Mini-math Div 3/4: Monday, October 26, 2020 (14 minutes)

(1) The motion of a particle is described by the position function

$$s = f(t) = 2t^3 - 13t^2 + 24t, \quad t \geq 0$$

where t is time measured in seconds and s is measured in metres.

(a) When is the particle at rest?
Solution: Differentiating,

$$v(t) = f'(t) = 6t^2 - 26t + 24$$

Setting $v = 0$,

$$0 = 2(3t^2 - 13t + 12) = 2(3t - 4)(t - 3)$$
$$t = \frac{4}{3}\,\text{s}, 3\,\text{s}$$

(b) When is the particle moving in the positive direction?
Solution: The particle is moving in the positive direction when $v > 0$, so $0 < t < 4/3$ or $t > 3$.

(c) Find the total distance travelled in the first 2 s to 2 decimal places.
Solution: We calculate

$$|f(4/3) - f(0)| = \frac{368}{27} \approx 13.6296\,\text{m}$$
$$|f(2) - f(4/3)| = \frac{44}{27} \approx 1.6296\,\text{m}$$

so the total distance travelled is 15.26 m.

(d) Find the acceleration of the particle as a function of time.
Solution: Differentiating,

$$a(t) = f''(t) = 12t - 26\,\text{m/s}^2$$

(e) When does the particle have 0 acceleration?

Solution: Solving,

$$0 = 12t - 26$$
$$t = \frac{26}{12} = \frac{13}{6}\,\mathrm{s}$$

(f) What is the particle's acceleration when it is at rest?

Solution: The particle is at rest for $t = \frac{4}{3}, 3$ by part (a), so by part (d), the acceleration at these times is

$$a(4/3) = -10\,\mathrm{m/s^2}$$
$$a(3) = 10\,\mathrm{m/s^2}$$

(2) The population of a bacteria colony after t hours is given by

$$n = 2t^3 + 6t^2 + 15t + 2000.$$

Find the rate of change of the population at time t.

Solution: The growth rate is given by

$$\frac{dn}{dt} = 6t^2 + 12t + 15$$

(3) Boyle's Law states that $PV = k$ where P is the pressure of a gas, V is the volume of the gas, and k is a constant. Find the rate of change of the pressure with respect to the volume.

Solution: Method 1: Solve $P = k/V$, so

$$\frac{dP}{dV} = -\frac{k}{V^2}$$

Method 2: Implicitly differentiate

$$\frac{dP}{dV} \cdot V + P = 0$$
$$\frac{dP}{dV} = -\frac{P}{V} = -\frac{k}{V^2}$$

where we used $PV = k$ in the last equality.