HO-06 Kecerdasan Buatan
Representasi Pengetahuan
Propositional Logic

Opim S Sitompul

Erna Budhiarti Nababan

Pendahuluan

Dalam perkuliahan ini sebagian besar materi yang akan dibahas adalah *Propositional Calculus*. Tidak seperti namanya materi ini tidak ada hubungannya dengan calculus.

Sebenarnya istilah "calculus" adalah nama *generic* bagi setiap area Matematik yang berkaitan perhitungan.

Pendahuluan (cont'd)

Sebagai contoh aritmatika sering disebut calculus of numbers (perhitungan bilangan). Dengan demikian *Propositional Calculus* adalah perhitungan proposional.

→ *Proposition* atau *statement* adalah kalimat deklaratif yang dapat bernilai True (T) atau False (F). Selanjutnya kita akan gunakan T atau F sebagai truth value dari statement.

Proposition

Example:

- The sentence "2+2 = 4" is a statement, since it can be either true or false. Since it happens to be a true statement, its truth value is T.
- The sentence "1 = 0" is also a statement, but its truth value is F.
- "It will rain tomorrow" is a proposition. For its truth value we shall have to wait for tomorrow.

Proposition (cont'd)

"Solve the following equation for x"

is not a statement, as it cannot be assigned any truth value whatsoever. (It is an imperative, or command, rather than a declarative sentence.)

"The number 5"

is not a proposition, since it is not even a complete sentence.

Propositional Logic

- Proposition adalah sebuah pernyataan, yang dalam bhs Inggris merupakan sebuah kalimat deklaratif. Setiap proposition dapat bernilai benar atau salah.
- Propositional logic merupakan suatu susunan argumen yang valid, khususnya yang menyangkut aturan-aturan yang membuktikan suatu kesimpulan dari sekumpulan premis atau asumsi
- Penting untuk diingat bahwa asumsi itu sendiri tidak harus benar untuk membuktikan asumsi yang mendasarinya adalah benar.
- Asumsi hanya perlu menyiratkan kesimpulan tapi ia tidak harus benar atau tidak benar.

Propositional Logic (cont'd)

- Sebuah proposition dikatakan sederhana atau atomic jika ia tidak mempunyai hubungan atau *quantifier* , dalam hal ini dinyatakan dengan sebuah huruf tunggal atau alphabet seperti *p* atau *q* .
- The truth-functional structure of such a statement can be represented by a truth table in which all possible truth values are displayed.

- Karena ketiga proposition masing-masing mempunyai dua nilai kemungkinan, maka terdapat $2^3 = 8$ baris kemungkinan

p	q	r
True	True	True
True	True	False
True	False	True
True	False	False
False	True	True
False	True	False
False	False	True
False	False	False

- Lima standar penghubung yang digunakan unruk menggabungkan proposition dari proposition tunggal, adalah:

1. **Negation (NOT)** $\sim p$
2. **Disjunction (OR)** $p \vee q$
3. **Conjunction (AND)** $p \wedge q$
4. **Conditional (IF…THEN…)** $p \rightarrow q$
5. **Biconditional (IF AND ONLY IF)** $p \leftrightarrow q$

1. Negation (Negasi)

- Sebagai contoh, jika p adalah proposition, "George Washington was born in 1732", then ~ p is the proposition "George Washington was not born in 1732". Tabel kebenaran untuk negasi adalah:

p	*~ p*
True	*False*
False	*True*

2. Disjunction (OR)

- Disebut juga sebagai inclusive OR dan akan bernilai true selama salah satu komponennya bernilai true.

Contoh:

"Students must take a statistics course or a logic course to graduate“

Artinya, mahasiswa akan memenuhi syarat kelulusan jika sudah mengambil mtkuliah statistik atau mtkuliah logic atau kedua-duanya

Disjunction (cont'd)

- Cat: $p \vee q$ akan bernilai false jika dan hanya jika kedua-dua p and q adalah false. OR adalah operator komutatif, artinya $p \vee q$ selalu sama dengan $q \vee p$

p	q	$p \vee q$
True	True	True
True	False	True
False	True	True
False	False	False

3. Conjunction (AND)

- Digunakan untuk menggabungkan dua pernyataan dengan pengertian penggabungan itu akan bernilai true jika dan hanya jika semua komponenny abernilai true.

