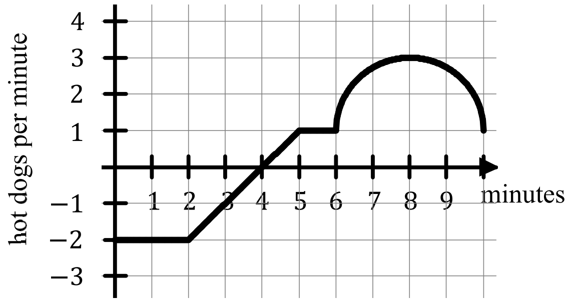
6.1 Accumulation of Change

Calculus Name: ______________________________ **CA #1**

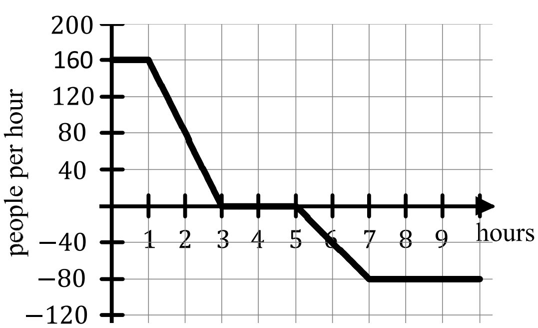
1. The graph below shows the rate at which hot dogs are on Mr. Kelly's plate. Assume there are 10 hot dogs on the plate at $t = 0$ minutes.



a. How many hot dogs are on Mr. Kelly's plate after six minutes?

b. How many hot dogs are on Mr. Kelly's plate after 10 minutes?

2. The graph below shows the rate of change of the number of people in a movie theater. Assume no one was in the theater at $t = 0$ hours.



a. How many people are in the theater after 3 hours?

b. How many people are in the theater after 10 hours?

Each function listed represents a rate of change. What are the units for the area under the curve?	
3. $g(t)$ is measured in ounces per second and t is measured in seconds.	4. $T(d)$ is measured in ℃ per day and d is measured in days.

1a. 5.5 hotdogs	1b. $9.5 + 2\pi$ hotdogs	2a. 320 people	2b. 0 people	3. ounces	4. ℃

Answers to 6.1 CA #1

6.2 Approximating Areas with Riemann Sums

Calculus Name: ______________________________ **CA #1**

Approximate the area under the curve using the given Riemann Sum.

1. $f(x) = \frac{1}{5}x^3 - x + 7$

   Midpoint Riemann Sum on the interval $[-1,2]$ with $n = 3$ subintervals.

2. $f(x) = 6x + 5$

   Left Riemann Sum on $[-2,2]$ with $n = 5$ subintervals.

3. $f(x) = -0.2x^2 - x + 12$

   Trapezoid approximation on the interval $[-1, 3]$ with $n = 4$ subintervals

4. Let $y(t)$ represent the weight loss per week of a contestant on the Biggest Loser, where y is a differentiable function of t. The table shows the weight loss per week recorded at selected times.

Time (week)	2	4	7	8	11
$\boldsymbol{y(t)}$ **(pounds/week)**	14	12	18	14	17

   a. Use the data from the table and a left Riemann Sum with four subintervals. Show the computations that lead to your answer.

   b. What does your answer represent in this situation?

5. Let $v(t)$ represent the rate of change of a hot air balloon over time, where v is a differentiable function of t. The table shows the rate of change at selected times. The balloons height at $t = 0$ was 50 meters.

Time (minutes)	0	4	6	9	11
$\boldsymbol{v(t)}$ **(meters/min)**	5.2	6.3	7.1	7.9	8.4

a. Use the data from the table and a trapezoidal approximation with four subintervals. Show the computations that lead to your answer.

b. What is the approximate height of the balloon at 11 minutes?

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6. A particle moves along a horizontal line with a positive velocity $v(t)$, where v is a differentiable function of t. The time t is measured in seconds, and the velocity is measured in cm/sec. The velocity of the particle at selected times is given in the table below.

