Mini-math Div 3/4: Monday, October 19, 2020 (10 minutes)

(1) The motion of a particle is described by the position function

$$s = f(t) = 2t^3 - 13t^2 + 24t, \quad t \geq 0$$

where t is time measured in seconds and s is measured in metres.

(a) When is the particle at rest?
Solution: Differentiating,

$$v(t) = f'(t) = 6t^2 - 26t + 24$$

Setting $v = 0$,

$$\begin{aligned} 0 &= 2(3t^2 - 13t + 12) = 2(3t-4)(t-3) \\ t &= \frac{4}{3}\,\mathrm{s}, 3\,\mathrm{s} \end{aligned}$$

(b) When is the particle moving in the positive direction?
Solution: The particle is moving in the positive direction when $v > 0$, so $0 < t < 4/3$ or $t > 3$.

(c) Find the total distance travelled in the first 2 s to 2 decimal places.
Solution: We calculate

$$\begin{aligned} |f(4/3) - f(0)| &= \frac{368}{27} \approx 13.6296\,\mathrm{m} \\ |f(2) - f(4/3)| &= \frac{44}{27} \approx 1.6296\,\mathrm{m} \end{aligned}$$

so the total distance travelled is 15.26 m.

(d) Find the acceleration of the particle as a function of time.
Solution: Differentiating,

$$a(t) = f''(t) = 12t - 26\,\mathrm{m/s^2}$$

(e) When does the particle have 0 acceleration?
Solution: Solving,

$$\begin{aligned} 0 &= 12t - 26 \\ t &= \frac{26}{12} = \frac{13}{6}\,\text{s} \end{aligned}$$

(f) What is the particle's acceleration when it is at rest?
Solution: The particle is at rest for $t = \frac{4}{3}, 3$ by part (a), so by part (d), the acceleration at these times is

$$\begin{aligned} a(4/3) &= -10\,\text{m/s}^2 \\ a(3) &= 10\,\text{m/s}^2 \end{aligned}$$

(2) The population of a bacteria colony after t hours is given by

$$n = 2t^3 + 6t^2 + 15t + 2000.$$

Find the rate of change of the population at time t.

Solution: The growth rate is given by

$$\frac{dn}{dt} = 6t^2 + 12t + 15$$

(3) Boyle's Law states that $PV = k$ where P is the pressure of a gas, V is the volume of the gas, and k is a constant. Find the rate of change of the pressure with respect to the volume.

Solution: Method 1: Solve $P = k/V$, so

$$\frac{dP}{dV} = -\frac{k}{V^2}$$

Method 2: Implicitly differentiate

$$\begin{aligned} \frac{dP}{dV} \cdot V + P &= 0 \\ \frac{dP}{dV} &= -\frac{P}{V} = -\frac{k}{V^2} \end{aligned}$$

where we used $PV = k$ in the last equality.