Mini-math AP Calculus BC: Friday, February 11, 2022 (12 minutes)

SOLUTIONS

1. (2 points) Determine whether the following series converges or diverges:

$$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$$

Solution: By inspection (or using the substitution $u = \ln x$),

$$\begin{aligned} \int_2^{\infty} \frac{1}{x(\ln x)^2}\, dx &= \lim_{b\to\infty} \int_2^b \frac{1}{x(\ln x)^2}\, dx = \lim_{b\to\infty} -\frac{1}{\ln x}\Big|_2^b \\ &= \lim_{b\to\infty} -\frac{1}{\ln b} + \frac{1}{\ln 2} = \frac{1}{\ln 2} < \infty \end{aligned}$$

By the integral test, $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$ converges.

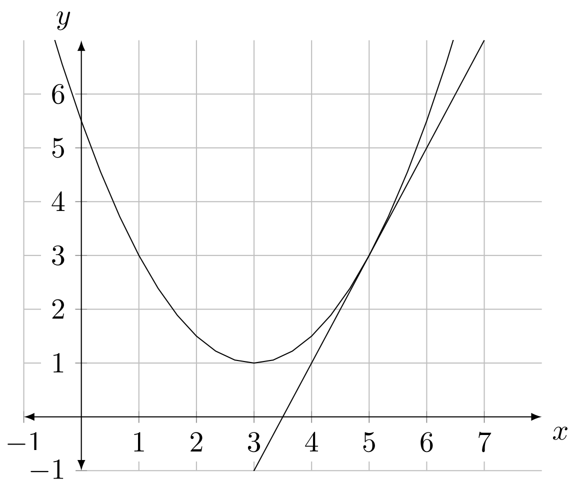
2. (2 points) If a function f is approximated by the third order Taylor polynomial $2 - 5(x-2) + 4(x-2)^2 - 3(x-2)^3$ centred at $x = 2$, what is $f'''(2)$?

Solution:

$$-3 = \frac{f'''(2)}{3!}$$

$$f'''(2) = -3 \cdot 3! = -18$$

3. (2 points) The figure below shows the graph of the differentiable function f and the line tangent to the graph of f at the point $(5, 3)$. Let g be the function given by $g(x) = \int_5^x f(t)\,dt$. Find the 2nd order Taylor polynomial for $g(x)$ centred at $a = 5$.



Solution: By the Fundamental Theorem of Calculus Part I, $g'(x) = f(x)$. Then

$$\begin{aligned} P_2(x) &= g(5) + g'(5)(x-5) + \frac{g''(5)}{2}(x-5)^2 \\ &= 0 + f(5)(x-5) + \frac{f'(5)}{2}(x-5)^2 \\ &= 3(x-5) + (x-5)^2 \end{aligned}$$

4. (2 points) Suppose we know the following bounds:

$$|f^{(2)}(c)| \le 2, \quad |f^{(3)}(c)| \le 5, \quad |f^{(4)}(c)| \le 3,$$

for any c on the interval $[0, 1]$. Use the Lagrange error bound to estimate the absolute value of the error in using a 3rd degree Maclaurin polynomial to approximate $f(0.1)$.

Solution:

$$|Error| \le \frac{|f^{(4)}(c)|}{4!}|0.1 - 0|^4 \le \frac{3}{24} \cdot \frac{1}{10000} = \frac{1}{80000}$$