NO CALCULATOR IS ALLOWED FOR THIS QUESTION.

Show all of your work, even though the question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.

Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which f(x) is a real number.

$$f\left(x
ight) = egin{cases} rac{x^2 + kx - 2}{3x^2 + 4x + 3} & ext{for } x < -2 \ x^3 + 2 & ext{for } -2 \leq x < 0 \ 0 & ext{for } x = 0 \ rac{2e^x}{2 - e^x} & ext{for } x > 0 \end{cases}$$

Let f be the function defined above, where k is a constant.

- (a) For what value of k, if any, is f continuous at x = -2? Justify your answer.
 - Please respond on separate paper, following directions from your teacher.
- (b) What type of discontinuity does f have at x=0? Give a reason for your answer.
 - Please respond on separate paper, following directions from your teacher.
- (c) Find all horizontal asymptotes to the graph of f. Show the work that leads to your answer.

AP Calculus BC

Please respond on separate paper, following directions from your teacher.

A GRAPHING CALCULATOR IS REQUIRED FOR THIS QUESTION.

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Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which f(x) is a real number.

Let f be a twice-differentiable function with f(1.5) = 3. The derivative of f is given by $f'\left(x
ight)=\left(7x-19
ight)\sin\left(x^2-4x+4
ight)$ for $1\leq x\leq 4$.

(a) Find all values of x in the interval 1 < x < 4 at which f has a critical point. Classify each as the location of a relative minimum, a relative maximum, or neither. Justify your answers.

Please respond on separate paper, following directions from your teacher.

AP Calculus BC

(b) Use the line tangent to the graph of f at x = 1.5 to approximate f(1.8).

Please respond on separate paper, following directions from your teacher.

On the interval $1.5 \le x \le 1.8$, f'(x) < 0 and f''(x) > 0. Is the approximation found in part (b) an overestimate or an underestimate for f(1.8)? Give a reason for your answer.

Please respond on separate paper, following directions from your teacher.

Using the Mean Value Theorem, explain why the average rate of change of f over the interval $1 \le x \le 4$ cannot equal 6.5.

Please respond on separate paper, following directions from your teacher.

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Functions f, g, and h are twice-differentiable functions with $g\left(2\right)=h\left(2\right)=4$. The line

AP Calculus BC

 $y=4+rac{2}{3}(x-2)$ is tangent to both the graph of g at x=2 and the graph of h at x=2.

- a) Find h'(2).
 - Please respond on separate paper, following directions from your teacher.
- b) Let a be the function given by $a\left(x\right)=3x^{3}h\left(x\right)$. Write an expression for $a'\left(x\right)$. Find $a'\left(x\right)$.
 - Please respond on separate paper, following directions from your teacher.
- c) The function h satisfies $h(x)=\frac{x^2-4}{1-(f(x))^3}$ for $x\neq 2$. It is known that $\lim_{x\to 2}h(x)$ can be evaluated using L'Hospital's Rule. Use $\lim_{x\to 2}h(x)$ to find f(2) and f'(2). Show the work that leads to your answers.
 - Please respond on separate paper, following directions from your teacher.
- d) It is known that $g(x) \le h(x)$ for 1 < x < 3. Let k be a function satisfying $g(x) \le k(x) \le h(x)$ for 1 < x < 3. Is k continuous at x = 2? Justify your answer.
 - Please respond on separate paper, following directions from your teacher.
- 4.

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AP Calculus BC

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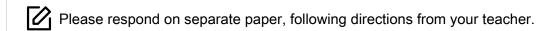
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\boldsymbol{x}	-1	0	1	2
$g\left(x\right)$	6	4	2	-1
$g'\left(x ight)$	-1	-7	-2	-3

The table above gives selected values for a differentiable and decreasing function g and its derivative. Let f be the function with f(1) = 0 and derivative given by $f'(x) = x \sin(x^2)$.

(a) Find $f''\left(e^{1.5}\right)$. Express your answer as a decimal approximation.



