Trigonometry — Related Rates

1. A 3-m ladder leans against a house on flat ground. The base of the ladder starts to slide away from the house at 2 m/s. At what rate is the angle between the ladder and the ground changing when the base is 1 m from the house?

   Solution: Let θ be the angle between the ladder and the ground and x be the distance between the foot of the ladder and the house. Then

$$\cos\theta = \frac{x}{3}$$
$$-\sin\theta \cdot \frac{d\theta}{dt} = \frac{1}{3}\cdot\frac{dx}{dt}$$

   At $x = 1$, $\cos\theta = \frac{1}{3}$, so $\sin\theta = \sqrt{1-(1/3)^2} = \frac{2\sqrt{2}}{3}$. Plugging in our information,

$$\frac{d\theta}{dt} = -\frac{1}{3}\cdot 2\cdot\frac{3}{2\sqrt{2}} = -\frac{1}{\sqrt{2}}$$

   The angle is decreasing at $\frac{1}{\sqrt{2}}$ radians/s.

2. If the height h of an isosceles triangle with base 4 m changes at a rate $\frac{dh}{dt} = 3$ m/s, how quickly is the angle opposite the base changing when $h = 2\sqrt{3}$ m?

   Solution: Let θ be the angle opposite the base. Then

$$\tan\frac{\theta}{2} = \frac{2}{h}$$
$$\sec^2\frac{\theta}{2}\cdot\frac{1}{2}\cdot\frac{d\theta}{dt} = -\frac{2}{h^2}\cdot\frac{dh}{dt}$$

   At $h = 2\sqrt{3}$, $\tan\frac{\theta}{2} = \frac{1}{\sqrt{3}}$, so $\frac{\theta}{2} = \frac{\pi}{6}$. Plugging in our information,

$$\sec^2\frac{\pi}{6}\cdot\frac{1}{2}\cdot\frac{d\theta}{dt} = -\frac{2}{12}\cdot 3 = -\frac{1}{2}$$
$$\frac{d\theta}{dt} = -\cos^2\frac{\pi}{6} = -\frac{3}{4}$$

   The angle is decreasing at 3/4 radians/s.