Forensics

Police arrive at the scene of a crime at 1:22AM. They immediately take and record the temperature of the body they find there: 32.2°C. By the time they finish the inspection of the crime scene, it is 2:34AM. They again take the temperature of the body, which has dropped to 29.6°C, and have it sent to the morgue. The air temperature at the crime scene is measured to be 22.8°C. Assuming the air temperature has remained constant, approximately when could the person have been murdered, to the nearest minute? Assume the person had a normal body temperature of 36.5°C to 37.5°C.

Solution: Let $T(t)$ be the temperature of the body t minutes after 1:22AM in °C.

$$\begin{aligned}\frac{dT}{dt} &= k(22.8 - T)\\ T &= 22.8 - Ae^{-kt}\end{aligned}$$

Using $T(0) = 32.2$, $A = 22.8 - 32.2 = -9.4$, so

$$T = 22.8 + 9.4e^{-kt}$$

Using $T(72) = 29.6$,

$$29.6 = 22.9 + 9.4(e^{-k})^{72} \quad \Longrightarrow \quad e^{-k} = \left(\frac{34}{47}\right)^{1/72} \quad \Longrightarrow \quad T = 22.8 + 9.4\left(\frac{34}{47}\right)^{t/72}$$

The lower bound for time of death is given by

$$37.5 = 22.8 + 9.4\left(\frac{34}{47}\right)^{t/72} \quad \Longrightarrow \quad t = 72\log_{34/47}\left(\frac{37.5 - 22.8}{9.4}\right) \approx -99.43$$

This corresponds to 11:43PM the previous night.
The upper bound for time of death is given by

$$36.5 = 22.8 + 9.4\left(\frac{34}{47}\right)^{t/72} \quad \Longrightarrow \quad t = 72\log_{34/47}\left(\frac{36.5 - 22.8}{9.4}\right) \approx -83.76$$

This corresponds to 11:58PM the previous night.
The time of death was between 11:43PM and 11:58PM the previous night.