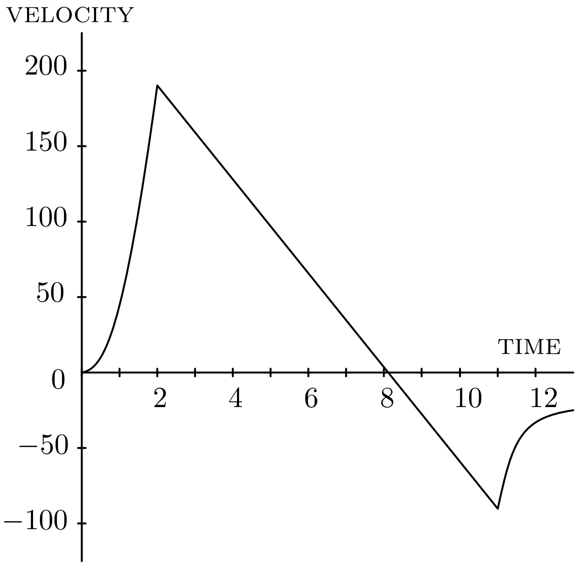
4.18 Sample A.P. Problems on Integrals

1090. The figure shows the graph of the velocity of a model rocket for the first 12 seconds after launch.

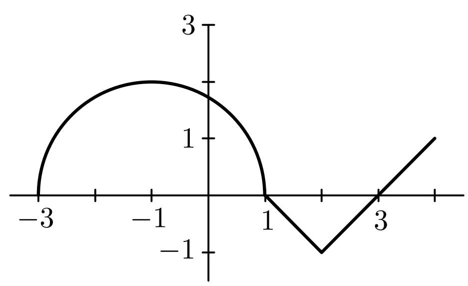


a) Assuming the rocket was launched from ground level, about how high did it go?

b) Assuming the rocket was launched from ground level, about how high was the rocket 12 seconds after launch?

c) What is the rocket's acceleration at $t = 6$ seconds? At $t = 2$ seconds?

1091. The graph of a function f consists of a semicircle and two line segments as shown below. Let $g(x) = \displaystyle\int_1^x f(t)\ dt$.



a) Find $g(1)$.

b) Find $g(3)$.

c) Find $g(-1)$.

d) Find all the values of x on the open interval $(-3, 4)$ at which g has a relative maximum.

e) Write an equation for the line tangent to the graph of g at $x = -1$.

f) Find the x-coordinate of each point of inflection of the graph of g on the open interval $(-3, 4)$.

g) Find the range of g.

1092. An automobile accelerates from rest at $1 + 3\sqrt{t}$ miles per hour per second for 9 seconds.

a) What is its velocity after 9 seconds?

b) How far does it travel in those 9 seconds?

1093. Find the function f with derivative $f'(x) = \sin x + \cos x$ whose graph passes through the point $(\pi, 3)$.

We have knowledge of the past but cannot control it; we may control the future but not have knowledge of it. —*Claude Shannon*

1094 (1989BC). Let f be a function such that $f''(x) = 6x + 8$.

a) Find $f(x)$ if the graph of f is tangent to the line $3x - y = 2$ at the point $(0, -2)$.

b) Find the average value of $f(x)$ on the closed interval $[-1, 1]$.

1095 (1999AB, Calculator). A particle moves along the y-axis with velocity given by $v(t) = t\sin(t^2)$ for $t \geq 0$.

a) In which direction (up or down) is the particle moving at time $t = 1.5$? Why?

b) Find the acceleration of the particle at time $t = 1.5$. Is the velocity of the particle increasing at $t = 1.5$?

c) Given that $y(t)$ is the position of the particle at time t and that $y(0) = 3$, find $y(2)$.

d) Find the total distance traveled by the particle from $t = 0$ and $t = 2$.

1096 (1990BC). Let f and g be continuous functions with the following properties:

i) $g(x) = A - f(x)$ where A is a constant

ii) $\displaystyle\int_1^2 f(x)\,dx = \int_2^3 g(x)\,dx$

iii) $\displaystyle\int_2^3 f(x)\,dx = -3A$

a) Find $\displaystyle\int_1^3 f(x)\,dx$ in terms of A.

b) Find the average value of $g(x)$ in terms of A over the interval $[1, 3]$.

c) Find the value of k if $\displaystyle\int_0^1 f(x+1)\,dx = kA$.

1097 (1994AB, Calculator). Let $F(x) = \displaystyle\int_0^x \sin(t^2)\,dt$ for $0 \leq x \leq 3$.

a) Use the trapezoidal rule with four equal subdivisions of the closed interval $[0, 1]$ to approximate $F(1)$.

b) On what interval is F increasing?

c) If the average rate of change of F on the closed interval $[1, 3]$ is k, find $\displaystyle\int_1^3 \sin(t^2)\,dt$ in terms of k.

