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Mini-math Div 3/4: Monday, December 14, 2020 (12 minutes)

1. Evaluate each of the following limits. You may use any of the three (generalized) Fundamental Trigonometric Limits without proof.

   (a) (2 points) $\displaystyle\lim_{x \to 0} \frac{\sin 4x}{\sin 3x}$

   > **Solution:** By the (generalized) First Fundamental Trigonometric Limit
   >
   > $$\lim_{x \to 0} \frac{\sin 4x}{\sin 3x} = \lim_{x \to 0} \frac{\sin 4x}{4x} \cdot \frac{3x}{\sin 3x} \cdot \frac{4}{3} = \frac{4}{3}$$

   (b) (2 points) $\displaystyle\lim_{x \to 0} \frac{\cos 2x}{x + 2\cos x}$

   > **Solution:** By continuity,
   >
   > $$\lim_{x \to 0} \frac{\cos 2x}{x + 2\cos x} = \frac{1}{0 + 2} = \frac{1}{2}$$

   (c) (2 points) $\displaystyle\lim_{x \to 0} \frac{\sin^2 x \cos x}{1 - \cos x}$

   > **Solution:** By continuity,
   >
   > $$\begin{aligned}
   > \lim_{x \to 0} \frac{\sin^2 x \cos x}{1 - \cos x} &= \lim_{x \to 0} \frac{\sin^2 x \cos x}{1 - \cos x} \cdot \frac{1 + \cos x}{1 + \cos x} \\
   > &= \lim_{x \to 0} \frac{\sin^2 x \cos x (1 + \cos x)}{1 - \cos^2 x} \\
   > &= \lim_{x \to 0} \frac{\sin^2 x \cos x (1 + \cos x)}{\sin^2 x} \\
   > &= \lim_{x \to 0} \cos x (1 + \cos x) = 1(1 + 1) = 2
   > \end{aligned}$$

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2. (a) (1 point) Find the derivative of $\sin x$ from first principles using the Newton quotient (you may use any of the three (generalized) Fundamental Trigonometric Limits without proof).

Solution:

$$\frac{d}{dx}(\sin x) = \lim_{h \to 0} \frac{\sin(x + h) - \sin x}{h}$$

$$= \lim_{h \to 0} \frac{\sin x \cos h + \sin h \cos x - \sin x}{h}$$

$$= \lim_{h \to 0} \frac{\sin x(\cos h - 1) + \sin h \cos x}{h}$$

$$= \sin x \cdot \lim_{h \to 0} \frac{\cos h - 1}{h} + \cos x \cdot \lim_{h \to 0} \frac{\sin h}{h}$$

$$= \sin x \cdot 0 + \cos x \cdot 1 = \cos x$$

(b) (1 point) Find the derivative of $\tan x$ using derivative rules (you may use the derivatives of $\sin x$ and $\cos x$ without proof).

Solution:

$$\frac{d}{dx}(\tan x) = \frac{d}{dx}\left(\frac{\sin x}{\cos x}\right)$$

$$= \frac{(\frac{d}{dx} \sin x) \cdot \cos x - (\frac{d}{dx} \cos x) \cdot \sin x}{(\cos x)^2}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x} = \sec^2 x$$