

Mini-math Div 3/4: Monday, January 9, 2023 (20 minutes)

SOLUTIONS

1. Write a differential equation that describes the following relationships. If necessary, use  $k$  as the constant of proportionality.

- (a) (2 points) The rate of change of population,  $P$ , with respect to time,  $t$ , is inversely proportional to the square root of time and directly proportional to the area,  $A$ , that the population covers.

**Solution:**

$$\frac{dP}{dt} = \frac{kA}{\sqrt{t}}$$

- (b) (2 points) The position of a particle is given by  $s(t)$ , where  $t$  is measured in seconds. Its acceleration is directly proportional to its position. When the particle is at position 4 units, its acceleration is 2 units/ $s^2$ .

**Solution:**

$$\frac{d^2s}{dt^2} = \frac{1}{2}s$$

2. (4 points) Determine the value of  $k$ , if any, for which  $y = \sin(2x) - k \sin(4x)$  would be a solution to the differential equation  $y'' + 4y = 3 \sin(4x)$ .

**Solution:**

$$\begin{aligned}y' &= 2 \cos(2x) - 4k \cos(4x) \\y'' &= -4 \sin(2x) + 16k \sin(4x)\end{aligned}$$

Then

$$[-4 \sin(2x) + 16k \sin(4x)] + 4 [\sin(2x) - k \sin(4x)] = 3 \sin(4x)$$

so  $16k - 4k = 3$ , giving  $k = 1/4$ .

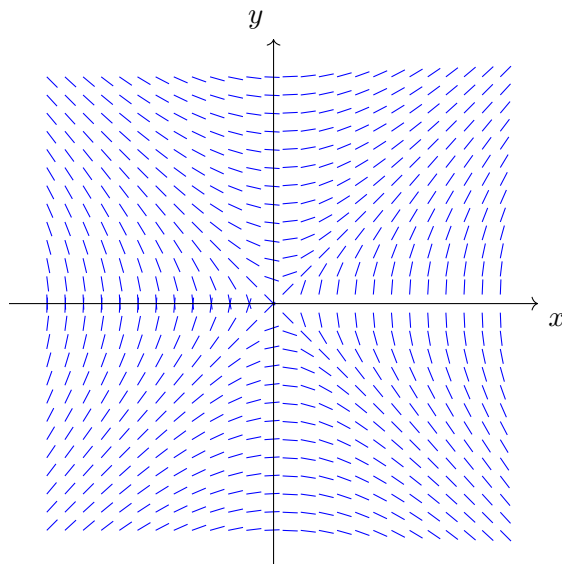
3. (2 points) (AP) What differential equation can the slope field to the right represent?

A.  $\frac{dy}{dx} = 1/x$

B.  $\frac{dy}{dx} = 1/y$

C.  $\frac{dy}{dx} = x/y$

D.  $\frac{dy}{dx} = y/x$



**Solution:** (C)

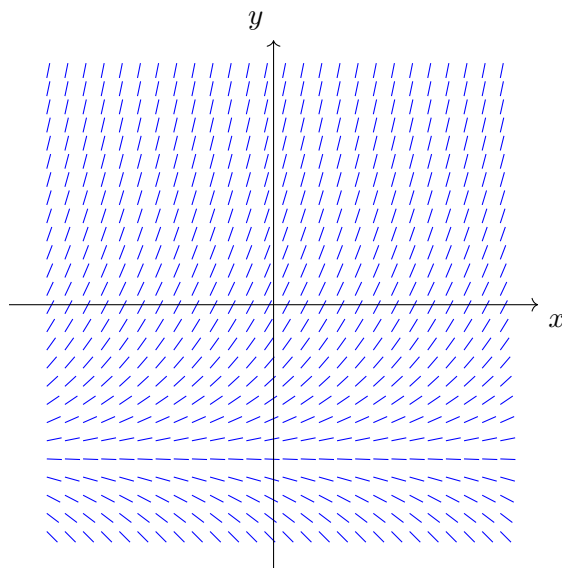
4. (2 points) (AP) The slope field for a certain differential equation is shown to the right. Which of the following could be a particular solution to the differential equation?

A.  $y = x^3$

B.  $y = \frac{1}{x+2}$

C.  $y = -2^x - 2$

D.  $y = e^{-x} - 2$



**Solution:** (C)

5. Consider the initial value problem  $\frac{dy}{dx} = 2x + y$  and  $y(1) = 2$ .

(a) (2 points) Find an approximation of  $y(1.2)$  using Euler's Method with two equal steps.

**Solution:**

$$y(1.1) \approx y(1) + y'(1)(1.1 - 1) = 2 + (2(1) + 2)(0.1) = 2.4,$$

$$y(1.2) \approx y(1.1) + y'(1.1)(1.2 - 1.1) \approx 2.4 + (2(1.1) + 2.4)(0.1) = 2.4 + (4.6)(0.1) = 2.86$$

(b) (2 points) Is your estimate in part (a) an overestimate or an underestimate?

**Solution:** At  $(1, 2)$ ,

$$\frac{d^2y}{dx^2} = 2 + \frac{dy}{dx} = 2 + 2x + y > 0$$

so the function is concave up. Therefore, the estimate will be an underestimate.

(Aside: the actual value is about 2.92842. If we used 10 equal steps, we would get the approximation 2.91397)