Mini-math AP Calculus BC: Friday, February 4, 2022 (8 minutes)

SOLUTIONS

1. (2 points) The continuous functions f and g and their derivatives take on the following values:

x	-2	-1	0	1	2
$f(x)$	-6	-1	3	-2	2
$f'(x)$	5	-2	4	-3	6
$g(x)$	3	-4	-2	5	4
$g'(x)$	-2	2	5	-4	3

If $\int_{-2}^{1} f'(x)g(x)\,dx = 7$, then what is $\int_{-2}^{1} f(x)g'(x)\,dx$?

Solution: By integration by parts,

$$\begin{aligned}\int_{-2}^{1} f(x)g'(x)\,dx &= f(x)g(x)\Big|_{-2}^{1} - \int_{-2}^{1} f'(x)g(x)\,dx \\ &= f(1)g(1) - f(-2)g(-2) - 7 \\ &= (-2)(5) - (-6)(3) - 7 = -10 + 18 - 7 = 1\end{aligned}$$

2. (2 points) Find $\displaystyle\int \frac{x^2+2x}{x^2+2x+2}\,dx$

Solution:

$$\begin{aligned}\int \frac{x^2+2x}{x^2+2x+2}\,dx &= \int \frac{x^2+2x+2-2}{x^2+2x+2}\,dx \\ &= \int 1 - \frac{2}{(x+1)^2+1}\,dx \\ &= x - 2\arctan(x+1) + C\end{aligned}$$

3. (2 points) Find $\int \frac{x^3+1}{x^2-1}\,dx$

Solution:

$$\begin{aligned}\int \frac{x^3+2}{x^2-1}\,dx = \int \frac{x(x^2-1)+x+2}{x^2-1}\,dx &= \int x + \frac{x+2}{(x+1)(x-1)}\,dx \\ &= \int \left(x + \frac{\frac{1}{-2}}{x+1} + \frac{\frac{3}{2}}{x-1} \right)\,dx \\ &= \frac{1}{2}x^2 - \frac{1}{2}\ln|x+1| + \frac{3}{2}\ln|x-1| + C\end{aligned}$$

4. (2 points) Find $\int_{-1}^{2} \frac{1}{x^2}\,dx$

Solution: Since the integrand is not continuous at $0 \in [-1,2]$,

$$\int_{-1}^{2} \frac{1}{x^2}\,dx = \lim_{b\to 0^-} \int_{-1}^{b} \frac{1}{x^2}\,dx + \lim_{b\to 0^+} \int_{-1}^{b} \frac{1}{x^2}\,dx$$

But

$$\lim_{b\to 0^-} \int_{-1}^{b} \frac{1}{x^2}\,dx = \lim_{b\to 0^-} \left.\frac{1}{x}\right|_{-1}^{b} = \lim_{b\to 0^-} \left(\frac{1}{b}+1\right) = -\infty$$

so the integral does not converge.