Number Bases

(1) In a strange land they have a slightly different place value system than our base ten system. For example, $4 \times 6 = 30$ and $4 \times 7 = 34$. Based on this system give the value of $5 \times 4 \times 7$.

(2) A base-two numeral consists of 15 digits all of which are ones. This number when tripled and written in base two, contains how many digits?

(3) The symbol 37_b represents a two-digit number written in base b. For instance, $37_8 = 31$, where the last number is written in base 10. Suppose that 37_b is exactly one-half of 73_b. What is b?

(4) The symbol $(x)_7$ means that the number is written in base 7; so, for instance, $(23)_7$ equals $2 \times 7 + 3 = 17$ in base 10. Suppose that a certain integer x can be written as a two-digit number in both bases 5 and 6, so that $x = (4z)_5$ and $x = (4y)_6$. What is x (in base 10)?

(5) The following addition problem is not correct if the numbers are interpreted as base 10 numbers. In what base is the problem correct?

$$66 + 87 + 85 + 48 = 132$$

(6) James did the following addition problem: $440 + 340 = 1000$. This problem is done correctly! In what base was James writing his numbers?

(7) Let x and y be digits in base 7 such that

$$1111_2 + 1111_3 + 1111_4 + 1111_5 + 1111_6 + 1111_7 + 1111_8 + 1111_9 = xyxyx_7 - 91$$

where $xyxyx_7$ is a five-digit number in base 7. Find x and y.

(8) A positive integer n can be written in base 7 as the three-digit number xyy where x and y are digits. The same number n can be written in base 6 as the three-digit number yxx. Find n.

(9) A three digit number xyz in base 7 when written in base 9 becomes the three-digit number zyx. What is the decimal representation of the number?

(10) In what base(s) is 165 a divisor of 561?

(11) Prove that, in any base, 10101 is divisible by 111.

(12) Find the base in which the following multiplication holds: $25 \times 314 = 10274$.

(13) 1580_{10} is 2145 in what base?