(b) Let h be the function defined by h(x) = f(g(3x)). Find h'(0). Express your answer as a decimal approximation.

Please respond on separate paper, following directions from your teacher.

(c) Write an equation for the line tangent to the graph of g^{-1} , the inverse function of g, at x=-1.

AP Calculus BC



Please respond on separate paper, following directions from your teacher.

The point (1,4) lies on the curve in the xy-plane given by the equation $f(x)+(g(x))^2=xy$. What is the value of $\frac{dy}{dx}$ at the point (1,4) ? Express your answer as a decimal approximation.



Please respond on separate paper, following directions from your teacher.

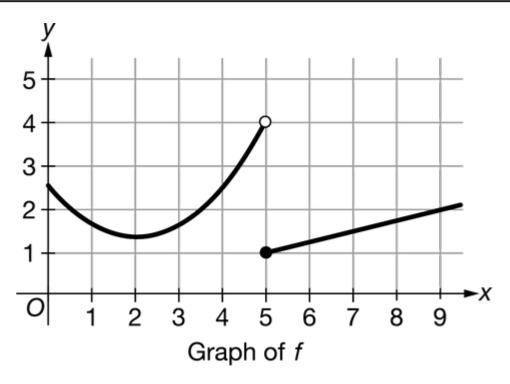
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AP Calculus BC



\boldsymbol{x}	3	3.5	4	4.5	5	5.5	6
f(x)	1.653	2.02	2.533	3.193	1	1.125	1.25

The graph of the function f and a table of selected values of f(x) are shown above. The graph of f has a horizontal tangent line at x = 2, is concave up for 0 < x < 5, and is linear for $x \ge 5$.

(a) Approximate the value of f'(4.5) using data from the table. Show the computations that lead to your answer.

Please respond on separate paper, following directions from your teacher.

(b) Is there a value of x, for 0 < x < 5, such that $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = 0$? Give a reason for your answer.

Please respond on separate paper, following directions from your teacher.

(c) For each of the following limits, find the value or explain why it does not exist.

- (i) $\lim_{h\to 0^+} \frac{f(5+h)-f(5)}{h}$
- (ii) $\lim_{h \to 0^-} \frac{f(5+h)-f(5)}{h}$
- (iii) $\lim_{h \to 0} \frac{f(5+h)-f(5)}{h}$



Please respond on separate paper, following directions from your teacher.

A particle moves along the x-axis so that its velocity at any time $t \ge 0$ is given by $v(t) = 3t^2 - 2t - 1$. The position x(t) is 5 for t = 2.

- For what values of $t,0 \le t \le 3$, is the particle's instantaneous velocity the same as its average velocity on the closed interval [0,3]?

Please respond on separate paper, following directions from your teacher.

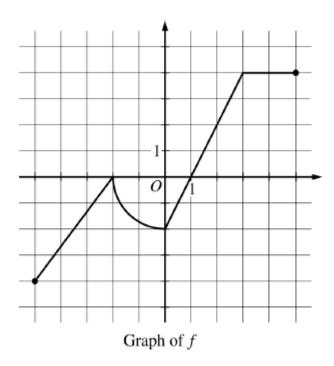
CALCULUS AB 7.

SECTION II, Part B

Time - 1 hour

Number of questions - 4

NO CALCULATOR IS ALLOWED FOR THESE QUESTIONS.



- 3. The graph of the function f, consisting of three line segments and a quarter of a circle, is shown above. Let g be the function defined by $g(x)=\int_1^x f(t)\,dt$.
- (a) Find the average rate of change of g from x = -5 to x = 5.
 - Please respond on separate paper, following directions from your teacher.
- (b) Find the instantaneous rate of change of g with respect to x at x = 3, or state that it does not exist.
 - Please respond on separate paper, following directions from your teacher.
- (c) On what open intervals, if any, is the graph of g concave up? Justify your answer.
 - Please respond on separate paper, following directions from your teacher.
- (d) Find all x-values in the interval -5 < x < 5 at which g has a critical point. Classify each critical point as the location of a local minimum, a local maximum, or neither. Justify your answers.

