CSC 411: Assignment #2

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Problem 1

The data set contains hand-written digits from 0 to 9 (Fig 1). Among these data, most data are labeled accurately. However, some data are hard to be distinguished and even human can't really predict the digit.(Fig 2)

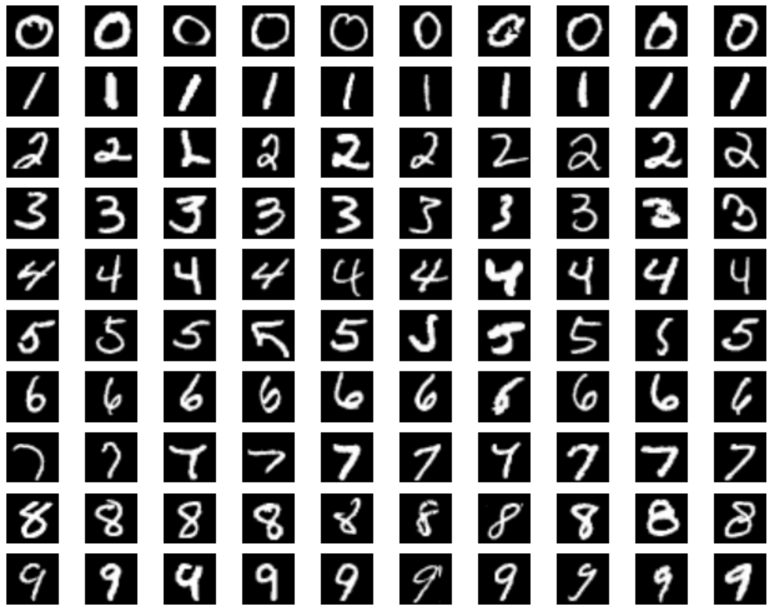
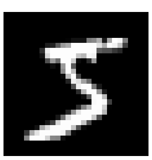
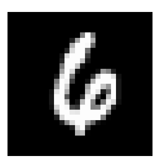


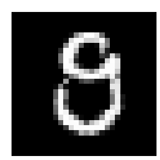
Figure 1: Full data



(a) 5 but looks like a 3



(b) 6 but looks like a 4



(c) 9 but looks like an 8

Figure 2: Inaccurate Labels

Problem 2

The output should be:

$$o^{(i)} = \sum_j w_j x_j^{(i)} + b^{(i)}$$

The listing of the implementation is as follows:

Listing 1: code for linear net output

```
def linear_forward(x, W):
    lin_output = np.dot(W.T, x)
    return softmax(lin_output)
```

Problem 3

1. Let the loss function C be defined as:

$$C = -\sum_i y^{(i)} log(p_i)$$

where p_i is:

$$p_i = \frac{e^{o_i}}{\sum_j e^{o_j}}$$

and o_i is:

$$o_i = \sum_j x_j^{(i)} w_{ij} + b_i$$

The gradient for the loss function with respect to the weight w_{ij} $\frac{\partial C}{\partial w_{ij}}$ is:

$$\frac{\partial C}{\partial w_{ij}} = \frac{\partial C}{\partial o_i}\frac{\partial o_i}{\partial w_{ij}}$$

Note:

$$\frac{\partial p_k}{\partial o_i} = \begin{cases} -p_k p_i & \text{if } k \neq i \\ p_i(1-p_i) & \text{if } k = i \end{cases}$$

Then:

$$\begin{aligned} \frac{\partial C}{\partial o_i} &= -\frac{\partial C}{\partial p_i}\frac{\partial p_i}{\partial o_i} + \sum_{k \neq i} \frac{\partial C}{\partial p_k}\frac{\partial p_k}{\partial o_i} \\ &= -\frac{y^{(i)}}{p_i} p_i(1-p_i) + \sum_{k \neq i} \frac{y^{(k)}}{p_k} p_k p_i \\ &= p_i y^{(i)} - y^{(i)} + \sum_{k \neq i} y^{(k)} p_i \\ &= \sum_k y^{(k)} p_i - y^{(i)} \\ &= p_i - y^{(i)} \end{aligned}$$

Also,

$$\frac{\partial o_i}{\partial w_{ij}} = x_j^{(i)}$$

Then

$$\begin{aligned} \frac{\partial C}{\partial w_{ij}} &= \frac{\partial C}{\partial o_i}\frac{\partial o_i}{\partial w_{ij}} \\ &= x_j^{(i)}(p_i - y^{(i)}) \end{aligned}$$

2. The vectorized implementation is as follows:

Listing 2: Vectorized gradient

```
def linear_forward(x, W):
    lin_output = np.dot(W.T, x)
    return softmax(lin_output)

def loss(x, W, y):
    p = linear_forward(x, W)
    return -np.sum(y * np.log(p)) / x.shape[1]

def dlossdw(x, W, y):
    p = linear_forward(x, W)
    return np.matmul((p - y), x.T).T
```

Problem 4

The learning curve is plotted in Fig 3, and the weights going to the output is in Fig 4. The learning curve is as expected: the loss for the training set decreases gradually and the loss for the test set first decreases then increases. The weights' visualization gives each handwritten digits form 0 to 9.

