

2020–2021 Winter Break Math Challenges

Due Jan. 11 in-class to your math teacher. If you are online or do not have a math teacher, submit to vincent.chan@renertschool.ca with the subject “2020-2021 Winter Break Math Challenges.”

Challenge 1: Digit puzzle

To ring in the new year, make the number 2021 in 9 different ways, each time by using copies of the same digit and the following operations (in addition to parentheses):

- Standard operations: $+$, $-$, \times , \div
- Negation: $-\square$
- Exponentiation of two numbers: \square^\square
- Square root of a number: $\sqrt{\square}$
- Factorial: $\square!$ (Note: you may use iterated factorial but not multi-factorial, so that $3!! = (3!)! = 6! = 720$, and **not** $3!! = 3 \times 1 = 3$.)
- Concatenation (i.e. “glueing”) of digits (only of the original digit used): dd

Your score for a particular digit is the number of copies you use, and your goal is to have the lowest score possible (**you will be competing against others in your division**).

For example, you can make 2021 by using copies of the digit “9” as follows:

$$2021 = \underbrace{\frac{9}{9} + \frac{9}{9} + \cdots + \frac{9}{9}}_{2021 \text{ times}}.$$

If you do it like this, you are using 4042 copies of 9, which is not good for you. A far more efficient way to do it is

$$2021 = 9 \times \sqrt{9} \times \left(99 - \left(\sqrt{9} + \frac{9}{9} \right)! \right) - \sqrt{9}! + \frac{9}{9} + \frac{9}{9}$$

which gets you there with only 12 copies (this is, of course, not optimal).

- (1) Using the digit 1
- (2) Using the digit 2
- (3) Using the digit 3
- (4) Using the digit 4
- (5) Using the digit 5
- (6) Using the digit 6
- (7) Using the digit 7
- (8) Using the digit 8
- (9) Using the digit 9

Challenge 2: Approximations

To say goodbye to the last year, here is a puzzle about 2020. How close can you get to 2020 using the digits 2, 0, 2, 0 exactly once? You may use the following operations (in addition to parentheses):

- Standard operations: $+$, $-$, \times , \div
- Negation: $-\square$
- Exponentiation of two numbers: \square^\square
- Square root of a number: $\sqrt{\square}$
- Factorial: $\square!$
 - Note: you may use iterated factorial but not multi-factorial, so that $3!! = (3!)! = 6! = 720$, and **not** $3!! = 3 \times 1 = 3$.
 - Hint: $0! = 1$
- Floor: $\lfloor \square \rfloor$ (this is the largest integer less than or equal to the input, so $\lfloor \pi \rfloor = 3$)

Notice you CANNOT use "glueing"!

(You will be competing against others in your division.)

Challenge 3: Art

Winter is a great time for beautiful images, especially in Calgary where we get so much sunlight. For this challenge, draw or paint a picture! It must be

