8.1 Average Value of a Function

Calculus Name: ______________________________

CA #1

Find the average value of each function on the given interval.		
1. $f(x) = \sqrt{x}$ on $[1, 9]$	2. $f(x) = \frac{1}{x^2}$ on $[1, 5]$	3. $f(x) = \cos(2x)$ on $\left[\frac{\pi}{3}, \pi\right]$

On the given interval, find the x-value where the function is equivalent to the average value on that interval.	
4. $f(x) = -2x + 1$ on $[0, 4]$	5. $f(x) = 2\sqrt{x}$ on $[0, 1]$

Find the average rate of change on the given interval.

6. $f(x) = \frac{1}{x-2}$ on $[-4, -1]$

7. $y = -x^2 + x + 2$ on $[-1, 2]$

Find where the instantaneous rate of change is equivalent to the average rate of change.

8. $y = -\frac{1}{2}x^2 + 2x - 1$ on $[1, 4]$

9. $y = -\sqrt{5x + 15}$ on $[-3, -1]$

Answers to 8.1 CA #1

1. $\frac{13}{6}$	2. $\frac{1}{5}$	3. $-\frac{3\sqrt{3}}{8\pi}$	4. $x = 2$	5. $x = \frac{4}{9}$	6. $-\frac{1}{18}$	7. 0	8. $\frac{5}{2}$	9. $-\frac{5}{2}$

8.2 Connecting Pos, Vel, Acc with Integrals

Calculus Name: ______________________________ **CA #1**

1. A particle moves along the x-axis for $t \geq 0$ with an acceleration of $a(t) = 12t + 6$ where t is time in seconds. The particle's velocity at $t = 3$ is 36 cm/sec. The initial position of the particle is 4 cm. What is the position of the particle when the velocity is zero?

2. A particle moves along the y-axis for $t \geq 0$ with a velocity of $v(t) = 12t^2 - 24t$. The particle's initial position is 10 cm. Find the position of the function at the particle's minimum velocity.

3. Mr. Brust leaves for a bike ride at 10:00 a.m. (time $t = 0$) and rides with velocity $v(t) = 20 - \frac{t}{5}$ miles per hour, where t is the number of hours since he started riding.
   a. Find $\int_1^2 v(t)\, dt$

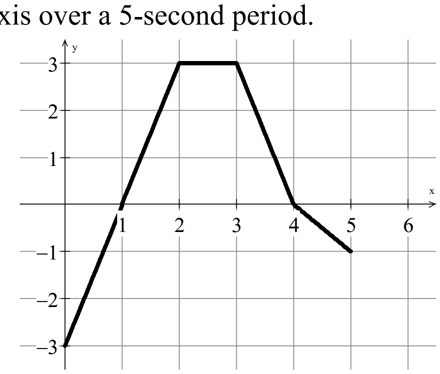
   b. Explain the meaning of your answer to part *a* in the context of this problem.

4. A particle's velocity along the x-axis is given by $v(t) = 5\cos t$.
   a. Find the particle's displacement on the interval $0 \leq t \leq \frac{3\pi}{2}$.

   b. If $s(0) = 3$, what is the particles position at $t = \frac{3\pi}{2}$?

5. The graph to the right shows the velocity of an object moving along the x-axis over a 5-second period.
   a) If the object started 10 meters to the right, where is the object after 3 seconds?

   b) Find the total distance traveled by the object over the 5-second period



6. A particle's velocity is given by $v(t) = 20 - 8t$, where t is measured in weeks, v is measured in inches per week, and $s(t)$ represents the particle's position.
   a. If $s(0) = 3$, what is the value of $s(3)$?

   b. What is the net change in distance over the first 10 weeks?

   c. What is the total distance traveled by the particle during the first 10 weeks? Show the set up AND your work.

7. **Calculator active.** A particle's velocity is given by $v(t) = e^{\sin t} \cos t$, where t is measured in months, v is measured in yards per month, and $s(t)$ represents the particle's position.
   a. If $s(0) = 5$, what is the value of $s(2\pi)$?

   b. What is the net change in distance over the first 8 months?

   c. What is the total distance traveled by the particle during the first 8 months? Show the set up.

