Name: _____ Mark: _____

Mini-math Div 3/4: Monday, March 8, 2021 (15 minutes)

1. Find the following general indefinite integrals.

   (a) (2 points) $\displaystyle\int \frac{u+3}{u^2+6u+8}\,du$

Solution: Let $v = u^2 + 6u + 8$, so that $dv = 2u + 6\,du$.

$$\begin{aligned}\int \frac{u+3}{u^2+6u+8}\,du &= \int \frac{1/2}{v}\,dv = \frac{1}{2}\ln|v| + C \\ &= \frac{1}{2}\ln|u^2+6u+8| + C\end{aligned}$$

   (b) (2 points) $\displaystyle\int \frac{e^{\tan\theta}}{\cos^2\theta}\,d\theta$

Solution: Let $u = \tan\theta$, so that $du = \sec^2\theta\,d\theta$.

$$\begin{aligned}\int \frac{e^{\tan\theta}}{\cos^2\theta}\,d\theta &= \int e^{\tan\theta}\sec^2\theta\,d\theta \\ &= \int e^u\,du = e^u + C \\ &= e^{\tan\theta} + C\end{aligned}$$

2. Evaluate the following definite integrals.

(a) (2 points) $\displaystyle\int_{-1}^{0} x(2x+1)^6\,dx$

Solution: Let $u = 2x+1$, so that $du = 2\,dx$, $-1 \mapsto -1$, and $0 \mapsto 1$. Notice that $x = \frac{1}{2}(u-1)$. By symmetry,

$$\begin{aligned}\int_{-1}^{0} x(2x+1)^6\,dx &= \int_{-1}^{1} \frac{u-1}{2}u^6 \cdot \frac{1}{2}\,du \\ &= \frac{1}{4}\int_{-1}^{1}(u^7 - u^6)\,du \\ &= \frac{1}{4}\left(0 - 2\int_{0}^{1} u^6\,du\right) \\ &= -\frac{1}{2}\cdot\frac{1}{7}u^7\Big|_0^1 \\ &= -\frac{1}{14}\end{aligned}$$

(b) (2 points) $\displaystyle\int_{e^2}^{e^3} \frac{1}{t(\ln t)^2}\,dt$

Solution: Let $u = \ln t$, so that $du = \frac{1}{t}\,dt$, $e^2 \mapsto 2$, and $e^3 \mapsto 3$.

$$\begin{aligned}\int_{e^2}^{e^3} \frac{1}{t(\ln t)^2}\,dt &= \int_2^3 \frac{1}{u^2}\,du \\ &= -\frac{1}{u}\Big|_2^3 \\ &= \frac{1}{2} - \frac{1}{3} \\ &= \frac{1}{6}\end{aligned}$$