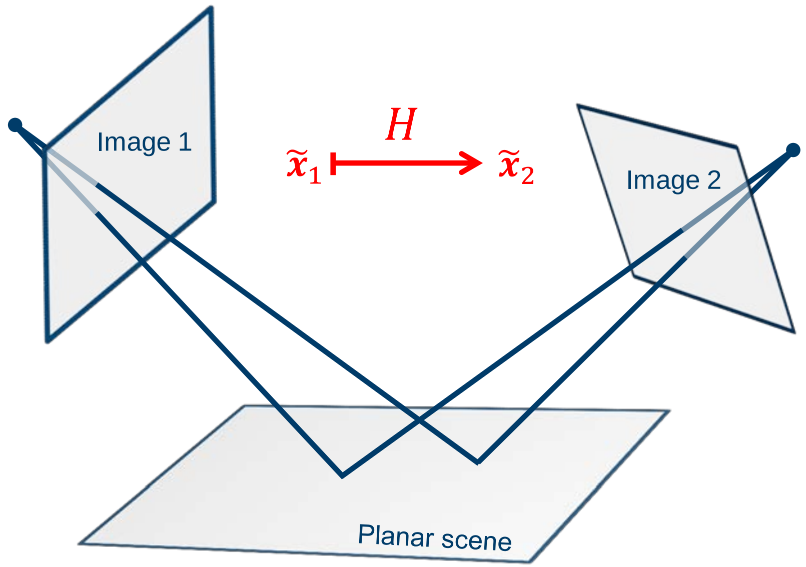
Lecture 3.3
Robust estimation with RANSAC

Thomas Opsahl

Motivation



- Two images, captured by perspective cameras, of the same planar scene are related by a homography H

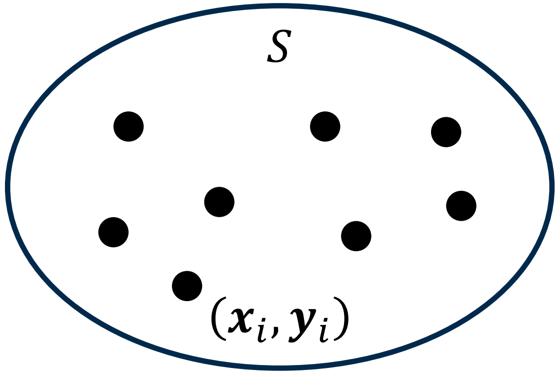
- This homography can be estimated from 4 or more point-correspondences between the images

- Point-correspondences can be established automatically
  - Find key points in both images
  - Represent key points by a descriptor (a vector of parameters)
  - Establish point correspondences by comparing descriptors

- The resulting set of point pairs typically contains several wrong correspondences

- A robust estimation method provides a good estimate of H despite the presence of erroneous correspondences (outliers)

RANdom SAmple Consensus - RANSAC



Observed data

$\longrightarrow$

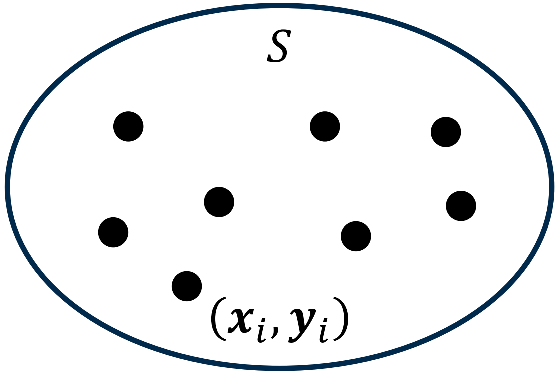
Estimation method

$$\boxed{\boldsymbol{y} = f(\boldsymbol{x}; \boldsymbol{\alpha})}$$

Mathematical model with parameters $\boldsymbol{\alpha} = (\alpha_1, \dots, \alpha_n)$

- RANSAC is an iterative method for estimating the parameters of a mathematical model from a set of observed data containing outliers

