*Exercises are taken from the recommended texts*

1. Find the coefficient of $x^7$ in the $(1+x)^{11}$.
2. Find the coefficient of $x^9$ in $(2-x)^{19}$.
3. Find the coefficient of $x^5$ in $(3x^2-2x)^7$.
4. Give a formula for the coefficient of $x^k$ in the expansion of $(1+1/x)^{20}$, where $k$ is an integer.
5. Give a formula for the coefficient of $x^k$ in the expansion of $(x^2-1/x)^{20}$, where $k$ is an integer.
6. Show that if $n$ and $k$ are integers with $1 \leq k \leq n$, then $\binom{n}{k} \leq n^k/2^{k-1}$.
7. Use a combinatorial argument to prove the identity

$$\binom{n}{r}\binom{r}{k} = \binom{n}{k}\binom{n-k}{r-k},$$

where $n, r$, and $k$ are nonnegative integers with $k \leq r \leq n$.

8. Prove Pascal's Identity:

$$\binom{n+1}{k} = \binom{n}{k-1} + \binom{n}{k}$$

9. Use Pascal's Identity to prove that:

$$\sum_{k=0}^{r}\binom{n+k}{k} = \binom{n+r+1}{r}.$$

Now prove the identity using a combinatorial argument.

10. Use a combinatorial argument to show that that if $n$ is a positive integer, then $\binom{2n}{2} = 2\binom{n}{2} + n^2$.
11. Use a combinatorial argument to show that $\sum_{k=1}^{n} k\binom{n}{k}^2 = n2^{n-1}$.
12. How many solutions are there to the equation

$$x_1 + x_2 + x_2 + x_4 = 17,$$

where $x_1, x_2, x_3$, and $x_4$ are nonnegative integers

13. *Ternary Digits* consist of the numbers 0, 1, and 2. How many strings of 10 ternary digits are there that contain exactly two 0s, three 1s and five 2s?

14. How many ways are there to choose eight coins from a coin box containing ten 50 pesewa coins, ten 1 cedi coins, and ten 2 cedi coins?

15. There are 10 questions on a discrete mathematics final exam. How many ways are there to assign scores to the problems if the sum of the scores is 100 and each question is worth at least five points?

16. Consider the multinomial theorem: If $n$ is a positive integer, then

$$(x_1 + x_2 + \cdots + x_m)^n = \sum_{n_1+n_2+\cdots+n_m=m} \frac{n!}{n_1!n_2!\cdots n_m!} x_1^{n_1} x_2^{n_2} \cdots x_m^{n_m}.$$

    Prove the multinomial theorem.

17. Find the expansion of $(x + y + z)^4$.