

11.16.3.15.2

EE24BTECH11019 - Dwarak A

Question:

If A and B are events such that

$$P(A) = \frac{1}{4} \quad (0.1)$$

$$P(B) = \frac{1}{2} \quad (0.2)$$

$$P(A \cap B) = \frac{1}{8} \quad (0.3)$$

Find

$$p(\neg A \cap \neg B) \quad (0.4)$$

Solution:

Define a random variable X that takes on four possible values corresponding to the four outcomes:

$$X = \begin{cases} 1, & \text{if only } A \text{ occurs} \\ 2, & \text{if only } B \text{ occurs} \\ 3, & \text{if both } A \text{ and } B \text{ occur} \\ 4, & \text{if neither } A \text{ nor } B \text{ occur} \end{cases} \quad (0.5)$$

Calculating probabilities of the 4 outcomes of X :

$$P(X = 1) = P(A \cap \neg B) = P(A) - P(A \cap B) = \frac{1}{4} - \frac{1}{8} = \frac{1}{8} \quad (0.6)$$

$$P(X = 2) = P(\neg A \cap B) = P(B) - P(A \cap B) = \frac{1}{2} - \frac{1}{8} = \frac{3}{8} \quad (0.7)$$

$$P(X = 3) = P(A \cap B) = \frac{1}{8} \quad (0.8)$$

$$P(X = 4) = P(\neg A \cap \neg B) = 1 - P(A \cup B) \quad (0.9)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \quad (0.10)$$

$$P(A \cup B) = \frac{1}{4} + \frac{1}{2} - \frac{1}{8} = \frac{5}{8} \quad (0.11)$$

Using (0.9) and (0.11):

$$P(X = 4) = P(\neg A \cap \neg B) = 1 - \frac{5}{8} = \frac{3}{8} \quad (0.12)$$

Thus, the Probability Mass Function (PMF) for X is:

$$p_X(x) = \begin{cases} \frac{1}{8}, & \text{if } x = 1 \\ \frac{3}{8}, & \text{if } x = 2 \\ \frac{1}{8}, & \text{if } x = 3 \\ \frac{3}{8}, & \text{if } x = 4 \end{cases} \quad (0.13)$$

$P(\neg A \wedge \neg B)$ corresponds to $P(X = 4)$

$$P(\neg A \wedge \neg B) = P(X = 4) = \frac{3}{8}. \quad (0.14)$$