

Binary prime catacomb

Imagine a catacomb in which each room is identified with a prime number. There is a path from A to B if you can change exactly 1 digit in the binary representation of A to obtain the binary representation of B , including the first leading 0. For instance, $2 = 10_2$ is connected to $3 = 11_2$ and vice versa, since you can change the last digit. We can also change the first 0 in $3 = 011_2$ to get $7 = 111_2$, or change the leading 1 to go back. Notice $19 = 10011_2$ can change to $3 = 11_2$ by changing the leading 1, but there is no direct path from 3 to 19.

- (1) Are there any numbers which can be reached starting from 2, but cannot return to 2?
- (2) Can 11 be reached starting from 2?
- (3) Does the catacomb reach infinitely many numbers, starting from 2?