Name: ______________________ Mark: ______________________

Mini-math Div 3/4: Monday, April 12, 2021 (15 minutes)

1. A particle moves along the x-axis so that the acceleration at any time t is given by $a(t) = 2t$. At time $t = 0$, the velocity of the particle is $v(0) = -4$ and at time $t = 1$, the position is $s(1) = 20$.

   (a) (2 points) What is the velocity as a function of t?

   Solution: $v(t) = t^2 + C$, but $v(0) = -4$ so $v(t) = t^2 - 4$.

   (b) (2 points) How far does the particle move from $t = 0$ to $t = 2$?

   Solution:

$$\begin{aligned}\int_0^2 |v(t)|\,dt &= \int_0^2 (4 - t^2)\,dt \\ &= \left(4t - \frac{1}{3}t^3\right)\Big|_0^2 = \frac{16}{3}\end{aligned}$$

2. (2 points) Suppose that the graph of $y = f(x)$ satisfies $\dfrac{dy}{dx} = xy$ for all x and that $f(1) = 5$. Find an equation of the line tangent to the graph of y at $(1, 5)$.

Solution: The slope of the tangent is given by $1 \cdot 5 = 5$, so by point-slope, the line is given by

$$y - 5 = 5(x - 1)$$

3. (4 points) Find the general solution to the differential equation

$$\frac{dy}{dx} = x + xy$$

Solution: If $1 + y \neq 0$, then

$$\begin{aligned}
\frac{dy}{dx} &= x + xy = x(1+y) \\
\frac{1}{1+y}\frac{dy}{dx} &= x \\
\int \frac{1}{1+y}\, dy &= \int x\, dx \\
\ln|1+y| &= \frac{1}{2}x^2 + C \\
|1+y| &= Ae^{x^2/2} && \text{where } A > 0 \\
y + 1 &= Ae^{x^2/2} && \text{where } A \neq 0 \\
y &= Ae^{x^2/2} - 1 && \text{where } A \neq 0
\end{aligned}$$

Including the case $1 + y = 0$ which works, we get the general solution

$$y = Ae^{x^2/2} - 1$$

for $A \in \mathbb{R}$.