10.1 Defining Convergent and Divergent Infinite Series

Calculus Name: ______________________________ **CA #1**

1. **Calculator active.** Given the infinite series: $1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \cdots$, find the sequence of partial sums $S_1, S_2, S_3, S_4,$ and S_5.

2. Find the nth partial sum for the infinite series $\sum_{n=1}^{\infty} \frac{1}{5^n}$.

3. The infinite series $\sum_{n=1}^{\infty} \frac{3}{4^{n+1}}$ has nth partial sum $S_n = \frac{1}{4} - \frac{1}{4^{n+1}}$. What is the sum of the series?

4. If the infinite series $\sum_{n=1}^{\infty} a^n$ has nth partial sum $S_n = \frac{4}{3}(4^n - 1)$ for $n \geq 1$. What is the sum of the series?

5. Does the series $\sum_{n=1}^{\infty} \left(\frac{1}{2n-1} - \frac{1}{2n+1}\right)$ converge or diverge? If it converges find its sum.

1. 1, 1.25, 1.3611, 1.4236, 1.4636	2. $S_n = \frac{1}{4}\left(1 - \frac{1}{5^n}\right)$	3. $\frac{1}{4}$	4. Diverges	5. Converges, sum = 1

Answers to 10.1 CA #1

10.2 Working with Geometric Series

Calculus

Name: ______________________________

CA #1

1. What is the sum of the infinite geometric series $11 + -\frac{11}{3} + \frac{11}{9} + -\frac{11}{27} + \cdots$?

2. What is the value of $\sum_{n=1}^{\infty} \frac{(-e)^{n+1}}{9^n}$?

3. Consider the series $\sum_{n=1}^{\infty} a_n$. If $\frac{a_{n+1}}{a_n} = \frac{1}{5}$ for all integers $n \geq 1$, and $\sum_{n=1}^{\infty} a_n = 50$, then $a_1 =$

4. **Calculator active.** If $f(x) = \sum_{n=1}^{\infty} \left(\cos^2 \frac{x}{2}\right)^n$, then $f(2.4) =$

5. For what value of a does the infinite series $\sum_{n=0}^{\infty} a\left(-\frac{3}{5}\right)^n$ equal 15?

1. $\frac{33}{4}$	2. $\frac{e^2}{9+e}$	3. 40	4. 0.1511	5. 24

Answers to 10.2 CA #1

10.3 The nth Term Test for Divergence

Calculus Name: ______________________________ **CA #1**

1. The nth-Term Test can be used to determine divergence for which of the following series?

   I. $\displaystyle\sum_{n=1}^{\infty} \frac{(n+1)^3}{3n^3 - 2n + 1}$ II. $\displaystyle\sum_{n=1}^{\infty} \frac{(n+1)^2}{2n^2 - 3n^3 + 1}$ III. $\displaystyle\sum_{n=1}^{\infty} \ln\frac{1}{n}$

   A. III only

   B. I and III only

   C. II and III only

   D. I, II, and III

Use the nth-Term Test for Divergence to determine if the series diverges.

2. $\displaystyle\sum_{n=0}^{\infty} \frac{\pi^{n+1}}{7^n}$	3. $\displaystyle\sum_{n=1}^{\infty} \frac{2(n-2)^2}{3(n+4)^2}$	4. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{e^n}$

5. Verify that the infinite series $\displaystyle\sum_{n=1}^{\infty} \frac{6^n + 1}{6^{n+1}}$ diverges by using the nth-Term Test for Divergence. Show the value of the limit.

1. B	2. Converges, Geometric Series, $r = \frac{\pi}{7}$	3. Diverges by nth-Term Test, $\lim\limits_{n\to\infty} a_n = \frac{2}{3}$	4. Converges, Geometric Series, $r = \frac{1}{e}$	5. Diverges by nth-Term Test, $\lim\limits_{n\to\infty} a_n = \frac{1}{6}$

Answers to 10.3 CA #1

10.4 Integral Test for Convergence

Calculus Name: ______________________________ **CA #1**

1. Use the Integral Test to determine the convergence or divergence of the series $\sum_{n=1}^{\infty} \frac{1}{n^5}$.

2. Confirm the Integral Test can be applied to the series $\frac{3}{2} + \frac{3}{5} + \frac{3}{10} + \cdots$ and use the Integral Test to determine the convergence or divergence of the series.

