Draft Quiz created April, 13, 2020

1. NO CALCULATOR IS ALLOWED FOR THIS QUESTION.

Show all of your work, even though the question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.

Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

$$f(x) = \begin{cases} \frac{x^2+kx-2}{3x^2+4x+3} & \text{for } x < -2 \\ x^3+2 & \text{for } -2 \le x < 0 \\ 0 & \text{for } x = 0 \\ \frac{2e^x}{2-e^x} & \text{for } x > 0 \end{cases}$$

Let f be the function defined above, where k is a constant.

(a) For what value of k, if any, is f continuous at $x = -2$? Justify your answer.

Please respond on separate paper, following directions from your teacher.

(b) What type of discontinuity does f have at $x = 0$? Give a reason for your answer.

Please respond on separate paper, following directions from your teacher.

(c) Find all horizontal asymptotes to the graph of f. Show the work that leads to your answer.

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Please respond on separate paper, following directions from your teacher.

Part A

At most 2 out of 3 points if mathematical notation for limits is missing or incorrect.

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3

The student response accurately includes all three of the criteria below.

- ☐ $\lim\limits_{x\to-2^-} f(x)$
- ☐ $\lim\limits_{x\to-2^+} f(x)$
- ☐ justification equating limit with $f(-2)$

Solution:

$$\lim_{x\to-2^-} f(x) = \lim_{x\to-2^-} \frac{x^2+kx-2}{3x^2+4x+3} = \frac{\lim\limits_{x\to-2^-}(x^2+kx-2)}{\lim\limits_{x\to-2^-}(3x^2+4x+3)}$$

$$= \frac{(-2)^2+k(-2)-2}{3(-2)^2+4(-2)+3} = \frac{2-2k}{7}$$

$$\lim_{x\to-2^+} f(x) = \lim_{x\to-2^+}\left(x^3+2\right) = (-2)^3+2 = -6$$

$$f(-2) = -6$$

f is continuous at $x=-2$ if $\lim\limits_{x\to-2^-} f(x) = \lim\limits_{x\to-2^+} f(x) = f(-2)$.

$$\frac{2-2k}{7} = -6 \Rightarrow 2-2k = -42 \Rightarrow k = 22$$

Part B

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Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2

The student response accurately includes both of the criteria below.

- ☐ $\lim_{x\to 0^-} f(x)$ and $\lim_{x\to 0^+} f(x)$
- ☐ answer

Solution:

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \left(x^3+2\right) = 2$$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \frac{2e^x}{2-e^x} = \frac{2e^0}{2-e^0} = 2$$

Because $\lim_{x\to 0} f(x) = 2$ and $f(0) = 0$, f has a removable discontinuity at $x = 0$.

Part C

At most 1 out of 2 points if both correct asymptotes are listed without reference to limits OR if one correct asymptote is listed without any incorrect asymptotes and/or reference to limits.

Students do not have to show the factorization to earn the second point.

Select a point value to view scoring criteria, solutions, and/or examples to score the response.

0	1	2

The student response accurately includes both of the criteria below.

- ☐ $\lim_{x\to\infty} f(x)$
- ☐ $\lim_{x\to-\infty} f(x)$

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Solution:

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{2e^x}{2-e^x} = \lim_{x\to\infty} \frac{2e^x}{-e^x} = \lim_{x\to\infty} (-2) = -2$$

$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \frac{x^2+kx-2}{3x^2+4x+3} = \lim_{x\to-\infty} \frac{x^2\left(1+\frac{k}{x}-\frac{2}{x^2}\right)}{x^2\left(3+\frac{4}{x}+\frac{3}{x^2}\right)}$$

$$= \lim_{x\to-\infty} \frac{1+\frac{k}{x}-\frac{2}{x^2}}{3+\frac{4}{x}+\frac{3}{x^2}} = \frac{\lim\limits_{x\to-\infty}\left(1+\frac{k}{x}-\frac{2}{x^2}\right)}{\lim\limits_{x\to-\infty}\left(3+\frac{4}{x}+\frac{3}{x^2}\right)}$$

$$= \frac{1+0-0}{3+0+0} = \frac{1}{3}$$

The graph of f has horizontal asymptotes at $y=-2$ and $y=\frac{1}{3}$.

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2. A GRAPHING CALCULATOR IS REQUIRED FOR THIS QUESTION.

You are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your question, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results. Your work must be expressed in standard mathematical notation rather than calculator syntax.

Show all of your work, even though the question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.

Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

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Let f be a twice-differentiable function with $f(1.5) = 3$. The derivative of f is given by $f'(x) = (7x - 19)\sin(x^2 - 4x + 4)$ for $1 \leq x \leq 4$.

(a) Find all values of x in the interval $1 < x < 4$ at which f has a critical point. Classify each as the location of a relative minimum, a relative maximum, or neither. Justify your answers.

Please respond on separate paper, following directions from your teacher.

(b) Use the line tangent to the graph of f at $x = 1.5$ to approximate $f(1.8)$.

Please respond on separate paper, following directions from your teacher.

On the interval $1.5 \leq x \leq 1.8$, $f'(x) < 0$ and $f''(x) > 0$. Is the approximation found in part (b) an overestimate or an underestimate for $f(1.8)$? Give a reason for your answer.

Please respond on separate paper, following directions from your teacher.

Using the Mean Value Theorem, explain why the average rate of change of f over the interval $1 \leq x \leq 4$ cannot equal 6.5.

Please respond on separate paper, following directions from your teacher.

Part A

Note: Sign charts are a useful tool to investigate and summarize the behavior of a function. By itself a sign chart for $f'(x)$ or $f''(x)$ is not a sufficient response for a justification.

A maximum of 1 out of 3 points is earned for reference to $f'(x) = 0$ and the three critical points without identification and/or justification, and no incorrect critical points are included.

Select a point value to view scoring criteria, solutions, and/or examples to score the response.

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0	1	2	3

The student response accurately includes all three of the criteria below.

- ☐ neither at $x = 2$ with justification
- ☐ relative minimum at $x = 2.714$ with justification
- ☐ relative maximum at $x = 3.772$ with justification

Solution:

$f'(x) = 0 \;\Rightarrow\; x = 2,\ x = 2.714,\ x = 3.772$

f has neither a relative minimum nor a relative maximum at $x = 2$ because f' does not change sign there.

f has a relative minimum at $x = 2.714$ because f' changes from negative to positive there.

f has a relative maximum at $x = 3.772$ because f' changes from positive to negative there.

Part B

A response of $3 + (-2.102934)(0.3)$ or $3 + (-8.5\sin(0.25))(0.3)$ earns both points.

The second point requires substitution of values without a simplified answer. Trigonometric function values do not need to be evaluated.

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2

The student response accurately includes both of the criteria below.

- ☐ form of tangent line approximation
- ☐ answer

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Solution:

$f'\,(1.5) = -8.5\sin(0.25) = -2.102934$

$f\,(1.8) \approx f\,(1.5) + f'\,(1.5)\,(0.3) = 2.369$

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✔

The student response accurately includes both of the criteria below.

- ☐ answer
- ☐ reason

Solution:

Because $f''\,(x) > 0$ for $1.5 \le x \le 1.8$, the graph of f is concave up for $1.5 \le x \le 1.8$.

Therefore, the tangent line approximation from part (b) is an underestimate for $f\,(1.8)$.

Part D

The first point is earned for using MVT to conclude that there should be a number c for $1 < c < 4$ such that $f'\,(c)$ equals the average rate of change of f over the interval $1 \le x \le 4$. The second point is earned for showing that this is not possible.

