Name: ______________________ Mark: ______________________

Mini-math Div 3/4: Monday, December 14, 2020 (12 minutes)

1. Find the derivative of y with respect to x in each of the following.

   (a) (2 points) $y = \sin(\cos^2 x)$

   Solution:

   $$\frac{dy}{dx} = \cos(\cos^2 x)(2\cos x)(-\sin x) = -2\cos(\cos^2 x)\sin x \cos x$$
   $$\text{or} \quad -\cos(\cos^2 x)\sin(2x)$$

   (b) (2 points) $y = x \sin 2x$

   Solution:

   $$\frac{dy}{dx} = \sin 2x + 2x \cos 2x$$

   (c) (2 points) $y = \dfrac{x}{\cos(x^2+1)}$

   Solution:

   $$\begin{aligned}\frac{dy}{dx} &= \frac{1 \cdot \cos(x^2+1) - x \cdot (-\sin(x^2+1)) \cdot (2x)}{\cos^2(x^2+1)} \\ &= \frac{\cos(x^2+1) + 2x^2 \sin(x^2+1)}{\cos^2(x^2+1)}\end{aligned}$$

2. Find the derivative of y with respect to x in each of the following.

(a) (2 points) $y = 2\tan x \sec x$

Solution:

$$\begin{aligned}\frac{dy}{dx} &= 2\sec^2 x \sec x + 2\tan x \sec x \tan x \\ &= 2\sec^3 x + 2\sec x \tan^2 x \quad \text{or} \quad 2\sec x(\sec^2 x + \tan^2 x) \\ &\qquad\qquad \text{or} \quad 2\sec x(2\sec^2 x - 1)\end{aligned}$$

(b) (2 points) $y = \cot^2 2x - \csc 2x$

Solution:

$$\begin{aligned}\frac{dy}{dx} &= 2(\cot 2x)(-\csc^2 2x)(2) - (-\csc 2x \cot 2x)(2) \\ &= -4\csc^2 2x \cot 2x + 2\csc 2x \cot 2x \quad \text{or} \quad 2\csc 2x \cot 2x(1 - 2\csc 2x)\end{aligned}$$

(c) (2 points) $\tan \dfrac{y}{x} = x$

Solution:

$$\begin{aligned}\sec^2\left(\frac{y}{x}\right) \cdot \frac{y' \cdot x - y}{x^2} &= 1 \\ \frac{dy}{dx} \cdot x - y &= x^2 \cos^2\left(\frac{y}{x}\right) \\ \frac{dy}{dx} &= x\cos^2\left(\frac{y}{x}\right) + \frac{y}{x} \quad \text{or} \quad \frac{\sin^2\left(\frac{y}{x}\right) + y}{x}\end{aligned}$$

3. (3 points) Find the equation of the line tangent to the given curve at the given point.

$$\sin y + \tan x = \sec 2y, \quad \text{at } \left(\frac{\pi}{4}, \frac{\pi}{6}\right)$$

Solution: Differentiating implicitly,

$$\cos y \cdot \frac{dy}{dx} + \sec^2 x = \sec 2y \tan 2y \cdot 2\frac{dy}{dx}$$

At the given point, we have

$$\begin{aligned} \frac{\sqrt{3}}{2} \cdot \frac{dy}{dx} + 2 &= 2 \cdot \sqrt{3} \cdot 2\frac{dy}{dx} \\ \frac{dy}{dx} &= \frac{2}{(7/2)\sqrt{3}} = \frac{4}{7\sqrt{3}} \end{aligned}$$

By the point-slope formula, the equation of the desired line is

$$y - \frac{\pi}{6} = \frac{4}{7\sqrt{3}}\left(x - \frac{\pi}{4}\right)$$