# CSC321: Assignment #5

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# Problem 1

Let $\mathbf{x}^{(t)}$ be a $2 \times 1$ vector containing $x_1$ and $x_2$ as the binary input at time $t$
Let $\mathbf{h}^{(t)}$ be a $3 \times 1$ vector containing $h_1$, $h_2$ and $h_3$ at time $t$ as hinted in the handout.
Let $y^{(t)}$ be a scaler of the output binary digit.

We let

$$\mathbf{U} = \begin{Bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{Bmatrix}$$

$$\mathbf{W} = \begin{Bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{Bmatrix}$$

$$\mathbf{b_h} = \begin{Bmatrix} -0.5 \\ -1.5 \\ -2.5 \end{Bmatrix}$$

$$\mathbf{v} = \begin{Bmatrix} 1 & -1 & 1 \end{Bmatrix}$$

$$b_y = -0.5$$

Then for all $t \geq 1$,

$$\mathbf{h}^{(t)} = \mathbf{U}\mathbf{x}^{(t)} + \mathbf{W}\mathbf{h}^{(t-1)} + \mathbf{b_h} \tag{1}$$

$$y^{(t)} = \mathbf{v}\mathbf{h}^{(t)} + b_y \tag{2}$$

Expanding Equation (1), we get

$$h_1^{(t)} = x_1^{(t)} + x_2^{(t)} + h_1^{(t-1)} - 0.5$$
$$h_2^{(t)} = x_1^{(t)} + x_2^{(t)} + h_2^{(t-1)} - 1.5$$
$$h_3^{(t)} = x_1^{(t)} + x_2^{(t)} + h_3^{(t-1)} - 2.5$$
$$y^{(t)} = h_1^{(t)} - h_2^{(t)} + h_3^{(t)} - 0.5$$

This satisfies the Truth table:

| $x_1$ | $x_2$ | $h_1^{(t-1)}$ | $h_1^{(t)}$ | $h_2^{(t-1)}$ | $h_2^{(t)}$ | $h_3^{(t-1)}$ | $h_3^{(t)}$ | y |
|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 | 0 | 1 | 0 | 1 |
| 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 1 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

# Problem 2

1.

$$
\begin{aligned}
\overline{h^{(t)}} &= 1 + \overline{i^{(t+1)}}\frac{\partial i^{(t+1)}}{\partial h^{(t+1)}} + \overline{f^{(t+1)}}\frac{\partial f^{(t+1)}}{\partial h^{(t+1)}} + \overline{o^{(t+1)}}\frac{\partial o^{(t+1)}}{\partial h^{(t+1)}} + \overline{g^{(t+1)}}\frac{\partial g^{(t+1)}}{\partial h^{(t+1)}} \\
&= 1 + \overline{i^{(t+1)}}\sigma^{-1}(w_{ix}x^{(t+1)} + w_{ih}h^{(t+1)})w_{ih} + \overline{f^{(t+1)}}\sigma^{-1}(w_{fx}x^{(t+1)} + w_{fh}h^{(t+1)})w_{fh} \\
&\quad + \overline{o^{(t+1)}}\sigma^{-1}(w_{ox}x^{(t+1)} + w_{oh}h^{(t+1)})w_{oh} + \overline{g^{(t+1)}}\tanh^{-1}(w_{gx}x^{(t+1)} + w_{gh}h^{(t+1)})w_{gh} \\
\overline{c^{(t)}} &= \overline{h^{(t)}}\frac{\partial h^{(t)}}{\partial c^{(t)}} + \overline{c^{(t+1)}}\frac{c^{(t+1)}}{c^{(t)}} \\
&= \overline{h^{(t)}}o^{(t)}\tanh^{-1}(c^{(t)}) + \overline{c^{(t+1)}}f^{(t)} \\
\overline{g^{(t)}} &= \overline{c^{(t)}}\frac{\partial c^{(t)}}{\partial g^{(t)}} \\
&= \overline{c^{(t)}}i^{(t)} \\
\overline{o^{(t)}} &= \overline{h^{(t)}}\frac{\partial h^{(t)}}{\partial o^{(t)}} \\
&= \overline{h^{(t)}}\tanh(c^{(t)}) \\
\overline{f^{(t)}} &= \overline{c^{(t)}}\frac{\partial c^{(t)}}{\partial f^{(t)}} \\
&= \overline{c^{(t)}}c^{(t-1)} \\
\overline{i^{(t)}} &= \overline{c^{(t)}}\frac{\partial c^{(t)}}{\partial i^{(t)}} \\
&= \overline{c^{(t)}}g^{(t)}
\end{aligned}
$$

2.

3.