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✔

The student response accurately includes both of the criteria below.

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- ☐ uses Mean Value Theorem to determine there should be a c such that $f'(c)$ equals the average rate of change of f
- ☐ explanation using Mean Value Theorem

Solution:

f' is differentiable for $1 \leq x \leq 4. \Rightarrow f$ is continuous for $1 \leq x \leq 4.$

Because the Mean Value Theorem can be applied to f on the interval $1 \leq x \leq 4$, there should be a number c for $1 < c < 4$ such that $f'(c)$ equals the average rate of change of f over the interval $1 \leq x \leq 4$.

However, because the equation $f'(x) = 6.5$ has no solution on the interval $1 \leq x \leq 4$, there can be no number c such that $f'(c) = 6.5$. Therefore, the average rate of change cannot equal 6.5.

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3. NO CALCULATOR IS ALLOWED FOR THIS QUESTION.

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Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

Functions f, g, and h are twice-differentiable functions with $g(2) = h(2) = 4$. The line $y = 4 + \frac{2}{3}(x - 2)$ is tangent to both the graph of g at $x = 2$ and the graph of h at $x = 2$.

a) Find $h'(2)$.

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Please respond on separate paper, following directions from your teacher.

b) Let a be the function given by $a(x) = 3x^3 h(x)$. Write an expression for $a'(x)$. Find $a'(2)$.

Please respond on separate paper, following directions from your teacher.

c) The function h satisfies $h(x) = \frac{x^2-4}{1-(f(x))^3}$ for $x \neq 2$. It is known that $\lim\limits_{x\to 2} h(x)$ can be evaluated using L'Hospital's Rule. Use $\lim\limits_{x\to 2} h(x)$ to find $f(2)$ and $f'(2)$. Show the work that leads to your answers.

Please respond on separate paper, following directions from your teacher.

d) It is known that $g(x) \leq h(x)$ for $1 < x < 3$. Let k be a function satisfying $g(x) \leq k(x) \leq h(x)$ for $1 < x < 3$. Is k continuous at $x = 2$? Justify your answer.

Please respond on separate paper, following directions from your teacher.

Part A

Select a point value to view scoring criteria, solutions, and/or examples to score the response.

0	1 ✔

The student response accurately includes the criteria below.

☐ · answer

Solution:

$h'(2) = \frac{2}{3}$

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

The first point is earned for a derivative in the form of the product rule, namely $a'(x) = p(x) \cdot h(x) + q(x) \cdot h'(x)$, where $p(x)$ and $q(x)$ are both polynomials.

Simplification is not required to earn the third point. Function values must be substituted into the expression.

0	1	2	3

The student response accurately includes **all three** of the criteria below.

- ☐ form of product rule
- ☐ $a'(x)$
- ☐ $a'(2)$

Solution:

$a'(x) = 9x^2 h(x) + 3x^3 h'(x)$

$a'(2) = 9 \cdot 2^2 h(2) + 3 \cdot 2^3 h'(2) = 36 \cdot 4 + 24 \cdot \frac{2}{3} = 160$

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

The first point is earned for showing $\lim\limits_{x \to 2} \frac{x^2-4}{1-(f(x))^3} = 4$. This work may appear anywhere in the response.

The second point is earned for calculating $f(2) = 0$ in the context of setting the denominator $1 - (f(x))^3$ equal to 0.

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The third point is earned for evidence of applying L'Hospital's Rule, which includes a new limit of a quotient where differentiation was attempted in both the numerator and denominator. Note that incorrect notation such as $\lim\limits_{x\to 2}\frac{x^2-4}{1-(f(x))^3}=\frac{0}{0}$ makes a response not eligible to earn the third point.

0	1	2	3	4 ✔

The student response accurately includes **all four** of the criteria below.

- ☐ $\lim\limits_{x\to 2}\frac{x^2-4}{1-(f(x))^3}=4$
- ☐ $f(2)$
- ☐ L'Hospital's Rule
- ☐ $f'(2)$

Solution:

Because h is differentiable, h is continuous, so $\lim\limits_{x\to 2}h(x)=h(2)=4.$

Also, $\lim\limits_{x\to 2}h(x)=\lim\limits_{x\to 2}\frac{x^2-4}{1-(f(x))^3}$, so $\lim\limits_{x\to 2}\frac{x^2-4}{1-(f(x))^3}=4.$

Because $\lim\limits_{x\to 2}\left(x^2-4\right)=0$, we must also have $\lim\limits_{x\to 2}\left(1-(f(x))^3\right)=0.$

Thus $\lim\limits_{x\to 2}f(x)=1.$

Because f is differentiable, f is continuous, so $f(2)=\lim\limits_{x\to 2}f(x)=1.$

Also, because f is twice differentiable, f' is continuous, so $\lim\limits_{x\to 2}f'(x)=f'(2)$ exists.

Using L'Hospital's Rule,

$$\lim_{x\to 2}\frac{x^2-4}{1-(f(x))^3}=\lim_{x\to 2}\frac{2x}{-3(f(x))^2f'(x)}=\frac{4}{-3(1)^2\cdot f'(2)}=4.$$

Thus $f'(2)=-\frac{1}{3}.$

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Part D

Select a point value to view scoring criteria, solutions, and/or examples to score the response.

The point does not require naming the squeeze theorem. The justification requires (1) stating g and h are continuous, (2) stating $\lim_{x\to 2} g(x) = 4$ and $\lim_{x\to 2} h(x) = 4$ and concluding that $\lim_{x\to 2} k(x) = 4$, and (3) stating $g(2) = 4 = h(2)$ and concluding $k(2) = 4$.

0	1

The student response accurately includes the criteria below.

☐ · continuous with justification

Solution:

Because g and h are differentiable, g and h are continuous, so $\lim_{x\to 2} g(x) = g(2) = 4$ and $\lim_{x\to 2} h(x) = h(2) = 4$.

Because $g(x) \le k(x) \le h(x)$ for $1 < x < 3$, it follows from the squeeze theorem that $\lim_{x\to 2} k(x) = 4$.

Also, $4 = g(2) \le k(2) \le h(2) = 4$, so $k(2) = 4$.

Thus k is continuous at $x = 2$.

4. A GRAPHING CALCULATOR IS REQUIRED FOR THIS QUESTION.

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x	-1	0	1	2
$g(x)$	6	4	2	-1
$g'(x)$	-1	-7	-2	-3

The table above gives selected values for a differentiable and decreasing function g and its derivative. Let f be the function with $f(1) = 0$ and derivative given by $f'(x) = x\sin(x^2)$.

(a) Find $f''(e^{1.5})$. Express your answer as a decimal approximation.

Please respond on separate paper, following directions from your teacher.

(b) Let h be the function defined by $h(x) = f(g(3x))$. Find $h'(0)$. Express your answer as a decimal approximation.

Please respond on separate paper, following directions from your teacher.

(c) Write an equation for the line tangent to the graph of g^{-1}, the inverse function of g, at $x = -1$.

Please respond on separate paper, following directions from your teacher.

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(d) The point $(1, 4)$ lies on the curve in the xy-plane given by the equation $f(x) + (g(x))^2 = xy$. What is the value of $\frac{dy}{dx}$ at the point $(1, 4)$? Express your answer as a decimal approximation.

Please respond on separate paper, following directions from your teacher.

Part A

A decimal approximation is required and must be accurate to three places after the decimal point.

Select a point value to view scoring criteria, solutions, and/or examples to score the response.