Time (sec)	0	2	4	6	8	10	12	14	16
$\boldsymbol{v(t)}$ **(cm/sec)**	21	18	15	23	27	31	35	32	29

a. Use the data from the table and a midpoint Riemann Sum with four subintervals. Show the computations that lead to your answer.

b. What does your answer represent in this situation?

Answers to 6.2 CA #1

1. 20.175	2. 10.4	3. 42
4. a. 124 b. The total pounds lost from week 2 to week 11.	5. a. 75.2 b. 125.2 meters	6. a. 416 b. The distance travelled by the particle from 0 to 16 seconds.

6.3 Summation Notation

Calculus — Name: ______________________________ **CA #1**

Write a definite integral that is equivalent to the given summation notation. The lower limit for the integral is also given to help you get started.

1. Integral's lower limit = 0

$$\lim_{n\to\infty}\sum_{k=1}^{n}\left(\frac{\pi}{4n}\right)\tan\left(\frac{\pi}{4n}k\right)$$

2. Integral's lower limit = −1

$$\lim_{n\to\infty}\sum_{k=1}^{n}\left(\frac{8}{n}\right)\left[4\left(-1+\frac{8k}{n}\right)\right]$$

Write a summation notation equivalent to the definite integral.

3. $\int_{-1}^{3} x^2\,dx$

4. $\int_{3}^{4} \ln x\,dx$

5. Which of the following expressions is equal to $\lim\limits_{n\to\infty}\frac{4}{n}\left(\left(1+\frac{4}{n}\right)^3+\left(1+\frac{8}{n}\right)^3+\left(1+\frac{12}{n}\right)^3+\cdots+\left(1+\frac{4n}{n}\right)^3\right)$?

(A) $\int_1^5 1+x^3\,dx$

(B) $\int_0^4 (1+x)^3\,dx$

(C) $\int_0^4 1+x^3\,dx$

(D) $\int_1^5 (1+x)^3\,dx$

---

6. The expression $\frac{2}{9}\left[\left(\frac{1}{3+\frac{2}{9}+1}\right)+\left(\frac{1}{3+\frac{4}{9}+1}\right)+\left(\frac{1}{3+\frac{6}{9}+1}\right)+\cdots+\left(\frac{1}{3+\frac{18}{9}+1}\right)\right]$ is a Riemann sum approximation of which of the following integrals?

(A) $\int_0^2 \frac{1}{x+1}dx$

(B) $\int_3^5 \frac{1}{x+1}dx$

(C) $\frac{1}{9}\int_0^2 \left(\frac{1}{3+x}\right)dx$

(D) $\int_0^2 \frac{1}{3+x}dx$

(E) $\frac{1}{9}\int_3^5 \frac{1}{2x+1}dx$

Answers to 6.3 CA #1

1. $\int_0^{\frac{\pi}{4}} \tan(x)\,dx$	2. $\int_{-1}^{7} 4x\,dx$	3. $\lim\limits_{n\to\infty}\sum_{k=1}^{n}\left(\frac{4}{n}\right)\left(-1+\frac{4k}{n}\right)^2$
4. $\lim\limits_{n\to\infty}\sum_{k=1}^{n}\left(\frac{1}{n}\right)\ln\left(3+\frac{k}{n}\right)$	5. $\int_0^4 (1+x)^3\,dx$	6. $\int_3^5 \frac{1}{x+1}dx$

6.4 Accumulation Functions

CA #1

Calculus

Name: ______________________________

Find $F'(x)$.

