

Numerical Integration

1. Consider the following table of values of $f(x)$:

x	0	1	2	3	4	5	6	7	8
$f(x)$	5	1	3	2	4	7	9	10	9

Approximate $\int_0^8 f(x) dx$ using the stated method.

- (a) Right Riemann sum with 8 equal subintervals

Solution:

$$\begin{aligned} &f(1) \cdot 1 + f(2) \cdot 1 + f(3) \cdot 1 + f(4) \cdot 1 + f(5) \cdot 1 + f(6) \cdot 1 + f(7) \cdot 1 + f(8) \cdot 1 \\ &= 1 + 3 + 2 + 4 + 7 + 9 + 10 + 9 = 45 \end{aligned}$$

- (b) Left Riemann sum with 8 equal subintervals

Solution:

$$\begin{aligned} &f(0) \cdot 1 + f(1) \cdot 1 + f(2) \cdot 1 + f(3) \cdot 1 + f(4) \cdot 1 + f(5) \cdot 1 + f(6) \cdot 1 + f(7) \cdot 1 \\ &= 5 + 1 + 3 + 2 + 4 + 7 + 9 + 10 = 41 \end{aligned}$$

- (c) Trapezoid Rule with 8 equal subintervals

Solution:

$$\begin{aligned} &\left(\frac{f(0) + f(1)}{2}\right) \cdot 1 + \left(\frac{f(1) + f(2)}{2}\right) \cdot 1 + \left(\frac{f(2) + f(3)}{2}\right) \cdot 1 + \left(\frac{f(3) + f(4)}{2}\right) \cdot 1 \\ &\quad + \left(\frac{f(4) + f(5)}{2}\right) \cdot 1 + \left(\frac{f(5) + f(6)}{2}\right) \cdot 1 + \left(\frac{f(6) + f(7)}{2}\right) \cdot 1 + \left(\frac{f(7) + f(8)}{2}\right) \cdot 1 \\ &= 3 + 2 + 5/2 + 3 + 11/2 + 8 + 19/2 + 19/2 = 43 \end{aligned}$$

- (d) Right Riemann sum with 4 equal subintervals

Solution:

$$f(2) \cdot 2 + f(4) \cdot 2 + f(6) \cdot 2 + f(8) \cdot 2 = 6 + 8 + 18 + 18 = 50$$

- (e) Left Riemann sum with 4 equal subintervals

Solution:

$$f(0) \cdot 2 + f(2) \cdot 2 + f(4) \cdot 2 + f(6) \cdot 2 = 10 + 6 + 8 + 18 = 42$$

- (f) Midpoint Rule with 4 equal subintervals

Solution:

$$f(1) \cdot 2 + f(3) \cdot 2 + f(5) \cdot 2 + f(7) \cdot 2 = 2 + 4 + 14 + 20 = 40$$

- (g) Trapezoid Rule with 4 equal subintervals

Solution:

$$\begin{aligned} & \left(\frac{f(0) + f(2)}{2} \right) \cdot 2 + \left(\frac{f(2) + f(4)}{2} \right) \cdot 2 + \left(\frac{f(4) + f(6)}{2} \right) \cdot 2 + \left(\frac{f(6) + f(8)}{2} \right) \cdot 2 \\ &= 8 + 7 + 13 + 18 = 46 \end{aligned}$$

- (h) Right Riemann sum with 2 equal subintervals

Solution: $f(4) \cdot 4 + f(8) \cdot 4 = 16 + 36 = 52$

- (i) Left Riemann sum with 2 equal subintervals

Solution: $f(0) \cdot 4 + f(4) \cdot 4 = 20 + 16 = 36$

- (j) Midpoint Rule with 2 equal subintervals

Solution: $f(2) \cdot 4 + f(6) \cdot 4 = 12 + 36 = 48$

- (k) Trapezoid Rule with 2 equal subintervals

Solution: $\left(\frac{f(0) + f(4)}{2} \right) \cdot 4 + \left(\frac{f(4) + f(8)}{2} \right) \cdot 4 = 18 + 26 = 44$

2. Consider the following table of values of $f(x)$:

x	0	2	3	6	9	11	15
$f(x)$	4	1	2	-2	4	6	10

Approximate $\int_0^{15} f(x) dx$ using the stated method.

- (a) Right Riemann Sum with 6 intervals

Solution:

$$\begin{aligned} & f(2) \cdot 2 + f(3) \cdot 1 + f(6) \cdot 3 + f(9) \cdot 3 + f(11) \cdot 2 + f(15) \cdot 4 \\ &= 2 + 2 - 6 + 12 + 12 + 40 = 62 \end{aligned}$$

- (b) Left Riemann Sum with 6 intervals

Solution:

$$\begin{aligned} & f(0) \cdot 2 + f(2) \cdot 1 + f(3) \cdot 3 + f(6) \cdot 3 + f(9) \cdot 2 + f(11) \cdot 4 \\ &= 8 + 1 + 6 - 6 + 8 + 24 = 41 \end{aligned}$$

- (c) Trapezoid Rule with 6 intervals

Solution:

$$\begin{aligned} & \left(\frac{f(0) + f(2)}{2} \right) \cdot 2 + \left(\frac{f(2) + f(3)}{2} \right) \cdot 1 + \left(\frac{f(3) + f(6)}{2} \right) \cdot 3 \\ & \quad + \left(\frac{f(6) + f(9)}{2} \right) \cdot 3 + \left(\frac{f(9) + f(11)}{2} \right) \cdot 2 + \left(\frac{f(11) + f(15)}{2} \right) \cdot 4 \\ &= 5 + 3/2 + 0 + 3 + 10 + 32 = 51.5 \end{aligned}$$

- (d) Midpoint Rule with 3 intervals

Solution:

$$\begin{aligned} & f(2) \cdot (3 - 0) + f(6) \cdot (9 - 3) + f(11) \cdot (15 - 9) \\ &= 3 - 12 + 36 = 27 \end{aligned}$$