Misc. algebra problems

1. Animals:

   (a) A farmer has some chickens and some pigs. There are 89 animals in total, and 286 legs in total. How many pigs are there?

   (b) A farmer has some chickens and some pigs. There are 104 animals in total, and 232 legs in total. How many pigs are there?

   (c) A farmer has some chickens and some pigs. There are 112 animals in total, and 360 legs in total. How many pigs are there?

   (d) A farmer has some chickens and some pigs. There are 104 animals in total, and 346 legs in total. How many pigs are there?

   (e) A farmer has some chickens and some pigs. There are 84 animals in total, and 230 legs in total. How many pigs are there?

   (f) A farmer has some chickens and some pigs. There are 106 animals in total, and 240 legs in total. How many chickens are there?

   (g) A farmer has some chickens and some pigs. There are 104 animals in total, and 352 legs in total. How many chickens are there?

   (h) A farmer has some chickens and some pigs. There are 108 animals in total, and 292 legs in total. How many pigs are there?

   (i) A farmer has some chickens and some pigs. There are 130 animals in total, and 460 legs in total. How many pigs are there?

   (j) A farmer has some chickens and some pigs. There are 51 animals in total, and 140 legs in total. How many pigs are there?

   (k) A farmer has some chickens and some pigs. There are 34 animals in total, and 116 legs in total. How many chickens are there?

   (l) A farmer has some chickens and some pigs. There are 125 animals in total, and 396 legs in total. How many pigs are there?

2. MacNuggets:

   (a) The famous restaurant MacDonal's sells their chicken MacNuggets in packs of 7 and 14. If there are 147 nuggets in 11 packs, how many packs of 14 are there?

   (b) The famous restaurant MacDonal's sells their chicken MacNuggets in packs of 7 and 7. If there are 210 nuggets in 30 packs, how many packs of 7 are there?

   (c) The famous restaurant MacDonal's sells their chicken MacNuggets in packs of 5 and 9. If there are 32 nuggets in 4 packs, how many packs of 9 are there?

   (d) The famous restaurant MacDonal's sells their chicken MacNuggets in packs of 10 and 14. If there are 216 nuggets in 20 packs, how many packs of 14 are there?

   (e) The famous restaurant MacDonal's sells their chicken MacNuggets in packs of 4 and 10. If there are 102 nuggets in 21 packs, how many packs of 10 are there?

   (f) The famous restaurant MacDonal's sells their chicken MacNuggets in packs of 10 and 16. If there are 222 nuggets in 15 packs, how many packs of 10 are there?

   (g) The famous restaurant MacDonal's sells their chicken MacNuggets in packs of 7 and 11. If there are 260 nuggets in 28 packs, how many packs of 7 are there?

   (h) The famous restaurant MacDonal's sells their chicken MacNuggets in packs of 9 and 13. If there are 349 nuggets in 33 packs, how many packs of 13 are there?

   (i) The famous restaurant MacDonal's sells their chicken MacNuggets in packs of 8 and 11. If there are 273 nuggets in 30 packs, how many packs of 11 are there?

   (j) The famous restaurant MacDonal's sells their chicken MacNuggets in packs of 6 and 12. If there are 264 nuggets in 28 packs, how many packs of 6 are there?

   (k) The famous restaurant MacDonal's sells their chicken MacNuggets in packs of 5 and 12. If there are 318 nuggets in 37 packs, how many packs of 5 are there?

   (l) The famous restaurant MacDonal's sells their chicken MacNuggets in packs of 8 and 5. If there are 188 nuggets in 31 packs, how many packs of 5 are there?

3. Tickets:

   (a) The admission to the Math Centre is $4.55 for a child and $6.85 for an adult. One day, the total admissions was $12,423.40 for 2370 people. How many adults were there?

   (b) The admission to the Math Centre is $3.00 for a child and $4.55 for an adult. One day, the total admissions was $5,215.45 for 1615 people. How many children were there?

   (c) The admission to the Math Centre is $3.25 for a child and $5.70 for an adult. One day, the total admissions was $13,408.60 for 3004 people. How many children were there?

   (d) The admission to the Math Centre is $3.05 for a child and $5.80 for an adult. One day, the total admissions was $13,171.65 for 2573 people. How many children were there?

   (e) The admission to the Math Centre is $3.05 for a child and $6.30 for an adult. One day, the total admissions was $15,385.30 for 2926 people. How many adults were there?

   (f) The admission to the Math Centre is $4.90 for a child and $7.45 for an adult. One day, the total admissions was $13,099.15 for 2296 people. How many adults were there?

   (g) The admission to the Math Centre is $4.30 for a child and $8.55 for an adult. One day, the total admissions was $22,297.45 for 3439 people. How many adults were there?

   (h) The admission to the Math Centre is $2.35 for a child and $5.20 for an adult. One day, the total admissions was $7,469.95 for 1858 people. How many adults were there?

