Fractions

Start with the number 1. Choose any natural number $n \geq 2$.

e.g. Choose n=2.

Step 1: Add 1/n.

e.g.

$$1 \stackrel{+}{\to} 1 + \frac{1}{2} = \frac{3}{2}$$

For following steps, you can choose to either add 1/n, or take the reciprocal of your current value.

e.g. From $\frac{3}{2}$, you can either add $\frac{1}{2}$ to get

$$\frac{3}{2} \stackrel{+}{\to} \frac{3}{2} + \frac{1}{2} = 2$$

or take the reciprocal to get

$$\frac{3}{2} \xrightarrow{r} \frac{2}{3}$$

Your goal: get back to 1 in the fewest number of steps. For a chosen number of n, we define a(n) to be the fewest number of steps to return to 1.

e.g.

$$1 \stackrel{+}{\rightarrow} \frac{3}{2} \stackrel{r}{\rightarrow} \frac{2}{3} \stackrel{r}{\rightarrow} \frac{3}{2} \stackrel{+}{\rightarrow} 2 \stackrel{r}{\rightarrow} \frac{1}{2} \stackrel{+}{\rightarrow} 1$$

takes 6 steps, so $a(4) \le 6$ (you can do better).

Challenges:

- (1) Provide the best upper bound you can for a(6).
- (2) Provide the best upper bound you can for a(1916).
- (3) Provide the best upper bound you can for a(103).