Name: ____________________ Mark: ____________________

Mini-math Div 3/4: Monday, December 14, 2020 (12 minutes)

1. (2 points) There is a function $f(x)$ such that $f'(x) = 0$ if and only if $x = -1, 2$, and whose **second derivative** is given by $f''(x) = \dfrac{2x^3 - 3x^2 + 5}{(x-1)^2}$. What does the Second Derivative Test tell you about the critical points $x = -1$ and $x = 2$?

Solution: We calculate:

$$f''(-1) = 0,$$
$$f''(2) = \frac{16 - 12 + 5}{1^2} = 9$$

so the Second Derivative Test is inconclusive for $x = -1$ and tells us that $x = 2$ is a local minimum.

2. Consider the continuous function whose **second derivative** is

$$f''(x) = (x-2)^3 \left(x - \frac{3}{5}\right)^2 (x+1)(x-1)^{1/3}$$

(a) (3 points) Find the interval(s) on which the original function f is concave up.

Solution: The critical points of the derivative are $-1, 3/5, 1, 2$.
On each subinterval, the derivative has the following sign:

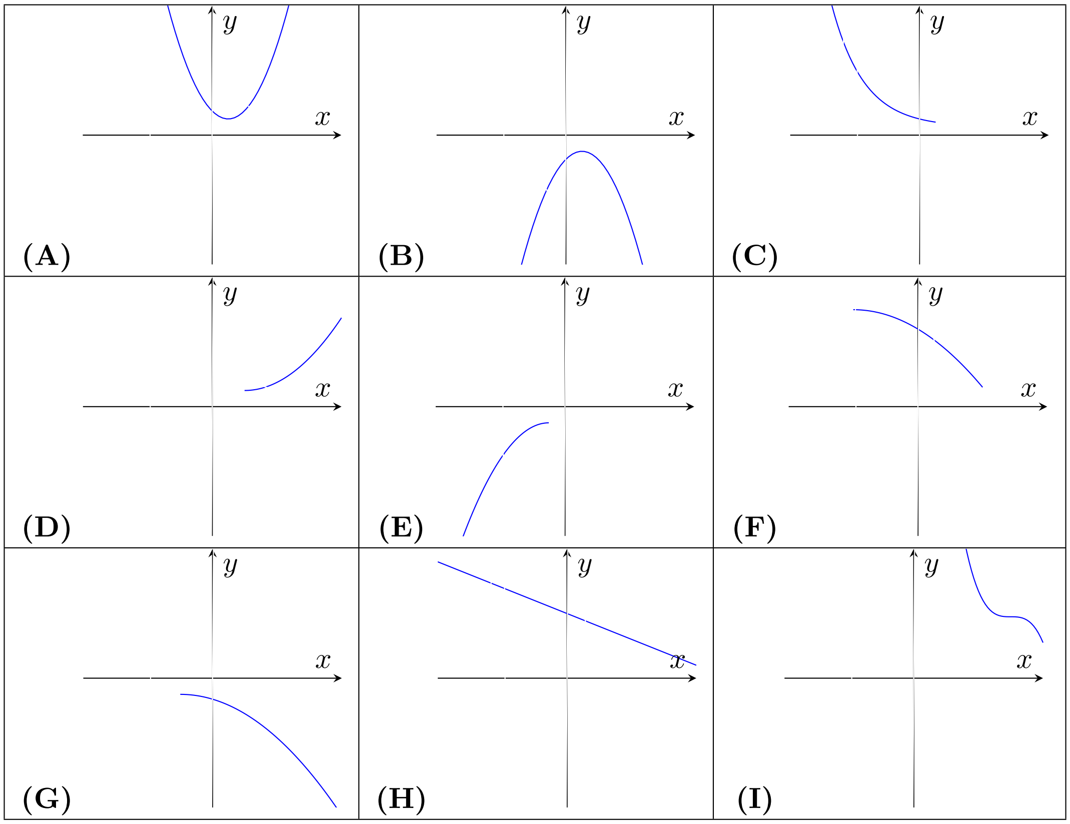
		-1		$3/5$		1		2	
$x+1$	$-$	|	$+$	|	$+$	|	$+$	|	$+$
$(x-3/5)^2$	$+$	|	$+$	|	$+$	|	$+$	|	$+$
$(x-1)^{1/3}$	$-$	|	$-$	|	$-$	|	$+$	|	$+$
$(x-2)^3$	$-$	|	$-$	|	$-$	|	$-$	|	$+$
$f''(x)$	$-$	|	$+$	|	$+$	|	$-$	|	$+$

Then $f(x)$ is concave up on $(-1, 3/5)$, $(3/5, 1)$, or $(2, \infty)$.

(b) (2 points) Find the points of inflection of f.

Solution: By part (a), $x = -1, 1, 2$ are points of inflection.

3. (2 points) Which of the following graphs of f could satisfy $f > 0$, $f' < 0$, and $f'' > 0$ for all points on its domain? Indicate **ALL** possibilities. You do not need to show your work for this question.



Solution: C is the only possibility.

This is why the rest do not work (not needed for marks):

- B, E, and G have $f < 0$.
- A and D have parts with $f' > 0$.
- F and I have parts with $f'' < 0$.
- H has $f'' = 0$ everywhere.