SASMS (Short Attention Span Math Seminars) 2021

Your final project is to prepare a 10-15 minute presentation on any mathematical topic you wish, as long as it is related to the material we learned in calculus in some way. Your presentation should come with a short, written synopsis (1-3 pages) to act as notes for the audience.

Rubric (all items out of 5 points):

- Synopsis
  - Mathematical relevancy
  - Ability to explain topic
  - Ability to explain mathematics
  - Format, neatness, mechanics

- Presentation
  - Creativity
  - Engagement
  - Audio/visual aid
  - Comprehensibility

You may choose any topic as long as you get approval from me in advance. Here are some ideas you may wish to explore, or which may help lead you to your own desired topic:

- Math-centred topics:
  - Normal distribution
  - Irrationality of π
  - Weierstrass function
  - Integrals which are "impossible" (ie nonelementary)
  - Wobbly table theorem
  - TI-83/TI-84 programs for solving calculus problems
  - Gabriel's horn
  - Ito calculus (very challenging, important in mathematical finance)
  - Discrete calculus
  - Hyperbolic trig

- Other science-centred topics:
  - Toughness (in material science)
  - Finite Element Analysis (in engineering)
  - Focusing problems for rocket launches
  - Optimizing mileage
  - How rainbows are formed
  - Calculus of voting
  - Applications to economics (e.g. marginal profit)
  - Pharmacokinetics (medicine)
  - Rocket science

- Topics which appear in AP Calculus BC:

  - Integration by parts (integration technique)
  - Partial fraction decomposition (integration technique)
  - Improper integrals
  - Euler's Method (differential equations)
  - Logistic models (differential equations)
  - Arc length (application of integration)
  - Integral test (series)
  - p-series (series)
  - Taylor polynomials (generalization of linear approximation, series)

- Specific problems (roughly in order of difficulty):

  - Mixing tank problems (differential equations)
  - Find the volume of a doughnut
  - Find the volume of intersection of 3 cylinders which are perpendicular to one another
  - Find $\displaystyle\lim_{x \to 0} \frac{\sin(\tan x) - \tan(\sin x)}{x^7}$
  - Find the smallest positive integer j such that for every polynomial $p(x)$ with integer coefficients and for every integer k, the integer

$$p^{(j)}(k) = \frac{d^j}{dx^j} p(x) \Big|_{x=k}$$

  (the jth derivative of $p(x)$ at k) is divisible by 2021.
  - Given a positive integer n, let $M(n)$ be the largest integer m such that

$$\binom{m}{n-1} > \binom{m-1}{n}.$$

  Evaluate $\displaystyle\lim_{n \to \infty} \frac{M(n)}{n}$
  - Find the minimum value of

$$|\sin x + \cos x + \tan x + \cot x + \sec x + \csc x|$$

  for x a real number
  - Evaluate $\displaystyle\int_0^1 \frac{\ln(x+1)}{x^2+1} \, dx$
  - Evaluate $\displaystyle\int_0^{\pi/4} \frac{x}{(\sin x + \cos x)\cos x} \, dx$
  - Does there exist an infinitely differentiable function which is either always non-increasing or always non-decreasing (that is, monotonic), with $\displaystyle\lim_{x \to \infty} f(x) = 0$ and $\displaystyle\lim_{x \to \infty} f'(x) \neq 0$?