

Mini-math Div 3/4: Friday, January 28, 2022 (8 minutes)

SOLUTIONS

1. (1 point) Choose the limit of the Riemann Sum that is the integral: $\int_2^4 \frac{1}{x+2} dx$

A. $\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{1}{\frac{k}{n} + 2} \cdot \left(\frac{2}{n}\right)$

C. $\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{1}{\frac{k}{n} + 4} \cdot \left(\frac{2}{n}\right)$

B. $\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{1}{\frac{2k}{n} + 2} \cdot \left(\frac{2}{n}\right)$

D. $\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{1}{\frac{2k}{n} + 4} \cdot \left(\frac{2}{n}\right)$

Solution: $\Delta x = \frac{4-2}{n} = \frac{2}{n}$, and $x_k = a + \Delta x = 2 + \frac{2k}{n}$, so if $f(x) = \frac{1}{x+2}$, we have $f(x_k) = \frac{1}{2+\frac{2k}{n}+2} = \frac{1}{\frac{2k}{n}+4}$.

(d) is correct.

2. (1 point) Choose the integral that is the limit of the Riemann Sum: $\lim_{n \rightarrow \infty} \sum_{k=1}^n \sin\left(1 + \frac{8k}{n}\right) \cdot \frac{4}{n}$

A. $\int_0^4 \sin(1+2x) dx$ B. $\int_1^5 \sin(1+x) dx$ C. $\int_1^5 \sin(1+2x) dx$ D. $\int_1^5 \sin x dx$

Solution:

$$\lim_{n \rightarrow \infty} \sum_{k=1}^n \sin\left(1 + \frac{8k}{n}\right) \cdot \frac{4}{n} = \lim_{n \rightarrow \infty} \sum_{k=1}^n \sin\left(1 + 2 \cdot \frac{4k}{n}\right) \cdot \frac{4}{n}$$

(a) is correct.

3. (1 point) Suppose f is a concave up function and the following are selected values of f :

x	0	1	3	4	6
$f(x)$	3	2	4	6	12

If we use the trapezoidal rule with 4 unequal subintervals to approximate $\int_0^6 f(x) dx$, then:

- A. $\int_0^6 f(x) dx \approx 31.5$ and this is an underestimate
- B. $\int_0^6 f(x) dx \approx 31.5$ and this is an overestimate
- C. $\int_0^6 f(x) dx \approx 63$ and this is an underestimate
- D. $\int_0^6 f(x) dx \approx 63$ and this is an overestimate

Solution:

$$(3 + 2)/2 \cdot 1 + (2 + 4)/2 \cdot 2 + (4 + 6)/2 \cdot 1 + (6 + 12)/2 \cdot 2 = \frac{63}{2} = 31.5$$

Since f is concave up, the trapezoidal rule is an overestimate.

(b) is correct.

4. (1 point) Suppose $V(x) = \int_0^{x^2} \sin t \, dt$. What is the derivative, $V'(x)$?

- A. $\cos x$
- B. $\sin x$
- C. $\sin x^2$
- D. $2x \sin x^2$

Solution: By FTC I,

$$V'(t) = \sin(x^2) \cdot \frac{d}{dx}(x^2) = 2x \sin(x^2)$$

(d) is correct.