SWITCH POINT ANALYSIS USING MCMC SAMPLING TECHNIQUES AND VARIATIONAL INFERENCE & IMAGE DENOISING

CS 726: ADVANCED MACHINE LEARNING

END-SEMESTER PROJECT

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Presentation Outline

Overview

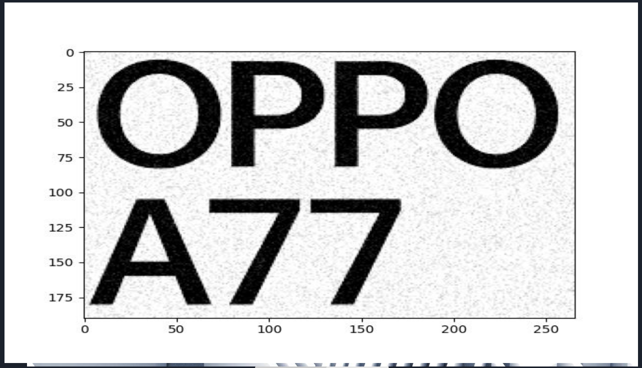
Algorithms

Implementation and Results

Changes and Comparison

Conclusions

Image Denoising by MCMC Sampling



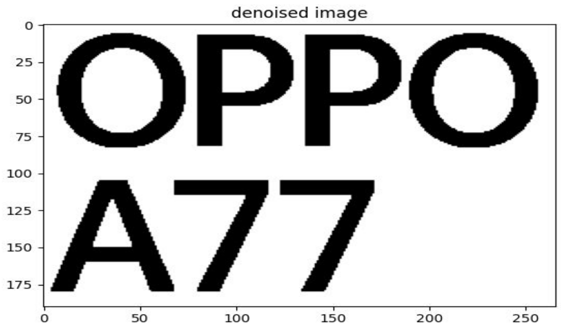


Image Denoising by MCMC Sampling

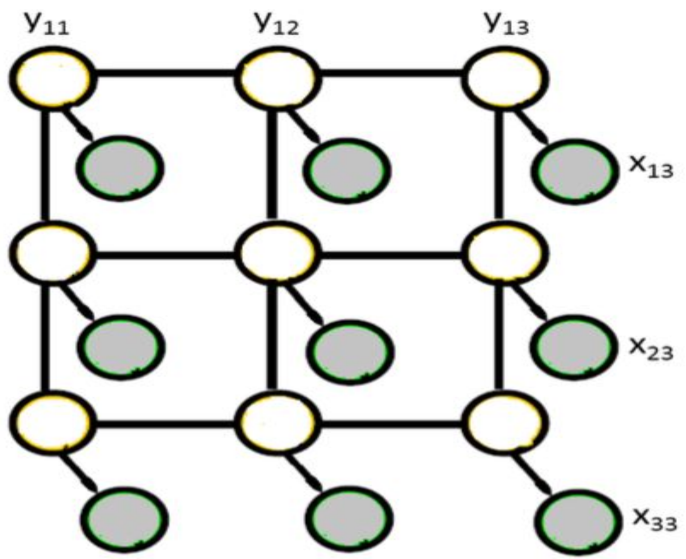
- Gibbs Sampling is a MCMC algorithm that generates a Markov chain of samples, each of which is calculated with its direct neighbors.
- Denoising problem: we are given a noisy image X and the goal is to restore it to the original image Y, which is unknown.
- Denoising can be treated as a probabilistic inference, where we perform maximum a posteriori (MAP) estimation by maximizing the a posteriori distribution p(Y|X)

$$log(P(Y|X) = log(P(X|Y) + log(P(Y)) - log(P(X))$$

Image Denoising by MCMC Sampling

- Assuming our posterior preference is black, the posterior we want to maximize is p(Y=1|Y_neighbors), where Y={yij} for i = 1, ..., N and j = 1, ..., M.
- The joint probability of Y and X is given as:

$$P(Y,X) = \frac{1}{Z} e^{\eta \sum \sum x_{ij} y_{ij} + \beta \sum y_{ij} y_{i'j'}}$$



X : noisy pixels
Y : “true” pixels

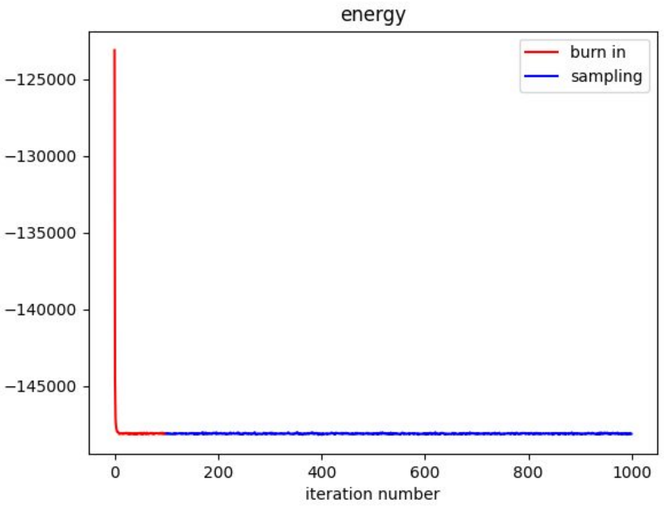
The loss function − log p(X|Y) − log p(Y)

