Mini-math Div 3/4: Thursday, September 29, 2022 (10 minutes)

SOLUTIONS

1. (1 point) Suppose $\int_{-2}^{5} (2f(x)+3)\,dx = 15$, and $\int_{3}^{5} f(x)\,dx = 10$. What is $\int_{-2}^{3} f(x)\,dx$?

A. -13 B. -4 C. 5 D. 7

Solution:

$$15 = 2\int_{-2}^{5} f(x)\,dx + \int_{-2}^{5} 3\,dx = 2\int_{-2}^{5} f(x)\,dx + 21 \quad \Rightarrow \quad \int_{-2}^{5} f(x)\,dx = \frac{15-21}{2} = -3,$$
$$\Rightarrow \int_{-2}^{3} f(x)\,dx = \int_{-2}^{5} f(x)\,dx - \int_{3}^{5} f(x)\,dx = -3 - 10 = -13$$

(a) is correct.

2. (1 point) Evaluate $\int_{1}^{4} \frac{x+4}{\sqrt{x}}\,dx$.

A. $-\frac{9}{4}$ B. 7 C. 11 D. $\frac{38}{3}$

Solution: Splitting up the integral,

$$\int_{1}^{4} \frac{x+4}{\sqrt{x}}\,dx = \int_{1}^{4} (x^{1/2}+4x^{-1/2})\,dx = \left(\frac{2}{3}x^{3/2} + 8x^{1/2}\right)\Big|_{1}^{4} = \frac{2}{3}(4^{3/2}-1^{3/2}) + 8(4^{1/2}-1^{1/2})$$
$$= \frac{2}{3}(2^3-1) + 8(2-1) = \frac{2}{3}\cdot 7 + 8 = \frac{14+24}{7} = \frac{38}{3}$$

(d) is correct.

3. (1 point) Evaluate $\int_{-1}^{1} x(x+1)^2\,dx$.

A. 0 B. $\frac{2}{3}$ C. $\frac{4}{3}$ D. 4

Solution:

$$\int_{-1}^{1} x(x+1)^2\,dx = \int_{-1}^{1} (x^3+2x^2+x)\,dx = \left(\frac{1}{4}x^4 + \frac{2}{3}x^3 + \frac{1}{2}x^2\right)\Big|_{-1}^{1}$$
$$= \frac{1}{4}(1^4-(-1)^4) + \frac{2}{3}(1^3-(-1)^3) + \frac{1}{2}(1^2-(-1)^2) = \frac{4}{3}$$

(c) is correct.

4. (1 point) Suppose $\int_1^5 f'(x)\,dx = 12$ and $f(5) = 3$. What is $f(1)$?

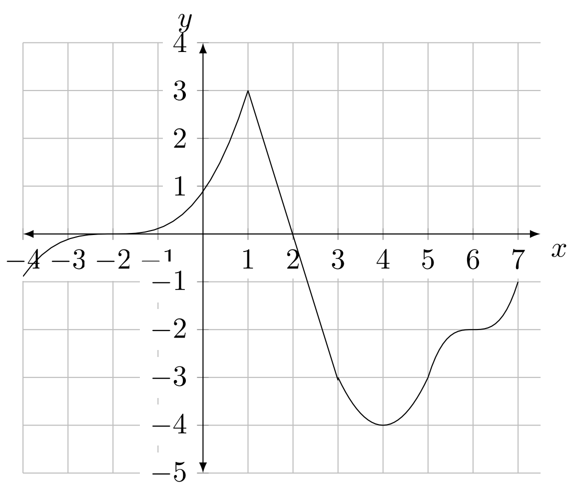
A. -15 B. -9 C. 9 D. 15

Solution: By FTC II,

$$12 = \int_1^5 f'(x)\,dx = f(x)\Big|_1^5 = f(5) - f(1) = 3 - f(1)$$
$$f(1) = 3 - 12 = -9$$

(b) is correct.

5. (1 point) The graph of f is below. Let $g(x) = \int_1^x f(t)\,dt$. At what value(s) of x in the interval $[-4, 7]$ does g have a point of inflection?



A. exactly one of -2 and 2

B. both -2 and 2

C. both 1 and 4

D. all of $-2, 5$ and 6

Solution: $g'(x) = f(x)$, so $g''(x) = f'(x)$. For a point of inflection, we need $g''(x) = f'(x)$ to change sign, so (c) is correct.