9.1 Parametric Equations

Calculus Name: ______________________________ **CA #1**

1. For $x = t^2 - 2t$ and $y = t + 1$, eliminate the parameter and write the corresponding rectangular equation.

2. If $x = 3\sin 2t$ and $y = 4e^{2t}$ then $\frac{dy}{dx} =$

3. A curve is described by the parametric equations $x(t) = t^2 + 3$ and $y(t) = t^3 + t^2 + 1$. Find an equation of the line tangent to this curve at the point determined by $t = 1$.

4. A curve is defined by the parametric equations $x(t) = t^3 + 1$ and $y(t) = t^2 + 10t$. For what values of t is the line tangent to this curve horizontal?

5. What is the slope of the tangent line to the curve defined parametrically by $x(t) = \sqrt{t}$ and $y(t) = \frac{1}{4}(t^2 - 4)$, $t \geq 0$ at the point $(2,3)$?

Answers to 9.1 CA #1

1. $x = y^2 - 4y + 3$	2. $\frac{4e^{2t}}{3\cos 2t}$	3. $y = \frac{5}{2}x - 7$	4. $t = -5$	5. 8

9.2 Second Derivatives of Parametric Equations

CA #1

Calculus Name: ______________________________

1. Given a curve defined by the parametric equations $x(t) = \sqrt{t}$ and $y(t) = 2t - 1$. Determine the open t-intervals on which the curve is concave up or down.

2. If $x(t) = 6t^2$ and $y(t) = t^3 - t$, what is $\frac{d^2y}{dx^2}$ in terms of t?

3. If $x(t) = t^2 - 5$ and $y(t) = t^{-1}$, find the slope and the concavity at the point $(-4,1)$.

4. If $x = \sin\theta$ and $y = 2\cos\theta$, what is $\frac{d^2y}{dx^2}$ in terms of θ?

5. If $\frac{dx}{dt} = 4$ and $\frac{dy}{dt} = \sin(3t)$, what is $\frac{d^2y}{dx^2}$ in terms of t?

Answers to 9.2 CA #1

1. $\frac{d^2y}{dx^2} = 4$, therefore the graph is concave up on its domain $t \geq 0$.	2. $\frac{d^2y}{dx^2} = \frac{3t^2+1}{144t^3}$	3. Slope: $-\frac{1}{2}$, Concave Up	4. $-2\sec^3\theta$	5. $\frac{3\cos(3t)}{16}$

9.3 Arc Length (Parametric Form)

Calculus Name: ________________________________ **CA #1**

What is the length of the curve defined by the parametric equations? Solve without the use of a calculator.

1. $x(t) = 7t + 1$ and $y(t) = 3 - 6t$ for the interval $-1 \le t \le 3$.

2. $x(t) = 4at^2$ and $y(t) = 4bt^2$, where a and b are constants. What is the length of the curve from $t = 0$ to $t = 1$?

3. $x(t) = 9\cos\theta$ and $y(t) = 9\sin\theta$ for the interval $0 \le \theta \le \frac{\pi}{2}$.

4. $x(\theta) = \cos\theta + \theta\sin\theta$ and $y(\theta) = \sin\theta - \theta\cos\theta$ on the interval $0 \le \theta \le \pi$.

5. Which of the following gives the length of the path described by the parametric equations $x = e^{2t}$ and $y = 1 - 2t$ from $0 \le t \le 3$?

A. $\int_0^3 \sqrt{4e^{2t} + 4}\, dt$ B. $\int_0^3 \sqrt{2e^{2t} + 2}\, dt$

C. $\int_0^3 \sqrt{4e^{4t} + 4}\, dt$ D. $\int_0^3 \sqrt{e^{4t} + 4}\, dt$

1. $4\sqrt{85}$	2. $4\sqrt{a^2+b^2}$	3. $\frac{9\pi}{2}$	4. $\frac{\pi^2}{2}$	5. C

Answers to 9.3 CA #1

9.4 Derivatives of Vector-Valued Functions

CA #1

Calculus Name: ______________________________

1. If f is a vector-valued function defined by $f(t) = \langle t\sin t\,, t\cos t\rangle$, then $f''(t) =$

2. At time t, $0 \le t \le 2\pi$, the position of a particle moving along a path in the xy-plane is given by the vector-valued function, $f(t) = \langle e^{2t}\cos t\,,\ e^{2t}\sin t\rangle$. Find the slope of the path of the particle at time $t = \frac{\pi}{2}$.

3. The position of a particle moving in the xy-plane is defined by the vector-valued function, $f(t) = \langle t^3 - 9t^2 + 1,\ 2t^3 - 15t^2 - 36t + 1\rangle$. For what value of t is the particle at rest?