3. Explain why the Integral Test does not apply to the series $\sum_{n=1}^{\infty} \frac{1}{e^{-n}}$.

4. Prove the Integral Test applies to the series $\sum_{n=1}^{\infty} \frac{1}{(n+1)^3}$. Determine the convergence or divergence of the series.

5. Use the Integral Test to determine if the series $\sum_{n=1}^{\infty} \frac{4n}{2n^2+1}$ converges or diverges.

Answers to 10.4 CA #1

1. $\int_1^{\infty} f(x)\,dx = \frac{1}{4}$, Series Converges	2. $\int_1^{\infty} f(x)\,dx = \frac{3\pi}{4}$, Series Converges	3. $f(x)$ is not a decreasing function for $x \geq 1$.
4. $\int_1^{\infty} f(x)\,dx = \frac{1}{8}$, Series Converges	5. $\int_1^{\infty} f(x)\,dx = \infty$, Series Diverges	

10.5 Harmonic and p-series

Calculus — Name: ______________________________ **CA #1**

1. Determine the convergence or divergence of the p-series $\sum_{n=1}^{\infty} n^{-2}$.

2. For what values of p will the infinite series $\sum_{n=1}^{\infty} \frac{1}{n^{1-p}}$ converge?

3. For what values of p will both infinite series $\sum_{n=1}^{\infty} \left(\frac{3}{p}\right)^n$ and $\sum_{n=1}^{\infty} \frac{1}{n^{5-p}}$ converge?

4. What are all values of p for which $\int_1^{\infty} x^{-(3p-2)}\,dx$ converges?

5. Which of the following is a divergent p-series?

A. $\sum_{n=1}^{\infty} n^{-\pi}$ B. $\sum_{n=1}^{\infty} \frac{1}{n}$ C. $\sum_{n=1}^{\infty} \left(\frac{e}{\pi}\right)^n$ D. $\sum_{n=1}^{\infty} \frac{1}{n^3}$

1. $p = 2 > 1$, convergent p-series	2. $p < 0$	3. $3 < p < 4$	4. $p > 1$	5. B

Answers to 10.5 CA #1

10.6 Comparison Tests for Convergence

Calculus Name: ______________________________ **CA #1**

1. Which of the following series converges?

(A) $\sum_{n=1}^{\infty} \frac{3n}{2n^2+1}$ (B) $\sum_{n=1}^{\infty} \frac{3n^2}{n+2n^2}$

(C) $\sum_{n=1}^{\infty} \left(\frac{\pi}{e}\right)^n$ (D) $\sum_{n=1}^{\infty} \frac{3n^2}{2n^3+3n}$ (E) $\sum_{n=1}^{\infty} \frac{n-1}{n4^n}$

2. Which of the following series can be used with the Limit Comparison Test to determine whether the series $\sum_{n=1}^{\infty} \frac{5^n}{7^n-n^2}$ diverges or converges?

(A) $\sum_{n=1}^{\infty} \frac{1}{n}$ (B) $\sum_{n=1}^{\infty} \frac{1}{5^n}$

(C) $\sum_{n=1}^{\infty} \frac{1}{7^n}$ (D) $\sum_{n=1}^{\infty} \left(\frac{5}{7}\right)^n$

3. Use the Comparison Test to determine the convergence or divergence of the series $\sum_{n=1}^{\infty} \frac{n-2}{n5^n}$. You must identify the series you are using for comparison.

4. Use the Comparison Test to determine the convergence or divergence of the series $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$. You must identify the series you are using for comparison.

5. Determine whether the series $\sum_{n=1}^{\infty} \frac{n5^n}{4n^4-3}$ converges or diverges. Identify the test for convergence used.

