

Name: _____

Mark: _____

Mini-math Div 3/4: Monday, January 25, 2021 (12 minutes)

1. Find the derivative of y with respect to x in each of the following.

(a) (2 points) $y = \arcsin x^3$

Solution:

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^6}} \cdot 3x^2 = \frac{3x^2}{\sqrt{1-x^6}}$$

(b) (2 points) $y = x \arctan x$

Solution:

$$\frac{dy}{dx} = \arctan x + \frac{x}{1+x^2}$$

(c) (2 points) $y = \arccos(\cos x)$

Solution: Simplify first: $y = x$, so

$$\frac{dy}{dx} = 1$$

(d) (2 points) $y - \arctan y = x$

Solution:

$$\begin{aligned} \frac{dy}{dx} - \frac{1}{1+y^2} \cdot \frac{dy}{dx} &= 1 \\ \frac{dy}{dx} \left(\frac{1+y^2}{1+y^2} - \frac{1}{1+y^2} \right) &= 1 \\ \frac{dy}{dx} &= \frac{1+y^2}{y^2} = 1 + \frac{1}{y^2} \end{aligned}$$

2. Find the derivative of y with respect to x in each of the following.

(a) (2 points) $y = e^{\cos^2 x}$

Solution:

$$\frac{dy}{dx} = e^{\cos^2 x} \cdot (2 \cos x)(-\sin x) \quad \text{or} \quad -e^{\cos^2 x} \sin 2x$$

(b) (2 points) $y = \frac{e^\pi}{1 - e^\pi}$

Solution:

$$\frac{dy}{dx} = 0$$

(c) (2 points) $y = \sqrt{e^{x^2} + 1}$

Solution:

$$\frac{dy}{dx} = \frac{2xe^{x^2}}{2\sqrt{e^{x^2} + 1}} = \frac{xe^{x^2}}{\sqrt{e^{x^2} + 1}}$$

(d) (2 points) $e^{xy} = x + y$

Solution:

$$e^{xy} \left(y + x \frac{dy}{dx} \right) = 1 + \frac{dy}{dx}$$
$$\frac{dy}{dx} = \frac{1 - ye^{xy}}{xe^{xy} - 1} \quad \text{or} \quad \frac{1 - y(x + y)}{x(x + y) - 1}$$