4. The vector-valued function f is defined by $f(t) = \langle 2e^{2t},\ te^{2t}\rangle$. Find $f'(1)$.

5. If h is the vector-valued function defined by $h(t) = \langle \sin\frac{t}{2}, \cos 3t\rangle$, then $h'(t) =$

1. $\langle 2\cos t - t\sin t\,, -2\sin t - t\cos t\rangle$	2. -2	3. $t = 6$	4. $\langle 4e^2,\ 3e^2\rangle$	5. $\langle \frac{1}{2}\cos\frac{t}{2}, -3\sin 3t\rangle$

Answers to 9.4 CA #1

9.5 Integrating Vector-Valued Functions

Calculus Name: ______________________________ **CA #1**

For problems 1-2, find the vector-valued function $f(t)$ that satisfies the given initial conditions.

1. $f(0) = \langle 1,\ 4\rangle$ and $f'(t) = \langle -4\cos 2t\,, -3\sin 3t\rangle$.

2. $f'(0) = \langle 4, 3\rangle$, $f(0) = \langle 2, 0\rangle$ and $f''(t) = \langle 8e^{2t}, 3e^{t}\rangle$.

3. The instantaneous rate of change of the vector-valued function $f(t)$ is given by $f'(t) = \langle 4t,\ 5\rangle$. If $f(1) = \langle 9, 7\rangle$ what is $f(2)$?

4. The position of a particle moving in the xy-plane is given by the parametric functions $x(t)$ and $y(t)$, where $\frac{dx}{dt} = 4\sin\frac{t}{2}$ and $\frac{dy}{dt} = 2\cos t$. The position of the particle is $(-2, 5)$ at time $t = 0$. What is the particle's position vector $\langle x(t), y(t)\rangle$?

5. **Calculator active.** At time $t \geq 0$, a particle moving in the xy-plane has a velocity vector given by $v(t) = \langle 2, 2^{-t^2}\rangle$. If the particle is at point $\left(1, \frac{1}{2}\right)$ at time $t = 0$, how far is the particle from the origin at time $t = 1$?

1. $\langle -2\sin 2t + 1, \cos 3t + 3\rangle$	2. $\langle 2e^{2t}, 3e^{t} - 3\rangle$	3. $\langle 15, 12\rangle$	4. $\langle -8\cos\frac{t}{2} + 6, 2\sin t + 5\rangle$	5. 3.274

Answers to 9.5 CA #1

9.6 Motion using Parametric and Vector-Valued Functions

Calculus Name: ______________________________ **CA #1**

1. A particle moving along a curve in the xy-plane has position $(x(t), y(t))$, at time $t \geq 0$, where $\frac{dx}{dt} = 3\cos(\pi t)$ and $\frac{dy}{dt} = 3t^2$. Find the speed of the particle at time $t = 2$.

2. For time $t \geq 0$, the position of a particle moving in the xy-plane is given by the parametric equations $x(t) = t + t^2$ and $y(t) = (3t+1)^{-1}$. What is the acceleration vector of the particle at time $t = 1$?

3. For time $t \geq 0$, the position of a particle moving in the xy-plane is given by the vector $\langle 2t^{-2},\ e^t \rangle$. What is the velocity vector of the particle at time $t = 3$.

4. **Calculator active.** The position of a particle at time $t \geq 0$ is given by $x(t) = \frac{\sqrt{t+1}}{3}$ and $y(t) = t^2 + 1$. Find the total distance traveled by the particle from $t = 0$ to $t = 2$.

5. **Calculator active.** The velocity vector a particle moving in the xy-plane has components given by $\frac{dx}{dt} = \sin 2t$ and $\frac{dy}{dt} = e^{\cos t}$. At time $t = 2$, the position of the particle is $(3, 2)$. What is the x-coordinate of the position vector at time $t = 3$?

1. $\sqrt{153}$	2. $\langle 2, \frac{9}{32} \rangle$	3. $\langle -\frac{4}{27}, e^3 \rangle$	4. 4.023	5. 2.193

Answers to 9.6 CA #1

9.7 Differentiating in Polar Form

Calculus Name: ______________________________ **CA #1**

1. Find the slope of the tangent line to the polar curve $r = 2 + 4\sin\theta$ at $\theta = \pi$.

2. A particle moves along the polar curve $r = 4 - 2\cos\theta$ so that $\frac{d\theta}{dt} = 4$. Find the value of $\frac{dr}{dt}$ at $\theta = \frac{\pi}{3}$.

3. For $0 \leq \theta \leq 2\pi$, find the values of θ for which the polar curve $r = 3\sin\theta$ **<u>might</u>** have a vertical tangent line. Then use a graphing utility to eliminate any of your possible answers.

4. A polar curve is given by the equation $r = 2\csc\theta + 3$ for $\theta \geq 0$. What is the instantaneous rate of change of r with respect to θ where $\theta = \frac{\pi}{4}$.

5. **Calculator active.** For a certain polar curve $r = f(\theta)$, it is known that $\frac{dx}{d\theta} = 3\cos\theta - 3\theta\sin\theta$ and $\frac{dy}{d\theta} = 3(\sin\theta + \theta\cos\theta)$. What is the value of $\frac{d^2y}{dx^2}$ at $\theta = 3$?

