

A function continuous only on the rationals

(EXTREMELY HARD) Does there exist a function $f : \mathbb{R} \rightarrow \mathbb{R}$ (i.e., the domain is the real numbers and every value is a real number) such that $f(x)$ is only continuous at rational numbers?