RANdom SAmple Consensus - RANSAC



Observed data

Estimation method

$\boxed{\boldsymbol{y} = f(\boldsymbol{x}; \boldsymbol{\alpha})}$

Mathematical model with parameters $\boldsymbol{\alpha} = (\alpha_1, \dots, \alpha_n)$

$\boxed{y = ax + b}$

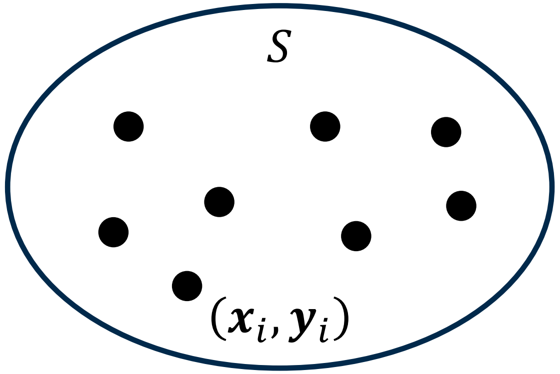
$\boxed{ax + by + c = 0}$

$\boxed{y = ax^2 + bx + c}$

$\boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1}$

- RANSAC is an iterative method for estimating the parameters of a mathematical model from a set of observed data containing outliers

RANdom SAmple Consensus - RANSAC



Observed data

$\xrightarrow{}$

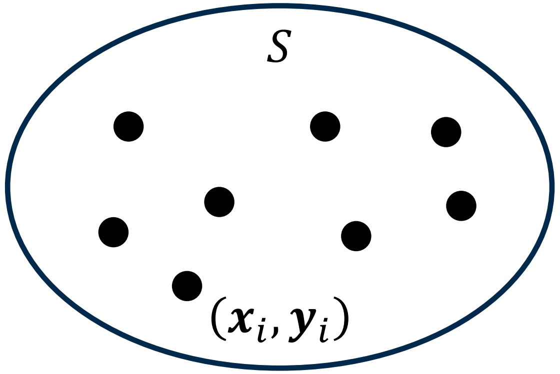
Estimation method

$$\boxed{\boldsymbol{y} = f(\boldsymbol{x}; \boldsymbol{\alpha})}$$

Mathematical model with parameters $\boldsymbol{\alpha} = (\alpha_1, \dots, \alpha_n)$

- RANSAC is an iterative method for estimating the parameters of a mathematical model from a set of observed data containing outliers
  - Robust method (handles up to 50% outliers)

RANdom SAmple Consensus - RANSAC



Observed data

$\longrightarrow$

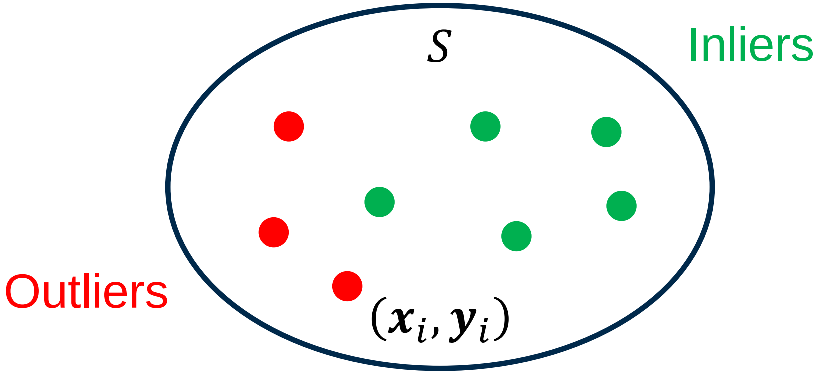
Estimation method

$$\boxed{\boldsymbol{y} = f(\boldsymbol{x}; \boldsymbol{\alpha})}$$

Mathematical model with parameters $\boldsymbol{\alpha} = (\alpha_1, \ldots, \alpha_n)$

- RANSAC is an iterative method for estimating the parameters of a mathematical model from a set of observed data containing outliers
  - Robust method (handles up to 50% outliers)
  - The estimated model is random but reasonable

