

**Problem 1** (Example).

$$\int_1^{mower} \frac{1}{u} du$$

**Solution:**

$$\int_1^{mower} \frac{1}{u} du = \ln |u| \Big|_1^{mower} = \ln |mower|$$

Problems 2-16 are great review.

**Problem 2.**

$$\int_{-8}^{-1} \left( \frac{2}{x^{1/3}} + 4x \right) dx$$

**Solution:**

$$\begin{aligned} \int_{-8}^{-1} \left( \frac{2}{x^{1/3}} + 4x \right) dx &= \left( 3x^{2/3} + 2x^2 \right) \Big|_{-8}^{-1} = 3((-1)^{2/3} - (-8)^{2/3}) + 2((-1)^2 - (-8)^2) \\ &= 3(1 - 4) + 2(1 - 64) = -9 - 126 = -135 \end{aligned}$$

**Problem 3.**

$$\int \frac{\sqrt{r} - 5}{\sqrt{r}} dr$$

**Solution:** Divide to get

$$\int \frac{\sqrt{r} - 5}{\sqrt{r}} dr = \int (1 - 5r^{-1/2}) dr = r - 10\sqrt{r} + C$$

**Problem 4.**

$$\int s(\sqrt{s} + 1) ds$$

**Solution:** Expand to get

$$\int s(\sqrt{s} + 1) ds = \int (s^{3/2} + s) ds = \frac{2}{5}s^{5/2} + \frac{1}{2}s^2 + C$$

**Problem 5.**

$$\int_{\pi/6}^{\pi/3} \sin(2x) dx$$

**Solution:** Method 1: Using  $u = 2x$ ,  $du = 2 dx$ ,  $\pi/6 \mapsto \pi/3$ ,  $\pi/3 \mapsto 2\pi/3$ ,

$$\int_{\pi/6}^{\pi/3} \sin(2x) dx = \int_{\pi/3}^{2\pi/3} \frac{1}{2} \sin u du = -\frac{1}{2} \cos u \Big|_{\pi/3}^{2\pi/3} = -\frac{1}{2} \left( -\frac{1}{2} - \frac{1}{2} \right) = \frac{1}{2}$$

Method 2: Integrating directly,

$$\int_{\pi/6}^{\pi/3} \sin(2x) dx = -\frac{1}{2} \cos(2x) \Big|_{\pi/6}^{\pi/3} = -\frac{1}{2} \left( -\frac{1}{2} - \frac{1}{2} \right) = \frac{1}{2}$$

**Problem 6.**

$$\int x^3 e^{x^4} dx$$

**Solution:** Using  $u = x^4$ ,

$$\int x^3 e^{x^4} dx = \int \frac{1}{4} e^u du = \frac{1}{4} e^{x^4} + C$$

**Problem 7.**

$$\int (2x - 1) \sin(4x) dx$$

**Solution:** Use  $f = 2x - 1$ ,  $g' = \sin(4x)$ , so  $f' = 2$ ,  $g = -\frac{1}{4} \cos(4x)$ . By IBP,

$$\begin{aligned} \int (2x - 1) \sin(4x) dx &= -\frac{2x - 1}{4} \cos(4x) + \frac{1}{2} \int \cos(4x) dx \\ &= -\frac{2x - 1}{4} \cos(4x) + \frac{1}{8} \sin(4x) + C \end{aligned}$$

**Problem 8.**

$$\int \frac{\sec \theta \tan \theta}{\sqrt{\sec \theta + 1}} d\theta$$

**Solution:** Using  $u = \sec \theta + 1$ ,  $du = \sec \theta \tan \theta d\theta$ , so

$$\int \frac{\sec \theta \tan \theta}{\sqrt{\sec \theta + 1}} d\theta = \int \frac{du}{\sqrt{u}} = 2u^{1/2} + C = 2\sqrt{\sec x + 1} + C$$

**Problem 9.**

$$\int \ln x^3 dx$$

**Solution:** Method 1:

Use  $f = \ln x^3, g' = 1$ , so  $f' = \frac{3x^2}{x^3} = \frac{3}{x}, g = x$ . By IBP,

$$\int \ln x^3 dx = x \ln x^3 - \int 3 dx = x \ln x^3 - 3x + C$$

Method 2:

Simplify first, then

$$\int \ln x^3 dx = \int 3 \ln x dx = 3(x \ln x - x) + C$$

**Problem 10.**

$$\int \frac{1}{t^2 - 6t + 10} dt$$

**Solution:**

$$\int \frac{1}{t^2 - 6t + 10} dt = \int \frac{1}{(t - 3)^2 + 1} dt = \arctan(t - 3) + C$$