0	1

The student response accurately includes a correct decimal approximation of $f''\left(e^{1.5}\right)$

Solution:

$f''\left(e^{1.5}\right) = 14.144$

Part B

The first and second points require evidence of applying the chain rule twice and no errors. At most 1 out of 3 points is earned for partial communication of chain rule with a maximum of one computational error. Substitution of function values is required. A decimal approximation is required and must be accurate to three places after the decimal point.

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3

The student response accurately includes all three of the criteria below.

- ☐ chain rule (2 points for full credit)
- ☐ answer

Solution:

$h'(x) = f'(g(3x)) \cdot g'(3x) \cdot 3$

$$\begin{aligned} h'(0) = f'(g(0)) \cdot g'(0) \cdot 3 &= f'(4) \cdot (-7) \cdot 3 \\ &= 4\sin 16 \cdot (-21) = -84\sin 16 \\ &= 24.184 \text{ (or } 24.183) \end{aligned}$$

Part C

The second point may be earned if response contains an incorrect value for slope based on one computational error.

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2 ✔

The student response accurately includes both of the criteria below.

- ☐ slope
- ☐ equation for tangent line

Solution:

Because $g(2) = -1$, $g^{-1}(-1) = 2$.

$$\left(g^{-1}\right)'(-1) = \frac{1}{g'\left(g^{-1}(-1)\right)} = \frac{1}{g'(2)} = \frac{1}{-3}$$

An equation for the tangent line is $y = 2 - \frac{1}{3}(x+1)$.

Part D

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At most 1 out of 2 implicit differentiation points is earned for correct implicit differentiation with a correct application of the chain rule on only one side of the equation OR a correct application of the chain rule on both sides of the equation with a maximum of one computational error.

Substitution of function values is required. The third point requires a decimal approximation that is accurate to three places after the decimal point.

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3

The student response accurately includes all three of the criteria below.

- ☐ implicit differentiation (2 points for full credit)
- ☐ answer

Solution:

$$\frac{d}{dx}\left(f(x)+(g(x))^2\right)=\frac{d}{dx}(xy)$$
$$\Rightarrow\ f'(x)+2g(x)\cdot g'(x)=y+x\frac{dy}{dx}$$
$$\Rightarrow\ f'(1)+2g(1)\cdot g'(1)=4+\frac{dy}{dx}$$
$$\Rightarrow\ \sin 1+2\cdot 2\cdot(-2)=4+\frac{dy}{dx}$$
$$\Rightarrow\ \frac{dy}{dx}=\sin 1-12=-11.159\ (\text{or }-11.158)$$

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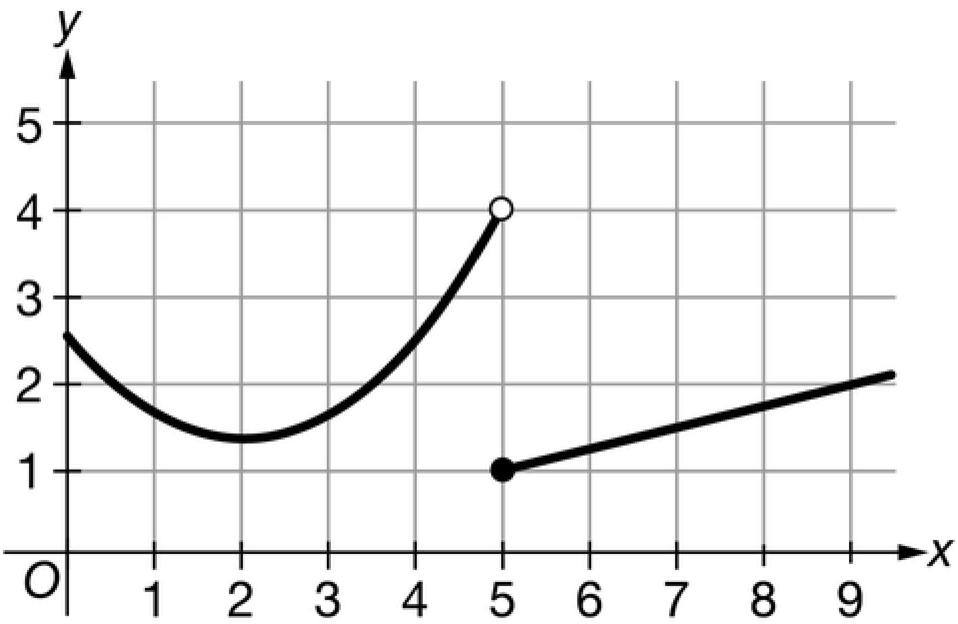

Graph of f

x	3	3.5	4	4.5	5	5.5	6
$f(x)$	1.653	2.02	2.533	3.193	1	1.125	1.25

The graph of the function f and a table of selected values of $f(x)$ are shown above. The graph of f has a horizontal tangent line at $x = 2$, is concave up for $0 < x < 5$, and is linear for $x \geq 5$.

(a) Approximate the value of $f'(4.5)$ using data from the table. Show the computations that lead to your answer.

Please respond on separate paper, following directions from your teacher.

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(b) Is there a value of x, for $0 < x < 5$, such that $\lim_{h \to 0} \frac{f(x+h)-f(x)}{h} = 0$? Give a reason for your answer.

Please respond on separate paper, following directions from your teacher.

(c) For each of the following limits, find the value or explain why it does not exist.

(i) $\lim_{h \to 0^+} \frac{f(5+h)-f(5)}{h}$

(ii) $\lim_{h \to 0^-} \frac{f(5+h)-f(5)}{h}$

(iii) $\lim_{h \to 0} \frac{f(5+h)-f(5)}{h}$

Please respond on separate paper, following directions from your teacher.

Part A

The first point is earned for a correct difference quotient over a subinterval *of* $3 \le x < 5$ *that contains* $x = 4.5$. *A response using* $(5,\ 1)$ as the right endpoint earns only 1 point for knowing to use a difference quotient; this response is not eligible for the second point because $x = 5$ is not in the relevant portion of the domain. The second point is earned for a correct expression that substitutes correct values from the table. The expression does not need to be simplified.

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2

The student response accurately includes both of the criteria below.

☐ difference quotient

☐ approximation

Solution:

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$f'(4.5) \approx \frac{f(4.5)-f(4)}{4.5-4} = \frac{3.193-2.533}{4.5-4} = \frac{0.660}{0.5} = 1.320$

Part B

The first point is for recognizing that the given expression is the derivative of f. The second point is for the correct answer (there is a value of x, for $0 < x < 5$) and communicating the connection between the derivative and the slope of the tangent line at $x = 2$.

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2

The student response accurately includes both of the criteria below.

- ☐ $\lim_{h \to 0} \frac{f(x+h)-f(x)}{h} = f'(x)$
- ☐ answer with reason

Solution:

$\lim_{h \to 0} \frac{f(x+h)-f(x)}{h} = f'(x)$

$f'(x) = 0$ at a point where the line tangent to the graph of f has slope 0. Since the graph of f has a horizontal tangent at $x = 2$, there is a value of x, for $0 < x < 5$, such that $\lim_{h \to 0} \frac{f(x+h)-f(x)}{h} = 0$.

Part C

To earn the first point, the response must report the slope of the linear section of the graph. To earn the second point, the response must explain that the difference in the numerator of the expression will approach 3 as h approaches 0.

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

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0	1	2	3

The student response accurately includes all three of the criteria below.