The weights are initialized to 0.5. This is because if the weights are initialized to 0, the network will not change at all. The learning rate is set to 0.00005. This is the best value for the network to change and does not overstep the minima

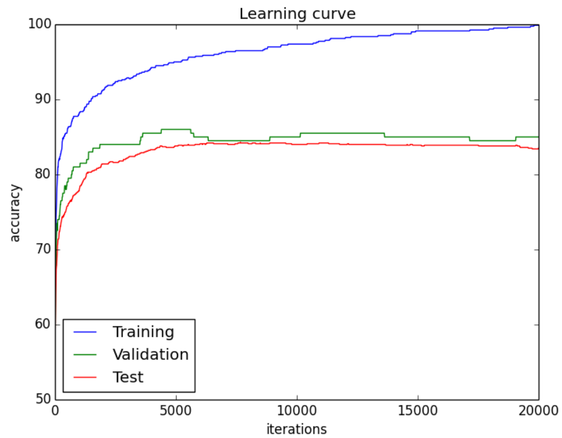


Figure 3: learning curve

Figure 4: weights

Problem 5

The new code for gradient descent with momentum is as follows:

Listing 3: Gradient descent with momentum

```
while i < max_iter and norm(W - prev_W) > eps:
    prev_W = W.copy()
    # W -= alpha * dlossdw(x_train, W, y_train)
    V = gamma * V + alpha * dlossdw(x_train, W, y_train)
    W -= V
    i += 1
```

The learning curve is plotted in Fig 5. Comparing the learning curve to Part 4, the training set learns faster and the test set first decreases and then keeps constant

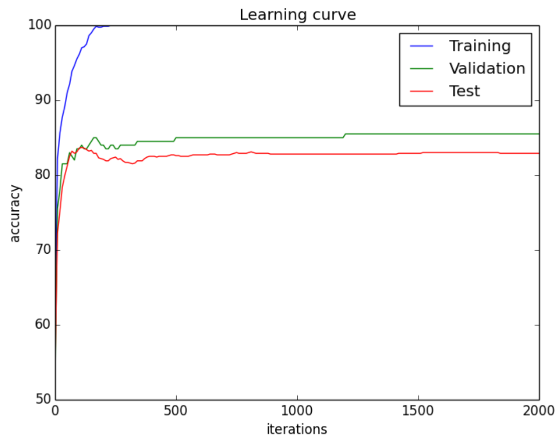


Figure 5: weights

Problem 6

1. The contour plot is given in Fig 6
2. The trajectory for "vanilla" gradient descent is the yellow line in the plot
3. The trajectory for momentum gradient descent is the green line in the plot
4. The "vanilla" gradient descent has more fluctuations due to the large learning rate. However, with momentum pointing to the constant optimum direction, the trajectory for momentum has less fluctuations. Also, to achieve the same learning effect (how close to the optimum), momentum gradient descent needs less steps.
5. The points were selected to be around the middle of the image. This is because that is where the weights matter the most. If the points were selected to be at the edge of the image, the weights won't matter because the edge does not contain anything that determines what the digit is.

   To produce a good visualization, the learning rate for both gradient descent must be tuned to be higher than normal, and the learning rate for "vanilla" gradient descent must be much higher to produce the fluctuation.

   A set of weights that won't produce a good visualization is chosen to be at the corner of the graph. The trajectories are not moving at all no matter how we change the initial values.

