

Machine Learning: Assignment 1

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1 Linear Regression

- Learning rate: 0.0001.
- stopping criteria: if modulus of the derivative of the cost function is less than 0.0005.
- Matrix formula used:

$$\frac{\partial J(\theta)}{\partial \theta} = \frac{1}{m} * (X^T X \theta - X^T Y)$$

$$J(\theta) = \frac{1}{2m} (Y - X\theta)^T (Y - X\theta)$$

- Where X is an (m x n) matrix, m = number of training examples, and n = number of features.

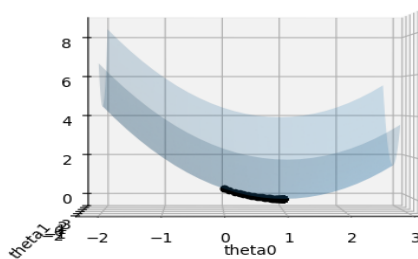


Figure 1: $J(\theta)$ as a function of θ_0, θ_1

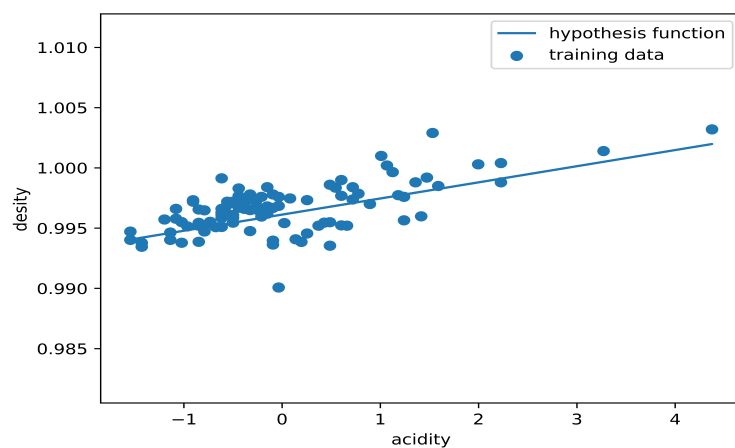


Figure 2: $h_{\theta}(x)$ and training data

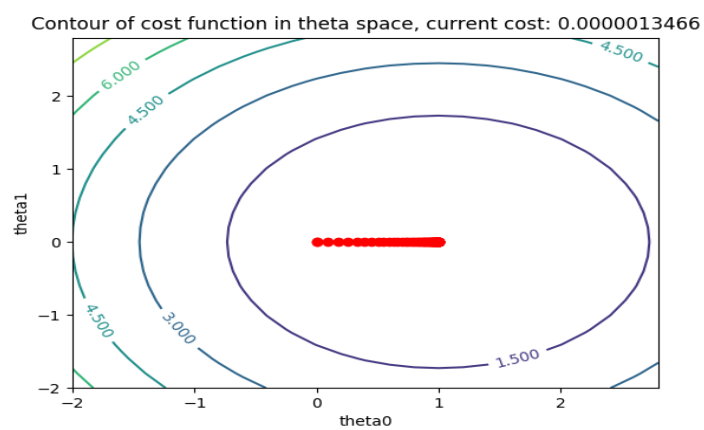


Figure 3: Question 1: Contour of cost function and iteration points

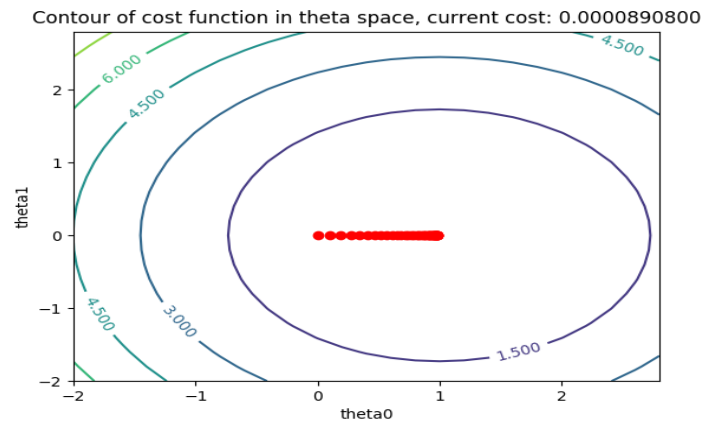


Figure 4: Question 1: Contour of cost function at learning rate: 0.1

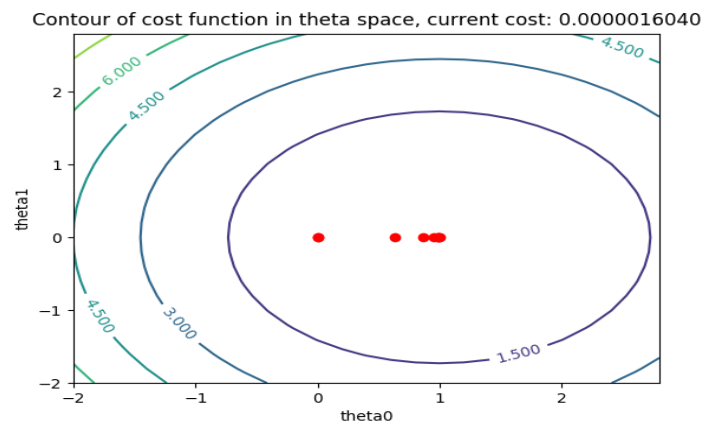


Figure 5: Question 1: Contour of cost function at learning rate: 0.001

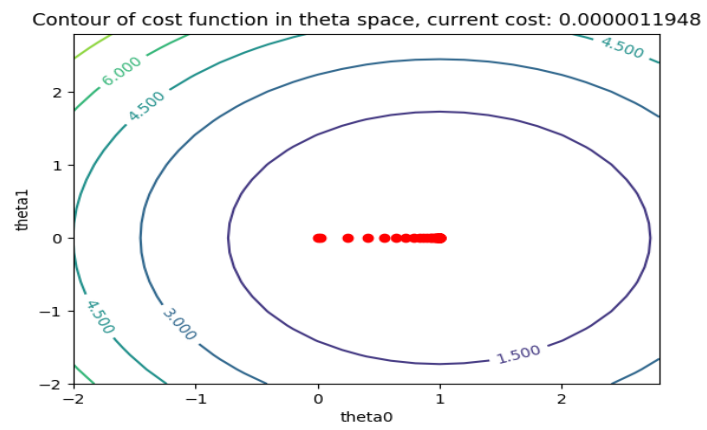


Figure 6: Question 1: Contour of cost function at learning rate: 0.025

2 Sampling and Stochastic Gradient Descent

2.1 Implementing stochastic gradient descent

| Batch size | θ_0 | θ_1 | θ_2 | epochs | opt. avg | history | error |
|------------|------------|------------|------------|--------|----------|---------|------------|
| 1 | 2.98841157 | 0.98489459 | 1.95629993 | 1 | NA | 5000 | 1.09743707 |
| 100 | 2.99857972 | 1.00004456 | 1.99967958 | 5 | 0.531 | 2000 | 0.98300276 |
| 10000 | 2.91204599 | 1.01900053 | 1.99320696 | 126 | 0.05 | 50 | 1.00538196 |
| 1000000 | 3.0002548 | 1.0002631 | 1.9995629 | 23753 | 0.0012 | 1 | 0.98300278 |
| NA | 3 | 1 | 2 | NA | NA | NA | 0.98294692 |

Table 1: Sampling and Stochastic Gradient Descent

2.2 Explanation

- As the batch size increases the number of epochs before converging increases.
- As the batch size decreases the speed of convergence increases.
- As the batch size increases, it converges to assumed hypothesis θ values, because it is going towards batch gradient descent as we increase the batch size.

