2021–2022 Winter Break Math Challenges

Challenge 1: Digit puzzle

To ring in the new year, make the number 2022 in 9 different ways, each time by using copies of the same digit and the following operations (in addition to parentheses):

- Standard operations: $+, -, \times, \div$
- Negation: $-\square$
- Exponentiation of two numbers: $\square^{\square}$
- Square root of a number: $\sqrt{\square}$
- Factorial: $\square!$ (Note: you may use iterated factorial but not muilti-factorial, so that $3!! = (3!)! = 6! = 720$, and **not** $3!! = 3 \times 1 = 3$.)
- Concatenation (i.e. "glueing") of digits (only of the original digit used): dd

Your score for a particular digit is the number of copies you use, and your goal is to have the lowest score possible.

For example, you can make 2022 by using copies of the digit "9" as follows:

$$2022 = \underbrace{\frac{9}{9} + \frac{9}{9} + \cdots + \frac{9}{9}}_{\text{2022 times}}.$$

If you do it like this, you are using 4044 copies of 9, which is not good for you. A far more efficient way to do it is

$$2022 = 99 \times 9 \times 9 - 9 \times 9 \times 9 \times 9 + 99 \times 9 - 99 \times \sqrt{9} - 9 \times \sqrt{9} - \sqrt{9}$$

which gets you there with only 17 copies (this is, of course, not optimal).

(1) Using the digit 1

(2) Using the digit 2

(3) Using the digit 3

(4) Using the digit 4

(5) Using the digit 5

(6) Using the digit 6

(7) Using the digit 7

(8) Using the digit 8

(9) Using the digit 9

Challenge 2: Digit puzzle 2

For this puzzle, you must make the number 2022 using the group of digits 2, 0, 2, 2. You may use as many groups as you wish, but your goal is to use as few as possible (the digits do not need to be in order). You may use the following operations (in addition to parentheses):

- Standard operations: $+, -, \times, \div$
- Negation: $-\square$
- Exponentiation of two numbers: $\square^{\square}$
- Square root of a number: $\sqrt{\square}$
- Factorial: $\square!$
  - Note: you may use iterated factorial but not multi-factorial, so that $3!! = (3!)! = 6! = 720$, and **not** $3!! = 3 \times 1 = 3$.
  - Hint: $0! = 1$

Notice you CANNOT use "glueing"!

For example, you can do it as follows:

$$2022 = \underbrace{\left(\frac{2}{2} + 2 + 0\right) + \left(\frac{2}{2} + 2 + 0\right) + \cdots + \left(\frac{2}{2} + 2 + 0\right)}_{\text{674 times}}$$

If you do it like this, you are using 674 groups of 2, 0, 2, 2, which is not good for you. A far more efficient way to do it is

$$2022 = (2+2)^{2+0!} \times (2+2+0!)^2 + (2+2)^2 \times (2+2+0!)^2 + (2+2+0!) \times 2 \times 2 + 2 + 2 \times 0 + 0$$

which gets you there with only 6 groups (this is, of course, not optimal).

Challenge 3: Approximations

How close can you get to 2022 using the digits $2, 0, 2, 2$ exactly once? You may use the following operations (in addition to parentheses):

- Standard operations: $+, -, \times, \div$
- Negation: $-\square$
- Exponentiation of two numbers: $\square^{\square}$
- Square root of a number: $\sqrt{\square}$
- Factorial: $\square!$
  - Note: you may use iterated factorial but not multi-factorial, so that $3!! = (3!)! = 6! = 720$, and **not** $3!! = 3 \times 1 = 3$.
  - Hint: $0! = 1$
- Floor: $\lfloor \square \rfloor$ (this is the largest integer less than or equal to the input, so $\lfloor \pi \rfloor = 3$)

Notice you CANNOT use "glueing"!