**Problem 11.**

$$\int_{-1}^1 \frac{1}{x^3} dx$$

**Solution:** This is an improper integral of type II - you cannot use FTC II or that this is an odd function over a symmetric interval. Instead, the discontinuity is at 0, so

$$\int_{-1}^1 \frac{1}{x^3} dx = \int_{-1}^0 \frac{1}{x^3} dx + \int_0^1 \frac{1}{x^3} dx$$

Considering the second integral,

$$\int_0^1 \frac{1}{x^3} dx = \lim_{c \rightarrow 0^+} \int_c^1 \frac{1}{x^3} dx = \lim_{c \rightarrow 0^+} \left. -\frac{1}{2x^2} \right|_c^1 = \lim_{c \rightarrow 0^+} \left( \frac{1}{2c^2} - \frac{1}{2} \right) = \infty$$

Therefore, the integral diverges.

**Problem 12.**

$$\int \frac{8x^2 - 2x + 3}{2x - 1} dx$$

**Solution:**

$$\begin{array}{r} 4x + 1 \\ 2x - 1 \overline{) 8x^2 - 2x + 3} \\ \underline{- 8x^2 + 4x} \phantom{+ 3} \\ 2x + 3 \\ \underline{- 2x + 1} \\ 4 \end{array}$$

$$\int \frac{8x^2 - 2x + 3}{2x - 1} dx = \int \left( 4x + 1 + \frac{4}{2x - 1} \right) dx = 2x^2 + x + 2 \ln |2x - 1| + C$$

**Problem 13.**

$$\int x^2 e^{2x} dx$$

**Solution:** Use  $f = x^2, g' = e^{2x}$ , so  $f' = 2x, g = \frac{1}{2}e^{2x}$ . By IBP,

$$\int x^2 e^{2x} dx = \frac{1}{2} x^2 e^{2x} - \int x e^{2x} dx$$

Use  $f = x, g' = e^{2x}$ , so  $f' = 1, g = \frac{1}{2}e^{2x}$ . By IBP again,

$$\int x^2 e^{2x} dx = \frac{1}{2} x^2 e^{2x} - \frac{1}{2} x e^{2x} + \frac{1}{2} \int e^{2x} dx = \frac{1}{2} x^2 e^{2x} - \frac{1}{2} x e^{2x} + \frac{1}{4} \int e^{2x} + C$$

**Problem 14.**

$$\int_e^{e^2} \frac{1}{x \ln x} dx$$

**Solution:** Method 1: Using  $u = \ln x, du = \frac{1}{x} dx, e \mapsto 1, e^2 \mapsto 2$ , so

$$\int_e^{e^2} \frac{1}{x \ln x} dx = \int_1^2 \frac{1}{u} du = \ln |u| \Big|_1^2 = \ln 2$$

Method 2: Integrating directly,

$$\int_e^{e^2} \frac{1}{x \ln x} dx = \ln \ln x \Big|_e^{e^2} = \ln \ln e^2 - \ln \ln e = \ln 2$$

**Problem 15.**

$$\int_0^4 \sqrt{3x + 4} dx$$

**Solution:** Method 1: Using  $u = 3x + 4, du = 3 dx, 0 \mapsto 4, 4 \mapsto 16$ , so

$$\int_0^4 \sqrt{3x + 4} dx = \int_4^{16} \frac{1}{3} \sqrt{u} du = \frac{2}{9} u^{3/2} \Big|_4^{16} = \frac{2}{9} (16^{3/2} - 4^{3/2}) = \frac{2}{9} (4^3 - 2^3) = \frac{112}{9}$$

Method 2: Integrating directly,

$$\int_0^4 \sqrt{3x + 4} dx = \frac{2}{9} \sqrt{3x + 4} \Big|_0^4 = \frac{2}{9} (16^{3/2} - 4^{3/2}) = \frac{2}{9} (4^3 - 2^3) = \frac{112}{9}$$

**Problem 16.**

$$\int \frac{x+1}{x(2x+1)} dx$$

**Solution:** If  $\frac{x+1}{x(2x+1)} = \frac{A}{x} + \frac{B}{2x+1}$ , the Heaviside cover-up method gives  $A = \frac{0+1}{2(0)+1} = 1$  and  $B = \frac{-\frac{1}{2}+1}{-\frac{1}{2}} = -1$ , so

$$\int \frac{x+1}{x(2x+1)} dx = \int \left( \frac{1}{x} + \frac{1}{2x+1} \right) dx = \ln|x| + \frac{1}{2} \ln|2x+1| + C$$