1. $F(x) = \int_4^x \frac{1}{\sqrt{t}}\,dt$

2. $F(x) = \int_3^x t^2\,dt$

3. $F(x) = \int_\pi^x \tan t\,dt$

4. $F(x) = \int_5^x \frac{1}{t}\,dt$

5. $F(x) = \int_{-1}^{2x} (1 - t^2)\,dt$

6. $F(x) = \int_e^{e^x} \ln t\,dt$

7. $F(x) = \int_9^{x^4} \sqrt{t}\,dt$

8. $F(x) = \int_0^{x^2 - x} t^2\,dt$

9. $F(x) = \int_{-\pi}^{\cos x} 2^t\,dt$

10. $F(x) = \int_{-x}^{x} \sin^2 t\,dt$

11. $F(x) = \int_{-x}^{3x^2} t^2\,dt$

12. $F(x) = \int_{x^2}^{x^4} \sqrt{t}\,dt$

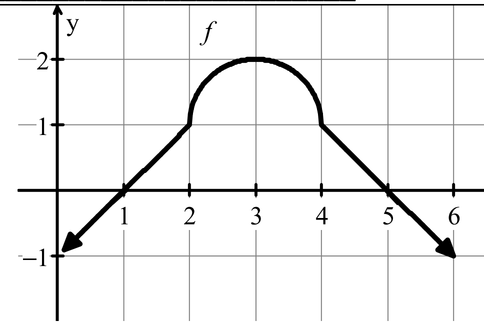
Answers to 9.1 CA #1

1. $\frac{1}{\sqrt{x}}$	2. x^2	3. $\tan x$	4. $\frac{1}{x}$	5. $2 - 8x^2$	6. xe^x	7. $4x^5$
8. $2x^5 - 5x^4 + 4x^3 - x^2$	9. $-\sin x\, 2^{\cos x}$	10. $\sin^2 x + \sin^2(-x)$	11. $54x^5 + x^2$	12. $4x^5 - 2x^2$		

6.5 Behavior of Accumulation Functions

CA #1

Calculus Name: ______________________________



1. Let $g(x) = \int_a^x f(t)\,dt$ with the graph of f shown above and a is a constant. Find the x-values of g regarding each of the following conditions.

a. Relative minimum(s)

b. Relative maximum(s)

c. Concave up

d. Concave down

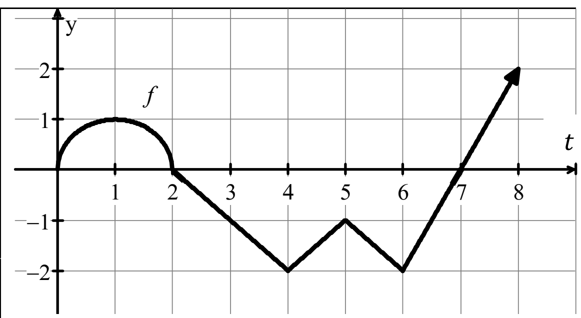
e. Increasing

f. Decreasing

g. Point(s) of inflection

h. If $g(1) = -5$, what is the maximum **value** of g on the interval $[0, 6]$?

i. Given $h(x) = \int_0^{\frac{x}{2}+3} f(t)\,dt$. Find the x-value where h has a relative minimum.



2. Let $g(x) = \int_a^x f(t)\,dt$ with the graph of f shown above and a is a constant. Find the x-values of g regarding each of the following conditions.

a. Relative minimum(s)

b. Relative maximum(s)

c. Concave up

d. Concave down

e. Increasing

f. Decreasing

g. Point(s) of inflection

h. If $g(4) = 3$, what is the minimum **value** of g on the interval $[0, 8]$?

i. Given $h(x) = \int_0^{2x-6} f(t)\,dt$. Find the x-value where h has a relative maximum.

