The ratio of girls to boys at a party was 27:31. After 9 boys arrived, the ratio of girls to boys at the party was 27:32. Find the number of boys at the start.

	Start	Change	End
girls:	$27x$	$+0$	$27y$
boys:	$31x$	$+9$	$32y$

$$\Rightarrow \begin{cases} 27x + 0 = 27y \\ 31x + 9 = 32y \end{cases} \Rightarrow x = y$$

Then $31x + 9 = 32x$

$9 = x$ $\quad \therefore 31 \cdot 9 = \boxed{279}$

The ratio of girls to boys at a party was 13:10. After 3 girls left and 3 boys arrived, the ratio of girls to boys at the party was 38:31. Find the number of girls at the start.

	Start	Change	End
girls:	$13x$	-3	$38y$
boys:	$10x$	$+3$	$31y$

$$\Rightarrow 13x - 3 = 38y \quad (1)$$

$$10x + 3 = 31y \quad (2)$$

* Can also use elimination or substitution

Method 1:

Adding, $23x = 69y$

$x = 3y$

Into (2), $10(3y) + 3 = 31y$

$30y + 3 = 31y$

$y = 3$

$\therefore 13x = 13(3y) = 13(3 \times 3)$

$= \boxed{117}$

Method 2:

Dividing,

$$\frac{13x - 3}{10x + 3} = \frac{38}{31}$$

$31(13x - 3) = 38(10x + 3)$

$403x - 93 = 380x + 114$

$23x = 207$

$x = 9$

$\therefore 13x = 13 \times 9 = \boxed{117}$

The ratio of girls to boys at a party was 3:4. After 510 girls arrive and 510 boys leave, the ratio becomes 5:1. How many girls at the end?

$$\frac{3x + 510}{4x - 510} = \frac{5}{1}$$

$3x + 510 = 5(4x - 510)$

$$3x + 510 = 20x - 2550$$

$$3060 = 17x$$

$$180 = x$$

$$\therefore \quad 3x + 510 = 3(180) + 510 = 540 + 510 = \boxed{1050}$$