Some mathmatic notation

$$n^{\underline{i}} = n * (n-1) * ... * (n-i+1)$$

Second-type Stirling Number:

$$\left\{ \begin{matrix} n \\ k \end{matrix} \right\}$$

Binomial Coeffcient

$$\binom{n}{k}$$

There is an egg-pain equality in Concrete Mathmatics.

$$n^k = \sum_i^k \left\{ \begin{matrix} k \\ i \end{matrix} \right\} * n^{\underline{i}} = \sum_i^k \left\{ \begin{matrix} k \\ i \end{matrix} \right\} * i! * \binom{n}{i}$$

Then, for each u

$$\begin{aligned} \sum_{v=1}^n dist(u,v)^k &= \sum_{v=1}^n \sum_i^k \left\{ \begin{matrix} k \\ i \end{matrix} \right\} * dist(u,v)^{\underline{i}} \\ &= \sum_i^k \left\{ \begin{matrix} k \\ i \end{matrix} \right\} * i! * \sum_{v=1}^n \binom{dist(u,v)}{i} \end{aligned}$$

So, all we have to do, is, for each u, to calculate its $\sum_{v=1}^n \binom{dist(u,v)}{i}$. Look at the summation closely, we found that,

$$\sum_{v=1}^n \binom{dist(u,v)}{i} = \sum_{v=1}^n \binom{dist(u,v)-1}{i} + \sum_{v=1}^n \binom{dist(u,v)-1}{i-1}$$

Thus, for each node u we can work out all $\sum_{v=1}^n \binom{dist(u,v)}{i}$ in $O(nk)$, through tree-type dynamic programming.

At last, calculate all $\sum_{v=1}^n dist(u,v)^k$ within $O(nk)$. Notice that we could construct stirling triangle, factorial table and pascal triangle in advance, with in $O(k^2)$.