RANdom SAmple Consensus - RANSAC



Observed data

$\longrightarrow$

Estimation method

$$\boxed{\boldsymbol{y} = f(\boldsymbol{x}; \boldsymbol{\alpha})}$$

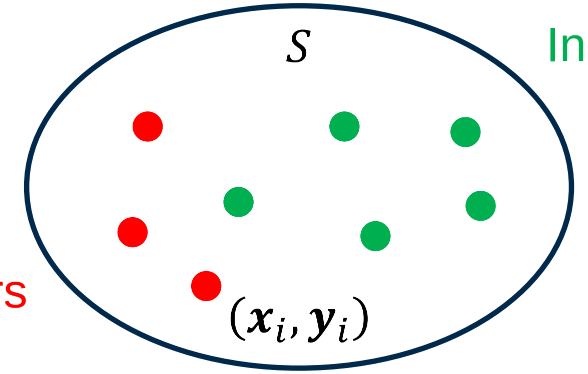
Mathematical model with parameters $\boldsymbol{\alpha} = (\alpha_1, \dots, \alpha_n)$

- RANSAC is an iterative method for estimating the parameters of a mathematical model from a set of observed data containing outliers
  - Robust method (handles up to 50% outliers)
  - The estimated model is random but reasonable
  - The estimation process divides the observed data into inliers and outliers

RANdom SAmple Consensus - RANSAC

Inliers

Outliers



Observed data

$\longrightarrow$

Estimation method

$\boxed{\boldsymbol{y} = f(\boldsymbol{x}; \boldsymbol{\alpha})}$

Mathematical model with parameters $\boldsymbol{\alpha} = (\alpha_1, \dots, \alpha_n)$

- RANSAC is an iterative method for estimating the parameters of a mathematical model from a set of observed data containing outliers
  - Robust method (handles up to 50% outliers)
  - The estimated model is random but reasonable
  - The estimation process divides the observed data into inliers and outliers
  - Usually an improved estimate of the model is determined based on the inliers using a less robust estimation method, e.g. least squares

Basic RANSAC

Objective
To robustly fit a model $\boldsymbol{y} = f(\boldsymbol{x}; \boldsymbol{\alpha})$ to a data set S containing outliers

Algorithm
1. Estimate the model parameters $\boldsymbol{\alpha}_{tst}$ from a randomly sampled subset of n data points from S

2. Determine the set of inliers $S_{tst} \subseteq S$ to be the data points within a distance t of the model

3. If this set of inliers is the largest so far, let $S_{IN} = S_{tst}$ and let $\boldsymbol{\alpha} = \boldsymbol{\alpha}_{tst}$

4. If $|S_{IN}| < T$, where T is some threshold value, repeat steps 1-3, otherwise stop

5. After N trials, stop

Basic RANSAC

Comments

- Typically the number of random samples, n, is the smallest number of data points required to estimate the model
- Assuming Gaussian noise in the data, the threshold value t should be in the region of 2σ were σ is the expected noise in the data set
- The threshold value T is set large enough to return a satisfactory inlier set, or simply omitted
- The maximal number of tests, N, can be chosen according to how certain we want to be of sampling at least one n-tuple with no outliers

  If p is the desired probability of sampling at least one $n$tuple with no outliers and ω is the probability of a random data point to be an inlier, then

$$N = \frac{log(1-p)}{log(1-\omega^n)}$$

  $p = 0.99$ is standard

Basic RANSAC

Comments

- $N = \frac{log(1-p)}{log(1-\omega^n)}$ with $p = 0.99$

ω

N	90	80	70	60	50
2	3	5	7	11	17
3	4	7	11	19	35
4	5	9	17	34	72
5	6	12	26	57	146
6	7	16	37	97	293
7	8	20	54	163	588
8	9	26	78	272	1177

n

- Typically we do not know the ratio of outliers in our data set, hence we do not know the probability ω or the number N

- Instead of operating with a larger than necessary N we can modify RANAC to adaptively estimate N as we perform the iterations

