

12.9.4.13

EE24BTECH11019 - Dwarak A

Question:

Find a particular solution satisfying the differential equation,

$$\cos\left(\frac{dy}{dx}\right) = a \quad (a \in \mathbb{R}) \quad (0.1)$$

given condition,

$$y(0) = 1 \quad (0.2)$$

Solution:

Theoretical Solution:

$$\frac{dy}{dx} = \cos^{-1} a \quad (0.3)$$

$$dy = (\cos^{-1} a) dx \quad (0.4)$$

Integration,

$$\int dy = \int (\cos^{-1} a) dx \quad (0.5)$$

$$y = (\cos^{-1} a) x + c \quad (0.6)$$

To find c substitute (0.2) in (0.6),

$$1 = (\cos^{-1} a)(0) + c \quad (0.7)$$

$$1 = 0 + c \quad (0.8)$$

$$c = 1 \quad (0.9)$$

Particular solution,

$$y = (\cos^{-1} a) x + 1 \quad (0.10)$$

Simulated Solution:

Trapezoidal rule,

$$\int_{x_0}^{x_n} f(x) dx \approx \frac{h}{2} \left(f(x_0) + 2 \sum_{i=1}^{n-1} f(x_i) + f(x_n) \right) \quad (0.11)$$

Discretization using trapezoidal rule, integrate $f(x) = \cos^{-1} a$ from x_n to x_{n+1} ,

$$y_{n+1} - y_n \approx \frac{h}{2} (f(x_n) + f(x_{n+1})) \quad (0.12)$$

We can write $f(x_{n+1})$ in terms of $f(x_n)$ using first principle of derivative. $f(x_{n+1}) = f(x_n) + hf'(x_n)$

$$y_{n+1} - y_n \approx \frac{h}{2} (f(x_n) + f(x_{n+1})) \quad (0.13)$$

$$y_{n+1} - y_n \approx \frac{h}{2} (2f(x_n) + hf'(x_n)) \quad (0.14)$$

$$y_{n+1} \approx y_n + h \cos^{-1} a + \frac{h^2}{2}(0) \quad (0.15)$$

Difference equation,

$$y_{n+1} \approx y_n + h \cos^{-1} a \quad (0.16)$$

$$x_{n+1} = x_n + h \quad (0.17)$$

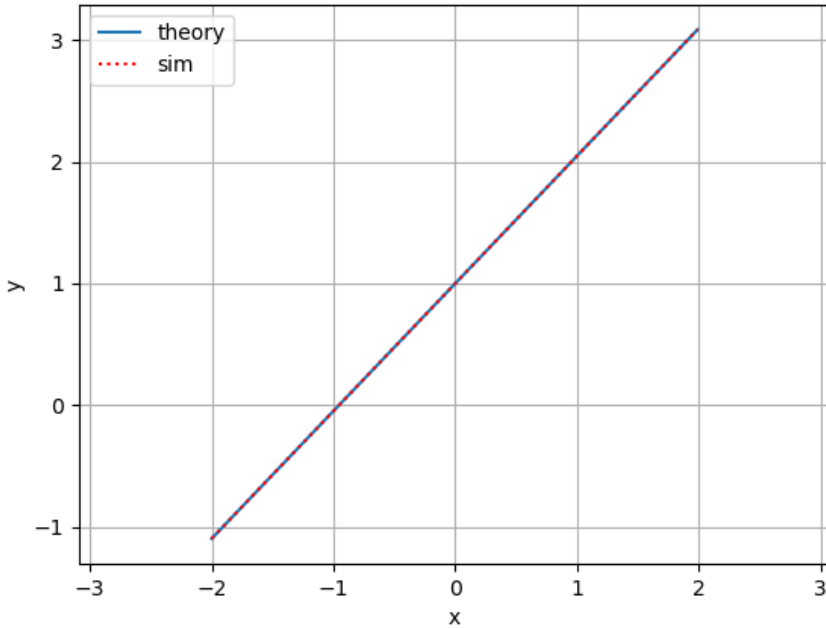


Fig. 0.1: Plot of the differential equation when $h = 0.01$, $a = 0.5$