AP Calculus BC

Please respond on separate paper, following directions from your teacher.

Consider the differential equation $\frac{dy}{dx} = \frac{3-x}{y}$.

- **8.** Let y = f(x) be the particular solution to the given differential equation for 1 x y = -2 is tangent to the graph of f. Find the x-coordinate of the point of tangency, and determine whether f has a local maximum, local minimum, or neither at this point. Justify your answer.

Please respond on separate paper, following directions from your teacher.

9. EALCULUS BC

SECTION II, Part A

Time - 30 minutes

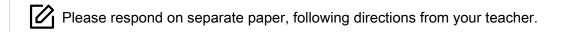
Number of questions - 2

A GRAPHING CALCULATOR IS REQUIRED FOR THESE QUESTIONS.

t (minutes)	0	1	5	6	8
g(t) (cubic feet per minute)	12.8	15.1	20.5	18.3	22.7

- 1. Grain is being added to a silo. At time t=0, the silo is empty. The rate at which grain is being added is modeled by the differentiable function g, where g(t) is measured in cubic feet per minute for $0 \le t \le 8$ minutes. Selected values of g(t) are given in the table above.
- (a) Using the data in the table, approximate g'(3). Using correct units, interpret the meaning of g'(3) in the context of the problem.

AP Calculus BC



- (b) Write an integral expression that represents the total amount of grain added to the silo from time t=0 to time t=8. Use a right Riemann sum with the four subintervals indicated by the data in the table to approximate the integral.
 - Please respond on separate paper, following directions from your teacher.
- (c) The grain in the silo is spoiling at a rate modeled by $w(t)=32\cdot\sqrt{\sin\left(\frac{\pi t}{74}\right)}$, where w(t) is measured in cubic feet per minute for $0\leq t\leq 8$ minutes. Using the result from part (b), approximate the amount of unspoiled grain remaining in the silo at time t=8.
 - Please respond on separate paper, following directions from your teacher.
- (d) Based on the model in part (c), is the amount of unspoiled grain in the silo increasing or decreasing at time t=6? Show the work that leads to your answer.
 - Please respond on separate paper, following directions from your teacher.

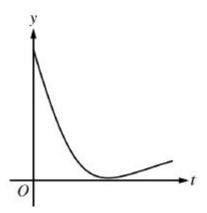
At time $t,t \ge 0$, the volume of a sphere is increasing at a rate proportional to the reciprocal of its radius. At t=0, the radius of the sphere is 1 and at t=15, the radius is 2. (The volume V of a sphere with a radius t=15, the ra

- **10.** Find the radius of the sphere as a function of *t*.
 - Please respond on separate paper, following directions from your teacher.
- 11. 5. During a chemical reaction, the function y=f(t) models the amount of a substance present, in grams, at time t seconds. At the start of the reaction (t=0), there are 10 grams of the substance present. The function y=f(t) satisfies the differential equation $\frac{dy}{dt}=-0.02y^2$.

(a) Use the line tangent to the graph of y = f(t) at t = 0 to approximate the amount of the substance remaining at time t = 2 seconds.

Please respond on separate paper, following directions from your teacher.

(b) Using the given differential equation, determine whether the graph of f could resemble the following graph. Give a reason for your answer.



Please respond on separate paper, following directions from your teacher.

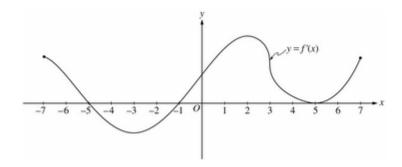
(c) Find an expression for y=f(t) by solving the differential equation $\frac{dy}{dt}=-0.02y^2$ with the initial condition f(0)=10.

Please respond on separate paper, following directions from your teacher.

(d) Determine whether the amount of the substance is changing at an increasing or a decreasing rate. Explain your reasoning.

Please respond on separate paper, following directions from your teacher.

