Mini-math AP Calculus BC: Friday, March 11, 2022 (8 minutes)

SOLUTIONS

1. (2 points) Write down (but do not evaluate) an integral which represents the length of the curve $y = \sin x^2$ from $x = 0$ to $x = \pi$.

Solution: First, note that $f'(x) = 2x \cos x^2$

$$L = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx = \int_0^\pi \sqrt{1 + (2x \cos x^2)^2}\, dx \quad (\approx 7.562)$$

2. (2 points) Suppose $g(x) = \int_x^{x^2} \sqrt{t^3 + 1}\, dt$. Write down (but do not evaluate) an integral which represents the length of the curve $y = g(x)$ from $x = 0$ to $x = 1$.

Solution: First, note that $g'(x) = 2x\sqrt{x^6 + 1} - \sqrt{x^3 + 1}$

$$L = \int_a^b \sqrt{1 + [g'(x)]^2}\, dx = \int_0^1 \sqrt{1 + (2x\sqrt{x^6 + 1} - \sqrt{x^3 + 1})^2}\, dx \quad (\approx 1.164)$$

3. (2 points) Write down (but do not evaluate) an integral which represents the length of the curve described by the parametric equations $x = \cos t$ and $y = \sin 2t$ from $t = 0$ to $t = \pi$.

Solution: First, note that $x'(t) = -\sin t$ and $y'(t) = 2\cos 2t$.s

$$L = \int_\alpha^\beta \sqrt{[x'(t)]^2 + [y'(t)]^2}\, dt = \int_0^\pi \sqrt{\sin^2 t + 4\cos^2 2t}\, dt \quad (\approx 4.715)$$

4. (2 points) Write down (but do not evaluate) an integral which represents the length of the curve described by the parametric equations $x = t^3/3$ and $y = t^2/2$ from $t = 0$ to $t = 1$. (Extra challenge: find the exact value.)

Solution: First, note that $x'(t) = t^2$ and $y'(t) = t$

$$L = \int_0^1 \sqrt{t^4 + t^2}\, dt \quad \left(= \frac{(t^2 + 1)^{3/2}}{3} \Bigg|_0^1 = \frac{2\sqrt{2} - 1}{3} \right)$$