Mini-math Div 3/4: Wednesday, October 14, 2020 (20 minutes)

(1) Find y' if $y = \dfrac{20x^2+21}{x}$.

Solution: $y = 20x + 21/x$, so $y' = 20 - 21/x^2$

(2) Find y' if $y = \dfrac{x}{20x^2+21}$

Solution: $y' = \dfrac{(20x^2+21) - 40x \cdot x}{(20x^2+21)^2} = \dfrac{21-20x^2}{(x^2+1)^2}$

(3) Find $\dfrac{df}{dt}$ if $f(t) = (t^2+1)\sqrt{t^2-1}$

Solution: $\dfrac{df}{dt} = 2t\sqrt{t^2-1} + \dfrac{t(t^2+1)}{\sqrt{t^2-1}} = \dfrac{3t^3-t}{\sqrt{t^2-1}}$

(4) Find $\dfrac{df}{dg}$ if $f(g) = \sqrt{\sqrt{g+1}+1}$ and $g(x) = x^2+1$

Solution: $\dfrac{df}{dg} = \dfrac{1}{2\sqrt{\sqrt{g+1}+1}} \cdot \dfrac{1}{2\sqrt{g+1}} = \dfrac{1}{4\sqrt{\sqrt{g+1}+1}\sqrt{g+1}}$

(5) Find an equation of the line tangent to the curve

$$xy + 7 = x^3 + y^3$$

at the point $(2, 1)$.

Solution: <u>Method 1:</u> Differentiating implicitly,

$$y + y'x = 3x^2 + 3y^2y'$$

At $(2, 1)$, we have

$$\begin{aligned} 1 + 2y' &= 12 + 3y' \\ y' &= -11 \end{aligned}$$

and so an equation of the line is

$$y - 1 = -11(x - 2)$$

<u>Method 2:</u> Differentiating implicitly,

$$\begin{aligned} y + y'x &= 3x^2 + 3y^2y' \\ y' &= \frac{3x^2 - y}{x - 3y^2} \end{aligned}$$

Then

$$\left.\frac{dy}{dx}\right|_{(x,y)=(2,1)} = \frac{3(4) - 1}{2 - 3} = -11$$

and so an equation of the line is

$$y - 1 = -11(x - 2)$$

(6) Find $\dfrac{d^2y}{dx^2}$ if $x + y^2 = 1$

Solution: We begin by finding the first derivative by differentiating implicitly:

$$\begin{aligned} 1 + 2yy' &= 0 \\ y' &= -\frac{1}{2y} = -\frac{1}{2}y^{-1} \end{aligned}$$

Differentiating again and using our result above

$$\frac{d^2y}{dx^2} = \frac{1}{2y^2} \cdot y' = -\frac{1}{4y^3}$$