CSC321: Assignment #5

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Problem 1

Let $\mathbf{x}^{(t)}$ be a 2×1 vector containing x_1 and x_2 as the binary input at time t
Let $\mathbf{h}^{(t)}$ be a 3×1 vector containing h_1, h_2 and h_3 at time t as hinted in the handout.
Let $y^{(t)}$ be a scaler of the output binary digit.

We let

$$\mathbf{U} = \begin{Bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{Bmatrix}$$

$$\mathbf{W} = \begin{Bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{Bmatrix}$$

$$\mathbf{b_h} = \begin{Bmatrix} -0.5 \\ -1.5 \\ -2.5 \end{Bmatrix}$$

$$\mathbf{v} = \begin{Bmatrix} 1 & -1 & 1 \end{Bmatrix}$$

$$b_y = -0.5$$

Then for all $t \geq 1$,

$$\mathbf{h}^{(t)} = \mathbf{U}\mathbf{x}^{(t)} + \mathbf{W}\mathbf{h}^{(t-1)} + \mathbf{b_h} \tag{1}$$

$$y^{(t)} = \mathbf{v}\mathbf{h}^{(t)} + b_y \tag{2}$$

Expanding Equation (1), we get

$$\begin{aligned} h_1^{(t)} &= x_1^{(t)} + x_2^{(t)} + h_1^{(t-1)} - 0.5 \\ h_2^{(t)} &= x_1^{(t)} + x_2^{(t)} + h_2^{(t-1)} - 1.5 \\ h_3^{(t)} &= x_1^{(t)} + x_2^{(t)} + h_3^{(t-1)} - 2.5 \\ y^{(t)} &= h_1^{(t)} - h_2^{(t)} + h_3^{(t)} - 0.5 \end{aligned}$$

This satisfies the Truth table:

x_1	x_2	$h_1^{(t-1)}$	$h_1^{(t)}$	$h_2^{(t-1)}$	$h_2^{(t)}$	$h_3^{(t-1)}$	$h_3^{(t)}$	y
0	0	0	0	0	0	0	0	0
0	0	1	1	1	0	1	0	1
0	1	0	1	0	0	0	0	1
0	1	1	1	1	1	1	0	0
1	0	0	1	0	0	0	0	1
1	0	1	1	1	1	1	0	0
1	1	0	1	0	1	0	0	0
1	1	1	1	1	1	1	1	1

Problem 2

1.

$$\begin{aligned}
\overline{h^{(t)}} &= 1 + \overline{i^{(t+1)}}\frac{\partial i^{(t+1)}}{\partial h^{(t+1)}} + \overline{f^{(t+1)}}\frac{\partial f^{(t+1)}}{\partial h^{(t+1)}} + \overline{o^{(t+1)}}\frac{\partial o^{(t+1)}}{\partial h^{(t+1)}} + \overline{g^{(t+1)}}\frac{\partial g^{(t+1)}}{\partial h^{(t+1)}} \\
&= 1 + \overline{i^{(t+1)}}\sigma^{-1}(w_{ix}x^{(t+1)} + w_{ih}h^{(t+1)})w_{ih} + \overline{f^{(t+1)}}\sigma^{-1}(w_{fx}x^{(t+1)} + w_{fh}h^{(t+1)})w_{fh} \\
&\quad + \overline{o^{(t+1)}}\sigma^{-1}(w_{ox}x^{(t+1)} + w_{oh}h^{(t+1)})w_{oh} + \overline{g^{(t+1)}}\tanh^{-1}(w_{gx}x^{(t+1)} + w_{gh}h^{(t+1)})w_{gh} \\
\overline{c^{(t)}} &= \overline{h^{(t)}}\frac{\partial h^{(t)}}{\partial c^{(t)}} + \overline{c^{(t+1)}}\frac{\partial c^{(t+1)}}{\partial c^{(t)}} \\
&= \overline{h^{(t)}}o^{(t)}\tanh^{-1}(c^{(t)}) + \overline{c^{(t+1)}}f^{(t)} \\
\overline{g^{(t)}} &= \overline{c^{(t)}}\frac{\partial c^{(t)}}{\partial g^{(t)}} \\
&= \overline{c^{(t)}}i^{(t)} \\
\overline{o^{(t)}} &= \overline{h^{(t)}}\frac{\partial h^{(t)}}{\partial o^{(t)}} \\
&= \overline{h^{(t)}}\tanh(c^{(t)}) \\
\overline{f^{(t)}} &= \overline{c^{(t)}}\frac{\partial c^{(t)}}{\partial f^{(t)}} \\
&= \overline{c^{(t)}}c^{(t-1)} \\
\overline{i^{(t)}} &= \overline{c^{(t)}}g^{(t)}
\end{aligned}$$

2.

$$\overline{w_{ix}} = \sum_t \overline{i^{(t)}}\sigma^{-1}(w_{ix}x^{(t)} + w_{ih}h^{(t)})x^t$$

3. This is because when $f^{(t)} = 1, i^{(t)} = 0$ and $o^{(t)} = 0$,

$$\begin{aligned}
\overline{c^{(t)}} &= \overline{h^{(t)}}o^{(t)}\tanh^{-1}(c^{(t)}) + \overline{c^{(t+1)}}f^{(t)} \\
&= \overline{c^{(t+1)}} \\
\overline{g^{(t)}} &= \overline{c^{(t)}}i^{(t)} \\
&= 0 \\
\overline{o^{(t)}} &= \overline{h^{(t)}}\tanh(c^{(t)}) \\
\overline{f^{(t)}} &= \overline{c^{(t)}}c^{(t-1)} \\
\overline{i^{(t)}} &= \overline{c^{(t)}}\frac{\partial c^{(t)}}{\partial i^{(t)}} \\
&= \overline{c^{(t)}}g^{(t)}
\end{aligned}$$

then $\overline{c^{(t)}} = \overline{c^{(t+1)}}$ stays the same