Answers to 10.6 CA #1

1. E	2. D	3. Converges by comparison to $\sum_{n=1}^{\infty} \frac{1}{5^n}$, a convergent geometric series.	4. Diverges by limit comparison with $\sum_{n=1}^{\infty} \frac{1}{n}$, a divergent harmonic series.	5. Diverges by limit comparison with $\sum_{n=1}^{\infty} \frac{5^n}{4n^3}$, which diverges by nth Term test.

10.7 Alternating Series Test

Calculus Name: ______________________________ **CA #1**

1. Explain why the Alternating Series Test does not apply to the series $\sum_{n=1}^{\infty} \frac{(-1)^n \cos(n\pi)}{n^2}$.

2. Determine the convergence or divergence of the series $\sum_{n=1}^{\infty} \frac{(-1)^n n}{\ln(n+1)}$.

3. Which of the following series converge?

   I. $\sum_{n=1}^{\infty} \frac{(-1)^n}{4^n}$ II. $\sum_{n=1}^{\infty} \frac{(-1)^n}{\pi^n}$ III. $\sum_{n=1}^{\infty} \frac{(-1)^n n^2}{1+n^2}$

   A. I only B. I and II only C. I and III only D. I, II, and III

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4. Which of the following statements are true about the series $\sum_{n=1}^{\infty} \frac{(-1)^n n!}{(2n-1)!}$?

   I. The series is alternating.

   II. $|a_{n+1}| \leq |a_n|$ for $n \geq 1$.

   III. $\lim_{n\to\infty} a_n = 0$

   A. I only B. I and II only C. I and III only D. I, II, and III

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5. Which of the following statements is true?

A. $\sum_{n=1}^{\infty} \frac{(-1)^n(1-n)}{n}$ converges by the Alternating Series Test.

B. $\sum_{n=1}^{\infty} \frac{(-1)^n(n+1)}{2n}$ converges by the Alternating Series Test.

C. $\sum_{n=1}^{\infty} \frac{(-1)^n n^2}{4\sqrt{n}}$ converges by the Alternating Series Test.

D. $\sum_{n=1}^{\infty} \frac{(-1)^n 2\sqrt{n}}{n}$ converges by the Alternating Series Test.

Answers to 10.7 CA #1

1. The Alternating Series Test does not apply because the series is not alternating.	2. The series diverges by the nth Term Test.	3. B	4. D	5. D

10.8 Ration Test

Calculus Name: ______________________________ **CA #1**

1. Use the Ratio Test to determine the convergence or divergence of the series $\sum_{n=1}^{\infty} \frac{n^4}{3^n}$.

2. If the Ratio Test is applied to the series $\sum_{n=0}^{\infty} \frac{6^n}{(n+1)^n}$, which of the following inequalities results, implying that the series converges?

A. $\lim_{n\to\infty} \frac{6^n}{(n+1)^n} < 1$ B. $\lim_{n\to\infty} \frac{6(n+1)^n}{(n+2)^{n+1}} < 1$ C. $\lim_{n\to\infty} \frac{6^{n+1}}{(n+1)^n} < 1$ D. $\lim_{n\to\infty} \frac{6^{n+1}}{(n+1)^{n+1}} < 1$

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3. If $a_n > 0$ for all n and $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = 5$, which of the following series converges?

A. $\sum_{n=1}^{\infty} \frac{a_n}{n^2}$ B. $\sum_{n=1}^{\infty} \frac{a_n}{2^n}$ C. $\sum_{n=1}^{\infty} \frac{a_n}{n^5}$ D. $\sum_{n=1}^{\infty} \frac{a_n}{7^n}$

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4. What are all values of $x > 0$ for which the series $\sum_{n=1}^{\infty} \frac{6n^3}{x^n}$ converges?

5. Which of the following series converge?

I. $\sum_{n=1}^{\infty} \frac{1}{n!}$ II. $\sum_{n=1}^{\infty} \frac{9^n}{n^5}$ III. $\sum_{n=1}^{\infty} \frac{5n}{2n-1}$

A. I only B. I and II only C. I and III only D. I, II, and III

1. Converges by Ratio Test	2. B	3. D	4. $x > 1$	5. A

Answers to 10.8 CA #1

10.9 Absolute or Conditional Convergence

Calculus Name: ______________________________ **CA #1**

1. For what values of x is the series $\sum_{n=0}^{\infty}(-1)^n(5x+1)^n$ absolutely convergent?

2. For what values of x is the series $\sum_{n=1}^{\infty}\frac{(5x-2)^n}{n}$ conditionally convergent?

A. $x > \frac{3}{5}$ B. $x = \frac{3}{5}$ C. $x = \frac{1}{5}$ D. $x < \frac{1}{5}$

3. Which of the following statements is true about the series $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^2\sqrt{n}}$.

