10.4.1.1.2

EE24BTECH11019 - Dwarak A

Question:

Find the roots of the quadratic equation:

$$x^2 - 2x = (-2)(3 - x) \tag{0.1}$$

Solution:

Rearranging terms,

$$x^2 - 2x = 2x - 6 \tag{0.2}$$

$$x^2 - 4x + 6 = 0 \tag{0.3}$$

Theoretical solution (Quadratic formula):

The roots are,

$$x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \tag{0.4}$$

$$= \frac{4 + \sqrt{16 - 24}}{2} \tag{0.5}$$

$$= 2 + \sqrt{2}i \tag{0.6}$$

$$x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \tag{0.7}$$

$$= \frac{4 - \sqrt{16 - 24}}{2} \tag{0.8}$$

$$= 2 - \sqrt{2}i \tag{0.9}$$

Computational solution (Newton-Raphson iterative method):

$$f(x) = x^2 - 4x + 6 \tag{0.10}$$

$$f'(x) = 2x - 4 \tag{0.11}$$

Difference equation,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \tag{0.12}$$

$$x_{n+1} = x_n - \frac{x_n^2 - 4x_n + 6}{2x_n - 4} \tag{0.13}$$

$$x_{n+1} = \frac{x_n}{2} - 1 + \frac{1}{x_n - 2} \tag{0.14}$$

Picking two initial guesses,

$$x_0 = 1 + i \text{ converges to } 2.0 + 1.4142135623730954i \tag{0.15}$$

$$x_0 = -1 - i \text{ converges to } 2.0000000000000733 + -1.4142135623729934i \tag{0.16}$$