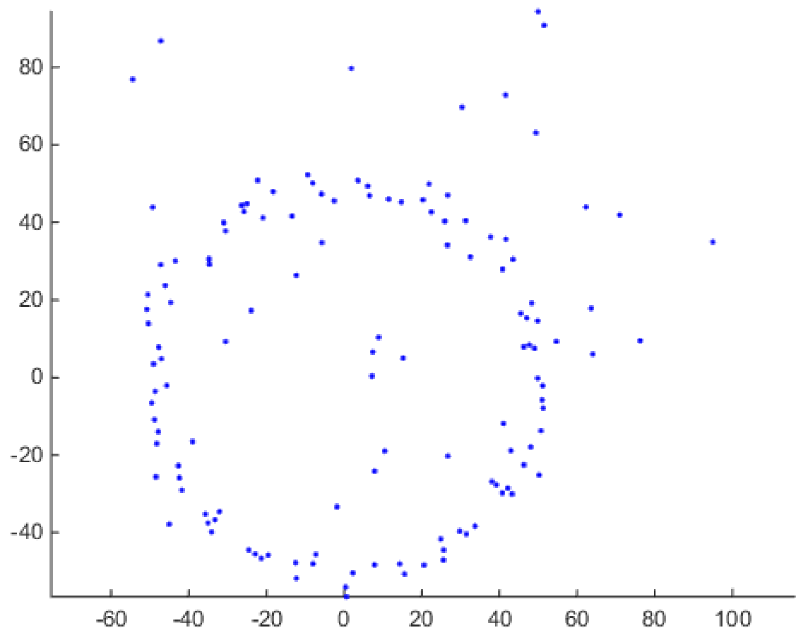
Adaptive RANSAC

Objective
To robustly fit a model $\boldsymbol{y} = f(\boldsymbol{x}; \boldsymbol{\alpha})$ to a data set S containing outliers

Algorithm

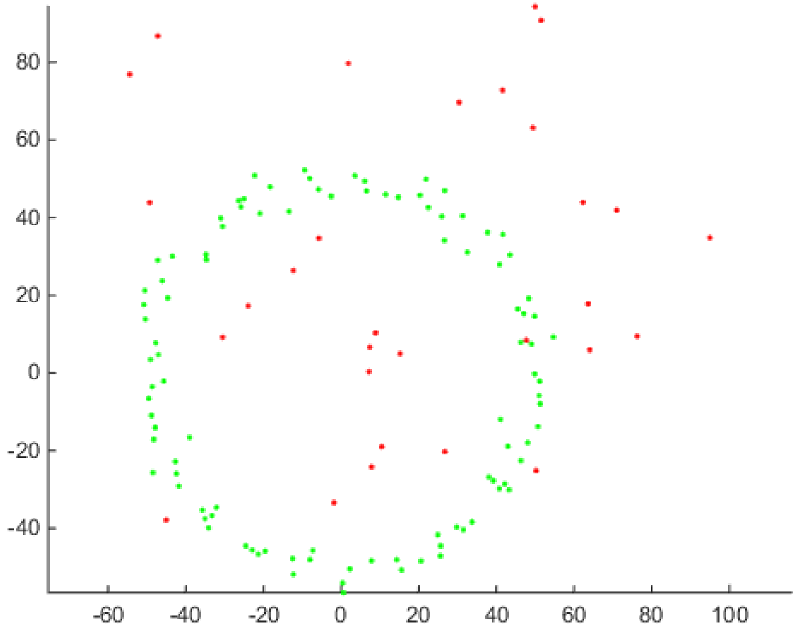
1. Let $N = \infty$, $S_{IN} = \emptyset$ and $\#iterations = 0$
2. while $N > \#iterations$ repeat 3-5
3. Estimate parameters $\boldsymbol{\alpha}_{tst}$ from a random n-tuple from S
4. Determine inlier set S_{tst}, i.e. data points within a distance t of the model $\boldsymbol{y} = f(\boldsymbol{x}; \boldsymbol{\alpha}_{tst})$
5. If $|S_{tst}| > |S_{IN}|$, set $S_{IN} = S_{tst}$, $\boldsymbol{\alpha} = \boldsymbol{\alpha}_{tst}$, $\omega = \frac{|S_{IN}|}{|S|}$ and $N = \frac{log(1-p)}{log(1-\omega^n)}$ with $p = 0.99$
   Increase $\#iterations$ by 1