- ☐ $\lim_{h \to 0^+} \frac{f(5+h)-f(5)}{h}$
- ☐ $\lim_{h \to 0^-} \frac{f(5+h)-f(5)}{h}$ does not exist with explanation
- ☐ $\lim_{h \to 0} \frac{f(5+h)-f(5)}{h}$ does not exist with explanation

Solution:

$\lim_{h \to 0^+} \frac{f(5+h)-f(5)}{h} = \frac{1}{4}$

$\lim_{h \to 0^-} \frac{f(5+h)-f(5)}{h}$ does not exist because the numerator approaches 3 as the denominator approaches 0.

$\lim_{h \to 0} \frac{f(5+h)-f(5)}{h}$ does not exist because the left-hand limit in (ii) does not exist.

—OR—

Because f is not continuous at $x = 5$ due to a jump discontinuity, f is not differentiable at $x = 5$. Since $\lim_{h \to 0} \frac{f(5+h)-f(5)}{h}$ is the definition of the derivative of f at $x = 5$, $\lim_{h \to 0} \frac{f(5+h)-f(5)}{h}$ does not exist.

A particle moves along the *x*-axis so that its velocity at any time $t \geq 0$ is given by $v(t) = 3t^2 - 2t - 1$. The position $x(t)$ is 5 for $t = 2$.

6. For what values of $t, 0 \leq t \leq 3$, is the particle's instantaneous velocity the same as its average velocity on the closed interval [0,3] ?

Please respond on separate paper, following directions from your teacher.

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Part B

1 point is earned for the correct average velocity = 5

avg.

$= \frac{18-3}{3} = 5$

1 point is earned for correctly setting *v*(*t*) equal to student's average velocity OR from student's *x*(*t*)

$3t^2 - 2t - 1 = 5$

1 point is earned for the correct answer

0/1 if not solving $3t^2 - 2t - 1 =$an avg. velocity in [0, 3]

$t = \frac{1+\sqrt{19}}{3}$

0	1	2	3

The student response earns three of the following points:

1 point is earned for the correct average velocity = 5

avg.

$= \frac{18-3}{3} = 5$

1 point is earned for correctly setting *v*(*t*) equal to student's average velocity OR from student's *x*(*t*)

$3t^2 - 2t - 1 = 5$

1 point is earned for the correct answer

0/1 if not solving $3t^2 - 2t - 1 =$ an avg. velocity in [0, 3]

$t = \frac{1+\sqrt{19}}{3}$

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7.

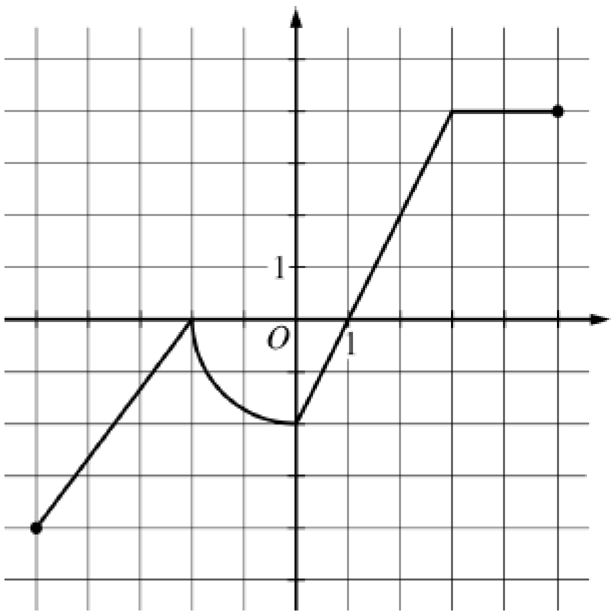
CALCULUS AB

SECTION II, Part B

Time - 1 hour

Number of questions - 4

NO CALCULATOR IS ALLOWED FOR THESE QUESTIONS.



Graph of f

3. The graph of the function *f,* consisting of three line segments and a quarter of a circle, is shown above. Let *g* be the function defined by $g(x) = \int_1^x f(t)\,dt.$

(a) Find the average rate of change of *g* from $x = -5$ to $x = 5$.

Please respond on separate paper, following directions from your teacher.

(b) Find the instantaneous rate of change of *g* with respect to *x* at $x = 3$, or state that it does not exist.

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Please respond on separate paper, following directions from your teacher.

(c) On what open intervals, if any, is the graph of *g* concave up? Justify your answer.

Please respond on separate paper, following directions from your teacher.

(d) Find all *x*-values in the interval $-5 < x < 5$ at which *g* has a critical point. Classify each critical point as the location of a local minimum, a local maximum, or neither. Justify your answers.

Please respond on separate paper, following directions from your teacher.

Part A

1 point is earned for: difference quotient

2 points are earned for: answer

$$\frac{g(5)-g(-5)}{5-(-5)} = \frac{12-(\pi+7)}{10} = \frac{5-\pi}{10}$$

0	1	2	3

The student response earns all of the following points:

1 point is earned for: difference quotient

2 points are earned for: answer

$$\frac{g(5)-g(-5)}{5-(-5)} = \frac{12-(\pi+7)}{10} = \frac{5-\pi}{10}$$

Part B

Draft Quiz created April, 13, 2020

1 point is earned for: answer

$g'(x) = f(x)$
$g'(3) = f(3) = 4$

The instantaneous rate of change of g at $x = 3$ is 4.

0	1

The student response earns all of the following points:

1 point is earned for: answer

$g'(x) = f(x)$
$g'(3) = f(3) = 4$

The instantaneous rate of change of g at $x = 3$ is 4.

Part C

2 points are earned for: intervals with justification

The graph of g is concave up on $-5 < x < -2$ and $0 < x < 3$, because $g'(x) = f(x)$ is increasing on these intervals.

0	1	2

The student response earns all of the following points:

2 points are earned for: intervals with justification

The graph of g is concave up on $-5 < x < -2$ and $0 < x < 3$, because $g'(x) = f(x)$ is increasing on these intervals.

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Part D

1 point is earned for: considers $f(x) = 0$

1 point is earned for: critical points at $x = -2$ and $x = 1$

1 point is earned for: answers with justifications

$g'(x) = f(x)$ is defined at all *x* with $-5 < x < 5$.

$g'(x) = f(x) = 0$ at $x = -2$ and $x = 1$.

Therefore, *g* has critical points at $x = -2$ and $x = 1$.

g has neither a local maximum nor a local minimum at $x = -2$ because g' does not change sign there.

g has a local minimum at $x = 1$ because g' changes from negative to positive there.

0	1	2	3 ✓

The student response earns all of the following points:

1 point is earned for: considers $f(x) = 0$

1 point is earned for: critical points at $x = -2$ and $x = 1$

1 point is earned for: answers with justifications

$g'(x) = f(x)$ is defined at all *x* with $-5 < x < 5$.

$g'(x) = f(x) = 0$ at $x = -2$ and $x = 1$.

Therefore, *g* has critical points at $x = -2$ and $x = 1$.

g has neither a local maximum nor a local minimum at $x = -2$ because g' does not change sign there.

g has a local minimum at $x = 1$ because g' changes from negative to positive there.

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Consider the differential equation $\frac{dy}{dx} = \frac{3-x}{y}$.

8. Let *y* = *f*(*x*) be the particular solution to the given differential equation for 1 x y = -2 is tangent to the graph of *f*. Find the *x*-coordinate of the point of tangency, and determine whether *f* has a local maximum, local minimum, or neither at this point. Justify your answer.

Please respond on separate paper, following directions from your teacher.