Answers to 8.2 CA #1

1. -40 cm	2. 2 cm	3a. 19.7 3b. During the 2^{nd} hour, Brust rode 19.7 miles.	4a. 5 units to the left. 4b. 2 units to the left.
5a. 13 meters to the right. 5b. 8 meters	6a. 27 6b. -200 inches 6c. 250 inches	7a. 5 7b. 1.6895 yards 7c. 6.4478 yards	

8.3 Applying Accumulation and Integrals

Calculus Name: ______________________________ **CA #1**

1. Mr. Sullivan can paint his tricycles at a rate of $r(t) = 50 - \frac{t}{2}$ square inches per minute, where t is the number of minutes since he started painting.
   a. Find $\int_0^5 r(t)\,dt$

   b. Explain the meaning of your answer to part *a* in the context of this problem.

2. A storm has washed away sand from a beach, causing the edge of the water to get closer to a nearby road. The rate at which the distance between the road and the edge of the water was changing during the storm is modeled by $r(t) = e^{-\cos t}$ feet per hour, t hours after the storm began. The edge of the water was 80 feet from the road when the storm began. If the storm lasted 5 hours, how far is the water from the road after the storm?

3. A store is having a 10-hour sale. The rate at which shoppers enter the store t hours after the sale begins is modeled by the function $E(t)$, which is measured as shoppers per hour. When the sale starts, there are already 30 shoppers in the store. Write, but do not solve, an equation involving an integral to find the time x when the amount of shoppers in the store is 150.

4.

t (minutes)	0	10	20	30	40	50	60
$W(t)$ (°F)	57	63	72	85	90	98	102

The temperature of water in Mr. Brust's hot tub at time t is modeled by a strictly increasing, twice-differentiable function W, where $W(t)$ is measured in degrees Fahrenheit and t is measured in minutes. The water is heated for 60 minutes, beginning at time $t = 0$. Values of $W(t)$ at selected times t are given in the table above.

a. Use the data in the table to evaluate $\int_{10}^{30} W'(t)\,dt$.

b. Using correct units, interpret the meaning of $\int_{10}^{30} W'(t)\,dt$ in the context of this problem.

5.

t (seconds)	0	2	3	7	8	10
$W'(t)$ (ounces per second)	0	4.2	8.3	10.7	13.5	14.1

On a cold winter morning, hot water is pouring into a large container for hot chocolate. The rate $W'(t)$ at which the water is being poured in the container at time t, $0 \leq t \leq 10$, is shown at selected values in the table above. The cup already had 10 ounces of cold water before pouring the hot water in.

a. Using correct units, interpret the meaning of $\int_0^3 W'(t)\,dt$ in the context of this problem.

b. Use a left Riemann sum, with the five subintervals indicated by the data in the table, to approximate $\int_0^{10} W'(t)\,dt$. Use appropriate units.

c. Using your answer from part (b), how much water is in the container after 10 seconds?

6. The rate at which Mr. Kelly eats hot dogs is given by $h(t)$, where h is measured in hot dogs per minute and t is measured in minutes since the start of lunch time. Using correct units, explain the meaning of $\int_0^3 h(t)\,dt$.

Answers to 8.3 CA #1

1a. $\frac{975}{4} = 243.75$ 1b. 243.75 square inches are painted during the first 5 minutes.	2. $80 - \int_0^5 r(t)\,dt = 72.668$ feet	3. $30 + \int_0^x E(t)\,dt = 150$
4a. 22 °F 4b. The temperature of the water has increased by 22 °F from the 10th minute to the 30th minute since the hot tub was turned on.	5a. How much water has been poured into the container during the first three seconds. 5b. 75.1 ounces 5c. 85.1 ounces	6. The number of hot dogs Mr. Kelly eats during the first three minutes of lunch.

8.4 Area Between Curves (with respect to x)

Calculus Name: ______________________________

CA #1

Find the area of the region bounded by the following graphs. Show your work.

