Mini-math AP Calculus BC: Friday, September 24, 2021 (15 minutes)

1. (1 point) Suppose S_n is the *n*th partial sum of the infinite series $\sum_{n=1}^{\infty} a_n$, where $a_n = \sin(n\pi/6)$. What is $S_{61} - S_{60}$?

Solution: $S_{61} - S_{60} = a_{61} = \sin(61\pi/6) = \sin(\pi/6) = 1/2$

2. (1 point) True or false: if $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=1}^{\infty} a_n$ converges.

Solution: False - for example, $a_n = 1/n$.

3. (1 point) For what values of k does $\sum_{n=1}^{\infty} 2^{kn}$ converge?

Solution: This is a geometric series with $r = 2^k$; we need $|2^k| < 1$, so k < 0.

4. (2 points) If $0 < |a| < b^2$, compute $\sum_{k=1}^{\infty} b \left(\frac{a}{b^2}\right)^k$.

Solution: This is a geometric series with $r = a/b^2$, so

$$\sum_{k=1}^{\infty} b(\frac{a}{b^2})^k = \frac{b \cdot \frac{a}{b^2}}{1 - \frac{a}{b^2}} = \frac{ab}{b^2 - a}$$