Seperti operator OR, operator AND juga adalah komutatif, $p \wedge q$ selalu sama dengan $q \wedge p$.

p	q	$p \wedge q$
True	True	True
True	False	False
False	True	False
False	False	False

- Jika dalam satu gabungan perposition digunakan lebih dari satu penghubung, maka prioritas urutannya adalah:

$$\sim\ ,\ \wedge,\ \vee,\ \rightarrow,\ \leftrightarrow$$

- Pengelompokkan simbol menggunakan <{[()]}> adalah untuk memperjelas urutan penyelesaian. Contoh, untuk menegasikan p and q dituliskan $\sim(p \wedge q)$

4. Conditional (IF…THEN…)

- Dasar pemikiran proposition ini adalah bahwa kebenaran kondisi yang dinyatakan dalam proposition p cukup untuk menjamin kebenaran dari proposition q.

  Salah satu inference rule yang membentuk inti Aristotelian logic adalah syllogism yang terdiri dari dua *premise* dan satu *conclusion.*

- premise: $p \rightarrow q$

  premise: p

  conclusion: $\therefore q$

Simbol $\therefore$ artinya adalah oleh karena itu.

Bentuk syllogism ini dikenal dengan Law of Detachment atau dalam bahasa Latin disebut modus ponendo ponen (**modus ponen**).

- Penulisan $p \rightarrow q$ (dibaca p implies q) artinya jika p is true, q **must** be true.

Contoh:

If roses are red and violets are blue, then sugar is sweet and so are you.
<u>Roses are red and violets are blue.</u>
Therefore, sugar is sweet and so are you.

Secara simbol dapat dituliskan

$$(p \land q) \rightarrow (r \land s)$$
$$\underline{(p \land q)}$$
$$\therefore (r \land s)$$

p	q	$p \rightarrow q$	$\sim p$	$\sim p \vee q$	$\sim q$	$\sim q \rightarrow \sim p$	$q \rightarrow p$
True	True	True	False	True	False	True	True
True	False	False	False	False	True	False	True
False	True	True	True	True	False	True	False
False	False	True	True	True	True	True	True

- Modus Tollen

$$p \to q$$

$$\underline{\sim q}$$

$$\therefore \sim p$$

5. Biconditional (IF AND ONLY IF)

- Adalah statement padanan (kesamaan).
- Ini berarti jika $p \leftrightarrow q$ nilai kebenaran p dan q adalah sama (identik)
- $p \leftrightarrow q$ equivalent dg $(p \rightarrow q) \wedge (q \rightarrow p)$

p	q	$p \leftrightarrow q$	$p \rightarrow q$	$q \rightarrow p$	$(p \rightarrow q) \wedge (q \rightarrow p)$
True	True	True	True	True	True
True	False	False	False	True	False
False	True	False	True	False	False
False	False	True	True	True	True

- Actually, we only **really** need two connectives, for example conjunction, conditional and biconditional can all be defined in terms of negation and disjunction.

- Conjuction $$p \wedge q \leftrightarrow \sim(\sim p \vee \sim q)$$
- Conditional $$p \rightarrow q \leftrightarrow \sim p \vee q$$
- Biconditional

$$p \leftrightarrow q \leftrightarrow (p \rightarrow q) \wedge (q \rightarrow p) \leftrightarrow \sim(\sim(\sim p \vee q) \vee \sim(\sim q \vee p))$$

Title	Rule
Commutative Property of OR	$p \vee q \Leftrightarrow q \vee p$
Commutative Property of AND	$p \wedge q \Leftrightarrow q \wedge p$
Associative Property of OR	$p \vee (q \vee r) \Leftrightarrow (p \vee q) \vee r$
Associative Property of AND	$p \wedge (q \wedge r) \Leftrightarrow (p \wedge q) \wedge r$
Distributive Property of OR	$p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$
Distributive Property of AND	$p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$
Double Negation	$p \Leftrightarrow \sim(\sim p)$ (The parentheses is added to avoid confusion.)
Definition of Implication	$p \rightarrow q \Leftrightarrow \sim p \vee q$
Definition of Equivalence	$p \leftrightarrow q \Leftrightarrow (\sim p \vee q) \wedge (\sim q \vee p)$
DeMorgan's Rule: Negation of an OR	$\sim(p \vee q) \Leftrightarrow \sim p \wedge \sim q$
DeMorgan's Rule: Negation of an AND	$\sim(p \wedge q) \Leftrightarrow \sim p \vee \sim q$

