

Probability

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February 6, 2025

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Problem Statement

Given:

$$P(A) = 0.25 \quad (1)$$

$$P(B) = 0.5 \quad (2)$$

$$P(AB) = 0.125 \quad (3)$$

Find:

$$P(A'B') \quad (4)$$

Boolean Algebra Axioms

Axioms:

$$A + A' = 1, \quad AA' = 0 \quad (5)$$

$$AB = BA, \quad A + B = B + A \quad (6)$$

$$(A + B) + C = A + (B + C) \quad (7)$$

$$(AB)C = A(BC) \quad (8)$$

$$A(B + C) = AB + AC \quad (9)$$

$$A + BC = (A + B)(A + C) \quad (10)$$

De Morgan's Theorems

De Morgan's Laws:

$$(A + B)' = A'B' \quad (11)$$

$$(AB)' = A' + B' \quad (12)$$

Derivation of $A + B$

Using the axiom:

$$A = A(B + B') \quad (13)$$

$$= AB + AB' \quad (14)$$

Similarly,

$$B = (A + A')B \quad (15)$$

$$= AB + A'B \quad (16)$$

Adding (14) and (16):

$$A + B = AB + AB' + A'B \quad (17)$$

Taking probability:

$$P(A + B) = P(AB) + P(A) - P(AB) + P(B) - P(AB) \quad (18)$$

$$= P(A) + P(B) - P(AB) \quad (19)$$

Computing $P(A'B')$

Using De Morgan's theorem:

$$(A + B)' = A'B' \quad (20)$$

Taking probability:

$$P(A'B') = 1 - P(A + B) \quad (21)$$

$$= 1 - (P(A) + P(B) - P(AB)) \quad (22)$$

$$= 1 - (0.25 + 0.5 - 0.125) \quad (23)$$

$$= 0.375 \quad (24)$$

Indicator Random Variables

Define:

$$X_1 = \begin{cases} 1, & A \\ 0, & A' \end{cases} \quad (25)$$

$$X_2 = \begin{cases} 1, & B \\ 0, & B' \end{cases} \quad (26)$$

$$X_3 = \begin{cases} 1, & AB \\ 0, & (AB)' \end{cases} \quad (27)$$

Probability Mass Function (PMF)

For X_1 :

$$p_{X_1}(n) = \begin{cases} p_1, & n = 1 \\ 1 - p_1, & n = 0 \end{cases} \quad (28)$$

Similarly for X_2, X_3 , with probabilities:

$$p_1 = 0.25, \quad p_2 = 0.50, \quad p_3 = 0.125 \quad (29)$$

Expected Value Approach

Define:

$$Y = 1 - X_1 - X_2 + X_3 \quad (30)$$

Compute expectation:

$$E(Y) = E(1 - X_1 - X_2 + X_3) \quad (31)$$

$$= 1 - E(X_1) - E(X_2) + E(X_3) \quad (32)$$

$$= 1 - 0.25 - 0.50 + 0.125 \quad (33)$$

$$= 0.375 \quad (34)$$

Graphical Representation

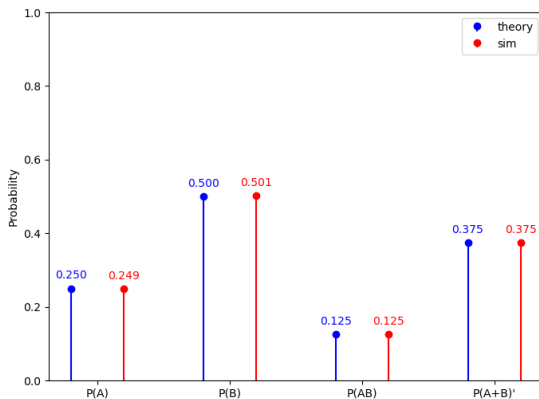


Figure: Graphical representation of probabilities