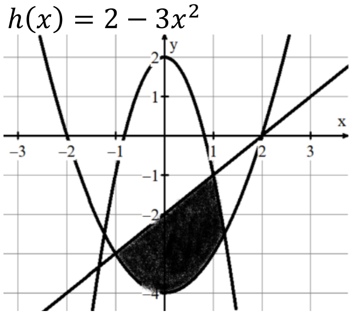
1. $y = x^2 - 4x - 5$ and $y = 2x - 5$

2. $y = 3x^2, y = 0, x = 1, x = 3$

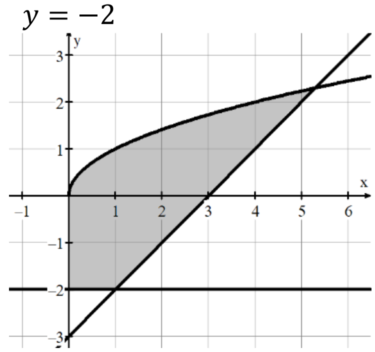
3. $y = \ln x$, $y = -\sqrt{x}$, and $x = 3$

Set up an integral(s) that represents the shaded region. Do not solve. Use a calculator if necessary to help find the lower and upper bounds.

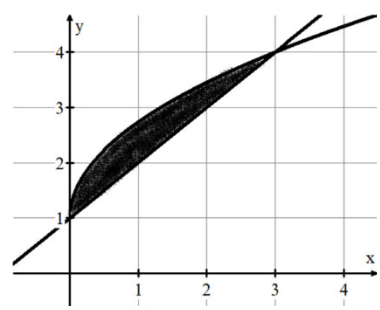
4. $f(x) = x^2 - 4,\ g(x) = x - 2,$
   $h(x) = 2 - 3x^2$



5. $y = \sqrt{x},\ y = x - 3, x = 0$ and $y = -2$

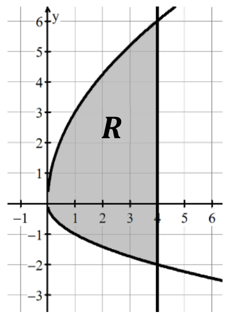


6. $f(x) = \sqrt{3x} + 1,\ g(x) = x + 1$

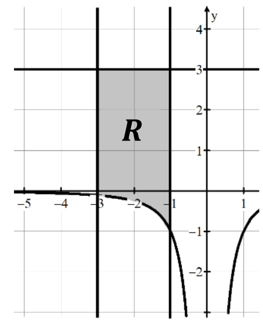


Let R be the region bounded by the given curves as shown in the figure. If the line $x = k$ divides R into two regions of equal area, find the value of k

7. $y = 3\sqrt{x}$, $y = -\sqrt{x}$ and $x = 4$



8. $y = -\frac{1}{x^2}$, $y = 3$, $x = -3$, and $x = -1$



Answers to 8.4 CA #1

1. 36	2. 26	3. $\int_{0.4948664}^{3} \ln x + \sqrt{x}\, dx = 4.3708$	4. $\int_{-1}^{1}(-x^2 + x + 2)\, dx + \int_{1}^{A}(-4x^2 + 6)\, dx$ where $A = 1.224744871$
5. $\int_0^1(\sqrt{x} + 2)\, dx + \int_1^A(\sqrt{x} - x + 3)\, dx$ where $A = 5.3027756$	6. $\int_0^3(\sqrt{3x} - x)\, dx$	7. $k \approx 2.5198$	8. $k \approx -1.9488$

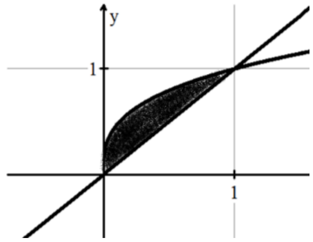
8.5 Area Between Curves (with respect to y)

CA #1

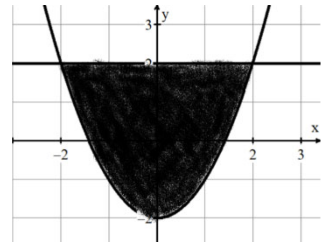
Calculus Name: ______________________________

For each region, set up an integral with respect to y that represents the area of the region. Do not solve.