Part A

1 point is earned for: *x* = 3

1 point is earned for the local minimum

1 point is earned for the justification

$$\frac{dy}{dx} = 0 when, x = 3$$
$$\left.\frac{d^2y}{dx^2}\right|_{(3,-2)} = \left.\frac{-y-y'(3-x)}{y^2}\right|_{(3,-2)} = \frac{1}{2},$$

so *f* has a local minimum at this point.

or

Because *f* is continuous for 1 x x = 3 on which *y* is negative to the left of *x* = 3 and $\frac{dy}{dx}$ is positive to the right of *x* = 3.Therefore f has a local minimum at *x* = 3.

0	1	2	3

The student response earns all of the following points:

1 point is earned for: *x* = 3

1 point is earned for the local minimum

1 point is earned for the justification

Draft Quiz created April, 13, 2020

$\frac{dy}{dx} = 0 when, x = 3$

$\left.\frac{d^2y}{dx^2}\right|_{(3,-2)} = \left.\frac{-y-y'(3-x)}{y^2}\right|_{(3,-2)} = \frac{1}{2},$

so *f* has a local minimum at this point.

or

Because *f* is continuous for 1 x x = 3 on which *y* is negative to the left of *x* = 3 and $\frac{dy}{dx}$ is positive to the right of *x* = 3.Therefore f has a local minimum at *x* = 3.

9.

CALCULUS BC

SECTION II, Part A

Time - 30 minutes

Number of questions - 2

A GRAPHING CALCULATOR IS REQUIRED FOR THESE QUESTIONS.

t (minutes)	0	1	5	6	8
$g(t)$ (cubic feet per minute)	12.8	15.1	20.5	18.3	22.7

1. Grain is being added to a silo. At time $t = 0$, the silo is empty. The rate at which grain is being added is modeled by the differentiable function g, where $g(t)$ is measured in cubic feet per minute for $0 \leq t \leq 8$ minutes. Selected values of $g(t)$ are given in the table above.

(a) Using the data in the table, approximate $g'(3)$. Using correct units, interpret the meaning of $g'(3)$ in the context of the problem.

Please respond on separate paper, following directions from your teacher.

(b) Write an integral expression that represents the total amount of grain added to the silo from time $t = 0$ to time $t = 8$. Use a right Riemann sum with the four subintervals indicated by the data in the table to approximate the integral.

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Please respond on separate paper, following directions from your teacher.

(c) The grain in the silo is spoiling at a rate modeled by $w(t) = 32 \cdot \sqrt{\sin\left(\frac{\pi t}{74}\right)}$, where $w(t)$ is measured in cubic feet per minute for $0 \le t \le 8$ minutes. Using the result from part (b), approximate the amount of unspoiled grain remaining in the silo at time $t = 8$.

Please respond on separate paper, following directions from your teacher.

(d) Based on the model in part (c), is the amount of unspoiled grain in the silo increasing or decreasing at time $t = 6$? Show the work that leads to your answer.

Please respond on separate paper, following directions from your teacher.

Part A

1 point is earned for: approximation

1 point is earned for: interpretation with units

$g'(3) \approx \frac{g(5)-g(1)}{5-1} = \frac{20.5-15.1}{4} = 1.35$

At time $t = 3$ minutes, the rate at which grain is being added to the silo is increasing at a rate of 1.35 cubic feet per minute per minute.

0	1	2

The student response earns all of the following points:
1 point is earned for: approximation

1 point is earned for: interpretation with units

$g'(3) \approx \frac{g(5)-g(1)}{5-1} = \frac{20.5-15.1}{4} = 1.35$

At time $t = 3$ minutes, the rate at which grain is being added to the silo is increasing at a rate of 1.35 cubic feet per minute per minute.

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Part B

1 point is earned for: integral expression

1 point is earned for: right Riemann sum

1 point is earned for: approximation

The total amount of grain added to the silo from time $t = 0$ to time $t = 8$ is $\int_0^8 g(t)\,dt$ cubic feet.

$$\int_0^8 g(t)\,dt \approx g(1)\cdot(1-0) + g(5)\cdot(5-1) + g(6)\cdot(6-5) + g(8)\cdot(8-6)$$
$$= 15.1\cdot 1 + 20.5\cdot 4 + 18.3\cdot 1 + 22.7\cdot 2 = 160.8$$

0	1	2	3

The student response earns all of the following points:

1 point is earned for: integral expression

1 point is earned for: right Riemann sum

1 point is earned for: approximation

The total amount of grain added to the silo from time $t = 0$ to time $t = 8$ is $\int_0^8 g(t)\,dt$ cubic feet.

$$\int_0^8 g(t)\,dt \approx g(1)\cdot(1-0) + g(5)\cdot(5-1) + g(6)\cdot(6-5) + g(8)\cdot(8-6)$$
$$= 15.1\cdot 1 + 20.5\cdot 4 + 18.3\cdot 1 + 22.7\cdot 2 = 160.8$$

Part C

1 point is earned for: integral

1 point is earned for: answer

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$$\int_0^8 w(t)\, dt = 99.051497$$

The approximate amount of unspoiled grain remaining in the silo at time $t = 8$ is $160.8 - \int_0^8 w(t)\, dt = 61.749$ (or 61.748) cubic feet.

0	1	2

The student response earns all of the following points:

1 point is earned for: integral

1 point is earned for: answer

$$\int_0^8 w(t)\, dt = 99.051497$$

The approximate amount of unspoiled grain remaining in the silo at time $t = 8$ is $160.8 - \int_0^8 w(t)\, dt = 61.749$ (or 61.748) cubic feet.

Part D

1 point is earned for: considers $g(6) - w(6)$

1 point is earned for: answer

$$g(6) - w(6) = 18.3 - 16.063173 = 2.236827 > 0$$

Because $g\,(6) - w\,(6) > 0$, the amount of unspoiled grain is increasing at time $t = 6$.

0	1	2

The student response earns all of the following points:

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1 point is earned for: considers $g(6) - w(6)$

1 point is earned for: answer

$g(6) - w(6) = 18.3 - 16.063173 = 2.236827 > 0$

Because $g(6) - w(6) > 0$, the amount of unspoiled grain is increasing at time $t = 6$.

At time $t, t \geq 0$, the volume of a sphere is increasing at a rate proportional to the reciprocal of its radius. At $t = 0$, the radius of the sphere is 1 and at $t = 15$, the radius is 2. (The volume V of a sphere with a radius r is $V = 4/3\pi r^3$.)

10. Find the radius of the sphere as a function of *t*.

Please respond on separate paper, following directions from your teacher.

