Mini-math Div 3/4: Friday, April 1, 2022 (12 minutes)

SOLUTIONS

1. (2 points) Solve the following differential equation:

$$\frac{dy}{dx} = xy\sin(x^2)\cdot \ln y$$

Solution:

$$\begin{aligned}
\int \frac{1}{y\ln y}\,dy &= \int x\sin x^2\,dx \\
\ln|\ln y| &= -\frac{1}{2}\cos x^2 + C_1 \\
|\ln y| &= e^{-\frac{1}{2}\cos x^2 + C_1} \\
\ln y &= \pm e^{-\frac{1}{2}\cos x^2 + C_1} \quad \text{or} \quad C_2 e^{-\frac{1}{2}\cos x^2} \\
y &= e^{\pm e^{-\frac{1}{2}\cos x^2 + C_1}} \quad \text{or} \quad e^{C_2 e^{-\frac{1}{2}\cos x^2}}
\end{aligned}$$

2. (2 points) During a chemical reaction, the rate of change of the amount of the chemical remaining is proportional to the amount remaining. At time $t = 0$, the amount of the chemical is 60 g. At time $t = 8$, the amount of the chemical is 12 g. At what time t is the amount of the chemical 4 g?

A. $\frac{4\sqrt{42}}{3}$ B. $\frac{28}{3}$ **C.** $\frac{8\ln 15}{\ln 5}$ D. $\frac{8\ln 4}{\ln 12}$

Solution:

$$P(t) = P_0 e^{kt} = P_0 (e^k)^t$$

$$P_1 = P_0 (e^k)^{t_1} \quad \Rightarrow \quad e^k = \left(\frac{P_1}{P_0}\right)^{1/t_1}$$

$$P(t) = P_0 \left(\frac{P_1}{P_0}\right)^{t/t_1}$$

$$P_2 = P_0 \left(\frac{P_1}{P_0}\right)^{t/t_1} \quad \Rightarrow \quad t = \frac{t_1 \ln(P_2/P_0)}{\ln(P_1/P_0)} = \frac{8\ln(1/15)}{\ln(1/5)} = \frac{8\ln 15}{\ln 5}$$

(C)

3. (2 points) (AP) Solve the following initial value problem:

$$\frac{dy}{dx} = y^2, \quad y(3) = -2$$

A. $y = \dfrac{1}{5/2 - x}$ for $x \neq 5/2$

B. $y = \dfrac{2}{5 - 2x}$ for $x > 5/2$

C. $y = -\dfrac{1}{x} - \dfrac{5}{3}$ for $x \neq 0$

D. $y = -\dfrac{5x + 3}{3x}$ for $x > 0$

Solution:

$$\int \frac{1}{y^2}\, dy = \int 1\, dx \quad \Rightarrow \quad -\frac{1}{y} = x + C,$$
$$\frac{1}{2} = 3 + C \quad \Rightarrow C = -\frac{5}{2},$$
$$y = \frac{1}{5/2 - x} = \frac{2}{5 - 2x}$$

Since the domain is the largest open interval which contains the initial condition, the domain is $x > 5/2$.

(B)