AP Calculus BC



The figure above shows the graph of f', the derivative of the function f, for $-7 \le x \le 7$. The graph of f' has horizontal tangent lines at x = -3, x = 2, and x = 5, and a vertical tangent line at x = 3.

- **12.** At what value of x, for $-7 \le x \le 7$, does f attain its absolute maximum? Justify your answer.
 - Please respond on separate paper, following directions from your teacher.
- **13.** Find all values of *x*, for -7 x f attains a relative minimum. Justify your answer.
 - Please respond on separate paper, following directions from your teacher.
- **14.** Find all values of x, for -7 x f''(x)
 - Please respond on separate paper, following directions from your teacher.
- 15. NO CALCULATOR IS ALLOWED FOR THIS QUESTION.

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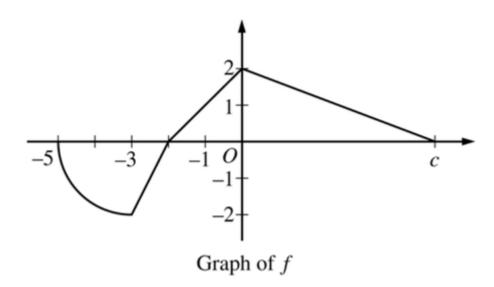
Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which f(x) is a real number.

Let f be an increasing function with f(0)=3. The derivative of f is given by $f'(x)=\cos(\pi x)+x^4+6$.

- (a) Find f''(-3).
 - Please respond on separate paper, following directions from your teacher.
- (b) Write an equation for the line tangent to the graph of $y = (f(x))^2$ at x = 0.
 - Please respond on separate paper, following directions from your teacher.
- (c) Let g be the function defined by $g\left(x
 ight)=f\left(\sqrt{2x^{2}+7}
 ight)$. Find $g'\left(3
 ight)$.
 - Please respond on separate paper, following directions from your teacher.
- (d) Let h be the inverse function of f. Find h'(3).
 - Please respond on separate paper, following directions from your teacher.

The derivative of a function f is given by $f'(x) = (x - 3)e^x$ for x > 0, and f(1) = 7.

- **16.** On what intervals, if any, is the graph of *f* both decreasing and concave up? Explain your reasoning.
 - Please respond on separate paper, following directions from your teacher.
- 17. 6. Consider the curve given by the equation $2(x-y)=3+\cos y$. For all points on the curve, $\frac{2}{3}\leq \frac{dy}{dx}\leq 2$.
 - (a) Show that $\frac{dy}{dx} = \frac{2}{2-\sin y}$.
 - Please respond on separate paper, following directions from your teacher.
 - (b) For $-\frac{\pi}{2} < y < \frac{\pi}{2}$, there is a point P on the curve through which the line tangent to the curve has slope 1. Find the coordinates of the point P.
 - Please respond on separate paper, following directions from your teacher.
 - (c) Determine the concavity of the curve at points for which $-\frac{\pi}{2} < y < \frac{\pi}{2}$. Give a reason for your answer.
 - Please respond on separate paper, following directions from your teacher.
 - (d) Let y=f(x) be a function, defined implicitly by $2(x-y)=3+\cos y$, that is continuous on the closed interval $[2,\ 2.1]$ and differentiable on the open interval $(2,\ 2.1)$. Use the Mean Value Theorem on the interval $[2,\ 2.1]$ to show that $\frac{1}{15} \leq f(2.1) f(2) \leq \frac{1}{5}$.
 - Please respond on separate paper, following directions from your teacher.



The function f is defined on the interval $-5 \le x \le c$, where c > 0 and f(c) = 0. The graph of f, which consists of three line segments and a quarter of a circle with center (-3,0) and radius 2, is shown in the figure above.

18. For $-5 \le x \le c$, let g be the function defined by $g(x) = \int_{-1}^{x} f(t)dt$. Find the x-coordinate of each point of inflection of the graph of g. Justify your answer.

Please respond on separate paper, following directions from your teacher.