

Mini-math AP Calculus BC: Friday, February 11, 2022 (8 minutes)

SOLUTIONS

1. (2 points) Determine whether the following series converges or diverges:

$$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$$

**Solution:** By inspection (or using the substitution  $u = \ln x$ ),

$$\begin{aligned} \int_2^{\infty} \frac{1}{x(\ln x)^2} dx &= \lim_{b \rightarrow \infty} \int_2^b \frac{1}{x(\ln x)^2} dx = \lim_{b \rightarrow \infty} \left. -\frac{1}{\ln x} \right|_2^b \\ &= \lim_{b \rightarrow \infty} -\frac{1}{\ln b} + \frac{1}{\ln 2} = \frac{1}{\ln 2} < \infty \end{aligned}$$

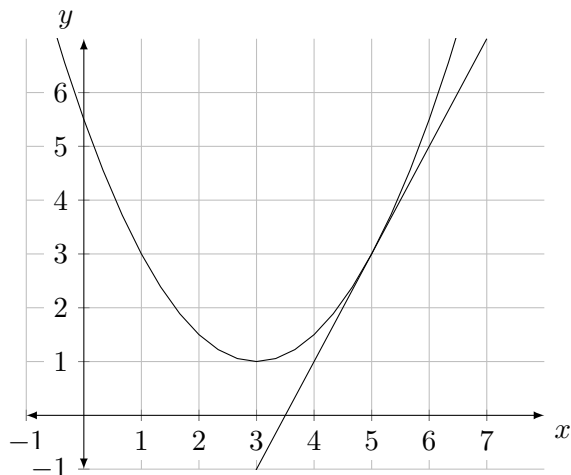
By the integral test,  $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$  converges.

2. (2 points) If a function  $f$  is approximated by the third order Taylor polynomial  $2 - 5(x - 2) + 4(x - 2)^2 - 3(x - 2)^3$  centred at  $x = 2$ , what is  $f'''(2)$ ?

**Solution:**

$$\begin{aligned} -3 &= \frac{f'''(2)}{3!} \\ f'''(2) &= -3 \cdot 3! = -18 \end{aligned}$$

3. (2 points) The figure below shows the graph of the differentiable function  $f$  and the line tangent to the graph of  $f$  at the point  $(5, 3)$ . Let  $g$  be the function given by  $g(x) = \int_5^x f(t) dt$ . Find the 2nd order Taylor polynomial for  $g(x)$  centred at  $a = 5$ .



**Solution:** By the Fundamental Theorem of Calculus Part I,  $g'(x) = f(x)$ . Then

$$\begin{aligned} P_2(x) &= g(5) + g'(5)(x - 5) + g''(5)(x - 5)^2 \\ &= 0 + f(5)(x - 5) + f'(5)(x - 5)^2 \\ &= 3(x - 5) + 2(x - 5)^2 \end{aligned}$$

4. (2 points) Suppose we know the following bounds:

$$|f^{(2)}(c)| \leq 2, \quad |f^{(3)}(c)| \leq 5, \quad |f^{(4)}(c)| \leq 3,$$

for any  $c$  on the interval  $[0, 1]$ . Use the Lagrange error bound to estimate the absolute value of the error in using a 3rd degree Maclaurin polynomial to approximate  $f(0.1)$ .

**Solution:**

$$|Error| \leq \frac{|f^{(4)}(c)|}{4!} |0.1 - 0|^4 \leq \frac{3}{24} \cdot \frac{1}{10000} = \frac{1}{80000}$$