

7567 digit puzzle

Make the number 7567 in 9 different ways, each time by using copies of the same digit and the following operations (in addition to parentheses):

- Standard operations: $+$, $-$, \times , \div
- Negation: $-\square$
- Exponentiation of two numbers: \square^\square
- Square root of a number: $\sqrt{\square}$
- Factorial: $\square!$ (Note: you may use iterated factorial but not multi-factorial, so that $3!! = (3!)! = 6! = 720$, and **not** $3!! = 3 \times 1 = 3$.)
- Concatenation (i.e. “glueing”) of digits (only of the original digit used): dd

Your score for a particular digit is the number of copies you use, and your goal is to have the lowest score possible.

For example, you can make 7567 by using copies of the digit “9” as follows:

$$7567 = \underbrace{\frac{9}{9} + \frac{9}{9} + \cdots + \frac{9}{9}}_{7567 \text{ times}}.$$

If you do it like this, you are using 15134 copies of 9, which is not good for you. A far more efficient way to do it is

$$7567 = 9 \times 9 \times \left(99 - \left(\sqrt{9} + \frac{9}{9} \right)! \right) + (9 + 9) \times \frac{(\sqrt{9}!)!}{9} + 9 \times \sqrt{9}! - \frac{9}{9} - \frac{9}{9}$$

which gets you there with only 17 copies (this is, of course, not optimal).

- (1) Using the digit 1
- (2) Using the digit 2
- (3) Using the digit 3
- (4) Using the digit 4
- (5) Using the digit 5
- (6) Using the digit 6
- (7) Using the digit 7
- (8) Using the digit 8
- (9) Using the digit 9