Propositional logic

- **Logical constants**: true, false
- **Propositional symbols**: P, Q, S, ... (**atomic sentences**)
- Wrapping **parentheses**: (…)
- Sentences are combined by **connectives**:

$\wedge$ ...and	[conjunction]
$\vee$ ...or	[disjunction]
$\Rightarrow$...implies	[implication / conditional]
$\Leftrightarrow$..is equivalent	[biconditional]
$\neg$ ...not	[negation]

- **Literal**: atomic sentence or negated atomic sentence

Examples of PL sentences

- P means “It is hot.”
- Q means “It is humid.”
- R means “It is raining.”
- $(P \wedge Q) \rightarrow R$

  “If it is hot and humid, then it is raining”
- $Q \rightarrow P$

  “If it is humid, then it is hot”
- A better way:

  Hot = “It is hot”

  Humid = “It is humid”

  Raining = “It is raining”

Propositional logic (PL)

- A simple language useful for showing key ideas and definitions
- User defines a set of propositional symbols, like P and Q.
- User defines the **semantics** of each propositional symbol:
  - P means "It is hot"
  - Q means "It is humid"
  - R means "It is raining"
- A sentence (well formed formula) is defined as follows:
  - A symbol is a sentence
  - If S is a sentence, then $\neg$S is a sentence
  - If S is a sentence, then (S) is a sentence
  - If S and T are sentences, then (S $\vee$ T), (S $\wedge$ T), (S $\rightarrow$ T), and (S $\leftrightarrow$ T) are sentences
  - A sentence results from a finite number of applications of the above rules

Some terms

- The meaning or **semantics** of a sentence determines its **interpretation**.
- Given the truth values of all symbols in a sentence, it can be "evaluated" to determine its **truth value** (True or False).
- A **model** for a KB is a "possible world" (assignment of truth values to propositional symbols) in which each sentence in the KB is True.

More terms

- A **valid sentence** or **tautology** is a sentence that is True under all interpretations, no matter what the world is actually like or how the semantics are defined. Example: “It’s raining or it’s not raining.”
- An **inconsistent sentence** or **contradiction** is a sentence that is False under all interpretations. The world is never like what it describes, as in “It’s raining and it’s not raining.”
- **P entails Q**, written P |= Q, means that whenever P is True, so is Q. In other words, all models of P are also models of Q.

TAUTOLOGI

- A compound proposition whose truth-values are all true is said to a **tautology**.
- The negation of a tautology has all false truth values and is called a **contradiction**.
- A tautology is true regardless of the truth values of its constituent atomic propositions just as a contradiction is false regardless of the truth values of its propositions.

- Since $\sim(p \wedge q)$ and $\sim p \vee \sim q$ have identical truth values $\sim(p \wedge q) \leftrightarrow \sim p \vee \sim q$ must be a tautology. Similarly, since $\sim(p \vee q)$ and $\sim p \wedge \sim q$ have identical truth values, $\sim(p \vee q) \leftrightarrow \sim p \wedge \sim q$ is a tautology.
- **Note:** The use of DeMorgan's Rules demonstrates that the negation of the tautology:

  $p \vee \sim p$ the contradiction $\sim p \wedge p$

  and the negation of the contradiction $p \wedge \sim p$

  is the tautology $\sim p \vee p$

- The compound proposition is a contradiction. It some sense it is the parent of all lies since it immediately and without shame purports to tell us something and then denies that same thing!
- If the compound propositions p and q always have has the same truth values, then the biconditional will always be a tautology. In sentential logic all theorems are tautologies and all tautologies are either axioms or theorems.