Part A

1 point is earned for dV dt = k r

1 point is earned for dV dt =4π r 2 dr dt

1 point is earned for separates variables

1 point is earned for integration (must have C)

1 point is earned for solving for C

1 point is earned for solving for k

1 point is earned for answer

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$\frac{dV}{dt} = \frac{k}{r}$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\frac{k}{r} = 4\pi r^2 \frac{dr}{dt}$

$k\ dt = 4\pi r^3 dr \int k\ dt = \int 4\pi r^3 \frac{dr}{dt}\ dt$

$kt + C = \pi r^4$

At t=0 , r=1 , so C=π

At t=15 , r=2 , so

15k+$\pi = 16\pi, k = \pi$

$\pi r^4 = \pi t + \pi$

$r = \sqrt[4]{t+1}$

0	1	2	3	4	5	6	7

The student response earns seven of the following points:

1 point is earned for dV dt = k r

1 point is earned for dV dt =4π r 2 dr dt

1 point is earned for separates variables

1 point is earned for integration (must have C)

1 point is earned for solving for C

1 point is earned for solving for k

1 point is earned for answer

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$\frac{dV}{dt} = \frac{k}{r}$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\frac{k}{r} = 4\pi r^2 \frac{dr}{dt}$

$k\ dt = 4\pi r^3 dr \int k\ dt = \int 4\pi r^3 \frac{dr}{dt}\ dt$

$kt + C = \pi r^4$

At t=0 , r=1 , so C=π

At t=15 , r=2 , so

15k+$\pi = 16\pi, k = \pi$

$\pi r^4 = \pi t + \pi$

$r = \sqrt[4]{t+1}$

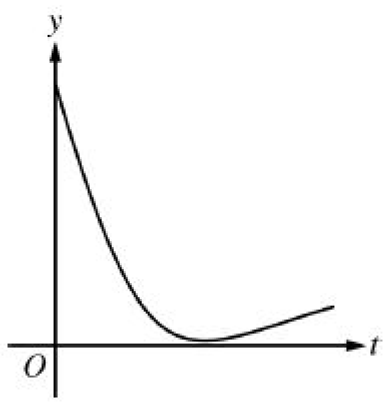
---

11. 5. During a chemical reaction, the function $y = f(t)$ models the amount of a substance present, in grams, at time *t* seconds. At the start of the reaction $(t = 0)$, there are 10 grams of the substance present. The function $y = f(t)$ satisfies the differential equation $\frac{dy}{dt} = -0.02y^2$.

(a) Use the line tangent to the graph of $y = f(t)$ at $t = 0$ to approximate the amount of the substance remaining at time $t = 2$ seconds.

Please respond on separate paper, following directions from your teacher.

(b) Using the given differential equation, determine whether the graph of *f* could resemble the following graph. Give a reason for your answer.



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Please respond on separate paper, following directions from your teacher.

(c) Find an expression for $y = f(t)$ by solving the differential equation $\frac{dy}{dt} = -0.02y^2$ with the initial condition $f(0) = 10$.

Please respond on separate paper, following directions from your teacher.

(d) Determine whether the amount of the substance is changing at an increasing or a decreasing rate. Explain your reasoning.

Please respond on separate paper, following directions from your teacher.

Part A

1 point is earned for: $y'(0)$

1 point is earned for: approximation

$y'(0) = -0.02\left(10^2\right) = -2$

An equation for the line tangent to the graph of $y = f(t)$ at $t = 0$ is $y = 10 - 2t$.

$y(2) \approx 10 - 2(2) = 6$ grams

0	1	2

The student response earns all of the following points:

1 point is earned for: $y'(0)$

1 point is earned for: approximation

$y'(0) = -0.02\left(10^2\right) = -2$

An equation for the line tangent to the graph of $y = f(t)$ at $t = 0$ is $y = 10 - 2t$.

$y(2) \approx 10 - 2(2) = 6$ grams

Part B

1 point is earned for: answer with reason

$\frac{dy}{dt} = -0.02y^2 \leq 0$, so the graph of *f* is nonincreasing.

The graph of *f* cannot resemble the given graph because the given graph is increasing on a portion of its domain.

0	1

The student response earns all of the following points:

1 point is earned for: answer with reason

$\frac{dy}{dt} = -0.02y^2 \leq 0$, so the graph of *f* is nonincreasing.

The graph of *f* cannot resemble the given graph because the given graph is increasing on a portion of its domain.

Part C

1 point is earned for: separation of variables

1 point is earned for: antiderivatives

1 point is earned for: constant of integration and uses initial condition

1 point is earned for: answer

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$$\int\left(-\frac{1}{y^2}\right)\,dy=\int 0.02\,dt$$
$\frac{1}{y}=0.02t+C$
$\frac{1}{10}=0.02(0)+C\Rightarrow C=0.1$
$\frac{1}{y}=0.02t+0.1\Rightarrow y=\frac{1}{0.02t+0.1}=\frac{50}{t+5}$

Note: this solution is valid for $t>-5$.

Note: max $2/4$ $[1\text{-}1\text{-}0\text{-}0]$ if no constant of integration

Note: $0/4$ if no separation of variables

0	1	2	3	4

The student response earns all of the following points:

1 point is earned for: separation of variables

1 point is earned for: antiderivatives

1 point is earned for: constant of integration and uses initial condition

1 point is earned for: answer

$$\int\left(-\frac{1}{y^2}\right)\,dy=\int 0.02\,dt$$
$\frac{1}{y}=0.02t+C$
$\frac{1}{10}=0.02(0)+C\Rightarrow C=0.1$
$\frac{1}{y}=0.02t+0.1\Rightarrow y=\frac{1}{0.02t+0.1}=\frac{50}{t+5}$

Note: this solution is valid for $t>-5$.

Note: max $2/4$ $[1\text{-}1\text{-}0\text{-}0]$ if no constant of integration

Note: $0/4$ if no separation of variables

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Part D

1 point is earned for: $\frac{d^2y}{dt^2}$

1 point is earned for: answer with reason

$$\begin{aligned}\frac{d^2y}{dt^2} &= -0.04y\frac{dy}{dt}\\ &= -\,0.04y(-0.02y^2)\\ &= 0.0008y^3\end{aligned}$$

Because $y > 0$, $0.0008y^3 > 0$.

The amount of substance is changing at an increasing rate.

— OR —

From part (c), $f(t) = \frac{50}{t+5}$, and from context, $t \geq 0$.

$f'(t) = \frac{-50}{(t+5)^2}$ and $f''(t) = \frac{100}{(t+5)^3} > 0$ for $t \geq 0$.

The amount of substance is changing at an increasing rate.

0	1	2

The student response earns all of the following points:

1 point is earned for: $\frac{d^2y}{dt^2}$

1 point is earned for: answer with reason

$$\begin{aligned}\frac{d^2y}{dt^2} &= -0.04y\frac{dy}{dt}\\ &= -\,0.04y(-0.02y^2)\\ &= 0.0008y^3\end{aligned}$$

Because $y > 0$, $0.0008y^3 > 0$.

The amount of substance is changing at an increasing rate.

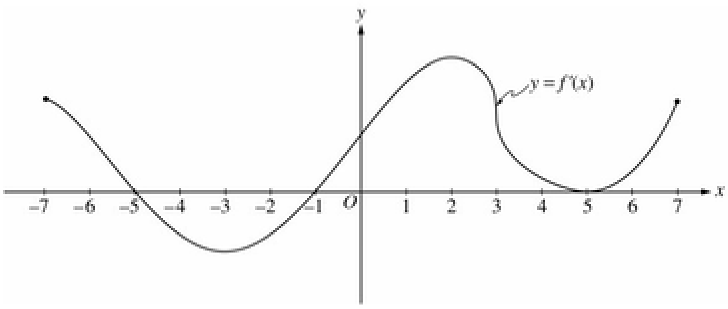
Draft Quiz created April, 13, 2020

— OR —

From part (c), $f(t) = \frac{50}{t+5}$, and from context, $t \geq 0$.

$f'(t) = \frac{-50}{(t+5)^2}$ and $f''(t) = \frac{100}{(t+5)^3} > 0$ for $t \geq 0$.

The amount of substance is changing at an increasing rate.



The figure above shows the graph of *f'*, the derivative of the function *f*, for $-7 \leq x \leq 7$. The graph of *f'* has horizontal tangent lines at *x* = -3, *x* = 2, and *x* = 5, and a vertical tangent line at *x* = 3.

---

12. At what value of *x*, for $-7 \leq x \leq 7$, does *f* attain its absolute maximum? Justify your answer.

Please respond on separate paper, following directions from your teacher.

