

12.8.3.1.2

EE24BTECH11019 - Dwarak A

Question:

Find the area under the given curves and given lines:

$$y = x^4 \quad (0.1)$$

$$x = 1 \quad (0.2)$$

$$x = 5 \quad (0.3)$$

$$y = 0 \quad (\text{x-axis}) \quad (0.4)$$

Solution:

Theoretical Solution:

Area between lines and curve A,

$$A = \int_1^5 x^4 dx \quad (0.5)$$

$$= \frac{x^5}{5} \Big|_{x=5} - \frac{x^5}{5} \Big|_{x=1} \quad (0.6)$$

$$= \frac{3125}{5} - \frac{1}{5} \quad (0.7)$$

$$= 624.8 \quad (0.8)$$

Simulated Solution (Trapezoidal Rule):

Taking trapezoid shaped strips of small area and adding them all up. Say we have to find the area of $y(x)$ from $x = x_0$ to $x = x_n$, discretize points on the x axis $x_0, x_1, x_2, \dots, x_n$ such that they are equally spaced with step-size h .

Sum of all trapezoidal areas is given by,

$$A = \frac{1}{2}h(y(x_1) + y(x_0)) + \frac{1}{2}h(y(x_2) + y(x_1)) + \dots + \frac{1}{2}h(y(x_n) + y(x_{n-1})) \quad (0.9)$$

$$= h \left[\frac{1}{2} (y(x_0) + y(x_n)) + y(x_1) + \dots + y(x_{n-1}) \right] \quad (0.10)$$

Let $A(x_n)$ be the area enclosed by the curve $y(x)$ from $x = x_0$ to $x = x_n$, (x_0, x_1, \dots, x_n) be equidistant points with step-size h .

$$A(x_n + h) = A(x_n) + \frac{1}{2}h(y(x_n + h) + y(x_n)) \quad (0.11)$$

We can repeat this till we get required area.

Discretizing the steps, making $A(x_n) = A_n, y(x_n) = y_n$ we get,

$$A_{n+1} = A_n + \frac{1}{2}h(y_{n+1} + y_n) \quad (0.12)$$

We can write y_{n+1} in terms of y_n using first principle of derivative. $y_{n+1} = y_n + hy'_n$

$$A_{n+1} = A_n + \frac{1}{2}h((y_n + hy'_n) + y_n) \quad (0.13)$$

$$A_{n+1} = A_n + \frac{1}{2}h(2y_n + hy'_n) \quad (0.14)$$

$$A_{n+1} = A_n + hy_n + \frac{1}{2}h^2y'_n \quad (0.15)$$

$$x_{n+1} = x_n + h \quad (0.16)$$

In the given question, $y_n = x_n^4$ and $y'_n = 4x_n^3$
General Difference Equation will be given by,

$$A_{n+1} = A_n + hy_n + \frac{1}{2}h^2y'_n \quad (0.17)$$

$$A_{n+1} = A_n + h(x_n^4) + \frac{1}{2}h^2(4x_n^3) \quad (0.18)$$

$$A_{n+1} = A_n + hx_n^4 + 2h^2x_n^3 \quad (0.19)$$

$$x_{n+1} = x_n + h \quad (0.20)$$

Iterating from $x_0 = 1$ till we reach $x_n = 5$ will return required area.

For $h = 0.05$, Area under the curve is 627.9241

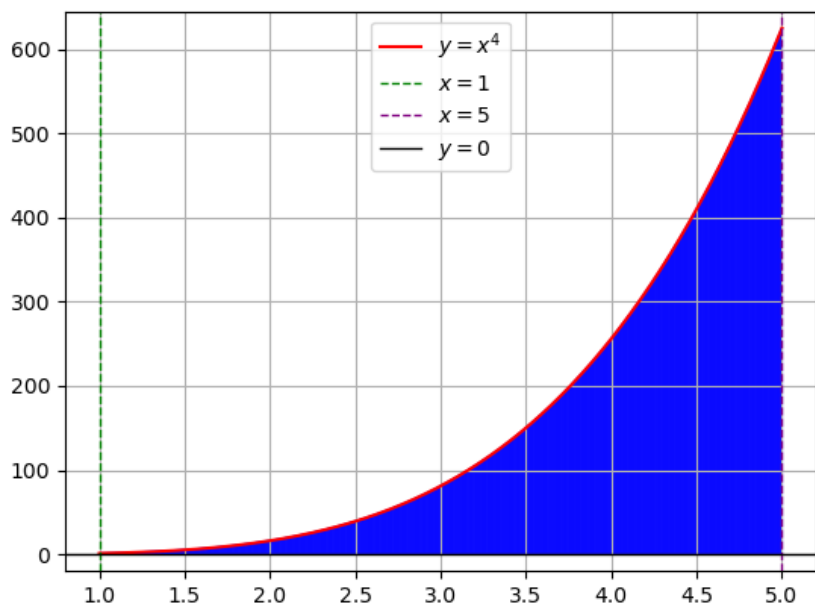


Fig. 0.1: Plot of the differential equation when $h = 0.01$