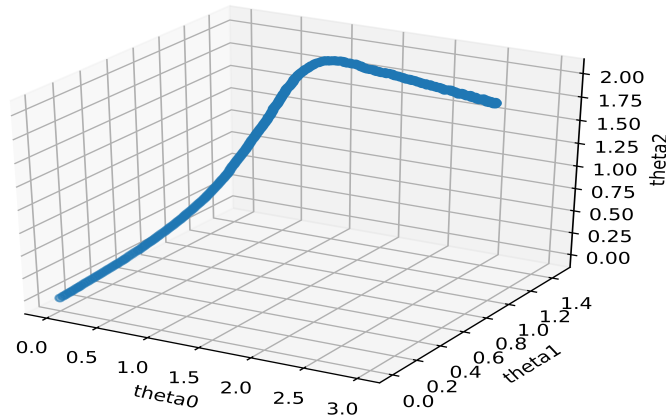


Figure 7: Question 2: batch size: 100

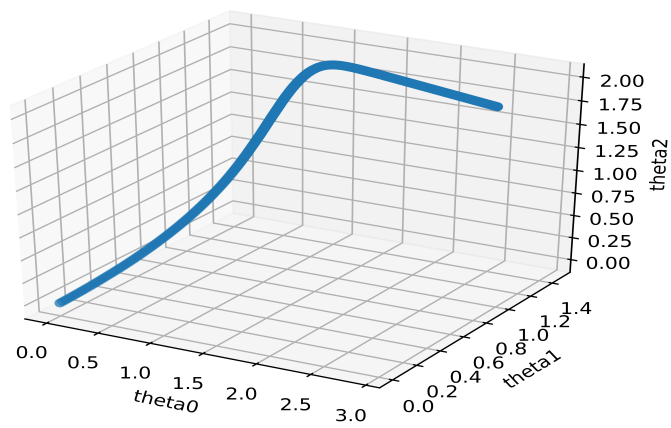


Figure 8: Question 2: Batch size: 10000

3 Logistic Regression

- θ : 0.40125316, 2.5885477, -2.72558849

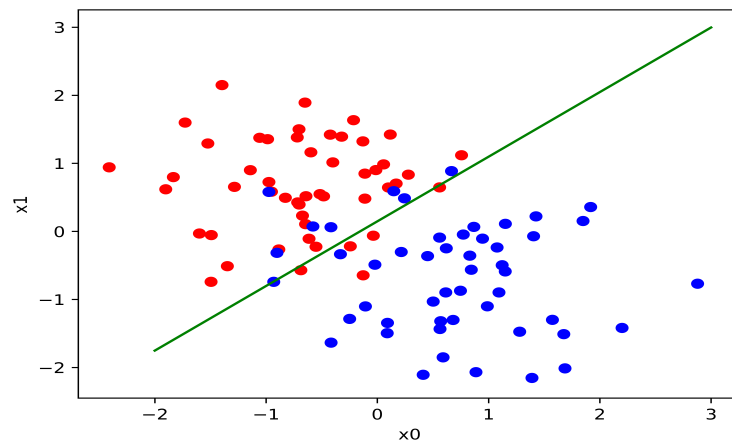


Figure 9: Question 3: Linear boundary separating the two classes

4 Gaussian Discriminant Analysis

$$\begin{vmatrix} 0.5 \end{vmatrix}$$

Table 2: ϕ

$$\begin{vmatrix} -0.75529433 \\ 0.68509431 \end{vmatrix}$$

Table 3: μ_0

$$\begin{vmatrix} 0.75529433 \\ -0.68509431 \end{vmatrix}$$

Table 4: μ_1

$$\begin{vmatrix} 0.38158978 & -0.15486516 \\ -0.15486516 & 0.64773717 \end{vmatrix}$$

Table 5: Σ_0

$$\begin{vmatrix} 0.47747117 & 0.1099206 \\ 0.1099206 & 0.41355441 \end{vmatrix}$$

Table 6: Σ_1

$$\begin{vmatrix} 0.42953048 & -0.02247228 \\ -0.02247228 & 0.53064579 \end{vmatrix}$$

Table 7: Σ

- Equation of the boundary separating the two classes:

$$x^T(\Sigma_1^{-1} - \Sigma_0^{-1})x + 2(\mu_0^T \Sigma_0^{-1} - \mu_1^T \Sigma_1^{-1})x + \mu_1^T \Sigma_1^{-1} \mu_1 - \mu_0^T \Sigma_0^{-1} \mu_0 + \log_e\left(\left(\frac{1-\phi}{\phi}\right)^2 \frac{|\Sigma_1|}{|\Sigma_0|}\right) = 0$$

- If we assume that our features shares the Σ matrix then we get a linear boundary and if we assume that the features can have different Σ s then we get a quadratic boundary. Looking at the training data, assuming the later case is a good choice.

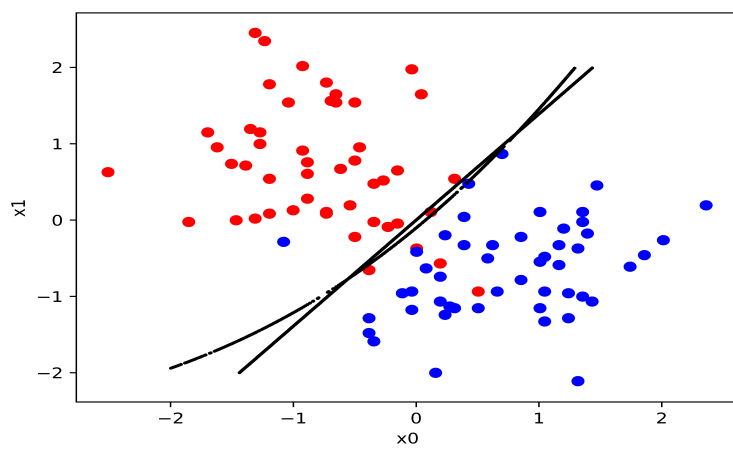


Figure 10: Question 4: Boundary separating the two classes