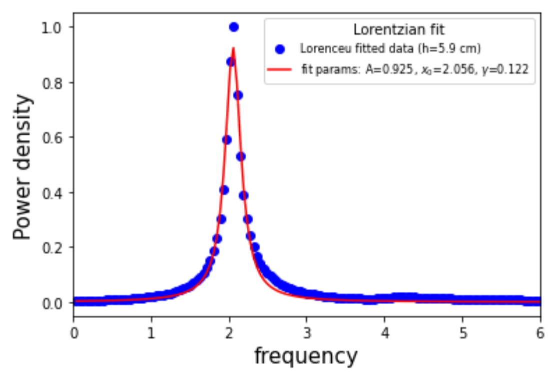

(b) Power spectrum fitted to Lorentzian function

Figure 6.

**For h=5.9cm,**



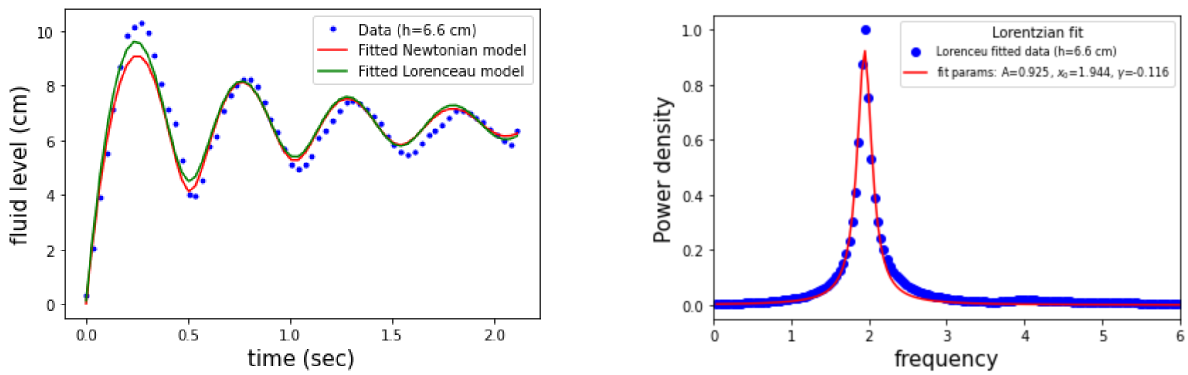

(a) Data fitted with Newton and Lorenceau model




(b) Power spectrum fitted to Lorentzian function

Figure 7.

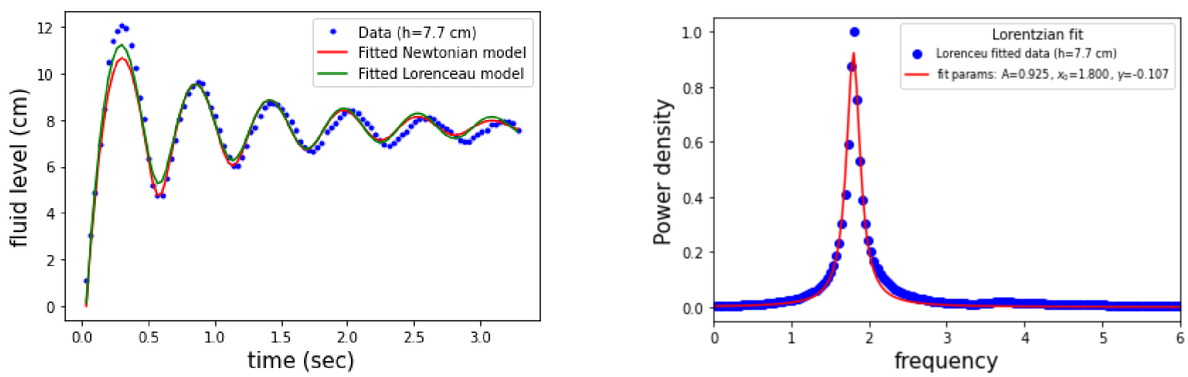
## For h=6.6cm,




(a) Data fitted with Newton and Lorenceau model (b) Power spectrum fitted to Lorentzian function

Figure 8.