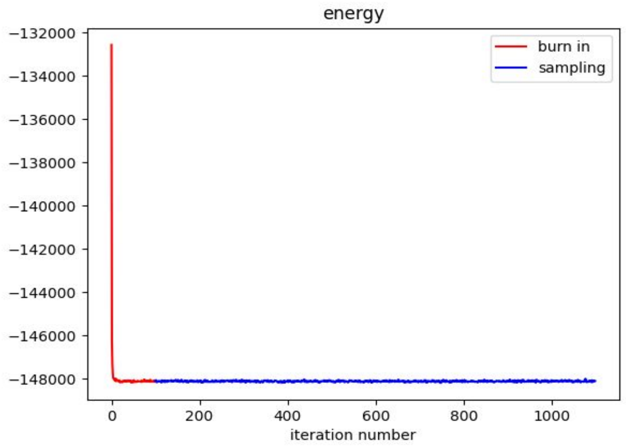
Results and Implementation

- GIBBS SAMPLING

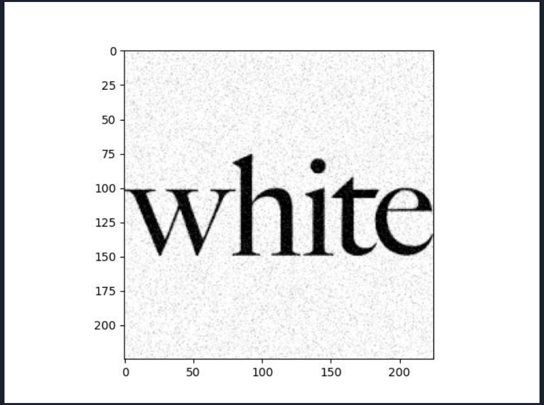
  MCMC algorithms generally have a burn-in period, during which samples may not be accurate. Therefore, samples are collected after the burn-in period, and used to estimate the posterior using the Monte Carlo method.

- METROPOLIS HASTING SAMPLING
  - Generate a candidate sample and accept/reject it based on the acceptance probability
  - the acceptance probability based on the energy difference between the current state and the proposed state (by flipping the current state),

