Numerical Integration

1. Consider the following table of values of $f(x)$:

x	0	1	2	3	4	5	6	7	8
$f(x)$	5	1	3	2	4	7	9	10	9

Approximate $\int_0^6 f(x)\,dx$ using the stated method.

(a) Right Riemann sum with 8 equal subintervals

Solution:

$$\begin{aligned} &f(1)\cdot 1 + f(2)\cdot 1 + f(3)\cdot 1 + f(4)\cdot 1 + f(5)\cdot 1 + f(6)\cdot 1 + f(7)\cdot 1 + f(8)\cdot 1 \\ &\quad = 1 + 3 + 2 + 4 + 7 + 9 + 10 + 9 = 45 \end{aligned}$$

(b) Left Riemann sum with 8 equal subintervals

Solution:

$$\begin{aligned} &f(0)\cdot 1 + f(1)\cdot 1 + f(2)\cdot 1 + f(3)\cdot 1 + f(4)\cdot 1 + f(5)\cdot 1 + f(6)\cdot 1 + f(7)\cdot 1 \\ &\quad = 5 + 1 + 3 + 2 + 4 + 7 + 9 + 10 = 41 \end{aligned}$$

(c) Trapezoid Rule with 8 equal subintervals

Solution:

$$\begin{aligned} &\left(\frac{f(0)+f(1)}{2}\right)\cdot 1 + \left(\frac{f(1)+f(2)}{2}\right)\cdot 1 + \left(\frac{f(2)+f(3)}{2}\right)\cdot 1 + \left(\frac{f(3)+f(4)}{2}\right)\cdot 1 \\ &\quad + \left(\frac{f(4)+f(5)}{2}\right)\cdot 1 + \left(\frac{f(5)+f(6)}{2}\right)\cdot 1 + \left(\frac{f(6)+f(7)}{2}\right)\cdot 1 + \left(\frac{f(7)+f(8)}{2}\right)\cdot 1 \\ &\quad = 3 + 2 + 5/2 + 3 + 11/2 + 8 + 19/2 + 19/2 = 43 \end{aligned}$$

(d) Right Riemann sum with 4 equal subintervals

Solution:

$$f(2)\cdot 2 + f(4)\cdot 2 + f(6)\cdot 2 + f(8)\cdot 2 = 6 + 8 + 18 + 18 = 50$$

(e) Left Riemann sum with 4 equal subintervals

Solution:

$$f(0)\cdot 2+f(2)\cdot 2+f(4)\cdot 2+f(6)\cdot 2=10+6+8+18=42$$

(f) Midpoint Rule with 4 equal subintervals

Solution:

$$f(1)\cdot 2+f(3)\cdot 2+f(5)\cdot 2+f(7)\cdot 2=2+4+14+20=40$$

(g) Trapezoid Rule with 4 equal subintervals

Solution:

$$\left(\frac{f(0)+f(2)}{2}\right)\cdot 2+\left(\frac{f(2)+f(4)}{2}\right)\cdot 2+\left(\frac{f(4)+f(6)}{2}\right)\cdot 2+\left(\frac{f(6)+f(8)}{2}\right)\cdot 2$$
$$=8+7+13+18=46$$

(h) Right Riemann sum with 2 equal subintervals

Solution: $f(4)\cdot 4+f(8)\cdot 4=16+36=52$

(i) Left Riemann sum with 2 equal subintervals

Solution: $f(0)\cdot 4+f(4)\cdot 4=20+16=36$

(j) Midpoint Rule with 2 equal subintervals

Solution: $f(2)\cdot 4+f(6)\cdot 4=12+36=48$

(k) Trapezoid Rule with 2 equal subintervals

Solution: $\left(\frac{f(0)+f(4)}{2}\right)\cdot 4+\left(\frac{f(4)+f(8)}{2}\right)\cdot 4=18+26=44$

2. Consider the following table of values of $f(x)$:

x	0	2	3	6	9	11	15
$f(x)$	4	1	2	-2	4	6	10

Approximate $\int_0^{15} f(x)\,dx$ using the stated method.

(a) Right Riemann Sum with 6 intervals

Solution:

$$\begin{aligned} & f(2)\cdot 2 + f(3)\cdot 1 + f(6)\cdot 3 + f(9)\cdot 3 + f(11)\cdot 2 + f(15)\cdot 4 \\ & \quad = 2 + 2 - 6 + 12 + 12 + 40 = 62 \end{aligned}$$

(b) Left Riemann Sum with 6 intervals

Solution:

$$\begin{aligned} & f(0)\cdot 2 + f(2)\cdot 1 + f(3)\cdot 3 + f(6)\cdot 3 + f(9)\cdot 2 + f(11)\cdot 4 \\ & \quad = 8 + 1 + 6 - 6 + 8 + 24 = 41 \end{aligned}$$

(c) Trapezoid Rule with 6 intervals

Solution:

$$\begin{aligned} & \left(\frac{f(0)+f(2)}{2}\right)\cdot 2 + \left(\frac{f(2)+f(3)}{2}\right)\cdot 1 + \left(\frac{f(3)+f(6)}{2}\right)\cdot 3 \\ & \quad + \left(\frac{f(6)+f(9)}{2}\right)\cdot 3 + \left(\frac{f(9)+f(11)}{2}\right)\cdot 2 + \left(\frac{f(11)+f(15)}{2}\right)\cdot 4 \\ & = 5 + 3/2 + 0 + 3 + 10 + 32 = 51.5 \end{aligned}$$

(d) Midpoint Rule with 3 intervals

Solution:

$$\begin{aligned} & f(2)\cdot(3-0) + f(6)\cdot(9-3) + f(11)\cdot(15-9) \\ & \quad = 3 - 12 + 36 = 27 \end{aligned}$$