

Mini-math AP Calculus BC: Friday, January 28, 2022 (6 minutes)

SOLUTIONS

1. (1 point) Find  $\int_0^2 x^2 e^x dx$

- A.  $\frac{8}{3}e^2$    B.  $-2e^2 - 2$    C.  $2e^2 - 2$    D.  $3e^2 - 1$

**Solution:** Using integration by parts twice,

$$\begin{aligned}\int_0^2 x^2 e^x dx &= x^2 e^x \Big|_0^2 - \int_0^2 2x e^x dx \\ &= x^2 e^x \Big|_0^2 - 2 \left( x e^x \Big|_0^2 - \int_0^2 e^x dx \right) \\ &= e^x (x^2 - 2x + 2) \Big|_0^2 = 2e^2 - 2\end{aligned}$$

(c) is correct.

2. (1 point) Find  $\int \frac{x-2}{x^3+3x^2+2x} dx$

- A.  $\frac{\frac{1}{2}x^2 - 2x}{\frac{1}{4}x^4 + x^3 + x^2} + C$   
 B.  $\ln|x| - 3\ln|x+1| + 2\ln|x+2| + C$   
 C.  $-\ln|x| + \frac{1}{3}\ln|x+1| + \frac{1}{2}\ln|x+2| + C$   
 D.  $\ln \left| \frac{(x+1)^3}{x(x+2)^2} \right| + C$

**Solution:**

$$\begin{aligned}\frac{x-2}{x(x+1)(x+2)} &= \frac{A}{x} + \frac{B}{x+1} + \frac{C}{x+2} \\ x-2 &= A(x+1)(x+2) + Bx(x+2) + Cx(x+1)\end{aligned}$$

If  $x = 0$ , then  $-2 = A(2)$ , so  $A = -1$ . If  $x = -1$ , then  $-3 = B(-1)(1)$ , so  $B = 3$ . If  $x = -2$ , then  $-4 = C(-2)(-1)$ , so  $C = -2$ . Then

$$\int \frac{x-2}{x^3+3x^2+2x} dx = \int \left( \frac{-1}{x} + \frac{3}{x+1} - \frac{2}{x+2} \right) dx = -\ln|x| + 3\ln|x+1| - 2\ln|x+2| + C$$

(d) is correct.

3. (1 point) Find  $\int_0^9 \frac{dx}{\sqrt{9-x}}$
- A. -6  
B. 3  
C. 6  
D. Does not exist because the  $\frac{1}{\sqrt{9-x}}$  is discontinuous on the interval

**Solution:**

$$\begin{aligned}\int_0^9 \frac{dx}{\sqrt{9-x}} &= \lim_{b \rightarrow 9^-} \int_0^b \frac{dx}{\sqrt{9-x}} dx \\ &= \lim_{b \rightarrow 9^-} -2\sqrt{9-x} \Big|_0^b \\ &= \lim_{b \rightarrow 9^-} -2\sqrt{9-b} + 2\sqrt{9} = 6\end{aligned}$$

(c) is correct.