9.2.8

EE24BTECH11019 - Dwarak A

Question:
Consider the differential equation

$$(y \sin y + \cos y + x)\, y' = y \tag{0.1}$$

Verify that

$$x = y - \cos y \tag{0.2}$$

is a solution for it
Solution:

Theoretical Solution:

$$y = \frac{dy}{dx}(y \sin y + \cos y + x) \tag{0.3}$$

$$\frac{dx}{dy} - \frac{x}{y} = \sin y + \frac{\cos y}{y} \tag{0.4}$$

Integrating factor,

$$\mu(y) = e^{\int -\frac{1}{y} dy} \tag{0.5}$$

$$\mu(y) = \frac{1}{y} \tag{0.6}$$

Integration,

$$\frac{x}{y} = \int \mu(y)\left(\sin y + \frac{\cos y}{y^2}\right) dy \tag{0.7}$$

$$\frac{x}{y} = \int \frac{\sin y}{y} dy + \int \frac{\cos y}{y^2} dy \tag{0.8}$$

$$\frac{x}{y} = \int \frac{\sin y}{y} dy + \cos y \int \frac{1}{y^2} dy - \int \frac{\sin y}{y} dy \tag{0.9}$$

$$\frac{x}{y} = -\frac{\cos y}{y} + c \tag{0.10}$$

$$x = cy - \cos y \tag{0.11}$$

Thus the equation (0.2) is a solution to the differential equation (0.1) when $c = 1$.

Algorithm (Forward Euler Method):

Definition of derivative,

$$f'(y) \approx \lim_{h\to 0} \frac{f(y+h) - f(y)}{h} \tag{0.12}$$

$$f(y+h) \approx f(y) + hf'(y) \tag{0.13}$$

$$\implies x_{n+1} \approx x_n + h\frac{dx}{dy}\Big|_{x=x_n, y=y_n} \tag{0.14}$$

Difference equation,

$$x_{n+1} \approx x_n + h\left(\sin y_n + \frac{\cos y_n + x_n}{y_n}\right) \tag{0.15}$$

$$y_{n+1} = y_n + h \tag{0.16}$$

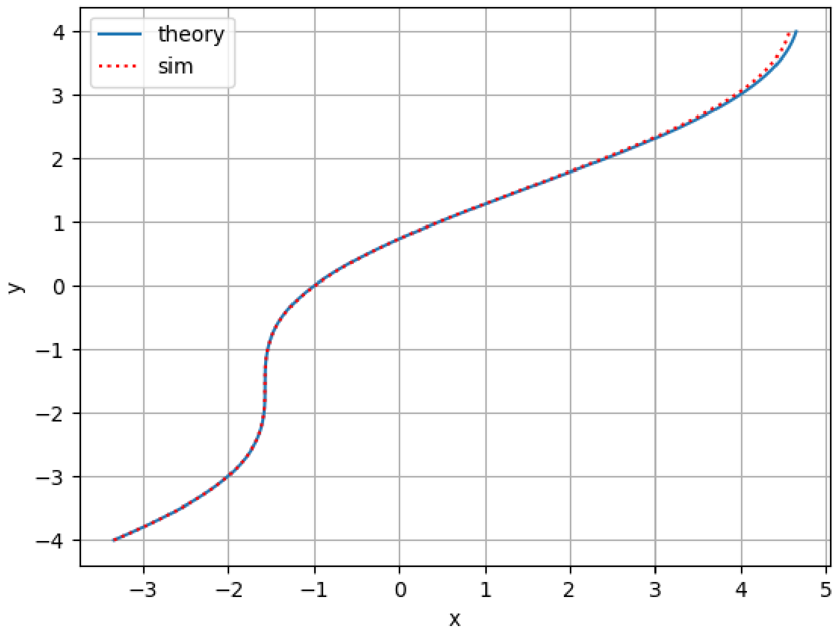

Fig. 0.1: Plot of the differential equation when $h = 0.01$