Minimum Arc Length Challenge

Give an example of a continuous function f that satisfies three conditions:

(1) $f(x) \geq 0$ on the interval $0 \leq x \leq 1$;

(2) $f(0) = 0$ and $f(1) = 0$;

(3) $\displaystyle\int_0^1 f(x)\,dx = 1$

Compute the arc length, L, for the function f (on $[0, 1]$). The goal is to minimize L given the three conditions above.

You may use `https://www.wolframalpha.com/` to compute your integrals and arc length to speed things up. Check the following links for examples for syntax:

`https://www.wolframalpha.com/input/?i=integral+of+%28-12%2F5%29%28x%5E3%2Bx%5E2-2x%29+from+x+%3D+0+to+1`

`https://www.wolframalpha.com/input/?i=arclength+of+%28%28-12%2F5%29%28x%5E3%2Bx%5E2-2x%29%29+from+0+to+1`

The above example of $f(x) = \left(-\dfrac{12}{5}\right)(x^3 + x^2 - 2x)$ has an arc length of $L \approx 3.27402\ldots$. You must do better!