2021-01-04

Monday, January 4, 2021   10:02 AM

Mental Math

1) $2381 + 7937 = 9000 + 1200 + 110 + 8$

$= 10318$

2) $85^2 = (80+5) \times 85$

$= 80 \times 85 + 5 \times 85$

$= 8 \times (80+5) \times 10 + 5 \times (80+5)$

$= (640 + 40) \times 10 + 400 + 25$

$= 7225$

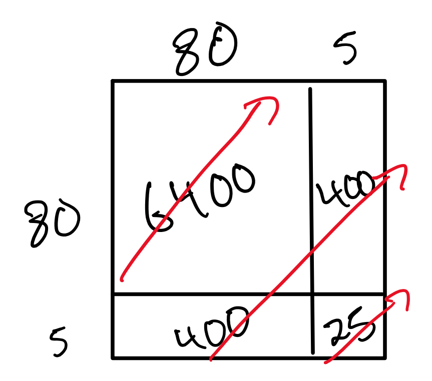
OR

$85^2 = 7225$

$8 \times (8+1)$

Why does this work? We will see soon. (*)

OR



$6400 + 800 + 25 = 7225$

OR

$(80+5) \times (80+5) = 80^2 + 80 \cdot 5 + 5 \cdot 80 + 5^2$

$= 80^2 + 2 \cdot 80 \cdot 5 + 5^2$

$(80 + 5)^2 = 80^2 + 2 \cdot 80 \cdot 5 + 5^2$

Generally, $(a+b)^2 = a^2 + 2ab + b^2$

(*) Any number ending with 5, say $\underline{a5}$, can be written as

$10 \times a + 5$

$\underline{a5}^2 = (10a+5)^2 = (10a)^2 + 2 \cdot 10a \cdot 5 + 5^2$

$= 100a^2 + 100a + 25$

$= 100\, a(a+1) + 25$

shift 2 places   1 more

Review A

1) Multiply (14/15) x (25/7). If necessary, express your answer as an improper fraction in simplest form.

$\frac{14}{15} \times \frac{25}{7} = \frac{2 \times 5}{3 \times 1} = \frac{10}{3}$

2) Apples cost \$1.05 each and oranges \$1.15 each. You buy four apples and three oranges and you pay with a \$10 bill. How much change does the seller give back to you? Write your answer in dollars and cents, with no units.

$4 \times 1.05 + 3 \times 1.15$

$= 7 + 0.20 + 0.45$

$= 7.65$

$10 - 7.65 = 2.35$

Check: $2.35 + 7.65 = 10$

3) Solve for y if 4y+2 = 7. If necessary, express your answer as an improper fraction in simplest form.

$4y + 2 - 2 = 7 - 2$

$4y = 5$

$\frac{4y}{4} = \frac{5}{4}$

$y = \frac{5}{4}$

4) Alice has an extensive library of books, but has not finished reading all of them. The ratio of finished books to unfinished books is 2:5. A week later, Alice has finished 24 books. The new ratio of finished books to unfinished books is then 4:7. How many books are in her library?

① What is not changing? The total.

2:5   2⬠ : 5⬠ → 7⬠

4:7   4△ : 7△ → 11△

② LCM

2 : 5   × 11   :   22 : 55

4 : 7   × 7   :   28 : 49

(22□ 55□ / 28□ 49□)

③ Difference is the given change

22□ → 28□

6□ = 24

④ Solve for □ (maybe)

□ = 4

⑤ Conclusion

Common mistake: · 2+5=7 □

· mixing up finished vs unfinished vs ~~total~~

· mixing up before / after

$22\square + 55\square = 77\square = 77 \times 4 = 308$

(Check)

2 pails and 3 buckets weighs 43 pezos. 3 pails and 7 buckets weights 82 pezos. How many pezos does a bucket weigh?

Review B

1) Compute the following product. Leave your answer as a fraction in simplest form (improper, if necessary).

$$\frac{1}{22} \times \frac{22}{34} \times \frac{34}{46} \times \frac{46}{58} \times \frac{58}{61} \times \frac{61}{73}$$

2) The ratio of girls to boys at a school was 49:34. After 10 girls left, the ratio of girls to boys at the party was 17:12. Find the number of children in the school after the girls left.

3) Solve for x if the following equations hold. Leave your answer as a fraction in simplest form (improper, if necessary).

$$28x + 17y = 271$$
$$35x + 22y = 168$$