**Mini-math Div 3/4: Friday, September 24, 2021 (15 minutes)**

**SOLUTIONS**

1. (2 points) Find the following limit, if it exists. If it does not, indicate if the limit is $-\infty, \infty$, or DNE. (No proof required)

$$\lim_{x\to 2^-} \frac{x-3}{x(x-2)}$$

**Solution:** This is of the form $\dfrac{(-)}{(+)(0^-)}$, so

$$\lim_{x\to 2^-} \frac{x-3}{x(x-2)} = \infty$$

2. (2 points) Find the horizontal and vertical asymptotes of the following function. (No proof required)

$$f(x) = \frac{(x-4)(x-1)(2x+3)}{(x-1)(2x-3)^2}$$

**Solution:** Horizontal asymptotes:

$$\lim_{x\to\pm\infty} f(x) = \lim_{x\to\pm\infty} \frac{(1-\frac{4}{x})(1-\frac{1}{x})(2+\frac{3}{x})}{(1-\frac{1}{x})(2-\frac{3}{x})^2} = \frac{1}{2},$$

so $y = \frac{1}{2}$ is the only horizontal asymptote.

Vertical asymptotes: Only $x = 3/2$ is a potential vertical asymptote, since $x = 1$ is a removable discontinuity.

$$\lim_{x\to 3/2^+} f(x) = \lim_{x\to 3/2^+} \frac{(x-4)(2x+3)}{(2x-3)^2} = -\infty$$

since this is of the form $\dfrac{(-)(+)}{0^+}$, so $x = 3/2$ is the only vertical asymptote.

3. (2 points) (AP) What conditions must be true in order for the Intermediate Value Theorem to guarantee a solution to the equation $f(x) = 4$ on the interval $[0, 3]$?

**Solution:** $f(x)$ must be continuous on $[0, 3]$, and 4 must be between $f(0)$ and $f(3)$.

4. (2 points) (AP) Find the horizontal and vertical asymptotes of the following function. (No proof required)

$$g(x) = \frac{e^{2x}\sin x - e^x + 2e}{e^{2x} - e}$$

**Solution:** Horizontal asymptotes:

$$\lim_{x\to-\infty} g(x) = \frac{2e}{-e} = -2,$$
$$\lim_{x\to\infty} g(x) = \lim_{x\to\infty} \frac{\sin x - \frac{1}{e^x} + \frac{2e}{e^x}}{1 - \frac{e}{e^{2x}}} \text{ DNE.}$$

so $y = -2$ is the only horizontal asymptote.

Vertical asymptotes: $e^{2x} - e = 0$ is a potential vertical asymptote. This occurs at $x = 1/2$.

$$\lim_{x\to 1/2^+} g(x) = \infty$$

since this is of the form $\dfrac{(+)}{0^+}$, so $x = 1/2$ is the only vertical asymptote.

5. (2 points) (AP) Find the horizontal and vertical asymptotes of the following function. (No proof required)

$$h(t) = \frac{t+1}{\sqrt{t^2-4}+2t}$$

**Solution:** Horizontal asymptotes:

$$\lim_{t\to-\infty} h(t) = \lim_{t\to-\infty} \frac{1+\frac{1}{t}}{-\sqrt{1-\frac{4}{t^2}}+2} = \frac{1}{-1+2} = 1,$$
$$\lim_{t\to\infty} h(t) = \lim_{t\to\infty} \frac{1+\frac{1}{t}}{\sqrt{1-\frac{4}{t^2}}+2} = \frac{1}{1+2} = \frac{1}{3}.$$

so $y = 1/3, 1$ are horizontal asymptotes.

Vertical asymptotes: Potential vertical asymptotes occur at $\sqrt{t^2-4}+2t = 0$, so

$$t^2 - 4 = 4t^2$$

This has no solution over the real numbers, so there are no vertical asymptotes.