Answers to 6.5 CA #1

1a. $x = 1$	1b. $x = 5$	1c. $(-\infty, 2)$ and $(2, 3)$	1d. $(3, 4)$ and $(4, \infty)$	1e. $(1, 5)$	1f. $(-\infty, 1)$ and $(5, \infty)$
1g. $x = 3$	1h. $-2 + \frac{\pi}{2}$	1i. $x = -4$	2a. $x = 7$	2b. $x = 2$	2c. $(0, 1)$, $(4, 5)$, and $(6, \infty)$
2d. $(1, 4)$ and $(5, 6)$	2e. $(0, 2)$ and $(7, \infty)$	2f. $(2, 7)$	2g. $x = 1, 4, 5, 6$	2h. -1	2i. $x = 4$

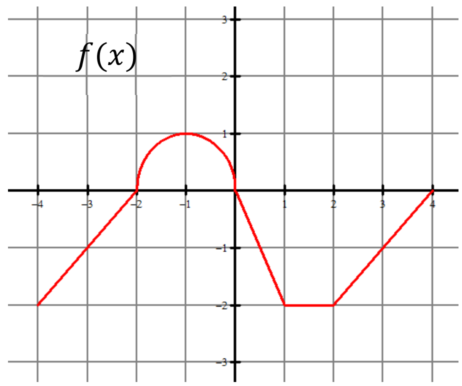
6.6 Properties of Definite Integrals

CA #1

Calculus Name: ______________________________

The graph of f consists of line segments and a semicircle. Evaluate each definite integral.

1.



a. $\int_{-4}^{-2} f(x)\,dx =$

b. $\int_{-2}^{0} 4f(x)\,dx =$

c. $\int_{4}^{0} f(x)\,dx =$

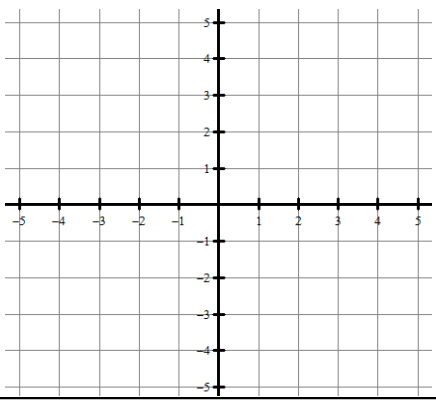
d. $\int_{-4}^{4} f(x)\,dx =$

e. $\int_{4}^{2} f(x)\,dx =$

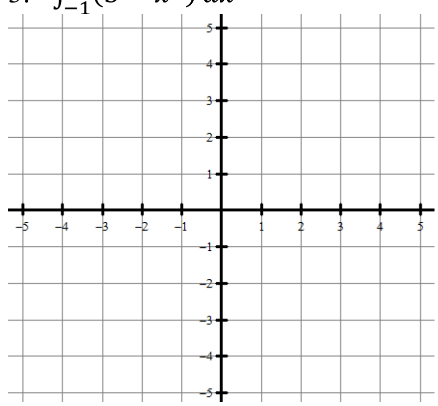
f. $\int_{-1}^{1} f(x)\,dx =$

Sketch a graph of the definite integral. Evaluate the integral with a graphing calculator.

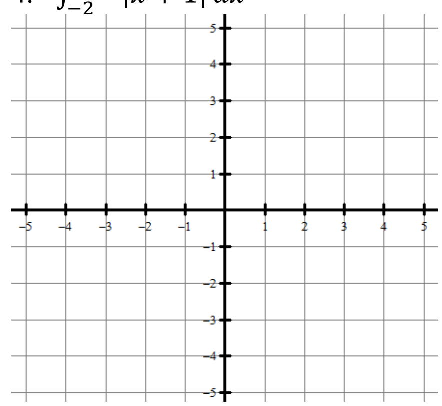
2. $\int_{-1}^{0} (x^3 - 1)\,dx =$



3. $\int_{-1}^{2} (5 - x^2)\,dx =$



4. $\int_{-2}^{3} -|x + 1|\,dx =$



Let f be a continuous functions that produces the following definite integral values.

$$\int_{-4}^{6} f(x)dx = 2 \text{ and } \int_{6}^{8} f(x)dx = -5$$

Find the following.

