12.9.6.14

EE24BTECH11019 - Dwarak A

Question:

Find the particular solution of the differential equation

$$\left(1+x^2\right)\frac{dy}{dx}+2xy=\frac{1}{1+x^2} \tag{0.1}$$

for the inital condition $y=0$, when $x=1$.

Solution:

Theoretical Solution:

$$\frac{dy}{dx}+\left(\frac{2x}{1+x^2}\right)y=\frac{1}{\left(1+x^2\right)^2} \tag{0.2}$$

Integrating factor,

$$\mu(x)=e^{\int \frac{2x}{1+x^2}dx} \tag{0.3}$$

$$\mu(x)=1+x^2 \tag{0.4}$$

General solution,

$$y\left(1+x^2\right)=\int \frac{1}{1+x^2}dx \tag{0.5}$$

$$y=\frac{\tan^{-1}x+c}{1+x^2} \tag{0.6}$$

where c is the constant of integration.

Substituting initial conditions, $x=1$ and $y=0$ in (0.6),

$$0=\frac{\tan^{-1}1+c}{1+1^2} \tag{0.7}$$

$$0=\frac{\frac{\pi}{4}+c}{2} \tag{0.8}$$

$$c=-\frac{\pi}{4} \tag{0.9}$$

Particular solution obtained by substituting (0.9) in (0.6),

$$y=\frac{\tan^{-1}x-\frac{\pi}{4}}{1+x^2} \tag{0.10}$$

Algorithm (Forward Euler Method):

Definition of derivative,

$$f'(x) \approx \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \tag{0.11}$$

$$f(x+h) \approx f(x) + hf'(x) \tag{0.12}$$

$$\implies y_{n+1} \approx y_n + h\frac{dy}{dx}\bigg|_{x=x_n, y=y_n} \tag{0.13}$$

Difference equation,

$$y_{n+1} \approx y_n + h\left(-\frac{2x_n y_n}{1+x_n^2} + \frac{1}{(1+x_n^2)^2}\right) \tag{0.14}$$

$$x_{n+1} = x_n + h \tag{0.15}$$

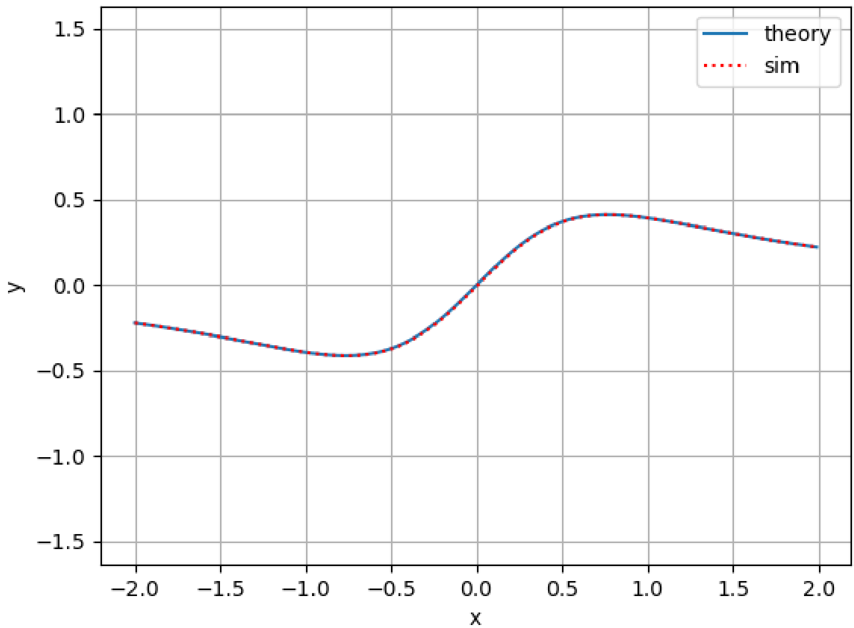

Fig. 0.1: Plot of the differential equation when $h = 0.01$