   (i) The admission to the Math Centre is $4.00 for a child and $7.45 for an adult. One day, the total admissions was $13,659.20 for 2297 people. How many children were there?

   (j) The admission to the Math Centre is $4.10 for a child and $5.60 for an adult. One day, the total admissions was $15,173.80 for 3008 people. How many adults were there?

   (k) The admission to the Math Centre is $1.25 for a child and $4.55 for an adult. One day, the total admissions was $8,563.20 for 2154 people. How many adults were there?

   (l) The admission to the Math Centre is $4.15 for a child and $5.45 for an adult. One day, the total admissions was $5,748.40 for 1190 people. How many children were there?

4. Apples:

   (a) Alice bought a total of 8.9 kg of apples, some of which cost \$2.10 per kg and some of which cost \$3.50 per kg. Alice spent \$22.19 in total. To the nearest gram, how many grams of the cheaper apples did she buy?

   (b) Alice bought a total of 13 kg of apples, some of which cost \$1.85 per kg and some of which cost \$3.25 per kg. Alice spent \$34.97 in total. To the nearest gram, how many grams of the more expensive apples did she buy?

   (c) Alice bought a total of 4 kg of apples, some of which cost \$2.00 per kg and some of which cost \$4.00 per kg. Alice spent \$11.6 in total. To the nearest gram, how many grams of the more expensive apples did she buy?

   (d) Alice bought a total of 11.2 kg of apples, some of which cost \$1.80 per kg and some of which cost \$3.20 per kg. Alice spent \$29.26 in total. To the nearest gram, how many grams of the more expensive apples did she buy?

   (e) Alice bought a total of 7.8 kg of apples, some of which cost \$2.80 per kg and some of which cost \$4.60 per kg. Alice spent \$32.64 in total. To the nearest gram, how many grams of the more expensive apples did she buy?

   (f) Alice bought a total of 7.3 kg of apples, some of which cost \$1.70 per kg and some of which cost \$2.50 per kg. Alice spent \$16.41 in total. To the nearest gram, how many grams of the cheaper apples did she buy?

   (g) Alice bought a total of 6 kg of apples, some of which cost \$2.80 per kg and some of which cost \$3.60 per kg. Alice spent \$19.76 in total. To the nearest gram, how many grams of the more expensive apples did she buy?

   (h) Alice bought a total of 9.2 kg of apples, some of which cost \$1.95 per kg and some of which cost \$3.55 per kg. Alice spent \$30.74 in total. To the nearest gram, how many grams of the more expensive apples did she buy?

   (i) Alice bought a total of 9.6 kg of apples, some of which cost \$1.60 per kg and some of which cost \$2.60 per kg. Alice spent \$21.36 in total. To the nearest gram, how many grams of the cheaper apples did she buy?

   (j) Alice bought a total of 11 kg of apples, some of which cost \$2.70 per kg and some of which cost \$4.50 per kg. Alice spent \$42.12 in total. To the nearest gram, how many grams of the cheaper apples did she buy?

   (k) Alice bought a total of 5.9 kg of apples, some of which cost \$2.70 per kg and some of which cost \$4.50 per kg. Alice spent \$19.17 in total. To the nearest gram, how many grams of the cheaper apples did she buy?

   (l) Alice bought a total of 7.6 kg of apples, some of which cost \$1.50 per kg and some of which cost \$2.10 per kg. Alice spent \$14.46 in total. To the nearest gram, how many grams of the more expensive apples did she buy?

Examples:

1. A farmer has some chickens and some pigs. There are 128 animals in total, and 372 legs in total. How many pigs are there?

   Solution:

   Method 1: (two variables)

   Let

   $$C = \text{number of chickens},$$
   $$P = \text{number of pigs}$$

   Then counting the number of animals and the number of legs,

   $$C + P = 128,$$
   $$2C + 4P = 372$$

   Since we are trying to find P, we eliminate C by multiplying the first equation by 2:

   $$2C + 2P = 256,$$
   $$2C + 4P = 372$$

   Subtracting the first equation from the second and dividing by 2,

   $$2P = 116$$
   $$P = 58$$

   So there are 58 pigs.

   Method 2: (one variable)

   Let

   $$P = \text{number of pigs}$$

   Then there are $128 - P$ chicken. Counting the number of legs,

   $$2(128 - P) + 4P = 372$$
   $$256 - 2P + 4P = 372,$$
   $$2P = 116$$
   $$P = 58$$

   So there are 58 pigs.

   Method 3: (no variables)

   Suppose all 128 animals are chicken. Then there are $2 \cdot 128 = 256$ legs, so we need $372 - 256 = 116$ more legs. Each chicken we replace with a pig gives 2 more legs, so we replace $116/2 = 58$ chicken with 58 pigs. Therefore, there are 58 pigs.

2. The famous restaurant MacDonal's sells their chicken MacNuggets in packs of 10 and 20. If there are 280 nuggets in 18 packs, how many packs of 10 are there?

