Mini-math Div 3/4: Friday, February 4, 2022 (10 minutes)

SOLUTIONS

1. (1 point) Suppose $\int_{-2}^{5}(2f(x)+3)\,dx = 15$, and $\int_{3}^{5} f(x)\,dx = 10$. What is $\int_{-2}^{3} f(x)\,dx$?

A. -13 B. -4 C. 5 D. 7

Solution:

$$15 = 2\int_{-2}^{5} f(x)\,dx + \int_{-2}^{5} 3\,dx = 2\int_{-2}^{5} f(x)\,dx + 21 \quad \Rightarrow \quad \int_{-2}^{5} f(x)\,dx = \frac{15-21}{2} = -3,$$
$$\Rightarrow \int_{-2}^{3} f(x)\,dx = \int_{-2}^{5} f(x)\,dx - \int_{3}^{5} f(x)\,dx = -3 - 10 = -13$$

(a) is correct.

2. (1 point) Evaluate $\int_{1}^{4} \frac{x+4}{\sqrt{x}}\,dx$.

A. $-\frac{9}{4}$ B. 7 C. 11 D. $\frac{38}{3}$

Solution: Splitting up the integral,

$$\int_{1}^{4} \frac{x+4}{\sqrt{x}}\,dx = \int_{1}^{4} (x^{1/2} + 4x^{-1/2})\,dx = \left(\frac{2}{3}x^{3/2} + 8x^{1/2}\right)\Big|_{1}^{4} = \frac{2}{3}(4^{3/2} - 1^{3/2}) + 8(4^{1/2} - 1^{1/2})$$
$$= \frac{2}{3}(2^3 - 1) + 8(2-1) = \frac{2}{3}\cdot 7 + 8 = \frac{14+24}{7} = \frac{38}{3}$$

(d) is correct.

3. (1 point) Evaluate $\int_{1}^{3} \frac{x+1}{x^2+2x-1}\,dx$.

A. $\frac{\ln 7}{2}$ B. $\frac{\ln 14 + \ln 2}{2}$ C. $\ln 14 - \ln 2$ D. $\ln 3$

Solution: Use $u = x^2+2x-1$, so that $du = (2x+2)\,dx = 2(x+1)\,dx$, $1 \mapsto 2$, and $3 \mapsto 14$. Then

$$\int_{1}^{3} \frac{x+1}{x^2+2x-1}\,dx = \int_{2}^{14} \frac{1}{u}\cdot\frac{1}{2} = \,du = \frac{1}{2}\ln|u|\Big|_{2}^{14} = \frac{1}{2}(\ln 14 - \ln 2) = \frac{\ln 7}{2}$$

(a) is correct.

4. (1 point) Suppose $\int_1^5 f'(x)\,dx = 12$ and $f(5) = 3$. What is $f(1)$?

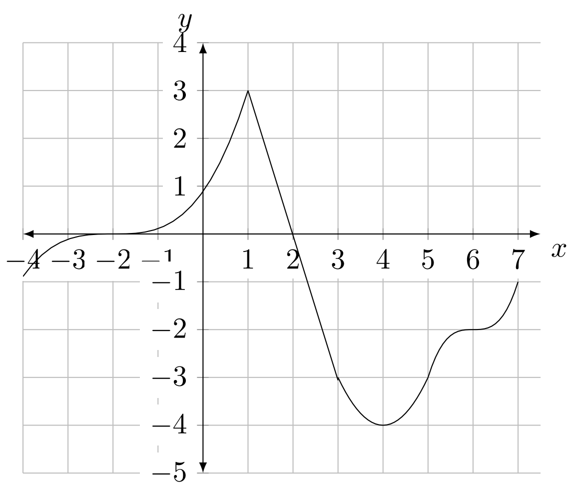
A. -15 B. -9 C. 9 D. 15

Solution: By FTC II,

$$12 = \int_1^5 f'(x)\,dx = f(x)\Big|_1^5 = f(5) - f(1) = 3 - f(1)$$
$$f(1) = 3 - 12 = -9$$

(b) is correct.

5. (1 point) (AP) The graph of f is below. Let $g(x) = \int_1^x f(t)\,dt$. At what value(s) of x in the interval $[-4, 7]$ does g have a point of inflection?



A. exactly one of -2 and 2

B. both -2 and 2

C. both 1 and 4

D. all of $-2, 5$ and 6

Solution: $g'(x) = f(x)$, so $g''(x) = f'(x)$. For a point of inflection, we need $g''(x) = f'(x)$ to change sign, so (c) is correct.