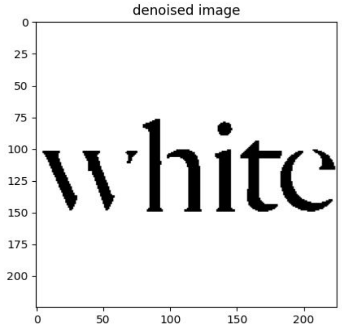




Results and Implementation

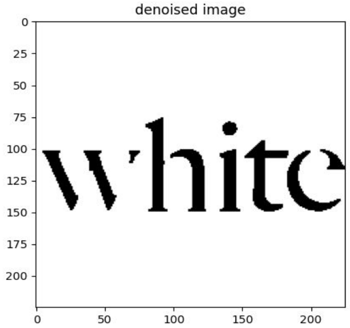


- GIBBS SAMPLING



- SSIM score between the actual and denoised image: 0.76

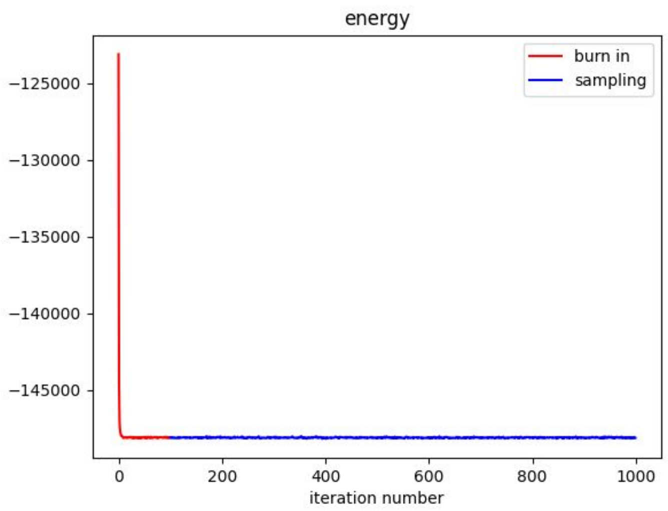
- METROPOLIS HASTING SAMPLING



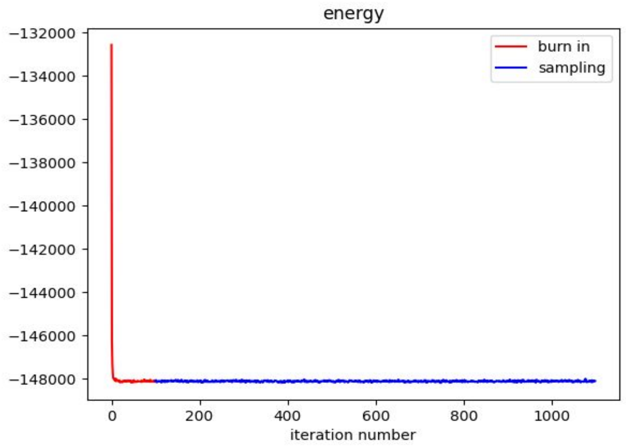
- SSSIM score between the actual and denoised image:0.81

Results and Implementation

- GIBBS SAMPLING



- METROPOLIS HASTING SAMPLING



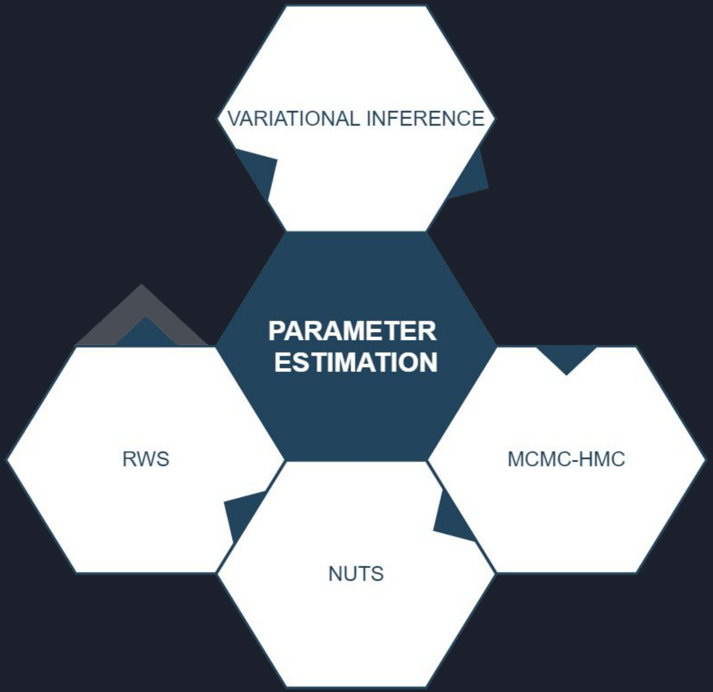
The SSIM score value for MH sampling was most of the times greater than the score of gibbs sampling but computational efficiency for gibbs is higher although.

Objective I[Switch Point Analysis]

Switch point analysis (Bayesian latent variable analysis)

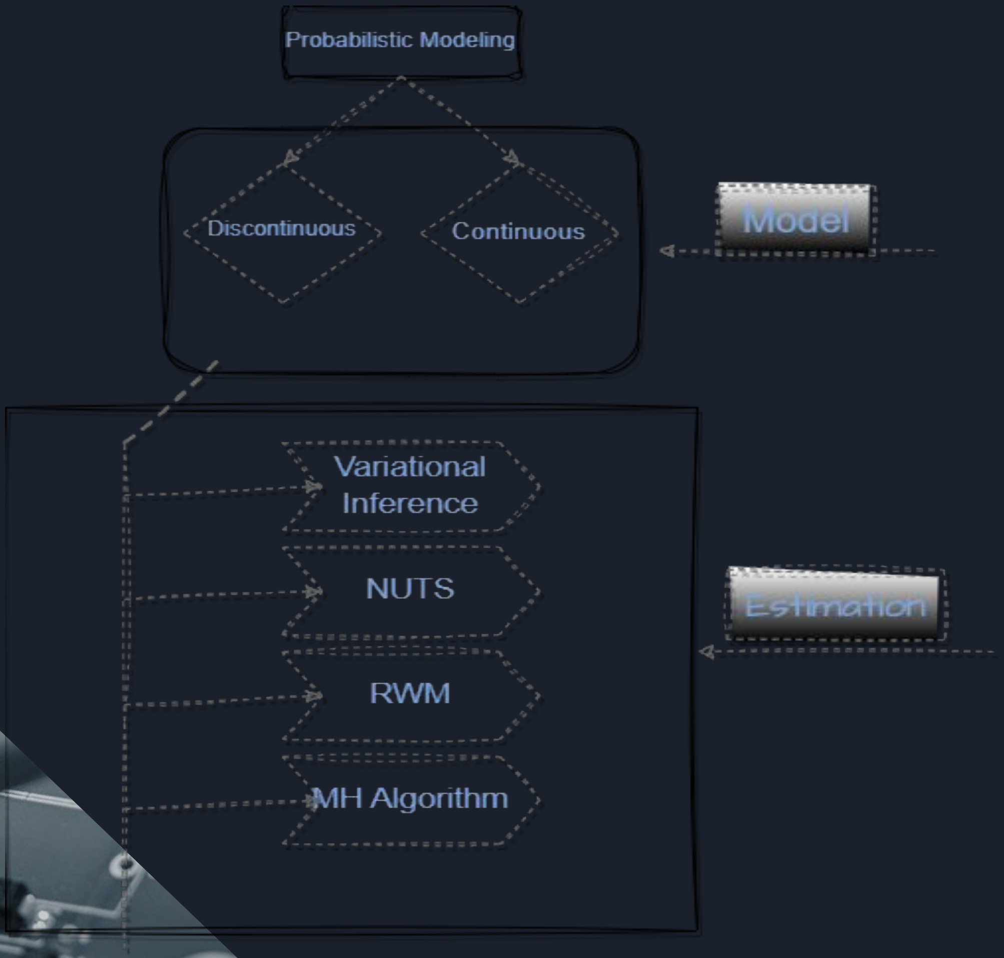
- *To infer the posterior probability distributions of the model parameters*
- *Maximize the likelihood of the observed data of Disaster Count*