- Thus, one can determine if a given proposition is an axiom or theorem by constructing its truth table. If the proposition is a tautology, it must be an axiom or theorem of sentential logic. This is illustrated in the following Excel spreadsheet, which establishes DeMorgan's Rules for negating conjunctions and disjunctions

Truth tables

And

p	q	$p \cdot q$
T	T	T
T	F	F
F	T	F
F	F	F

Or

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

If . . . then

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Not

p	$\sim p$
T	F
F	T

Truth tables II

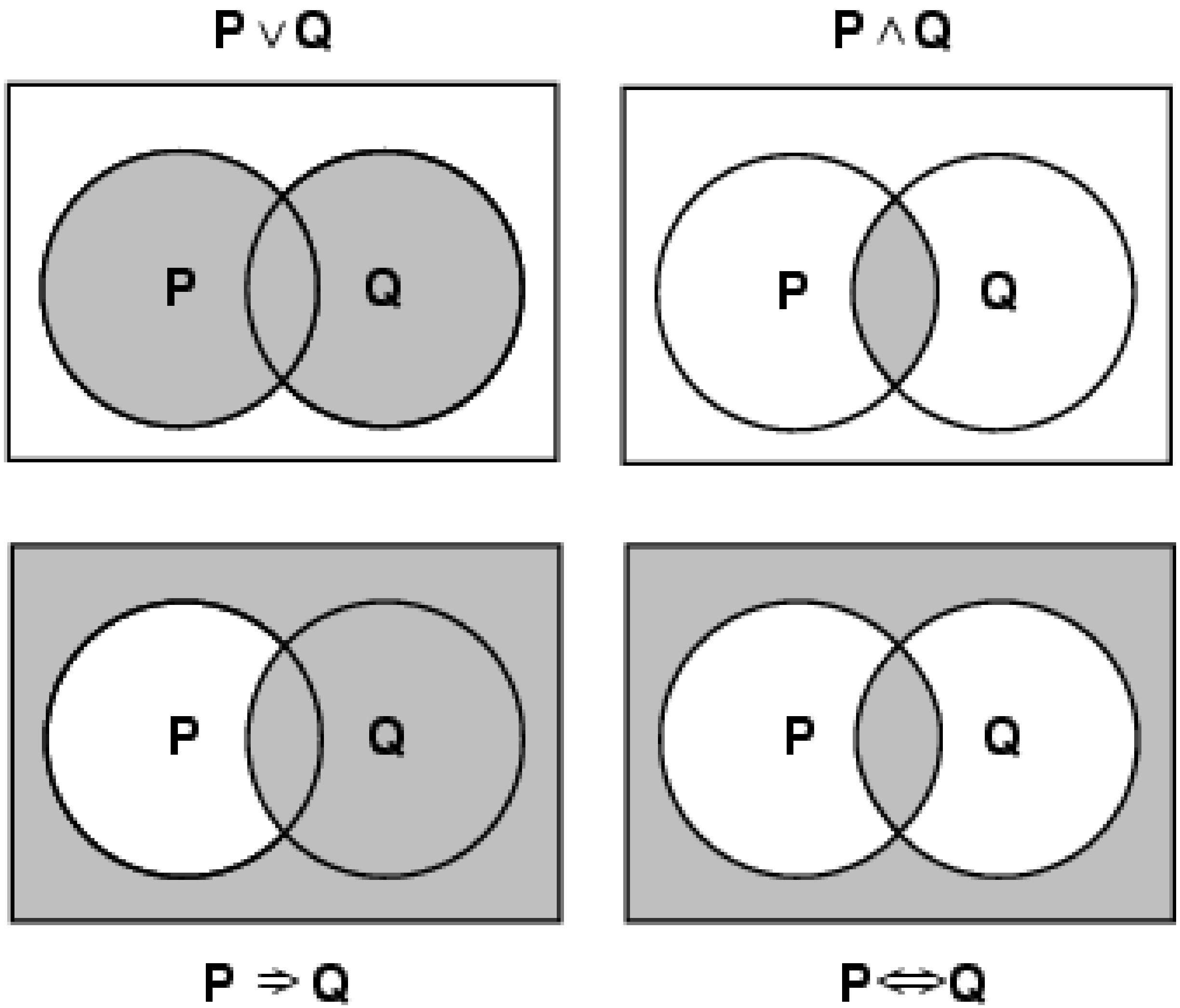
The five logical connectives:

