Name: ______________________ Mark: ______________________

Mini-math Div 3/4: Monday, November 16, 2020 (10 minutes)

(1) True or false: If $f(x)$ is defined on $[a, b]$ and $x = c \in (a, b)$ is a global maximum, then it is a local maximum.

Solution: True: if $f(x) \geq f(y)$ for all $x \in [a, b]$, then certainly $f(x) \geq f(y)$ for all x near c (within $[a, b]$). (Likewise, a global minimum is a local minimum.)

(2) True or false: If $f(x)$ is defined on $[a, b]$, then it must have a global maximum on $[a, b]$.

Solution: False: Consider

$$f(x) = \begin{cases} \frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

defined on $[-1, 1]$. Clearly, this function has no global maximum (nor does it have a global minimum).

(3) True or false: If $f(x)$ is continuous and defined on (a, b), then it must have a global maximum on $[a, b]$.

Solution: False: Consider $f(x) = x^2$ on $(0, 1)$. This function has no maximum (nor does it have a global minimum).

(4) (2 marks) Consider the function $f(x) = \dfrac{x^2 + 3}{x - 1}$. Find the intervals on which f is increasing.

Solution: By quotient rule,

$$f'(x) = \frac{2x(x-1) - 1(x^2+3)}{(x-1)^2} = \frac{x^2 - 2x - 3}{(x-1)^2} = \frac{(x+1)(x-3)}{(x-1)^2}$$

The critical points are $-1, 1, 3$. On each subinterval, the derivative has the following sign:

		-1		1		3	
$x+1$	$-$	|	$+$	|	$+$	|	$+$
$(x-1)^2$	$+$	|	$+$	|	$+$	|	$+$
$x-3$	$-$	|	$-$	|	$+$	|	$+$
$f'(x)$	$+$	|	$-$	|	$+$	|	$+$

(You can also omit the $(x-1)^2$ term, since it does not affect the sign of the expression.)

Then f is increasing on $(-\infty, -1) \cup (1, 3) \cup (3, \infty)$.