

Optimization problems

0.5 mark for variables

1 mark restriction function(s)

1 mark optimization function

1 mark reducing optimizing function to single variable

1 mark for derivative of optimizing function

0.5 mark for checking CP of form f' DNE

1 mark for checking CP of form $f' = 0$

1 mark for classification using one of the three methods (if applicable)

1 mark for solution (eg other dimensions, time, etc)

1. (8 points) Find the dimensions of the rectangle of maximum area with sides parallel to the axes that can be inscribed in the ellipse $4x^2 + y^2 = 16$. Give your answer as an exact value — no approximations.

2. (8 points) Alice is standing at the bank of a river which is 85 m wide. She sees Bob 250 m downstream, on the opposite shore, lying in distress. She can swim at 2 m/s and can run at 8 m/s, and plans to swim to the opposite shore at some point, then run the remaining distance to Bob. At what point should she swim to on the opposite shore in order to minimize the time it takes to reach Bob? (Give your answer in m, to 3 decimal places.)

3. (8 points) We want to construct a cylindrical can. Due to the manufacturing process, the side of the can costs 0.6 cents per cm^2 to produce and the top and bottom costs 1.3 cents per cm^2 to produce. If the can must have a volume of 540 cm^3 , determine the dimensions of the can (to the nearest hundredth of a centimetre) that will minimize the cost of the can.