Name: ______________________ Mark: ______________________

Mini-math Div 3/4: Monday, January 11, 2020 (10 minutes)

1. Evaluate each of the following limits. You may use any of the three (generalized) Fundamental Trigonometric Limits without proof.

   (a) (2 points) $\lim_{x\to 0} \dfrac{\sin 4x}{\sin 3x}$

   Solution: By the (generalized) First Fundamental Trigonometric Limit

   $$\lim_{x\to 0} \frac{\sin 4x}{\sin 3x} = \lim_{x\to 0} \frac{\sin 4x}{4x} \cdot \frac{3x}{\sin 3x} \cdot \frac{4}{3} = \frac{4}{3}$$

   (b) (2 points) $\lim_{x\to 0} \dfrac{\cos 2x}{x + 2\cos x}$

   Solution: By continuity,

   $$\lim_{x\to 0} \frac{\cos 2x}{x + 2\cos x} = \frac{1}{0+2} = \frac{1}{2}$$

   (c) (2 points) $\lim_{x\to 0} \dfrac{\sin^2 x \cos x}{1 - \cos x}$

   Solution: By continuity,

   $$\begin{aligned}
   \lim_{x\to 0} \frac{\sin^2 x \cos x}{1 - \cos x} &= \lim_{x\to 0} \frac{\sin^2 x \cos x}{1 - \cos x} \cdot \frac{1 + \cos x}{1 + \cos x} \\
   &= \lim_{x\to 0} \frac{\sin^2 x \cos x (1 + \cos x)}{1 - \cos^2 x} \\
   &= \lim_{x\to 0} \frac{\sin^2 x \cos x (1 + \cos x)}{\sin^2 x} \\
   &= \lim_{x\to 0} \cos x (1 + \cos x) = 1(1+1) = 2
   \end{aligned}$$

2. (a) (1 point) Find the derivative of $\sin x$ from first principles using the Newton quotient (you may use any of the three (generalized) Fundamental Trigonometric Limits without proof).

Solution:

$$\begin{aligned}
\frac{d}{dx}(\sin x) &= \lim_{h\to 0}\frac{\sin(x+h)-\sin x}{h}\\
&= \lim_{h\to 0}\frac{\sin x\cos h+\sin h\cos x-\sin x}{h}\\
&= \lim_{h\to 0}\frac{\sin x(\cos h-1)+\sin h\cos x}{h}\\
&= \sin x\cdot\lim_{h\to 0}\frac{\cos h-1}{h}+\cos x\cdot\lim_{h\to 0}\frac{\sin h}{h}\\
&= \sin x\cdot 0+\cos x\cdot 1=\cos x
\end{aligned}$$

(b) (1 point) Find the derivative of $\tan x$ using derivative rules (you may use the derivatives of $\sin x$ and $\cos x$ without proof).

Solution:

$$\begin{aligned}
\frac{d}{dx}(\tan x) &= \frac{d}{dx}\left(\frac{\sin x}{\cos x}\right)\\
&= \frac{(\frac{d}{dx}\sin x)\cdot\cos x-(\frac{d}{dx}\cos x)\cdot\sin x}{(\cos x)^2}\\
&= \frac{\cos^2 x+\sin^2 x}{\cos^2 x}\\
&= \frac{1}{\cos^2 x}=\sec^2 x
\end{aligned}$$