Mini-math Div 3/4: Friday, September 23, 2022 (8 minutes)

SOLUTIONS

1. (1 point) Choose the limit of the Riemann Sum that is the integral: $\displaystyle\int_2^4 \frac{1}{x+2}\,dx$

   A. $\displaystyle\lim_{n\to\infty}\sum_{k=1}^{n} \frac{1}{\frac{k}{n}+2}\cdot\left(\frac{2}{n}\right)$

   B. $\displaystyle\lim_{n\to\infty}\sum_{k=1}^{n} \frac{1}{\frac{2k}{n}+2}\cdot\left(\frac{2}{n}\right)$

   C. $\displaystyle\lim_{n\to\infty}\sum_{k=1}^{n} \frac{1}{\frac{k}{n}+4}\cdot\left(\frac{2}{n}\right)$

   D. $\displaystyle\lim_{n\to\infty}\sum_{k=1}^{n} \frac{1}{\frac{2k}{n}+4}\cdot\left(\frac{2}{n}\right)$

Solution: $\Delta x = \frac{4-2}{n} = \frac{2}{n}$, and $x_k = a + \Delta x = 2 + \frac{2k}{n}$, so if $f(x) = \frac{1}{x+2}$, we have $f(x_k) = \frac{1}{2+\frac{2k}{n}+2} = \frac{1}{\frac{2k}{n}+4}$.

(d) is correct.

2. (1 point) Choose the integral that is the limit of the Riemann Sum: $\displaystyle\lim_{n\to\infty}\sum_{k=1}^{n} \sin\left(1+\frac{8k}{n}\right)\cdot\frac{4}{n}$

   A. $\displaystyle\int_0^4 \sin(1+2x)\,dx$ B. $\displaystyle\int_1^5 \sin(1+x)\,dx$ C. $\displaystyle\int_1^5 \sin(1+2x)\,dx$ D. $\displaystyle\int_1^5 \sin x\,dx$

Solution:

$$\lim_{n\to\infty}\sum_{k=1}^{n} \sin\left(1+\frac{8k}{n}\right)\cdot\frac{4}{n} = \lim_{n\to\infty}\sum_{k=1}^{n} \sin\left(1+2\cdot\frac{4k}{n}\right)\cdot\frac{4}{n}$$

(a) is correct.

3. (1 point) Suppose f is a concave up function and the following are selected values of f:

x	0	1	3	4	6
$f(x)$	3	2	4	6	12

If we use the trapezoidal rule with 4 unequal subintervals to approximate $\int_0^6 f(x)\,dx$, then:

A. $\int_0^6 f(x)\,dx \approx 31.5$ and this is an underestimate

B. $\int_0^6 f(x)\,dx \approx 31.5$ and this is an overestimate

C. $\int_0^6 f(x)\,dx \approx 63$ and this is an underestimate

D. $\int_0^6 f(x)\,dx \approx 63$ and this is an overestimate

Solution:

$$(3+2)/2 \cdot 1 + (2+4)/2 \cdot 2 + (4+6)/2 \cdot 1 + (6+12)/2 \cdot 2 = \frac{63}{2} = 31.5$$

Since f is concave up, the trapezoidal rule is an overestimate.

(b) is correct.

4. (1 point) Suppose $V(x) = \int_0^{x^2} \sin t\,dt$. What is the derivative, $V'(x)$?

A. $\cos x$

B. $\sin x$

C. $\sin x^2$

D. $2x \sin x^2$

Solution: By FTC I,

$$V'(t) = \sin(x^2) \cdot \frac{d}{dx}(x^2) = 2x\sin(x^2)$$

(d) is correct.