Mini-math Div 3/4: Wednesday, November 24, 2021 (15 minutes)

SOLUTIONS

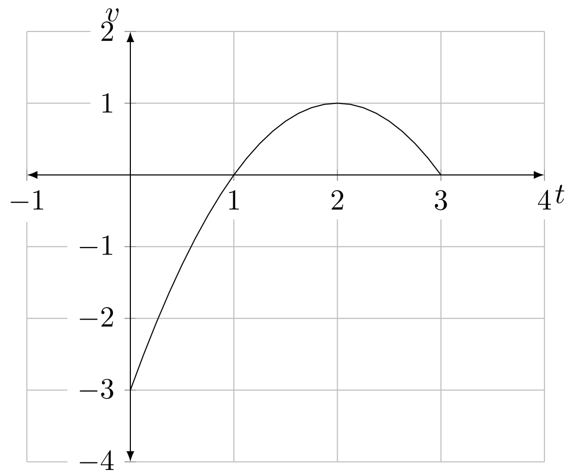
1. (1 point) If a particle is at rest, then
   A. its displacement is 0
   B. its acceleration is 0
   C. its acceleration is non-zero
   D. none of the above

> **Solution:** (D)
>
> If a particle is at rest, then the velocity is 0. But the value of a function and the value of its derivatives are not tied together, so (a), (b), and (c) are all false.

2. (1 point) (AP) Suppose $f(t), v(t)$, and $a(t)$ represents a particle's position, velocity, and acceleration, respectively. The particle is moving away from its starting position when:
   A. $f(t)$ and $v(t)$ have the same sign
   B. $f(t)$ and $a(t)$ have the same sign
   C. $v(t)$ and $a(t)$ have the same sign
   D. none of the above

> **Solution:** (A)
>
> If the particle is in the positive direction and moving in the positive direction or in the negative direction and moving in the negative direction, then it is moving away from the starting position. (C) would be the case that the particle is speeding up.

3. (2 points) The graph below shows the velocity of a particle over time.



If $f(t)$ represents the position of the particle at time t, write an expression which calculates the total distance travelled in the first 2 seconds.

Solution:

$$[f(0) - f(1)] + [f(2) - f(1)]$$

(Since the particle's velocity is negative for $0 \leq t < 1$ and positive for $2 < t \leq 2$, the particle moves to the left for $0 \leq t < 1$ and to the right for $2 < t \leq 2$.)

4. (2 points) Renal function is an indication of the state of the kidneys. Creatine clearance rate is a useful measure of this, and is given by the formula

$$C = \frac{U \cdot V}{S \cdot t}$$

where C is creatine clearance in mL/min, U is urine creatine in mg/dL, V is urine volume in mL, S is serum creatine in mg/dL, and t is collection time in min. Find the rate of change of U with respect to V if the other quantities are constant.

Solution: Method 1: Solve $U = CSt/V$, so

$$\frac{dP}{dV} = -\frac{CSt}{V^2}$$

Method 2: Implicitly differentiate

$$0 = \frac{\frac{dU}{dV}V + U}{ST}$$

$$\frac{dU}{dV} = -\frac{U}{V} = -\frac{CSt}{V^2}$$

where we used $C = \frac{UV}{St}$ in the last equality.

5. (2 points) (AP) Let f be a differentiable function such that $f(-1) = 5$, $f'(-1) = -\frac{1}{2}$, and it is concave up on the interval $(-3, 0)$. If we use the tangent line to the graph of $f(x)$ at $x = -1$ to approximate the value of $f(-1.2)$, what is the approximate value and is it an over or underestimate?

Solution: The linear approximation is

$$y = f(a) + f'(a)(x - a) = 5 - \frac{1}{2}(x + 1)$$

so

$$f(-1.2) \approx 5 - \frac{1}{2}(-1.2 + 1) = 5 + 0.1 = 5.1$$

Since f is concave up on this interval, this estimate is an underestimate.

6. (2 points) Particle A moves on the x-axis, with position $x(t)$ and velocity $\frac{dx}{dt}$. Particle B moves on the y-axis, with position $y(t)$ and velocity $\frac{dy}{dt}$. The distance z between particles A and B is given by $z^2 = x^2 + y^2$. At a certain instant, $x = 3$ cm, $y = 4$ cm, particle A is moving left at 3 cm/s, and particle B is moving up at 2 cm/s. How fast is is z changing and are the particles getting closer together or further apart?

Solution:

$$\begin{aligned} z\frac{dz}{dt} &= x\frac{dx}{dt} + y\frac{dy}{dt} \\ 5\frac{dz}{dt} &= 3(-3) + 4(2) \\ \frac{dz}{dt} &= -\frac{1}{5} \end{aligned}$$

The particles are getting closer together at a rate of 1/5 cm/s.

7. (AP) Find the following limits, if they exist. You may use l'Hôpital's Rule where applicable, but **you must indicate why it is applicable**.

(a) (2 points) $\lim_{x\to 0} \dfrac{\sin x}{\ln(2e^x - 1)}$

Solution: Since $\lim_{x\to 0} \sin x = 0 = \lim_{x\to 0} \ln(2e^x - 1)$, l'Hôpital's Rule is applicable:

$$\lim_{x\to 0} \frac{\sin x}{\ln(2e^x - 1)} = \lim_{x\to 0} \frac{\cos x}{\frac{1}{2e^x - 1} \cdot 2e^x} = \frac{1}{2}$$

(b) (2 points) Let $f(x) = 2x + 3e^{-x}$ and g be a differentiable function with $g'(x) = 3 + \frac{1}{x}$. It is known that $\lim_{x\to\infty} g(x) = \infty$. Find $\lim_{x\to\infty} \dfrac{f(x)}{g(x)}$.

Solution: Since $\lim_{x\to\infty} f(x) = \infty = \lim_{x\to\infty} g(x)$, l'Hôpital's Rule is applicable:

$$\lim_{x\to\infty} \frac{f(x)}{g(x)} = \lim_{x\to\infty} \frac{f'(x)}{g'(x)} = \lim_{x\to\infty} \frac{2 - 3e^{-x}}{3 + \frac{1}{x}} = \frac{2}{3}$$