Playing with Pattern Problems and Puzzles

For each of the following, write an explanation that describes how to solve this problem in detail, such that any average 6th grader can understand and follow along.

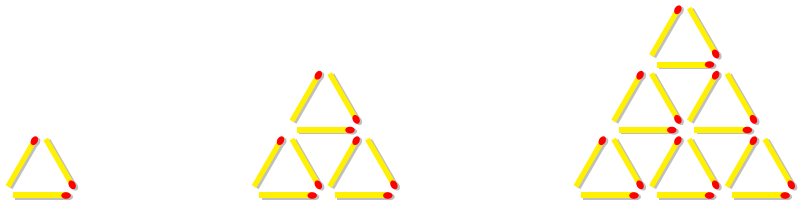
(1) Jordan is exploring a pattern shown below:

n	1	2	3	4	$\cdots$
t_n	8	11	14	17	$\cdots$

(a) What should the 20th number be?

(b) Bonus: Write an algebraic expression which gives the value of the nth term, t_n. Then, write an explanation that describes how to solve this problem in detail, such that any average 6th grader can understand and follow along.

(2) Consider the following pattern of 3 figures:



(a) How many small triangles (made up of 3 matchsticks) are in the 10th figure?

(b) How many matchsticks are in the 10th figure?

(c) Bonus 1: Write an algebraic expression which gives the number of small triangles in the nth figure.

(d) Bonus 2: Write an algebraic expression which gives the number of **matchsticks** in the nth figure.

(3) Suppose you see a pattern $t_1, t_2, \ldots$ such that there is a common difference between consecutive terms: $d = t_2 - t_1 = t_3 - t_2 = t_4 - t_3 = \ldots$.

(a) Write an algebraic expression which gives the value of the nth term, t_n, only in terms of t_1, n, and d.

(b) Write an algebraic expression which gives the value of the **sum** of the first n terms, S_n, only in terms of t_1, n, and d.

(4) Suppose you see a pattern $t_1, t_2, \ldots$ such that there is a common ratio between consecutive terms: $r = t_2/t_1 = t_3/t_2 = t_4/t_3 = \ldots$.

(a) Write an algebraic expression which gives the value of the nth term, t_n, only in terms of t_1, n, and r.

(b) Write an algebraic expression which gives the value of the **sum** of the first n terms, S_n, only in terms of t_1, n, and d. Then, find what the sum tends to as n goes to infinity, noting when your formula is valid.

(5) Suppose you see a pattern $t_1, t_2, \ldots$ such that each term is obtained by multiplying corresponding terms of an arithmetic sequence with those of a geometric sequence.

(a) Write an algebraic expression which gives the value of the **sum** of the first n terms, S_n, only in terms of t_1, n, and d. Then, find what the sum tends to as n goes to infinity, noting when your formula is valid.

(b) Calculate

$$\sum_{k=1}^{\infty} kr^k$$

and use it to answer the question: what is the expected number of coin flips before we get the first head?