Related Rates

SOLUTIONS

1 mark for diagram, if appropriate - otherwise, mark gets lumped into rest

1 mark for variables

1 mark for equation

1 mark for derivative

1 mark for plugging in given values

1 mark for finding necessary values (otherwise, previous mark is worth 2)

1 mark for solving for desired rate

1 mark for final answer, including units and correct contextual interpretation

1. (8 points) A woman 1.5 m tall walks toward a lamppost whose light is 4 m above the ground. If she walks at a speed of 2 m/s, at what rate is the tip of the shadow moving when she is 10 m from the lamppost (in m/s, to 3 decimal places) and in which direction?

Solution: Let x_s, x_p, x denote the distance from the tip of the shadow to the person, from the person to the lamppost, and the tip of the shadow to the lamppost respectively. By similar triangles,

$$\frac{x_s}{1.5} = \frac{x_p + x_s}{4}$$
$$x_s = \frac{1.5x_p}{2.5} = 0.6x_p$$

Notice $x = x_p + x_s$, so

$$x = x_p + 0.6x_p = 1.6x_p$$
$$\frac{dx}{dt} = 1.6\frac{dx_p}{dt} = 1.6 \cdot (-2) = -3.200\,\mathrm{m/s}$$

The tip of the shadow is moving 3.200 m/s towards the lamppost.

2. (8 points) A water trough is 10 m long and a cross-section has the shape of a triangle that is 4 m wide and 1 m tall, with the point towards the bottom. If water is being pumped in at a rate of $15\,\mathrm{m}^3/\mathrm{min}$, how fast is the water level changing when the water is $20\,\mathrm{cm}$ deep (in m/min, to 3 decimal places) and is it rising or lowering?

Solution: Let the height of the water be h and the width be b at a general time. By similar triangles,

$$\frac{b}{4} = \frac{h}{1} \qquad \Longrightarrow \qquad b = 4h$$

The volume is given by

$$V = \frac{1}{2}bh(10) = \frac{1}{2}(4h)h(10) = 20h^2$$

so

$$\begin{aligned} \frac{dV}{dt} &= 40h\frac{dh}{dt} \\ \frac{dh}{dt}\Big|_{h=0.2} &= \frac{V'}{40(0.2)} = \frac{15}{8} = 1.875\,\mathrm{m/min} \end{aligned}$$

The water level is rising at 1.875 m/min.

3. (8 points) Two cars start moving from the same point. One travels north at 80 km/h and the other travels west at 60 km/h. At what rate is the distance between the cars changing 2 hours later (in km/h, to 3 decimal places) and are the cars getting closer together or further apart?

Solution: Let x be the distance from the starting point to the westward card and y be the distance from the starting point to the northward car. Let D be the distance between the cars, so that

$$D^2 = x^2 + y^2$$

Differentiating,

$$2D\frac{dD}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$$

After 2 hours, $x = 120$, $y = 160$, and $D = \sqrt{x^2 + y^2} = 200$, so

$$\begin{aligned} 2D\frac{dD}{dt} &= 2x\frac{dx}{dt} + 2y\frac{dy}{dt} \\ \frac{dD}{dt} &= \frac{1}{200}\left((120)(60) + (160)(80)\right) = 100.000\,\mathrm{km/h} \end{aligned}$$

The cars are getting further apart at a rate of 100.000 km/h.