Mini-math Div 3/4: Monday, September 21, 2020

(1) True or false: The value of $\lim_{x\to a} f(x)$ is $f(a)$, assuming $f(a)$ is defined.

Solution: False - only for continuous functions.

(2) True or false: $\lim_{x\to a} f(x)$ can only exist if the left and right limits exist are are equal.

Solution: True.

(3) What method would you use to solve

$$\lim_{h\to 0} \frac{(2+h)^2-4}{h}?$$

For an extra half point, what is the limit?

Solution: Expand, simplify, and reduce h. Alternatively: factor as difference of squares, then simplify and reduce. Answer: 4

(4) What method would you use to solve

$$\lim_{x\to 9} \frac{\sqrt{x}-3}{x-9}?$$

For an extra half point, what is the limit?

Solution: factor and reduce, or rationalize and reduce. Answer: 1/6

(5) What method would you use to solve

$$\lim_{x\to 2} \frac{x^2+x+1}{3x^2+1}?$$

For an extra half point, what is the limit?

Solution: Plug it in. Answer: 7/13

(6) What method would you use to solve

$$\lim_{x\to 2} \frac{|x-2|}{x-2}?$$

For an extra half point, what is the limit?

Solution: Split into left and right limits, evaluate the absolute value. Answer: DNE

Long solution:

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} \frac{-(x-2)}{x-2} = -1,$$
$$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} \frac{x-2}{x-2} = 1$$

so the limit DNE.

(7) Where is the following function discontinuous? Identify the type of discontinuity, if any.

$$f(x) = \begin{cases} 0 & \text{if } x \le 0 \\ x & \text{if } 0 < x < 2 \\ 1 & \text{if } x \ge 2 \end{cases}$$

Solution: Only discontinuous at 2; jump discontinuity

Long solution: Since $f(x)$ is continuous on each piece, we need only worry about where the function stitches together.

At $x = 0$:

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} 0 = 0,$$
$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} x = 0$$

so $\lim_{x\to 0} f(x) = 0$. Since $f(0) = 0$ also, f is continuous at $x = 0$.

At $x = 2$:

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} x = 2,$$
$$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} 1 = 1$$

so $\lim_{x\to 2} f(x)$ DNE. So $f(x)$ has a (jump) discontinuity at $x = 2$.

(8) If $s(t)$ represents the position of a particle at time t, write an expression which represents the velocity of the particle at time $t = a$.

Solution:

$$\lim_{h\to 0} \frac{s(a+h) - s(a)}{h} \quad \text{or} \quad \lim_{t\to a} \frac{s(t) - s(a)}{t - a}$$

(9) What method would you use to solve

$$\lim_{n\to\infty} \frac{3n^2 + 1}{2n^2 - 4n + 1}?$$

For an extra half point, what is the limit?

Solution: Divide by highest power of the denominator. Answer: 3/2

(10) Find the sum of

$$\sum_{n=2}^{\infty} 2 \cdot \frac{1}{3^n}$$

Solution:

$$\frac{2/3^2}{1 - 1/3} = \frac{2}{9-3} = \frac{2}{6} = \frac{1}{3}$$