Example



- Fit a circle $(x - x_0)^2 + (y - y_0)^2 = r^2$ to these data points by estimating the 3 parameters x_0, y_0 and r

Example



Circle + Gaussian noise
Random points

- Fit a circle $(x - x_0)^2 + (y - y_0)^2 = r^2$ to these data points by estimating the 3 parameters x_0, y_0 and r

- The data consists of some points on a circle with Gaussian noise and some random points

Example

- **Least-squares approach**
  Separate observables from parameters:

$$(x-x_0)^2+(y-y_0)^2=r^2$$

$$x^2-2xx_0+x_0^2+y^2-2yy_0+y_0^2=r^2$$

$$2xx_0+2yy_0+r^2-x_0^2-y_0^2=x^2+y^2$$

$$\begin{bmatrix} x & y & 1 \end{bmatrix}\begin{bmatrix} 2x_0 \\ 2y_0 \\ r^2-x_0^2-y_0^2 \end{bmatrix}=\begin{bmatrix} x^2+y^2 \end{bmatrix}$$

$$\begin{bmatrix} x & y & 1 \end{bmatrix}\begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix}=\begin{bmatrix} x^2+y^2 \end{bmatrix}$$

- So for each observation (x_i, y_i) we get one equation

$$\begin{bmatrix} x_i & y_i & 1 \end{bmatrix}\begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix}=\begin{bmatrix} {x_i}^2+{y_i}^2 \end{bmatrix}$$

- From all our N observations we get a system of linear equations

$$\begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ & \vdots & \\ x_N & y_N & 1 \end{bmatrix}\begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix}=\begin{bmatrix} x_1^2+y_1^2 \\ x_2^2+y_2^2 \\ \vdots \\ x_N^2+y_N^2 \end{bmatrix}$$

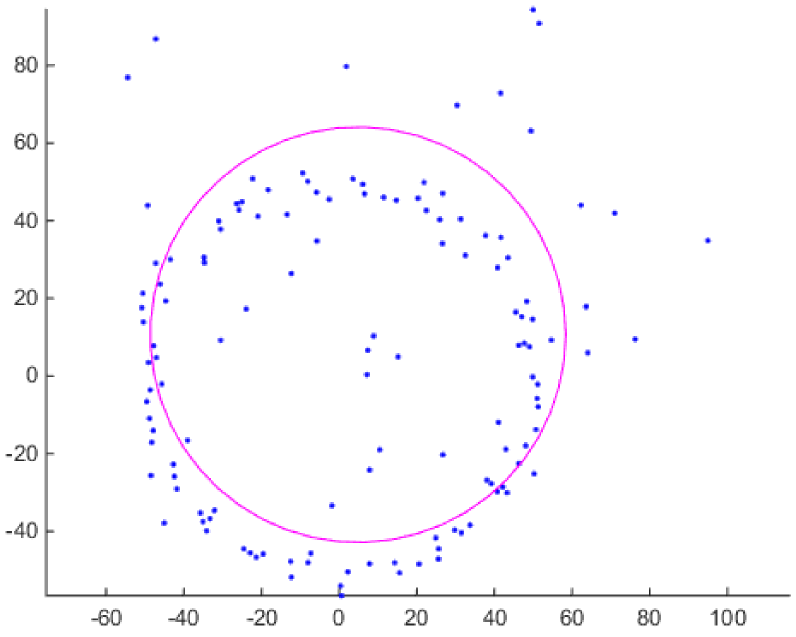
$$A\boldsymbol{p}=\boldsymbol{b}$$

Example

- We can solve this using the pseudo inverse $\boldsymbol{p} = (A^T A)^{-1} A^T \boldsymbol{b}$
  - This is the solution that minimize $||A\boldsymbol{p} - \boldsymbol{b}||$