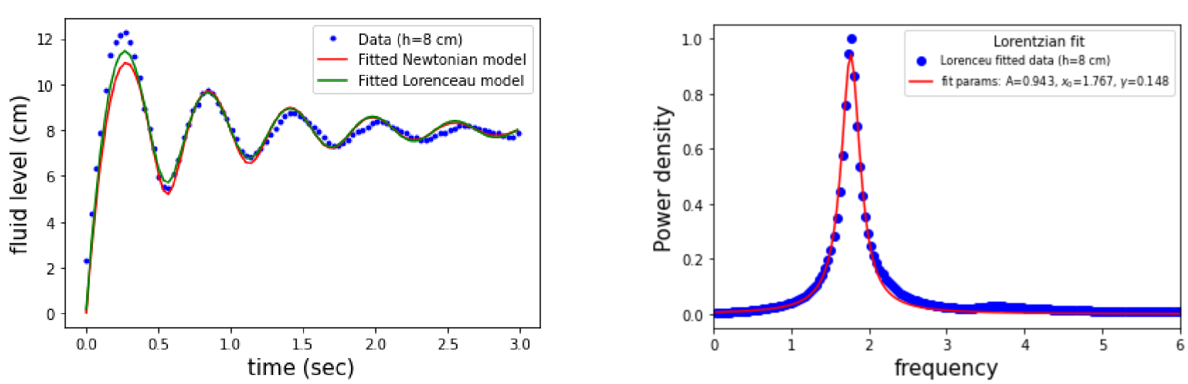
## For h=7.7cm,




(a) Data fitted with Newton and Lorenceau model (b) Power spectrum fitted to Lorentzian function

Figure 9.

## For h=8cm,




(a) Data fitted with Newton and Lorenceau model (b) Power spectrum fitted to Lorentzian function

Figure 10.

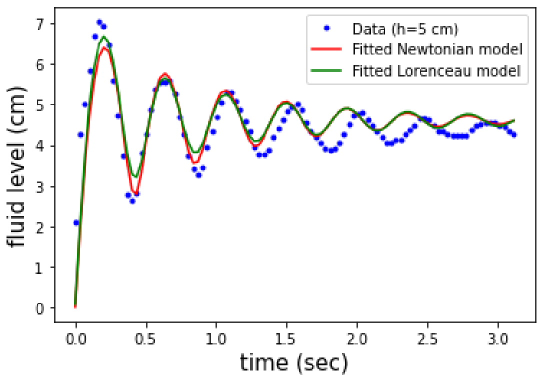
| h (cm) | b' ($s^{-1}$) | $\Omega$ | Theoretical frequency(Hz) | Frequency from power spectrum(Hz) | Error in frequency(%) |
|---|---|---|---|---|---|
| 5 | 11.443 | 0.0927 | 2.228 | 2.234 | -0.272 |
| 5.9 | 9.47 | 0.0651 | 2.051 | 2.056 | -0.245 |
| 6.6 | 11.362 | 0.0651 | 1.939 | 1.944 | -0.245 |
| 7.7 | 11.689 | 0.0651 | 1.796 | 1.799 | -0.245 |
| 8 | 19.73 | 0.1065 | 1.762 | 1.766 | -0.283 |

Table I
OUTPUT DATA TABULATED

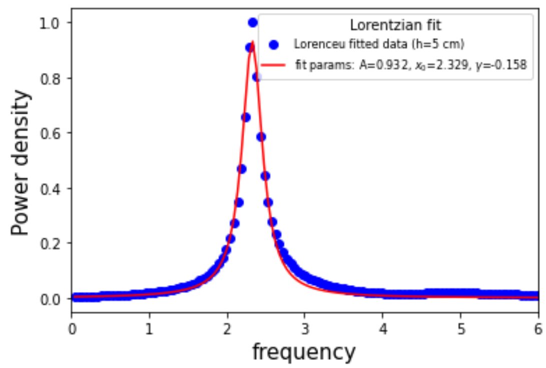
### *B. Water with glass tube (r=0.55 cm, $\rho$=1 $g/cm^3$)*

