12.9.2.8

EE24BTECH11019 - Dwarak A

Question:
Find a particular solution satisfying the differential equation,

$$\cos\left(\frac{dy}{dx}\right) = a \quad (a \in \mathbb{R}) \tag{0.1}$$

given condition,

$$y(0) = 1 \tag{0.2}$$

Solution:

Theoretical Solution:

$$\frac{dy}{dx} = \cos^{-1} a \tag{0.3}$$

$$dy = \left(\cos^{-1} a\right) dx \tag{0.4}$$

Integration,

$$\int dy = \int \left(\cos^{-1} a\right) dx \tag{0.5}$$

$$y = \left(\cos^{-1} a\right) x + c \tag{0.6}$$

To find c substitute (0.2) in (0.6),

$$1 = \left(\cos^{-1} a\right)(0) + c \tag{0.7}$$

$$1 = 0 + c \tag{0.8}$$

$$c = 1 \tag{0.9}$$

Particular solution,

$$y = \left(\cos^{-1} a\right) x + 1 \tag{0.10}$$

Simulated Solution:
Trapezoidal rule,

$$\int_{x_0}^{x_n} f(x)dx \approx \frac{h}{2}\left(f(x_0) + 2\sum_{i=1}^{n-1} f(x_i) + f(x_n)\right) \tag{0.11}$$

Discretization using trapezoidal rule, integrate $f(x) = \cos^{-1} a$ from x_n to x_{n+1},

$$y_{n+1} - y_n \approx \frac{h}{2}\left(f(x_n) + f(x_{n+1})\right) \tag{0.12}$$

We can write $f(x_{n+1})$ in terms of $f(x_n)$ using first principle of derivative. $f(x_{n+1}) = f(x_n) + hf'(x_n)$

$$y_{n+1} - y_n \approx \frac{h}{2}\left(f\left(x_n\right) + f\left(x_{n+1}\right)\right) \tag{0.13}$$

$$y_{n+1} - y_n \approx \frac{h}{2}\left(2f\left(x_n\right) + hf'\left(x_n\right)\right) \tag{0.14}$$

$$y_{n+1} \approx y_n + h\cos^{-1} a + \frac{h^2}{2}(0) \tag{0.15}$$

Difference equation,

$$y_{n+1} \approx y_n + h\cos^{-1} a \tag{0.16}$$

$$x_{n+1} = x_n + h \tag{0.17}$$

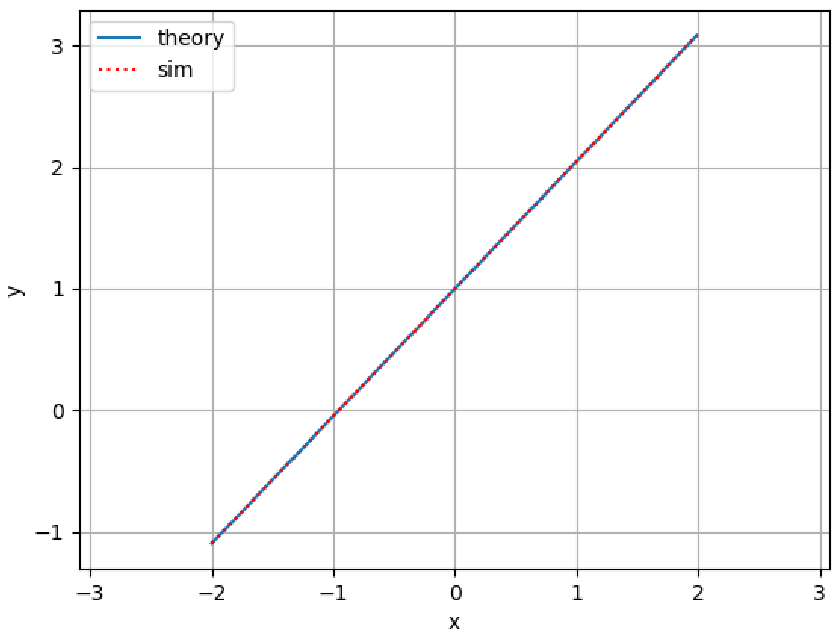

Fig. 0.1: Plot of the differential equation when $h = 0.01$, $a = 0.5$