## Boomerang fractions

Start with the number 1. Choose any natural number at least 2.

e.g. Choose 2.

Step 1: Add the reciprocal of your starting number.

e.g.

$$1 \xrightarrow{+} 1 + \frac{1}{2} = \frac{3}{2}$$

For following steps, you can choose to either add the reciprocal of your starting number, or take the reciprocal of your current value.

e.g. From $\frac{3}{2}$, you can either add $\frac{1}{2}$ to get

$$\frac{3}{2} \xrightarrow{+} \frac{3}{2} + \frac{1}{2} = 2$$

or take the reciprocal to get

$$\frac{3}{2} \xrightarrow{r} \frac{2}{3}$$

Your goal: get back to 1 in the fewest number of steps.

e.g.

$$1 \xrightarrow{+} \frac{3}{2} \xrightarrow{r} \frac{2}{3} \xrightarrow{r} \frac{3}{2} \xrightarrow{+} 2 \xrightarrow{r} \frac{1}{2} \xrightarrow{+} 1$$

takes 6 steps (you can do better).

Challenges:

(1) Is the number of steps always finite for any starting number?

(2) What is the fewest number of steps required for starting numbers of 2 through 16?

(3) How good of an upper bound for the fewest number of steps in the case of a starting number which is prime can you get? What about a prime power? What about other numbers?