A. The series converges conditionally.

B. The series converges absolutely.

C. The series converges but neither conditionally nor absolutely.

D. The series diverges.

4. Determine whether the series $\sum_{n=1}^{\infty}\frac{(-1)^n}{n+5}$ converges absolutely, converges conditionally, or diverges.

5. Determine whether the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(n+1)!}$ converges absolutely, converges conditionally, or diverges.

Answers to 10.9 CA #1

1. $-\frac{2}{5} < x < 0$	2. C	3. B	4. Converges Conditionally	5. Converges Absolutely

10.10 Alternating Series Error Bound

Calculus Name: ______________________________ **CA #1**

1. If the series $\sum_{n=1}^{\infty}(-1)^n \frac{1}{2n+1}$ is approximated by the partial sum with 50 terms, what is the alternating series error bound?

2. Approximate and interval for the sum of the convergent alternating series $\sum_{n=1}^{\infty}\frac{(-1)^n 2}{n^2}$ using the Alternating Series Error Bound the first 6 terms.

3. The series $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{\sqrt{n}}$ converges to S. Based on the alternating series error bound, what is the least number of terms to guarantee a partial sum that is within 0.02 of S?

4. If the series $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{5}{n}$ is approximated by $S_k = \sum_{n=1}^{k}(-1)^{n+1}\frac{5}{n}$, what is the least value of k for which the alternating series error bound guarantees that $|S - S_k| < 0.001$?

(A) 999 (B) 1000 (C) 4999 (D) 5000

5. Determine the least number of terms necessary to approximate the sum of the series $\sum_{n=1}^{\infty}\frac{(-1)^n 3}{4^n}$ with an error less than 10^{-3}.

Answers to 10.10 CA #1

1. $\frac{1}{103}$	2. $-1.656 \le S \le -1.607$	3. 2500	4. D	5. 5

10.12 Lagrange Error Bound

Calculus Name: ______________________________ **CA #1**

1. The fourth-degree Maclaurin polynomial for $\cos x$ is given by $1 - \frac{x^2}{2!} + \frac{x^4}{4!}$. Use the Lagrange error bound to estimate the error in using this polynomial to approximate $\cos \frac{\pi}{3}$.

2. The function f has derivatives of all orders for all real numbers and $f^{(4)}(x) = e^{\sin x}$. If the third-degree Taylor Polynomial for f about $x = 0$ is used to approximate f on $[0,1]$, what is the Lagrange error bound for the maximum error on $[0,1]$?

3. Assume a third-degree Taylor Polynomial about $x = 2$ is used for the approximation f and $\left|f^{(4)}(x)\right| \leq 12$ for all $x \geq 1$. Which of the following represents the Lagrange error bound in the approximation of $f(2.5)$?

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{1}{16}$ (D) $\frac{1}{32}$

4. Determine the degree of the Taylor Polynomial about $x = 0$ for $f(x) = e^x$ required for the error in the approximation of $f(0.8)$ to be less than 0.005.

5.

x	$f(x)$	$f'(x)$	$f''(x)$	$f'''(x)$	$f^{(4)}(x)$
2	112	164	214	312	345

Let f be a function having derivatives of all orders for $x > 0$. Selected values for the first four derivatives of f are given for $x = 2$. Use the Lagrange error bound to show that the third-degree Taylor Polynomial for f about $x = 2$ approximates $f(1.9)$ with an error less than 0.002.

Answers to 10.12 CA #1

1. 0.0105	2. 0.0967	3. D	4. $n = 5$	5. $R_3 = 0.0014375 < 0.002$

10.13 Radius and Interval of Convergence

Calculus Name: ______________________________ **CA #1**

Find the interval of convergence for each power series.

1. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n (x+4)^n}{n}$

2. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n\, n!\, (x-4)^n}{3^n}$

3. What is the radius of convergence of the power series $\displaystyle\sum_{n=0}^{\infty} \frac{(x+2)^n}{2^n}$?

4. What is the interval of convergence for the power series $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n+1}(-kx)^{n-1}$, where k is a positive integer?

5. If the power series $\displaystyle\sum_{n=0}^{\infty} a_n (x-4)^n$ converges at $x = 7$ and diverges at $x = 8$, which of the following must be true?

   I. The series converges at $x = 1$.

   II. The series converges at $x = 2$.

   III. The series diverges at $x = 0$.

(A) I only (B) II only (C) I and II only (D) II and III only

Answers to 10.13 CA #1

1. $-5 < x \leq -3$	2. $x = 4$	3. 2	4. $-\frac{1}{k} < x < \frac{1}{k}$	5. B

10.14 Finding Taylor or Maclaurin Series

Calculus Name: ______________________________ **CA #1**

1. What is the coefficient of x^6 in the Taylor Series about $x = 0$ for the function $f(x) = \frac{e^{3x^2}}{4}$?

2. Write the first four non-zero terms for the Taylor Series for the function $f(x) = 2x\cos x$ about $x = 0$?

3. What is the sum of the series $1 - \frac{3^2}{2!} + \frac{3^4}{4!} - \frac{3^6}{6!} + \cdots + \frac{(-1)^n 3^{2n}}{(2n)!}$?

(A) $\ln 3$ (B) e^3 (C) $\sin 3$ (D) $\cos 3$

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4. Write the first four non-zero terms in the Maclaurin Series for the function $f(x) = x\sin 2x$.

5. Which of the following is the Maclaurin Series for the function f defined by $f(x) = 1 + x^2 + \cos x$?

(A) $2 + \frac{x^2}{2} + \frac{x^4}{24} + \cdots$ (B) $2 + \frac{3x^2}{2} + \frac{x^4}{24} + \cdots$ (C) $1 + x + x^2 - \frac{x^3}{6} + \cdots$ (D) $2 + x + \frac{3x^2}{2} + \frac{x^3}{6} + \cdots$

1. $\frac{9}{8}$	2. $2x - x^3 + \frac{x^5}{12} - \frac{x^7}{360}$	3. D	4. $2x^2 - \frac{2^3x^4}{3!} + \frac{2^5x^6}{5!} - \frac{2^7x^8}{7!}$	5. A

Answers to 10.14 CA #1

10.15 Representing Functions as Power Series

Calculus Name: ______________________________ **CA #1**

1. What is the coefficient of x^5 in the Taylor series for the function $f(x) = e^x \sin x$ about $x = 0$?

2. If the function f is defined by $f(x) = \sum_{n=0}^{\infty} \frac{x^{2n}}{n!}$, then $f'(x) = ?$ Write the first four nonzero terms and the general term of the Taylor series about $x = 0$.

3. Let f be the function defined by $f(x) = e^{3x}$. Find the Maclaurin series for the derivative f'. Write the first four nonzero terms and the general term.

4. Find the third-degree Taylor Polynomial for $f(x) = \sin x \cos x$ about $x = 0$.

5. If $f'(x) = \frac{4}{1+x}$ and $f(0) = 0$, write the first four nonzero terms and the general term of the Maclaurin series for $f(x)$.

Answers to 10.15 CA #1

1. $-\frac{1}{30}$	2. $f'(x) = 2x + 2x^3 + x^5 + \frac{x^7}{3} + \cdots + \frac{2nx^{2n-1}}{n!}$	3. $f'(x) = 3 + 9x + \frac{27x^2}{2} + \frac{27x^3}{2} + \cdots + \frac{3n(3x)^{n-1}}{n!}$
4. $T = x - \frac{2}{3}x^3$	5. $f(x) = 4x - 2x^2 + \frac{4}{3}x^3 - x^4 + \cdots + \frac{(-1)^n 4x^{n+1}}{n+1}$	