

Four Dice complexity - maximum score

In this puzzle, your goal is to make a target number using each of four starting digits exactly once. A solution gets 1 point. You may use the following operations (in addition to parentheses):

- Standard operations: $+$, $-$, \times , \div
- Exponentiation of two numbers: \square^\square
- Square root of a number: $\sqrt{\square}$
- Factorial: $\square!$

Complexity bonus: for each of the operations \div , \square^\square , $\sqrt{\square}$, $\square!$,

- For the first usage of the operation, you earn an additional point.
- For each subsequent use of the operation, you earn half as many points as the previous time.
- No bonus is earned for $\sqrt{0}$, $\sqrt{1}$, $1!$, $2!$, or using square roots/factorials on the exponent of 0 or 1, or using square roots/factorials on two equal numbers to make 0 via subtraction or 1 via division (to avoid adding points “for free”). More generally, if a solution still works by removing one or more square roots and/or factorials without changing anything else, the extra operators do not gain bonus points.

Example 1: Making 4 with 3, 4, 4, 9 as follows

$$4 = 4! \div \sqrt{\sqrt{3! \times 4! \times 9}}$$

earns 1 point for a valid solution, 1 bonus point for using division once, $1 + \frac{1}{2} = 1.5$ bonus points for using square root twice, and $1 + \frac{1}{2} + \frac{1}{4} = 1.75$ bonus points for using factorial three times, for a total of 5.25 points.

Example 2: Making 6 with 1, 3, 3, 4 as follows

$$6 = 3 + 3 \div \sqrt{1^{4!!}}$$

earns 1 point for a valid solution, 1 bonus point for using division once, and 1 bonus point for using exponentiation once, for a total of 3 points. Notice no bonus points are awarded for using the factorials, since they are used on the exponent of 1, nor are any bonus points awarded for using the square root, since it is the square root of 1.

Example 3: Making 5 with 1, 3, 3, 6 as follows

$$5 = 1 + 4 + 3!!! - 6!!$$

earns 1 point for a valid solution, and 1 bonus point for using factorial once (to make $3! = 6$). Notice no bonus points are awarded for using the additional factorials, since they are used on two equal numbers (both 6) to make 0 via subtraction.

Challenge: Find four starting digits, a target number between 0 and 20 inclusive, and a solution which maximizes your score.