From equation (9),
Calculated value of omega = 0.067

**For h=5cm,**



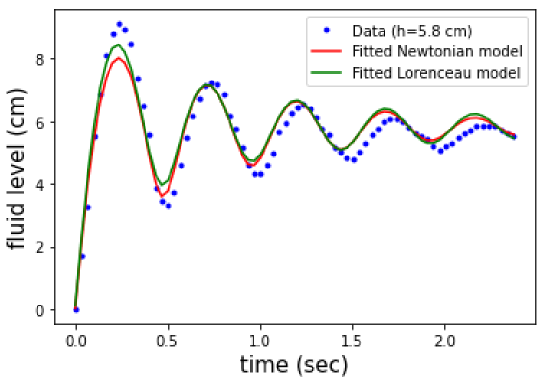

(a) Data fitted with Newton and Lorenceau model



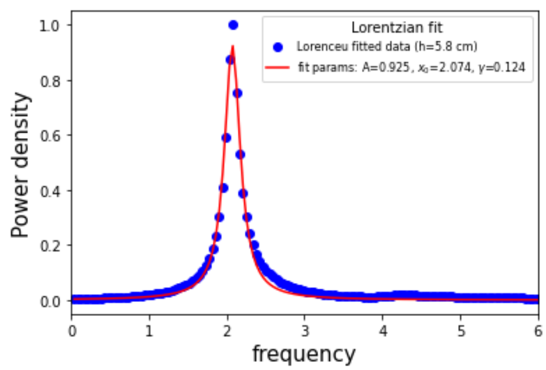

(b) Power spectrum fitted to Lorentzian function

Figure 11.

**For h=5.8cm,**



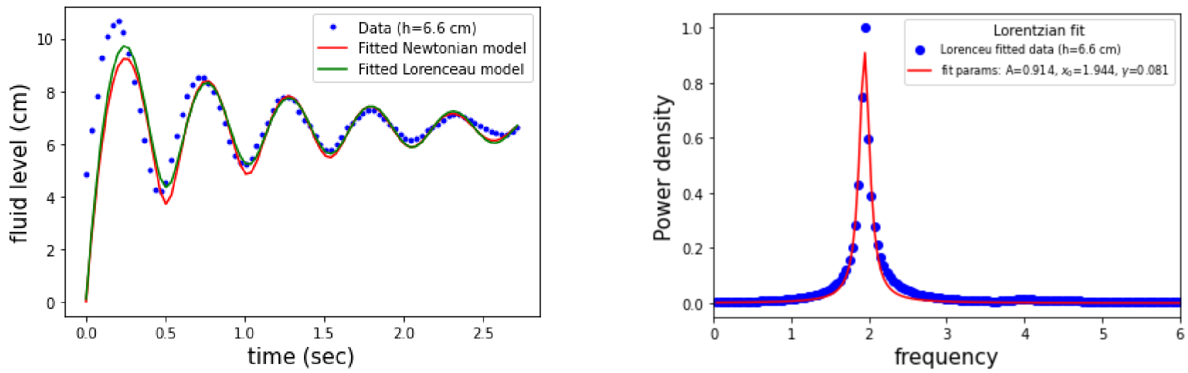

(a) Data fitted with Newton and Lorenceau model




(b) Power spectrum fitted to Lorentzian function

Figure 12.

