Fractions

Start with the number 1. Choose any natural number $n \geq 2$.

e.g. Choose $n = 2$.

Step 1: Add $1/n$.

e.g.

$$1 \xrightarrow{+} 1 + \frac{1}{2} = \frac{3}{2}$$

For following steps, you can choose to either add $1/n$, or take the reciprocal of your current value.

e.g. From $\frac{3}{2}$, you can either add $\frac{1}{2}$ to get

$$\frac{3}{2} \xrightarrow{+} \frac{3}{2} + \frac{1}{2} = 2$$

or take the reciprocal to get

$$\frac{3}{2} \xrightarrow{r} \frac{2}{3}$$

Your goal: get back to 1 in the fewest number of steps. For a chosen number of n, we define $a(n)$ to be the fewest number of steps to return to 1.

e.g.

$$1 \xrightarrow{+} \frac{3}{2} \xrightarrow{r} \frac{2}{3} \xrightarrow{r} \frac{3}{2} \xrightarrow{+} 2 \xrightarrow{r} \frac{1}{2} \xrightarrow{+} 1$$

takes 6 steps, so $a(4) \leq 6$ (you can do better).

Challenges:

(1) Provide the best upper bound you can for $a(6)$.

(2) Provide the best upper bound you can for $a(1916)$.

(3) Provide the best upper bound you can for $a(103)$.

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