5. $\int_{-4}^{6} 5f(x)dx =$

6. $\int_{-4}^{8} f(x)dx =$

7. $\int_{8}^{6} f(x)dx =$

Let f be a continuous functions that produces the following definite integral values.

$$\int_{0}^{3} f(x)dx = -4 \text{ and } \int_{3}^{7} f(x)dx = 2$$

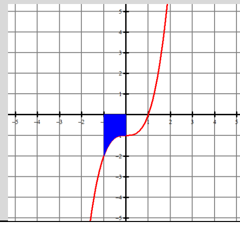
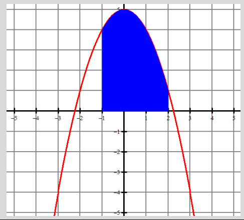
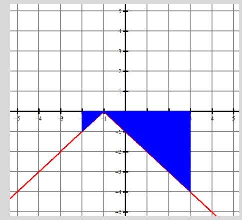
Find the following.

8. $\int_{3}^{0} f(x)dx =$

9. $\int_{3}^{3} f(x)dx =$

10. $\int_{3}^{7} 6f(x)dx =$

Answers to 6.6 CA #1

1. a. -2 b. 2π c. 5 d. $\frac{\pi}{2} - 7$ e. 2 f. $\frac{\pi}{4} - 1$	2. -1.25 	3. 12 	4. -8.5 	5. 10 6. -3 7. 5 8. 4 9. 0 10. 12

6.7 Definite Integrals

Calculus　　　　　　　　　　　　　　　　Name: ______________________________　　**CA #1**

Find the value of the definite integral.

1. $\int_0^4 (4x+5)\,dx$

2. $\int_{-1}^{2} \left(3x^2 - \frac{4}{x^2} + 1\right) dx$

3. $\int_4^{16} -\sqrt{x}\,dx$

4. $\int_{-\pi}^{-\frac{\pi}{2}} (1-\cos x)\,dx$

5. $\int_0^{\pi} (3-\sin x)\,dx$

Use the given information to find the value of the function.

6. If $g'(x) = \cos x$ and $g(\pi) = 7$, then $g\left(\frac{3\pi}{2}\right) =$

7. Let $h(x)$ be an antiderivative of $x^2 - 2x$. If $h(-3) = 4$, then $h(1) =$

8. Let f be a differentiable function such that $f(2) = 6$ and $f'(x) = 3x^2 - x$. What is the value of $f(3)$?

1. 52	2. 18	3. $-\frac{112}{3}$	4. $\frac{\pi}{2}+1$	5. $3\pi - 2$	6. 6	7. $21\frac{1}{3} = \frac{64}{3}$	8. 22.5

Answers to 6.7 CA #1

6.8 Indefinite Integrals

Calculus   Name: ______________________________

CA #1

Find the following indefinite integrals.

1. $\int \left(2x^2 - \frac{3}{x} + 2^x\right) dx$	2. $\int \left(\frac{x^7 - 2x}{x^2}\right) dx$	3. $\int \sqrt{x}\left(x - \sqrt[4]{x}\right) dx$
4. $\int \left(\frac{8x^2 + 2x - 3}{x}\right) dx$	5. $\int \left(\frac{1}{x} - e^x\right) dx$	6. $\int \frac{1}{x\sqrt{x}} dx$

Find the function that satisfies the given conditions.

7. $s'(t) = 8t^2 + 6t - 1$ and $s(3) = 50$

8. $\frac{dy}{dx} = 2e^x + \sin x$ and $y(0) = 2$

9. $f''(x) = 3x^2 - 8x$ and $f'(-2) = -20$ and $f(1) = 3$

Answers to 6.8 CA #1

1. $\frac{2}{3}x^3 - 3\ln\lvert x\rvert + \frac{2^x}{\ln 2} + C$	2. $\frac{1}{6}x^6 - 2\ln\lvert x\rvert + C$	3. $\frac{2}{5}x^{\frac{5}{2}} - \frac{4}{7}x^{\frac{7}{4}} + C$
4. $4x^2 + 2x - 3\ln\lvert x\rvert + C$	5. $\ln\lvert x\rvert - e^x + C$	6. $-\frac{2}{\sqrt{x}}$
7. $s(t) = \frac{8}{3}t^3 + 3t^2 - t - 46$	8. $2e^x - \cos x + 1$	9. $f(x) = \frac{1}{4}x^4 - \frac{4}{3}x^3 + 4x + \frac{1}{12}$

6.9 Integrating Using Substitution

Calculus

Name: ______________________________

CA #1

Find the indefinite integrals.

