Name: ____________________ Mark: ____________________

Mini-math Div 3/4: Monday, December 7, 2020 (12 minutes)

1. Determine all horizontal and vertical asymptotes of the following functions, or indicate there are no such asymptotes. (Show your work for vertical asymptotes via a sign analysis)

   (a) (2 points) $f(x) = \dfrac{3x^2 - 9x}{x^2 - 2x - 3}$

   Solution: Notice

   $$\lim_{x\to\pm\infty} f(x) = \lim_{x\to\pm\infty} \frac{3 - \frac{9}{x}}{1 - \frac{2}{x} - \frac{3}{x^2}} = 3$$

   so $y = 3$ is the only horizontal asymptote.
   Notice $f(x) = \dfrac{3x(x-3)}{(x-3)(x+1)}$, so $x = 3$ and $x = -1$ are potential vertical asymptotes.
   Near -1^-, the fraction is of the form $\frac{(-)(-)}{(-)(0^-)}$, so $\displaystyle\lim_{x\to-1^-} f(x) = \infty$, while

   $$\lim_{x\to 3} f(x) = \lim_{x\to 3} \frac{3x}{x+1} = \frac{9}{4}$$

   so only $x = -1$ is a vertical asymptote.

   (b) (2 points) $f(x) = \dfrac{2x^3 + 10x}{x^2 + 3x + 2}$

   Solution: Notice

   $$\lim_{x\to\pm\infty} f(x) = \lim_{x\to\pm\infty} \frac{2x + \frac{10}{x}}{1 + \frac{3}{x} + \frac{2}{x^2}} = \pm\infty$$

   so there are no horizontal asymptotes.
   Notice $f(x) = \dfrac{2x(x^2+5)}{(x+2)(x+1)}$, so $x = -2$ and $x = -1$ are potential vertical asymptotes.
   Near -2^-, the fraction is of the form $\frac{(-)(+)}{(0^-)(-)}$, so $\lim_{x\to-2^-} f(x) = -\infty$, while near -1^-, the fraction is of the form $\frac{(-)(+)}{(+)(0^-)}$, so $\lim_{x\to-1^-} f(x) = \infty$. Then both $x = -2, -1$ are vertical asymptotes.

2. (2 points) Consider the function $f(x) = x^4 + 6x^3 + 12x^2 + x + 1$. Find the interval(s) on which f is concave up.

Solution: Differentiating twice,

$$f'(x) = 4x^3 + 18x^2 + 24x + 1,$$
$$f''(x) = 12x^2 + 36x + 24$$

We find the critical points: $f''(x)$ always exists, and

$$0 = 12x^2 + 36x + 24 = 12(x^2 + 3x + 2) = 12(x+1)(x+2)$$

so the critical points are $x = -2, -1$.

On each subinterval, the second derivative has the following sign:

		-2		-1	
$x+2$	$-$	|	$+$	|	$+$
$x+1$	$-$	|	$-$	|	$+$
$f''(x)$	$+$	|	$-$	|	$+$

Then $f(x)$ is concave up on $(-\infty, -2)$ and $(-1, \infty)$.