10.4.1.1.2

EE24BTECH11019 - Dwarak A

Question:
Find the roots of the quadratic equation:

$$x^2 - 2x = (-2)(3 - x) \tag{0.1}$$

Solution:
Rearranging terms,

$$x^2 - 2x = 2x - 6 \tag{0.2}$$
$$x^2 - 4x + 6 = 0 \tag{0.3}$$

Theoretical solution (Quadratic formula):
The roots are,

$$x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \tag{0.4}$$
$$= \frac{4 + \sqrt{16 - 24}}{2} \tag{0.5}$$
$$= 2 + \sqrt{2}i \tag{0.6}$$
$$x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \tag{0.7}$$
$$= \frac{4 - \sqrt{16 - 24}}{2} \tag{0.8}$$
$$= 2 - \sqrt{2}i \tag{0.9}$$

Computational solution:
(1) Eigenvalues of Companion Matrix:
The roots of a polynomial equation $x^n + b_{n-1}x^{n-1} + \cdots + b_2x^2 + b_1x + b_0 = 0$ is given by finding eigenvalues of the companion matrix (C).

$$C = \begin{pmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \vdots & 1 \\ -b_0 & -b_1 & -b_2 & \dots & -b_{n-1} \end{pmatrix} \tag{0.10}$$

The solution given by the code is,

$$x_1 = 2.00000000 + 1.41421356i \tag{0.11}$$
$$x_2 = 2.00000000 - 1.41421356j \tag{0.12}$$

(2) Newton-Raphson iterative method:

$$f(x) = x^2 - 4x + 6 \tag{0.13}$$

$$f'(x) = 2x - 4 \tag{0.14}$$

Difference equation,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \tag{0.15}$$

$$x_{n+1} = x_n - \frac{x_n^2 - 4x_n + 6}{2x_n - 4} \tag{0.16}$$

$$x_{n+1} = \frac{x_n}{2} - 1 + \frac{1}{x_n - 2} \tag{0.17}$$

Picking two initial guesses,

$$x_0 = 1 + i \text{ converges to } 2.0 + 1.4142135623730954i \tag{0.18}$$

$$x_0 = -1 - i \text{ converges to } 2.0000000000000733 + -1.4142135623729934i \tag{0.19}$$