1. $\int x(x^2+3)^5\, dx$

2. $\int \sin(x)\cos^3(x)\, dx$

3. $\int \frac{2x-1}{x^2-x+5}\, dx$

4. $\int \cos x \sqrt{\sin x}\, dx$

5. $\int \frac{x^3}{(5x^4+2)^3}\, dx$

Evaluate the definite integrals.

6. $\int_0^1 x(x^2+1)^3 dx$

7. $\int_0^2 3x^2\sqrt{x^3+1}\, dx$

8. $\int_1^2 e^{1-x} dx$

9. $\int_1^e \frac{(1+\ln x)^2}{x}\,dx$

10. $\int_0^{\frac{\pi}{8}} \tan(2x)\sec^2(2x)\,dx$

Answers to 6.9 CA #1

1. $\frac{1}{12}(x^2+3)^6+C$		2. $-\frac{1}{4}\cos^4 x+C$	3. $\ln\lvert x^2-x+5\rvert+C$		4. $\frac{2}{3}(\sin x)^{\frac{3}{2}}+C$
5. $\frac{-1}{40(5x^4+2)^2}+C$	6. $\frac{15}{8}$	7. $\frac{52}{3}$	8. $1-\frac{1}{e}$	9. $\frac{7}{3}$	10. $\frac{1}{4}$

6.10 Integrating with Long Division and Completing the Square

Calculus Name: ______________________________ **CA #1**

Find the indefinite integral.

1. $\int \frac{6x^2}{x+1} dx$

2. $\int \frac{1}{x^2+6x+10} dx$

3. $\int \frac{5x^2-31x-20}{5x+4} dx$

4. $\int \frac{20x^3-4x^2-67x+44}{10x-7} dx$

5. $\int \frac{1}{x^2-12x+36}\,dx$

6. $\int \frac{1}{\sqrt{-x^2-4x-3}}\,dx$

Answers to 6.10 CA #1

1. $3x^2 - 6x + 6\ln\lvert x+1\rvert + C$	2. $\tan^{-1}(x+3) + C$	3. $\frac{1}{2}x^2 - 7x + \frac{8}{5}\ln\lvert 5x+4\rvert + C$
4. $\frac{2}{3}x^3 + \frac{1}{2}x^2 - 6x + \frac{1}{5}\ln\lvert 10x-7\rvert + C$	5. $-\frac{1}{(x-6)} + C$	6. $\sin^{-1}(x+2) + C$

6.11 Integration by Parts

Calculus Name: ______________________________ **CA #1**

Find the integral.

1. $\int x \sec^2 x \, dx$	2. $\int x \cos x \, dx$
3. $\int_1^2 x \ln x \, dx$	4. $\int 3x \ln x^2 \, dx$

5. $\int x \cos 4x \, dx$

6. The function f has a continuous derivative. The table gives the values of f and its derivatives for $x = 2$ and $x = 7$. If $\int_2^7 f(x)\, dx = 10$, what is the value of $\int_2^7 2xf'(x)\, dx$?

x	$f(x)$	$f'(x)$
2	3	5
7	9	-4

Answers to 6.11 CA #1

1. $x\tan x + \ln\lvert\cos x\rvert + C$	2. $x\sin x + \cos x + C$	3. $2\ln 2 - \frac{3}{4}$
4. $\frac{3}{2}x^2\ln x^2 - \frac{3}{2}x^2 + C$	5. $\frac{x}{4}\sin 4x + \frac{1}{16}\cos 4x + C$	6. 94

6.12 Linear Partial Fractions

Calculus Name: ______________________________ **CA #1**

Evaluate using partial fractions.