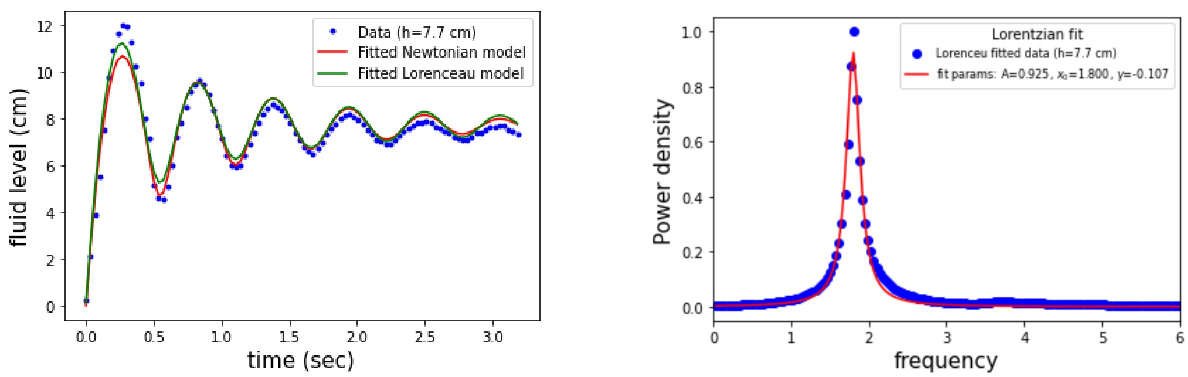
## For h=6.6cm,




(a) Data fitted with Newton and Lorenceau model (b) Power spectrum fitted to Lorentzian function

Figure 13.

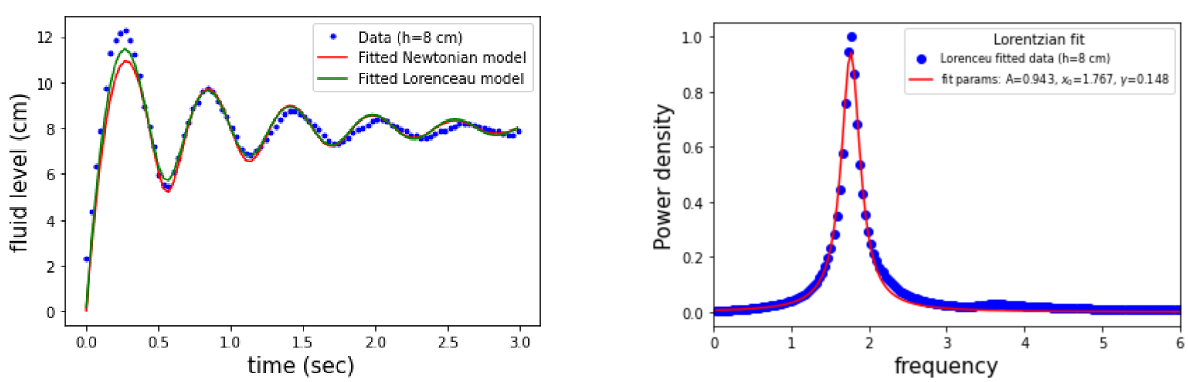
## For h=7.7cm,




(a) Data fitted with Newton and Lorenceau model (b) Power spectrum fitted to Lorentzian function

Figure 14.

## For h=8cm,



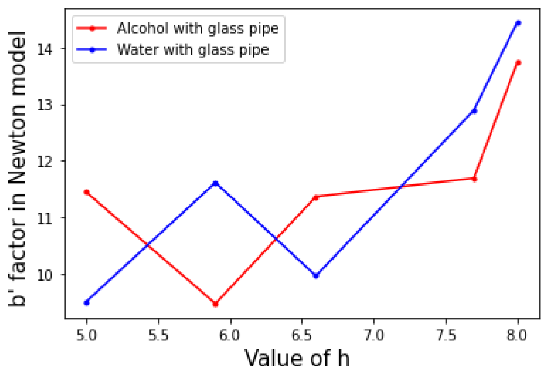

(a) Data fitted with Newton and Lorenceau model (b) Power spectrum fitted to Lorentzian function

Figure 15.

