2020–2021 Winter Break Math Challlenges
Due Jan. 11 in-class to your math teacher

Challenge 1: Digit puzzle

To ring in the new year, make the number 2021 in 9 different ways, each time by using copies of the same digit and the following operations (in addition to parentheses):

- Standard operations: $+, -, \times, \div$
- Negation: $-\square$
- Exponentiation of two numbers: $\square^{\square}$
- Square root of a number: $\sqrt{\square}$
- Factorial: $\square!$ (Note: you may use iterated factorial but not muilti-factorial, so that $3!! = (3!)! = 6! = 720$, and **not** $3!! = 3 \times 1 = 3$.)
- Concatenation (i.e. "glueing") of digits (only of the original digit used): dd

Your score for a particular digit is the number of copies you use, and your goal is to have the lowest score possible (**you will be competing against others in your division**).

For example, you can make 2021 by using copies of the digit "9" as follows:

$$2021 = \underbrace{\frac{9}{9} + \frac{9}{9} + \cdots + \frac{9}{9}}_{2021 \text{ times}}.$$

If you do it like this, you are using 4042 copies of 9, which is not good for you. A far more efficient way to do it is

$$2021 = 9 \times \sqrt{9} \times \left(99 - \left(\sqrt{9} + \frac{9}{9}\right)!\right) - \sqrt{9}! + \frac{9}{9} + \frac{9}{9}$$

which gets you there with only 12 copies (this is, of course, not optimal).

(1) Using the digit 1

(2) Using the digit 2

(3) Using the digit 3

(4) Using the digit 4

(5) Using the digit 5

(6) Using the digit 6

(7) Using the digit 7

(8) Using the digit 8

(9) Using the digit 9

Challenge 2: Approximations

To say goodbye to the last year, here is a puzzle about 2020. How close can you get to 2020 using the digits $2, 0, 2, 0$ exactly once? You may use the following operations (in addition to parentheses):

- Standard operations: $+, -, \times, \div$
- Negation: $-\square$
- Exponentiation of two numbers: $\square^{\square}$
- Square root of a number: $\sqrt{\square}$
- Factorial: $\square!$
  - Note: you may use iterated factorial but not multi-factorial, so that $3!! = (3!)! = 6! = 720$, and **not** $3!! = 3 \times 1 = 3$.
  - Hint: $0! = 1$
- Floor: $\lfloor \square \rfloor$ (this is the largest integer less than or equal to the input, so $\lfloor \pi \rfloor = 3$)

Notice you CANNOT use "glueing"!

(**You will be competing against others in your division.**)

Challenge 3: Art

Winter is a great time for beautiful images, especially in Calgary where we get so much sunlight. For this challenge, draw or paint a picture! It must be

(1) winter-themed, and

(2) math-related.

You must also write a short paragraph explaining how your art satisfies these two points. You will be judged on:

(1) the aesthetic and technical quality of the art,

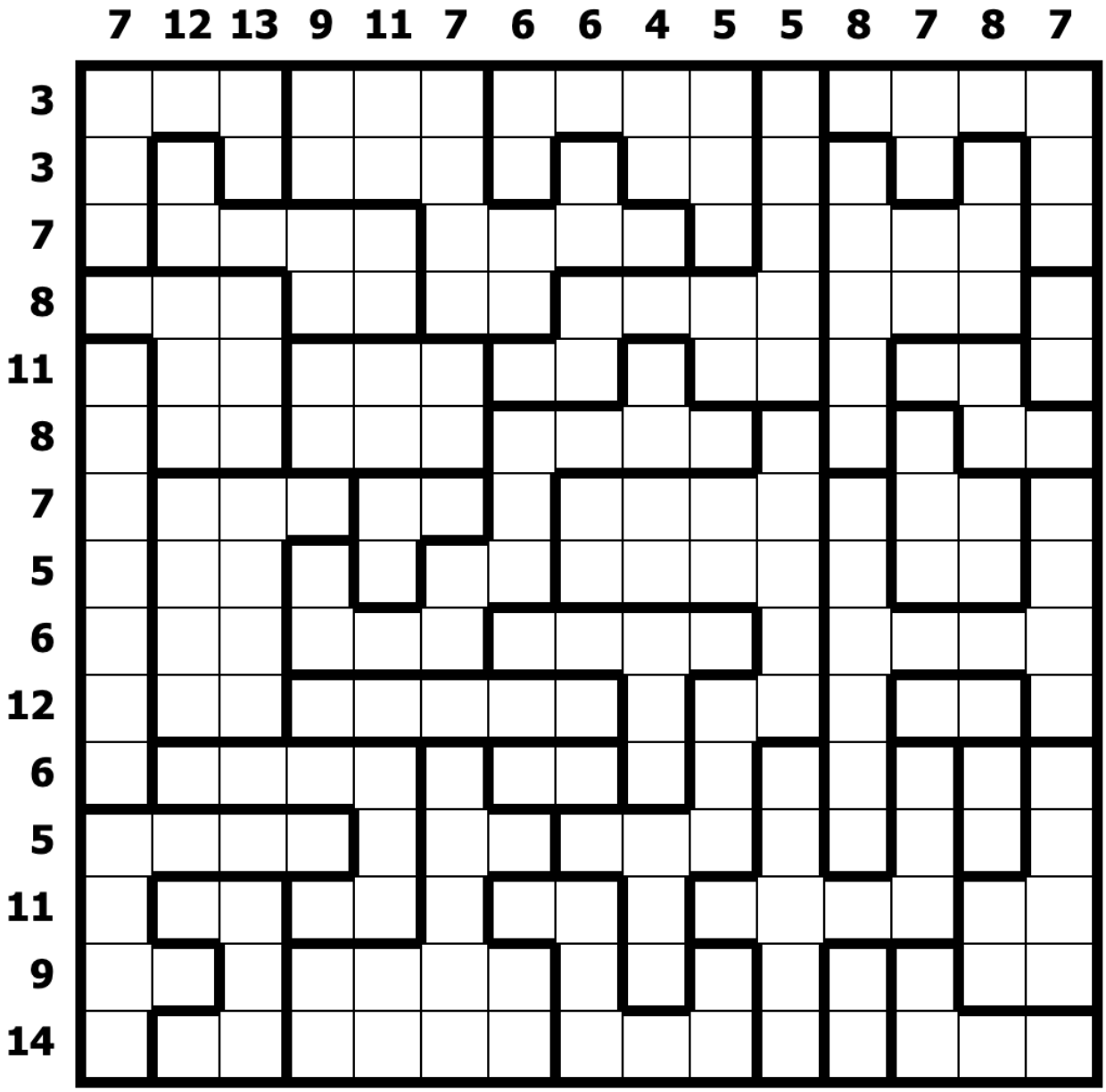
(2) the depth of connection to mathematics, and

(3) the quality of writing.

(**You will be competing against others in your division.**)

Challenge 4: Aquarium

In Aquarium, you'll find "cages," which are denoted by the bold outlines. You must shade some cells (indicating water) such that within a cage, all cells at the same row or below of a shaded cell are also shaded. The numbers outside the grid indicate how many cells are shaded in the respective row or column. A cage can be completely blank, completely shaded, or partially shaded. Solve the following Aquarium!



Challenge 5: Suguru

In Suguru, you'll find "cages," which are denoted by the bold outlines. Each cage needs to be filled with unique digits counting up from 1, so that a 1-cell cage only has a 1, a 2-cell cage has a 1 and a 2, a 3-cell cage has a 1, 2, and a 3, and so forth. Cells that are adjacent (touching) cannot contain the same number, including diagonally. Solve the following Suguru!