1. $\int \frac{1}{x^2-36}\,dx$

2. $\int \frac{4x+1}{(2x+1)(x-2)}\,dx$

3. $\int \frac{3}{x(x+3)}\,dx$

4. $\int_0^1 \frac{x-1}{x^2-x-2}\,dx$

5. $\int \frac{1}{(2x+1)(1-x)}\, dx$

6. $\int \frac{1}{(x+1)(x+2)(x+3)}\, dx$

Answers to 6.12 CA #1

1. $\frac{1}{12}\ln\left\|\frac{x-6}{x+6}\right\| + C$	2. $\frac{1}{5}\ln\|2x+1\| + \frac{9}{5}\ln\|x-2\| + C$	3. $\ln\left\|\frac{x}{x+3}\right\| + C$
4. $\frac{1}{3}\ln 2$	5. $\frac{1}{3}\ln\left\|\frac{2x+1}{1-x}\right\| + C$	6. $\ln\left\|\frac{\sqrt{(x+1)(x+3)}}{x+2}\right\| + C$

6.13 Improper Integrals

Calculus Name: ______________________________ **CA #1**

Evaluate each integral.

1. $\int_0^4 \frac{1}{\sqrt{4-x}}\,dx$

2. $\int_2^\infty x^{-3}\,dx$

3. $\int_{-\infty}^0 e^{3x}\,dx$

4. $\int_0^3 \frac{1}{\sqrt{9-x^2}}\,dx$

5. $\int_0^4 \frac{1}{x^{4/3}}\ dx$

6. $\int_{-\infty}^{\infty} 2xe^{-x^2}\ dx$

Answers to 6.13 CA #1

1. -4	2. $\frac{1}{8}$	3. $\frac{1}{3}$	4. $\frac{\pi}{2}$	5. Diverges	6. 0

6.14 Selecting Techniques for Antidifferentiation

CA #1

Calculus Name: ______________________________

Find the indefinite integral.

1. $\int x\left(e^2 - \sqrt{x}\right)\,dx$

2. $\int \frac{6x^2-63x+74}{x-9}\,dx$

3. $\int \frac{\sec^2 x}{\sqrt{\tan x}}\,dx$

4. $\int \left(\frac{4x^2-3x+6}{x}\right)dx$

5. $\int \frac{(\ln x)^5}{x}\,dx$

6. $\int \frac{1}{x^2-4x+5}\,dx$

7. $\int \frac{1}{\sqrt{1-x^2}}\,dx$

8. $\int \frac{e^x}{4-e^x}\,dx$

Evaluate the definite integral.

9. $\int_0^4 (4x+5)\,dx$

10. $\int_0^1 \frac{x}{(x^2+1)^3}\,dx$

11. $\int_{-\pi}^{-\frac{\pi}{2}} (1-\cos x)\,dx$

12. $\int_0^1 e^{-2x}\,dx$

13. $\int_{-\frac{1}{3}}^{\frac{1}{3}} \frac{1}{1+9t^2}\,dt$

Answers to 6.14 CA #1

1. $\frac{e^2}{2}x^2 - \frac{2}{5}x^{\frac{5}{2}} + C$	2. $3x^2 - 9x - 7\ln\lvert x-9\rvert + C$	3. $2\sqrt{\tan x} + C$

4. $2x^2 - 3x + 6\ln\lvert x\rvert + C$	5. $\frac{[\ln x]^6}{6} + C$	6. $\tan^{-1}(x-2) + C$	7. $\sin^{-1}(x) + C$

8. $-\ln\lvert 4-e^x\rvert + C$	9. 52	10. $\frac{3}{16}$	11. $\frac{\pi}{2} + 1$	12. $\frac{1}{2} - \frac{1}{2e^2}$	13. $\frac{\pi}{6}$