1098 (1991BC). A particle moves on the x-axis so that its velocity at any time $t \geq 0$ is given by $v(t) = 12t^2 - 36t + 15$.

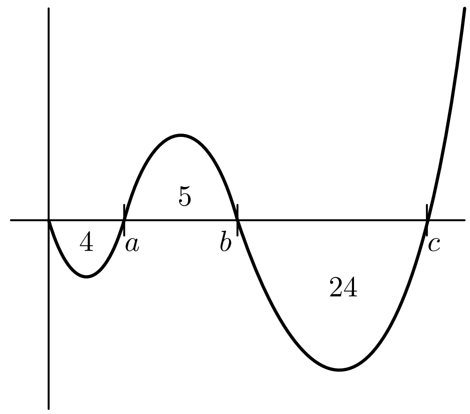
a) Find the position $x(t)$ of the particle at any time $t \geq 0$.

b) Find all values of t for which the particle is at rest.

c) Find the maximum velocity of the particle for $0 \leq t \leq 2$.

d) Find the total distance traveled by the particle from $t = 0$ to $t = 2$.

1099. A particle moves along the x-axis. Its initial position at $t = 0$ sec is $x(0) = 15$. The graph below shows the particle's velocity $v(t)$. The numbers are areas of the enclosed figures.



a) What is the particle's displacement between $t = 0$ and $t = c$?

b) What is the total distance traveled by the particle in the same time period?

c) Give the positions of the particle at times a, b, and c.

d) Approximately where does the particle achieve its greatest positive acceleration on the interval $[0, b]$? On $[0, c]$?

1100 (1987BC). Let f be a continuous function with domain $x > 0$ and let F be the function given by $F(x) = \int_1^x f(t)\,dt$ for $x > 0$. Suppose that $F(ab) = F(a) + F(b)$ for all $a > 0$ and $b > 0$ and that $F'(1) = 3$.

a) Find $f(1)$.

b) Prove that $aF'(ax) = F'(x)$ for every positive constant a.

c) Use the results from parts (a) and (b) to find $f(x)$. Justify your answer.

1101 (1999AB, Calculator). The rate at which water flows out of a pipe, in gallons per hour, is given by a differentiable function R of time t. The table below shows the rate as measured every 3 hours for a 24-hour period.

t (hours)	0	3	6	9	12	15	18	21	24
$R(t)$ (gal/hr)	9.6	10.4	10.8	11.2	11.4	11.3	10.7	10.2	9.6

a) Use a midpoint Riemann sum with 4 subdivisions of equal length to approximate the value of $\int_0^{24} R(t)\,dt$. Using correct units, explain the meaning of your answer in terms of water flow.

b) Is there some time t, $0 < t < 24$, such that $R'(t) = 0$? Justify your answer.

c) The rate of the water flow $R(t)$ can be approximated by $Q(t) = \frac{1}{79}(768 + 23t - t^2)$. Use $Q(t)$ to approximate the average rate of water flow during the 24-hour time period. Indicate units of measure.

E. H. Moore was presenting a paper on a highly technical topic to a large gathering of faculty and graduate students from all parts of the country. When half way through he discovered what seemed to be an error (though probably no one else in the room observed it). He stopped and re-examined the doubtful step for several minutes and then, convinced of the error, he abruptly dismissed the meeting – to the astonishment of most of the audience. It was an evidence of intellectual courage as well as honesty and doubtless won for him the supreme admiration of every person in the group – an admiration which was in no ways diminished, but rather increased, when at a later meeting he announced that after all he had been able to prove the step to be correct. —*H. E. Slaught*

4.19 Multiple Choice Problems on Integrals

1102 (AP). For any real number b, $\int_0^b |2x|\ dx$ is

A) $-b|b|$ B) b^2 C) $-b^2$ D) $b|b|$ E) None of these

1103 (AP). Let f and g have continuous first and second derivatives everywhere. If $f(x) \leq g(x)$ for all real x, which of the following must be true?

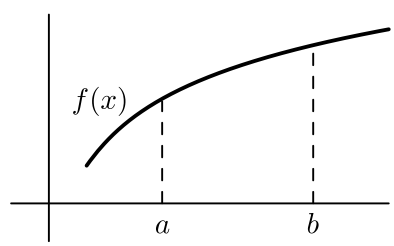
I) $f'(x) \leq g'(x)$ for all real x
II) $f''(x) \leq g''(x)$ for all real x
III) $\int_0^1 f(x)\ dx \leq \int_0^1 g(x)\ dx$

A) None B) I only C) III only D) I and II E) I, II, and III

1104 (AP). Let f be a continuous function on the closed interval $[0, 2]$. If $2 \leq f(x) \leq 4$, then the greatest possible value of $\int_0^2 f(x)\ dx$ is

A) 0 B) 2 C) 4 D) 8 E) 16

1105 (AP). If f is the continuous, strictly increasing function on the interval $[a, b]$ as shown below, which of the following must be true?