- *Computing P(X|Z) --->*



Overview(Continued...)

METHODOLOGY



Overview(Continued...)

PROBABILISTIC MODELLING

1. Discontinuous model

$$(D_t|s,e,l) \sim \text{Poisson}(r_t),$$
$$\text{with } r_t = \begin{cases} e & \text{if } t < s \\ l & \text{if } t \geq s \end{cases}$$
$$s \sim \text{Discrete Uniform}(t_l, t_h)$$
$$e \sim \text{Exponential}(r_e)$$
$$l \sim \text{Exponential}(r_l)$$

2. Continuous model

$$(D_t|s,e,l) \sim \text{Poisson}(r_t),$$
$$\text{with } r_t = e + \frac{1}{1+\exp(s-t)}(l-e)$$
$$s \sim \text{Uniform}(t_l, t_h)$$
$$e \sim \text{Exponential}(r_e)$$
$$l \sim \text{Exponential}(r_l)$$

Estimation & Algorithms

- MCMC-Metropolis-Hasting (MH)

  Works by starting with an initial value and proposing a new value based on a proposal distribution. It allows for sampling from complex distributions that may not have a closed-form solution.

- MCMC-Random Walk Metropolis (RWM)

  Variant of the Metropolis-Hastings algorithm in which the proposal distribution is centred on the current state and the proposal step is defined by a fixed variance

- MCMC-No-U-Turn Sampler (NUTS)

  Sophisticated gradient-based MCMC method that extends the Hamiltonian Monte Carlo (HMC) algorithm by automatically tuning the step size and the number of leapfrog steps during sampling

Algorithms(Continued...)

MCMC-Random Walk Metropolis (RWM)

- It is variant of the Metropolis-Hastings algorithm in which the proposal distribution is centred on the current state and the proposal step is defined by a fixed variance
- Simple to construct and requires little adjustment
- Robustness and capacity to successfully explore the parameter space in the setting of the models addressed in this project, which have relatively simple posterior distributions.

Algorithms(Continued...)

MCMC-No-U-Turn Sampler (NUTS)

- The main idea behind NUTS is to build a binary tree of leapfrog steps in each iteration, growing the tree until a ”U-turn” condition is detected.
- The algorithm first samples a random momentum from a Gaussian distribution, then initializes the binary tree with the current state and momentum.
- Efficient exploration of complex, high-dimensional posterior distributions compared to the standard HMC algorithm because of adaptive nature of the tree expansion.

Algorithms(Continued…)

Variational Inference

- Variational Inference (VI) aims to maximize the Evidence Lower Bound (ELBO) loss function
- By maximizing the ELBO, we can sample from the posterior
- ELBO is a balancing act between the prior and the optimal point estimates of Z that would maximize the likelihood of the observed data X

Implementations and Results

WE WILL SWITCH TO REPORT TO SHOW OUR RESULTS FOR THIS PART BECAUSE OF EXCESSIVE AMOUNT OF GRAPHS GENERATED!!

References

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https://www.tensorflow.org/probability/examples/Bayesian_Switchpoint_Analysis

All code for the project can be found in this git repo:
https://github.com/Dwivedi07/CS726_ENDSEMPROJECT/tree/main

Thank you!