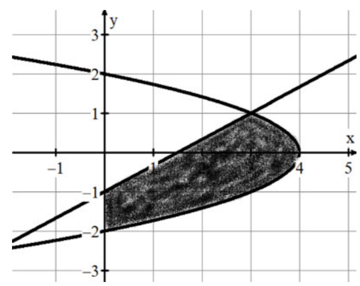
1. $y = \sqrt[3]{x},\ y = x$



2. $y = x^2 - 2$ and $y = 2$



3. $x = 4 - y^2,\ y = \frac{2}{3}x - 1, x = 0$



Set up the integral(s) that give the area of the region bounded by the given equations. Show the equivalent set up with respect to x as well as with respect to y.

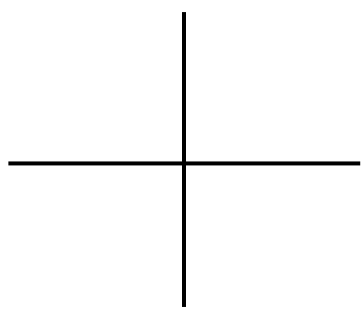
4. $y = x^3, y = x$

1st quadrant only!

with respect to x

Sketch a graph here in the middle!

with respect to y



Find the area of the region bounded by the following curves. Set up your integrals with respect to y. A calculator is allowed to evaluate the integral.

5. $y = 1 - x^2,\ y = \sqrt{x} - 1$ and $y = -\sqrt{x} - 1$.

Answers to 8.5 CA #1

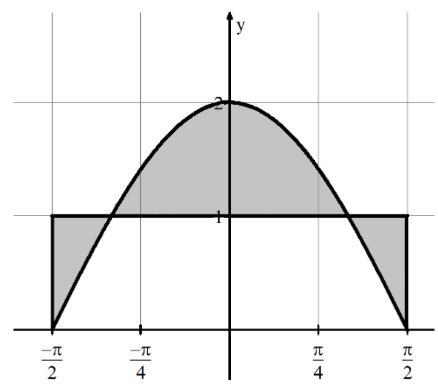
1. $\int_0^1 (y - y^3)\,dy$	2. $\int_{-2}^{2} (2\sqrt{y+2})\,dy$	3. $\int_{-2}^{-1}(4 - y^2)\,dy + \int_{-1}^{1}\left(-y^2 - \frac{3}{2}y + \frac{5}{2}\right)dy$
4a. $\int_0^1 (x - x^3)\,dx$ 4b. $\int_0^1 (\sqrt[3]{y} - y)\,dy$		5. $\int_{-2.35321}^{0} \left[\sqrt{1-y} - (y+1)^2\right] dy = 2.2675$

8.6 Area – More than Two Intersections

Calculus Name: ______________________________ **CA #1**

The given functions create boundaries for multiple regions.	
1. $y = (x-1)^3 - 1$ and $y = 2x - 3$	2. $y = -2x^3 + 3x^2 + 5x$, $y = x^2 - 1$
a. Find x-values of the points of intersection, and label them from smallest to largest as A, B, and C.	a. Find x-values of the points of intersection, and label them from smallest to largest as A, B, and C.
$A =$ $B =$ $C =$	$A =$ $B =$ $C =$
b. Set up integrals	b. Set up integrals

3. The figure shows the graph of $y = 2\cos(x)$, and the line $y = 1$, for $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$. Write a set of integrals that represents the sum of all the areas of the shaded regions. Use exact values for your boundaries, not rounded decimals.



Answers to 8.6 CA #1

1a. $A = -0.414$ $B = 1$ $C = 2.414$	1b. $\int_A^B ((x-1)^3 - 2x + 2)\,dx + \int_B^C (2x - 2 - (x-1)^3)\,dx$	2a. $A = -1$ $B = -0.2247$ $C = 2.2247$
2b. $\int_A^B (2x^3 - 2x^2 - 5x - 1)\,dx +$ $\int_B^C (-2x^3 + 2x^2 + 5x + 1)\,dx$	3. $\int_{-\frac{\pi}{2}}^{-\frac{\pi}{3}} (1 - 2\cos x)\,dx + \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} (2\cos x - 1)\,dx +$ $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} (1 - 2\cos x)\,dx$	