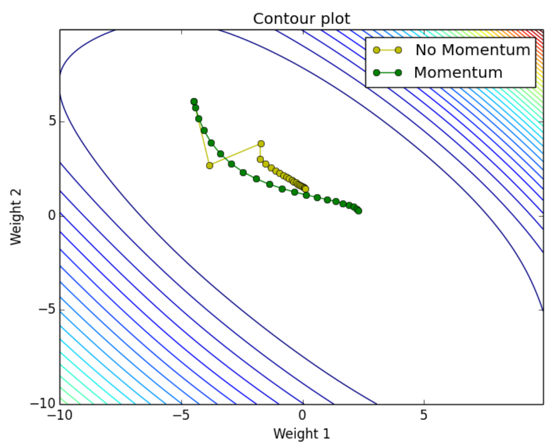


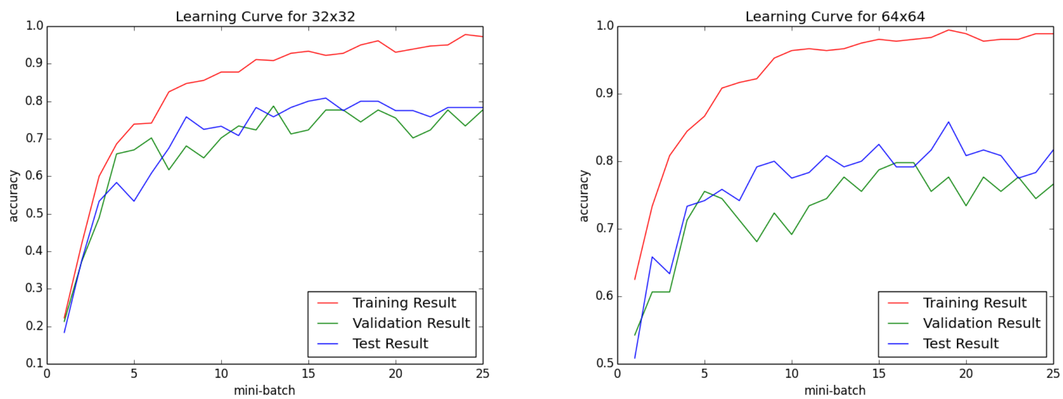
Figure 6: part (a-c)

Problem 7

Problem 8

The final test result for resolution of 32×32 has an accuracy of 83.3% and for resolution of 64×64 has an accuracy of 80%. The learning curves of the two resolutions are shown in Fig 7.

The results are trained by a fully connected, three-layered (input, hidden, output) network. The weights and bias are randomized. The activation function for input and output layer is linear, and for the hidden layer is ReLU. The network uses Adam as its optimizer. The hyper-parameters are picked through repeated experiments. The final hyper-parameters are: alpha = 0.001, number of hidden neurons = 36, epoch = 5, number of mini-batch in each epoch = 5, and the number of iterations = 1000



(a) 32×32 (b) 64×64

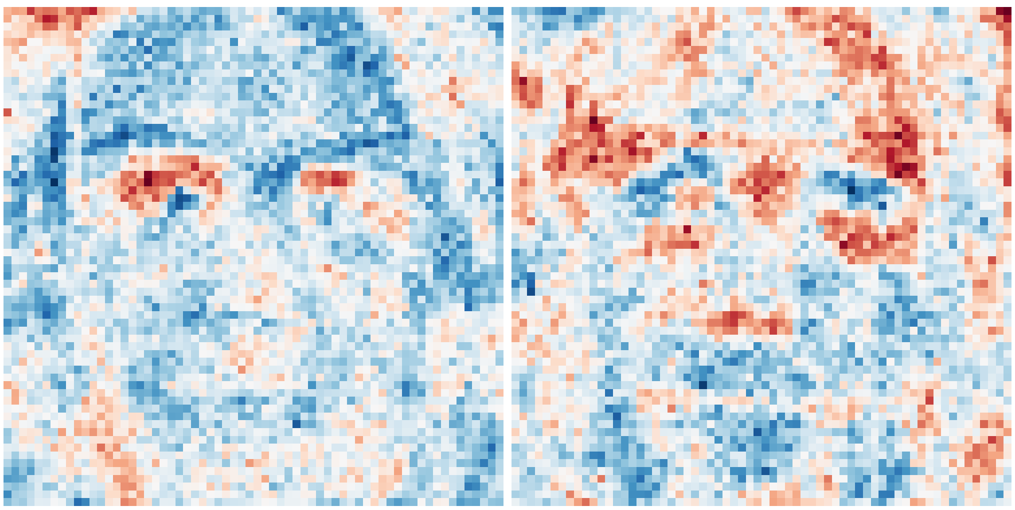
Figure 7: learning curves

Problem 9

The actors are chosen to be Harmon and Hader. Their hidden units are visualized as in Fig 8.
The weight units are chosen using the following snippet of code:

Listing 4: Code for choosing featuring weight unit

```
x = Variable(torch.from_numpy(W),
        requires_grad=False).type(dtype_float)
y = model(x).data.numpy()
for k in range(y.shape[0]):
print k, actor_names[np.argmax(y[k, :])]
```



(a) Actor1: Harmon (b) Actor2: Hader

Figure 8: Weight Visualizations

Problem 10