

Mini-math Div 3/4: Friday, March 25, 2022 (12 minutes)

SOLUTIONS

1. Write a differential equation that describes the following relationships. If necessary, use k as the constant of proportionality.

- (a) (2 points) The rate of change of population, P , with respect to time, t , is inversely proportional to the square root of time and directly proportional to the area, A , that the population covers.

Solution:

$$\frac{dP}{dt} = \frac{kA}{\sqrt{t}}$$

- (b) (2 points) The position of a particle is given by $s(t)$, where t is measured in seconds. Its acceleration is directly proportional to its position. When the particle is at position 4 units, its acceleration is 2 units/ s^2 .

Solution:

$$\frac{d^2s}{dt^2} = \frac{1}{2}s$$

2. (4 points) Determine the value of k , if any, for which $y = \sin(2x) - k \sin(4x)$ would be a solution to the differential equation $y'' + 4y = 3 \sin(4x)$.

Solution:

$$\begin{aligned}y' &= 2 \cos(2x) - 4k \cos(4x) \\y'' &= -4 \sin(2x) + 16k \sin(4x)\end{aligned}$$

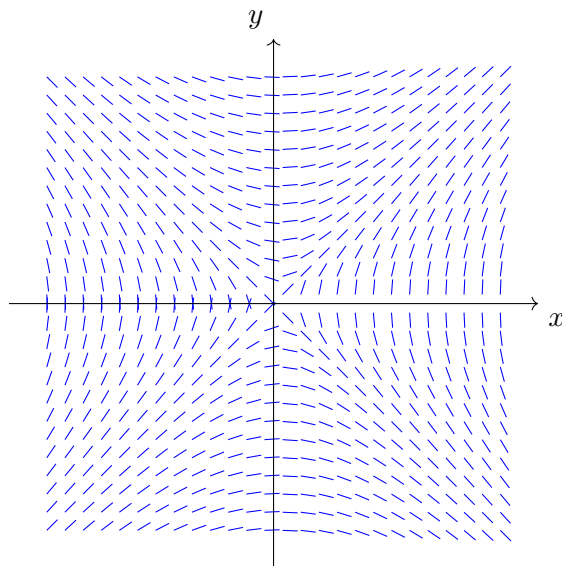
Then

$$[-4 \sin(2x) + 16k \sin(4x)] + 4 [\sin(2x) - k \sin(4x)] = 3 \sin(4x)$$

so $16k - 4k = 3$, giving $k = 1/4$.

3. (2 points) (AP) What differential equation can the slope field to the right represent?

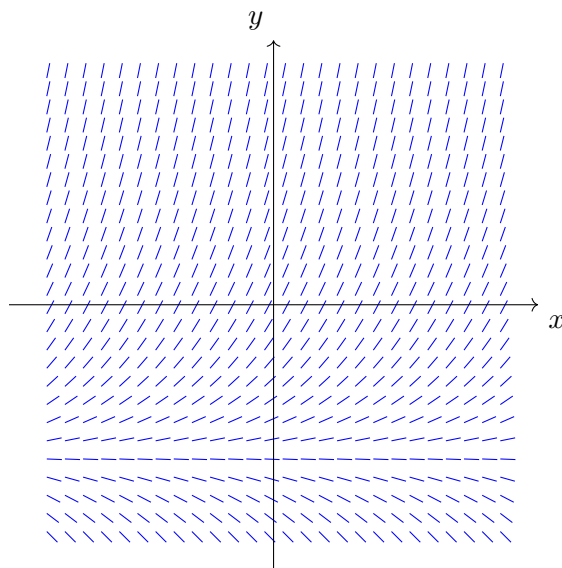
- A. $\frac{dy}{dx} = 1/x$
- B. $\frac{dy}{dx} = 1/y$
- C. $\frac{dy}{dx} = x/y$
- D. $\frac{dy}{dx} = y/x$



Solution: (D)

4. (2 points) (AP) The slope field for a certain differential equation is shown to the right. Which of the following could be a particular solution to the differential equation?

- A. $y = x^3$
- B. $y = \frac{1}{x+2}$
- C. $y = -2^x - 2$
- D. $y = e^{-x} - 2$



Solution: (C)