MATH 359: Tutorial III 24/25

Exercises are taken from the recommended texts

1. Use the Quine-McCluskey method to simplify the expressions below:
   (a) $xyz + xy\overline{z} + \overline{x}yz + \overline{x}y\overline{z}$
   (b) $xyz + x\overline{y}z + x\overline{y}\,\overline{z} + \overline{x}yz + \overline{x}y\overline{z} + \overline{x}\,\overline{y}\,\overline{z}$
   (c) $wxyz + wx\overline{y}z + w\overline{x}yz + \overline{w}x\overline{y}z + \overline{w}\,\overline{x}y\overline{z} + \overline{w}\,\overline{x}\,\overline{y}z$.
2. There are 128 different ASCII characters. How many strings of five ASCII characters contain the character @?
3. How many license plates can be made using three uppercase letters followed by three digits?
4. How many strings of eight uppercase English letters are there
   (a) if letters can be repeated?
   (b) if no letter can be repeated?
   (c) that start with `X`, if letters can be repeated?
   (d) that start with `X`, if no letter can be repeated?
   (e) that start and end with `X`, if letters can be repeated?
5. How many different functions are there from a set with 10 elements a set with 2 elements? How about 3 elements?
6. A palindrome is a string which when reversed is identical to the original string (an example is `madam`). How many bit strings of length n are palindromes?
7. In how many ways can a photographer at a wedding arrange six people in a row, including the bride and the groom, if
   (a) the bride must be next to the groom?
   (b) the bride is not next to the groom?
   (c) the bride is positioned somewhere to the left of the groom?
8. How many positive integers not exceeding 100 are divisible by either 4 or by 6?
9. A bowl contains 10 red balls and 10 blue balls. A person selects balls at random without looking at them.
   (a) How many balls must be selected in order to be sure of having at least three balls of the same color?
   (b) How many balls must be selected in order to be sure of having at least three blue balls?

10. Let n be a positive integer. Show that in any set of n consecutive integers, there is exactly one divisible by n.

11. Let d be a positive integer. Show that among any group of $d + 1$ (not necessarily consecutive) integers, there are two with exactly the same remainder when they are divided by d.

12. There are nine students in a discrete mathematics class.

    (a) Show that the class must have at least five male students or at least five female students.

    (b) Show that the class must have at least three male students or at least seven female students.

13. In how many ways can five runners finish a race if no ties are allowed?

14. How many bit strings of length 10 contain

    (a) exactly four `1`s?

    (b) at most four `1`s?

    (c) at least four `1`s?

15. How many permutations of the letters `ABCDEFGH` contain

    (a) the string `ED`?

    (b) the string `CE`?

    (c) the strings `BA` and `FGH`?