Part D

1 point is earned for the answer

1 point is earned for identifies *x* = -5 and *x* = 7 as candidates

- or -

indicates that the graph of *f* increase, decreases, then increases

1 point is earned for justifies $f(7) > f(-5)$

$x = 7$

The absolute maximum must occur at $x = -5$ or at an endpoint.

$f(-5) > f(-7)$ because f is increasing on (-7, -5)

The graph of f' shows that the magnitude of the negative change in f from $x = -5$ to $x = -1$ is smaller than the positive change in f from x=-1 to $x = 7$. Therefore the net change in f is positive from $x = -5$ to $x = 7$, and f(7) > f(-5). So f(7) is the absolute maximum.

0	1	2	3

The student response earns all of the following points:

1 point is earned for the answer

1 point is earned for identifies $x = -5$ and $x = 7$ as candidates

- or -

indicates that the graph of f increase, decreases, then increases

1 point is earned for justifies $f(7) > f(-5)$

$x = 7$

The absolute maximum must occur at $x = -5$ or at an endpoint.

$f(-5) > f(-7)$ because f is increasing on (-7, -5)

The graph of f' shows that the magnitude of the negative change in f from $x = -5$ to $x = -1$ is smaller than the positive change in f from x=-1 to $x = 7$. Therefore the net change in f is positive from $x = -5$ to $x = 7$, and f(7) > f(-5). So f(7) is the absolute maximum.

13. Find all values of x, for -7 x f attains a relative minimum. Justify your answer.

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Please respond on separate paper, following directions from your teacher.

Part A

1 point is earned for the answer

1 point is earned for the justification

x = -1

$f'(x)$ changes from negative to positive at x = -1

0	1	2

The student response earns all of the following points:

1 point is earned for the answer

1 point is earned for the justification

x = -1

$f'(x)$ changes from negative to positive at x = -1

---

14. Find all values of x, for -7 x f''(x)

Please respond on separate paper, following directions from your teacher.

Part C

1 point is earned for: (-7, -3)

1 point is earned for (2, 3) U (3, 5)

$f''(x)$ exists and f' is decreasing on the intervals

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(-7, -3), (2, 3), and (3, 5)

0	1	2

The student response earns all of the following points:

1 point is earned for: (-7, -3)

1 point is earned for (2, 3) U (3, 5)

f''(x) exists and *f'* is decreasing on the intervals

(-7, -3), (2, 3), and (3, 5)

---

15. NO CALCULATOR IS ALLOWED FOR THIS QUESTION.

Show all of your work, even though the question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.

Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

Let f be an increasing function with $f(0) = 3$. The derivative of f is given by $f'(x) = \cos(\pi x) + x^4 + 6$.

(a) Find $f''(-3)$.

Please respond on separate paper, following directions from your teacher.

(b) Write an equation for the line tangent to the graph of $y = (f(x))^2$ at $x = 0$.

Please respond on separate paper, following directions from your teacher.

(c) Let g be the function defined by $g(x) = f\left(\sqrt{2x^2 + 7}\right)$. Find $g'(3)$.

Please respond on separate paper, following directions from your teacher.

(d) Let h be the inverse function of f. Find $h'(3)$.

Please respond on separate paper, following directions from your teacher.

Part A

The second point cannot be earned without the first point. The second point does not require a simplified answer; trigonometric function values do not need to be evaluated.Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2

The student response accurately includes both of the criteria below.

- ☐ chain rule
- ☐ answer

Solution:

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$f''(x) = -\pi \sin(\pi x) + 4x^3$

$f''(-3) = -\pi \sin(-3\pi) + 4(-3)^3 = -108$

Part B

The second point may be earned based on a derivative expression that includes application of power rule and chain rule with a maximum of one computational error. Trigonometric function values do not need to be evaluated.The third point may be earned if response contains a slope that is consistent with the derivative expression.Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3

The student response does not accurately include any of the criteria below.

- ☐ $\frac{d}{dx}\left((f(x))^2\right)$
- ☐ slope
- ☐ equation for tangent line

Solution:

At $x = 0$, $y = (f(0))^2 = 3^2 = 9$.

$\frac{d}{dx}\left((f(x))^2\right) = 2f(x)f'(x)$

$\frac{d}{dx}\left((f(x))^2\right)\Big|_{x=0} = 2f(0)f'(0) \quad = 2(3)\ (\cos 0 + 6) = 6(\cos 0 + 6) = 42$

An equation for the tangent line is y equals, $y = 9 + 42x$.

Part C

The first and second points require evidence of applying the chain rule twice and no errors. At most 1 out of 3 points is earned for partial communication of chain rule with a maximum of one computational error.

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Substitution of function values is required. Trigonometric function values do not need to be evaluated. A simplified answer is not required.Select a point value to view scoring criteria, solutions, and/or examples to score the response.

0	1	2	3

The student response accurately includes all three of the criteria below.

- ☐ chain rule (2 points for full credit)
- ☐ answer

Solution:

$$g'(x) = f'\left(\sqrt{2x^2+7}\right) \cdot \frac{4x}{2\sqrt{2x^2+7}} = f'\left(\sqrt{2x^2+7}\right) \cdot \frac{2x}{\sqrt{2x^2+7}}$$

$$g'(3) = f'\left(\sqrt{2 \cdot 3^2+7}\right) \cdot \frac{2 \cdot 3}{\sqrt{2 \cdot 3^2+7}} = f'(5) \cdot \frac{6}{5} = (\cos(5\pi) + 631) \cdot \frac{6}{5} = (630)\,\frac{6}{5} = 756$$

Part D

Substitution of function values is required. Trigonometric function values do not need to be evaluated.Select a point value to view scoring criteria, solutions, and/or examples to score the response.

0	1

The student response accurately includes the criteria below.

- ☐ answer

Solution:

Because $f(0) = 3$, $h(3) = 0$.

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$$h'(3) = \frac{1}{f'(h(3))} = \frac{1}{f'(0)} = \frac{1}{(\cos\ 0+6)} = \frac{1}{7}$$

The derivative of a function f is given by $f'(x) = (x - 3)e^x$ for $x > 0$, and $f(1) = 7$.

16. On what intervals, if any, is the graph of f both decreasing and concave up? Explain your reasoning.

Please respond on separate paper, following directions from your teacher.

Part B

The response can earn up to 3 points:

2 point: For $f''(x)$

1 point: For correct answer with reason

$f''(x) = e^x + (x - 3)e^x = (x - 2)e^x$

$f''(x) > 0$ for x > 2

$f'(x)$

Therefore, the graph of f is both decreasing and concave up on the interval 2 x

0	1	2	3

The response earns all three of the following points:

2 point: For $f''(x)$

1 point: For correct answer with reason

$f''(x) = e^x + (x - 3)e^x = (x - 2)e^x$

$f''(x) > 0$ for x > 2

$f'(x)$

Therefore, the graph of f is both decreasing and concave up on the interval 2 x

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17. 6. Consider the curve given by the equation $2(x-y) = 3 + \cos y$. For all points on the curve, $\frac{2}{3} \leq \frac{dy}{dx} \leq 2$.

(a) Show that $\frac{dy}{dx} = \frac{2}{2 - \sin y}$.

Please respond on separate paper, following directions from your teacher.

(b) For $-\frac{\pi}{2} < y < \frac{\pi}{2}$, there is a point *P* on the curve through which the line tangent to the curve has slope 1. Find the coordinates of the point *P*.