P	Q	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \Rightarrow Q$	$P \Leftrightarrow Q$
False	*False*	*True*	*False*	*False*	*True*	*True*
False	*True*	*True*	*False*	*True*	*True*	*False*
True	*False*	*False*	*False*	*True*	*False*	*False*
True	*True*	*False*	*True*	*True*	*True*	*True*

A complex sentence:

P	H	$P \vee H$	$(P \vee H) \wedge \neg H$	$((P \vee H) \wedge \neg H) \Rightarrow P$
False	*False*	*False*	*False*	*True*
False	*True*	*True*	*False*	*True*
True	*False*	*True*	*True*	*True*
True	*True*	*True*	*False*	*True*

Models of complex sentences



Inference rules

- **Logical inference** is used to create new sentences that logically follow from a given set of predicate calculus sentences (KB).
- An inference rule is **sound** if every sentence X produced by an inference rule operating on a KB logically follows from the KB. (That is, the inference rule does not create any contradictions)
- An inference rule is **complete** if it is able to produce every expression that logically follows from (is entailed by) the KB. (Note the analogy to complete search algorithms.)

Sound rules of inference

- Here are some examples of sound rules of inference
  - *A rule is sound if its conclusion is true whenever the premise is true*
- Each can be shown to be sound using a truth table

RULE	**PREMISE**	**CONCLUSION**
Modus Ponens	$A, A \rightarrow B$	B
And Introduction	A, B	$A \wedge B$
And Elimination	$A \wedge B$	A
Double Negation	$\neg\neg A$	A
Unit Resolution	$A \vee B, \neg B$	A
Resolution	$\mathbf{A \vee B, \neg B \vee C}$	$\mathbf{A \vee C}$

Soundness of modus ponens

A	B	$A \to B$	OK?
True	True	True	√
True	False	False	√
False	True	True	√
False	False	True	√

Soundness of the resolution inference rule

α	β	γ	$\alpha \vee \beta$	$\neg\beta \vee \gamma$	$\alpha \vee \gamma$
False	*False*	*False*	*False*	*True*	*False*
False	*False*	*True*	*False*	*True*	*True*
False	*True*	*False*	*True*	*False*	*False*
<u>*False*</u>	<u>*True*</u>	<u>*True*</u>	<u>*True*</u>	<u>*True*</u>	<u>*True*</u>
<u>*True*</u>	<u>*False*</u>	<u>*False*</u>	<u>*True*</u>	<u>*True*</u>	<u>*True*</u>
<u>*True*</u>	<u>*False*</u>	<u>*True*</u>	<u>*True*</u>	<u>*True*</u>	<u>*True*</u>
True	*True*	*False*	*True*	*False*	*True*
<u>*True*</u>	<u>*True*</u>	<u>*True*</u>	<u>*True*</u>	<u>*True*</u>	<u>*True*</u>

Proving things

- A **proof** is a sequence of sentences, where each sentence is either a premise or a sentence derived from earlier sentences in the proof by one of the rules of inference.
- The last sentence is the **theorem** (also called goal or query) that we want to prove.
- Example for the "weather problem" given above.

1 Humid	Premise	"It is humid"
2 Humid$\rightarrow$Hot	Premise	"If it is humid, it is hot"
3 Hot	Modus Ponens(1,2)	"It is hot"
4 (Hot$\wedge$Humid)$\rightarrow$Rain	Premise	"If it's hot & humid, it's raining"
5 Hot$\wedge$Humid	And Introduction(1,2)	"It is hot and humid"
6 Rain	Modus Ponens(4,5)	"It is raining"

Horn sentences

- A **Horn sentence** or **Horn clause** has the form:

  $P1 \wedge P2 \wedge P3 \ldots \wedge Pn \rightarrow Q$

  or alternatively

  $\neg P1 \vee \neg P2 \vee \neg P3 \ldots \vee \neg Pn \vee Q$

  $(P \rightarrow Q) = (\neg P \vee Q)$

  where Ps and Q are non-negated atoms
- To get a proof for Horn sentences, apply Modus Ponens repeatedly until nothing can be done
- We will use the Horn clause form later