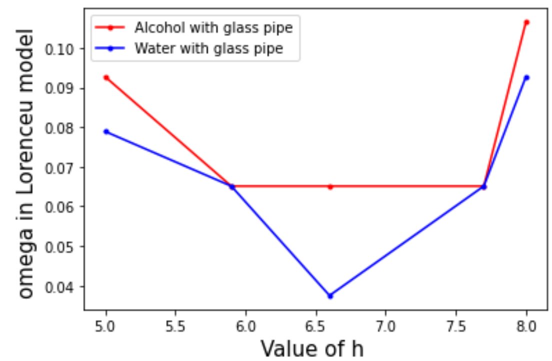
| h (cm) | b' $(s^{-1})$ | $\Omega$ | Theoretical frequency(Hz) | Frequency from power spectrum(Hz) | Error in frequency(%) |
|---|---|---|---|---|---|
| 5 | 9.503 | 0.0789 | 2.323 | 2.329 | -0.26 |
| 5.9 | 11.61 | 0.0651 | 2.069 | 2.07 | -0.245 |
| 6.6 | 9.96 | 0.0375 | 1.939 | 1.943 | -0.235 |
| 7.7 | 12.892 | 0.0651 | 1.796 | 1.799 | -0.245 |
| 8 | 14.44 | 0.0927 | 1.762 | 1.766 | -0.272 |

Table II
OUTPUT DATA TABULATED

### C. *Comparison between water and diluted alcohol*




(a) Comparison study of b' between water and alcohol

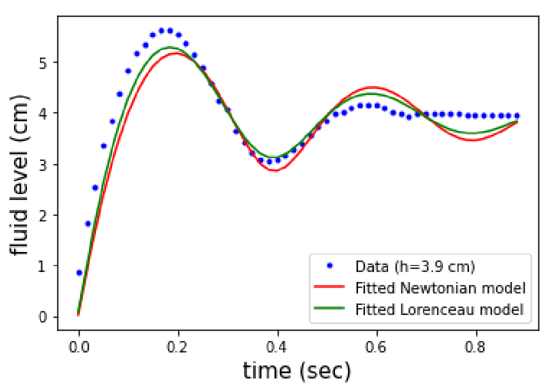



(b) Comparison study of $\Omega$ between water and alcohol

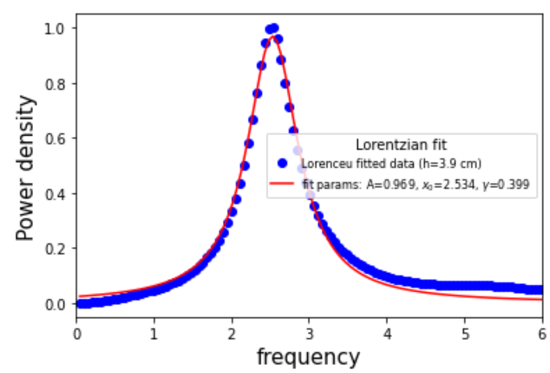
Figure 16.

