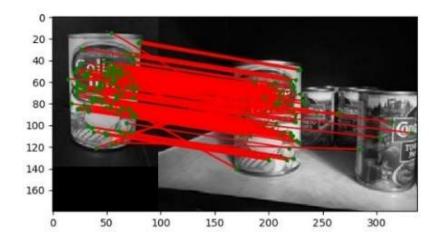
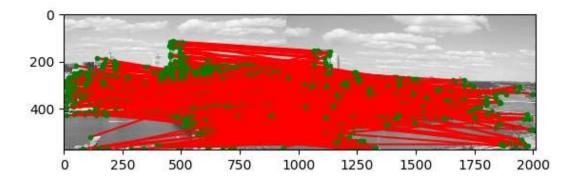
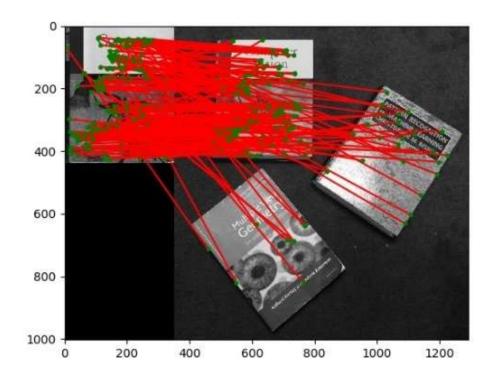
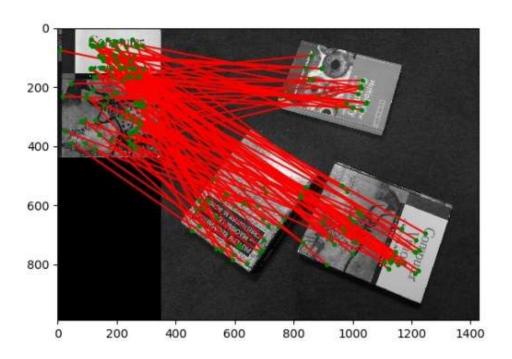
Q1.5 Keypoint Detection Result

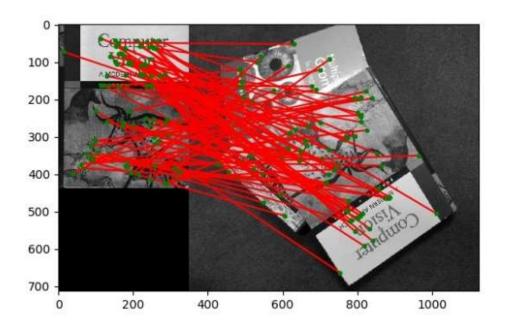


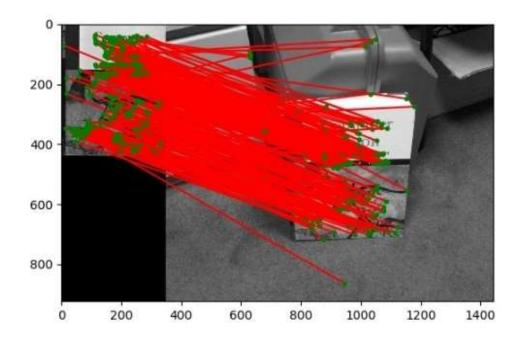




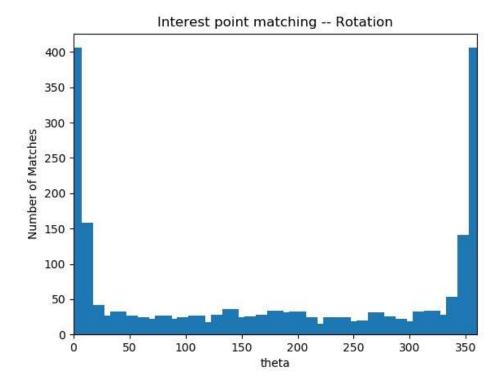








The number of correct matches is great when there is not much rotation of the template. As you can see from the chicken broth example and pf textbook examples. But when the source image has a rotation of the template, the BRIEF descriptor fails because the test patch is not flipping.



The BRIEF descriptor depends on randomly generated test patches. When the patch is small, or only translation is involved, the BRIEF matches is great. This is because local pixels are highly linearly correlated. This is even true for small scale rotation and affine transform. However, when large rotation or affine warp is involved, we cannot approximate the value of local pixels with high fidelity. So even there exists a perfect match, there is a great chance that the BRIEF descriptor fails to give a match response.

3. Planar homography theory

(a)

$$\lambda_{n} \begin{bmatrix} \widetilde{\chi_{n}} \\ \widetilde{y_{n}} \\ 1 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} \widetilde{u_{n}} \\ \widetilde{v_{n}} \\ 1 \end{bmatrix}, for \ n = 1: N$$

Form linear system $\mathbf{Ah} = 0$, s.t. $||\mathbf{h}||_2^2 = 1$.

$$\begin{bmatrix} 0 & 0 & 0 & -u_1 & -v_1 & -1 & y_1u_1 & y_1v_1 & y_1 \\ u_1 & v_1 & 1 & 0 & 0 & 0 & -x_1u_1 & -x_1v_1 & -x_1 \\ 0 & 0 & 0 & -u_2 & -v_2 & -1 & y_2u_2 & y_2v_2 & y_2 \\ u_2 & v_2 & 1 & 0 & 0 & 0 & -x_2u_2 & -x_2v_2 & -x_2 \\ u_2 & v_2 & 1 & 0 & 0 & 0 & -x_2u_2 & -x_2v_2 & -x_2 \\ 0 & 0 & 0 & -u_N & -v_N & -1 & y_Nu_N & y_Nv_N & y_N \\ u_N & v_N & 1 & 0 & 0 & 0 & -x_Nu_{1N} & -x_Nv_{1N} & -x_N \end{bmatrix} \begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{33} \\ h_{31} \\ h_{32} \\ h_{33} \end{bmatrix} = 0$$

- (b) 9 elements
- (c) At least 4 point pairs.
- (d)

$$\mathbf{h} = argmin ||A\mathbf{h}||_{2}^{2}, s.t. ||\mathbf{h}||_{2}^{2} = 1$$

 $h = argmin \frac{h^T A^T A h}{h^T h}$, take the first derivative and set it to zero. (Lagrange multipliers)

$$\frac{\partial}{\partial h} (\mathbf{h}^T \mathbf{A}^T \mathbf{A} \mathbf{h} + \lambda (1 - \mathbf{h}^T \mathbf{h})) = 0$$
$$2\mathbf{A}^T \mathbf{A} \mathbf{h} - 2\lambda \mathbf{h} = 0$$

 $error = \mathbf{h}^T A^T A \mathbf{h} / \mathbf{1} = \mathbf{h}^T \lambda \mathbf{h}$, the solution must be the eigenvector corresponds to the smallest eigenvalues. This is also equivalent to perform single value decomposition of matrix A. U,S,Vt = SVD(A), where the last column of V is the linear h array. We can even use symmetric transfer error function to improve our homography.