Example

- We can solve this using the pseudo inverse $\boldsymbol{p} = (A^T A)^{-1} A^T \boldsymbol{b}$
  - This is the solution that minimize $||A\boldsymbol{p} - \boldsymbol{b}||$



- NOT GOOD! All points are treated equally, so the random points shifts the estimated circle away from the desired solution

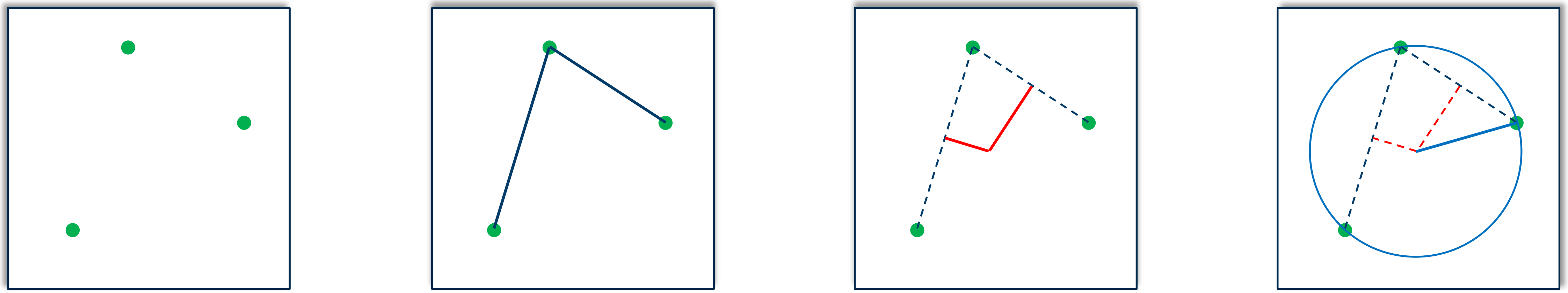
Example

- To estimate the circle using RANSAC, we need two things
  1. A way to estimate a circle from n-points, where n is as small as possible
  2. A way to determine which of the points are inliers for an estimated circle

Example

- To estimate the circle using RANSAC, we need two things
  1. **A way to estimate a circle from n-points, where n is as small as possible**
  2. A way to determine which of the points are inliers for an estimated circle

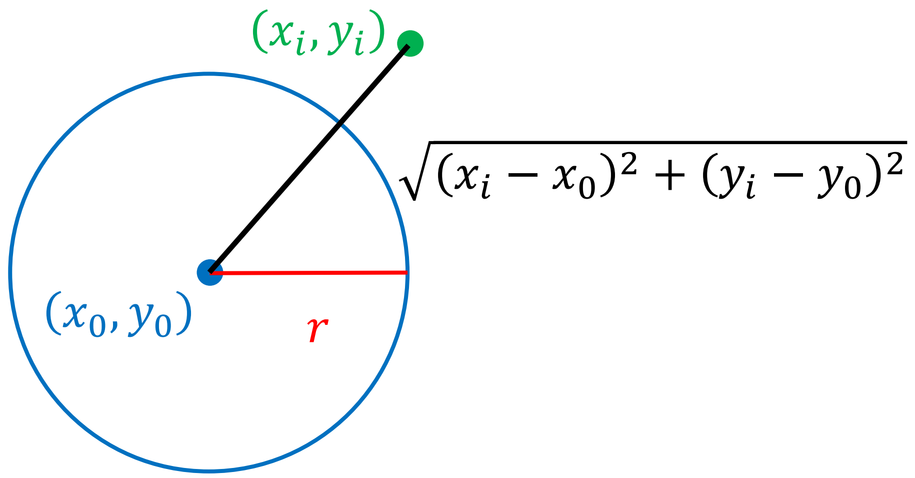
- The smallest number of points required to determine a circle is 3, i.e. $n = 3$, and the algorithm for computing the circle is quite simple



Example

- To estimate the circle using RANSAC, we need two things
  1. A way to estimate a circle from n-points, where n is as small as possible
  2. **A way to determine which of the points are inliers for an estimated circle**