Entailment and derivation

- **Entailment: KB |= Q**
  - Q is entailed by KB (a set of premises or assumptions) if and only if there is no logically possible world in which Q is false while all the premises in KB are true.
  - Or, stated positively, Q is entailed by KB if and only if the conclusion is true in every logically possible world in which all the premises in KB are true.
- **Derivation: KB |- Q**
  - We can derive Q from KB if there is a proof consisting of a sequence of valid inference steps starting from the premises in KB and resulting in Q

Two important properties for inference

Soundness: If KB |- Q then KB |= Q

- If Q is derived from a set of sentences KB using a given set of rules of inference, then Q is entailed by KB.
- Hence, inference produces only real entailments, or any sentence that follows deductively from the premises is valid.

Completeness: If KB |= Q then KB |- Q

- If Q is entailed by a set of sentences KB, then Q can be derived from KB using the rules of inference.
- Hence, inference produces all entailments, or all valid sentences can be proved from the premises.

Propositional logic is a weak language

- Hard to identify "individuals" (e.g., Mary, 3)
- Can't directly talk about properties of individuals or relations between individuals (e.g., "Bill is tall")
- Generalizations, patterns, regularities can't easily be represented (e.g., "all triangles have 3 sides")
- First-Order Logic (abbreviated FOL or FOPC) is expressive enough to concisely represent this kind of information

  FOL adds relations, variables, and quantifiers, e.g.,

  - *"Every elephant is gray":* $\forall$ x (elephant(x) $\rightarrow$ gray(x))
  - *"There is a white alligator":* $\exists$ x (alligator(X) ^ white(X))

Example

- Consider the problem of representing the following information:
  - Every person is mortal.
  - Confucius is a person.
  - Confucius is mortal.
- How can these sentences be represented so that we can infer the third sentence from the first two?

Example II

- In PL we have to create propositional symbols to stand for all or part of each sentence. For example, we might have:

  P = “person”; Q = “mortal”; R = “Confucius”

- so the above 3 sentences are represented as:

  $P \rightarrow Q$; $R \rightarrow P$; $R \rightarrow Q$

- Although the third sentence is entailed by the first two, we needed an explicit symbol, R, to represent an individual, Confucius, who is a member of the classes “person” and “mortal”
- To represent other individuals we must introduce separate symbols for each one, with some way to represent the fact that all individuals who are “people” are also “mortal”

The "Hunt the Wumpus" agent

- Some atomic propositions:

  S12 = There is a stench in cell (1,2)

  B34 = There is a breeze in cell (3,4)

  W22 = The Wumpus is in cell (2,2)

  V11 = We have visited cell (1,1)

  OK11 = Cell (1,1) is safe.

  etc

- Some rules:

  (R1) $\neg S11 \rightarrow \neg W11 \wedge \neg W12 \wedge \neg W21$

  (R2) $\neg S21 \rightarrow \neg W11 \wedge \neg W21 \wedge \neg W22 \wedge \neg W31$

  (R3) $\neg S12 \rightarrow \neg W11 \wedge \neg W12 \wedge \neg W22 \wedge \neg W13$

