AI chapter 7 homework problems

7.1 Describe the wumpus world according to the properties of task environments listed in Chapter 2 (pages 40-42).

Fully observable		Partially observable	x
Deterministic	x	Stochastic	
Episodic		Sequential	x
Static	x	Dynamic	
Discrete	x	Continuous	
Single agent	x	Multiagent	

7.2 Suppose the agent has progressed to the point shown in Figure 7.4(a), having perceived nothing in [1,1], a breeze in [2,1], and a stench in [1,2]. and is now concerned with the contents of [1,3], [2,2], and [3,1]. Each of these can contain a pit and at most one can contain a wumpus. Following the example of Figure 7.5, construct the set of possible worlds. (You should find 32 of them.) Mark the worlds in which the KB is true and those in which each of the following sentences is true:
α_2 = "There is no pit in [2,2]."
α_3 = "There is a wumpus in [1,3]."
Hence show that KB $\models \alpha_2$ and KB $\models \alpha_3$.

? [1,3]			
Stench [1,2]	? [2,2]		
- [1,1]	Breeze [2,1]	? [3,1]	

KB:

W1. There must be a wumpus in either [1, 3] or [2, 2]. $(W_{1,3} \wedge \neg W_{2,2}) \vee (\neg W_{1,3} \wedge W_{2,2})$
(Stench in [1, 2])

W2. There cannot be a wumpus in [2, 2] or [3, 1]. $\neg W_{2,2} \wedge \neg W_{3,1}$
(No stench in [2,1]

W3. The wumpus is in [1, 3], and not in [2, 2] or [3, 1]. $W_{1,3} \wedge \neg W_{2,2} \wedge \neg W_{3,1}$
(W1 and W2)

P1. There must be a pit in [2, 2] or [3, 1] or both. $P_{2,2} \wedge P_{3,1}$
(Breeze in [2, 1])

P2. There cannot be a pit in [1, 3] or [2, 2]. $\neg P_{1,3} \wedge \neg P_{2,2}$
(No breeze in [1, 2])

P3. There is a pit in [3, 1] and no pit in [1, 3] or [2, 2]. $P_{3,1} \wedge \neg P_{1,3} \wedge \neg P_{2,2}$
(P4 and P5)

Wumpus in [1,3], [2,2], or [3,1]	pit in [1,3]	pit in [2,2]	pit in [3,1]	W3 is true	P3 is true	KB is true	α_2 = "There is no pit in [2,2]."	α_3 = "There is a wumpus in [1,3]."
no	0	0	0	0	0	0	1	0
no	0	0	1	0	1	0	1	0
no	0	1	0	0	0	0	0	0
no	0	1	1	0	0	0	0	0
no	1	0	0	0	0	0	1	0
no	1	0	1	0	0	0	1	0
no	1	1	0	0	0	0	0	0
no	1	1	1	0	0	0	0	0
[1,3]	0	0	0	1	0	0	1	1
[1,3]	0	0	1	1	1	1	1	1
[1,3]	0	1	0	1	0	0	0	1
[1,3]	0	1	1	1	0	0	0	1
[1,3]	1	0	0	1	0	0	1	1
[1,3]	1	0	1	1	0	0	1	1
[1,3]	1	1	0	1	0	0	0	1
[1,3]	1	1	1	1	0	0	0	1
[2,2]	0	0	0	0	0	0	1	0
[2,2]	0	0	1	0	1	0	1	0
[2,2]	0	1	0	0	0	0	0	0
[2,2]	0	1	1	0	0	0	0	0
[2,2]	1	0	0	0	0	0	1	0
[2,2]	1	0	1	0	0	0	1	0
[2,2]	1	1	0	0	0	0	0	0
[2,2]	1	1	1	0	0	0	0	0
[3,1]	0	0	0	0	0	0	1	0
[3,1]	0	0	1	0	1	0	1	0
[3,1]	0	1	0	0	0	0	0	0
[3,1]	0	1	1	0	0	0	0	0
[3,1]	1	0	0	0	0	0	1	0
[3,1]	1	0	1	0	0	0	1	0
[3,1]	1	1	0	0	0	0	0	0
[3,1]	1	1	1	0	0	0	0	0

Conclusions:

Since α_2 is true in every model (row) where the KB is true (W3 and P3 are true), (KB $\models \alpha_2$). It can be concluded that α_2 = "There is no pit in [2,2]."

Since α_3 is true in every model (row) where the KB is true (W3 and P3 are true), (KB $\models \alpha_3$). It can be concluded that α_3 = "There is a wumpus in [1,3]."

7.4 Prove each of the following assertions:

a. α is valid if and only if True $\models$ α.

b. For any α, False $\models$ α.

c. $\alpha \models \beta$ if and only if the sentence $(\alpha \rightarrow \beta)$ is valid.

d. $\alpha \equiv \beta$ if and only if the sentence $(\alpha \leftrightarrow \beta)$ is valid.

e. $\alpha \models \beta$ if and only if the sentence $(\alpha \wedge \neg\beta)$ is unsatisfiable.

