

Mini-math Div 3/4: Monday, September 21, 2020

- (1) True or false: The value of $\lim_{x \rightarrow a} f(x)$ is $f(a)$, assuming $f(a)$ is defined.

Solution: False - only for continuous functions.

- (2) True or false: $\lim_{x \rightarrow a} f(x)$ can only exist if the left and right limits exist and are equal.

Solution: True.

- (3) What method would you use to solve

$$\lim_{h \rightarrow 0} \frac{(2+h)^2 - 4}{h}?$$

For an extra half point, what is the limit?

Solution: Expand, simplify, and reduce h . Alternatively: factor as difference of squares, then simplify and reduce. Answer: 4

- (4) What method would you use to solve

$$\lim_{x \rightarrow 9} \frac{\sqrt{x} - 3}{x - 9}?$$

For an extra half point, what is the limit?

Solution: factor and reduce, or rationalize and reduce. Answer: 1/6

- (5) What method would you use to solve

$$\lim_{x \rightarrow 2} \frac{x^2 + x + 1}{3x^2 + 1}?$$

For an extra half point, what is the limit?

Solution: Plug it in. Answer: 7/13

- (6) What method would you use to solve

$$\lim_{x \rightarrow 2} \frac{|x - 2|}{x - 2}?$$

For an extra half point, what is the limit?

Solution: Split into left and right limits, evaluate the absolute value. Answer: DNE

- (7) Where is the following function discontinuous? Identify the type of discontinuity, if any.

$$f(x) = \begin{cases} 0 & \text{if } x \leq 0 \\ x & \text{if } 0 < x < 2 \\ 1 & \text{if } x \geq 2 \end{cases}$$

Solution: Only discontinuous at 2; jump discontinuity

Long solution: Since $f(x)$ is continuous on each piece, we need only worry about where the function stitches together.

At $x = 0$:

$$\begin{aligned}\lim_{x \rightarrow 0^-} f(x) &= \lim_{x \rightarrow 0^-} 0 = 0, \\ \lim_{x \rightarrow 0^+} f(x) &= \lim_{x \rightarrow 0^+} x = 0\end{aligned}$$

so $\lim_{x \rightarrow 0} f(x) = 0$. Since $f(0) = 0$ also, f is continuous at $x = 0$.

At $x = 2$:

$$\begin{aligned}\lim_{x \rightarrow 2^-} f(x) &= \lim_{x \rightarrow 2^-} x = 2, \\ \lim_{x \rightarrow 2^+} f(x) &= \lim_{x \rightarrow 2^+} 1 = 1\end{aligned}$$

so $\lim_{x \rightarrow 2} f(x)$ DNE. So $f(x)$ has a (jump) discontinuity at $x = 2$.

- (8) If $s(t)$ represents the position of a particle at time t , write an equation which represents the velocity of the particle at time $t = a$.

Solution:

$$\lim_{h \rightarrow 0} \frac{s(a+h) - s(a)}{h} \quad \text{or} \quad \lim_{t \rightarrow a} \frac{s(t) - s(a)}{t - a}$$

- (9) What method would you use to solve

$$\lim_{n \rightarrow \infty} \frac{3n^2 + 1}{2n^2 - 4n + 1}?$$

For an extra half point, what is the limit?

Solution: Divide by highest power of the denominator. Answer: $3/2$

- (10) Find the sum of

$$\sum_{n=2}^{\infty} 2 \cdot \frac{1}{3^n}$$

Solution:

$$\frac{2/3^2}{1 - 1/3} = \frac{2}{9 - 3} = \frac{2}{6} = \frac{1}{3}$$