### D. *Water with plastic tube (r=0.523 cm, $\rho$= 1$g/cm^3$)*

From equation (9),
Calculated value of omega = 0.0679
**For h=3.9cm,**



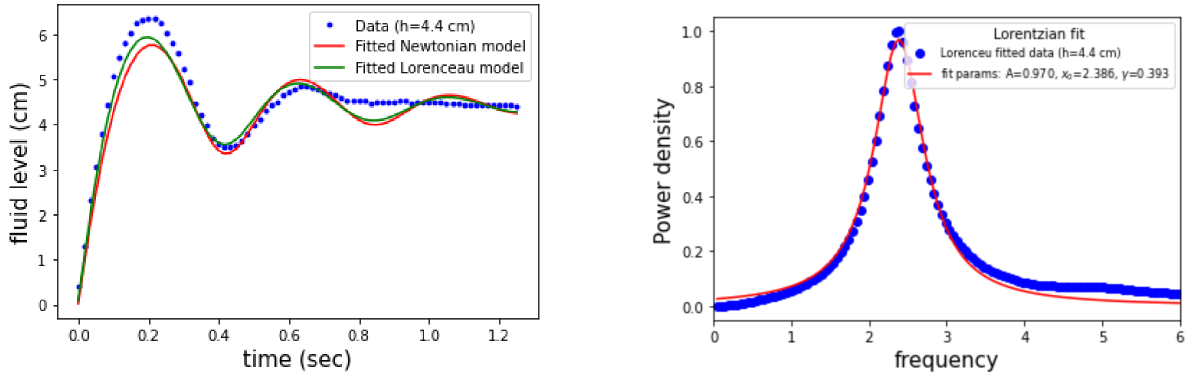

(a) Data fitted with Newton and Lorenceau model




(b) Power spectrum fitted to Lorentzian function

Figure 17.

## For h=4.4cm,

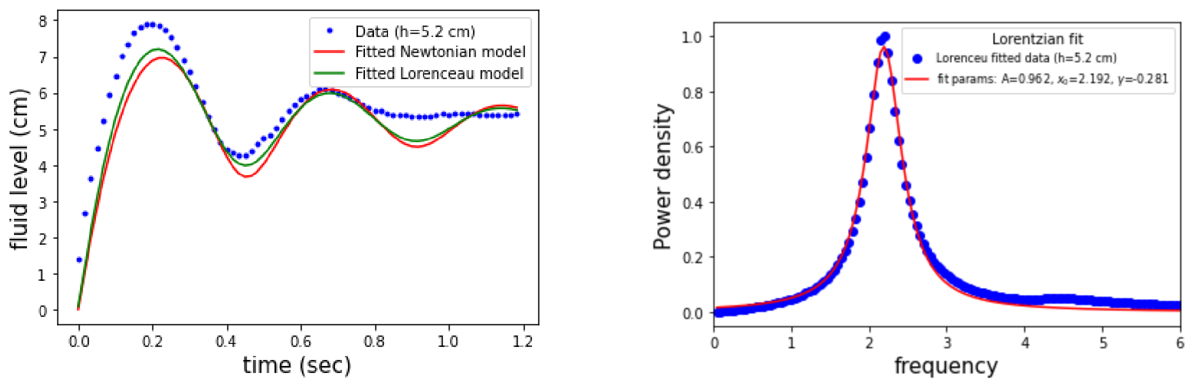



(a) Data fitted with Newton and Lorenceau model

(b) Power spectrum fitted to Lorentzian function

Figure 18.

## For h=5.2cm,




(a) Data fitted with Newton and Lorenceau model

(b) Power spectrum fitted to Lorentzian function

Figure 19.

| h (cm) | b' ($s^{-1}$) | Ω | Theoretical frequency(Hz) | Frequency from power spectrum(Hz) | Error in frequency(%) |
|---|---|---|---|---|---|
| 3.9 | 15.716 | 0.258 | 2.375 | 2.385 | -0.449 |
| 4.4 | 13.479 | 0.161 | 2.228 | 2.235 | -0.32 |
| 5.2 | 13.976 | 0.189 | 2.185 | 2.192 | -0.344 |

Table III

OUTPUT DATA TABULATED

## VI. Discussion

All the plots are given above and the output from the code have been tabulated. Some points to be noted from all the results of the experiment:-

### *A. Comparison of Newton model and Lorenceau model*

As discussed above, the experimental data was fitted to the models using chi-square fit method. It was not that effective as we had to give guess for the parameters. I tried to include many guesses (through a loop) to find a the desired parameter for the minimum chi-square. When the value of minimum chi-square was compared between the two fitted models, Lorenceau model had the most closest plot to the experimental data, in most of the cases. This proves our statement that Lorenceau model is more accurate than Newton model.

