Mini-math Div 3/4: Friday, October 8, 2021 (15 minutes)

SOLUTIONS

1. (1 point) True or False: If a function is not differentiable at a point $x = a$, then a is not in the domain of f.

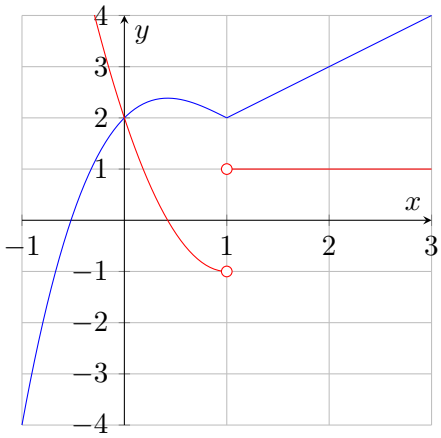
Solution: False.

2. (1 point) True or False: If f and g are differentiable at $x = a$, so is $\dfrac{f}{g}$.

Solution: False.

3. (2 points) Sketch the derivative of the following function on the same graph. Be sure to indicate, if any, approximate zeros, asymptotes, and general shape of the graph of the derivative.

Solution:



4. (2 points) Compute

$$\lim_{x \to 1} \frac{10^x - 10}{x - 1}$$

Solution: This limit is the derivative of $f(x) = 10^x$ using first principles, so

$$\lim_{x \to 1} \frac{10^x - 10}{x - 1} = \frac{d}{dx} 10^x \Big|_{x=1} = 10^x \ln 10 \Big|_{x=1} = 10 \ln 10$$

5. (2 points) Differentiate: $f(x) = \dfrac{x^2 + \sqrt{x} - \sin x}{x^2}$

Solution: First, $f(x) = 1 + x^{-3/2} - \sin x/x^2$, so

$$f'(x) = -\frac{3}{2}x^{-5/2} - \frac{x^2\cos x - 2x\sin x}{x^4} = -\frac{3}{2}x^{-5/2} - \frac{x\cos x - 2\sin x}{x^3}$$

6. (2 points) Differentiate: $f(x) = 2^x \sec x$

Solution:

$$f'(x) = 2^x \ln 2 \sec x - \sec x \tan x 2^x$$

7. (3 points) (AP) Consider differentiable functions $f(x)$ and $g(x)$ which have the following values and derivatives:

x	$-\frac{\pi}{6}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{5\pi}{6}$
$f(x)$	1	2	-1	-2	4
$g(x)$	-2	1	-3	-1	2
$f'(x)$	3	-2	2	1	-1
$g'(x)$	2	-3	1	-2	3

Find $h'(\pi/6)$ if $h(x) = \dfrac{g(x)\sin x}{f(x)}$

Solution:

$$h'(x) = \frac{g'(x)\cos x \cdot f(x) - f'(x)\cdot g(x)\sin x}{[f(x)]^2},$$

$$h'(\pi/6) = \frac{(-3)(\frac{\sqrt{3}}{2})\cdot 2 - (-2)\cdot 1\cdot \frac{1}{2}}{2^2}$$

$$= \frac{-3\sqrt{3}+1}{4}$$