Mini-math Div 3/4: Friday, September 15, 2022 (18 minutes)

SOLUTIONS

1. (1 point) Given $\int_1^7 f(x)\,dx = 4$, $\int_{-1}^7 f(x)\,dx = -3$, and $\int_1^5 f(x)\,dx = 6$, find $\int_{-1}^5 (2f(x)+3)\,dx$

   A. -2 B. 15 C. 16 D. 17 E. 28

Solution:

$$\int_{-1}^5 f(x)\,dx = \int_{-1}^7 f(x)\,dx - \int_1^7 f(x)\,dx + \int_1^5 f(x)\,dx = -3 - 4 + 6 = -1$$

so

$$\int_{-1}^5 (2f(x)+3)\,dx = 2(-1) + \int_{-1}^5 3\,dx = -2 + 18 = 16$$

(E) is correct.

2. (1 point) Using the substitution $u = x^3 - 2$, $\int_{-2}^3 x^2(x^3-2)^3\,dx$ is equal to which of the following?

   A. $3\int_{-10}^{25} u^3\,du$

   B. $\int_{-10}^{25} u^3\,du$

   C. $\frac{1}{3}\int_{-10}^{25} u^3\,du$

   D. $\int_{-2}^{3} u^3\,du$

   E. $\frac{1}{3}\int_{-2}^{3} u^3\,du$

Solution: Changing the bounds of integration, $-2 \mapsto (-2)^3 - 2 = -8 = 2 = -10$ and $3 \mapsto 3^3 - 2 = 27 - 2 = 25$. Since $u = x^3 - 2$, we have $du = 3x^2\,dx$, so $x^2\,dx = \frac{1}{3}\,du$.

(C) is correct.

3. (1 point) $\displaystyle\int_0^1 \frac{2x-3}{x^2-5x+6}\,dx$ is

A. $\ln\left(\frac{16}{27}\right)$ B. $\ln 8$ C. $\ln 27$ D. $\ln 432$ E. divergent

Solution:

$$\int_0^1 \frac{2x-3}{x^2-5x+6}\,dx = \int_0^1 \frac{2x-3}{(x-2)(x-3)}\,dx = \int_0^1 \left(\frac{-1}{x-2} + \frac{3}{x-3}\right)dx$$
$$= \left(-\ln|x-2| + 3\ln|x-3|\right)\Big|_0^1 = 4\ln 2 - 3\ln 3 = \ln\frac{16}{27}$$

(A) is correct.

4. (1 point) $\displaystyle\int_1^\infty xe^{-x^2}\,dx$ is

A. $-\frac{1}{e}$ B. $\frac{1}{2e}$ C. $\frac{1}{e}$ D. $\frac{2}{e}$ E. divergent

Solution:

$$\int_1^\infty xe^{-x^2}\,dx = \lim_{b\to\infty}\int_1^b xe^{-x^2}\,dx = \lim_{b\to\infty} -\frac{1}{2}e^{-x^2}\Big|_1^b = \lim_{b\to\infty} -\frac{1}{2}(e^{-b^2} - e^{-1}) = \frac{1}{2e}$$

(D) is correct.

5. (1 point) $\displaystyle\int_1^8 t^{-2/3}\,dt =$

A. -3 B. -1 C. $\dfrac{93}{160}$ D. 1 E. 3

Solution:

$$\int_1^8 t^{-2/3}\,dt = 3t^{1/3}\Big|_1^8 = 3(2-1) = 3$$

(E) is correct.

6. (1 point) To the right is a graph of the function $f(x)$. Suppose $g(x) = \displaystyle\int_a^x f(t)\,dt$ and $g(1) = 5$. What is the minimum value of $g(x)$ on $[-6, 2]$?

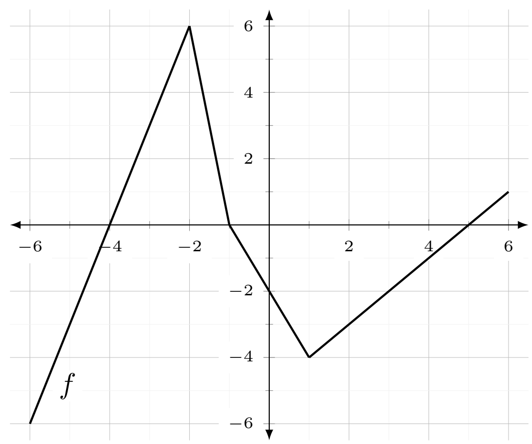
A. -8

B. -5

C. -4

D. -3

E. -2



Solution: The minimum occurs at a point where $g' = f$ changes from negative to positive ($x = -4$) or at an endpoint. By the Net Change Theorem, $g(b) = g(1) + \int_1^b g'(t)\,dt = 5 + \int_1^b f(t)\,dt$. We test:

$$g(-6) = 5 + 2 - 9 + 6 = 4$$
$$g(-4) = 5 + 2 - 9 = -2$$
$$g(2) = 5 - 3.5 = 1.5$$

(E) is correct.

7. (2 points) Find $\displaystyle\int \frac{dx}{\sqrt{-x^2+4x-3}}$

Solution:

$$\int \frac{dx}{\sqrt{-x^2+4x-3}} = \int \frac{dx}{\sqrt{-(x-2)^2+4-3}} = \int \frac{dx}{\sqrt{1-(x-2)^2}} = \arcsin(x-2)+C$$

8. (2 points) Find $\displaystyle\int (3x-1)\sin x\,dx$

Solution: Use

$$\begin{aligned} f &= 3x-1 \qquad & g' &= \sin x\,dx \\ f' &= 3\,dx & g &= -\cos x \end{aligned}$$

so that

$$\int (3x-1)\sin x\,dx = -(3x-1)\cos x - \int 3(-\cos x)\,dx = (1-3x)\cos x + 3\sin x + C$$