

Function Composition

Consider the functions

$$f(x) = 1 - x \quad \text{and} \quad g(x) = \frac{1}{x}$$

We can compose these functions in several ways:

$$f(f(x)), \quad f(g(x)), \quad g(f(x)), \quad \text{and} \quad g(g(x))$$

We can then go further and compose these new functions with themselves and with the original functions f and g , in many ways. (Find their simplified forms in terms of x — remember, two very different formulae may represent the same function.)

- (1) Find all possible functions created in this way.
- (2) How is each function composed from f and g originally?
- (3) For what real number are all of these functions simultaneously defined?