Name: ______________________ Mark: ______________________

Mini-math Div 3/4: Monday, November 30, 2020 (12 minutes)

Find the optimizing function (in terms of a single variable) for the following optimization problems. You do not need to simplify the function, nor do you need to solve the optimization question.

1. (2 points) Find the largest value of $(x+2)(y-3)$ if $2x+3y=4$

> **Solution:** The constraint is $y = \frac{1}{3}(4-2x)$, so the optimizing function is
>
> $$f(x) = (x+2)\left(\frac{1}{3}(4-2x) - 3\right)$$
>
> OR: The constraint is $x = \frac{1}{2}(4-3y)$, so the optimizing function is
>
> $$f(y) = \left(\frac{1}{2}(4-3y)+2\right))\,(y-3)$$
>
> Aside: the answer is $1/24$, which occurs at $x = -9/4$, $y = 17/6$.

2. (2 points) What is the shortest distance between the point $(3,1)$ and the curve $y = x^2+2$

> **Solution:** If the point on the curve which is closest to $(3,1)$ is (x,y), then the constraint is $y = x^2+2$. The optimizing function is
>
> $$f(x) = d^2 = (x-3)^2 + (x^2+1)^2$$
>
> Aside: the answer is ≈ 2.7391, which occurs at $x \approx 0.7351$.

3. (2 points) 2500 cm^2 of material is available to make a cylindrical can with an open top. Find the largest possible volume of the can.

Solution: The constraint is $A = \pi r^2 + 2\pi r h$, so the optimizing function is

$$V = \pi r^2 h = \pi r^2 \cdot \left(\frac{2500 - \pi r^2}{2\pi r} \right)$$

Aside: the answer is $\dfrac{125000}{3\sqrt{3\pi}}$ cm^3, which occurs at $r = h = 50/\sqrt{3\pi}$.

4. (2 points) A cable television company is laying cable in an area with underground utilities. Two subdivisions are located on opposite sides of Terren Creek, which is 88 m wide. The company has to connect points P and Q with cable, where P is on the south bank while Q is 120 m downstream from P, on the opposite shore. It costs \$35/m to lay the cable underground and \$65/m to lay the cable underwater. At what point should the company lay the point to on the opposite shore in order to minimize the cost of the cable?

Solution: Let x be the distance downstream of the point directly opposite the river from P. Then the cable must run underwater for $\sqrt{88^2 + x^2}$ m and underground for $120 - x$ m.

The optimizing function is

$$C(x) = 35(120 - x) + 65\sqrt{88^2 + x^2}$$

Aside: the answer is $x = 154\sqrt{2/15} \approx 56.233$ m.