8.7 Volumes with Cross Sections: Squares and Rectangles

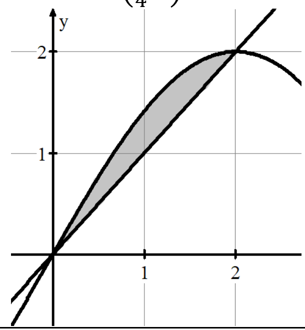
CA #1

Calculus Name: ______________________________

The bounded region shown for each problem represents the base of a solid. Find the volume of each solid based on the given cross sections. Set up the integral(s) first, then use a calculator to evaluate.

1. Square cross sections perpendicular to the x-axis.

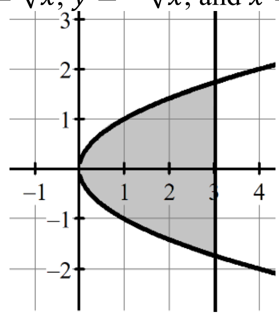
$y = 2\sin\left(\frac{\pi}{4}x\right)$ and $y = x$



2. Square cross sections perpendicular to the y-axis.

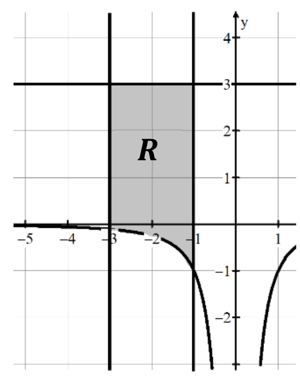
3. Square cross sections perpendicular to the x-axis.

$y = \sqrt{x}$, $y = -\sqrt{x}$, and $x = 3$

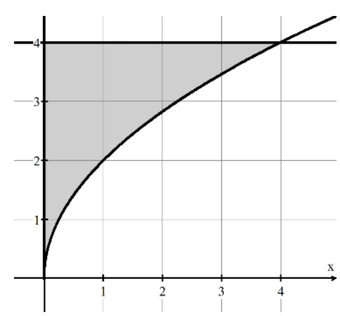


4. Square cross sections perpendicular to the y-axis.

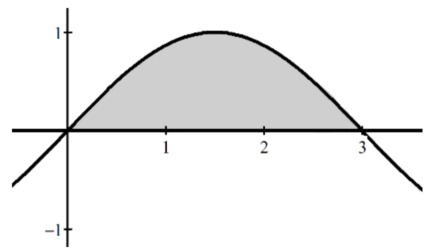
5. Let R be the region bounded by the graphs $y = -\frac{1}{x^2}$, $y = 3$, $x = -3$, and $x = -1$ as shown in the figure. The cross sections perpendicular to the x-axis are rectangles whose height is twice the width.



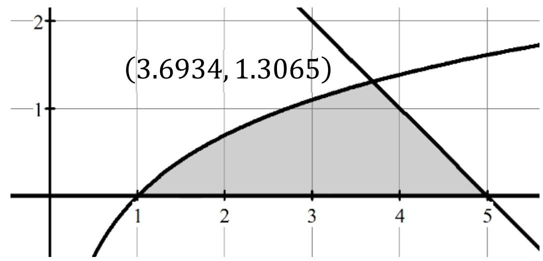
6. The base of a solid is the region bounded by the y-axis, the graph of $y = 2\sqrt{x}$ and the horizontal line $y = 4$. For the solid, each cross section perpendicular to the y-axis is a rectangle whose height is 3.



7. $y = \sin\left(\frac{\pi}{3}x\right)$ and the x-axis as shown in the figure. Each cross section perpendicular to the x-axis is a rectangle whose height is 4 times its width. What is the volume?



8. The x-axis $y = \ln x$, $y = 0$, and $y = 5 - x$. Each cross section perpendicular to the y-axis is a rectangle whose height is 6 times its width. What is the volume?



9. The graphs of $y = x^2 - 4$ and $y = 4 - 2x$ create a bounded region that represents the base of a solid. The cross sections of this solid are perpendicular to the x-axis and form squares. Find the volume of the solid.

