

9.2.8

EE24BTECH11019 - Dwarak A

Question:

Consider the differential equation

$$(y \sin y + \cos y + x) y' = y \quad (0.1)$$

Verify that

$$x = y - \cos y \quad (0.2)$$

is a solution for it

Solution:

Theoretical Solution:

$$y = \frac{dy}{dx} (y \sin y + \cos y + x) \quad (0.3)$$

$$\frac{dx}{dy} - \frac{x}{y} = \sin y + \frac{\cos y}{y} \quad (0.4)$$

Integrating factor,

$$\mu(y) = e^{\int -\frac{1}{y} dy} \quad (0.5)$$

$$\mu(y) = \frac{1}{y} \quad (0.6)$$

Integration,

$$\frac{x}{y} = \int \mu(y) \left(\sin y + \frac{\cos y}{y^2} \right) dy \quad (0.7)$$

$$\frac{x}{y} = \int \frac{\sin y}{y} dy + \int \frac{\cos y}{y^2} dy \quad (0.8)$$

$$\frac{x}{y} = \int \frac{\sin y}{y} dy + \cos y \int \frac{1}{y^2} dy - \int \frac{\sin y}{y} dy \quad (0.9)$$

$$\frac{x}{y} = -\frac{\cos y}{y} + c \quad (0.10)$$

$$x = cy - \cos y \quad (0.11)$$

Thus the equation (0.2) is a solution to the differential equation (0.1) when $c = 1$.

Algorithm (Forward Euler Method):

Definition of derivative,

$$f'(y) \approx \lim_{h \rightarrow 0} \frac{f(y+h) - f(y)}{h} \quad (0.12)$$

$$f(y+h) \approx f(y) + hf'(y) \quad (0.13)$$

$$x_{n+1} \approx x_n + h \left. \frac{dx}{dy} \right|_{x=x_n, y=y_n} \quad (0.14)$$

Difference equation,

$$x_{n+1} \approx x_n + h \left(\sin y_n + \frac{\cos y_n + x_n}{y_n} \right) \quad (0.15)$$

$$y_{n+1} = y_n + h \quad (0.16)$$

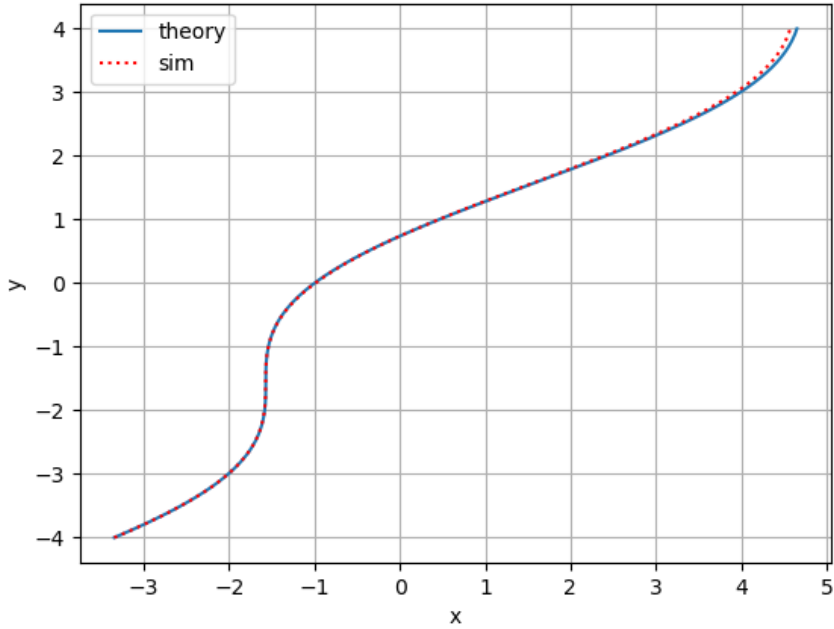


Fig. 0.1: Plot of the differential equation when $h = 0.01$