AP CALCULUS BC	YouTube Live Virtual Lessons	Mr. Bryan Passwater Mr. Anthony Record
Topic: All Units	**Free Response Question Stem Types** **Verbal Interpretations, Explanations & Reasoning**	**Date:** April 30, 2020

Free Response Questions Stem Types: Verbal

2020 FRQ Practice Problem BC1

t (minutes)	0	1	4	6	10
$E'(t)$ (students/minute)	21	18	8	3	1

BC1: When Mr. Passwater starts his live online help session for his AP Calculus students, there are 25 students in the session. For $0 \leq t \leq 10$ minutes, students enter the online session at a rate modeled by the differentiable function, $E'(t)$, where $E'(t)$ is decreasing and measured in students per minute.

(**a**) Use the data in the table to approximate $E''(5)$. Using correct units, interpret the meaning of $E''(5)$ in the context of the problem.

$$E''(5) \approx \frac{E'(6)-E'(4)}{6-4} = \frac{(3)-(8)}{2} = -\frac{5}{2}$$

At $t=5$ minutes, the rate students enter the online session is changing at a rate of $-\frac{5}{2}$ students per minute per minute.

(**b**) Is there a time t, $0 < t < 10$, at which $E''(t) = -2$? Justify your answer.

$E'(t)$ is differentiable and continous on the interval $0 < t < 10$ so by the Mean Value Theorem garantees there is at least one number $t = c$ in $0 < t < 10$ such that

$$E''(c) = \frac{E'(10)-E'(0)}{10-0} = \frac{1-21}{10} = -2.$$

The problem has been restated.

t (minutes)	0	1	4	6	10
$E'(t)$ (students/minute)	21	18	8	3	1

BC1: When Mr. Passwater starts his live online help session for his AP Calculus students, there are 25 students in the session. For $0 \le t \le 10$ minutes, students enter the online session at a rate modeled by the differentiable function, $E'(t)$, where $E'(t)$ is decreasing and measured in students per minute.

(**c**) Using correct units, explain the meaning of $\int_0^{10} E'(t)dt$ in the context of the problem. Use a right Riemann with the four subintervals indicated in the table to estimate $\int_0^{10} E'(t)dt$.

$\int_0^{10} E'(t)dt$ is the total number of students that enter the online session between $t=0$ minutes and $t=10$ minutes.

$$\int_0^{10} E'(t)dt \approx \left[E'(1)(1)+E'(4)(3)+E'(6)(2)+E'(10)(4)\right]$$

$$=\left[(18)(1)+(8)(3)+(3)(2)+(1)(4)\right]=\left[(18)+(24)+(6)+(4)\right]$$

$= 52$ students entered during the 10 minute time interval

(**d**) Is the approximation in part (b) an overestimate or underestimate of $\int_0^{10} E'(t)dt$? Give a reason for your answer.

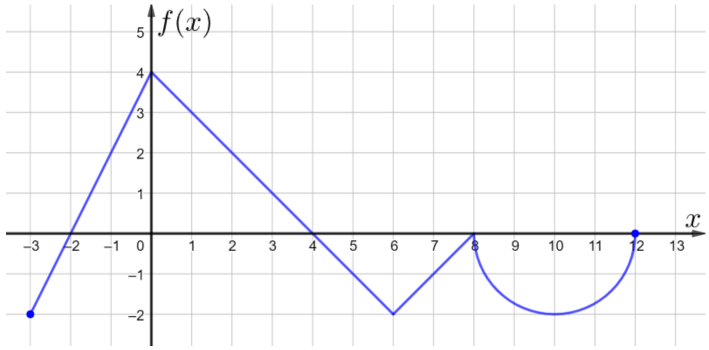
$E'(t)$ is decreasing for $0 \le t \le 10$ so the right Riemann sum approximation for $\int_0^{10} E'(t)dt$ is an underestimate.

(**e**) A tangent line to the graph of $y = E(t)$ at $t = 0$ is used to approximate $E(1)$. Does this approximation overestimate or underestimate $E(1)$? Give a reason for your answer.

The approximation is overestimate because $E'(t)$ is decreasing which means $E(t)$ is concave down and the tangent line lies above the curve.

Free Response Questions Stem Types: Verbal

2020 FRQ Practice Problem BC2



BC2: The function f is defined on the closed interval $[-3, 12]$. The graph of f, shown in the figure above, consists of three line segments and a semicircle centered at $(10, 0)$. Let g be the function defined by

$$g(x) = \int_4^x f(t)dt.$$

(**a**) Find all value(s)of x on the open interval $-3 < x < 12$ for which the function g has a local maximum. Justify your answer.

$g'(x) = f(x)$ There is a local maximum at $x = 4$ because $g'(x) = f(x)$ changes from positive to negative at $x = 4$.

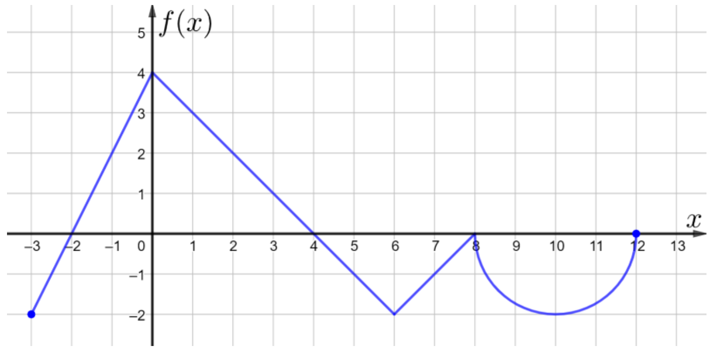
(**b**) On what open intervals contained in $-3 < x < 12$ is the graph of g both concave up and decreasing? Explain your reasoning.

