Mini-math Div 3/4: Monday, October 25, 2021 (15 minutes)

SOLUTIONS

1. Find $\dfrac{dy}{dx}$ in the following cases:

   (a) (2 points) $y = xg(\sin x)$

   Solution:

   $$\frac{dy}{dx} = g(\sin x) + g'(\sin x)\cos x$$

   (b) (2 points) $y = e^{e^{\tan x}}$

   Solution:

   $$\frac{dy}{dx} = e^{e^{\tan x}} e^{\tan x} \sec^2 x$$

   (c) (2 points) (AP) $y = \frac{\arctan x}{x}$

   Solution:

   $$\frac{dy}{dx} = \frac{\frac{x}{x^2+1} - \arctan x}{x^2} \quad \text{or} \quad \frac{1 - (x^2+1)\arctan x}{(x^2+1)x^2}$$

   (d) (2 points) $x^2 y - x = \sin(x + y)$

   Solution:

   $$2xy + x^2\frac{dy}{dx} - 1 = \cos(x+y)\left(1 + \frac{dy}{dx}\right)$$
   $$\frac{dy}{dx} = \frac{\cos(x+y) - 2xy + 1}{x^2 - \cos(x+y)}$$

2. (2 points) Find y'' if $y = \tan x^2$.

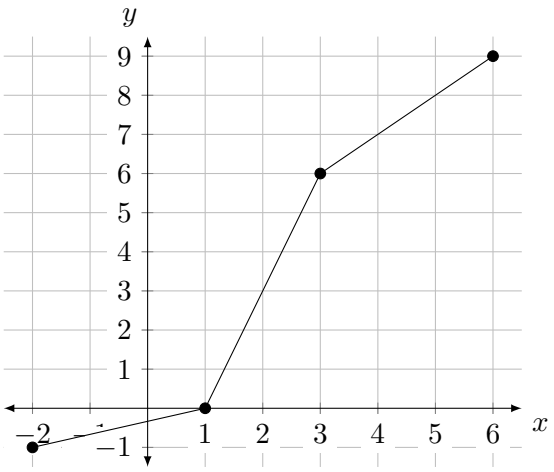
   Solution:

   $$y' = 2x\sec^2(x^2)$$
   $$y'' = 2\sec^2(x^2) + 2x \cdot 2\sec(x^2) \cdot \sec(x^2)\tan(x^2) \cdot 2x$$
   $$= 2\sec^2(x^2) + 8x^2\sec^2(x^2)\tan(x^2)$$

3. (2 points) The differentiable function $f(x)$ has the following values and derivatives:

x	$-2/3$	$-1/3$	3	6	7	8
$f(x)$	1	2	-9	-11	5	3
$f'(x)$	3	4	9	10	7	8

The function $g(x)$ is shown below:



Graph of g

If $h(x) = f(g(x))$, find the equation of the tangent line to $h(x)$ at $x = 2$.

Solution: $h'(2) = f'(g(2))g'(2) = f'(3) \cdot (3) = (9) \cdot (3) = 27$

Then the equation of the tangent line is, by point-slope,

$$y + 9 = 27(x - 2)$$

4. (2 points) (AP) Suppose f and g are differentiable functions where $g(x) = f^{-1}(x)$ for all x. Suppose further that

$$f(-9) = 7, \quad f(8) = 6, \quad f(6) = -9, \quad f(7) = 8$$
$$f'(-9) = 6, \quad f'(8) = 4, \quad f'(6) = -4, \quad f'(7) = 3$$

Find $g'(-9)$.

Solution:

$$g'(-9) = \frac{1}{f'(g(-9))} = \frac{1}{f'(6)} = \frac{1}{-4}$$