Algebraic "Facts" Challenge

Here are the properties of integers discussed in class:

1. Additive associativity: for any elements a, b, c,
$$a + (b + c) = (a + b) + c$$
2. Additive commutativity: for any elements a, b,
$$a + b = b + a$$
3. Additive identity: there exists an element z such that for any element a,
$$a + z = a$$
We denote z as 0.
4. Additive inverse: for any element a, there exists a element b such that
$$a + b = 0$$
We denote b as $-a$.
5. Multiplicative associativity: for any elements a, b, c,
$$a(bc) = (ab)c$$
6. Multiplicative commutativity: for any numbers a, b,
$$ab = ba$$
7. Multiplicative identity: there exists an element u such that for any element a,
$$au = a$$
We denote $u = 1$.
8. Distributivity: for any elements a, b, c,
$$a(b + c) = ab + ac$$

Prove the following "facts" using the above properties:

1. 0 is the unique additive identity.
2. For any elements a, b, c: if $a + b = a + c$, then $b = c$.
3. Additive inverses are unique.
4. For any element a: $0a = 0$ (So 0 times anything is 0.)
5. For any elements a, b: $(-a)b = -(ab)$ (In particular, a negative times a positive is negative.)
6. For any elements a, b: $(-a)(-b) = ab$ (In particular, a negative times a negative is positive.)