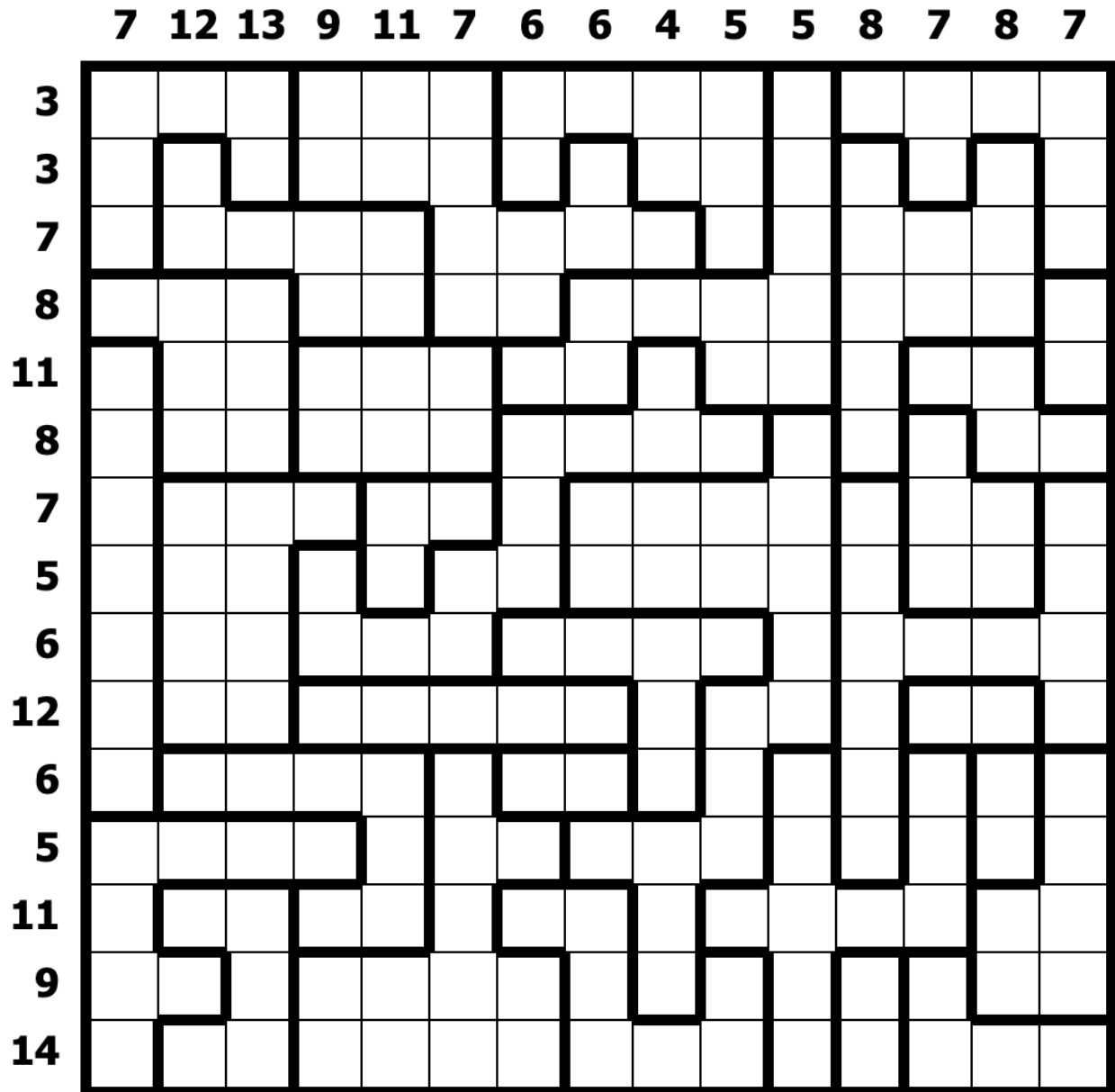
- (1) winter-themed, and
- (2) math-related.

You must also write a short paragraph explaining how your art satisfies these two points. You will be judged on:

- (1) the aesthetic and technical quality of the art,
- (2) the depth of connection to mathematics, and
- (3) the quality of writing.

(You will be competing against others in your division.)

In Aquarium, you'll find "cages," which are denoted by the bold outlines. You must shade some cells (indicating water) such that within a cage, all cells at the same row or below of a shaded cell are also shaded. The numbers outside the grid indicate how many cells are shaded in the respective row or column. A cage can be completely blank, completely shaded, or partially shaded. (Go to <https://www.puzzle-aquarium.com/> to practice.) Solve the following Aquarium!



Challenge 5: Suguru

In Suguru, you'll find "cages," which are denoted by the bold outlines. Each cage needs to be filled with unique digits counting up from 1, so that a 1-cell cage only has a 1, a 2-cell cage has a 1 and a 2, a 3-cell cage has a 1, 2, and a 3, and so forth. Cells that are adjacent (touching) cannot contain the same number, including diagonally. (Go to <https://krazydad.com/tablet/suguru/> to practice.) Solve the following Suguru!

		2			5		5							4
4														
								3			4			
4													2	
								5						
			5											
5														
	2										2			
										5	3		4	
					4								3	1
2								3		5		4		