## EE24BTECH11019 - Dwarak A

# **Question:**

Find a particular solution satisfying the differential equation,

$$\cos\left(\frac{dy}{dx}\right) = a \quad (a \in \mathbb{R}) \tag{0.1}$$

given condition,

$$y(0) = 1 (0.2)$$

#### **Solution:**

### **Theoretical Solution:**

$$\frac{dy}{dx} = \cos^{-1} a \tag{0.3}$$

$$dy = \left(\cos^{-1} a\right) dx \tag{0.4}$$

Integration,

$$\int dy = \int \left(\cos^{-1} a\right) dx \tag{0.5}$$

$$y = \left(\cos^{-1} a\right) x + c \tag{0.6}$$

To find c substitute (0.2) in (0.6),

$$1 = (\cos^{-1} a)(0) + c \tag{0.7}$$

$$1 = 0 + c \tag{0.8}$$

$$c = 1 \tag{0.9}$$

Particular solution,

$$y = (\cos^{-1} a)x + 1 \tag{0.10}$$

#### **Simulated Solution:**

Trapezoidal rule,

$$\int_{x_0}^{x_n} f(x)dx \approx \frac{h}{2} \left( f(x_0) + 2 \sum_{i=1}^{n-1} f(x_i) + f(x_n) \right)$$
 (0.11)

Discretization using trapezoidal rule, integrate  $f(x) = cos^{-1}a$  from  $x_n$  to  $x_{n+1}$ ,

$$y_{n+1} - y_n \approx \frac{h}{2} (f(x_n) + f(x_{n+1}))$$
 (0.12)

We can write  $f(x_{n+1})$  in terms of  $f(x_n)$  using first principle of derivative.  $f(x_{n+1}) = f(x_n) + hf'(x_n)$ 

$$y_{n+1} - y_n \approx \frac{h}{2} (f(x_n) + f(x_{n+1}))$$
 (0.13)

$$y_{n+1} - y_n \approx \frac{h}{2} (2f(x_n) + hf'(x_n))$$
 (0.14)

$$y_{n+1} \approx y_n + h \cos^{-1} a + \frac{h^2}{2}(0)$$
 (0.15)

Difference equation,

$$y_{n+1} \approx y_n + h \cos^{-1} a {(0.16)}$$

$$x_{n+1} = x_n + h (0.17)$$

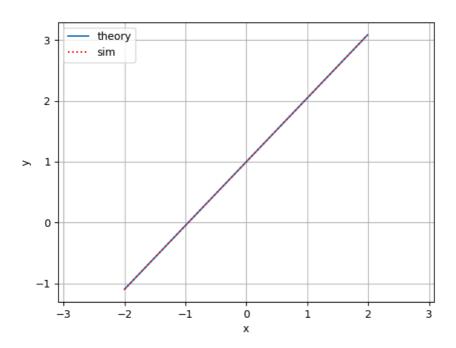


Fig. 0.1: Plot of the differential equation when h = 0.01, a = 0.5