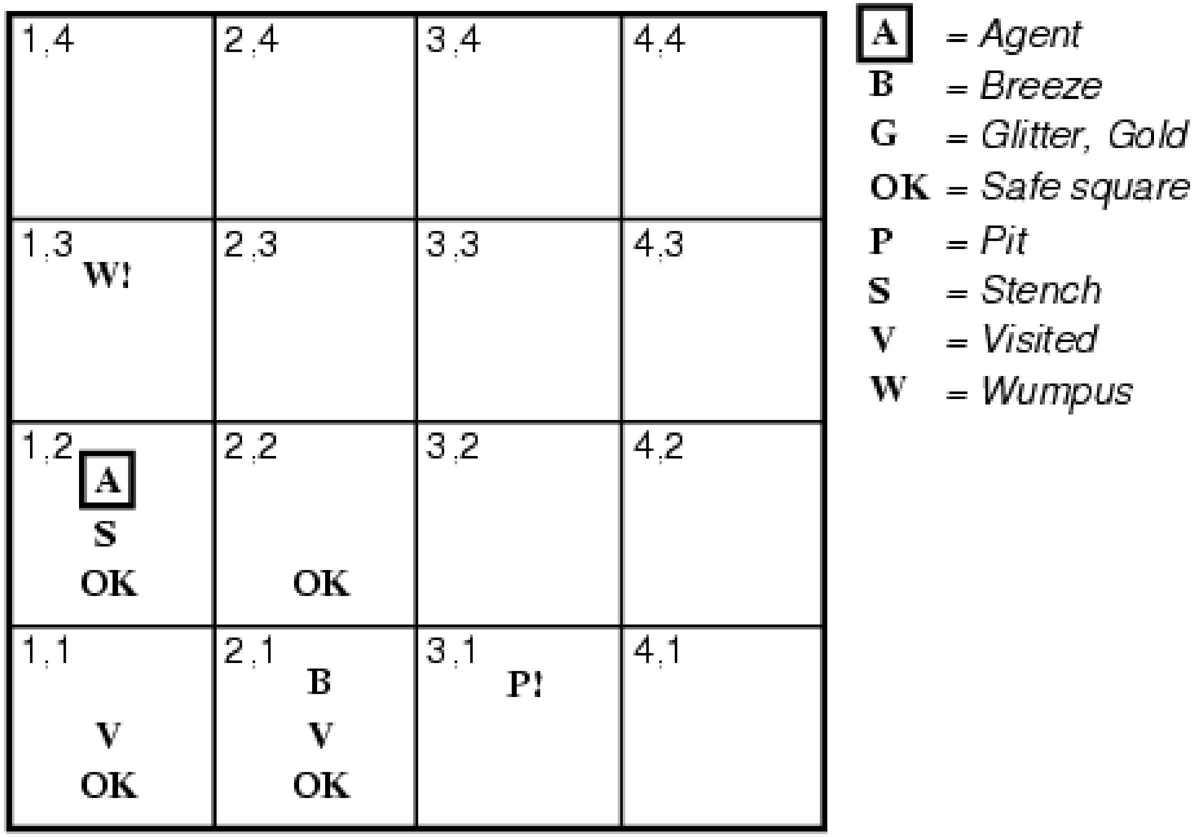
  (R4) $S12 \rightarrow W13 \vee W12 \vee W22 \vee W11$

  etc

- Note that the lack of variables requires us to give similar rules for each cell

After the third move

- We can prove that the Wumpus is in (1,3) using the four rules given.
- See R&N section 7.5



Proving W13

- Apply MP with $\neg S11$ and R1:
  $\neg W11 \wedge \neg W12 \wedge \neg W21$
- Apply And-Elimination to this, yielding 3 sentences:
  $\neg W11, \neg W12, \neg W21$
- Apply MP to ~S21 and R2, then apply And-elimination:
  $\neg W22, \neg W21, \neg W31$
- Apply MP to S12 and R4 to obtain:
  $W13 \vee W12 \vee W22 \vee W11$
- Apply Unit resolution on ($W13 \vee W12 \vee W22 \vee W11$) and $\neg W11$:
  $W13 \vee W12 \vee W22$
- Apply Unit Resolution with ($W13 \vee W12 \vee W22$) and $\neg W22$:
  $W13 \vee W12$
- Apply UR with ($W13 \vee W12$) and $\neg W12$:
  W13
- QED

Problems with the propositional Wumpus hunter

- Lack of variables prevents stating more general rules
  - We need a set of similar rules for each cell
- Change of the KB over time is difficult to represent
  - Standard technique is to index facts with the time when they're true
  - This means we have a separate KB for every time point

Summary

- The process of deriving new sentences from old one is called **inference**.
  - **Sound** inference processes derives true conclusions given true premises
  - **Complete** inference processes derive all true conclusions from a set of premises
- A **valid sentence** is true in all worlds under all interpretations
- If an implication sentence can be shown to be valid, then—given its premise—its consequent can be derived
- Different logics make different **commitments** about what the world is made of and what kind of beliefs we can have regarding the facts
  - Logics are useful for the commitments they do not make because lack of commitment gives the knowledge base engineer more freedom
- **Propositional logic** commits only to the existence of facts that may or may not be the case in the world being represented
  - It has a simple syntax and simple semantics. It suffices to illustrate the process of inference
  - Propositional logic quickly becomes impractical, even for very small worlds