

Name: \_\_\_\_\_

Mark: \_\_\_\_\_

**Mini-math Div 3/4: Monday, January 18, 2021 (12 minutes)**

1. Find the derivative of  $y$  with respect to  $x$  in each of the following.

(a) (2 points)  $y = \sin(\cos^2 x)$

**Solution:**

$$\begin{aligned}\frac{dy}{dx} &= \cos(\cos^2 x)(2 \cos x)(-\sin x) = -2 \cos(\cos^2 x) \sin x \cos x \\ \text{or} \quad & -\cos(\cos^2 x) \sin(2x)\end{aligned}$$

(b) (2 points)  $y = x \sin 2x$

**Solution:**

$$\frac{dy}{dx} = \sin 2x + 2x \cos 2x$$

(c) (2 points)  $y = \frac{x}{\cos(x^2 + 1)}$

**Solution:**

$$\begin{aligned}\frac{dy}{dx} &= \frac{1 \cdot \cos(x^2 + 1) - x \cdot (-\sin(x^2 + 1)) \cdot (2x)}{\cos^2(x^2 + 1)} \\ &= \frac{\cos(x^2 + 1) + 2x^2 \sin(x^2 + 1)}{\cos^2(x^2 + 1)}\end{aligned}$$

2. Find the derivative of  $y$  with respect to  $x$  in each of the following.

(a) (2 points)  $y = 2 \tan x \sec x$

**Solution:**

$$\begin{aligned}\frac{dy}{dx} &= 2 \sec^2 x \sec x + 2 \tan x \sec x \tan x \\ &= 2 \sec^3 x + 2 \sec x \tan^2 x \quad \text{or} \quad 2 \sec x (\sec^2 x + \tan^2 x) \\ &\quad \text{or} \quad 2 \sec x (2 \sec^2 x - 1)\end{aligned}$$

(b) (2 points)  $y = \cot^2 2x - \csc 2x$

**Solution:**

$$\begin{aligned}\frac{dy}{dx} &= 2(\cot 2x)(-\csc^2 2x)(2) - (-\csc 2x \cot 2x)(2) \\ &= -4 \csc^2 2x \cot 2x + 2 \csc 2x \cot 2x \quad \text{or} \quad 2 \csc 2x \cot 2x (1 - 2 \csc 2x)\end{aligned}$$

(c) (2 points)  $\tan \frac{y}{x} = x$

**Solution:**

$$\begin{aligned}\sec^2 \left( \frac{y}{x} \right) \cdot \frac{y' \cdot x - y}{x^2} &= 1 \\ \frac{dy}{dx} \cdot x - y &= x^2 \cos^2 \left( \frac{y}{x} \right) \\ \frac{dy}{dx} &= x \cos^2 \left( \frac{y}{x} \right) + \frac{y}{x} \quad \text{or} \quad \frac{\sin^2 \left( \frac{y}{x} \right) + y}{x}\end{aligned}$$

3. (3 points) Find the equation of the line tangent to the given curve at the given point.

$$\frac{1}{2} + \sin y + \tan x = \sec 2y, \quad \text{at } \left(\frac{\pi}{4}, \frac{\pi}{6}\right)$$

**Solution:** Differentiating implicitly,

$$\cos y \cdot \frac{dy}{dx} + \sec^2 x = \sec 2y \tan 2y \cdot 2 \frac{dy}{dx}$$

At the given point, we have

$$\begin{aligned} \frac{\sqrt{3}}{2} \cdot \frac{dy}{dx} + 2 &= 2 \cdot \sqrt{3} \cdot 2 \frac{dy}{dx} \\ \frac{dy}{dx} &= \frac{2}{(7/2)\sqrt{3}} = \frac{4}{7\sqrt{3}} \end{aligned}$$

By the point-slope formula, the equation of the desired line is

$$y - \frac{\pi}{6} = \frac{4}{7\sqrt{3}} \left(x - \frac{\pi}{4}\right)$$