Answers to 8.7 CA #1

1. $\int_0^2 \left(2\sin\left(\frac{\pi}{4}x\right) - x\right)^2 dx \approx 0.182$	2. $\int_0^2 \left(y - \frac{4}{\pi}\sin^{-1}\left(\frac{y}{2}\right)\right)^2 dy \approx 0.182$	3. $\int_0^3 \left(2\sqrt{x}\right)^2 dx = 18$
4. $\int_{-\sqrt{3}}^{\sqrt{3}} (3 - y^2)^2\, dy = 16.6276$	5. $\int_{-3}^{-1} 2\left(3 + \frac{1}{x^2}\right)^2 dx \approx 44.6419$	6. $\int_0^4 3\left(\frac{y^2}{4}\right) dx = 16$
7. $\int_0^3 4\left(\sin\left(\frac{\pi}{3}x\right)\right)^2 dx = 6$	8. $\int_0^{1.3065} 6(5 - y - e^y)^2\, dy \approx 51.1368$	9. $\int_{-4}^{2} (-x^2 - 2x + 8)^2\, dx = 259.2$

8.8 Volumes with Cross Sections: Triangles and Semicircles

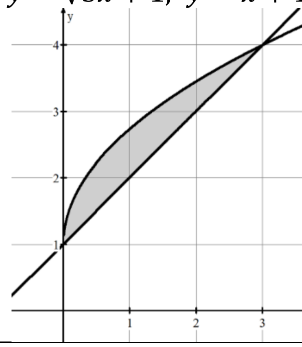
CA #1

Calculus Name: ______________________________

The bounded region shown for each problem represents the base of a solid. Find the volume of each solid based on the given cross sections. Set up the integral(s) first, then use a calculator to evaluate.

1. Semicircle cross sections perpendicular to the x-axis.

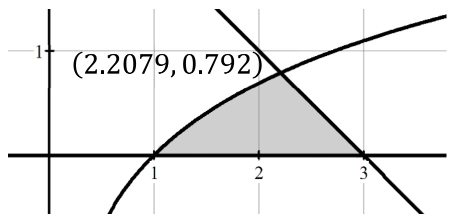
$y = \sqrt{3x} + 1,\ y = x + 1$



2. Equilateral triangle cross sections perpendicular to the y-axis.

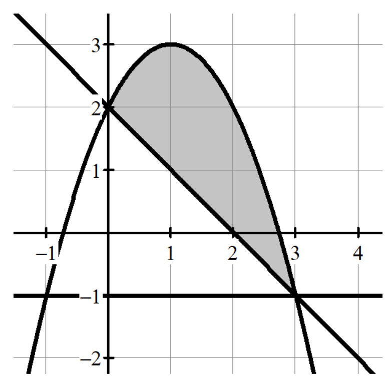
3. Isosceles right triangle cross sections perpendicular to the x-axis.

$y = \ln x\,, y = 3 - x$ and the x-axis

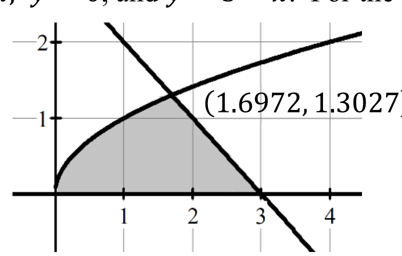


4. Semicircle cross sections perpendicular to the y-axis.

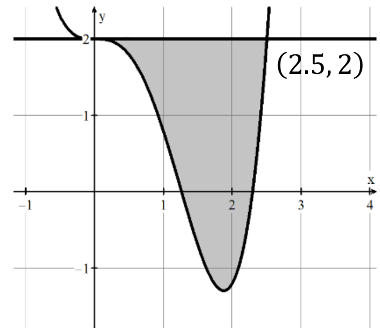
5. A region is bounded by $y = -x^2 + 2x + 3$ and $y = 2 - x$ as shown in the figure. The cross sections perpendicular to the x-axis are isosceles right triangles. Set up the integral, but do not evaluate.