- The distance from a point (x_i, y_i) to a circle $(x - x_0)^2 + (y - y_0)^2 = r^2$ is given by $\left|\sqrt{(x_i - x_0)^2 + (y_i - y_0)^2} - r\right|$



Example

- To estimate the circle using RANSAC, we need two things
  1. A way to estimate a circle from n-points, where n is as small as possible
  2. **A way to determine which of the points are inliers for an estimated circle**

- The distance from a point (x_i, y_i) to a circle $(x - x_0)^2 + (y - y_0)^2 = r^2$ is given by $\left|\sqrt{(x_i - x_0)^2 + (y_i - y_0)^2} - r\right|$

- So for a threshold value t, we say that (x_i, y_i) is an inlier if $\left|\sqrt{(x_i - x_0)^2 + (y_i - y_0)^2} - r\right| < t$

Example

Objective

To robustly fit the model $(x - x_0)^2 + (y - y_0)^2 = r^2$ to our data set $S = \{(x_i, y_i)\}$

Algorithm

1. Let $N = \infty$, $S_{IN} = \emptyset$, $p = 0.99$, $t = 2 \cdot expected\ noise$ and $\#iterations = 0$

2. while $N > \#iterations$ repeat 3-5

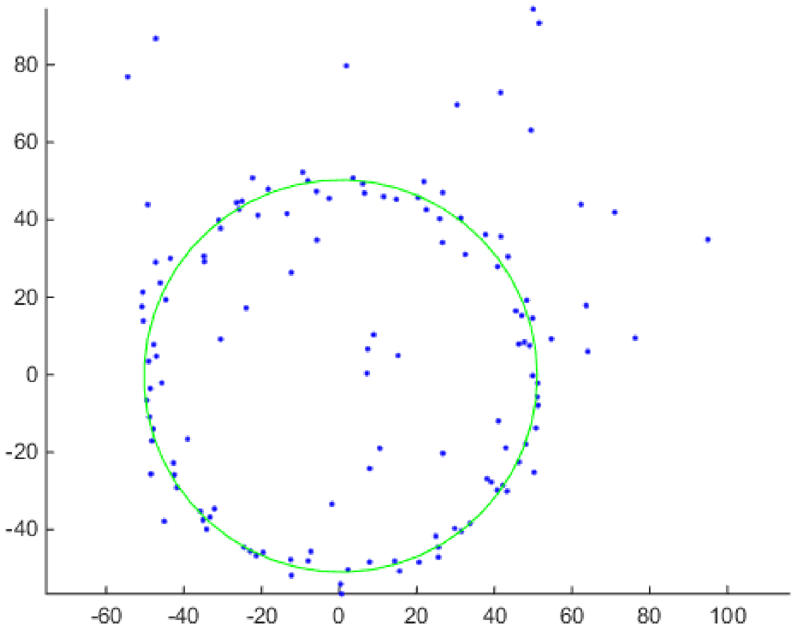
3. Estimate parameters $(x_{tst}, y_{tst}, r_{tst})$ from three random points from S

4. Determine inlier set $S_{tst} = \left\{(x_i, y_i) \in S \text{ such that } \left|\sqrt{(x_i - x_0)^2 + (y_i - y_0)^2} - r\right| < t \right\}$

5. If $|S_{tst}| > |S_{IN}|$, set $S_{IN} = S_{tst}$,$(x_0, y_0, r) = (x_{tst}, y_{tst}, r_{tst})$, $\omega = \frac{|S_{IN}|}{|S|}$ and $N = \frac{log(1-p)}{log(1-\omega^n)}$
   Increase $\#iterations$ by 1