### *B. Deviation of omega from the theoretical value*

We have calculated the value of omega i.e Poiseuille friction from equation (9), but as we have seen from the fit that the experimental value deviates from the theoretical one. This is because all our calculation in Lorenceau model was based on the assumption of laminar flow. But in reality the fluid also has turbulent flow which is not taken into account. This can be shown by the calculation of reynolds number, which shows it is above 2000 (approximately 2420). This explains the deviation.

### *C. Alcohol solution and water comparison*

With respect to damping co-efficient b', there is not much to notice. But in Poiseuille friction $\Omega$, the alcohol seems to have more value of $\Omega$ than water.
While in plastic straw, the value of b' and $\Omega$ is considerably higher in comparison with the glass pipe which is expected.(Table II and III)

### *D. Change in damping co-efficient and viscous friction co-efficient with height h*

There was not any given direct relation between b',$\Omega$ and h. So I went ahead to plot them out of curiosity. Figure 16 (a) and (b) shows b' vs h and $\Omega$ vs h respectively for two different liquids.
In b' vs h graph, b' seemed to have increased with h but there is no as such trend in case of $\Omega$ vs h. Both the plots show non-linear form which is expected.

### *E. Error in fitted model*

As seen from the plots of fitted models, the curvefit is not that accurate and at some points the data deviates too much. This can be due to the following reasons:-

- The pipe was hold by a hand, so it was shaky at certain periods of time.
- The camera of the mobile phone is restricted to 30 fps recording, so it was a little blurry while tracking the liquid level in the tracker software.
- As the straw was held manually, it was not exactly perpendicular to liquid level as it should be theoretically. This is a human error.

### *F. Error in frequency calculation*

The table I,II,III shows the theoretical and experimental natural frequencies; and also shows the error. It shows that the error percentage is very low as 0.27 percent average. This means the theoretical and experimental values are quite close to each other.

## VII. Conclusion

The plots and and analysis has been done successfully. The interesting part about the whole experiment is that how the factors works or contributes to the damped oscillation. Even ideally (when the factors like viscous friction, damping coefficient are absent), still the oscillation would not be harmonic. It will have a characteristic sharp turn below y=0.

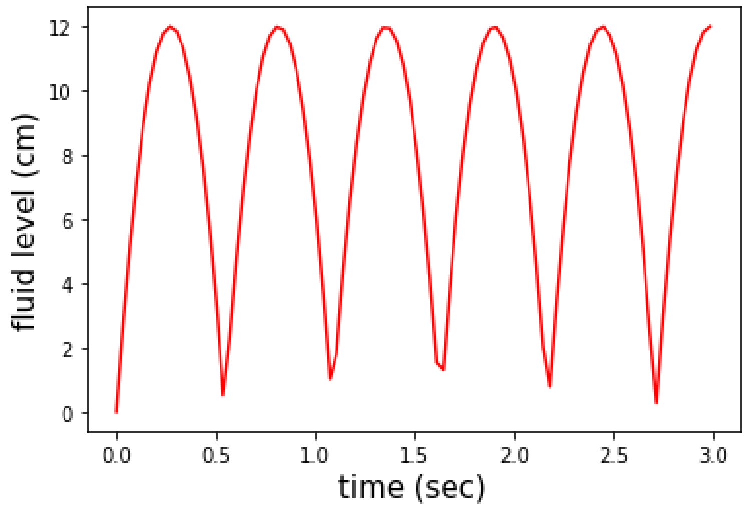



Figure 20. b' = 0 in Newton model

But considering small oscillations limit, it shows the Hooke's law which gives the natural frequency. Newton and Lorenceau model studies the factors and tries to be as accurate as possible to the experimental data. Lorenceau data provides much more complexity as the contributing factors weigh in the force equation.
We further studied the frequencies in the oscillation using Fourier Transform. We verified the natural frequency found from the experiment to the theoretical value. All the errors and discrepancies are discussed in the above section. In future, if I get to work on this project, I will go on further, trying out other types fluids, types of straws and comparing them along the radius.

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