

RANSAC With Likelihood Sampling

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Abstract

RANSAC (Random Sample Consensus) has been a popular algorithm in computer vision for fitting a model to data points containing outliers [2]. Traditional RANSAC algorithm uses random sampling to choose data points, and use these data points to generate models [4]. Instead of sampling randomly, we explore the possibility of sampling based on the likelihoods of the data point to be inliers. A previously developed form of RANSAC, BAYSAC [1], is used in the experiment. Experiments were conducted on the noisy circle finding problem [3] in order to demonstrate the differences and similarities between BAYSAC and RANSAC.

1. Introduction

1.1. RANSAC

RANSAC (Random Sample Consensus) is mainly used in computer vision for fitting a model to data points containing outliers [2]. In general, RANSAC algorithm follows the algorithm given in Algorithm 1:

Algorithm 1 RANSAC($num_iter, threshold$)

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1:  $t = 0, \mathbb{I} = \{\}$ 
2: while  $t < num\_iter$  and  $\mathbb{I}.size() < desired\_size$  do
3:   Randomly Sample dataset  $H_i$  of size  $(n + 1)$ .
4:   Fit a polynomial model of degree  $n$ .
5:   if  $\forall j \in H_i, dist(model, j) < threshold$  then
6:      $\mathbb{I} = \mathbb{I} \cup H_i$ .
7:   end if
8:    $t++$ 
9: end while
10: return the fitted model using all points in  $\mathbb{I}$ .
```

1.2. Problem Definition

The problem of noisy circle finding problem is defined as follows. Given two input arrays of pixel locations where $X = [x_1, x_2, \dots, x_n]$, $Y = [y_1, y_2, \dots, y_n]$, $n \geq 3$, find

a best-fit circle with center (x, y) , and its radius r [3]. An example of the problem is shown in Figure 1

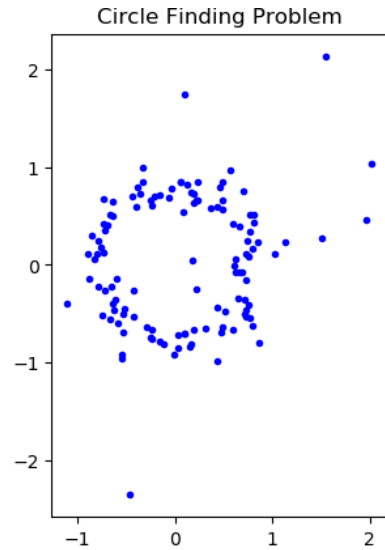


Figure 1. Example of Circle Finding Problem

2. Related Work

2.1. Noisy Circle Finding Problem

The noisy circle finding problem has been a classic exercise for computer vision students and an ongoing research topic for scholars. There are many approaches to solving this problem, including RANSAC, Patch Matching, Neural Network, etc.

For our experiment, we adopted and extended on an existing RANSAC circle detection project [5]. The original project uses RANSAC with a twist. Instead of counting the number of inliers, the author chooses the model that has the minimum distance to all data points. We modified this part back to counting the number of inliers, and extended the class to implement the BAYSAC algorithm described in Section 2.2.

2.2. BAYSAC

The BAYSAC algorithm [1] first assign all points' likelihoods to be inliers to 0.5. For each iteration, it updates the points' likelihoods using Equation (1), where $P_t(i \in \mathbb{I})$ indicates the probability of point i being in the inliers set \mathbb{I} at iteration t ; $P(H_t \subseteq \mathbb{I})$ indicates the probability of the sample H at iteration t being a subset of the inlier set \mathbb{I} . I.e. for all points in the sample $j \in H_t, j \in \mathbb{I}$.

$$P_t(i \in \mathbb{I}) = \begin{cases} \frac{P_{t-1}(i \in \mathbb{I})P(H_t \not\subseteq \mathbb{I} | i \in \mathbb{I})}{P(H_t \not\subseteq \mathbb{I})} & i \in H_t \\ P_{t-1}(i \in \mathbb{I}) & i \notin H_t \end{cases} \quad (1)$$

To calculate $P(H_t \not\subseteq \mathbb{I} | i \in \mathbb{I})$ and $P(H_t \not\subseteq \mathbb{I})$, Equation (2) and Equation (3) are used.

$$\begin{aligned} P(H_t \not\subseteq \mathbb{I}) &= 1 - P(H_t \subseteq \mathbb{I}) \\ &= 1 - \prod_{j \in H_t} P_{t-1}(j \in \mathbb{I}) \end{aligned} \quad (2)$$

$$\begin{aligned} P(H_t \not\subseteq \mathbb{I} | i \in \mathbb{I}) &= 1 - P(H_t \subseteq \mathbb{I} | i \in \mathbb{I}) \\ &= 1 - \prod_{j \in H_t; j \neq i} P_{t-1}(j \in \mathbb{I}) \\ &= 1 - \frac{P(H_t \subseteq \mathbb{I})}{P_{t-1}(i \in \mathbb{I})} \end{aligned} \quad (3)$$

Finally, combining Equations (2, 3) with Equation (1), we derive Equation (4)

$$P_t(i \in \mathbb{I}) = \begin{cases} \frac{P_{t-1}(i \in \mathbb{I}) - P(H_t \subseteq \mathbb{I})}{1 - P(H_t \subseteq \mathbb{I})} & i \in H_t \\ P_{t-1}(i \in \mathbb{I}) & i \notin H_t \end{cases} \quad (4)$$

3. Experiment

3.1. Algorithm

Combining the two related work described in the Section 2, the final BAYSAC algorithm can be written as Algorithm 2, where $\text{dist}(\text{model}, j)$ is the L2 distance defined by Equation 5.

$$\text{dist}(x_c, y_c, r_c, x_j, y_j) = |\sqrt{(x_j - x_c)^2 + (y_j - y_c)^2} - r_c| \quad (5)$$

4. Conclusions

Algorithm 2 BAYSAC(*num_iter*, *threshold*)

```

1:  $t = 0, \mathbb{I} = \{\}$ 
2: while  $t < \text{num\_iter}$  and  $\mathbb{I}.\text{size}() < \text{desired\_size}$  do
3:   Sample  $H_t$  of  $(n + 1)$  points ordered by highest
      $P_{t-1}(i \in \mathbb{I})$ 
4:   Fit a polynomial model of degree  $n$ .
5:   if  $\forall j \in H_t, \text{dist}(\text{model}, j) < \text{threshold}$  then
6:      $\mathbb{I} = \mathbb{I} \cup H_t$ .
7:   end if
8:   update all points'  $P_t(i \in \mathbb{I})$  base on Equation 4
9:    $t++$ 
10: end while
11: return the fitted model using all points in  $\mathbb{I}$ .

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References

- [1] Tom Botterill, Steven Mills, and Richard D Green. New conditional sampling strategies for speeded-up ransac. In *BMVC*, pages 1–11. Citeseer, 2009.
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- [5] Bae Seong-hyun. Ransac-circle-python, Jan. 2019. GitHub repository.