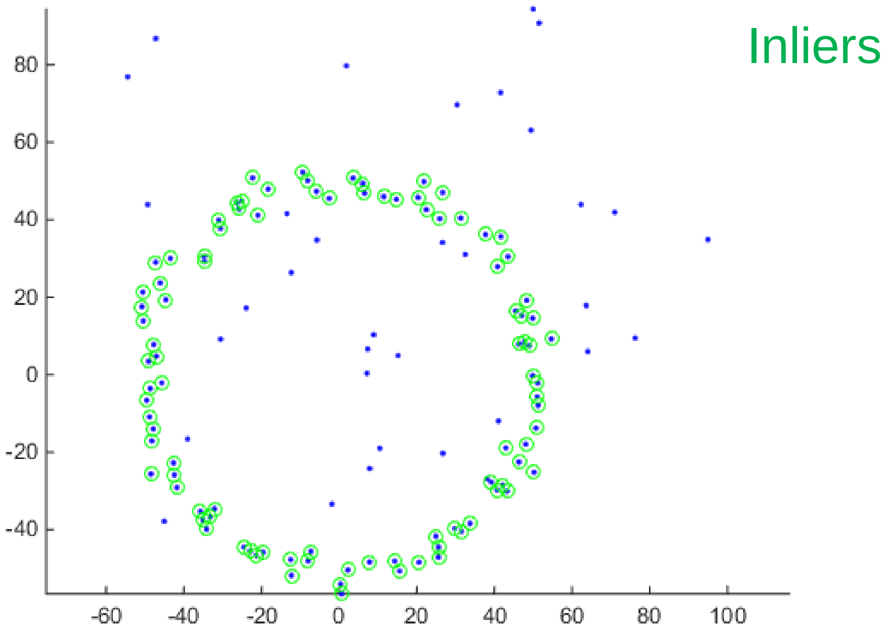
Example

- The RANSAC algorithm evaluates many different circles and returns the circle with the largest inlier set



Example

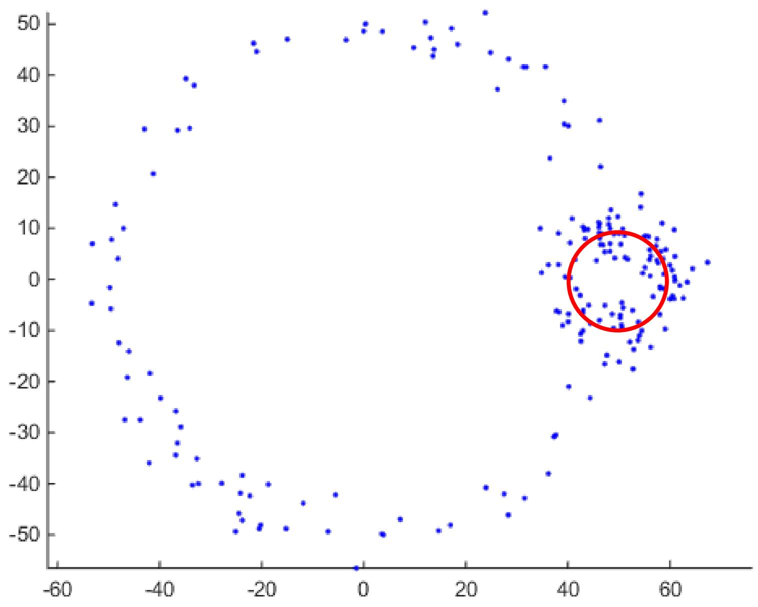
- The RANSAC algorithm evaluates many different circles and returns the circle with the largest inlier set



- Inliers can be used to get an improved estimate of the circle

Robust estimation

- RANSAC is not perfect…



- Several other robust estimation methods exist
  - Least Median Squares (LMS)
  - Preemptive RANSAC
  - PROgressive Sample and Consensus (PROSAC)
  - M-estimator Sample and Consensus (MSAC)
  - Maximum Likelihood Estimation Sample and Consensus (MLESAC)
  - Randomized RANSAC (R-RANSAC)
  - KALMANSAC
  - +++

Summary

- RANSAC
  - A robust iterative method for estimating the parameters of a mathematical model from a set of observed data containing outliers
  - Separates the observed data into “inliers” and “outliers”
  - Very useful if we want to use better, but less robust, estimation methods
  - Not perfect

- Additional reading
  - Szeliski: 6.1.4

- Homework?
  - Implement a RANSAC algorithm for estimating a line