   Solution:

   Method 1: (two variables)

   Let

   $$s = \text{number of packs of 10},$$
   $$b = \text{number of packs of 20}$$

   Then counting the number of packs and the number of nuggets,

   $$s + b = 18,$$
   $$10s + 20b = 280$$

   Since we are trying to find s, we eliminate b by multiplying the first equation by 20:

   $$20s + 20b = 360,$$
   $$10s + 20b = 280$$

   Subtracting the second equation from the first and dividing by 10,

   $$10s = 80$$
   $$s = 10$$

   So there are 8 packs of 10.

   Method 2: (one variable)

   Let

   $$s = \text{number of packs of 10}$$

   Then there are $18 - s$ packs of 20. Counting the number of nuggets,

   $$10s + 20(18 - s) = 280$$
   $$10s + 360 - 20s = 280,$$
   $$-10s = -80$$
   $$s = 8$$

   So there are 8 packs of 10.

   Method 3: (no variables)

   Suppose all 18 packs are packs of 20. Then there are $20 \cdot 18 = 360$ nuggets, so we need $360 - 280 = 80$ fewer nuggets. Each pack of 20 we replace with a pack of 10 gives 10 fewer nuggets, so we replace $80/10 = 8$ packs of 20 with packs of 10. Therefore, there are 8 packs of 10.

3. The admission to the Math Centre is \$3.55 for a child and \$4.65 for an adult. One day, the total admissions was \$3,819.85 for 953 people. How many adults were there?

   Solution:

   Method 1: (two variables)

   Let

   $$c = \text{number of children},$$
   $$a = \text{number of adults}$$

   Then counting the number of people and the total admissions,

   $$c + a = 953,$$
   $$3.55c + 4.65a = 3819.85$$

   Since we are trying to find a, we eliminate c by multiplying the first equation by 3.55:

   $$3.55c + 3.55a = 3383.15,$$
   $$3.55c + 4.65a = 3819.85$$

   Subtracting the second equation from the first and dividing by 1.1,

   $$1.1a = 436.7$$
   $$a = 397$$

   So there were 397 adults.

   Method 2: (one variable)

   Let

   $$a = \text{number of adults}$$

   Then there were $953 - a$ children. Counting the total admissions,

   $$\begin{aligned} 3.55(953 - a) + 4.65a &= 3819.85 \\ 3383.15 - 3.55a + 4.65a &= 3819.85 \\ 1.1a &= 436.7 \\ a &= 397 \end{aligned}$$

   So there were 397 adults.

   Method 3: (no variables)

   Suppose all 953 of the people were children. Then there was $3.55 \cdot 953 = 3383.15$ in admissions, so we need \$3819.85 − \$3383.15 = \$436.7 more in admissions. Each child we replace with an adult increases the admissions by \$4.65 − \$3.55 = \$1.10, so we replace $436.7/1.1 = 397$ children with adults. Therefore, there are 397 adults.

4. Alice bought a total of 6.8 kg of apples, some of which cost \$1.65 per kg and some of which cost \$3.25 per kg. Alice spent \$15.38 in total. To the nearest gram, how many grams of the more expensive apples did she buy?

   Solution:

   Method 1: (two variables)

   Let

   $$C = \text{amount of cheaper apples bought (in kg)},$$
   $$E = \text{amount of more expensive apples bought (in kg)}$$

   Then counting the total amount of apples and the total cost,

   $$C + E = 6.8,$$
   $$1.65C + 3.25E = 15.38$$

   Since we are trying to find E, we eliminate C by multiplying the first equation by 1.65:

   $$1.65C + 1.65E = 11.22,$$
   $$1.65C + 3.25E = 15.38$$

   Subtracting the first equation from the second and dividing by 1.6,

   $$1.6E = 4.16$$
   $$E = 2.6$$

   So Alice bought 2600 g of the more expensive apples.

   Method 2: (one variable)

   Let

   $$E = \text{amount of more expensive apples bought (in kg)}$$

   Then Alice bought $6.8 - E$ kg of the cheaper apples. Counting the total cost,

   $$1.65(6.8 - E) + 3.25E = 15.38$$
   $$11.22 - 1.65E + 3.25E = 15.38$$
   $$1.6E = 4.16$$
   $$E = 2.6$$

   So Alice bought 2600 g of the more expensive apples.

   Method 3: (no variables)

   Suppose all 6.8 kg of the apples were the cheaper apples. This would cost$1.65 \cdot 6.8 = 11.22$, so we need \$15.38 − \$11.22 = \$4.16 more. Each kg of cheaper apples we replace with a kg of more expensive apples increases the total cost by \$3.25 − \$1.65 = \$1.60, so we replace $4.16/1.6 = 2.6$ kg of cheaper apples with more expensive apples. Therefore, Alice bought 2600 g of the more expensive apples.