PROBLEMS

Click here to submit problems proposals as well as solutions, comments and generalizations to any problem in this section.

To facilitate their consideration, solutions should be received by **September 30, 2025**.

5051. *Proposed by Giuseppe Fera.*

A climber starts at altitude 0 at time 0. Until he reaches the mountain top, every second he tosses a biased coin that gives heads with probability p such that $0 < p < 1/2$ and tails with probability $1 - p$. If the coin shows heads, the climber moves up one meter; otherwise, he either moves down one meter or he remains at altitude 0. The mountain top is at an altitude N meters, where N is a positive integer. Find the average climbing time to the top.

5052. *Proposed by Tatsunori Irie.*

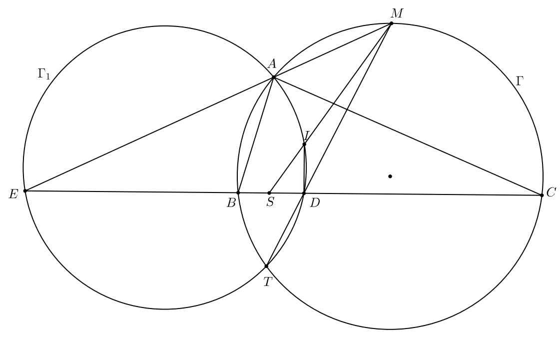
Let p be a prime number with $p \geq 3$ and let m be a natural number that is relatively prime to p and that is not congruent to 1 modulo p. Also, let n be an integer with $n \geq 2$. Define

$$N = \frac{(1+p)^{p^{m-1}} - 1}{p^{m-1}} + p(m-1).$$

Determine whether it is possible for N, when expressed in base n, to be a p-digit number consisting solely of the digit 1.

5053. *Proposed by Michel Bataille.*

Let triangle ABC with $AB \neq AC$ be inscribed in circle Γ and let D be the projection of its incenter I onto BC. Let M be the midpoint of the arc BC of Γ containing A and let the line MI intersect BC at S. Prove that the line AS, the line MD, Γ and the circumcircle of ΔAID have a common point.



5054. *Proposed by Eugen J. Ionascu, modified by the Editorial Board.*

Find the smallest possible number of 1's in the binary representation of a positive integer which is a multiple of 2025.

5055. *Proposed by Bing Jian.*

In triangle ABC, let AD be the altitude from vertex A to side BC. Let $DE \perp AB$ and $DF \perp AC$, with points E and F lying on AB and AC, respectively. Let points P and Q lie on the line AB and the line AC, respectively, such that $DP \parallel AC$ and $DQ \parallel AB$. Prove that the lines PQ, EF, and BC are concurrent or parallel.

5056. *Proposed by Mihaela Berindeanu.*

If $a_n = \frac{1}{1^3}+\frac{1}{2^3}+\cdots+\frac{1}{n^3}$ $\forall n \in \mathbb{N}^*$, show that $\frac{1}{a_1^2}+\frac{1}{8a_2^2}+\frac{1}{27a_3^2}+\cdots+\frac{1}{n^3a_n^2} < \frac{6}{5}$.

5057. *Proposed by Ovidiu Furdui and Alina Sîntămărian.*

Evaluate

$$\lim_{n\to\infty} n(-1)^n \sum_{k=1}^{n} \frac{(-1)^k}{k(n-k)!}.$$

5058. *Proposed by Vasile Cîrtoaje.*

Let $a_1, a_2, \ldots, a_n$ be positive real numbers such that at most one of them is less than 1 and $a_1^3 + a_2^3 + \cdots + a_n^3 = n$. Prove that

$$a_1^2 + a_2^2 + \cdots + a_n^2 \geq a_1 + a_2 + \cdots + a_n.$$

5059. *Proposed by Tatsunori Irie, modified by the Editorial Board.*

Let T be the Fermat-Torricelli point of triangle ABC with angles less than 120 degrees, that is T is the point such that the sum of the three distances from each of the three vertices of the triangle to the point is the smallest possible. Prove that Area(ABC) is less than the area of an equilateral triangle with sides equal to $TA + TB + TC$.

5060. *Proposed by Nguyen Van Huyen.*

Find the smallest constant k such that the inequality

$$2(a+b+c+abc) + k[(ab-1)^2 + (bc-1)^2 + (ca-1)^2] \geqslant (a+1)(b+1)(c+1),$$

holds for all non-negative real numbers $a,\ b,\ c.$

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