Please respond on separate paper, following directions from your teacher.

(c) Determine the concavity of the curve at points for which $-\frac{\pi}{2} < y < \frac{\pi}{2}$. Give a reason for your answer.

Please respond on separate paper, following directions from your teacher.

(d) Let $y = f(x)$ be a function, defined implicitly by $2(x-y) = 3 + \cos y$, that is continuous on the closed interval $[2,\ 2.1]$ and differentiable on the open interval $(2,\ 2.1)$. Use the Mean Value Theorem on the interval $[2,\ 2.1]$ to show that $\frac{1}{15} \leq f(2.1) - f(2) \leq \frac{1}{5}$.

Please respond on separate paper, following directions from your teacher.

Part A

1 point is earned for: implicit differentiation

1 point is earned for: verification

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$\frac{d}{dx}(2(x-y)) = \frac{d}{dx}(3+\cos y)$
$2 - 2\frac{dy}{dx} = (-\sin y)\frac{dy}{dx}$
$2 = (2-\sin y)\frac{dy}{dx}$
$\frac{dy}{dx} = \frac{2}{2-\sin y}$

0	1	2

The student response earns all of the following points:

1 point is earned for: implicit differentiation

1 point is earned for: verification

$\frac{d}{dx}(2(x-y)) = \frac{d}{dx}(3+\cos y)$
$2 - 2\frac{dy}{dx} = (-\sin y)\frac{dy}{dx}$
$2 = (2-\sin y)\frac{dy}{dx}$
$\frac{dy}{dx} = \frac{2}{2-\sin y}$

Part B

1 point is earned for: $\frac{dy}{dx} = 1$

1 point is earned for: answer

$\frac{dy}{dx} = \frac{2}{2-\sin y} = 1 \Rightarrow \sin y = 0 \Rightarrow y = 0$
$2(x-0) = 3 + \cos 0 \Rightarrow 2x = 4 \Rightarrow x = 2$

Point *P* has coordinates $(2,\ 0)$.

0	1	2

Draft Quiz created April, 13, 2020

The student response earns all of the following points:

1 point is earned for: $\frac{dy}{dx} = 1$

1 point is earned for: answer

$\frac{dy}{dx} = \frac{2}{2-\sin y} = 1 \Rightarrow \sin y = 0 \Rightarrow y = 0$
$2(x-0) = 3 + \cos 0 \Rightarrow 2x = 4 \Rightarrow x = 2$

Point *P* has coordinates $(2,\ 0)$.

Part C

2 points are earned for: $\frac{d^2y}{dx^2}$

1 point is earned for: answer with reason

$\frac{d^2y}{dx^2} = \frac{-2}{(2-\sin y)^2}(-\cos y)\frac{dy}{dx} = \frac{4\cos y}{(2-\sin y)^3}$

$\frac{d^2y}{dx^2} > 0$ for all *y* in the interval $-\frac{\pi}{2} < y < \frac{\pi}{2}$.

Therefore, the curve is concave up for $-\frac{\pi}{2} < y < \frac{\pi}{2}$.

0	1	2	3 ✔

The student response earns all of the following points:

2 points are earned for: $\frac{d^2y}{dx^2}$

1 point is earned for: answer with reason

$\frac{d^2y}{dx^2} = \frac{-2}{(2-\sin y)^2}(-\cos y)\frac{dy}{dx} = \frac{4\cos y}{(2-\sin y)^3}$

$\frac{d^2y}{dx^2} > 0$ for all *y* in the interval $-\frac{\pi}{2} < y < \frac{\pi}{2}$.

Therefore, the curve is concave up for $-\frac{\pi}{2} < y < \frac{\pi}{2}$.

Part D

1 point is earned for: applies Mean Value Theorem

1 point is earned for: verification

By the Mean Value Theorem, for some value *c* in the interval $(2,\ 2.1)$, $f'(c) = \frac{f(2.1) - f(2)}{0.1}$.

For all points on the curve, $\frac{2}{3} \leq f'(x) \leq 2$.

Thus, $\frac{2}{3} \leq \frac{f(2.1) - f(2)}{0.1} \leq 2 \Rightarrow \frac{1}{15} \leq f(2.1) - f(2) \leq \frac{1}{5}$.

0	1	2 ✔

The student response earns all of the following points:

1 point is earned for: applies Mean Value Theorem

1 point is earned for: verification

By the Mean Value Theorem, for some value *c* in the interval $(2,\ 2.1)$, $f'(c) = \frac{f(2.1) - f(2)}{0.1}$.

For all points on the curve, $\frac{2}{3} \leq f'(x) \leq 2$.

Thus, $\frac{2}{3} \leq \frac{f(2.1) - f(2)}{0.1} \leq 2 \Rightarrow \frac{1}{15} \leq f(2.1) - f(2) \leq \frac{1}{5}$.

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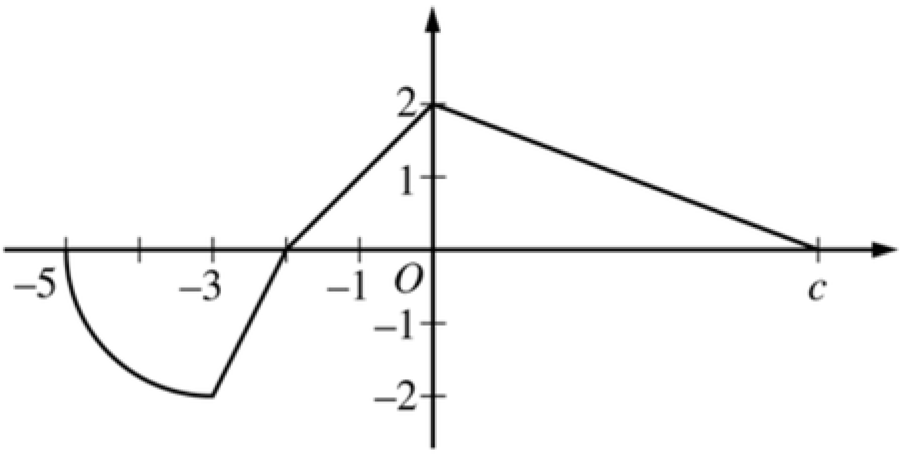

Graph of f

The function f is defined on the interval $-5 \leq x \leq c$, where c > 0 and $f(c)$ = 0. The graph of f, which consists of three line segments and a quarter of a circle with center (-3,0) and radius 2, is shown in the figure above.

18.

For −5 ≤ x ≤ c, let g be the function defined by $g(x) = \int_{-1}^{x} f(t)dt$. Find the x-coordinate of each point of inflection of the graph of g. Justify your answer.

Please respond on separate paper, following directions from your teacher.

Part B

1 point(s) earned for: g'(x) = f(x)

g'(x) = f(x)

1 point(s) earned for: identifies x = -3 and x =0

The graph of g has a point of inflection at x = −3 because g′ = f changes from decreasing to increasing at this point

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1 point(s) earned for: justification

The graph of g has a point of inflection at $x = 0$ because $g' = f$ changes from increasing to decreasing at this point

0	1	2	3

Student response earns 3 of the following 3 point(s)

1 point(s) earned for: $g'(x) = f(x)$

$g'(x) = f(x)$

1 point(s) earned for: identifies $x = -3$ and $x =0$

The graph of g has a point of inflection at $x = -3$ because $g' = f$ changes from decreasing to increasing at this point

1 point(s) earned for: justification

The graph of g has a point of inflection at $x = 0$ because $g' = f$ changes from increasing to decreasing at this point