

Mini-math Div 3/4: Friday, February 11, 2022 (18 minutes)

SOLUTIONS

1. (2 points) $\int \frac{2}{x^2 + 2x + 2} dx$

Solution: By completing the square,

$$\begin{aligned}\int \frac{2}{x^2 + 2x + 2} dx &= \int \frac{2}{(x+1)^2 + 1} dx = \int \frac{1}{\left(\frac{x+1}{2}\right)^2 + 1} dx \\ &= \sqrt{2} \arctan\left(\frac{x+1}{\sqrt{2}}\right) + C\end{aligned}$$

2. (2 points) $\int \frac{\cos^3 \theta - 1}{\cos^2 \theta} d\theta$

Solution:

$$\begin{aligned}\int \frac{\cos^3 \theta - 1}{\cos^2 \theta} d\theta &= \int (\cos \theta - \sec^2 \theta) d\theta \\ &= \sin \theta - \tan \theta + C\end{aligned}$$

3. (2 points) $\int_{-2}^0 x \sqrt{\frac{x}{2} + 1} dx$

Solution: Let $u = x/2 + 1$, so that $du = (1/2) dx$, $-2 \mapsto 0$, and $0 \mapsto 1$. Notice that $x = 2(u - 1)$. Then

$$\begin{aligned}\int_{-2}^0 x \sqrt{\frac{x}{2} + 1} dx &= \int_0^1 2(u-1)\sqrt{u} \cdot 2 du \\ &= 4 \int_0^1 (u^{3/2} - u^{1/2}) du \\ &= 4 \left(\frac{2}{5} u^{5/2} - \frac{2}{3} u^{3/2} \right) \Big|_0^1 \\ &= 4 \left(\frac{2}{5} - \frac{2}{3} \right) = -\frac{16}{15}\end{aligned}$$

4. (2 points) $\int \frac{1}{(1+y^2) \arctan y} dy$

Solution: Let $u = \arctan y$, so that $du = \frac{1}{1+y^2} dy$. Then

$$\int \frac{1}{(1+y^2) \arctan y} dy = \int \frac{1}{u} du = \ln |u| + C = \ln |\arctan y| + C$$

5. (2 points) $\int \frac{3x^2 + 5x}{x+2} dx$

Solution: By polynomial division,

$$\int \frac{3x^2 + 5x}{x+2} dx = \int \left(3x - 1 + \frac{2}{x+2} \right) dx = \frac{3}{2}x^2 - x + 2 \ln |x+2| + C$$

6. (2 points) $\int_1^2 (u-1)^3 du$

Solution:

$$\int_1^2 (u-1)^3 du = \frac{1}{4}(u-1)^4 \Big|_1^2 = \frac{1}{4}(1^4 - 0) = \frac{1}{4}$$