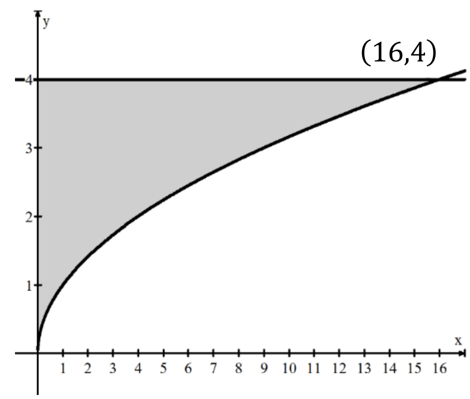
6. The base of a solid is the region bounded by the y-axis, the graphs of $y = \sqrt{x},\ y = 0$, and $y = 3 - x$. For the solid, each cross section perpendicular to the y-axis is a semicircle. Set up the integral, but do not evaluate.



7. A region is bounded by $y = 0.8x^4 - 2x^3 + 2$ and $y = 2$ as shown in the figure. Each cross section perpendicular to the x-axis is an equilateral triangle. Set up the integral, but do not evaluate.



8. The region bounded by the y-axis, the graph of $y = \sqrt{x}$ and the line $y = 4$ is shown. For the solid, each cross section perpendicular to the y-axis is a semicircle. Set up the integral, but do not evaluate.



9. The graphs of $y = x^2 - x - 3$ and $y = x$ create a bounded region that represents the base of a solid. The cross sections of this solid are perpendicular to the x-axis and form semicircles. Find the volume of the solid. Set up the integral, but do not evaluate.

Answers to 8.8 CA #1

1. $\frac{\pi}{8}\int_0^3 \left(\sqrt{3x} - x\right)^2 dx \approx 0.353$	2. $\int_1^4 \frac{\sqrt{3}}{4}\left(y - 1 - \frac{(y-1)^2}{3}\right)^2 dy \approx$ 0.3897	3. $\frac{1}{2}\int_1^{2.2079} (\ln x)^2\, dx +$ $\frac{1}{2}\int_{2.2079}^{3} (3 - x)^2\, dx \approx 0.2345$
4. $\frac{\pi}{8}\int_0^{0.792} (3 - y - e^y)^2\, dy \approx 0.4648$	5. $\frac{1}{2}\int_0^3 (-x^2 + 3x + 1)^2\, dx$	6. $\frac{\pi}{8}\int_0^{1.3027} (3 - y - y^2)^2\, dy$
7. $\frac{\sqrt{3}}{4}\int_0^{2.5} (-0.8x^4 + 2x^3)^2\, dx$	8. $\frac{\pi}{8}\int_0^4 (y^4)\, dy$	9. $\frac{\pi}{8}\int_{-1}^{3} (-x^2 + 2x + 3)^2\, dx$

8.9 Disc Method: Revolve Around x or y Axis

CA #1

Calculus Name: ______________________________

For each problem, sketch the area bounded by the equations and revolve it around the x-axis. Find the volume of the solid formed by this revolution. Leave your answers in terms of π.

1. $y = -x + 4,\ x = 1,\ y = 0$

2. $y = -\sqrt{x},\ x = 2,\ x = 3$

Same instructions as above but use a calculator and round to three decimals.

3. $y = 2 - x^2,\ x = 0$

4. $y = \sqrt{16 - x^2},\ x = -1,\ y = 0$

Same instructions as above but revolve around the y-axis now. Leave your answers in terms of π.

5. $y = \sqrt{16 - x^2},\ x \geq 0,\ y = 0$

6. $y = x^3,\ x = 0,\ y = 8$

Answers to 8.9 CA #1

1. $\pi \int_1^4 (-x+4)^2\,dx = 9\pi$	2. $\pi \int_2^3 x\,dx = \frac{5}{2}\pi$	3. $\pi \int_0^{\sqrt{2}} (2-x^2)^2\,dx = 9.478$
4. $\pi \int_{-1}^4 (16-x^2)\,dx = 183.2596$	5. $\pi \int_0^4 (16-y^2)\,dy = \frac{128}{3}\pi$	6. $\pi \int_0^8 \left(\sqrt[3]{y}\right)^2 dy = \frac{96}{5}\pi$

8.10 Disc Method: Revolve Around Other Axes

CA #1

Calculus Name: ______________________________

Setup the integral that gives the volume of the solid formed from revolving the bounded region about the given line. Set up the integral, but do not evaluate.