I) $\int_a^b f(x)\ dx < f(b)(b-a)$
II) $\int_a^b f(x)\ dx > f(a)(b-a)$
III) $\int_a^b f(x)\ dx = f(c)(b-a)$ for some c in $[a, b]$.

A) I only B) II only C) III only D) I and II E) I, II, and III

1106 (AP). Which of the following definite integrals is *not* equal to zero?

A) $\int_{-\pi}^{\pi} \sin^3 x\ dx$ B) $\int_{-\pi}^{\pi} x^2 \sin x\ dx$ C) $\int_0^{\pi} \cos x\ dx$ D) $\int_{-\pi}^{\pi} \cos^3 x\ dx$ E) $\int_{-\pi}^{\pi} \cos^2 x\ dx$

Archimedes will be remembered when Aeschylus is forgotten, because languages die and mathematical ideas do not. —*G. H. Hardy*

1107. $\displaystyle\int_{\pi/6}^{\pi/2} \cot x \, dx =$

A) $\ln \frac{1}{2}$ B) $\ln 2$ C) $\frac{1}{2}$ D) $\ln(\sqrt{3}-1)$ E) None of these

1108. $\displaystyle\int_{-2}^{3} |x+1| \, dx =$

A) $\frac{5}{2}$ B) $\frac{17}{2}$ C) $\frac{9}{2}$ D) $\frac{11}{2}$ E) $\frac{13}{2}$

1109. $\displaystyle\int_{1}^{2} (3x-2)^3 \, dx =$

A) $\frac{16}{3}$ B) $\frac{63}{4}$ C) $\frac{13}{3}$ D) $\frac{85}{4}$ E) None of these

1110. $\displaystyle\int_{\pi/4}^{\pi/2} \sin^3\theta \cos\theta \, d\theta =$

A) $\frac{3}{16}$ B) $\frac{1}{8}$ C) $-\frac{1}{8}$ D) $-\frac{3}{16}$ E) $\frac{3}{4}$

1111. $\displaystyle\int_{0}^{1} \frac{e^x}{(3-e^x)^2} \, dx =$

A) $3\ln(e-3)$ B) 1 C) $\dfrac{1}{3-e}$ D) $\dfrac{e-1}{2(3-e)}$ E) $\dfrac{e-2}{3-e}$

1112. $\displaystyle\int_{-1}^{0} e^{-x} \, dx =$

A) $1-e$ B) $\dfrac{1-e}{e}$ C) $e-1$ D) $1-\dfrac{1}{e}$ E) $e+1$

1113. $\displaystyle\int_{0}^{1} \frac{x}{x^2+1} \, dx =$

A) $\frac{\pi}{4}$ B) $\ln\sqrt{2}$ C) $\frac{1}{2}(\ln 2 - 1)$ D) $\frac{3}{2}$ E) $\ln 2$

Anyone who cannot cope with mathematics is not fully human. At best he is a tolerable subhuman who has learned to wear shoes, bathe, and not make messes in the house. —*Robert A. Heinlein*

1114. The acceleration of a particle moving along a straight line is given by $a = 6t$. If, when $t = 0$ its velocity $v = 1$ and its distance $s = 3$, then at any time t the position function is given by

A) $s = t^3 + 3t + 1$

B) $s = t^3 + 3$

C) $s = t^3 + t + 3$

D) $s = \frac{1}{3}t^3 + t + 3$

E) $s = \frac{1}{3}t^3 + \frac{1}{2}t^2 + 3$

1115. If the displacement of a particle on a line is given by $s = 3 + (t - 2)^4$, then the number of times the particle changes direction is

A) 0 B) 1 C) 2 D) 3 E) None of these

1116. $\displaystyle\int_0^{\pi/2} \cos^2 x \sin x \; dx =$

A) -1 B) $-\frac{1}{3}$ C) 0 D) $\frac{1}{3}$ E) 1

1117. $\displaystyle\int_0^1 (3x^2 - 2x + 3) \; dx =$

A) 0 B) 5 C) 3 D) 8 E) None of these

1118. $\displaystyle\int_1^e \left(x - \frac{1}{2x} \right) \; dx =$

A) $\frac{1}{2}e^2$ B) $\frac{1}{2}e^2 + 1$ C) $\frac{1}{2}(e^2 + 1)$ D) $\frac{1}{2}(e^2 - 1)$ E) None of these

1119. $\displaystyle\int_0^1 (2 - 3x)^5 \; dx =$

A) $-\frac{1}{2}$ B) $\frac{1}{6}$ C) $\frac{1}{2}$ D) $-\frac{1}{18}$ E) None of these

Work is the greatest thing in the world, so we should always save some of it for tomorrow. —*Don Herald*