A nowhere continuous function

(Hard) Does there exist a function $f: \mathbb{R} \to \mathbb{R}$ (i.e., the domain is the real numbers and every value is a real number) such that f(x) is not continuous at any $a \in \mathbb{R}$?