$$13x - 17y = -415$$
$$6x - 15y = -306$$

Method 1a: (Elimination) We try to eliminate the y variable. To do so, we need to ensure that both equations have the same coefficient of y. In general, we use the least common multiple of the current coefficients, but we can also just use their product. Multiplying the first equation by 15 and the second by 17, we get

$$195x - 255y = -6225$$
$$102x - 255y = -5202$$

(Note that we didn't actually need to multiply 15×17 for the coefficient of y; we know that they will cancel out regardless of the actual value.) Now we're ready to eliminate the unknown y, by subtracting the equations:

$$93x = -6225 - (-5202) = -1023$$

Then we are ready to solve for x by dividing by its coefficient:

$$x = \frac{-1023}{93} = -11$$

Finally, we can find y by plugging our newly found value of x into any equation and solving. Here, we will use the second given equation:

$$\begin{aligned} 6(-11) - 15y &= -306 \\ -15y &= -306 + 6(11) \\ -15y &= -240 \\ y &= \frac{-240}{-15} = 16 \end{aligned}$$

Therefore, our answer is $x = -11, y = 16$.

(Note: at this point, a great check is to plug in both x and y into both given equations to verify that both are true.)

Method 1b: (Elimination) We try to eliminate the y variable, but first simplify the equations as much as possible before we begin. In this case, the second equation can be simplified by dividing by 3:

$$13x - 17y = -415$$
$$2x - 5y = -102$$

To eliminate y, we need to ensure that both equations have the same coefficient of y. In general, we use the least common multiple of the current coefficients, but we can also just use their product. Multiplying the first equation by 5 and the second by 17, we get

$$65x - 85y = -2075$$
$$34x - 85y = -1734$$

(Note that we didn't actually need to multiply 5×17 for the coefficient of y; we know that they will cancel out regardless of the actual value.) Now we're ready to eliminate the unknown y, by subtracting the equations:

$$31x = -2075 - (-1734) = -341$$

Then we are ready to solve for x by dividing by its coefficient:

$$x = \frac{-341}{31} = -11$$

Finally, we can find y by plugging our newly found value of x into any equation and solving. Here, we will use the (simplified) second given equation:

$$\begin{aligned} 2(-11) - 5y &= -102 \\ -5y &= -102 + 2(11) \\ -5y &= -80 \\ y &= \frac{-80}{-5} = 16 \end{aligned}$$

Therefore, our answer is $x = -11, y = 16$.

(Note: at this point, a great check is to plug in both x and y into both given equations to verify that both are true.)

Method 2: (Elimination) We try to eliminate the x variable. To do so, we need to ensure that both equations have the same coefficient of x. In general, we use the least common multiple of the current coefficients, but we can also just use their product. Multiplying the first equation by 6 and the second by 13, we get

$$\begin{aligned} 78x - 102y &= -2490 \\ 78x - 195y &= -3978 \end{aligned}$$

(Note that we didn't actually need to multiply 6×13 for the coefficient of x; we know that they will cancel out regardless of the actual value.) Now we're ready to eliminate the unknown x, by subtracting the equations:

$$93y = -2490 - (-3978) = 1488$$

Then we are ready to solve for y by dividing by its coefficient:

$$y = \frac{1488}{93} = 16$$

Finally, we can find x by plugging our newly found value of y into any equation and solving. Here, we will use the second given equation:

$$\begin{aligned} 6x - 15(16) &= -306 \\ 6x &= -306 + 15(16) \\ 6x &= -66 \\ x &= \frac{-66}{6} = -11 \end{aligned}$$

Therefore, our answer is $x = -11, y = 16$.

(Note: at this point, a great check is to plug in both x and y into both given equations to verify that both are true.)

Method 3: (Substitution) We try to solve for y in terms of x and then substitute. Either equation works for this first step, but here, we will use the second given equation:

$$\begin{aligned} 6x - 15y &= -306 \\ -15y &= -306 - 6x \\ y &= \frac{306 + 6x}{15} \end{aligned}$$

At this point, we can go ahead and substitute this value into the equation we didn't use yet (the first one), but you should generally simplify your fractions to make things easier:

$$y = \frac{102 + 2x}{5}$$

Now, we substitute into the first equation and solve:

$$\begin{aligned} 13x - 17\left(\frac{102 + 2x}{5}\right) &= -415 \\ 13x - \frac{1734}{5} - \frac{34}{5}x &= -415 \\ \frac{31}{5}x &= -415 + \frac{1734}{5} = -\frac{341}{5} \\ x &= -\frac{341}{5} \times \frac{5}{31} = 11 \end{aligned}$$

Finally, we can find y by plugging our newly found value of x into any equation and solving. Here, we will use the second given equation:

$$\begin{aligned} 6(-11) - 15y &= -306 \\ -15y &= -306 + 6(11) \\ -15y &= -240 \\ y &= \frac{-240}{-15} = 16 \end{aligned}$$

Therefore, our answer is $x = -11, y = 16$.

(Note: at this point, a great check is to plug in both x and y into both given equations to verify that both are true.)

Method 4: (Substitution) We try to solve for x in terms of y and then substitute. Either equation works for this first step, but here, we will use the second given equation:

$$\begin{aligned} 6x - 15y &= -306 \\ 6x &= -306 + 15y \\ x &= \frac{-306 + 15x}{6} = \frac{-102 + 5y}{2} \end{aligned}$$

Notice in this method, we simplified to make things easier. Now, we substitute into the first equation and solve:

$$\begin{aligned} 13\left(\frac{-102+5y}{2}\right) - 17y &= -415 \\ -\frac{1326}{2} + \frac{65}{2}y - 17y &= -415 \\ \frac{31}{2}y &= 248 \\ y &= 248 \times \frac{2}{31} = 16 \end{aligned}$$

Finally, we can find x by plugging our newly found value of y into any equation and solving. Here, we will use the second given equation:

$$\begin{aligned} 6x - 15(16) &= -306 \\ 6x &= -306 + 15(16) \\ 6x &= -66 \\ x &= \frac{-66}{6} = -11 \end{aligned}$$

Therefore, our answer is $x = -11, y = 16$.

(Note: at this point, a great check is to plug in both x and y into both given equations to verify that both are true.)