Mini-math AP Calculus BC: Friday, March 25, 2022 (8 minutes)

SOLUTIONS

1. (2 points) Write down (but do not evaluate) an integral which represents the area inside $r = 2 - \cos\theta$ for $0 \le \theta \le \pi$.

Solution:

$$A = \int_0^\pi \frac{1}{2} r^2 \, d\theta = \int_0^\pi \frac{1}{2}(2 - \cos\theta)^2 \, d\theta \quad \left(= \frac{9\pi}{4}\right)$$

2. (2 points) Write down (but do not evaluate) an integral which represents the area inside $r_1 = 2\sin\theta$ and outside $r_2 = 2\sqrt{3} - 2\sin\theta$.

Solution: Solving $2\sin\theta = 2\sqrt{3} - 2\sin\theta$, we get $\sin\theta = \sqrt{3}/2$, so $\theta = \pi/3, 2\pi/3$. On $[\pi/3, 2\pi/3]$, $r_1 \ge r_2$.

$$A = \int_{\pi/3}^{2\pi/3} \frac{1}{2}(r_{outter}^2 - r_{inner}^2) \, d\theta = \frac{1}{2} \int_{\pi/3}^{2\pi/3} [(2\sin\theta)^2 - (2\sqrt{3} - 2\sin\theta)^2] \, d\theta \quad \left(= 4\sqrt{3} - 2\pi\right)$$

3. (2 points) Write down (but do not evaluate) an integral which represents the area outside $r_1 = 2\sin\theta$ and inside $r_2 = 2\sqrt{3} - 2\sin\theta$.

Solution: We want the portion where $r_2 \ge r_1$, so we need to be outside $[\pi/3, 2\pi/3]$. Although we could use two separate intervals $[0, \pi/3]$ and $[2\pi/3, 2\pi]$, we can make use of periodicity and instead just use $[2\pi/3, 7\pi/3]$.

$$A = \int_{2\pi/3}^{7\pi/3} \frac{1}{2}(r_{outter}^2 - r_{inner}^2) \, d\theta = \frac{1}{2} \int_{\pi/3}^{2\pi/3} [(2\sqrt{3} - 2\sin\theta)^2 - (2\sin\theta)^2] \, d\theta \quad \left(= 8\sqrt{3} + 10\pi\right)$$