(Solutions later)

7.5 Consider a vocabulary with only four propositions, A, B, C, and D. How many models are there for the following sentences?

a. $(A \wedge B) \vee (B \wedge C)$

$2^3 = 8$ models

b. $A \vee B$

$2^2 = 4$ models

c. $A \leftrightarrow B \leftrightarrow C$

$2^3 = 8$ models

7.8 Decide whether each of the following sentences is valid, unsatisfiable, or neither. Verify your decisions using truth tables or the equivalence rules of Figure 7.11.

$$
\begin{array}{rcll}
(\alpha \land \beta) & \equiv & (\beta \land \alpha) & \text{commutativity of } \land \\
(\alpha \lor \beta) & \equiv & (\beta \lor \alpha) & \text{commutativity of } \lor \\
((\alpha \land \beta) \land \gamma) & \equiv & (\alpha \land (\beta \land \gamma)) & \text{associativity of } \land \\
((\alpha \lor \beta) \lor \gamma) & \equiv & (\alpha \lor (\beta \lor \gamma)) & \text{associativity of } \lor \\
\neg(\neg\alpha) & \equiv & \alpha & \text{double-negation elimination} \\
(\alpha \Rightarrow \beta) & \equiv & (\neg\beta \Rightarrow \neg\alpha) & \text{contraposition} \\
(\alpha \Rightarrow \beta) & \equiv & (\neg\alpha \lor \beta) & \text{implication elimination} \\
(\alpha \Leftrightarrow \beta) & \equiv & ((\alpha \Rightarrow \beta) \land (\beta \Rightarrow \alpha)) & \text{biconditional elimination} \\
\neg(\alpha \land \beta) & \equiv & (\neg\alpha \lor \neg\beta) & \text{de Morgan} \\
\neg(\alpha \lor \beta) & \equiv & (\neg\alpha \land \neg\beta) & \text{de Morgan} \\
(\alpha \land (\beta \lor \gamma)) & \equiv & ((\alpha \land \beta) \lor (\alpha \land \gamma)) & \text{distributivity of } \land \text{ over } \lor \\
(\alpha \lor (\beta \land \gamma)) & \equiv & ((\alpha \lor \beta) \land (\alpha \lor \gamma)) & \text{distributivity of } \lor \text{ over } \land
\end{array}
$$

Figure 7.11 Standard logical equivalences. The symbols α, β, and γ stand for arbitrary sentences of propositional logic.

a. Smoke → Smoke

¬Smoke v Smoke	implication elimination
TRUE	definition of v

Valid

Smoke	Smoke → Smoke
0	1
1	1

b. Smoke → Fire

¬Smoke v Fire　　implication elimination

Neither (satisfiable)

Smoke	Fire	Smoke → Fire
0	0	1
0	1	1
1	0	0
1	1	1

Neither (satisfiable)

c. (Smoke → Fire) → (¬Smoke → ¬Fire)

(¬Smoke v Fire) → (Smoke v ¬Fire)	implication elimination
¬(¬Smoke v Fire) v (Smoke v ¬Fire)	implication elimination
(¬ (¬Smoke) ^ ¬Fire)) v (Smoke v ¬Fire)	de Morgan
(Smoke ^ ¬Fire) v (Smoke v ¬Fire)	double-negation elimination
(Smoke ^ ¬Fire) v Smoke v ¬Fire	associativity of v

Neither (satisfiable)

Smoke	Fire	¬Smoke	¬Fire	(Smoke → Fire)	(¬Smoke → ¬Fire)	entire
0	0	1	1	1	1	1
0	1	1	0	1	0	0
1	0	0	1	0	1	1
1	1	0	0	1	1	1

d. Smoke v Fire v ¬Fire

Smoke v TRUE	definition of v
TRUE	definition of v

Valid

Smoke	Fire	¬Fire	Smoke v Fire v ¬Fire
0	0	1	1
0	1	0	1
1	0	1	1
1	1	0	1

Valid

e. ((Smoke ^ Heat) → Fire) ↔ ((Smoke → Fire) v (Heat → Fire))

Smoke	Heat	Fire	(Smoke ^ Heat)	((Smoke ^ Heat) → Fire)	(Smoke → Fire)	(Heat → Fire)	((Smoke → Fire) v (Heat → Fire))	entire
0	0	0	0	1	1	1	1	1
0	0	1	0	1	1	1	1	1
0	1	0	0	1	1	0	1	1
0	1	1	0	1	1	1	1	1
1	0	0	0	1	0	1	1	1
1	0	1	0	1	1	1	1	1
1	1	0	1	0	0	0	0	1
1	1	1	1	1	1	1	1	1

Valid

f. (Smoke → Fire) → ((Smoke ^ Heat) → Fire)

(¬Smoke v Fire) → (¬(Smoke ^ Heat) v Fire)	implication elimination
¬(¬Smoke v Fire) v (¬(Smoke ^ Heat) v Fire)	implication elimination
(Smoke ^ ¬Fire) v (¬Smoke v ¬Heat) v Fire)	de Morgan
(Smoke ^ ¬Fire) v ¬Smoke v ¬Heat v Fire	associativity of v
******	
TRUE	definition of v

Valid

Smoke	Heat	Fire	Smoke → Fire)	((Smoke ^ Heat) → Fire)	entire
0	0	0	1	1	1
0	0	1	1	1	1
0	1	0	1	1	1
0	1	1	1	1	1
1	0	0	0	1	1
1	0	1	1	1	1
1	1	0	0	0	1
1	1	1	1	1	1

Valid

g. Big v Dumb v (Big → Dumb)

Big v Dumb v (¬Big v Dumb)	implication elimination
Big v Dumb v ¬Big v Dumb	associativity of v
Big v ¬Big v Dumb v Dumb	commutativity of v
TRUE v Dumb	definition of v
TRUE	definition of v

Valid

Big	Dumb	(Big → Dumb)	Big v Dumb v (Big → Dumb)
0	0	1	1
0	1	1	1
1	0	0	1
1	1	1	1

Valid

h. (Big ^ Dumb) v ¬Dumb

Big	Dumb	(Big ^ Dumb)	¬Dumb	(Big ^ Dumb) v ¬Dumb
0	0	0	1	1
0	1	0	0	0
1	0	0	1	1
1	1	1	0	1

Neither (satisfiable)