$g(x)$ is concave up and decreasing when $g'(x) = f(x)$ is increasing and negative.
$g(x)$ is concave up and decreasing on three intervals $-3 < x < -2$ and $6 < x < 8$ and $10 < x < 12$.

(**c**) For $-3 < x < 12$, find all values of x for which the graph of g has a point of inflection. Explain your reasoning.

The graph of g has a point of inflection when $g'(x) = f(x)$ changes from increasing to decreasing or vice versa which also means $g'(x) = f(x)$ has an extrema.
The graph of g has a point of inflection when $x = 0, 6, 8,$ and 10.

The problem has been restated.



BC2: The function f is defined on the closed interval $[-3, 12]$. The graph of f, shown in the figure above, consists of three line segments and a semicircle centered at $(10, 0)$. Let g be the function defined by $g(x) = \int_4^x f(t)dt$.

(**d**) Determine the minimum value of g on the closed interval $[-3, 12]$. Justify your answer.

local mimimum candidate: $x=-2$ endpoint candidates: $x=-3,12$

x	$g(x)=\int_4^x f(t)dt$
-3	$g(-3)=\int_4^{-3} f(t)dt=-\int_{-3}^{4} f(t)dt=-\left[-\frac{1}{2}(1)(2)+\frac{1}{2}(6)(4)\right]=-[(-1)+(12)]=-11$
-2	$g(-2)=\int_4^{-2} f(t)dt=-\int_{-2}^{4} f(t)dt=-\left[\frac{1}{2}(6)(4)\right]=-12$
12	$g(12)=\int_4^{12} f(t)dt=-\left[\frac{1}{2}(2)(4)+\frac{1}{2}\pi(2)^2\right]=-[(4)+2\pi]=-4-2\pi\approx-10$

The minimumvalue of g on $[-3,12]$ is $g(-2)=-12$

(**e**) Find $\lim\limits_{x\to3}\dfrac{g(2x)+x-1}{1-e^{f(x+1)}}$.

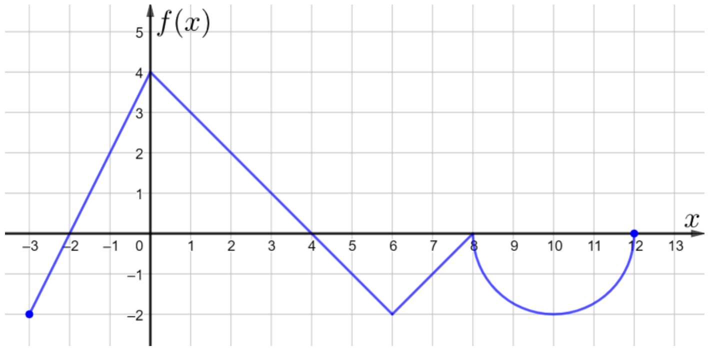
$$\lim_{x\to3}\left(g(2x)+x-1\right)=g(6)+3-1=-\frac{1}{2}(2)(2)+3-1=0$$

$$\lim_{x\to3}\left(1-e^{f(x+1)}\right)=1-e^{f(4)}=1-e^0=1$$

$\lim\limits_{x\to3}\dfrac{g(2x)+x-1}{1-e^{f(x+1)}}$ produces the indeterminant form $\dfrac{0}{0}$ so we can apply l'Hospital's Rule

$$\lim_{x\to3}\frac{g(2x)+x-1}{1-e^{f(x+1)}}=\underbrace{\lim_{x\to3}\frac{g'(2x)(2)+1}{-e^{f(x+1)}f'(x+1)}}_{\text{l'Hospital's Rule}}=\frac{g'(6)(2)+1}{-e^{f(4)}f'(4)}=\frac{f(6)(2)+1}{-e^{(0)}(-1)}=\frac{(-2)(2)+1}{1}=-3$$

The problem has been restated.



BC2: The function f is defined on the closed interval $[-3, 12]$. The graph of f, shown in the figure above, consists of three line segments and a semicircle centered at $(10, 0)$. Let g be the function defined by

$$g(x) = \int_4^x f(t)dt.$$

(f) Let $H(x) = \begin{cases} \dfrac{x}{g(x)} & x < 8 \\ -2\ln|e + f(x)| & x \geq 8 \end{cases}$. Is $H(x)$ continuous at $x = 8$? Why or why not?

$$H(8) = -2\ln|e + f(8)| = -2\ln|e + (0)| = -2\ln|e| = -2(1) = -2$$

$$\lim_{x\to 8^-} H(x) = \lim_{x\to 8^-} \frac{x}{g(x)} = \frac{8}{g(8)} = \frac{8}{\int_4^8 f(t)\,dt} = \frac{8}{\left[-\frac{1}{2}(4)(2)\right]} = \frac{8}{-4} = -2$$

$$\lim_{x\to 8^+} H(x) = \lim_{x\to 8^-} \left(-2\ln|e + f(x)|\right) = \lim_{x\to 8^-} \left(-2\ln|e + (0)|\right) = -2$$

$H(x)$ is continuous at $x = 8$ because $\lim_{x\to 8} H(x) = H(8)$.