Name: _____ Mark: _____

Mini-math Div 3/4: Monday, February 1, 2021 (15 minutes)

1. (2 points) Let f be a differentiable function such that:

$$f(4) = 6, \quad f(8) = 4, \quad f'(4) = -3, \quad f'(8) = -5.$$

Suppose the function $g(x) = f^{-1}(x)$ is differentiable for all x. What is $g'(4)$?

Solution:

$$g'(4) = \frac{1}{f'(g(4))} = \frac{1}{f'(8)} = -\frac{1}{5}$$

2. (4 points) Find the equation of the line tangent to the given curve at the given point.

$$e^x + \ln(x + y) = x + 1, \quad \text{at } (0, 1)$$

Solution: Differentiating implicitly,

$$e^x + \frac{1}{x+y} \cdot \left(1 + \frac{dy}{dx}\right) = 1$$

At the given point, we have

$$\begin{aligned} 1 + \frac{1}{1} \cdot \left(1 + \frac{dy}{dx}\right) &= 1 \\ 1 + \frac{dy}{dx} &= 0 \\ \frac{dy}{dx} &= -1 \end{aligned}$$

By the point-slope formula, the equation of the desired line is

$$y - 1 = -x$$

3. (2 points) Find $f(t)$ if $f(t) = 2^{3^x}$.

Solution:

$$f'(t) = 2^{3^x} \ln 2 \cdot \frac{d}{dx}(3^x) = 2^{3^x} (\ln 2) \cdot 3^x \ln 3$$

4. Find the derivative of y with respect to x in each of the following.

   (a) (2 points) $y = \ln|x^4 - 4|$

   Solution:

$$\frac{dy}{dx} = \frac{1}{x^4 - 4} \cdot 4x^3 = \frac{4x^3}{x^4 - 4}$$

   (b) (2 points) $y = \log_2\left(\dfrac{\sin x}{2^x}\right)$

   Solution: It is easier to simplify first: $y = \log_2 \sin x - \log_2 2^x = \log_2 \sin x - x$, so

$$\frac{dy}{dx} = \frac{1}{\sin x \ln 2} \cdot \cos x - 1 = \frac{\cot x}{\ln 2} - 1$$

5. (4 points) Find the derivative of y with respect to x in the following via logarithmic differentiation. **You do not need to simplify your final expression, and may express your answer in terms of both y and x.**

$$y = \sqrt{\frac{(x+1)^2(2x-1)^3}{x}}$$

Solution: Taking logarithm of both sides and using the logarithm laws,

$$\begin{aligned}
\ln y &= \ln \sqrt{\frac{(x+1)^2(2x-1)^3}{x}} \\
\ln y &= \frac{1}{2}\ln(x+1)^2 + \frac{1}{2}\ln(2x-1)^3 - \frac{1}{2}\ln x \\
\ln y &= \ln(x+1) + \frac{3}{2}\ln(2x-1) - \frac{1}{2}\ln x \\
\frac{1}{y}\cdot\frac{dy}{dx} &= \frac{1}{x+1} + \frac{3}{2}\cdot\frac{2}{2x-1} - \frac{1}{2}\cdot\frac{1}{x} \\
\frac{dy}{dx} &= y\left(\frac{1}{x+1} + \frac{3}{2x-1} - \frac{1}{2x}\right)
\end{aligned}$$