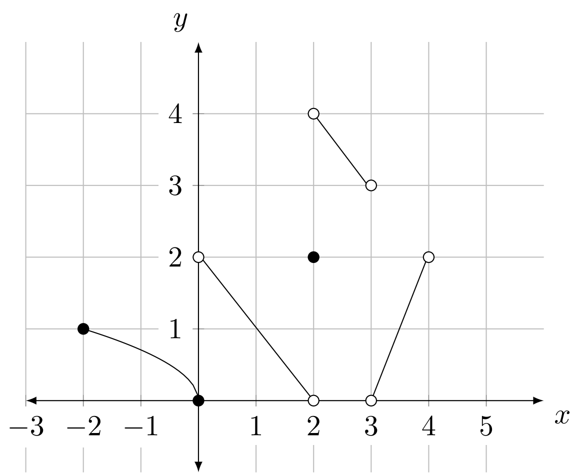
**Mini-math Div 3/4: Friday, September 17, 2021 (15 minutes)**

**SOLUTIONS**

1. Consider the below graph of a function $g(x)$, which consists of straight line segments.




Graph of $g$

(a) (1 point) Determine $\lim_{x \to 2} g(x)$, if it exists. If it does not, explain why it does not.

> **Solution:** $\lim_{x \to 2^-} g(x) = 0$ but $\lim_{x \to 2^+} g(x) = 4$, so $\lim_{x \to 2} g(x)$ does not exist.

(b) (1 point) (AP) Determine $\lim_{x \to 2} g(g(x))$, if it exists. If it does not, explain why it does not.

> **Solution:**
>
> $$\lim_{x \to 2^-} g(g(x)) = \lim_{x \to 0^+} g(x) = 2,$$
> $$\lim_{x \to 2^+} g(g(x)) = \lim_{x \to 4^-} g(x) = 2,$$
>
> so $\lim_{x \to 2} g(g(x)) = 2$.

2. (1 point) True or false: The value of $\lim_{x \to a} f(x)$ is $f(a)$, assuming $f(a)$ is defined.

> **Solution:** False - only for continuous functions.

3. (1 point) True or false: $\lim_{x\to a} f(x)$ can only exist if the left and right limits exist and are equal.

**Solution:** True.

4. What method would you use to solve the following limits?

(a) (1 point) $\lim_{h\to 0} \frac{(3+h)^2 - 9}{h}$?

**Solution:** Expand, simplify, and reduce $h$. Alternatively: factor as difference of squares, then simplify and reduce.
e.g.

$$\lim_{h\to 0} \frac{(3+h)^2 - 9}{h} = \lim_{h\to 0} \frac{9 + 6h + h^2 - 9}{h} = \lim_{h\to 0} \frac{6h + h^2}{h} = \lim_{h\to 0} 6 + h = 6$$

(b) (1 point) $\lim_{x\to 4} \frac{\sqrt{8-x} - 2}{1 - \sqrt{5-x}}$

**Solution:** Rationalize both the numerator and denominator.
e.g.

$$\begin{aligned}\lim_{x\to 4} \frac{\sqrt{8-x} - 2}{1 - \sqrt{5-x}} &= \lim_{x\to 4} \frac{\sqrt{8-x} - 2}{1 - \sqrt{5-x}} \cdot \frac{(\sqrt{8-x} + 2)(1 + \sqrt{5-x})}{(\sqrt{8-x} + 2)(1 + \sqrt{5-x})} \\ &= \lim_{x\to 4} \frac{(8-x) - 4}{1 - (5-x)} \cdot \frac{1 + \sqrt{5-x}}{\sqrt{8-x} + 2} = \lim_{x\to 4} -\frac{1 + \sqrt{5-x}}{\sqrt{8-x} + 2} = -\frac{1}{2}\end{aligned}$$

Value of limit: $-1/2$

(c) (1 point) $\lim_{x\to 1} \frac{x^2 + 5x + 6}{x^2 - 5x + 6}$

**Solution:** Plug it in.
e.g.

$$\lim_{x\to 1} \frac{x^2 + 5x + 6}{x^2 - 5x + 6} = \frac{1 + 5 + 6}{1 - 5 + 6} = \frac{12}{2} = 6$$

(d) (1 point) $\lim_{x \to -4} \frac{x^2+3x-4}{x|x+4|}$

**Solution:** Split into left and right limits, evaluate the absolute value.
e.g.

$$\lim_{x \to -4^-} f(x) = \lim_{x \to -4^-} \frac{(x+4)(x-1)}{x(-(x+4))} = -5/4,$$
$$\lim_{x \to -4^+} f(x) = \lim_{x \to -4^+} \frac{(x+4)(x-1)}{x(x+4)} = 5/4$$

so the limit DNE.

5. (1 point) (AP) Suppose $g(x) \leq f(x) \leq h(x)$ for all $x$ except for $x = a$. What additional conditions are necessary to guarantee that $\lim_{x \to a} f(x)$ exists?

**Solution:** If $\lim_{x \to a} g(x) = \lim_{x \to a} h(x)$ exists, Squeeze Theorem guarantees a solution.

6. (3 points) Where is the following function discontinuous? Identify the type of discontinuity, if any.

$$f(x) = \begin{cases} \frac{6}{x+3} & \text{if } x < 0 \\ 3 & \text{if } x = 0 \\ x+2 & \text{if } x > 0 \end{cases}$$

**Solution:** $\frac{4}{x+2}$ is discontinuous at $-2$. Since $\lim_{x \to -2^+} \frac{4}{x+2} = \infty$, this is an infinite discontinuity.
At 0,

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} \frac{6}{x+3} = 2,$$
$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} x+2 = 2,$$

so $\lim_{x \to 0} f(x) = 2$, but $f(0) = 3$, so this is a removable discontinuity.