1. $y = 2x^2,\ x = 2,\ y = 0$ about the line $x = 2$.

2. $y = x^2,\ x = -2,\ y = 1$ and revolve about the line $x = -2$.

3. $y = x^2$ and $y = 4$ and revolve about the line $y = 4$.

4. $y = x,\ y = 0,\ x = 6$ and revolve about the line $x = 6$.

5. $y = x - 1$, $y = 3$ and $x = 6$ and revolve about the line $y = 3$.

6. $y = \sqrt{x},\ x = 0,\ x = 9,\ y = -2$ about the line $y = -2$.

Answers to 8.10 CA #1

1. $\int_0^8 \pi\left(2 - \sqrt{\frac{y}{2}}\right)^2 dy$	2. $\int_1^4 \pi(-2 + \sqrt{y})^2\, dy$	3. $\int_{-2}^{2} \pi(4 - x^2)^2\, dx$
4. $\int_0^6 \pi(6 - y)^2\, dy$	5. $\int_4^6 \pi(x - 4)^2\, dx$	6. $\int_0^9 \pi(\sqrt{x} + 2)^2\, dx$

8.11 Washer Method: Revolve Around the x- or y-axis

Calculus Name: ______________________________ **CA #1**

For each problem, sketch the area bounded by the equations and revolve it around the axis indicated. Find the volume of the solid formed by this revolution. A calculator is allowed, so round to three decimal places.

1. $y = x^2 + 4,\ x = -1, x = 1$, and $y = 3$. Revolve around the x-axis.

2. $y = \frac{2}{x},\ x = 4$, and $y = 3$. Revolve around the y-axis.

3. $y = x^2$ and $y = 2x$. Revolve around the x-axis.

4. Same region as #3, but revolve around the y-axis.

Answers to 8.11 CA #1

1. $V = \pi \int_{-1}^{1} [(x^2+4)^2 - 9]\, dx \approx 61.994$	2. $V = \pi \int_{\frac{1}{2}}^{3} \left(16 - \frac{4}{y^2}\right) dy \approx 104.72$
3. $V = \pi \int_0^2 (4x^2 - x^4)\, dx \approx 13.404$	4. $V = \pi \int_0^4 \left(y - \frac{1}{4}y^2\right) dy \approx 8.3775$

8.12 Washer Method: Revolve Around Other Axes

CA #1

Calculus Name: ______________________________

A region S is bounded by the graphs of $y = x - 1$, $x = 0$, and $y = 3$.

1. Sketch the graph and find the area of region S.

2. Let S be the base of a solid with cross sections perpendicular to the x-axis that form a square. Find the volume of this solid. [Use a calculator after you set up the integral.]

3. Let S be the base of a solid with cross sections perpendicular to the y-axis that form a semi-circle. Find the volume of this solid. [Use a calculator after you set up the integral.]

Write the equation for the "big radius" and the "little radius" for the solid of revolution when revolving S around the given line. Then setup the integral to find the volume of the solid formed. **DO NOT EVALUATE.**

4. The line $y = -1$.

$R =$

$r =$

$V =$

5. The line $y = 5$.

$R =$

$r =$

$V =$

6. The line $x = -1$.

$R =$

$r =$

$V =$

Answers to 8.12 CA #1

1. $A = \int_0^4 [3 - (x-1)]\,dx = 8$	2. $V = \int_0^4 [3 - (x-1)]^2\,dx = 21.333$	3. $V = \frac{\pi}{2}\int_{-1}^3 \left(\frac{y+1}{2}\right)^2 dy = 8.3775$
4. $V = \pi\int_0^4 (16 - x^2)\,dx$	5. $V = \pi\int_0^4 (x-1-5)^2 - (-2)^2\,dx$	6. $V = \pi\int_{-1}^3 (y+2)^2 - (1)^2\,dy$