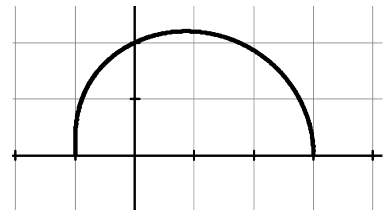
1. $-\frac{1}{2}$	2. $4\sqrt{3}$	3. $\frac{\pi}{4}, \frac{3\pi}{4}$	4. $-2\sqrt{2}$	5. -1.299

Answers to 9.7 CA #1

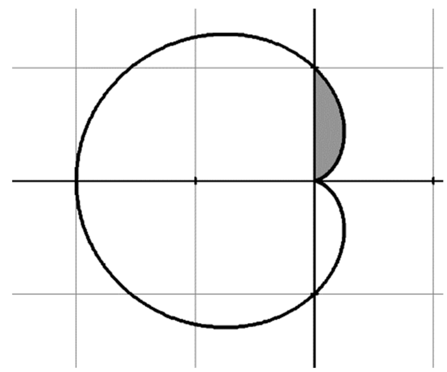
9.8 Area Bounded by a Polar Curve

Calculus Name: ______________________________ **CA #1**

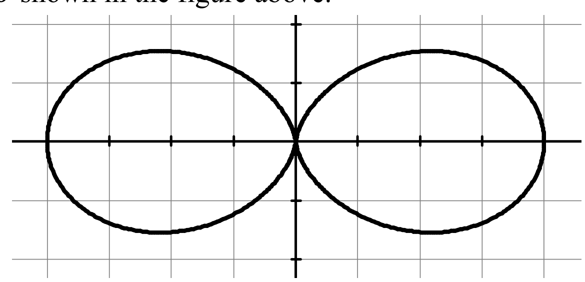
1. The graph to the right shows the polar curve $r = 2 + \cos\theta$ for $0 \leq \theta \leq \pi$. What is the area of the region bounded by the curve and the x-axis?



2. Find the area of the shaded region for the polar curve $r = 1 - \cos\theta$.



3. Find the total area enclosed by the polar curve $r = 2 + 2\cos 2\theta$ shown in the figure above.



4. Write do not solve, an integral expression that represents the area enclosed by the smaller loop of the polar curve $r = 1 - 2\sin\theta$.

5. Find the limits of integration required to find the area of one petal of the polar graph $r = 4\sin 3\theta$ in the second quadrant.

1. 7.069	2. 0.178	3. 18.850	4. $\frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}}(1 - 2\sin\theta)^2\,d\theta$	5. $\frac{2\pi}{3}, \pi$

Answers to 9.8 CA #1

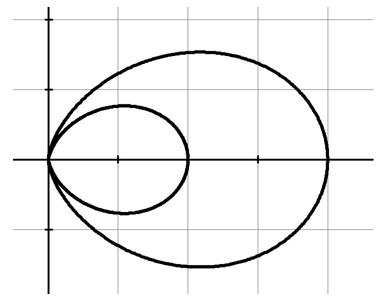
9.9 Area Bounded by Two Polar Curves

CA #1

Calculus — Name: ________________________________

1. What is the total area between the polar curves $r = 2\sin 3\theta$ and $r = 5\sin 3\theta$.

2. The figure to the right shows the graphs of the polar curves $r = 2\cos^2\theta$ and $r = 4\cos^2\theta$ for $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$. Which of the following integrals gives the area of the region bounded between the two polar curves?



   A. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2\theta\, d\theta$

   B. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 6\cos^4\theta\, d\theta$

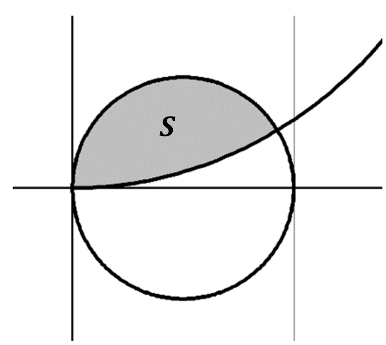
   C. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 2\cos^4\theta\, d\theta$

   D. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 2\cos^2\theta\, d\theta$

3. Find the total area in the first quadrant of the common interior of $r = 4\sin 2\theta$ and $r = 2$.

4. Find the area of the common interior of the polar graphs $r = 3\cos\theta$ and $r = 3\sin\theta$.

5. Let S be the region in the 1st Quadrant bounded above by the graph of the polar curve $r = \cos\theta$ and bounded below by the graph of the polar curve $r = \frac{7}{2}\theta$, as shown in the figure. The two curves intersect when $\theta = 0.275$. What is the area of S?



1. 16.493	2. B	3. 2.457	4. 1.284 or $\frac{9\pi}{8} - \frac{9}{4}$	5. 0.301

Answers to 9.9 CA #1